

THE PERFORMANCE OF SMALL SCALE TIDAL
POWER PLANTS

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Report No. MITSG 82-9
Index No. NOAA/NA 79AA-D-00101/R/T-3
June 1982

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Part 1: The Performance of Small
Scale Tidal Power

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ACKNOWLEDGEMENT

This work was sponsored by the M.I.T. Sea Grant College Program under grant number NA 79 AA-D-00101 from the Office of Sea Grant, National Oceanic and Atmospheric Administration, U.S. Department of Commerce.

The authors have benefitted from attendance at meetings sponsored by the Fundy Environmental Studies Committee and the National Research Council of Canada.

RELATED REPORT

Fay, James A. and Smachlo, Mark A. (1982).
CAPITAL COST OF SMALL SCALE TIDAL POWER
PLANTS. MITSG 82-10, Sea Grant College
Program, Massachusetts Institute of
Technology, Cambridge.

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ABSTRACT

Small scale tidal power plants - having electric power between one and a hundred megawatts approximately - possess several attractive economic and environmental benefits. The dynamical behavior of such systems is calculated in terms of dimensionless variables and parameters so that the size of the system is inconsequential (except for one parameter related to the slope of the walls of the tidal basin). Two measures of system performance are defined: capacity factor (ratio of average to rated power) and effectiveness (ratio of average to ideal tidal power). It was found that improving both parameters is mutually incompatible so that an economic analysis will determine the optimum values of the system design and performance parameters. The effects of variation of tidal range and basin shape were determined. Using typical variable flow properties of low head hydro turbines, a favorable design head could be determined from the analysis. It was found that the change in the area of the intertidal zone relative to the surface area of the tidal pond is greater for small as compared to large systems, possibly leading to proportionately greater environmental effects. A comparison of the performance of several tidal power plant designs with the methodology of this paper showed generally good agreement with the dimensionless performance parameters and only a modest difference among them over several orders of magnitude in size of power plant.

PREFACE

This study was initiated because of current interest in the possibility of small scale tidal power projects which might be located in the Gulf of Maine, especially the Bay of Fundy. While there is a continuing study of a large scale facility to be located in the Minas Basin (Nova Scotia) of the Bay of Fundy, recent studies of much smaller sites in Cobscook Bay (Maine) have raised the question of whether a number of small facilities might be preferable to a "megaproject". A pilot project at Annapolis Royal (Nova Scotia), soon to be completed, typifies a small scale tidal power plant.

Without prejudice to the large vs. small argument, this and an accompanying study develop a generic approach to the preliminary design and costing of small scale tidal power projects. They provide quantitative estimates of the technical performance and capital costs of such facilities as well as the hydrodynamic parameters of the tidal pond which will influence some environmental effects. A more definitive and precise characterization of hydrodynamic, environmental and economic factors would require a site-specific study.

We hope that the results of these analyses will be useful in the assessment of the advantages and disadvantages, both economic and environmental, of small scale tidal power and will provide a basis for screening of candidate sites and proposed projects.

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1. Introduction

The development of tidal power has lagged behind that of other renewable sources primarily because of unfavorable economics, the limited number of favorable sites and the uncertain environmental effects of the projects which have been investigated. Most proposals have considered large scale projects in which tidal basins of ten thousand hectares would be enclosed by barrages to generate thousands of megawatts of electric power. (The largest operating tidal plant at La Rance, France, generates 240 MW from a 1300 hectare estuarine pond.) The large capital cost and lengthy construction time of such systems, which cannot be constructed in incremental or staged fashion, deters their consideration.

Little attention has been given to small scale tidal power plants (e.g., 1-100 MW electric power) which would possess some of the desirable features of renewable energy sources, such as solar, wind and low head hydropower. When added to existing networks, such systems can be constructed quickly in incremental fashion with small economic and environmental risks. They can often be located close to the point of end consumption, reducing transmission costs. A system of small scale tidal plants could produce nearly uniform power throughout the diurnal cycle. Potential sites are much more numerous than for large scale installations. The economies of multiple construction of facilities may be favorable.

There are drawbacks to the use of tidal power. The allocation of a tidal flow region to power production interferes with or precludes other uses, such as navigation, fishing, etc. Small scale projects will

have comparatively greater effects on intertidal areas than large scale plants. Unit costs may be larger because of the smaller scale.

Nevertheless, the potential for net benefits of small scale tidal power plants has increased interest in them. A 17.8 MW tidal plant is under construction at Annapolis Royal, Nova Scotia, on the Bay of Fundy and a design study of a 12 MW plant in Cobscook Bay, Maine, has been completed. The tidal range and coastal configuration of eastern Maine and New Brunswick is favorable to small scale tidal installations.

Because most tidal power plant studies have been site specific, there is no general method for quickly assessing the potential of a number of small scale candidate sites. Also, it is not clear which is the most economical size of power plant for a given size of the resource. The purpose of this paper is to present a generic method of determining the performance of a tidal power plant as a function of the relative sizes of the tidal pond, the turbine and the sluiceway. The measures of performance are those which would be most directly useful in an economic analysis and optimization of the plant design, which is not considered at this point. The application of this method to a particular site would permit determination of the turbine and sluiceway characteristics needed to achieve a desired (and attainable) performance measure, which would subsequently form the basis of an economic analysis of that site.

Our approach is to analyze the system behavior in terms of dimensionless variables which are scale independent. As will be seen, there are four principal dimensionless parameters which affect the system dynamics. These parameters measure the relative size or properties of the system components. Two principal measures of performance are advanced: capacity factor (time averaged power divided by rated power)

and effectiveness (average power divided by the ideal power available from the tidal pond). Other performance measures are determined in the process, such as maximum and minimum pond levels which are useful in environmental impact assessment, and ratio of the rated turbine head to the tidal amplitude.

The effect of each of the four parameters on performance is studied in sequence for a realistic range of each, and their significance to the performance measures is assessed. Off-design performance caused by variations in the lunar cycle is also investigated. Both single effect and double effect systems are treated. Finally, a comparison is made of this method with prior studies of plant design.

The economic significance of the performance measures is readily understood. The ratio of the economic benefit (power) to the cost of the turbine will be proportional to the capacity factor. For a given resource (tidal pool), as the size of the power plant is increased so will the average power (although usually in lessened proportion). On the other hand, the other major capital cost, that of the barrage, will be practically independent of the size of the power plant. The ratio of the value of the power production to the fixed cost of the barrage will be proportional to the effectiveness. The economic optimum design will thus depend upon the relative values of these variables and the unit costs of construction, but requires additional analysis not included herein.

2. Dynamical Model of Tidal Power System

For small scale systems, it seems likely that only a single pool operating in a single effect (outflow or inflow) or double effect mode

would be economic since it can produce more power from the resource than more elaborate schemes (Bernshtein 1965). The benefits of more even power production could be more easily attained by a system of single pool plants operating in different modes. Thus a single pool scheme only will be considered here, but the three modes of operation will be analyzed.

The tidal pool surface area A will change with surface elevation Z above mean sea level. Following Swales and Wilson (1968), we will assume a linear relationship between A and Z :

$$A = A_0 + \ell Z \quad (2.1)$$

in which A_0 is the tidal pond surface area at mean sea level ($Z = 0$) and the characteristic length ℓ is determined from the rate of increase of A with Z at mean sea level:

$$\ell \equiv (dA/dZ)_{Z=0} \quad (2.2)$$

Where the variation of A with Z is very non-linear, it may be desirable to define A_0 and ℓ in such a manner as to give the best fit about the average level in the pond, which will be different from mean sea level.

We next assume that the sea level Z_t outside the pond is unaffected by the flow through the turbine or sluiceway and is a simple sinusoidal function of time:

$$Z_t = H_t \sin (2\pi t/T) \quad (2.3)$$

in which t is the time measured from flood midtide, T is the tidal period and H_t is the tidal amplitude (half the tidal range). We consider the tidal amplitude H_t and period T to vary with the diurnal, lunar and solar cycles, but otherwise disregard the additional components of tidal motion which are used in the accurate representation of tidal levels.

The variation of H_t with the lunar cycle is the most important effect on tidal plant performance, and will be considered below.

