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STRESS INTENSITY FACTORS FOR ANISOTROPIC FRACTURE TEST SPECIMENS OF SEVERAL GEOMETRIES

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ABSTRACT

Stress intensity factors have been obtained for single-edge-notched, double-edge-notched, and double cantilever beam fracture toughness test specimens using a two-dimensional hybrid stress model finite element analysis. The degree of anisotropy is shown to have a significant effect on the stress intensity factor in some cases, with differing effects for different specimen shapes; however, effects of anisotropy are relatively constant for varying crack lengths of a given shape. An experimental K-calibration for the double cantilever beam specimen is in good agreement with the analytical prediction, and the effect of geometry on the applied load to cause crack propagation is accurately predicted by the analysis.

INTRODUCTION

The opening mode stress intensity factor for sharp cracks in isotropic bodies of many geometries can be obtained by several techniques [1,2]. Recent advances using a modified mapping-collocation technique have indicated that the degree of anisotropy of the material may significantly influence the stress intensity factor in some cases [3,4]. The relationship between the strain energy release rate and the stress intensity factor for anisotropic materials has also been obtained [5,6], and confirmed experimentally for a unidirectional composite with the crack oriented parallel to the fibers [7].

This paper gives the results of a hybrid stress finite element analysis of finite dimension single-edge-notched, double-edge-notched, and double cantilever beam type specimens, including the effects of fiber orientation and crack length aspect ratio. The analytical results are compared to an experimental K-calibration of the double cantilever beam specimen, and the use of the K-calibration in predicting the effects of specimen geometry on the specimen fracture load is discussed.

THE HYBRID STRESS FINITE ELEMENT ANALYSIS

The hybrid stress method of finite element analysis is characterized by the use of an assumed stress field in the element and the principle of minumum complimentary energy [8]. The stress distribution is selected such that the equilibrium equations are satisfied within the element, and the inter-element boundary displacements are interpolated functions of the nodal displacements to ensure continuity. The ability to select the most appropriate stress function is of particular importance for the crack problem since the correct singular terms can be included for those elements near to the crack tip, increasing the accuracy and efficiency of the analysis.

The assumed stress function for the crack problem, in matrix form, is expressed as

$$\underset{\sim}{\sigma} = \underset{\sim}{P}\underset{\sim}{\beta} + \underset{\sim}{P}_s\underset{\sim}{\beta}_s \tag{1}$$

where $\underset{\sim}{P}\underset{\sim}{\beta}$ is a simple polynomial satisfying the homogeneous stress equilibrium equations and $\underset{\sim}{P}_s\underset{\sim}{\beta}_s$ corresponds to the singular term due to the presence of the crack [9]. The singular term will satisfy both the equilibrium and compatibility equations. The magnitudes of the stress parameters $\underset{\sim}{\beta}_s$ which are among the unknowns of the final matrix equations are, in fact, the stress intensity factors to be evaluated.

The details of formulation for the analysis are given in references [8] and [9], and are not described in detail here.

The stress intensity factor, K_I, calculated by this method is essentially for an ideally sharp crack embedded in a homogeneous laminated plate. Free boundaries, specimen geometry, aspect ratio of crack length and the degree of anisotropy all are expected to affect the value of the stress intensity factor, and must be accounted for before fracture mechanics is applied to laboratory testing and practical applications. For anisotropic laminated composites, the correction factor Y for finite dimension fracture toughness test specimens is a functional form of the above variables as

$$Y = Y(c/w,\ A_{ij},\ \alpha)$$

where c/w = crack length aspect ratio

A_{ij} = elastic material constants

α = geometry of specimen

Based on the analytically determined K, the K-calibration of an anisotropic laminate with different geometries can be evaluated as

$$Y(c/w,\ A_{ij},\ \alpha) = K_I/(\sigma\sqrt{c}) \tag{2}$$

where σ is the applied stress and c is the crack length.

EXPERIMENTAL K-CALIBRATION TECHNIQUE

The K-calibration was determined experimentally following the procedure suggested by Irwin and Kies [10]. The K-calibration is calculated from the critical opening mode strain energy release rate, G_I, which is determined experimentally by measurement of the specimen compliance, λ, for various crack lengths. The value of G_I is given by

$$G_I = 1/2\ P_a^2 \frac{\partial\lambda}{\partial c} \qquad (3)$$

where P_a is the applied load, and $\partial\lambda/\partial c$ is determined from a polynomial curve fit to the λ vs. c data. The opening mode stress intensity factor, K_I, is then determined from G_I by [5]

$$K_I^2 = G_I / \{ [\frac{A_{11}A_{22}}{2}]^{1/2} [(\frac{A_{22}}{A_{11}})^{1/2} + \frac{2A_{12}+A_{66}}{2A_{11}}]^{1/2} \} \qquad (4)$$

where 1 is the direction parallel to the crack, and the A_{ij} are elastic compliances from the stress-strain relations

$$\underset{\sim}{\varepsilon} = \underset{\sim}{A}\underset{\sim}{\sigma}$$

RESULTS

Figures 1 and 2 show the finite element mesh for typical double cantilever beam specimens and edge-notched tension specimens which were loaded by a uniform tensile stress at the end. Investigation of mesh size requirements reported in Reference [9] indicates that the sizes chosen should give stress intensity factors with less than one percent error; for the double cantilever beam specimen, an increase in crack tip mesh dimension of a factor of two resulted in less than 0.5% change in the calculated stress intensity factor.

The effects of the degree of anisotropy on the stress intensity factor were investigated by analyzing edge-notched-tension specimens with elastic moduli representative of medium modulus graphite/epoxy of balanced ply configuration $(\theta/-\theta/\theta/-\theta/-\theta/\theta/-\theta/\theta)$, where θ is the angle between the fiber direction and the load direction. Figure 3 gives the elastic constants of the anisotropic laminates calculated by an energy based semi-empirical micromechanical analysis following Chamis [11], using the fiber and matrix properties given in Table 1.

