

**DEVELOPMENT OF AUTONOMOUS VEHICLES FOR
LONG TERM OCEAN EDDY OBSERVATION**

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DEVELOPMENT OF AUTONOMOUS VEHICLES FOR LONG TERM OCEAN EDDY OBSERVATION

Summary

There is a pressing need for developing a capability for long term observation of the whole ocean water column, and in particular its unsteady large scale features, known as ocean eddies. These eddies range in size from a few kilometers to a few hundred kilometers in diameter and their role is singularly important for momentum, nutrient and heat transport in the ocean. Their presence almost everywhere in the oceans, and their irregular drifting over long periods of time requires a new capability not presently available. Surface observation from satellites has proven valuable, the properties and dynamics, however, of these eddies throughout the water column have been found to be of great importance to a number of physical processes, while they remain largely unknown.

Autonomous underwater vehicles have been developed to a point that they offer a unique alternative to existing observation methods in the ocean, uniquely suitable for observing dynamically evolving phenomena, requiring a certain degree of intelligence from the observing sensor.

Given the large spatial extent of an ocean eddy (order one hundred kilometers in diameter); the relatively large ocean depth (typically a few kilometers); the long time scale (a few weeks to a few months) required to observe these patterns; and the dynamic and largely unpredictable eddy evolution, it becomes clear that the use of a single vehicle is not a feasible solution. Instead, several (tens or hundreds) of relatively inexpensive vehicles are required, each self-powered and equipped with autonomous control.

The principal issues in developing this capability are: the design of the vehicles so they can be inexpensive for massive use; and the simultaneous control of these vehicles so that they can be used to map systematically a specific feature of the ocean. In this report we concentrate on certain important issues of vehicle control; then we proceed to outline a novel scheme for controlling a large number of vehicles, which is robust, flexible and very simple to implement.

We illustrate the concepts through application to the problem of observing a Gulf Stream eddy.

1 Introduction

Developing unmanned underwater vehicles for performing underwater operations, and designing "smart" instruments, adopted to a specific ocean environment to measure sea properties and monitor a large area, are the major current challenges in marine exploration and utilization.

Given the immense size of the ocean, the classical method of observing the sea from ships of opportunity, or from the small fleet of oceanographic vessels available, is clearly restricting seriously any attempt to obtain a global understanding of the ocean circulation and processes. This understanding is crucial, as recent discoveries have demonstrated, such as the "El Nino" effects off the west coast of Central America, and the current controversy over the CO_2 absorptive capability of the ocean, which will determine the long term effects of the greenhouse warming of the planet. Satellites have provided a valuable means for long term ocean observation, their measurements, however, are restricted to the first few centimeters below the ocean surface, and correlation of surface observables with the processes below is a task left to acoustical mapping techniques and to underwater devices with long-range capabilities.

Another significant phenomenon, requiring extensive observation through the use of smart underwater probes, is the presence of large scale vortices practically everywhere in the ocean. Such vortices were known to exist near major currents, such as the Gulf Stream [Lugt 1983, Wunsch 1981, Pickard and Emery 1982]. It was only recently, however, that oceanographers discovered through direct observation that such vortices exist everywhere, and in fact cover the entire ocean surface, embedded in the general circulation [Wunsch 1981]. This discovery that the ocean is covered with vortices of all sizes, from 10 up to 200 kilometers in diameter, and lasting from a few weeks up to three years [Pickard and Emery 1982], changed our view of the water column, making it a dynamic one. Eddies can transport heat and momentum, as well as floating plants and animals, to regions where they would not otherwise exist.

The need for continuous observation of such vortical patterns led to the present study for the development of underwater vehicles capable of optimum motion within a rotating gyre, for long term observation of the flow and its properties (Triantafyllou 1988). One of the principal limitations of underwater vehicles is their powering. The development of efficient, power-dense batteries is a slow process, and for immediate progress one has to turn to other improvements, such as increasing the efficiency of the propulsors. Even a very efficient vehicle, however, can not cover the three dimensional extent of a large eddy fast enough so as to map its properties globally and as they evolve in time. The necessity for a *grid of vehicles* to provide detailed information in a moving frame of reference (as the eddy moves erratically in the ocean, its boundaries constantly changing with time) becomes obvious after some reflection on the problem.

2 Inexpensive Vehicle Design

Presently, autonomous underwater vehicles (AUV) are still in a developmental phase and are not mass-produced. Also, most vehicles have been build as technological extensions of tethered, remotely operated vehicles (ROV). To replace an ROV, the AUV must have considerable capabilities for observation, computation and powering. If instead one sees the AUV as a more sophisticated version of a self-powered sensor, the requirements are considerably reduced. To this effect the recently developed *Layered Control Theory* (Brooks 1986) is of substantial help, because it allows the development of autonomous control with very simple, even crude sensors; requires very limited computational capability and operates without the need of a "world map", i.e. neither requiring a detailed a priori description of the environment in which it will operate, nor building and storing such a map through observations on the site. One may compare the resulting low-level autonomous behaviour of such systems to the behaviour of insects, which although equipped with relatively primitive sensors and reasoning capability can still function very well in an unknown environment. We will not pursue the description of such algorithms herein, since they are given in Brooks (1986), we note, however, that Sea Grant-sponsored research addresses specifically the development of vehicles equipped with layered control (Belingham et al 1989).

This low-level autonomous control equips the vehicles with a survival capability, as well as a primitive capability for decision making; the methodology, also, keeps the price of the vehicle low: neither expensive sensors for navigation, nor substantial computational capabilities are required. The vehicle itself has modest requirements; the designer can reserve most of the power and the computer and sensor capabilities for observing the ocean properties, and not simply to control and navigate the vehicle.

Powering the vehicle is still a major issue, and all efforts must be made to improve the battery capability and the efficiency of the vehicle. To this effect imitating the superb performance of fish is a most challenging technological development (currently we are pursuing a study of fish-like locomotion of AUV, Triantafyllou 1990). However, when several vehicles are available, one can exploit the fact that each vehicle can largely follow the local flow, hence saving substantially in fuel: obviously no power is required to just follow the flow when it is steady; the vehicle reserves its power only for transient conditions, caused by dynamic changes in the eddy, or accidentally reaching the edge of the eddy, or simply being ordered to observe a different part of the flow.

The remaining difficulty is in controlling these (possibly hundreds of) vehicles simultaneously, and gathering the obtained data in a central unit for transfer to an earth station. A solution for this control problem, developed in this study, is through the use of effective chains of vehicles. The word *chain* is used here with the specific physical system in mind, since the control algorithm builds a virtual chain out of the vehicles, as explained in the sequel. Such a scheme has considerable advantages of simplicity of implementation, and inexpensive production; while it is

best suited for the ocean environment, where communication is acoustical, because it requires a very short range transmission capability.

3 Proposed Multi-Vehicle System: A Virtual Chain

We will demonstrate the proposed concept through an example: Consider N vehicles within a large eddy, profiling vertically the ocean column (figure 1). The order of N is typically 100. The purpose is to keep the vehicles on a vertical line (we could, of course, have defined any other shape), each vehicle at a distance h from the other, all moving at speed $U(t)$ following a surface buoy, which is remotely controlled (for example from a satellite). Obviously $U(t)$ is to be kept small to reduce powering requirements, exploiting the natural circulation of the eddy. One envisions $U(t)$ as small most of the time, with corrections commanded from the buoy to the chain from time to time. Several such profilers can provide a simultaneous global picture of the eddy.

The principal aim is to keep the navigation and control hardware on each component vehicle as simple as possible, including the acoustics hardware. For this reason, the listening capabilities of each vehicle are very limited in range (i.e. distance from the source of the sound). We quantify this listening range as at least equal to $2h$, for reasons that will be obvious. Each vehicle is equipped with an equally limited emission capability (since the range is limited, the frequency can be high, resulting in accurate positioning).