The volume flow rate of sea water through the turbine or sluice gates will be a function of the head difference H between the tidal pond and the sea:

$$H = |Z - Z_t| = |Z - H_t \sin(2\pi t/T)| \quad (2.4)$$

For the turbine, the volume flow rate Q_t is expressed in the functional form:

$$Q_t = Q_0 f\{H/H_0\} \quad (2.5)$$

in which Q_0 and H_0 are the rated volume flow rate and head, respectively, of the turbine and the dimensionless function f defines how the flow rate is varied when the ratio of actual head H to rated head H_0 is different from one. As described below, f is determined by the turbine design and the method by which the turbine is operated so as to maintain rated power over as wide a range of head H as is possible. For the sluiceway volume flow rate Q_s , we write:

$$Q_s = A_s (2gH)^{1/2} \quad (2.6)$$

in which A_s is the effective flow area of the sluiceway, which will be generally greater than the actual area. For a submerged venturi sluiceway, A_s would be constant during the period of its use.

We next write the dynamical equation for the tidal pond level:

$$A \, dZ/dt = Q_t h_t + Q_s h_s \quad (2.7)$$

where the functions h_t and h_s are 0, -1 or +1 when the turbine or sluiceway is inoperative or used during outflow or inflow, respectively.

Eq. (2.7) may be integrated in combination with Eqs. (2.1) - (2.6) to

determine the response of the tidal pond to the operation of the turbine and sluiceway. In addition to the variable Z , the ideal turbine power P may also be calculated from:

$$P = \rho g Q_t H \quad (2.8)$$

Before proceeding to a typical calculation, it is necessary to model the turbine flow function $f \{H/H_0\}$ of Eq. (2.5). Since it is expected that a turbine would drive either a synchronous or induction generator at constant speed, turbine power would be regulated by varying the turbine flow so as to match the power capability of the generator. When the head H is near the rated value of H_0 , the turbine Q would be adjusted to maintain rated power so that $QH = Q_0 H_0$. But as the flow rate is increased when the head decreases, a limit is reached when the wicket gates are wide open, beyond which no further increase is possible. The head at this limit is denoted by VH_0 . Further decreases in head will be accompanied by decreasing flow rate Q in proportion to $H^{1/2}$. At some lower head, denoted by MH_0 , the turbine power would become very small and the turbine would be shut down. We therefore choose the following form for the function f of Eq. (2.5):

$$\begin{aligned} f &= 0 && \text{if } H/H_0 \leq M \\ &= V^{-3/2} (H/H_0)^{1/2} && \text{if } M \leq H/H_0 \leq V \\ &= H_0/H && \text{if } V \leq H/H_0 \end{aligned} \quad (2.9)$$

Turbine flow functions of similar form are discussed by Bernshtein (1965). Two examples of tidal power plant turbine flow functions are shown in Fig. 1, together with their approximate but satisfactory representations by Eq. (2.9). For subsequent calculations, we use $M = 0.3$ and $V = 0.8$.

For the purpose of cost analysis and preliminary plant design, it is desirable to estimate turbine size as a function of rated power and head. A useful empirical relation can be derived from a compilation of lowhead turbine characteristics (Smachlo 1982) using an assumed turbine-generator efficiency of 80%:

$$P_o / \rho D_o^2 (gH_o)^{3/2} = 0.79 \pm 0.15 \quad (2.10)$$

in which D_o and P_o are the runner diameter and rated power, respectively. This relationship is independent of the ratio H_o/D_o over the range $10^{-1} < H_o/D_o < 10^3$.

It is desirable to express the dynamical performance of the tidal power plant in terms of dimensionless variables, which we define as follows:

$$\begin{aligned} \hat{A} &\equiv A/A_o & \hat{Q}_t &\equiv Q_t/Q_o \\ \hat{H} &\equiv H/H_t & \hat{Q}_s &\equiv Q_s/Q_o \\ \hat{P} &\equiv P/\rho g Q_o H_o & \hat{Z} &\equiv Z/H_t \\ \hat{t} &\equiv t/T & \hat{Z}_t &= Z_t/H_t \end{aligned} \quad (2.11)$$

In terms of these variables, Eqs. (2.1) - (2.9) take the form:

$$\begin{aligned} d\hat{Z}/d\hat{t} &= \beta(\hat{Q}_t h_t + \hat{Q}_s h_s)/(1 + \lambda\hat{Z}) \\ \hat{H} &= |\hat{Z} - \sin 2\pi\hat{t}| \\ \hat{Q}_s &= \gamma(\psi\hat{H})^{1/2} \\ \hat{Q}_t &= (\psi\hat{H})^{-1} & \text{if } \psi\hat{H} \geq V \\ &= V^{-3/2}(\psi\hat{H})^{1/2} & \text{if } M \leq \psi\hat{H} \leq V \\ &= 0 & \text{if } \psi\hat{H} \leq M \\ \hat{P} &= \psi\hat{Q}_t\hat{H} \end{aligned} \quad (2.12)$$

where

$$\begin{aligned}
 \beta &\equiv Q_0 T / A_0 H_t \\
 \gamma &\equiv A_s (2gH_t)^{1/2} / Q_0 \\
 \psi &\equiv H_t / H_0 \\
 \lambda &\equiv \lambda H_t / A_0
 \end{aligned}
 \tag{2.13}$$

The four parameters of Eq. (2.13) appear in the set (2.12), together with V and M which are regarded as fixed values determined by the turbine internal flow design. Of these, β measures the turbine design flow rate compared to the natural tidal flow rate into the pond and γ the sluice flow rate compared with the turbine flow rate. The ratio of tidal amplitude to rated head is ψ . Unlike the variable parameters β , γ and ψ which depend upon the turbine and sluice characteristics, the parameter λ depends only upon the tidal pond volume characteristics and is thus a property of the site.

While ψ is explicitly a variable parameter of the system, it is clear that the design head of the turbine ought to be related to an average value of the variable head experienced during operation. We therefore apply a constraint to the solution of Eq. (2.12) by requiring that the flow-averaged head equal the design head,

$$\begin{aligned}
 \int HQ_t dt &= H_0 \int Q_t dt \\
 \text{or} \quad \psi \int_0^1 \hat{H} \hat{Q}_t d\hat{t} &= \psi \int_0^1 \hat{P} d\hat{t} = \int_0^1 \hat{Q}_t d\hat{t}
 \end{aligned}
 \tag{2.14}$$

We shall see later on that additional constraints on \hat{Q}_t will be required for large values of β if degradation of system performance is to be avoided.

The variation of pond and sea levels and turbine power during a single tidal cycle is shown in Fig. 2 for both single effect outflow and double effect operation, assuming typical values of β and γ and $\lambda = 0$. For single effect outflow operation, the mean pond level is raised above the mean sea level whereas these two mean levels are the same in double effect operation. It is also noteworthy that the turbine operates at rated power most of the time that it is in operation.

3. System Performance

An obvious measure of the performance of the system is the average amount of power produced during a tidal cycle. One dimensionless measure of this average power is the capacity factor η , the ratio of average power to rated power:

$$\eta \equiv (P_o T)^{-1} \int_0^T P dt = \int_0^1 \hat{P} d\hat{t} \quad (3.1)$$

Assuming that turbine capital costs are proportional to turbine power, the ratio of revenues from the electric power produced to the annualized capital cost of the turbine would be proportional to the capacity factor.

While the capacity factor measures the degree of utilization of the turbine, it does not reflect how well the power available in the tidal pond is utilized. To investigate the latter we first determine the ideal power available in a tidal pond by multiplying the weight of fluid in the tidal volume, $2\rho g A_o H_t$, by the amount by which its mass center is lowered (or raised), $(1 \pm \lambda/3)H_t$, when it is drained (or filled) at low (or high) tide, to find the ideal work W done during outflow (or inflow):

$$W = 2\rho g A_0 H_t^2 (1 \pm \lambda/3) \quad (3.2)$$

The ideal tidal power I is found by summing W for both outflow and inflow and dividing by the tidal period T :

$$I = 4\rho g A_0 H_t^2 / T \quad (3.3)$$

It is clear from Eq. (3.2) that, given the choice of outflow or inflow operation in the single effect mode, the former is to be preferred since there is more tidal energy available during outflow than inflow by the factor $(3+\lambda)/(3-\lambda)$.

Finally, we define the effectiveness ϵ as the ratio of average power to the ideal tidal power:

$$\begin{aligned} \epsilon &\equiv \int_0^T P dt / I \\ &= \eta \beta / 4\psi \end{aligned} \quad (3.4)$$

in which the equality follows from use of Eqs. (3.1) and (2.13).

Both η and ϵ are functions of the principal variable parameters β and γ , both implicitly and explicitly as expressed in Eqs. (3.1), (3.4) and (2.14), the latter expressing the implicit dependence of ψ on β and γ . Thus, given a tidal pond (and hence λ), the performance measures η and ϵ will depend principally upon the size of the turbine (β) and sluiceway (γ).