Figures 4 and 5 give the variation of the K-calibration parameter, Y, with fiber orientation for single and double-edge-notched tension specimens. The fiber orientation has a significant effect on the K-calibration with a different relationship observed for each specimen. Both curves indicate a minimum value of Y at approximately $\theta = \frac{7}{24}\pi$, and a value equal to the isotropic case [12] at $\theta = \frac{7}{36}\pi$.

Figures 6 and 7 compare isotropic and 0°/90° graphite/epoxy results for various crack length to specimen width ratios for the elastic constants given in Table 2. Although the values of Y differ for the double-edge-notched specimen, the shape of the curve is similar for both cases.

Figure 8 gives the analytical and experimental K-calibration for a double cantilever beam specimen of E-glass/polyester typical of marine applications. The moduli given in Table 2 were measured using strain gages except for E_{xy} which was calculated from laminate theory

[13]; other characteristics of the material are also listed in Table 2. Further details of the experimental K-calibration are given in Reference [14]. Figure 8 indicates good agreement between the analytical K-calibration using measured elastic constants, and the experimental K-calibration, with significant deviations from the isotropic solutions of Srawley [15] which assumes an infinitely long specimen, Kanninen [16], and the present finite element solution using isotropic elastic constants.

For the E-glass/polyester material studied, a crack can be propagated in a stable manner down the full length of the specimen, colinear with the original notch. The crack propagates in a stable manner, with increasing applied deflection necessary for continuous propagation of the crack. Figure 9 gives the results of several test specimens, indicating that the value of load necessary for crack extension is consistent with a constant K_I criterion using the analytical K-calibration, over a broad range of crack lengths.

DISCUSSION

The use of a two-dimensional analysis for the K-calibration fails to account for three-dimensional effects, particularly the interlaminar shear effects, which may be significant for some ply orientations [17,18,19]. Recent results of a three-dimensional analysis using a similar finite element technique [19] indicate that the in-plane stresses over a relatively large domain near the crack tip vary significantly from those determined by two-dimensional analyses. The free-edge effects along the specimen edges and crack flanks may also produce significant deviations for small specimens.

The common use of isotropic K-calibrations in fracture toughness testing for composite materials clearly results in errors both in the absolute magnitude of K_I and in the relative magnitude for different specimen shapes. However, the errors are not large in most cases, and the relative values of K_I for various crack lengths for a single specimen shape appear to be consistent. Caution should be exercised in assessing the implications of the agreement between analytical and experimental results for a propagating crack in the E-glass/polyester specimens. Although the results do imply that crack propagation occurs in the absence of any general inelastic behavior of the specimen, the validity of the assumptions of linear elastic fracture mechanics is not implicit. Recent results [19, 20] indicate that a naturally propagating crack in many fibrous composites may be so blunted by the growth of subcracking zones at various angles to the main crack as to completely eliminate any $1/\sqrt{r}$ singular stress field at the main crack tip. The appearance of a propagating crack in the E-glass/polyester material is indicated in Figure 10, in which such zones are readily apparent.

CONCLUSIONS

The stress intensity factor for specimens of finite dimension is significantly affected by the degree of anisotropy of the material. The effect of anisotropy differs for specimens of different shape, but is relatively constant for varying crack lengths within a particular specimen shape. The value of the stress intensity factor obtained from the hybrid stress finite element analysis is in good agreement with the value obtained by an experimental compliance calibration for an E-glass/polyester laminate; the effect of specimen goemetry on the load necessary for crack propagation is also predicted accurately by the analysis.

ACKNOWLEDGEMENTS

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TABLE I

ASSUMED FIBER AND MATRIX PROPERTIES FOR GRAPHITE/EPOXY

Properties of graphite fibers: (1 is direction parallel to fiber axis)

$E_{11} = 32.60 \times 10^6$ lb/in^2

$E_{22} = E_{23} = 2.0 \times 10^6$ lb/in^2

$\nu_{12} = \nu_{13} = .20$

$\nu_{23} = 0.25$

$G_{12} = G_{13} = 2.0 \times 10^6$ lb/in^2

$G_{23} = 0.8 \times 10^6$ lb/in^2

Properties of Matrix:

$E_{11} = E_{22} = E_{33} = 0.78 \times 10^6$ lb/in^2

$\nu_{12} = \nu_{23} = \nu_{13} = 0.35$

Fiber volume fraction: 0.55

TABLE II

ELASTIC CONSTANTS FOR K-CALIBRATIONS

Elastic Constant	E-glass/Polyester*	(90/0/90/0/90/0/90) Graphite/Epoxy
E_{xx} (10^6 psi)	1.79	8.79
E_{yy} (10^6 psi)	2.66	11.20
E_{xy} (10^6 psi)	0.48	3.90
ν_{xy}	0.19	0.035

*E-glass/polyester characteristics:

Average thickness = 0.216 in.

Average fiber volume fraction = 0.305

Fabrication Method: hand layup

Laminate Construction: M/R/M/R/M, where

M: 1-1/2 oz/ft^2 chopped fiber mat

R: 18 oz/yd^2 woven roving style 779, warp direction parallel to y-axis. (Stevens Fiberglass Co.)

Matrix: Laminac 4155 polyester (American Cyanamid Co.)

