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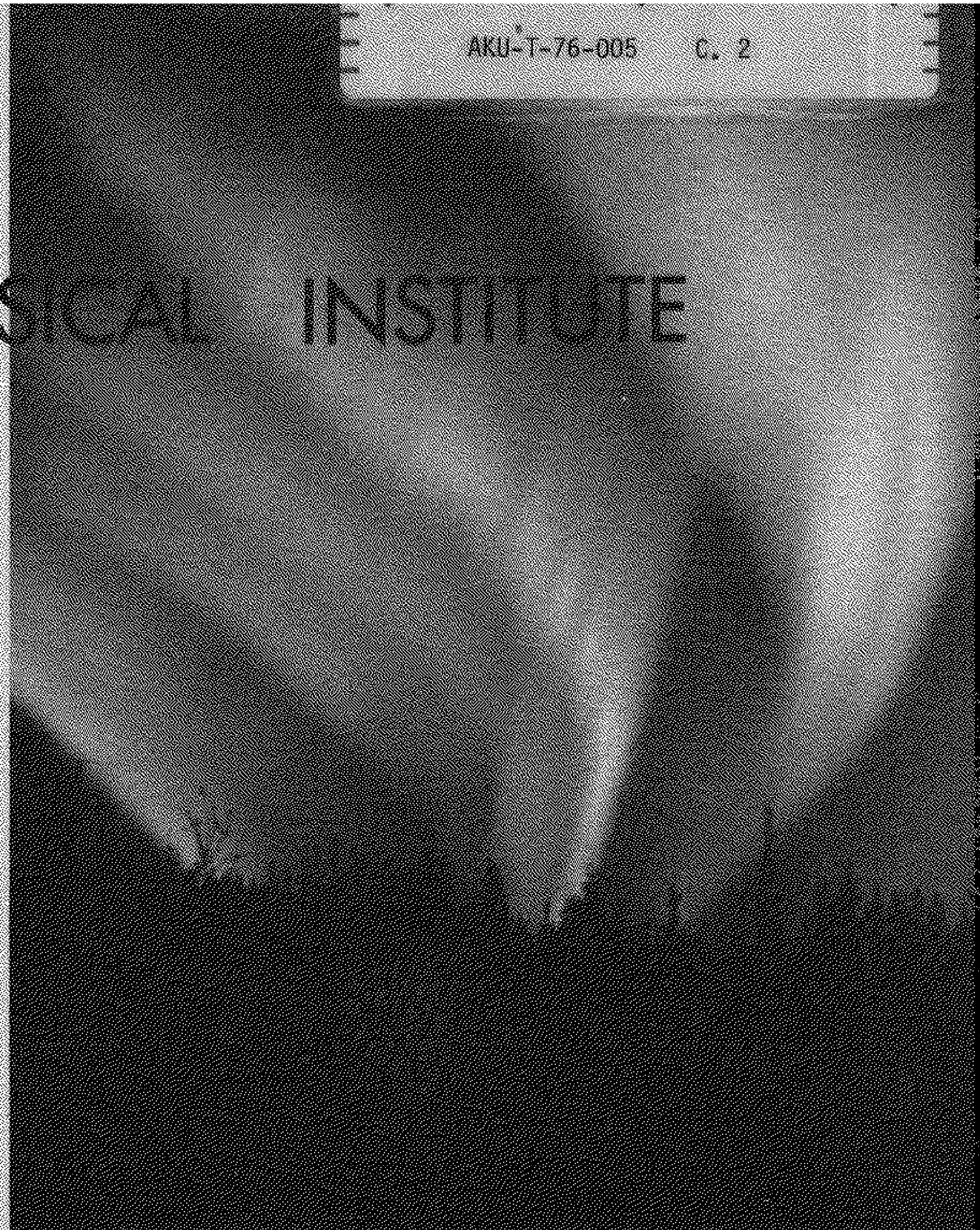

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A COUPLED HEAT AND SALT TRANSPORT MODEL FOR SUB-SEA PERMAFROST

by

W. D. Harrison and T. E. Osterkamp

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Geophysical Institute
University of Alaska
Fairbanks, Alaska 99701

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ABSTRACT

The sub-sea permafrost regime off much of Alaska's arctic coast can be understood by considering the response of land-formed permafrost to changing temperature and salinity conditions associated with shoreline recession. Sea-bed temperatures seem to be negative in much of the Beaufort Sea, but inundated permafrost will still thaw downward from the sea bed if the sea water is above its freezing temperature. The process cannot be understood within the framework of conventional heat transport models because of the key role played by salt. This is illustrated by a simple coupled heat and salt transport model, solved in closed form, in which heat and mass are transported by diffusion. The solution is a generalization of the Stefan solution for growth of an ice cover. It illustrates how the thawing rate depends almost entirely on salt transport properties at a sea-bed temperature of -1°C, on thermal properties at +1°C, and on both at intermediate temperatures. The calculated thawing rates are so slow in this diffusion model that the significance of pore liquid motion is suggested.