The control algorithm is based on the following concept: each vehicle responds only to errors in its horizontal and vertical position with respect to the vehicle immediately above and below. This is achieved through coding (in frequency) of the acoustical emission of each vehicle. Since the range is small, coding can be repeatable, roughly every three vehicles, so there is no substantial complexity and book-keeping required. The control law is based on an equivalent finite-difference approximation of a chain hanging under constant tension, i.e. the n th vehicle having mass m , added mass m_a , drag coefficient c_d , projected area A_p , is governed by the following equation in the horizontal plane:

$$(m + m_a) \frac{d^2 y_n}{dt^2} + \frac{1}{2} \rho A_p U_r |U_r| = T_n(t) \quad (1)$$

where ρ is the water density, U_r is the relative velocity between the vehicle and the fluid, and the thrust T_n is given as:

$$T_n = \alpha (y_{n-1} - y_n) + \alpha (y_{n+1} - y_n) \quad (2)$$

with α a control constant. The direct measurables are relative distances, i.e. $y_{n+1} - y_n$ and $y_{n-1} - y_n$, hence

equation (2) can be implemented easily. The principal issues in obtaining vertical and horizontal vertical distances are summarized in Appendix 1.

Similar equations are implemented for the vertical plane, except that one has to modify the drag equation to account for the coupling in drag; the essence of the control law, however, remains the same. Also, the control constant in the vertical plane will invariably be larger than in the horizontal plane, in complete analogy with the properties of a chain.

Equations (1) and (2) are the essence of defining a *virtual chain*. These equations are exactly equivalent to the finite-difference approximation of the linearized transverse dynamics of a chain. One must add the boundary conditions, i.e. that the top vehicle has on the upper side the floating buoy, which transmits commands y_b to the chain:

$$T_1 = \alpha(y_b - y_1) + \alpha(y_2 - y_1) \quad (3)$$

and that the bottom vehicle should: (a) not expect to receive any acoustical signal from the bottom side, and (b) an equivalent heavy weight ($Wh^2 \gg \alpha$) should be (fictitiously) added at the bottom to keep the chain straight:

$$T_N = (\alpha + Wh^2)(y_{N-1} - y_N) \quad (4)$$

Provided there is sufficient thrust in the vehicles to follow top commands, the scheme above is guaranteed to be stable; with natural frequencies given by the chain natural frequencies ω_n :

$$\omega_n = \frac{n\pi}{N} \sqrt{\frac{\alpha}{m}} \quad (5)$$

The drag-induced damping normally eliminates these natural frequencies in a chain (Triantafyllou et al 1985). In this problem this depends on the value of α , hence it may be necessary to add a velocity-dependent term to equation (2):

$$T_n = \alpha(y_{n-1} - y_n) + \alpha(y_{n+1} - y_n) + \beta \frac{dy_n}{dt} \quad (6)$$

where β is a control constant.

4 Properties of a Virtual Vehicle Chain

We will study some of the properties of a virtual chain by considering an example. We let the buoy perform harmonic oscillations, $y_b = \text{Re}[e^{i\omega t}]$, where $\text{Re}[x]$ stands for real part of x ; then in the steady state all vehicles will perform a steady oscillation as well, $y_n = \text{Re}[Y_n e^{i\omega t}]$. Let the number of vehicles be N , and omit the nonlinear drag for simplicity. Then the amplitudes Y_n can be found as:

$$Y_n = [-\omega^2 + 2i\zeta\omega + 1]^{-n} \quad (7)$$

where $Y_0 = 1$ and $Y_{N+1} = 0$. The solution is shown for particular numerical values in figure 2.

We see that high damping is desirable and that α should be made such as to keep the natural frequency of the system, $\sqrt{1-2\zeta^2}$ away from the excitation frequency.

A natural extension of this concept would be to have the vehicles spread out in the horizontal plane as in figure 3, and their dynamics could then be described as *the virtual membrane*. However, we will retain the same name, *virtual chain*, for all such constructions of multiple vehicles, and we will refer to schemes such as shown in figure 3 as *two-dimensional virtual chains*.

The equations governing the control law of the two-dimensional virtual chain are the finite-difference approximations to the equation of a membrane.

5 Leading Vehicles: the Leader

Commands to the chain may be transmitted from one or several points along the chain. For example, the top vehicle, or buoy may be commanded directly from a satellite, or make its own autonomous decisions. The other vehicles are an integral part of a vertical chain, hence following the top commands. The same response would have also been obtained by commanding the M th vehicle. Similarly, the top and bottom vehicles may move independently; the other vehicles will form a shape connecting these two points, which depends on the control law and the environmental excitation.

Central to the effective chain concept, then, is the notion of a *leader*, a vehicle which is part of the chain and moves intelligently, causing the rest to follow. As noted above, more than one leader may exist; in fact for redundancy, or when more than one functions are required from the chain, different vehicles may become the leading vehicles according to some pre-determined plan. Caution should be exercised to avoid conflicts of command, causing a break up of the chain.

6 Transient Behaviour

The behaviour of effective chains as defined above is guaranteed to be good because the algorithms guarantee a closed-loop passive system. There are conditions, however, when a chain must revert to a different shape to accommodate survival conditions. For example (figure 4) a vertical chain in a two-dimensional environment encountering a substantial obstacle, can not avoid the obstacle while retaining the chain form. This requires an *emergency mode* of operation: for example all vehicles, temporarily group together in a different configuration, that of a horizontal chain, returning to the shape of a vertical chain once the obstacle has been avoided.

7 Real Time Data Transmission

One of the issues to be resolved in controlling several vehicles is the transmission of the data as they are being collected to a central unit, at the surface or some other fixed location. If one employs the same short range acoustic capability used for control, then bottle-necking occurs near the top when the information from all the vehicles below must be passed through.

Alternatively, long-range acoustic transmission can be used for communicating directly from each vehicle to the surface buoy. This places significant payload requirements to the surface buoy, which must be equipped for receiving signals simultaneously from several vehicles; and each vehicle must be equipped with both a short-range (control) and long-range (data transmission) capability.

Although data transmission is a payload problem, i.e. it may be viewed as an integral part of the data collection process and not specific to the effective chain concept, advantages may be obtained if the two issues, control and communication, are tackled simultaneously.

8 Application: Multiple Vehicles in a Vortex

Figure 5 shows a cross-section of measured velocities in the Gulf Stream (Worthington 1954). Figure 6 shows measured temperatures in a cold Gulf Stream Ring (vortex) (Richardson 1978). We consider as an example the feasibility of having 50 vehicles profiling vertically a vortex from the surface down to a depth of 1000 m. The distance between vehicles is constant at 20 m. It is typical that the velocity gradient will be at most 2 m/sec from the top to the bottom vehicle. Usually only a few vehicles near the top and the bottom are expected to encounter strong velocity shear.

We will establish certain upper limits on the range and endurance of such vehicles. Following a study by Bradley (1991), we will consider the effect of velocity of operation on the life and endurance of a streamlined vehicle, equipped with batteries. We consider an axially symmetric, streamlined shape for which the drag coefficient is

explicitly available (Hoerner 1958). We employ the coefficient c_d based on the projected area of the vehicle, rather than the wetted surface coefficient. We employ this coefficient, because the present analysis is immediately extensible to bluff-body shaped vehicles. The drag coefficient is expressed as a function of the slenderness ratio (length to diameter ratio l/d) and the Reynolds number Rn . l denotes the length of the vehicle, d its diameter; while $Rn = (ul)/\nu$, with u the velocity of the vehicle, and ν the kinematic viscosity of water. When $Rn < 10^5$ then:

$$c_d = 0.45 \frac{d}{l} + c_f \left\{ 3 \frac{l}{d} + 3 \sqrt{\frac{d}{l}} \right\} \quad (8)$$

$$c_f = \frac{1.328}{\sqrt{Rn}}$$

and when $Rn > 10^7$, then

$$c_d = c_f \left\{ 3 \frac{l}{d} + 4.5 \sqrt{\frac{d}{l}} + 21 \left(\frac{d}{l} \right)^2 \right\} \quad (9)$$

$$\frac{1}{\sqrt{c_f}} = 3.46 \log_{10}(Rn) - 5.6$$

while for $10^5 < Rn < 10^7$, the value of c_d is given approximately by (9) for $Rn = 10^7$.