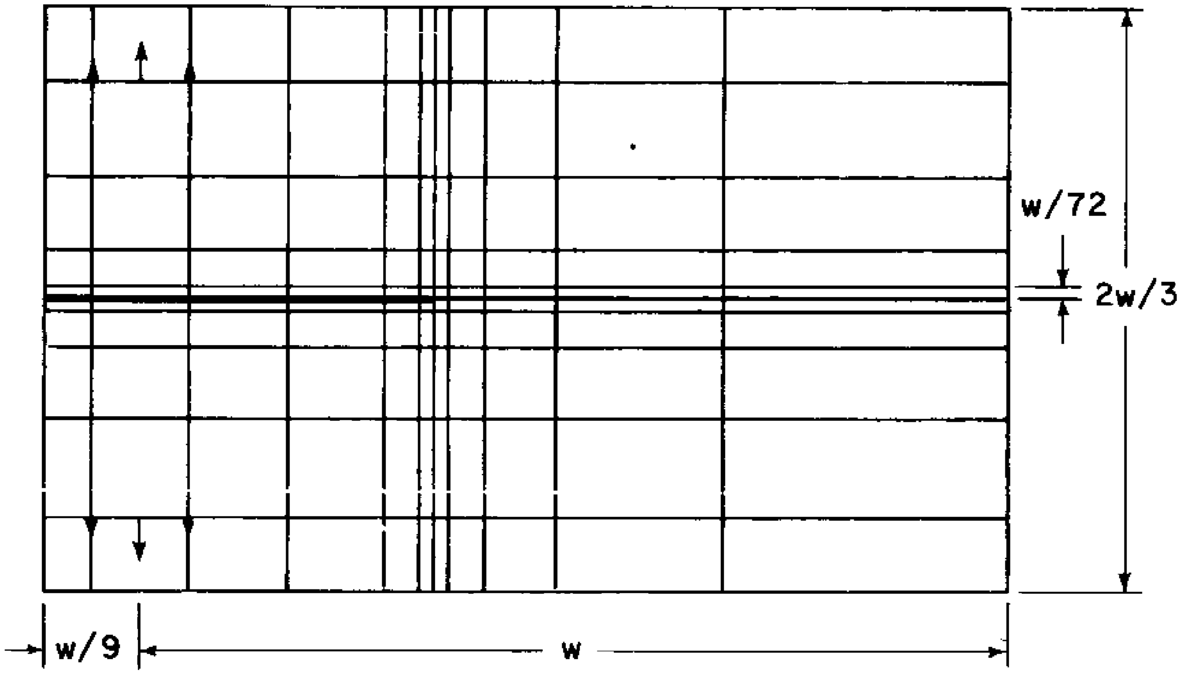

FIGURE 1.

TYPICAL FINITE ELEMENT MESH FOR THE DOUBLE CANTILEVER BEAM SPECIMEN.

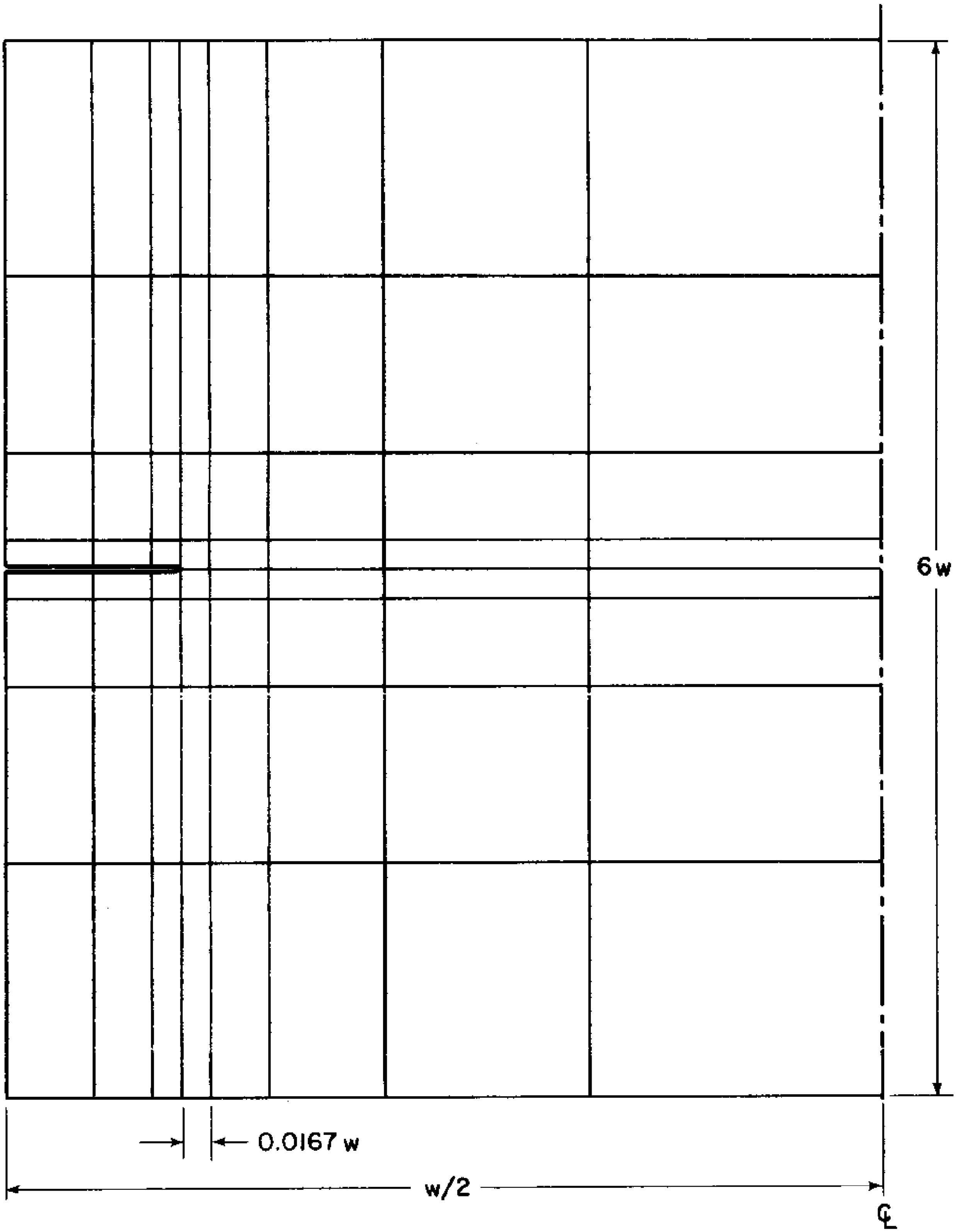

FIGURE 2.

TYPICAL FINITE ELEMENT MESH FOR THE DOUBLE-EDGE-NOTCHED TENSION SPECIMEN.