We first examine the dependence of the capacity factor η on the sluiceway size parameter γ . In Fig. 3 we show this dependence for several values of β . Almost independent of β , we find that no significant improvement in capacity factor will ensue from increasing γ beyond about 5. For this reason we will use this value in subsequent analyses.

We next turn to the dependence of η and ϵ on the turbine size parameter β , shown in Fig. 4. We note that the capacity factor decreases with increasing β because the more rapid drawdown of the tidal pool decreases the duration of power generation. However, the fraction of the ideal tidal pond power utilized (ϵ) increases with β toward the limit of 0.5.

In arriving at the results of Fig. 4 it was necessary to apply an additional constraint for large values of β . The turbine start was delayed beyond the point at which the head reached MH_0 , as given in Eq. (2.9). A starting head value was selected which maximized ϵ . This starting head increased with β , as shown in Fig. 5. For large values of β , the turbine ran in a pulsed mode, operating over a short portion of the tidal cycle just preceding low tide (for outflow). Note also in Fig. 5 the slight change in design head as β increases.

An alternative method of presentation of the system performance is shown in Fig. 6 where the relationship between the capacity factor η and effectiveness ϵ is displayed for both single effect outflow and double effect systems. In this representation β is not shown explicitly, but increases with increasing ϵ and decreasing η . In the limit of $\beta = \infty$, $\epsilon = 0.5$ and 1.0 for single and double effect systems, respectively. Fig. 6 shows quite clearly that maximizing both η and ϵ are anti-thetical goals. The choice of η and hence ϵ will depend upon an economic optimization of the overall plant design.

It is possible to operate in the double effect mode with use of a sluiceway although its extra cost may not be justified. However, the improvements in η and ϵ are quite small (Smachlo 1982) and are unlikely to compensate for the increased capital cost.

An extensive set of performance calculations is tabulated by Smachlo (1982).

4. Effects of Tidal Volume Shape

The parameter λ (Eqs. 2.13 and 2.2) measures the degree to which the tidal pond surface area increases with rising pond level. For $\lambda = 1$, the pond surface area is zero at low tide and at high tide is twice the mean tidal area. Thus $\lambda = 1$ represents the most extreme condition which is likely to be encountered.

We have investigated the effect of varying λ between 0 and 1 upon the performance of single effect outflow plants. As might be expected, both η and ϵ increase with λ , although not dramatically so (see Fig. 7). This improvement stems from the increased average surface area (compared to $\lambda = 0$) in the tidal pond, whose mean level for single effect outflow operation is always above the mean tidal level. Because of this small dependence on λ , we consider the previous calculations for $\lambda = 0$ to be nearly correct for the typical values of λ likely to be encountered in attractive sites.

In double effect systems the mean pond level is very close to the mean tidal level and λ has virtually no effect upon η or ϵ (Smachlo 1982).

5. Effects of Variation of the Tidal Amplitude

In this analysis we have been using as a reference height the tidal amplitude H_t , which we assume to be an annual mean amplitude at the site. During the lunar and annual cycles other tidal amplitudes H'_t , larger or smaller than H_t , will be experienced. How will the performance be altered by a value of H'_t different from H_t ?

The calculation of such effects from Eqs. (2.12) and (2.13) is straightforward. Those variables and parameters which depend upon H_t by definition are modified by substitution of H'_t but the physical variables

Q_0 , H_0 , A_s , A_0 and ϵ are held fixed by the values of β , γ , ψ and λ appropriate to the mean tidal amplitude H_t . However, the definition of ϵ is not modified, but is based upon H_t .

The results of such a calculation are shown in Fig. 8 in which the tidal amplitude ratio H'_t/H_t has been varied over a factor of two, which is typical of the lunar cycle range. Both η and ϵ are seen to vary almost linearly with H'_t/H_t . Thus the monthly averaged values of η and ϵ will be nearly the same as those values for the monthly mean tidal amplitude.

In making these calculations it was found necessary to modify the constraints on turbine startup when H'_t/H_t was less than unity. In these cases the startup head was chosen to maximize the capacity factor.

6. Effects on the Intertidal Zone

A major environmental effect of a tidal power plant is the alteration of the natural cycle of flooding and draining of the intertidal zone of the tidal pond. The normal high and low tide levels will be changed and the regular periodic variation of tidal pond level will be modified, as shown for example in Fig. 2. It can be seen that a principal effect is the diminishment of the range of tidal motion in the pond and hence a reduction in the area of the intertidal zone.

In the undisturbed state, the area of the intertidal zone (projected onto a horizontal plane) can be found from Eqs. (2.1) and (2.13):

$$2\epsilon H_t = 2\lambda A_0 \quad (6.1)$$

With a tidal power plant in operation, this area is reduced in proportion

to the ratio of the tidal range in the pond to the undisturbed tidal range $2H_t$. We should therefore examine how the maximum and minimum pond surface level Z is modified by the tidal plant operation.

The variation of maximum, mean and minimum tidal pond levels for single and double effect systems is shown in Fig. 9 as a function of the turbine capacity compared to the pond size (β). For single effect outflow systems, increasing the relative turbine capacity decreases the mean pond level and increases the tidal range toward their undisturbed values, but on average the tidal level is higher than in the undisturbed state. On the other hand, a double effect system always maintains the same mean level as in the sea, but the range increases with increasing turbine capacity. For a value of $\beta = 5$, the intertidal zone would be reduced by about 25% below its undisturbed values. For $\beta = 2$, which may be the most economical design, the intertidal zone would be reduced to about one third or less of its original area.

There are, of course, other environmental effects associated with the disturbance to the natural tidal motion within the pond, such as the interference with the access by finfish populations, reduced volume flow of sea water into the pond, impediments to navigation, etc.

7. Comparison with Existing and Proposed Tidal Plant Performance

To provide a basis of comparison between the methodology used in this analysis and the results of specific designs we have reviewed the published data on several existing and proposed designs. Only one of these, a plant under construction at Annapolis Royal (McLean 1980), could be regarded as small, having an electric power rating of 17.8 MW. The most notable operating plant, located at La Rance (Cotillon 1974), is

perhaps of intermediate size (240 MWe). Rather extensive studies of potential large scale plants in the upper Bay of Fundy (Furst and Swales 1978 and Simeons 1980) have identified three sites, Shepody Bay, Cumberland Basin and Cobequid Bay, (denoted respectively as A6, A8 and B9) for which performance has been calculated. Because the basic principles of our analysis, as expressed in dimensionless form, are independent of size, we can compare the values of the dimensionless parameters and variables derived from our analysis with those determined from the dimensional properties reported in the published studies.

In Table 1 we first list the principal dimensional parameters of each power plant. In cases of multiple turbines and sluiceways we aggregate the turbine design flow rate Q_0 , sluiceway effective area A_S and rated electric power. Next we list the values of the dimensionless parameters β , γ , ψ and the performance parameters η and ϵ . To evaluate ϵ , which is defined in terms of the ideal turbine power (Eq. 2.8) rather than the electric power, we use an overall turbogenerator power efficiency of 80%. Note that, despite the large range of the dimensional variables, there are not great differences among the dimensionless quantities.

Next we calculate values of η^* and ϵ^* using the methodology of this paper and the values of β , γ and ψ of Table 1 and $\lambda = 0$, but with an important exception. Since ψ has been specified for each plant, the constraint of Eq. (2.14) is voided. (In these calculations, $M = 0.3$ and $V = 0.8$, except for the La Rance facility for which $M = 0.5$ and $V = 0.7$, see Fig. 1.) The values of η^* and ϵ^* so calculated, shown in Table 1, are indeed quite close to the values determined from the published data. In this respect the results of our methodology are consistent with those of other small and large scale design studies.

However, our methodology provides a procedure for determining the most desirable design head (i.e., ψ) by using the constraint of Eq. (2.14). We have therefor recalculated the performance of the plants listed in Table 1, employing this constraint and the values of β and γ shown in the table to determine ψ^{**} , η^{**} and ϵ^{**} , listed at the bottom of Table 1. Comparing these values with those of ψ , η and ϵ above, we note satisfactory agreement for the Bay of Fundy sites but considerable differences for the La Rance and Annapolis Royal facilities, except for the values of effectiveness. For these two power plants the designers have chosen a greater design head (compared to tidal amplitude) than our methodology would select, especially for the La Rance turbines, which results in a lower capacity factor because the turbines are utilized for a shorter fraction of the tidal cycle. A greater design head should result in a lower cost turbine, which might offset a lower capacity factor. It is possible that cost or other factors have influenced the design choices in these two plants.