Let J denote the energy density of the batteries (energy per unit weight). Typical values of J are: $J = 0.112 \times 10^6$ Joule/kg for lead acid batteries; $J = 0.443 \times 10^6$ Joule/kg for alkaline batteries; and a high value of $J = 1.55 \times 10^6$ Joule/kg for Lithium-Bromium cells, which however are expensive. As typical for applications, with relatively low cost and high energy density, we will employ here the value for the alkaline batteries.

If we denote by α the fraction of the available displacement which will be used by the batteries, and by s_g the specific gravity of syntactic foam (equal roughly to 0.625), then the mass of the batteries, M_b , is equal to $M_b = \rho V \alpha (1 - s_g)$, where ρ is the density of seawater, and V is the volume of the vehicle. If c_p is the prismatic coefficient of the vehicle (usually with a value between 0.6 and 0.8), and A_p denotes the projected area, i.e. $A_p = \pi d^2$, then $V = c_p A_p l$.

Let W_p denote the "hotel" load, i.e. all the power consumed by the computer, the sensors and all other non-propulsive elements on the vehicle (including all payload). Then, if R denotes the maximum range of the vehicle, η the propulsive efficiency, and u the average velocity, energy balance requires that:

$$\frac{1}{2\eta} \rho A_p u^2 R + W_p \frac{R}{u} = J \alpha (1 - s_g) \rho V \quad (10)$$

resulting in the following expression for R (Bradley 1991):

$$R = \{J\alpha(1-s_g)\rho V\} / \left\{ \frac{1}{2}\eta\rho A_p u^2 R + W_p \frac{R}{u} \right\} \quad (11)$$

The maximum operational life, T , is found as $T=R/u$. Figures 7 through 16 show the range and operational life of streamlined vehicles of varying length and under various conditions of operation. The range increases with size (compare for example figures 7 and 11), since the total available energy is proportional to the volume (length cube), while the drag increases as the projected area (length square); with a minor additional effect from the Reynolds number. Also, there is an optimal length to diameter ratio, of about 2.5 (Hoerner 1978), which provides a minimal drag coefficient (compare for example figure 13 with figures 11 and 15). This minimum, however, is rather flat and may be modified by appendages. One notes the substantial effect of hotel load on the maximum achievable range (figures 7, 9, 11, 13, 15); otherwise, in the absence of hotel load, the vehicle achieves a very large range for very small speeds. The operational time, on the other hand, is a monotonically decreasing function of the speed (figures 8, 10, 12, 14, 16).

These figures have been prepared with ideal conditions (100% propulsive efficiency, no appendage loads, no side velocity components); they serve however to demonstrate the long ranges and lifes achievable at speeds which are very reasonable for operating within a vortex, i.e. around $u=1$ (m/s).

Assuming, then, a vehicle of length $l=2$ m, with a hotel load of $W_p=10$ (W), we find (figure 13) at a speed of 2 (m/s) a maximum range of 1,100 km and a maximum life of 200 hr.

8.1 vehicle interchange

One must find ways to spread the energy requirements evenly among the vehicles. As pointed above, vehicles near the top and bottom of the chain are expected to encounter the strongest shear currents, hence they must expend considerable energy overcoming the drag forces, while other vehicles along the chain have a much lower load. Still, the life of the operation is determined by the first vehicle to exhaust its power source. As a result, we consider the *operation of vehicle interchange* to be essential for balancing out the power requirements of all the vehicles of the chain.

We define the *operation of vehicle interchange*, as shown in figure 17, to be the operation of having the n th and $n+1$ vehicles switch places. This is an isolated operation, involving only two vehicles at a time and propagating from the top to the bottom, i.e. it is envisaged that first the 1st and 2nd vehicles exchange positions in the chain; then the 2nd and 3rd; and so on until the last vehicle is reached, when it is repeated from the top. This exchange is a slow

process (roughly one exchange every hour), which need not be accurate in time; requiring very little energy, while guaranteeing statistically a balancing of the power requirements for each vehicle.

As already mentioned, this operation of vehicle interchange is borrowed from nature: when migrating birds fly, one leads the way creating through the flapping of its wings a trail of vortices, which are favourable to the rest of the flock, making their flight easier; the leading bird switches places at regular intervals with another bird, thus distributing the overall load evenly among them.

8.2 maximum achievable range and operational life

As suggested by figure 13, the optimal operating speed of the vehicle considered herein (2 m in length) with hotel load 10 (W) is equal to 0.8 (m/s). This velocity, of course is with respect to the already moving fluid; hence a more meaningful quantity is the operational life, estimated at about 1000 hr. Certain vehicles, of course, will be subjected to more adverse conditions, since they must overcome additional drag forces wherever there is strong velocity shear. Using the values in figure 5, we estimate that each vehicle will spend the equivalent of operating 2/3 of its time at a speed of 0.8 m/s, and 1/3 at a speed of 2 m/s. This results in a maximum range of 1,900 km and a maximum life of 700 hr (1 month), a substantial improvement over the value of 200 hr (8 days) found above, when vehicles did not interchange places.

The values found above are obtained under ideal conditions; we estimate that if these values are divided by a factor of about 2, realistic figures are obtained, i.e. a range of 1,000 km and operational life of 400 hr (15 days). Given that an eddy has a diameter of about 100 km, this provides an acceptably detailed picture of the eddy in real time.

9 Summary

The basic conclusion of the study is that multiple vehicle arrangements are essential in order to map large ocean features, such as the ocean eddies. It is impossible to obtain a global view of an eddy through the use of a single vehicle, because the eddy evolves dynamically (and erratically) and it would require an enormous amount of energy or time in order to provide such an overall picture. By employing several vehicles, one can exploit the natural flow of the eddy and accomplish long term observation with relatively low power levels.

A significant element in the design of these multiple vehicles is that they must be viewed as sensors equipped with some low level intelligence and autonomy. The layered control theory is of significant help in building a simple robust control scheme for the vehicles, with minimal requirements in both initial cost and power for operation.

A central issue in the operation of such vehicles, then, is the simultaneous control of a large number of autonomous vehicles to achieve mapping of a specific ocean feature. The proposed control scheme of a virtual chain creates an equivalent passive system, hence guaranteeing stability with minimal requirements, good command following with minimal acoustic transducer requirements, and simplicity and adaptability of design.

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APPENDIX 1: Measurement of relative position between vehicles

We consider, for example, the case of a virtual chain of vehicles moving within a plane, in a nominally vertical configuration (figure 1). Each vehicle is assumed to be capable of measuring at each instant of time:

1. the direction of the vertical
2. the relative distance from the vehicle above it (by interrogating)
3. the relative velocity from the vehicle above it (by interrogating)
4. the angle of the position vector of the vehicle above it with respect to the vertical
5. the relative distance from the vehicle below it (by interrogating)
6. the relative velocity from the vehicle above it (by interrogating)
7. the angle of the position vector of the vehicle above it with respect to the vertical
8. the static pressure

The static pressure is an additional measurement, needed to establish that a vehicle is not too close to the free surface, or beyond the depth of operability. From the remaining six measurements the vehicle can deduce the relative horizontal and vertical distances with respect to the vehicles above it and below it, which are essential for implementing the control of the virtual chain scheme.

It should be noted that all these measurements are possible through existing software, especially since the required acoustic range is quite short. In fact, with good Ultra Short Base Line (USBL) systems operating at 15 kHz, the accuracy of bearing is within a tenth of a degree, and less than 1 ms in time of travel.