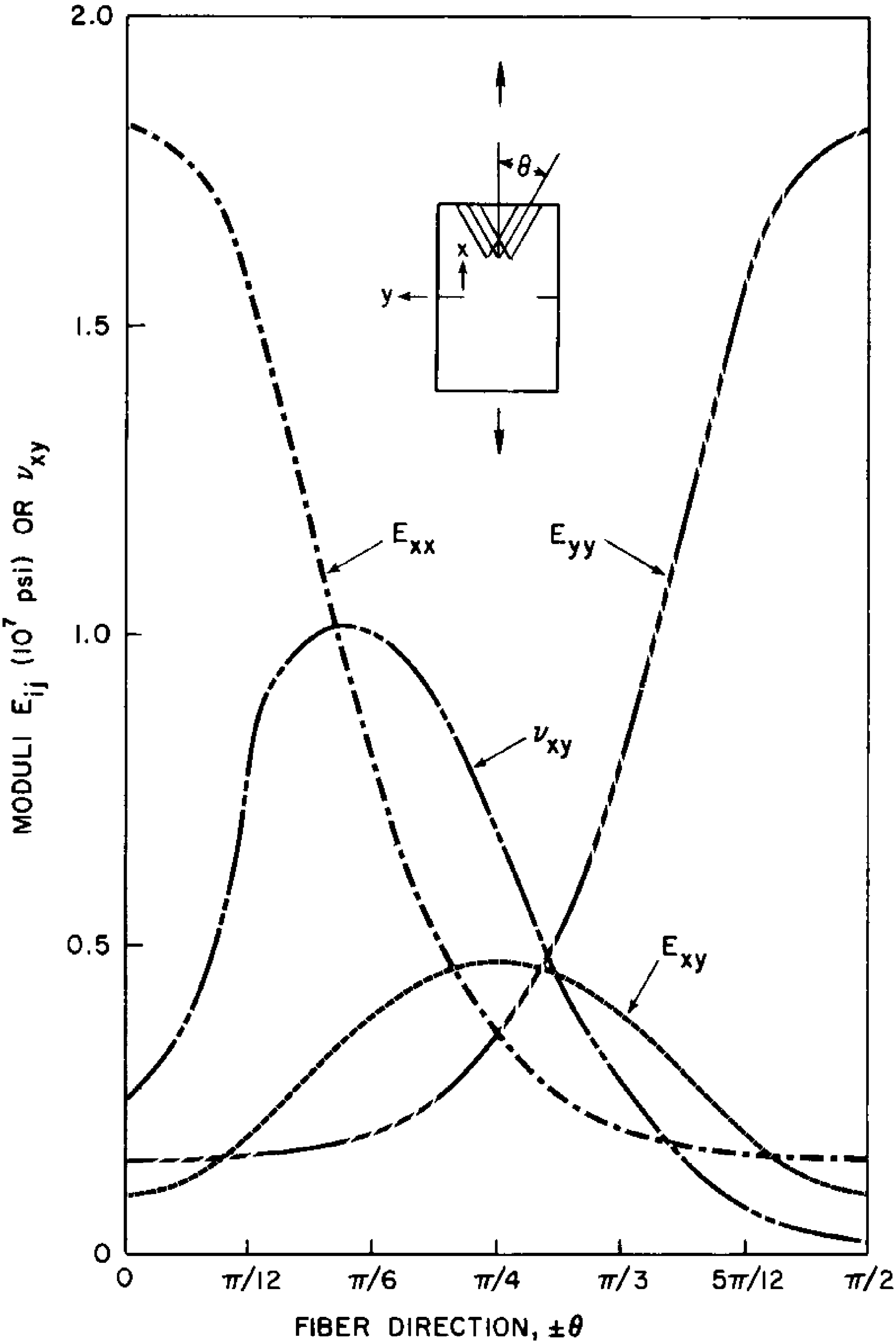

FIGURE 3.

ELASTIC MODULI FOR 8-PLY GRAPHITE/EPOXY LAMINATES OF BALANCED CONSTRUCTION.