In making this comparison we have used the values of β and γ selected by the designers. It is very likely that these values were chosen on the basis of minimizing the cost of electric power generation, as there is no physical or engineering constraint which determines their optimum values.

8. Conclusions

The dynamical behavior of tidal power plants can be described in terms of dimensionless variables and parameters which, with one exception (λ), are independent of the size of the system. Thus it is possible to compare the performance of existing or proposed tidal power plants and to estimate quickly the potential performance of a plant located at a

site of any size.

Two performance parameters are advanced as being quite useful. The first, the capacity factor, which measures the ratio of average to rated ideal turbine power, lies in the range of 0.25 to 0.35 for most single effect tidal plant designs. The second, called the effectiveness, is the ratio of average ideal turbine power to the ideal tidal power of the site. For single effect plants its value usually lies within the range of 0.20 to 0.35. While it would be desirable economically to maximize both of these performance measures simultaneously, increasing one inevitably results in a decrease in the other, so that a compromise is required.

The analysis, which assumes typical turbine characteristics of variable wicket and/or runner pitch, defines an optimum design head which is found to be approximately 1.25 times the tidal half amplitude. In order to maximize the effectiveness, however, turbine starting head must be increased for cases with relatively large installed power.

It was found that the effects of variation of tidal range throughout the lunar cycle average out to equal the performance at the mean tidal range. Also, the effects of tidal basin wall shape were not found to be substantial for typical cases.

A major environmental effect will be the reduction in the area of the intertidal zone along the perimeter of the tidal pond, which could be as great as a factor of three for typical plant designs. In addition, for single effect systems, the mean level of the tidal pond is raised above that of the sea.

In comparing our method of analysis with the results of design studies and (in one case) operating performance of an existing tidal plant, good agreement was found for the capacity factor and effectiveness. However,

some designs use a greater turbine design head than our methodology would suggest, presumably for economical reasons.

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10. Nomenclature

A	Surface area of tidal pond
A_0	Value of A at mean tidal level
A_s	Effective flow area of sluiceway, Eq. (2.6)
D_0	Turbine runner diameter
H	Head
H_0	Turbine design head
H_t	Tidal amplitude (one-half of peak to peak)
I	Ideal tidal power, Eq. (3.3)
M	Turbine flow cut-off parameter, Eq. (2.9)
P	Turbine ideal power, Eq. (2.8)
P_0	Turbine design ideal power
Q_0	Turbine design volume flow rate
Q_s	Sluiceway volume flow rate
Q_t	Turbine volume flow rate
T	Tidal period
V	Turbine part load flow parameter, Eq. (2.9)
W	Tidal pond ideal work, Eq. (3.2)
Z	Tidal pond elevation above mean sea level
Z_t	Sea surface elevation above mean sea level
f	Normalized turbine flow rate, Eq. (2.5)
g	Acceleration of gravity
h_s	Sluiceway flow function, Eq. (2.7)
h_t	Turbine flow function, Eq. (2.7)
ℓ	dA/dZ at mean sea level
t	Time
β	Ratio of turbine flow to tidal pond flow, Eq. (2.13)

Nomenclature (continued)

γ	Ratio of sluiceway flow to turbine flow, Eq. (2.13)
ϵ	Effectiveness, Eq. (3.4)
λ	Dimensionless pond wall slope parameter, Eq. (2.13)
η	Capacity factor, Eq. (3.1)
ψ	Ratio of tidal amplitude to design head, Eq. (2.13)
ρ	Density of sea water

Superscript

Dimensionless variable, Eq. (2.11)

Table 1
COMPARISON OF TIDAL POWER PLANT PERFORMANCE

	S i t e L o c a t i o n					
	Half Moon Cove	Anna- polis Royal	La Rance	Bay of Fundy		
				A8	A6	B9
A_o (10^6 m ²)	2.2	4.8	12.9	73	128	175
H_t (m)	2.77	3.14	4.25	4.9	4.8	5.9
Q_o (m ³ /s)	285	378	3240	25700	36800	78200
H_o (m)	4.3	5.5	8	6.5	6.5	7.5
Power (MWe)	12	17.8	240	1150	1640	4030
A_s (m ²)	284	230	1530	5970	7470	14900
ψ	0.64	0.57	0.53	0.75	0.74	0.79
β	2.11	1.11	2.64	3.20	2.68	3.39
γ	3.03	4.73	4.31	2.29	1.97	2.06
η	0.35	0.321	0.225	0.341	0.315	0.358
ϵ	0.29	0.171	0.331	0.318	0.250	0.338
η^*	0.34	0.363	0.279	0.307	0.305	0.307
ϵ^*	0.28	0.176	0.347	0.327	0.276	0.329
ψ^{**}	0.80	0.80	1.30	0.80	0.80	0.80
η^{**}	0.44	0.443	0.659	0.332	0.359	0.323
ϵ^{**}	0.29	0.159	0.329	0.329	0.295	0.338

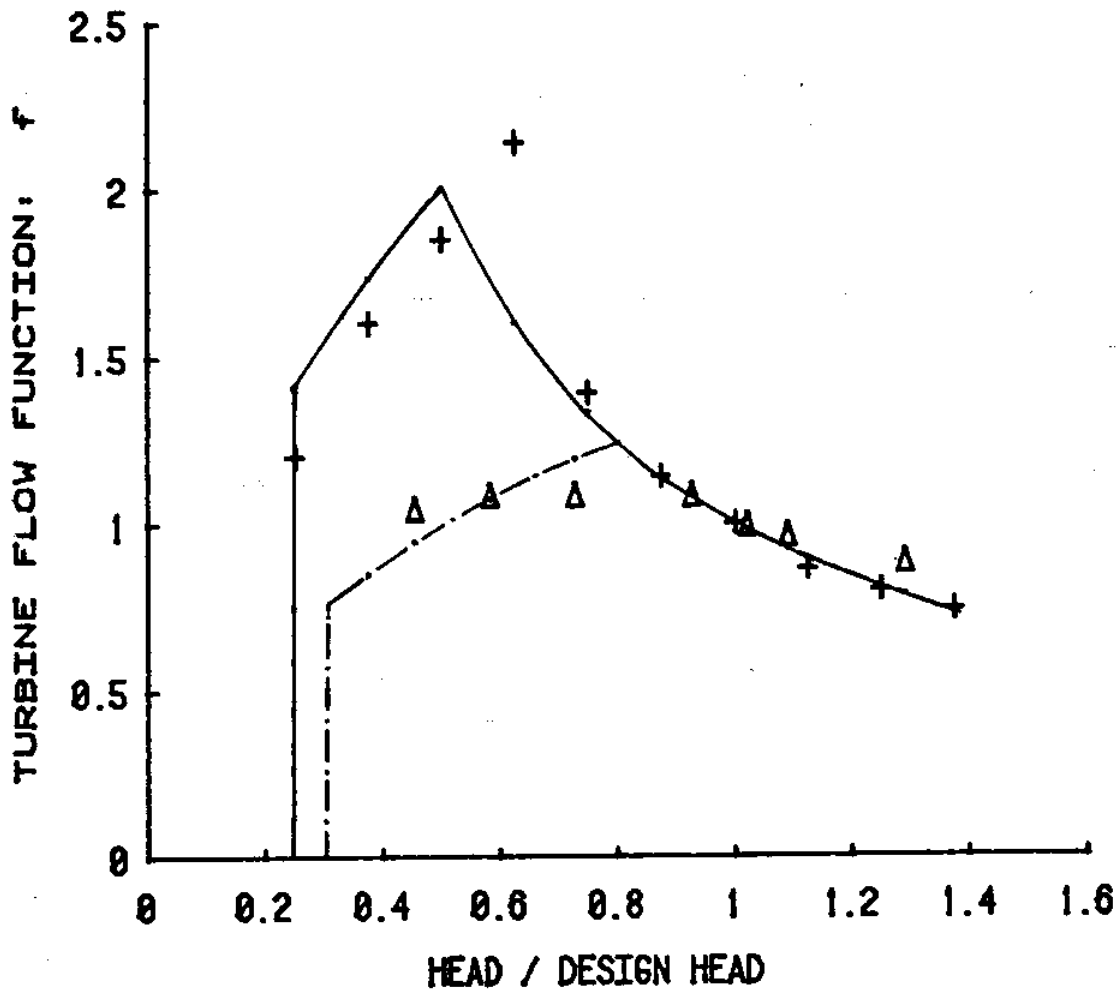


Fig. 1 Comparison of the turbine flow function f (Eq. 2.5) for the La Rance (+) and Annapolis Royal (Δ) power plants with the representation of Eq (2.9). For La Rance (—) and Annapolis Royal (---), $M = 0.25$ and 0.3 and $V = 0.5$ and 0.8 , respectively.