INTRODUCTION

There is little doubt that permafrost is common under the northern seas of Alaska, Canada and the Soviet Union. Although our knowledge about it is still quite limited, a number of experimental studies have been undertaken. Examples are Lachenbruch and others, 1962; MacKay, 1972; Molochuskin, 1973; Lewellen, 1974; Brewer, 1975; Hunter and Judge, 1975; Rogers and others, 1975; Judge and others, 1976; Osterkamp and Harrison, 1976. The work presented here is theoretical. Its use at this stage is not so much for detailed numerical prediction as for the insight it provides into processes which determine the sub-sea regime, and which should be studied as part of the experimental work. It will be seen how salt plays a key role in these processes.

At the scientific core of the sub-sea permafrost problem is the question of how permafrost, formed on land, responds to changing temperature and salinity boundary conditions after it is inundated by the sea. Shoreline recession seems to be the rule in the Beaufort Sea, not only from the melting of the Wisconsin ice 5,000 to 10,000 years ago, but also from present day active erosion of the coastline at a rate amounting to several meters per year in some places (Lewellen, 1970). As the shore line retreats we can imagine at least five stages in the evolution of the "surface" (no distinction being made between emergent land surface and shallow sea bed) temperature and salinity boundary conditions (Osterkamp, 1975). Stage 1 is the initial condition on land, stage 2 is beach, stage 3 is water less than 2 m in depth where the sea ice freezes to the bottom in winter, stage 4 is slightly deeper water where the ice does not freeze to the bottom but poor circulation causes the winter water to be highly saline, and stage 5 is deeper water in which circulation is sufficient to maintain normal sea water salinity all year. Other complications, such as ocean transgression into a fresh water lake, the effect of relatively warm river water, or reworking and deposition of shallow sediments, may exist.

In an equilibrium situation the thickness of the permafrost, usually defined as the depth to the 0°C temperature contour (Muller, 1947), is determined by the thermal conductivity, the geothermal heat flux, and the surface temperature (assuming there is no liquid motion). With shoreline recession the permafrost surface temperature becomes warmer, and the permafrost slowly thins until equilibrium is re-established; tens of thousands of years would be required in thick ice-rich permafrost at a location such as Prudhoe Bay. In the process, any ice in the permafrost is melted from the bottom by geothermal heat. If the sea-bed temperature is positive, the new equilibrium thickness is zero, and in the process of achieving it the permafrost will thaw from the top as well as the bottom (see MacKay, 1972). The question of what happens at the top if the sea-bed temperature is negative is the problem addressed here. It is a highly relevant problem, because the state of the permafrost near the sea bed is of considerable engineering and environmental importance, and the mean annual bottom temperature in much of the Beaufort Sea is probably negative (MacKay, 1972; Lewellen, 1974; Osterkamp and Harrison, 1976).

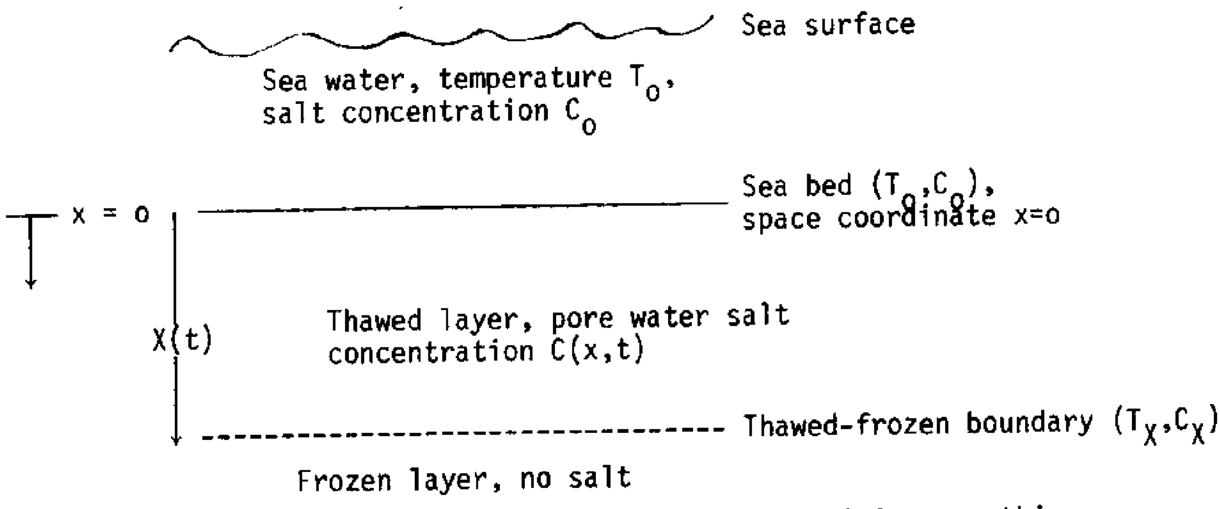
If the sea water maintains its temperature above that at which it begins to freeze, about -1.8°C, it will melt any ice in contact with it. A sub-bottom thawed layer will develop in any previously ice-bearing permafrost, with temperature and pore water salinity gradients in it. The rate of thawing may be primarily controlled by the rate that salt, rather than heat, penetrates the layer. Because of the role of salt, the process cannot be understood within the framework of a conventional thermal model. For the same reason, the conventional definition of permafrost, ground material below 0°C for several years (Muller, 1947), is less useful than on land.