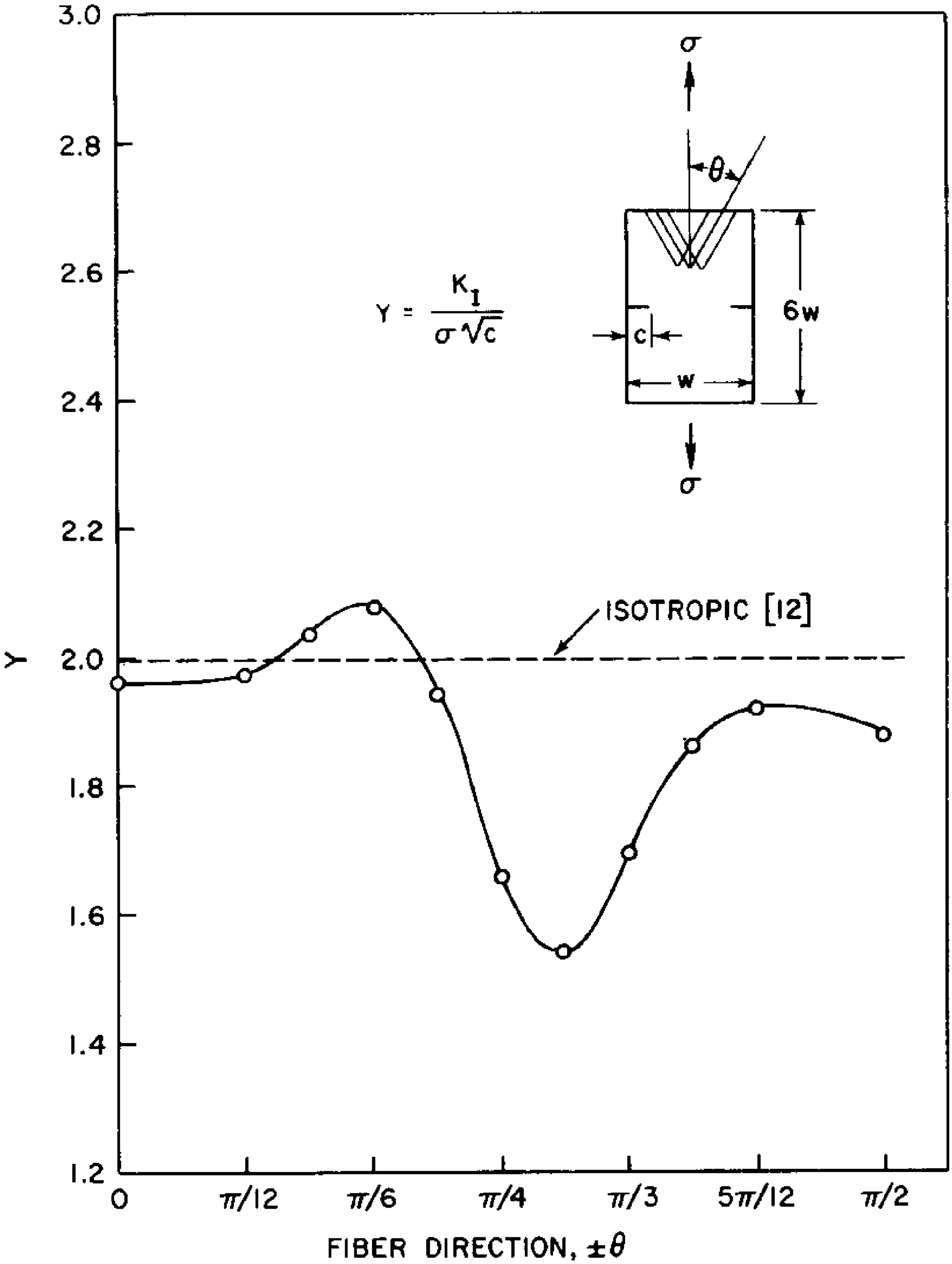

FIGURE 4.

K-CALIBRATION FOR VARIOUS PLY ORIENTATIONS (DOUBLE-EDGE-NOTCHED SPECIMENS, GRAPHITE/EPOXY, 2c/w = 0.3).

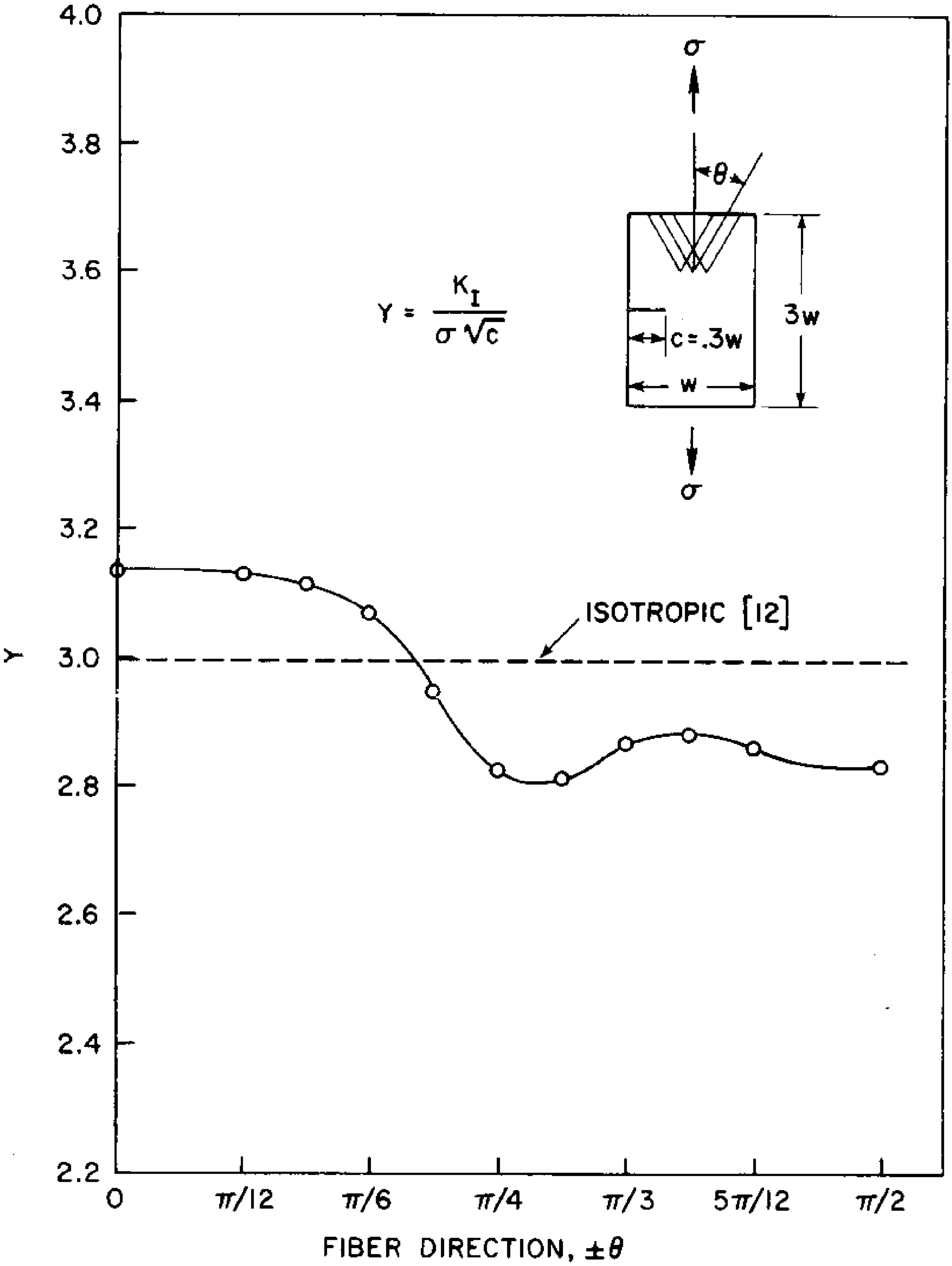

FIGURE 5.