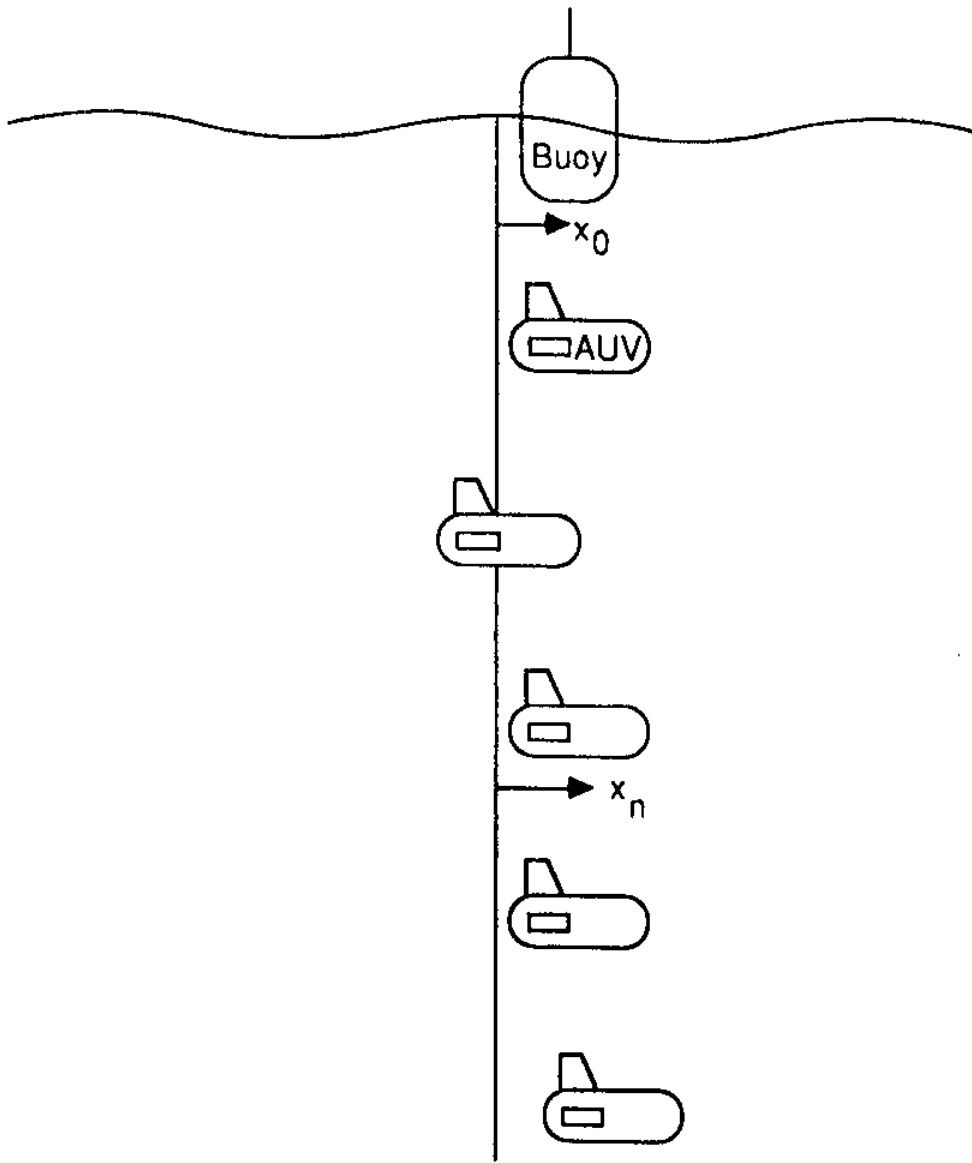
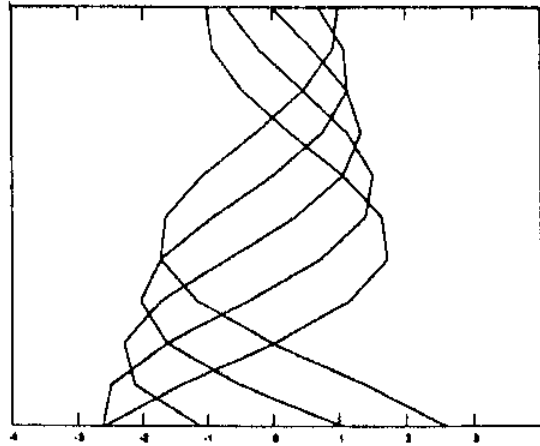
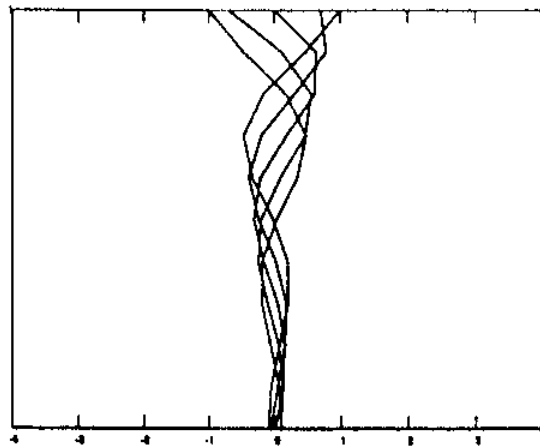


Figure 1: One-Dimensional, Vertical Virtual Chain



$\omega = 0.5, \zeta = 0.5$



$\omega = 0.5, \zeta = 1.0$

Figure 2: Response of the vehicle array in the case $\beta/\alpha=0$.