Although the importance of salt seems obvious enough, the mechanism of its transport in the ground, or its possible presence there before ocean transgression, are important questions ultimately to be answered experimentally, and the answers will depend on the location. In the meantime the basic ideas can be illustrated fairly well by a simple theoretical model which can be solved in closed form, the predictions of which can be taken as a starting point for comparison with experimental data. In this model the initial condition is a thick layer of frozen, ice-saturated, salt-free, homogeneous permafrost. At time zero this material is suddenly covered with sea water at a temperature and concentration which remain constant thereafter. Only one spatial dimension is considered. The key physical assumptions are: (1) The pore liquid generated in the sediment as the ice melts does not move. Heat is then transported only by molecular conduction and salt by molecular diffusion; the heat transported by the moving salt can be neglected. (2) There is no salt sink in the thawed layer; in other words, no salt is adsorbed on soil particle surfaces. Some other assumptions and approximations are: The specific volume change of the ice upon melting is neglected; the influence of the soil particle surfaces on melting temperature is neglected; a sharp boundary exists between the thawed and frozen material. There is some field evidence for the last assumption (Osterkamp and Harrison, 1976).

MODEL WITH THERMAL PROCESSES NEGLECTED

When the sea-bed temperature is negative, the inundated permafrost beneath can only be thawed if both salt and heat are transported to it. (The influence of soil particle surfaces on melting temperature is neglected, as noted above.) Because the diffusivity of salt in thawed sediment is roughly three orders of magnitude less than that of heat, the thawing rate may be controlled by the rate that salt rather than heat diffuses. Therefore, we first consider a simple model in which thermal processes are neglected entirely. This is completely opposite from the situation on land, where the permafrost regime can usually be understood within the framework of a purely thermal model.

The model is illustrated by the following sketch:



If all the salt ions are assumed to have the same diffusivity κ_s, this simple model is described by a diffusion equation,

$$\kappa_s \frac{\partial^2 C}{\partial x^2} = \frac{\partial C}{\partial t},$$

where C is the mass concentration of salt per unit volume of pore liquid, x and t are the space and time coordinates, and x is measured positive downward from the sea bed. This equation is assumed to apply in the thawed layer; C is assumed to remain zero in the frozen region. Initially,

$$C(x,0) = 0$$
$$X(0) = 0$$

where X(t) is the thickness of the thawed layer. After t = 0 the condition at the sea bed, the upper boundary of the thawed layer, is

$$C(0,t>0) = C_o$$

where C_o is the salt concentration of sea water. At the lower boundary of the thawed layer (x = X)

$$\frac{dX}{dt} = -\kappa_s \left[\frac{1}{C}\frac{\partial C}{\partial x}\right]_X \tag{1}$$

where the subscript X implies evaluation at this boundary. This is the statement that as the ice melts, salt diffuses into the liquid so formed. (The flux is $-\kappa_s \frac{\partial C}{\partial x}$, and the total C dX has to be transported in time dt.)

The concentration C_X at the lower boundary of the thawed layer must also be specified to determine the problem. Since the thaw rate is controlled by the salt transport rate in this model, and therefore very slow, there should be adequate time for heat conduction to maintain the temperature of this thawed-frozen boundary at about the same value as the sea bed temperature T_o. Because two phases are present here, the salt concentration C_X is determined in terms of the temperature by the requirement of chemical equilibrium. Since the phase equilibrium concentration varies approximately linearly with temperature, this gives

$$\frac{C_X}{C_o} \approx \frac{T_o}{T_f}\,, \tag{2}$$

where T_f is the freezing temperature of sea water with concentration C_o.

The governing equation, the initial conditions, and the conditions at the upper boundary are satisfied by a solution of the form

$$\frac{C(x,t)}{C_o} = 1 - A \operatorname{erf} \frac{x}{\sqrt{4\kappa_s t}}$$

$\left(\operatorname{erf} u \equiv \frac{2}{\sqrt{\pi}} \int_0^u e^{-v^2}\, dv\right)$; the thickness X of the thawed layer increases as

$$X = \sqrt{4\kappa_s}\;\lambda\;\sqrt{t} \tag{3}$$

where A and λ are constants. λ could also be defined to include the factor $\sqrt{4\kappa_s}$, but the form used is more convenient since λ is non-dimensional. A and λ are found from the conditions at the thawed-frozen boundary. Equation (2) gives

$$A = \frac{(1 - T_o/T_f)}{\operatorname{erf} \lambda}\,,$$

so that

$$\frac{C(x,t)}{C_o} = 1 - \frac{(1 - T_o/T_f)}{\operatorname{erf} \lambda} \operatorname{erf} \frac{x}{\sqrt{4\kappa_s t}}\,. \tag{4}$$

Equation (1) gives

$$\sqrt{\pi}\ \lambda\ e^{\lambda^2}\ \mathrm{erf}\ \lambda \equiv \Phi(\lambda) = (\frac{T_f}{T_o} - 1) \tag{5}$$

The function Φ is defined for convenience. The solution for λ can be read off a plot of Φ against its argument, once the ratio T_f/T_o is specified.