K-CALIBRATION FOR VARIOUS PLY ORIENTATIONS (SINGLE-EDGE-NOTCHED SPECIMENS, GRAPHITE/EPOXY, c/w = 0.3).

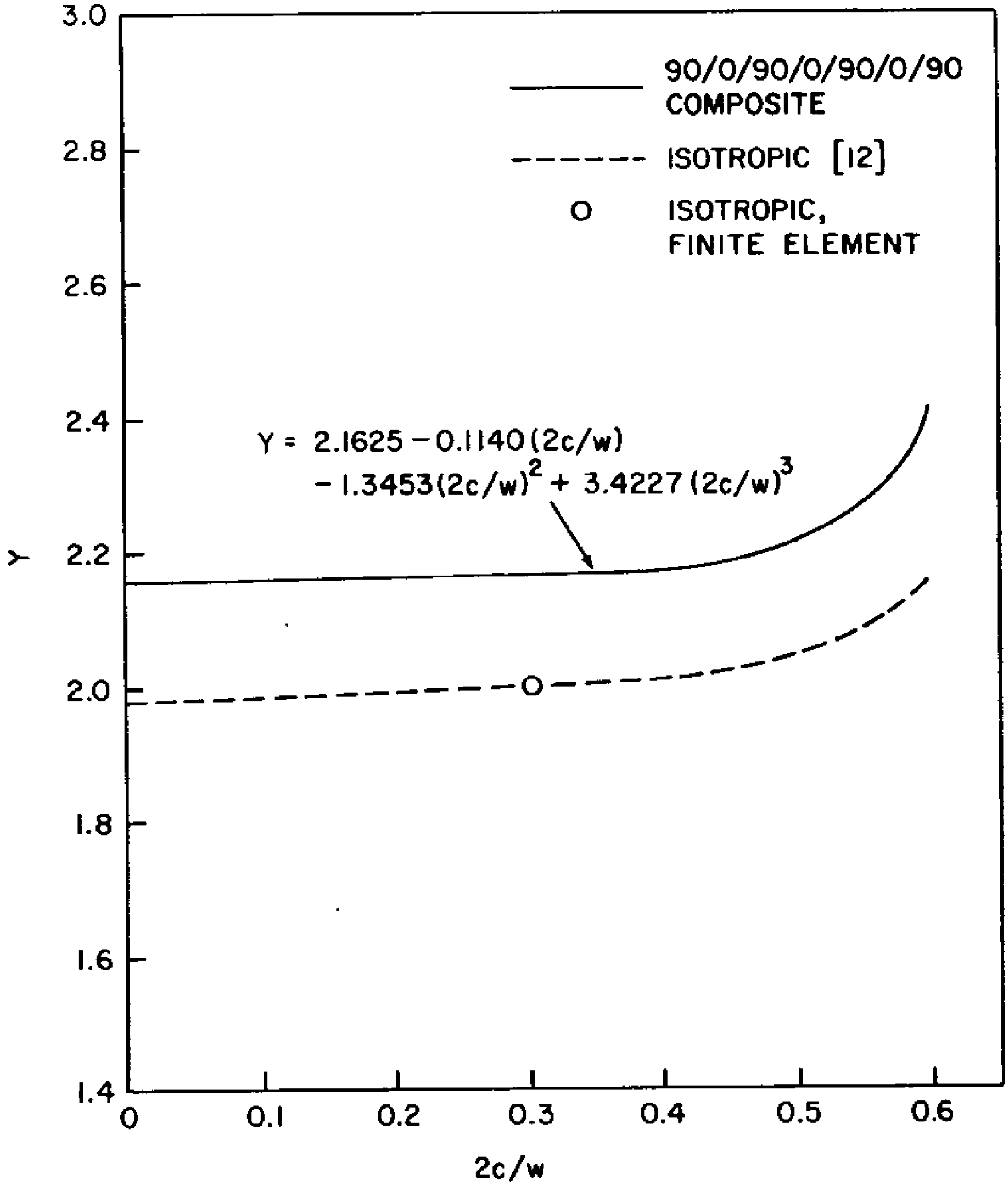

FIGURE 6.

K-CALIBRATION FOR DOUBLE-EDGE-NOTCHED SPECIMEN WITH DIFFERENT CRACK ASPECT RATIOS, 0°/90° GRAPHITE/EPOXY.