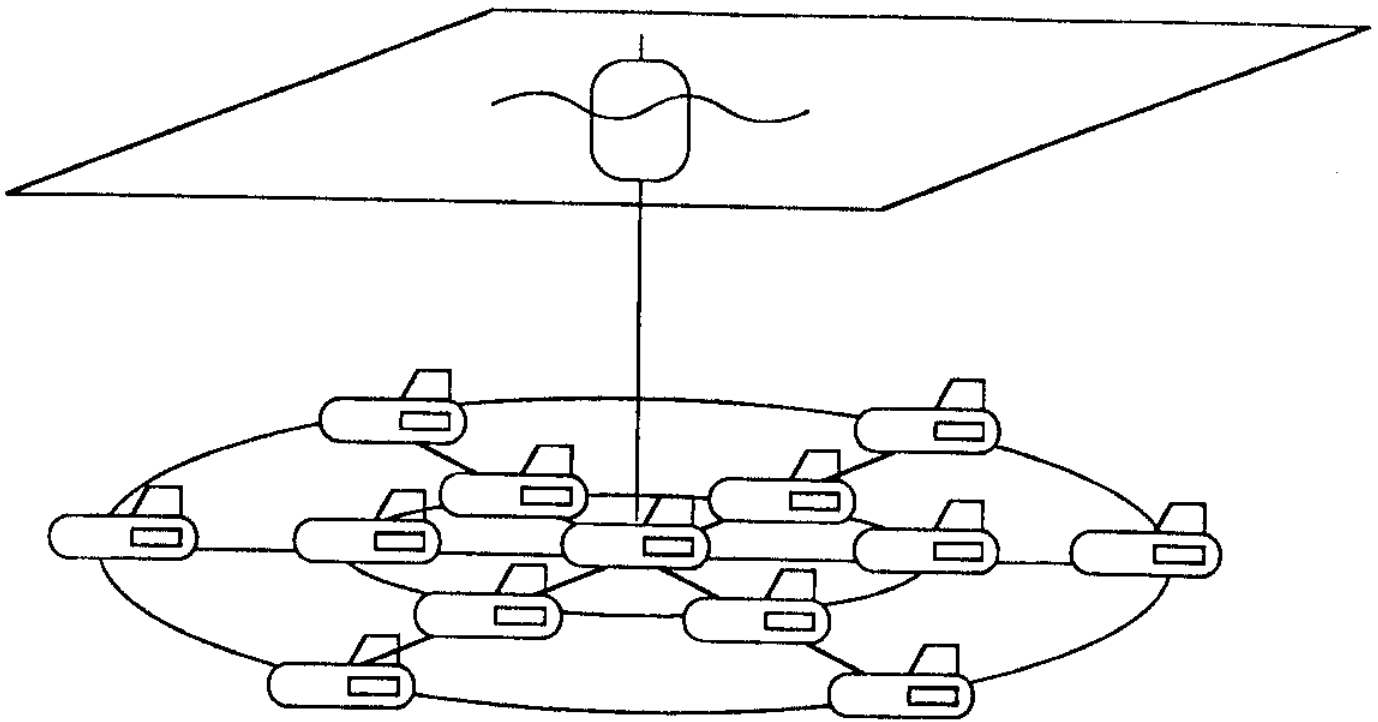


Figure 3: The Two Dimensional Virtual Chain

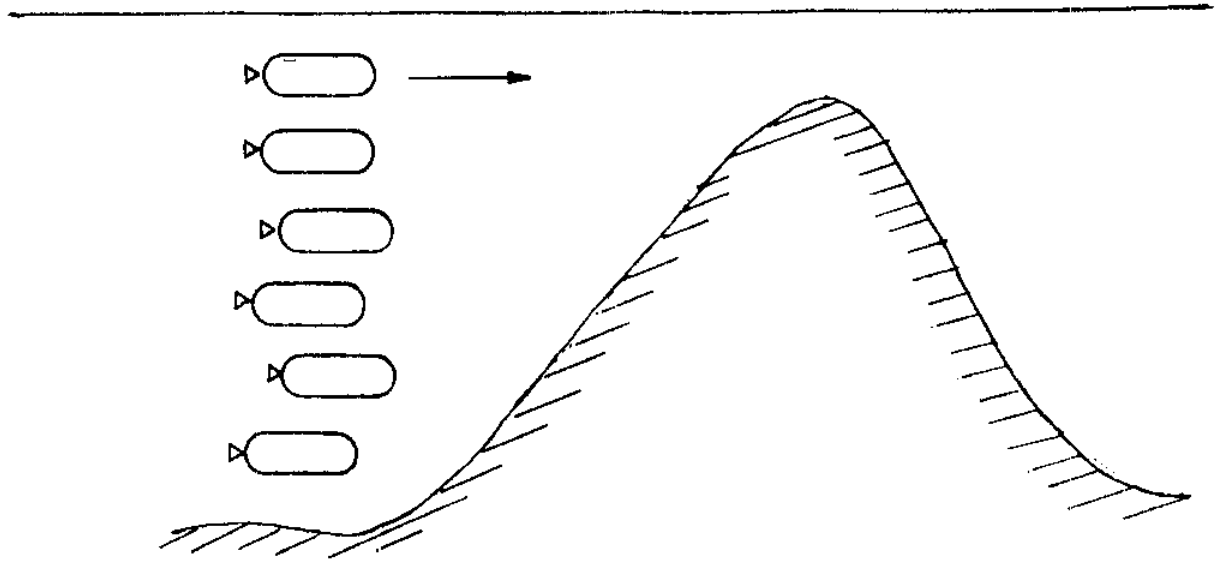


Figure 4: Obstacle requiring emergency maneuvers, breaking the chain

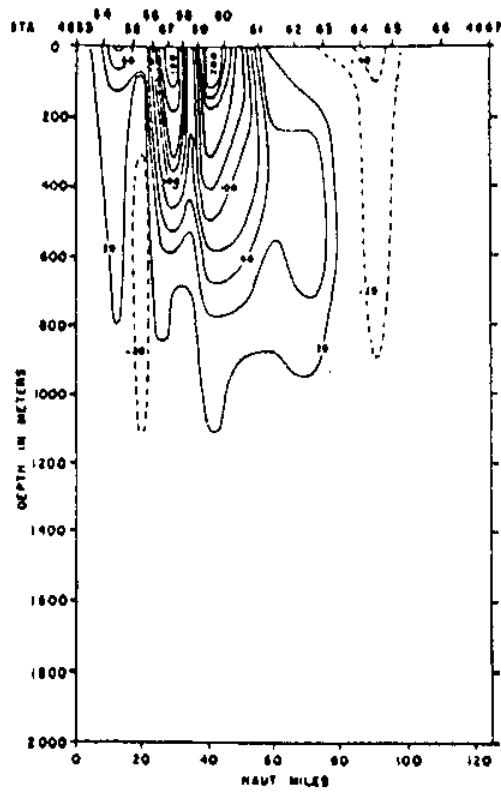


Figure 5: Cold ring measured velocity distribution

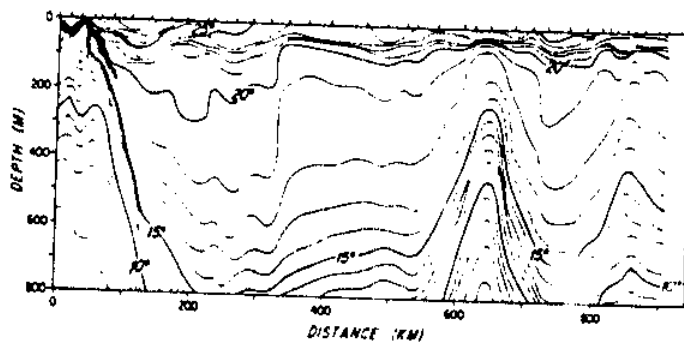


Figure 6: Cold ring measured temperature distribution

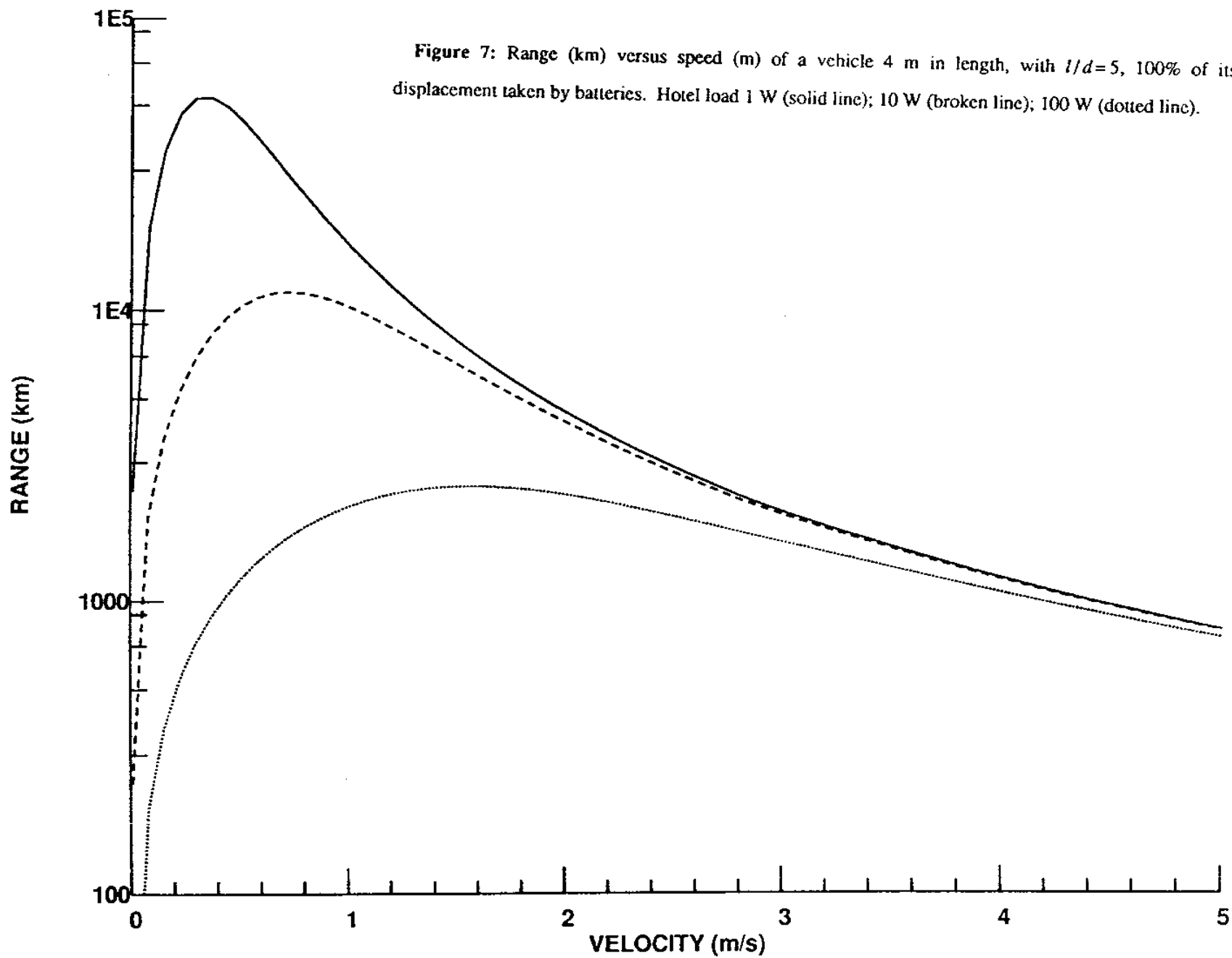


Figure 8: Operational life (hr) versus speed (m), under same conditions as in figure 7.