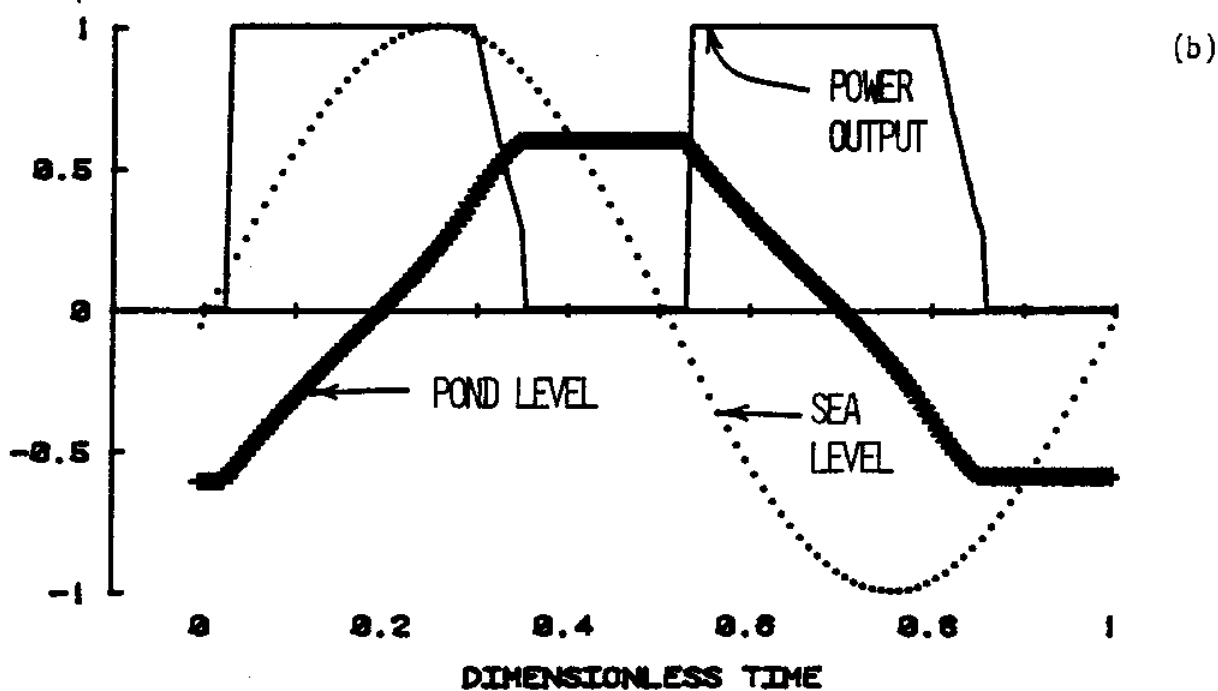
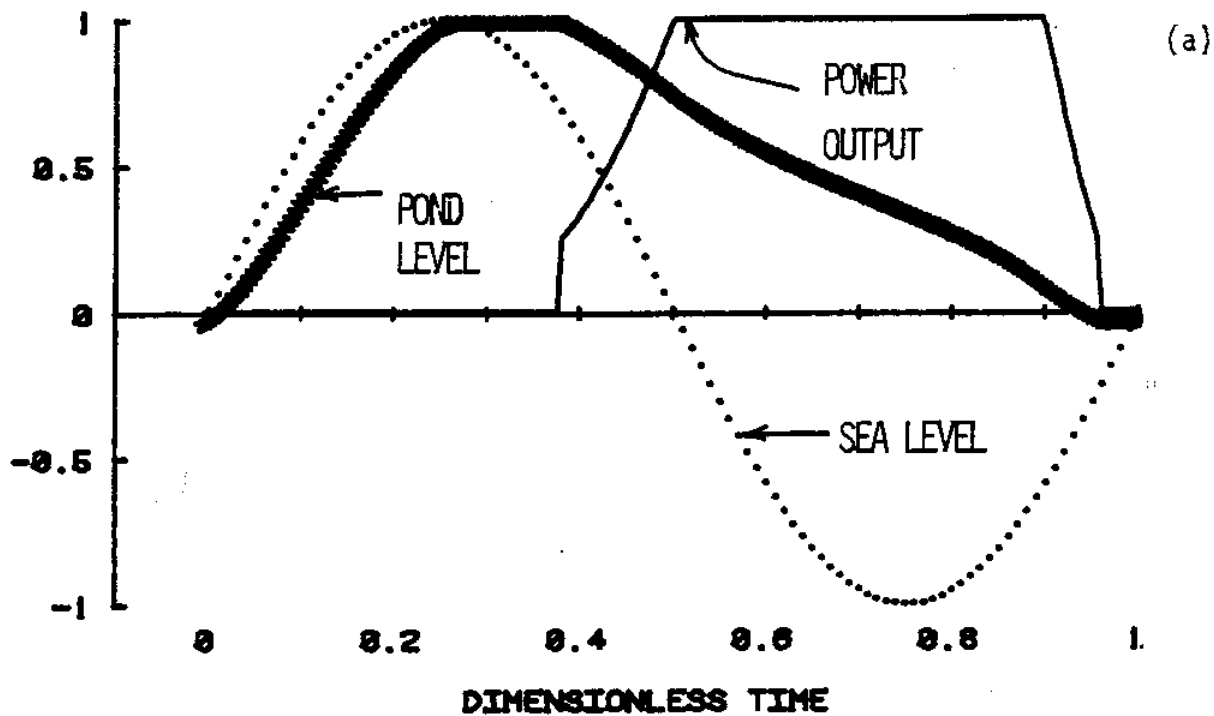


Fig. 2 Typical system response for (a) single effect ($\beta = 2, \gamma = 5, \lambda = 0$) and (b) double effect ($\beta = 4, \gamma = 0, \lambda = 0$) tidal plant operation.

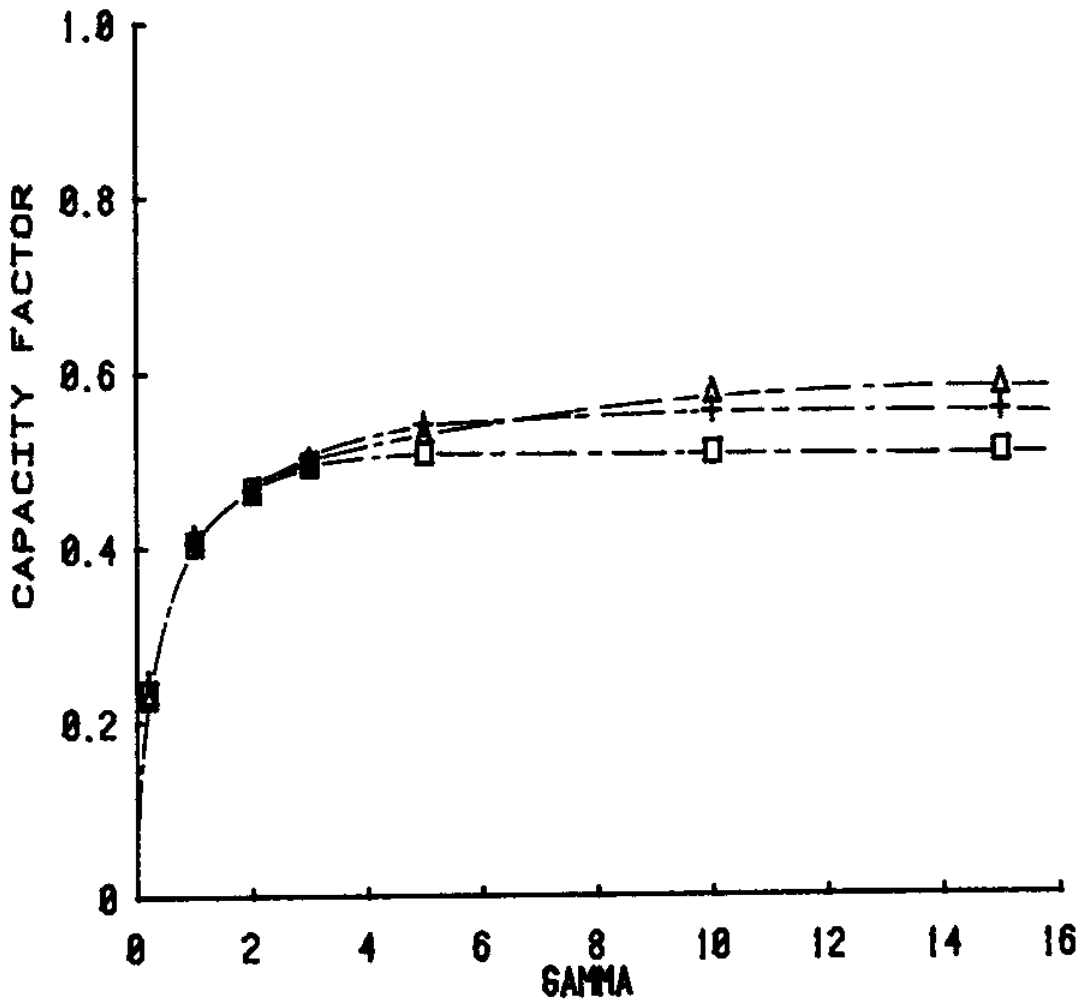


Fig. 3 The dependence of capacity factor η on the sluiceway size parameter γ for single effect operation (Δ : $\beta = 0.5$; + : $\beta = 1$; \square : $\beta = 2$) with $\lambda = 0$.