We shall consider numerical values representative of Prudhoe Bay, where field studies are in progress. Temperatures are T_f = -1.8°C and T_o = -1.0°C, from which λ = 0.567 by equation (5). The salt diffusivity κ_s is not known but $\kappa_s = 1.0 \times 10^{-2}\ m^2\ a^{-1}$ is reasonable; this is 0.4 times the value for NaCl in free liquid at 0°C. Other values of κ_s might be chosen (Stoessell and Hanor, 1975; Li and Gregory, 1974, for example), but our conclusions would be unchanged. Equations (3), (4) and (2) become

$$X = 0.1135\ \sqrt{t} \tag{6a}$$

$$\frac{C(x,t)}{C_o} = 1.000 - 0.770\ \mathrm{erf}\ \frac{x}{0.200\sqrt{t}} \tag{6b}$$

$$\frac{C_X}{C_o} = 0.555 \tag{6c}$$

in meter-year units.

MODEL WITH THERMAL PROCESSES INCLUDED

Although the mechanism of permafrost thawing in a negative sea bed temperature regime is illustrated by the previous model, further insight can be gained by generalizing it to include thermal processes as well. Solutions can still be found in closed form, if prior to t = 0 the permafrost is at the constant temperature T_i at all depths. In the thawed region ($x < X$) the temperature T and concentration C are described by

$$\kappa_1 \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

$$\kappa_s \frac{\partial C^2}{\partial x^2} = \frac{\partial C}{\partial t},$$

and in the frozen region ($x > X$) by

$$\kappa_2 \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

$$C = 0,$$

where κ_1 and κ_2 are the thermal diffusivities in the thawed and frozen regions, respectively, and κ_s is the diffusivity of salt. Initially

$$T(x,0) = T_i$$

$$X(0) = 0$$

where X is the thickness of the thawed layer. After $t = 0$ the condition at the sea bed, the upper boundary of the thawed layer , is

$$T(0,t>0) = T_o$$

$$C(0,t>0) = C_o$$

where T_o and C_o are temperature and salinity at the sea bed. At the lower boundary of the thawed layer ($x = X$), the thawed-frozen interface,

$$\frac{dX}{dt} = -\frac{K_1}{h}\left[\frac{\partial T(x<X)}{\partial x}\right]_X + \frac{K_2}{h}\left[\frac{\partial T(x>X)}{\partial x}\right]_X \qquad (7)$$

where K_1 and K_2 are the thermal conductivities in the thawed and frozen regions respectively, and h is the volumetric latent heat released on melting. This is the statement that the thawing rate is the ratio of the total heat flux into the boundary to the latent heat. Also at the thawed-frozen boundary,

$$\frac{dX}{dt} = -\kappa_s \left[\frac{1}{C}\frac{\partial C}{\partial x}\right]_X \tag{1}$$

as before, and

$$\frac{C_X}{C_o} = \frac{T_X}{T_f} \tag{8}$$

which is similar to equation (2), except that the boundary temperature T_X is no longer assumed to be equal to the sea bed temperature T_o. In what follows, it is convenient to replace the thermal properties (κ_1, κ_2, K_2) by (α, β, γ):

$$\sqrt{\kappa_1} \equiv \alpha\sqrt{\kappa_s} \tag{9a}$$

$$\sqrt{\kappa_2} \equiv \beta\sqrt{\kappa_s} \tag{9b}$$

$$K_2 \equiv \gamma K_1. \tag{9c}$$

If the temperature T_X and therefore the concentration C_X at the thawed-frozen boundary should remain constant as thawing proceeds, the problem would be easily solved. The temperature solution would be essentially that of Stefan and Neumann (Carslaw and Jaeger, 1959, Chapter XI), and the concentration solution essentially that of the previous section. Although it does not seem obvious on physical grounds that T_X and C_X should be constant, it turns out to be so, as can be verified *a posteriori*.

The Stefan temperature solution is of the form

$$T(x<X,t) = T_o + \left[\frac{(T_X - T_o)}{\text{erf } \Lambda/\alpha}\right] \text{erf } \frac{x}{\alpha\sqrt{4\kappa_s t}} \tag{10a}$$

in the thawed layer ($x < X$), and

$$T(x>X,t) = T_i - \left[\frac{(T_i - T_X)}{\text{erfc } \Lambda/\beta}\right] \text{erfc } \frac{x}{\beta\sqrt{4\kappa_s t}} \tag{10b}$$

($\text{erfc } u \equiv 1 - \text{erf } u$) in the frozen region ($x > X$), with the thickness of the thawed layer X increasing as

$$X = \sqrt{4\kappa_s}\ \Lambda\ \sqrt{t}\ . \tag{11}$$

Λ is a constant and the factor $\sqrt{4\kappa_s}$ is included for convenience. The quantities in square brackets are also constants, determined by the requirement that $T(x<X,t) = T(x>X,t) = T_X$ at the boundary. Λ is determined as a function of T_X by equation (7), which, after some manipulation, yields

$$\frac{T_X^* - T_o^*}{\alpha^2\Phi(\Lambda/\alpha)} + \frac{\gamma}{\beta^2}\frac{(T_X^* - T_i^*)}{\Psi(\Lambda/\beta)} = \varepsilon \tag{12}$$