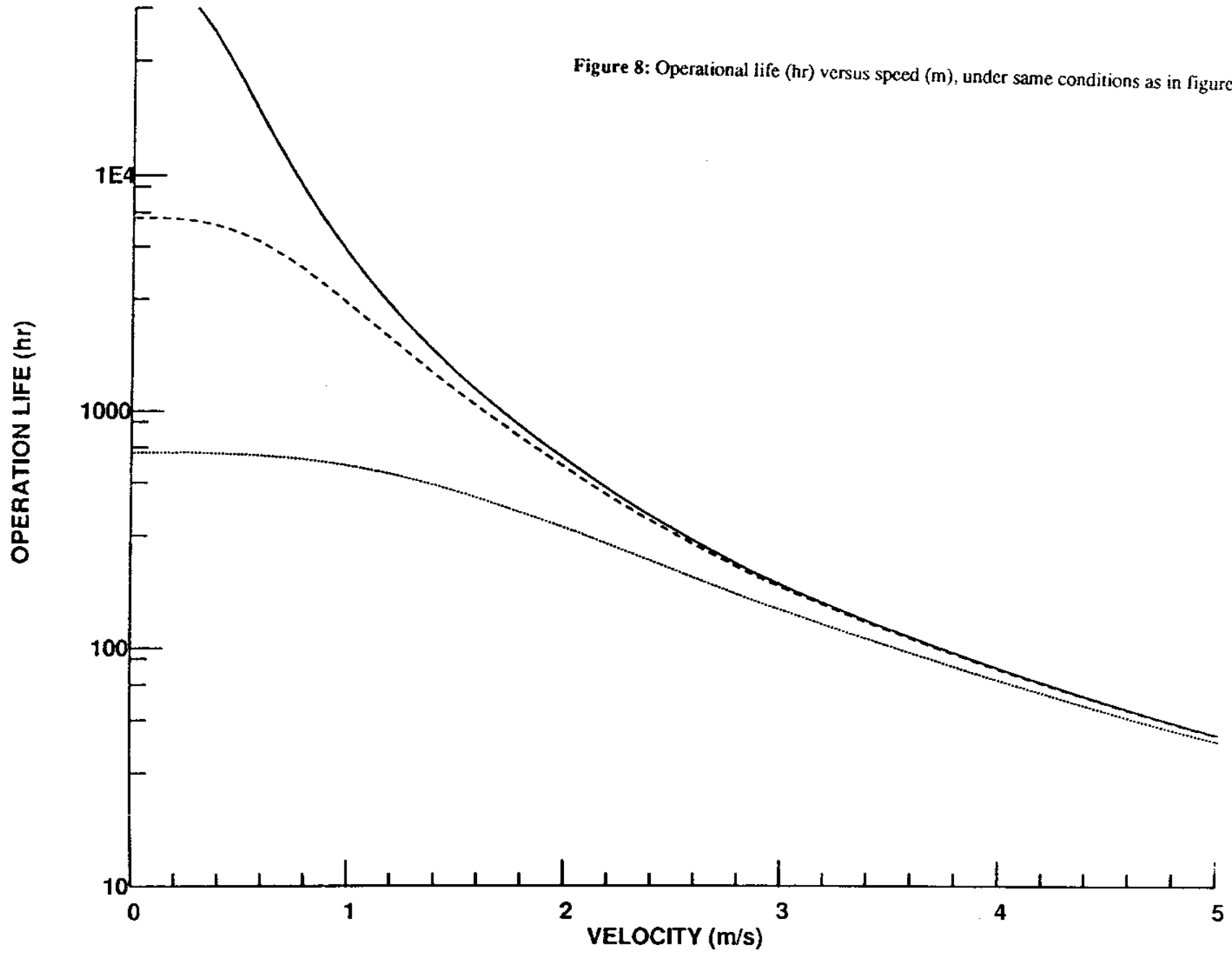
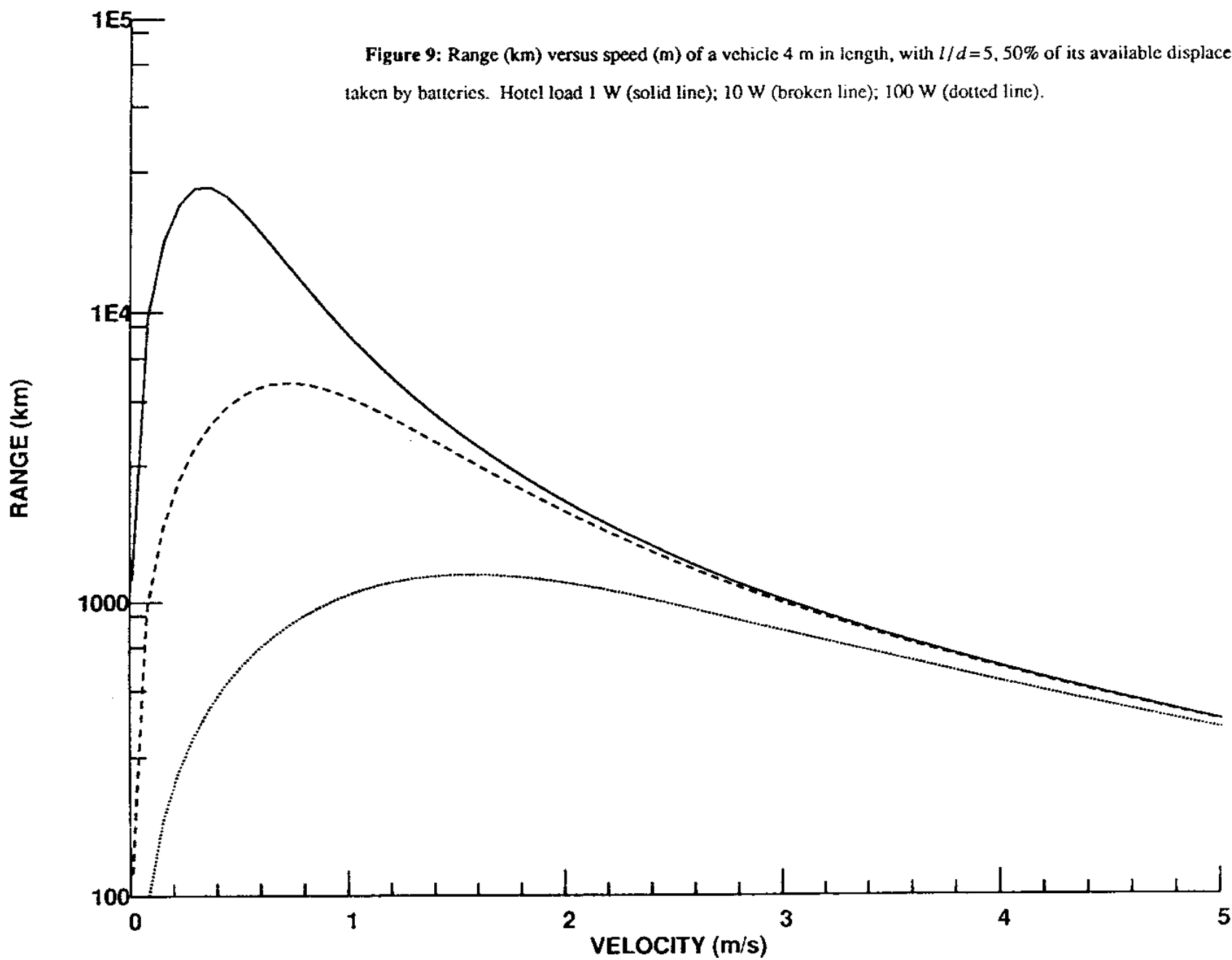


Figure 9: Range (km) versus speed (m) of a vehicle 4 m in length, with $l/d=5$, 50% of its available displacement taken by batteries. Hotel load 1 W (solid line); 10 W (broken line); 100 W (dotted line).