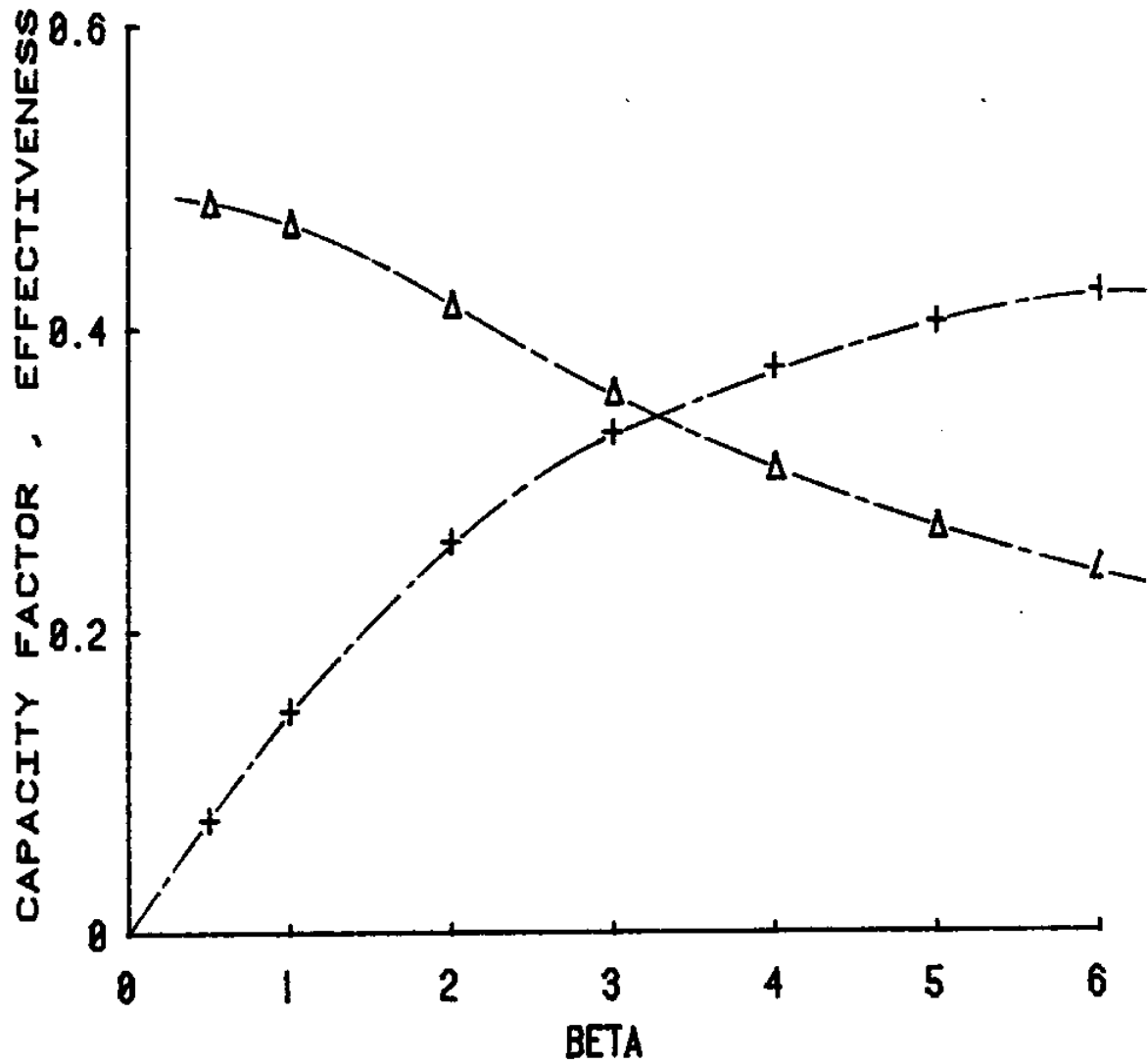


Fig. 4 The variation of capacity factor (Δ) and effectiveness (+) with increasing β (size of turbine compared to the tidal pond size), for single effect operation with $\gamma = 5$, $\lambda = 0$.

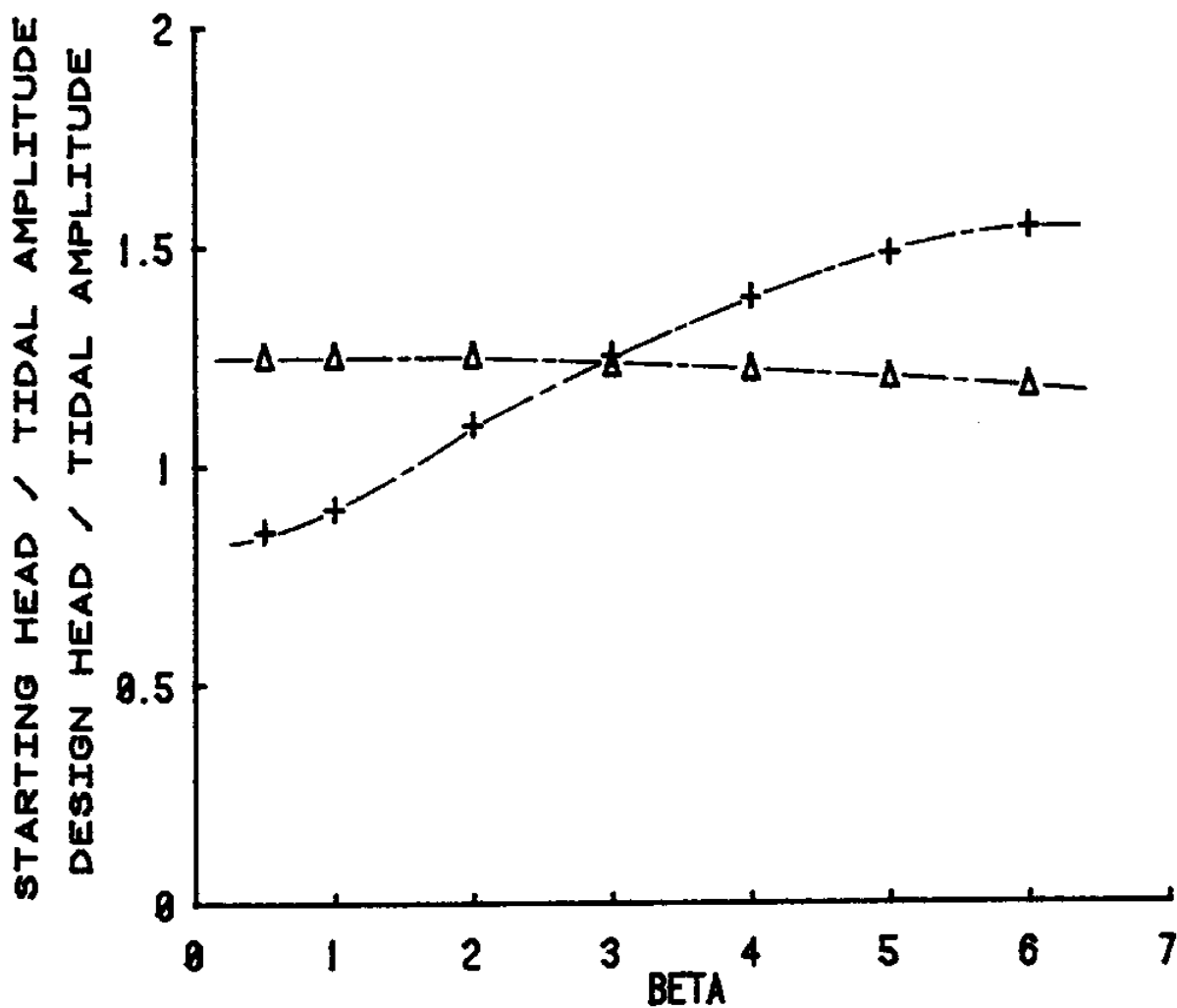


Fig. 5 The increasing starting head (+) resulting from the constraint which maximizes the effectiveness ϵ . (Conditions as for Fig. 4). Note also the corresponding variation in design head(Δ).