where non-dimensional temperatures, denoted by an asterisk, are defined in terms of the freezing temperature T_f by

$$T^* \equiv \frac{T}{T_f}, \tag{13}$$

a non-dimensional parameter ε is defined by

$$\varepsilon \equiv \frac{h\kappa_s}{(-K_1 T_f)}, \tag{14}$$

and the two functions Φ and Ψ are defined by

$$\Phi(u) \equiv \sqrt{\pi}\ u\ e^{u^2}\ \text{erf } u \ \rightarrow\ 2u^2 \text{ if } u \ll 1 \tag{15a}$$

$$\Psi(u) \equiv \sqrt{\pi}\, u\, e^{u^2} \operatorname{erfc} u \;\rightarrow\; \sqrt{\pi}\, u \text{ if } u \ll 1. \tag{15b}$$

These definitions permit equation (12) to be written in its simple and non-dimensional form.

In the Stefan problem T_X is specified, and equation (12) can be solved for Λ to complete the solution of the problem. In our problem T_X is an unknown. However, from the concentration solution,

$$\frac{C(x,t)}{C_o} = 1 - \frac{(1 - T_X/T_f)}{\operatorname{erf} \Lambda} \operatorname{erf} \frac{x}{\sqrt{4\kappa_s t}} \tag{16}$$

$$\Phi(\Lambda) = \left(\frac{1}{T_X^*} - 1\right), \tag{17}$$

which is found just as in the previous section except that T_X is not assumed to be equal to T_o, we obtain another relation, equation (17), between Λ and T_X*. Equations (12) and (17) can now be solved simultaneously for Λ and T_X*. Eliminating T_X* we find

$$\Phi(\Lambda) = \frac{1 + \gamma\, (\alpha/\beta)^2 \dfrac{\Phi(\Lambda/\beta)}{\Psi(\Lambda/\beta)}}{T_o^* + \varepsilon\alpha^2\Phi(\Lambda/\alpha) + \gamma\, (\alpha/\beta)^2 \dfrac{\Phi(\Lambda/\alpha)}{\Psi(\Lambda/\beta)}\, T_i^*} - 1 \tag{18}$$

Once this is solved numerically for Λ, T_X^* is given by equation (17),

$$T_X^* = \frac{1}{\Phi(\Lambda) + 1} \tag{19}$$

and the solution is complete.

The Stefan theory must follow as a limiting case of our theory. If $\kappa_s \rightarrow 0$, no salt can enter the thawed layer, so at the thawed-frozen boundary $C_X \rightarrow 0$, and therefore $T_X \rightarrow 0$ by equation (8). Solution of equation (12) with $T_X = 0$ thus yields the Stefan result. That the dependence on κ_s is gone, as it must be, can be verified by replacing the variable Λ by $\sqrt{\kappa_1/\kappa_s}\,\Lambda'$.

We shall use numerical values representative of Prudhoe Bay. The salt diffusivity $\kappa_s \approx 1.0 \times 10^{-2}$ m^2 a^{-1} as in the previous section. The thermal diffusivities κ_1, κ_2 and conductivities K_1, K_2 are estimated using the method described by Gold and Lachenbruch (1973), assuming a volumetric water content of 40% and a rock conductivity of 24×10^7 J m^{-1} a^{-1} deg^{-1}. The latter value is characteristic of randomly-oriented quartz, which is the dominant mineral in the sediments of interest (Osterkamp and Harrison, 1976). The results are $\kappa_1 = 28$, $\kappa_2 = 69$ m^2 a^{-1} and $K_1 = 8.3 \times 10^7$, $K_2 = 14.5 \times 10^7$ J m^{-1} a^{-1} deg^{-1}. The volumetric latent heat $h = 1.34 \times 10^8$ J m^{-3}, which reflects the assumed water content. The temperatures are $T_f = -1.8$, $T_0 = -1.0$, $T_i = -9.0$ °C. From equation (9) it follows that $\alpha = 52.9$, $\beta = 83.0$, $\gamma = 1.747$; from equation (14), that $\epsilon = 0.897 \times 10^{-2}$; and from equation (13), that $T_0^* = 0.555$, $T_i^* = 5.00$.

For these values it is valid to use the small argument approximations for Φ and Ψ (equation 15) in equation (18), which becomes

$$\Phi(\Lambda) \approx \frac{1 + \frac{2}{\sqrt{\pi}}\frac{\gamma}{\beta}\Lambda}{T_0^* + \frac{2}{\sqrt{\pi}}\frac{\gamma}{\beta}T_i^*\Lambda + 2\epsilon\Lambda^2} - 1. \tag{20}$$

This resembles the corresponding equation (5) in the previous section when thermal processes were neglected. In fact, one might estimate that as a first approximation $\Lambda \approx \lambda = 0.567$, the value determined in the previous section. Λ can then be replaced by this value on the right hand side of equation (20), which can then be solved for a better value of Λ from a plot of Φ, just as in the previous section. Iterating once or twice, one finds $\Lambda = 0.514$. Then by equation (19), $T_X{}^* = 0.612$.