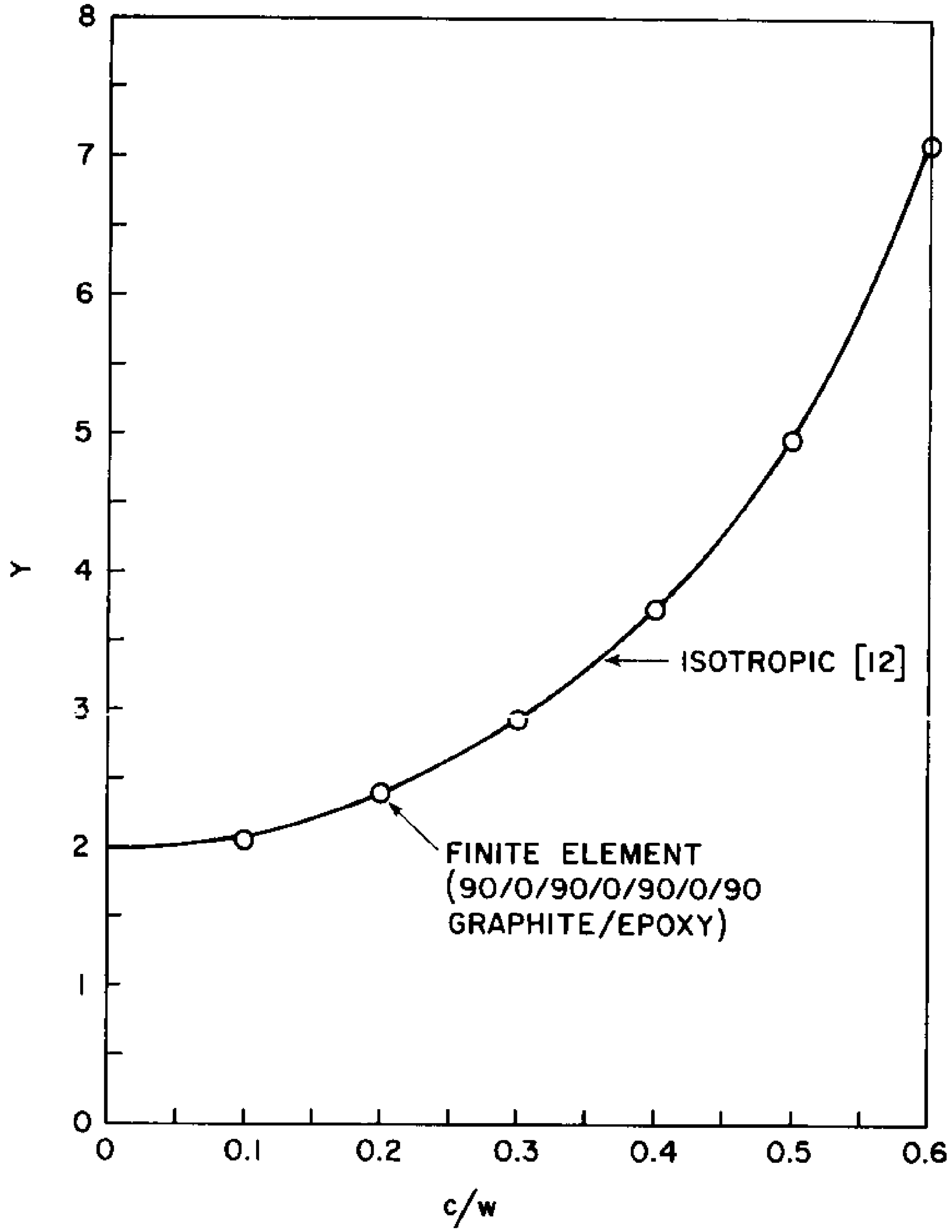

FIGURE 7.

K-CALIBRATION FOR SINGLE-EDGE-NOTCHED SPECIMEN WITH DIFFERENT CRACK ASPECT RATIOS, 0°/90° GRAPHITE/EPOXY.

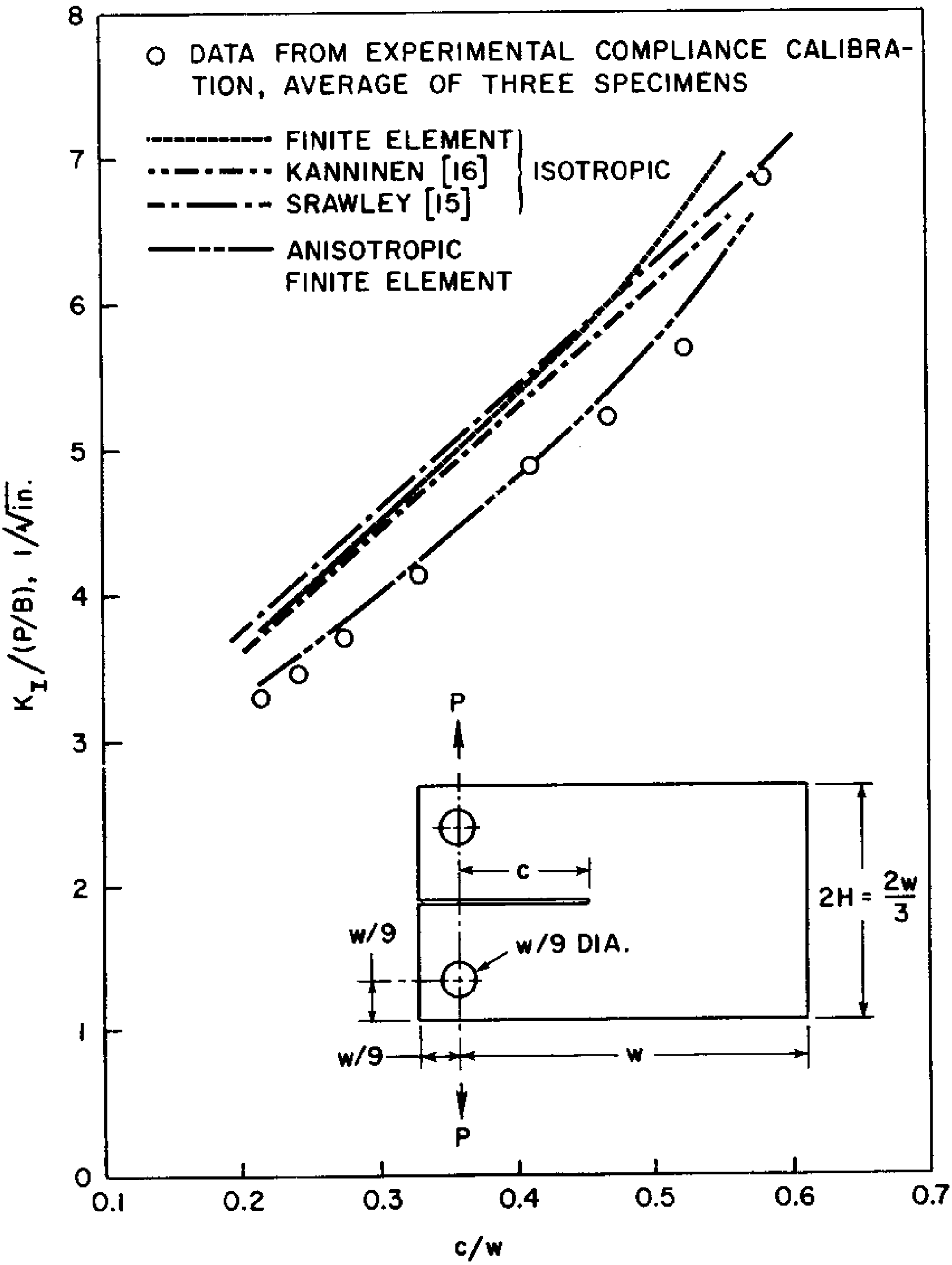

FIGURE 8.