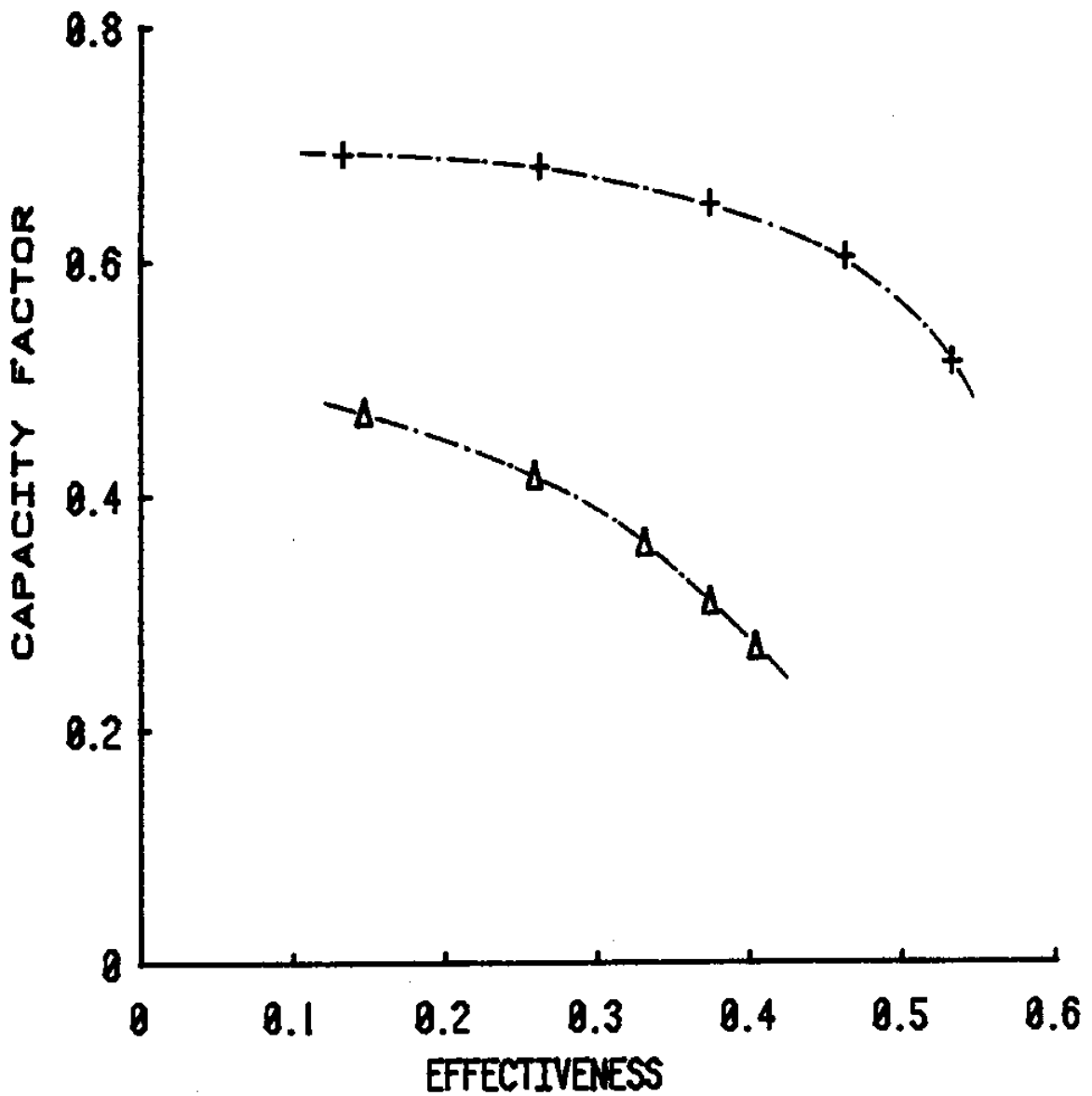


Fig. 6 The Relationship between capacity factor η and effectiveness ϵ for single effect (Δ) and double effect ($+$) operation. Note that double effect values are not double the single effect values.

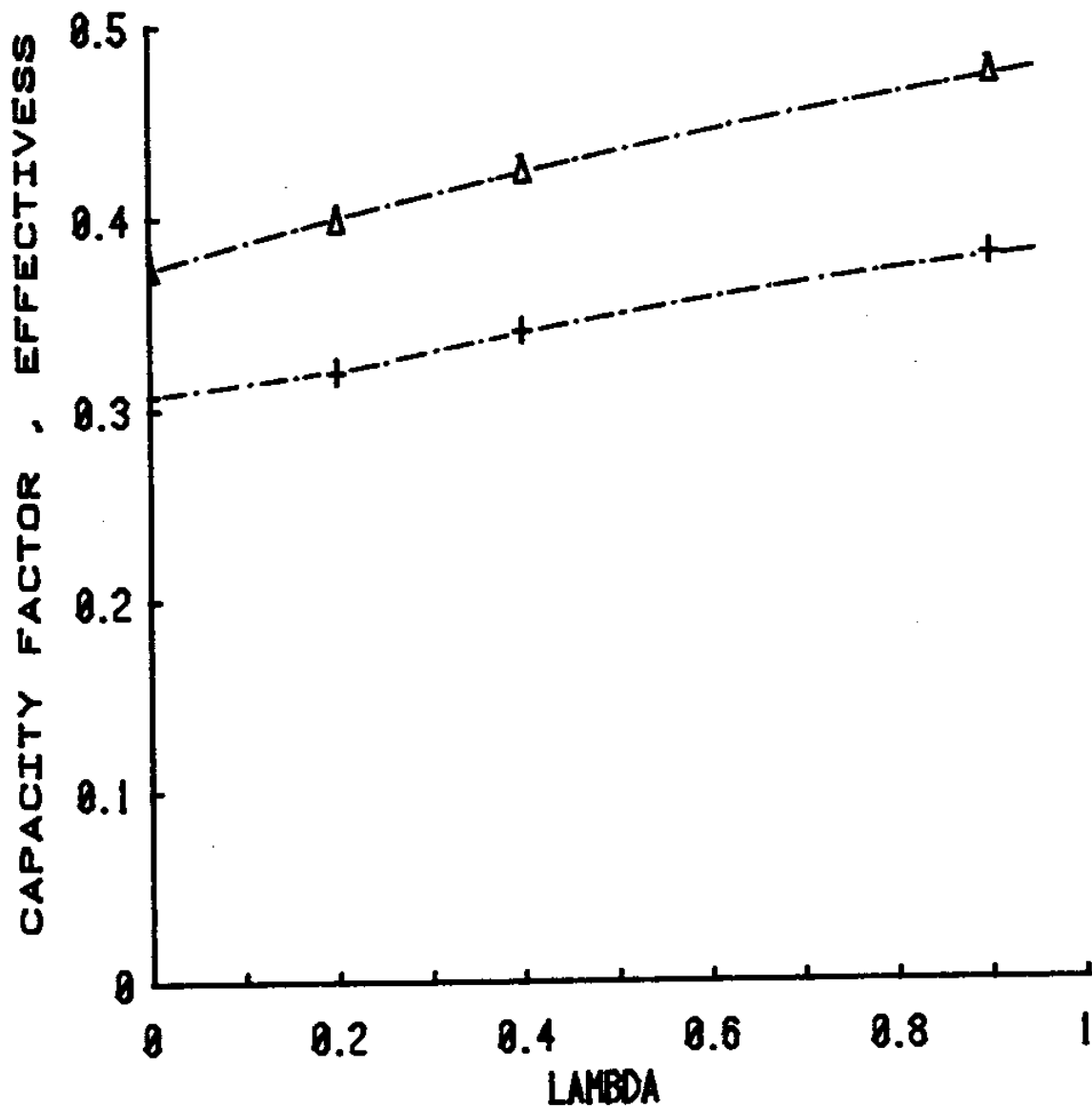


Fig. 7 The effect of tidal pond volume shape factor λ on capacity factor (+) and effectiveness (Δ) for single effect outflow operation with $\beta = 4$, $\gamma = 5$.