Given the numerical values under consideration, equation (10a) can be simplified. Because $x < X$ and $X = \sqrt{4\kappa_s}\,\Lambda\,\sqrt{t}$, it follows that $\frac{x}{\alpha\sqrt{4\kappa_s t}} < \frac{\Lambda}{\alpha} \ll 1$. Therefore $\operatorname{erf}\frac{x}{\alpha\sqrt{4\kappa_s t}} \approx \frac{2}{\sqrt{\pi}}\,\frac{x}{\alpha\sqrt{4\kappa_s t}}$, using the small argument approximation for the error function. Equations (11), (10), (19), (16) and (8) finally become

$$X = 0.1028\ \sqrt{t} \tag{21a}$$

$$T(x<X,t) \approx -1.000 - 1.01\ \frac{x}{\sqrt{t}} \tag{21b}$$

$$T(x>X,t) = -9.00 + 7.96\ \operatorname{erfc}\frac{x}{16.61\sqrt{t}} \tag{21c}$$

$$T_X = -1.104 \tag{21d}$$

$$\frac{C(x,t)}{C_o} = 1.000 - 0.724\ \operatorname{erf}\frac{x}{0.200\sqrt{t}} \tag{21e}$$

$$\frac{C_X}{C_o} = 0.614 \tag{21f}$$

in meter-year-degree units. These are plotted in Figures 1 and 2. It is understood that although numbers are given to several figures in equations (6) and (21), they are subject to large uncertainties.

DISCUSSION

Comparison of equations (6) and (21) shows that the thawing rate and the concentration in the pore water are not greatly changed by the inclusion of thermal processes. The decrease in the thawing rate is about 10%. Therefore, for the particular sea-bed temperature of -1°C

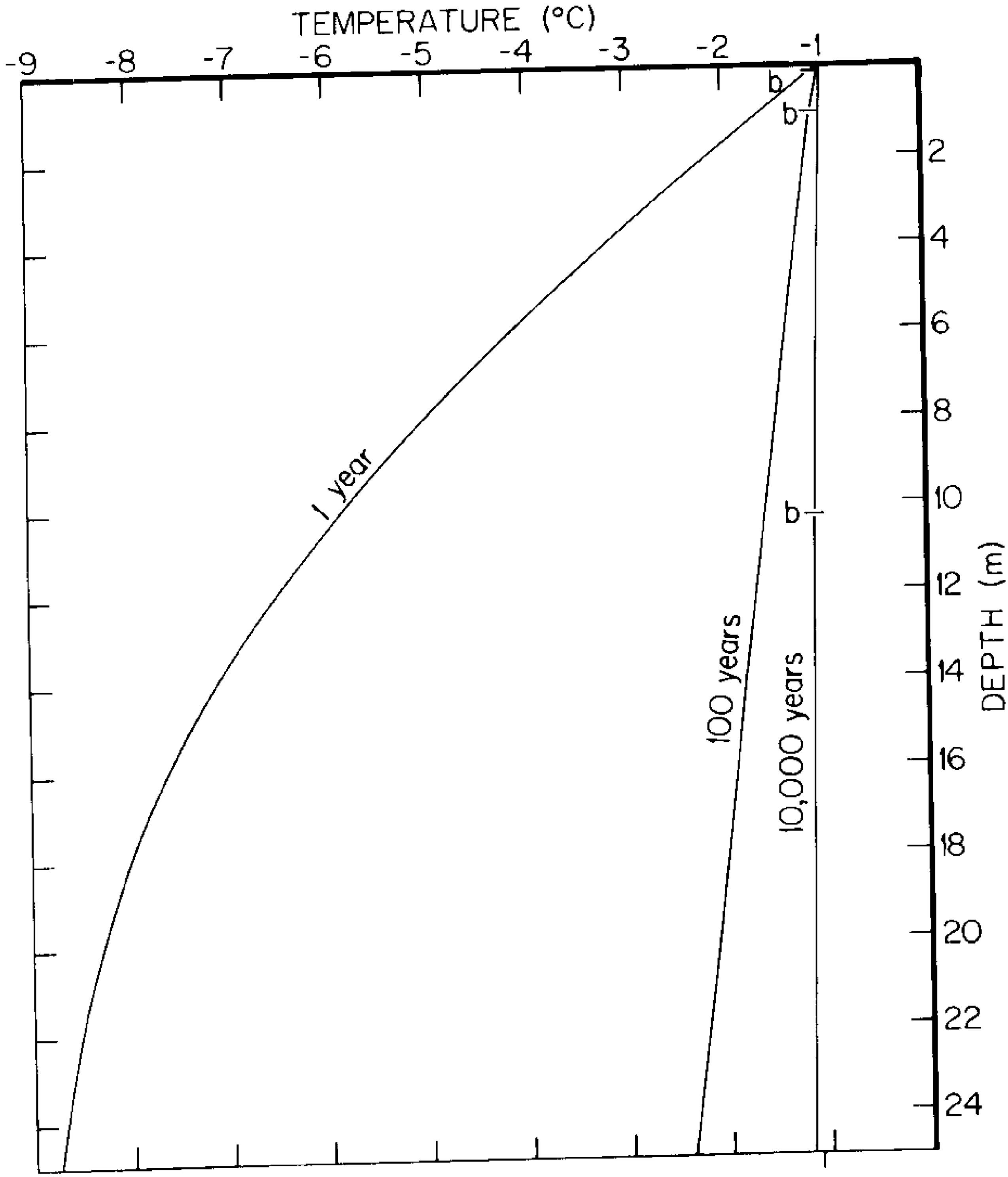


Figure 1. Depth dependence of temperature at 1, 100 and 10,000 years after ocean transgression. The position of the thawed-frozen boundary at each time is indicated by the symbol b. The temperature at the boundary is constant at -1.10°C. Seasonal temperature variations are neglected.