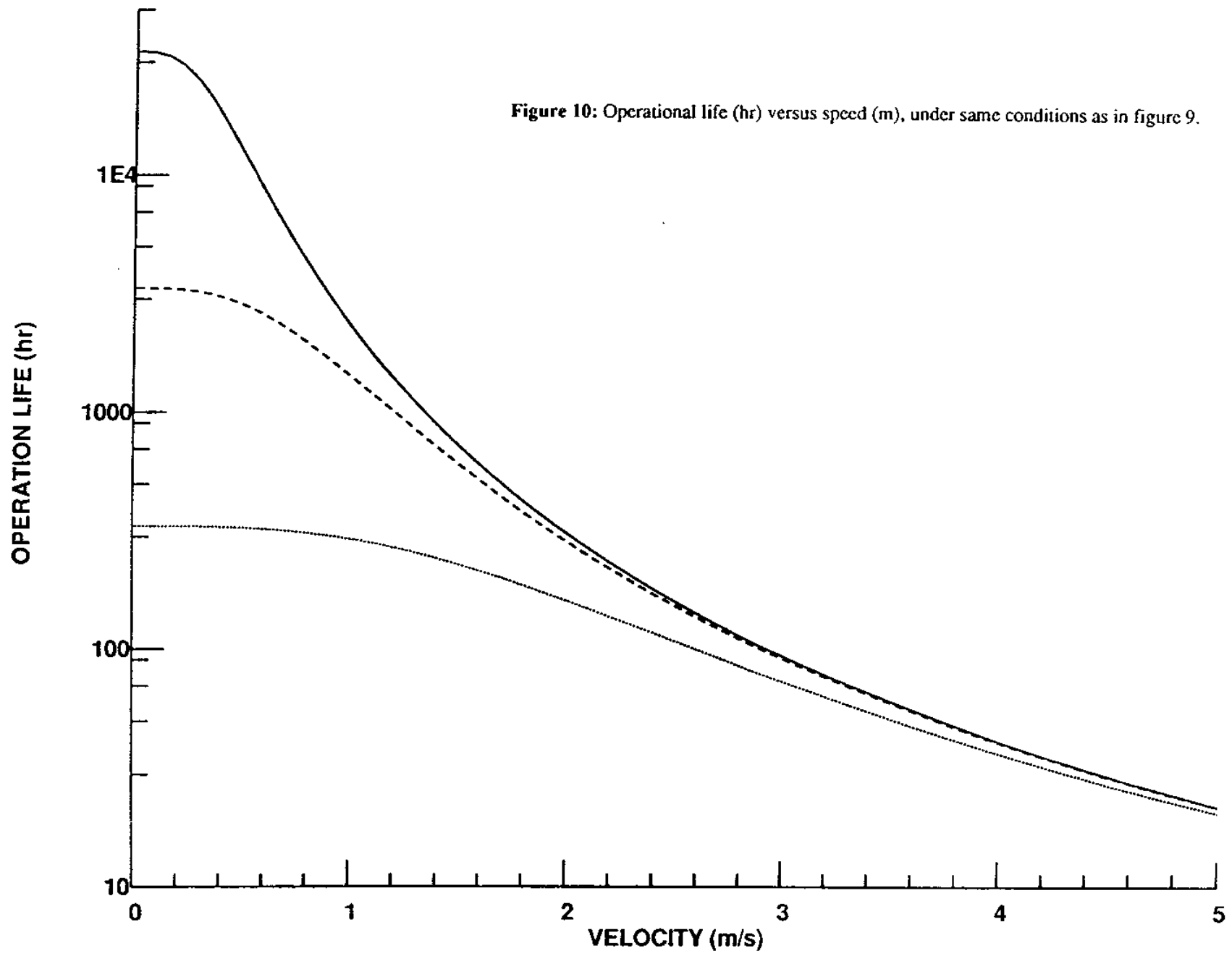
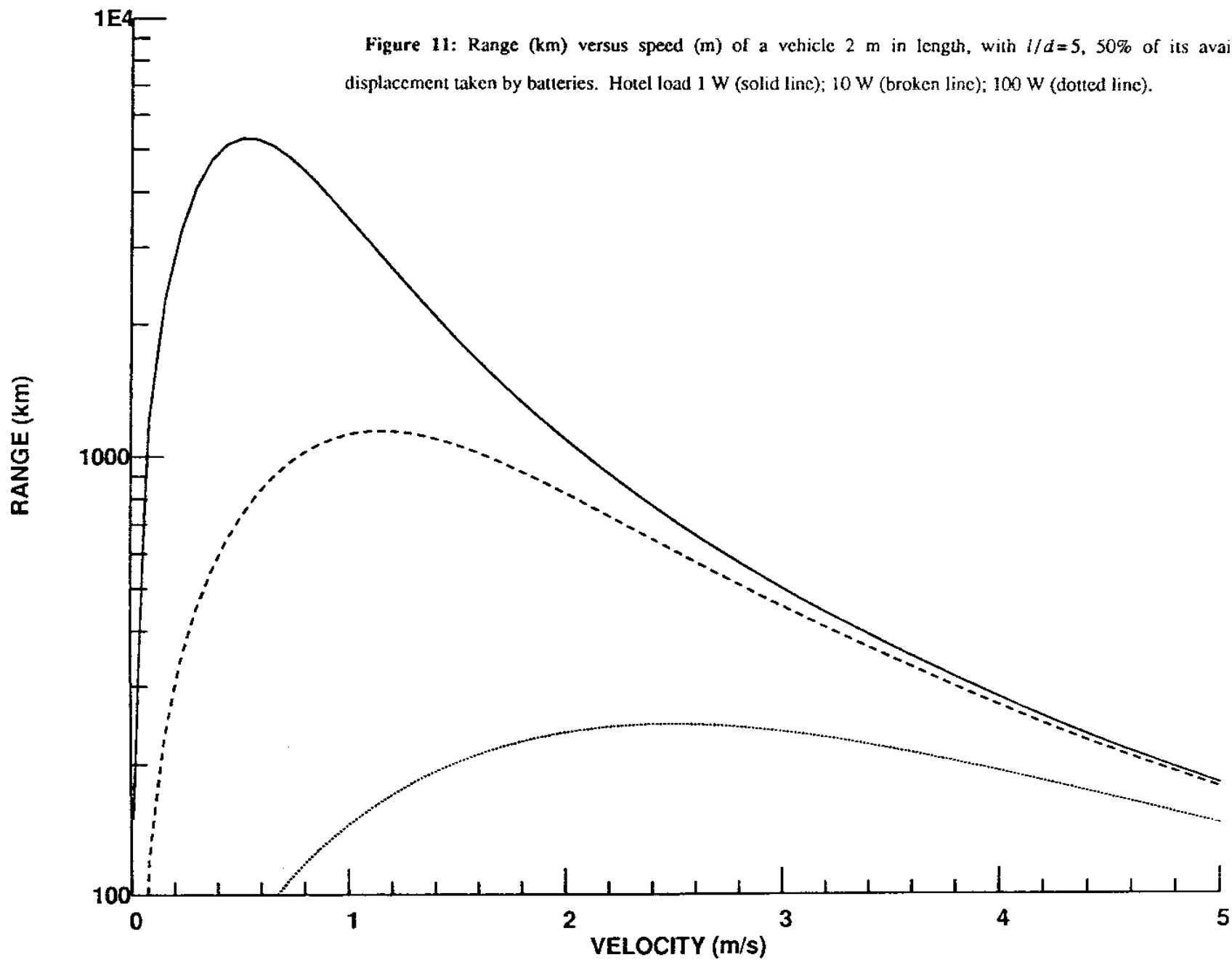


Figure 10: Operational life (hr) versus speed (m), under same conditions as in figure 9.