K-CALIBRATIONS FOR E-GLASS/POLYESTER (w = 4.5 in.).

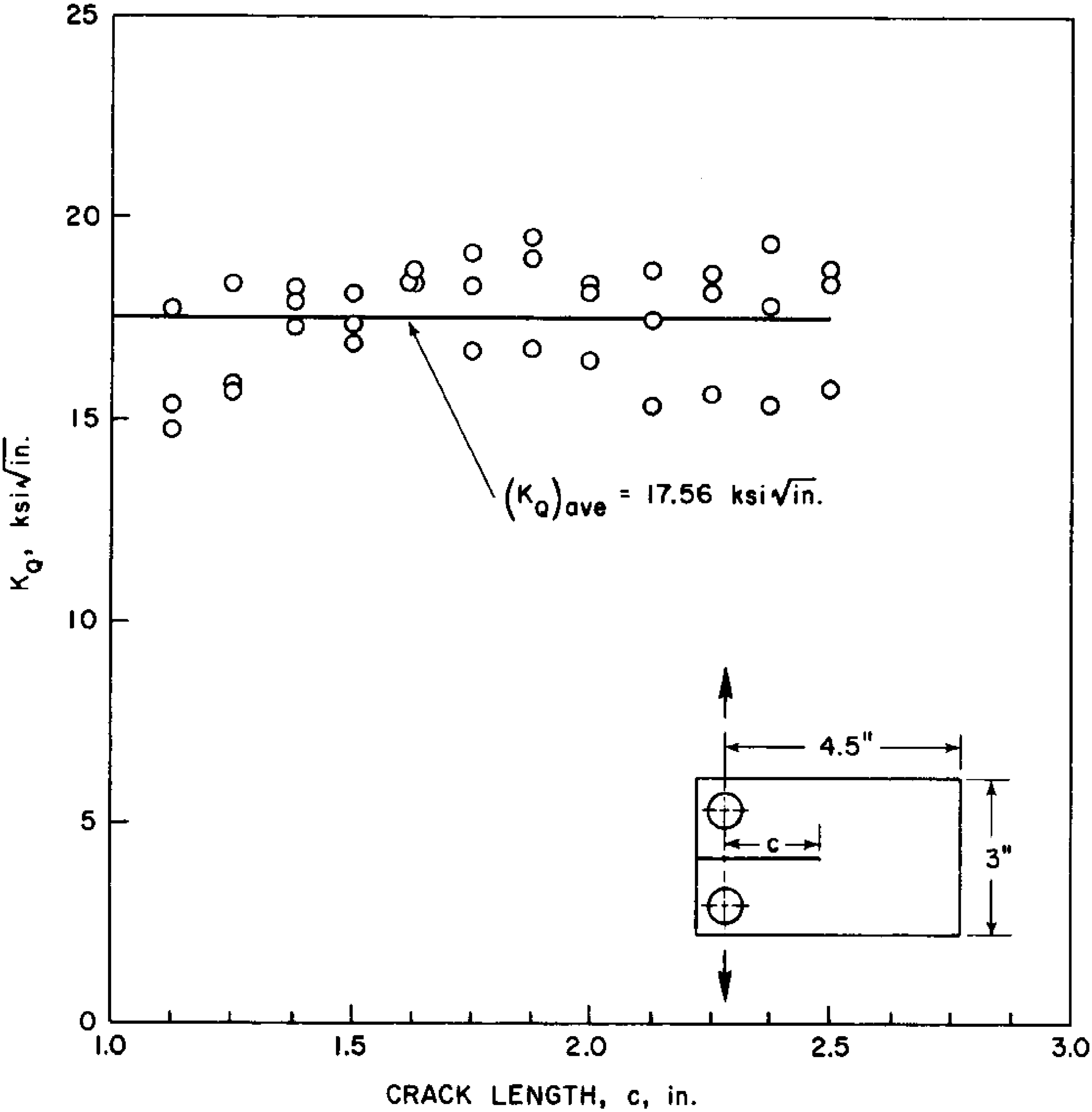

FIGURE 9.

K_Q vs. CRACK LENGTH FOR E-GLASS/POLYESTER USING THE FINITE-ELEMENT K-CALIBRATION.

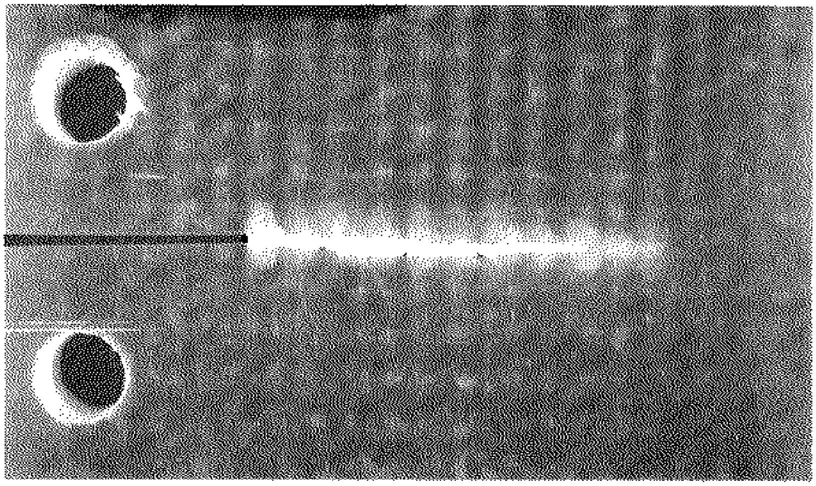

FIGURE 10.

CRACK PROPAGATION IN E-GLASS/POLYESTER DOUBLE CANTILEVER BEAM SPECIMEN.