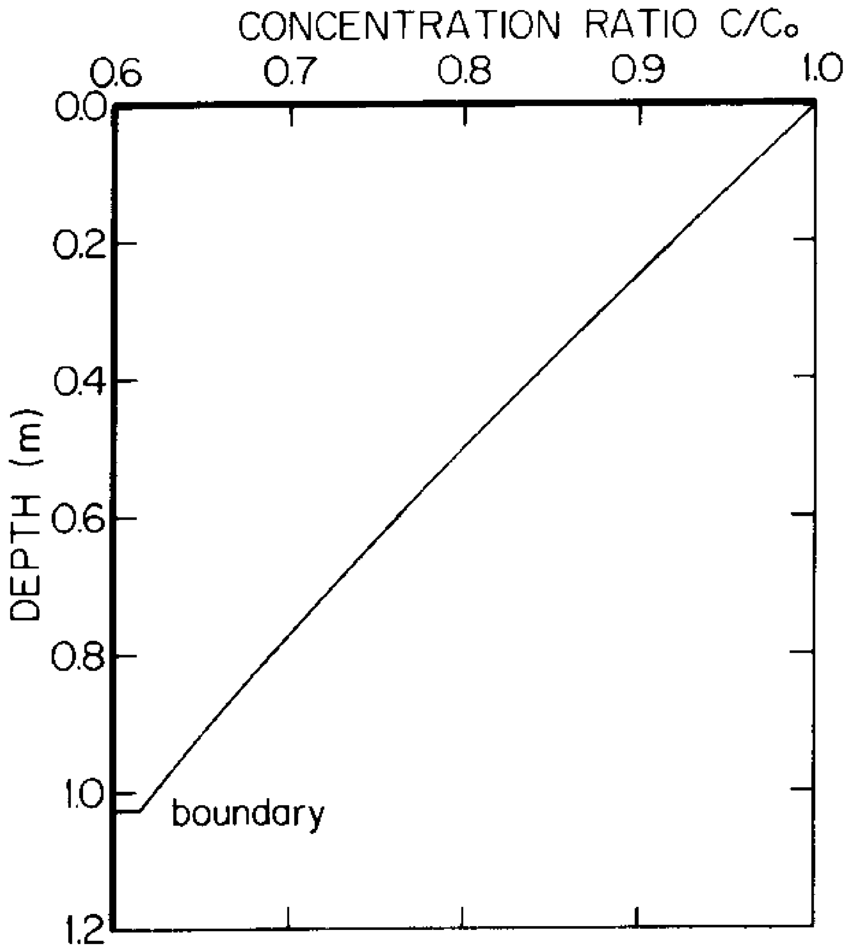


Figure 2. Depth dependence of salt concentration of pore water in thawed layer calculated from the coupled model. C/C_o is the ratio of the concentration to that of sea water. The position of the thawed-frozen boundary is indicated; $C/C_o = 0.614$ there. The depth scale applies at a time 100 years after ocean transgression. To find the dependence after 1 year, multiply the depth scale by 1/10; after 10,000 years, by 10.

considered, the thawing rate in this diffusive model is predominantly under chemical rather than thermal control. In general, the situation varies from total chemical control at sea bed temperatures close to the freezing point of sea water, to total thermal control at sufficiently positive temperatures. In the former limit only the mass transport property κ_s enters the results; in the latter, only the thermal properties κ_1, κ_2, K_1, K_2 and h. The thermal limit can be calculated by conventional Stefan theory with thawed-frozen boundary temperature T_X = 0°C. Figure 3 shows how the thawing rate and boundary temperature T_X depend on the sea-bed temperature. At a sea-bed temperature of +1°C the rate is about six times faster than at -1°C, T_X is 0°C, and the thermal limit applies. The predictions of the T_X = 0°C Stefan theory are indicated by the broken line, which predicts the same thawing rate at a sea-bed temperature of +0.1°C as our theory predicts at -1°C.

The simple model considered here illustrates the coupling between heat and mass transfer processes. In many such processes this coupling appears in the governing equations as advective terms; for example, a velocity times temperature gradient term in the heat equation. These terms are negligible in this particular model, which includes diffusion only, and the coupling is via the phase equilibrium condition relating temperature and concentration at the thawed-frozen boundary.