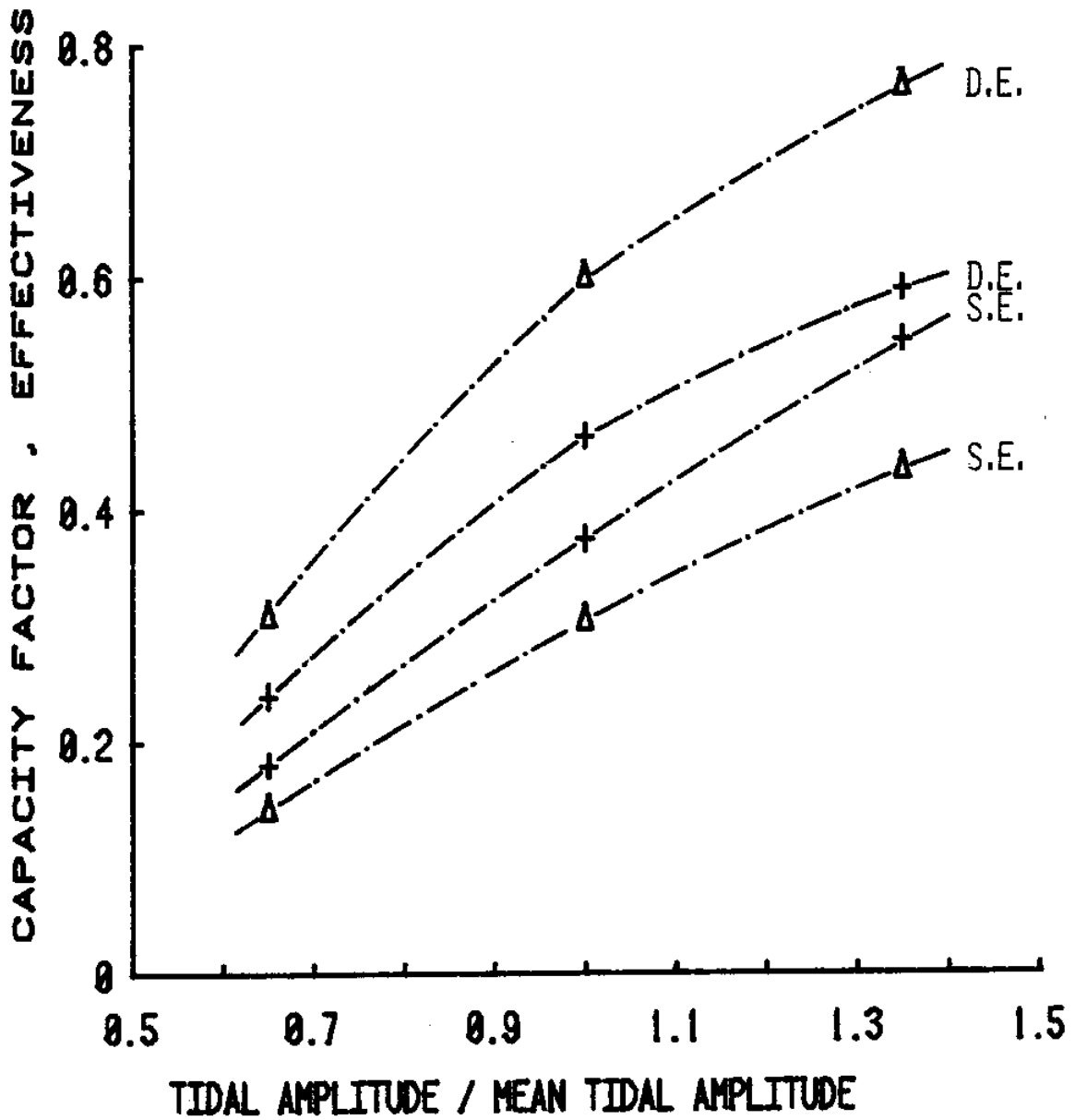


Fig. 8 The effect of tidal amplitude ratio H_t'/H_t on capacity factor (Δ) and effectiveness (+) for single effect ($\beta = 4, \gamma = 5, \lambda = 0$) and double effect ($\beta = 4, \gamma = \lambda = 0$) operation.

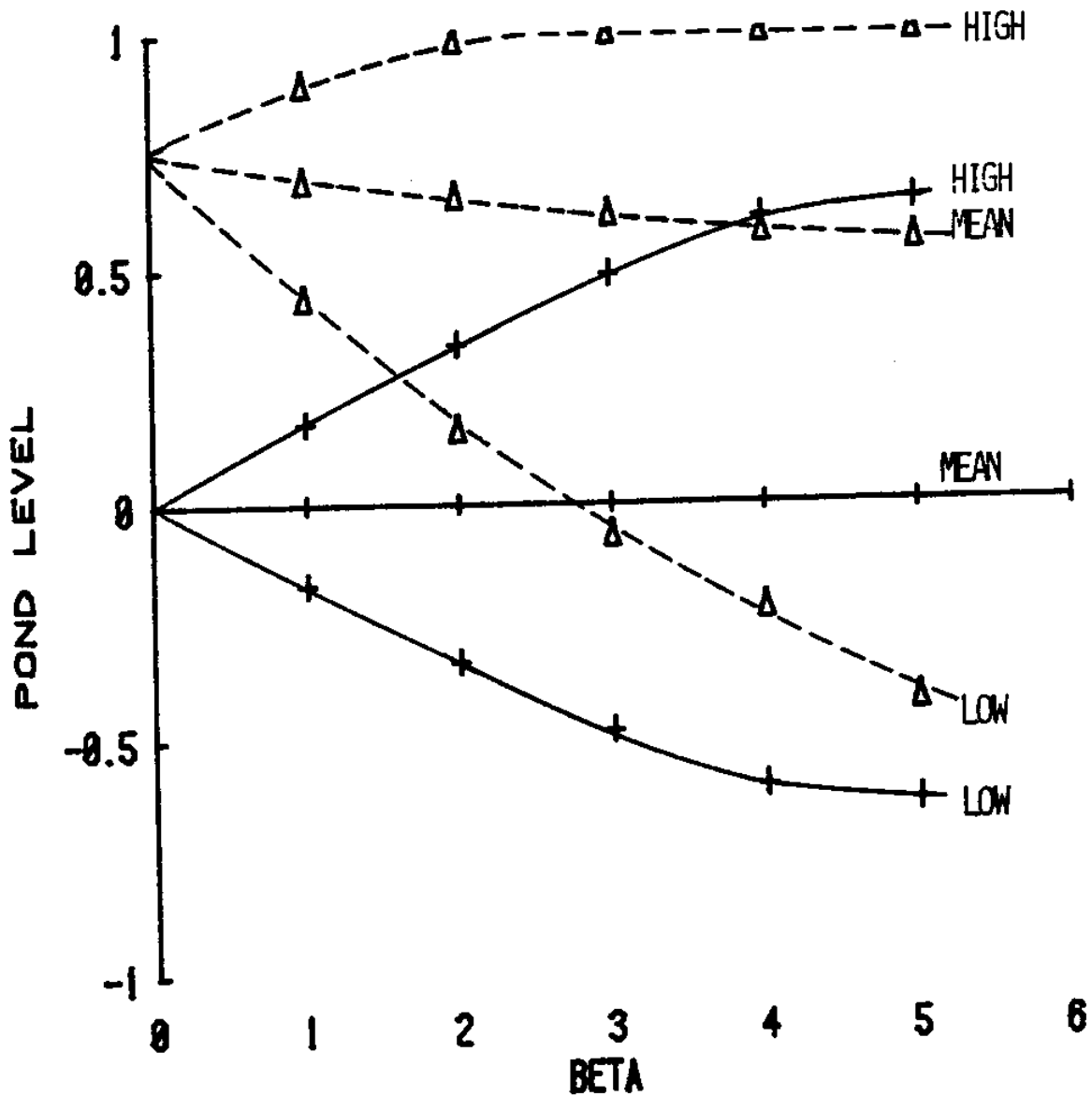


Fig. 9 The variation of maximum, mean and minimum tidal pond levels as a function of the turbine size parameter β for a single effect (Δ) outflow system ($\gamma = 5, \lambda = 0$) and a double effect (+) system ($\gamma = 0, \lambda = 0$).