One obvious factor that has been neglected in order to get a solution in closed form is geothermal heat flow. This means that the behavior of the temperature solution is incorrect at large depths, although it should be reasonably good for fairly small depths and times. Our approach would obviously break down if geothermal heat were to entirely melt the permafrost from the bottom over the period of interest, which

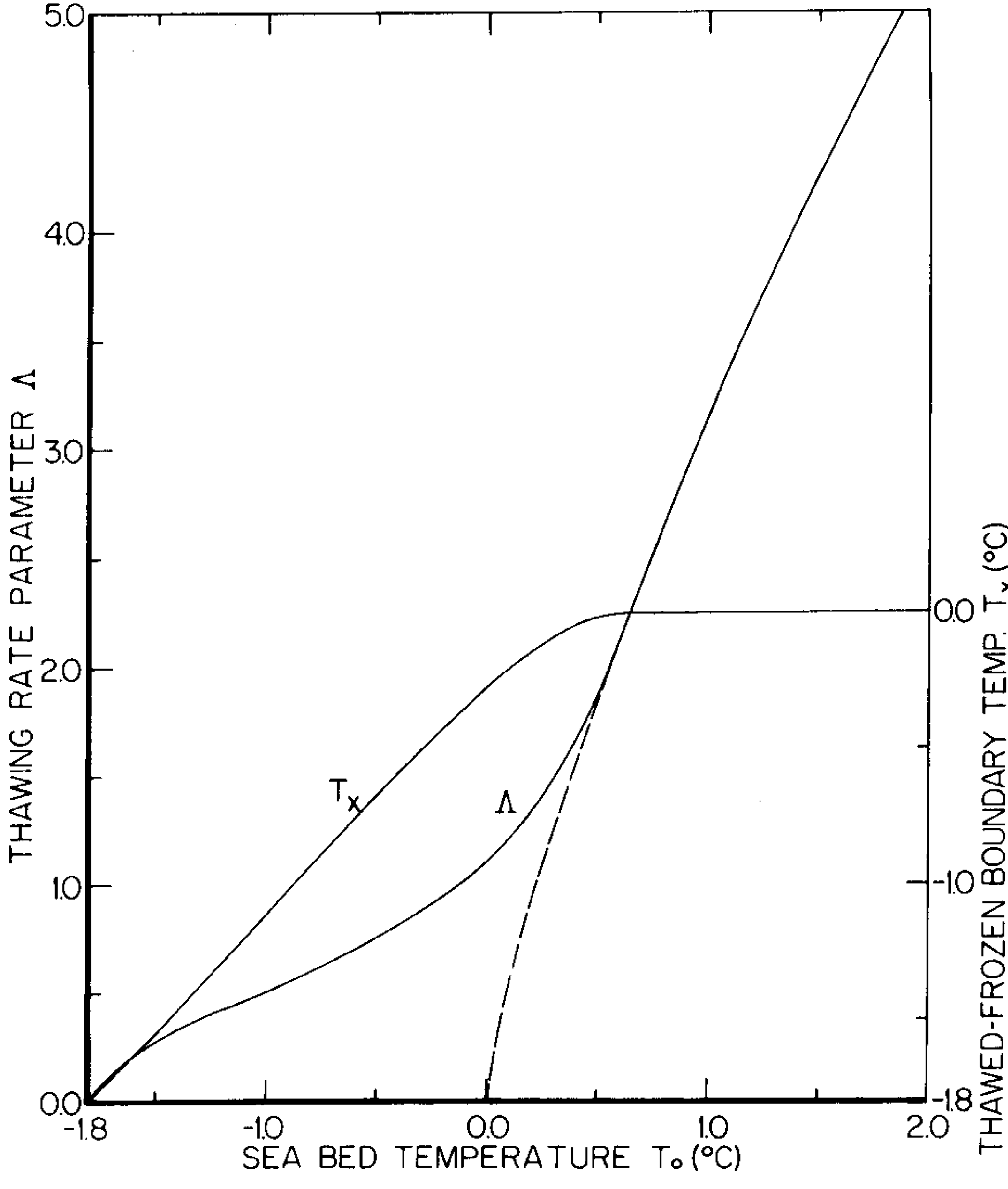


Figure 3. Dependence of thawing rate parameter Λ and thawed-frozen boundary temperature T_x on sea bed temperature T_o. The thawing rate is proportional to Λ obtained by conventional Stefan theory when salt transport is neglected ($T_x = 0$).

is probably less than 10,000 years near Prudhoe Bay, since it is unlikely that the near-shore areas have been inundated any longer. During this time geothermal heat should thaw only the bottom 100 m or so of the 600 m total permafrost thickness (Gold and Lachenbruch, 1973); the thickness of the thawed layer at the -1°C sea bed is 10 m in our model. It is not likely that geothermal heat affects this thickness much, since it is primarily controlled by the rate that salt diffuses down from the sea bed. The geothermal effect eventually must change the thawed-frozen boundary temperature somewhat from that calculated in our model, but in such a way that the heat flux into the boundary is roughly the same.

The rate of thawing from the sea bed, about 10 m in 10,000 years, is very small in this simple diffusion model. Experiments now underway indicate that the thawed layer is typically much thicker than this (Osterkamp and Harrison, 1976). Our diffusion model therefore suggests that we should look for other mechanisms of salt transport such as liquid motion. This simple theory is really a stepping stone to a more comprehensive one, which is best developed as more data become available. Nevertheless, the simple theory (1) illustrates the important role of salt, (2) suggests that pore liquid motion is important, and (3) can be used to check numerical computation schemes as they are developed.

ACKNOWLEDGMENTS

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