Figure 11: Range (km) versus speed (m) of a vehicle 2 m in length, with $l/d=5$, 50% of its available displacement taken by batteries. Hotel load 1 W (solid line); 10 W (broken line); 100 W (dotted line).



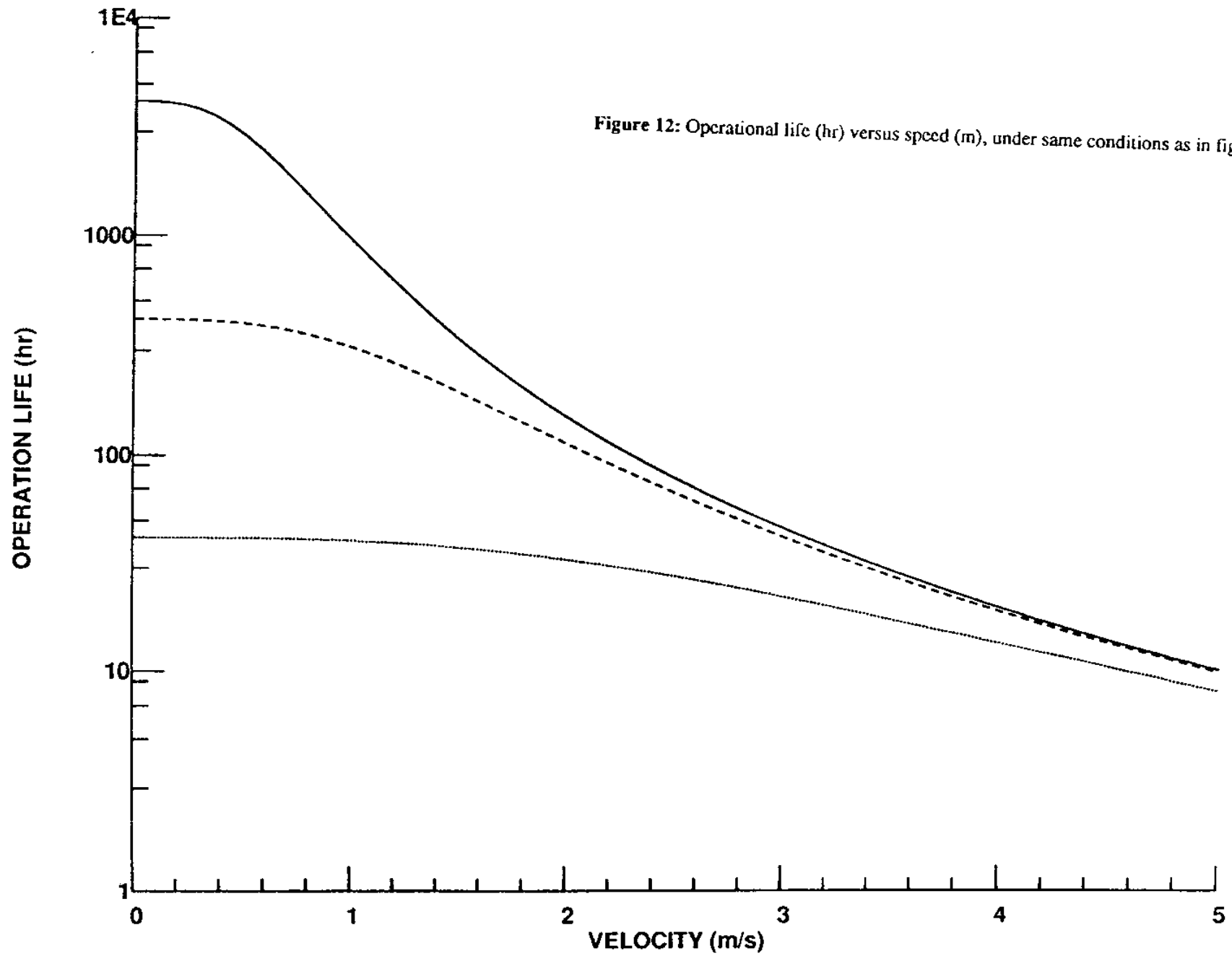
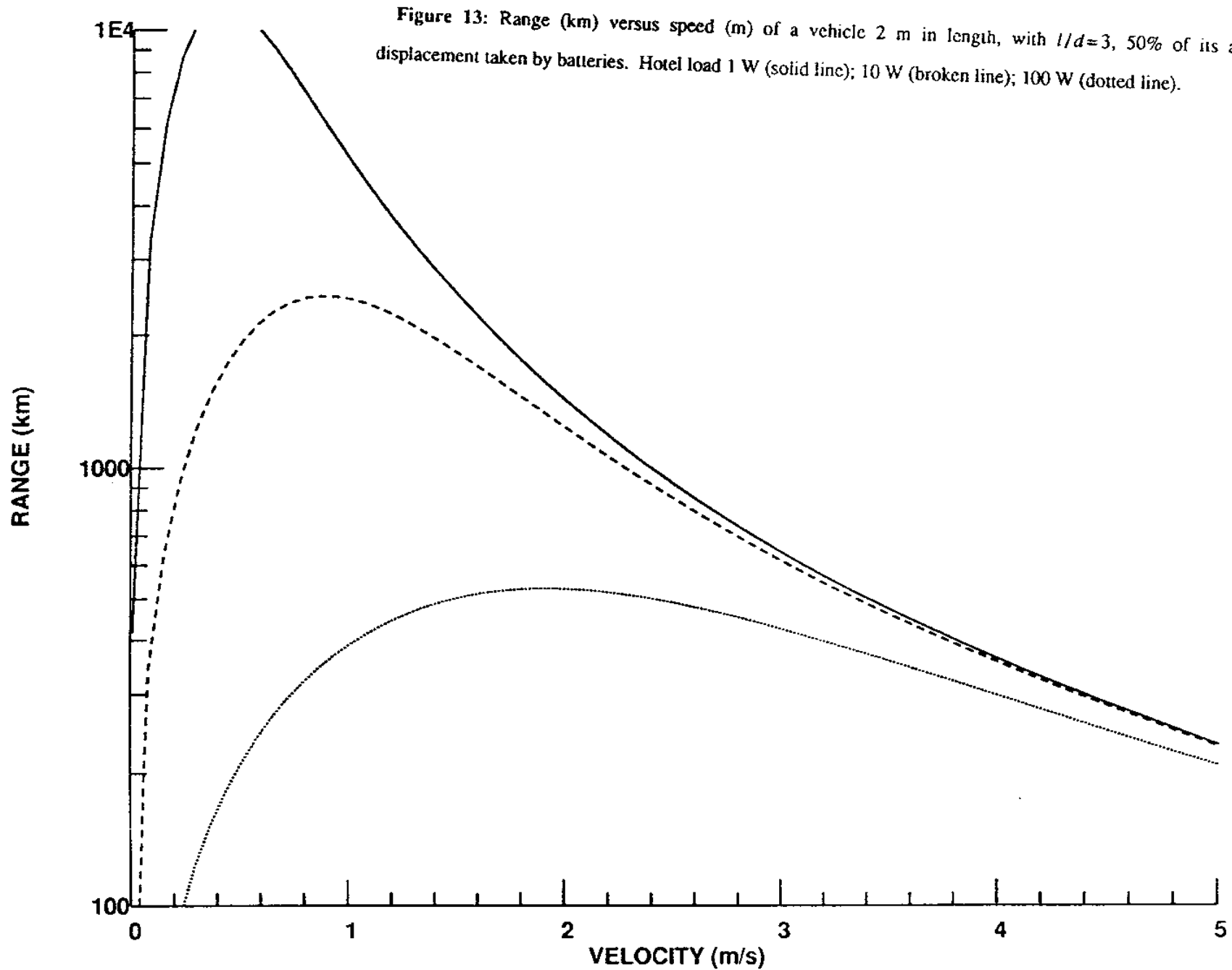


Figure 12: Operational life (hr) versus speed (m), under same conditions as in figure 11.



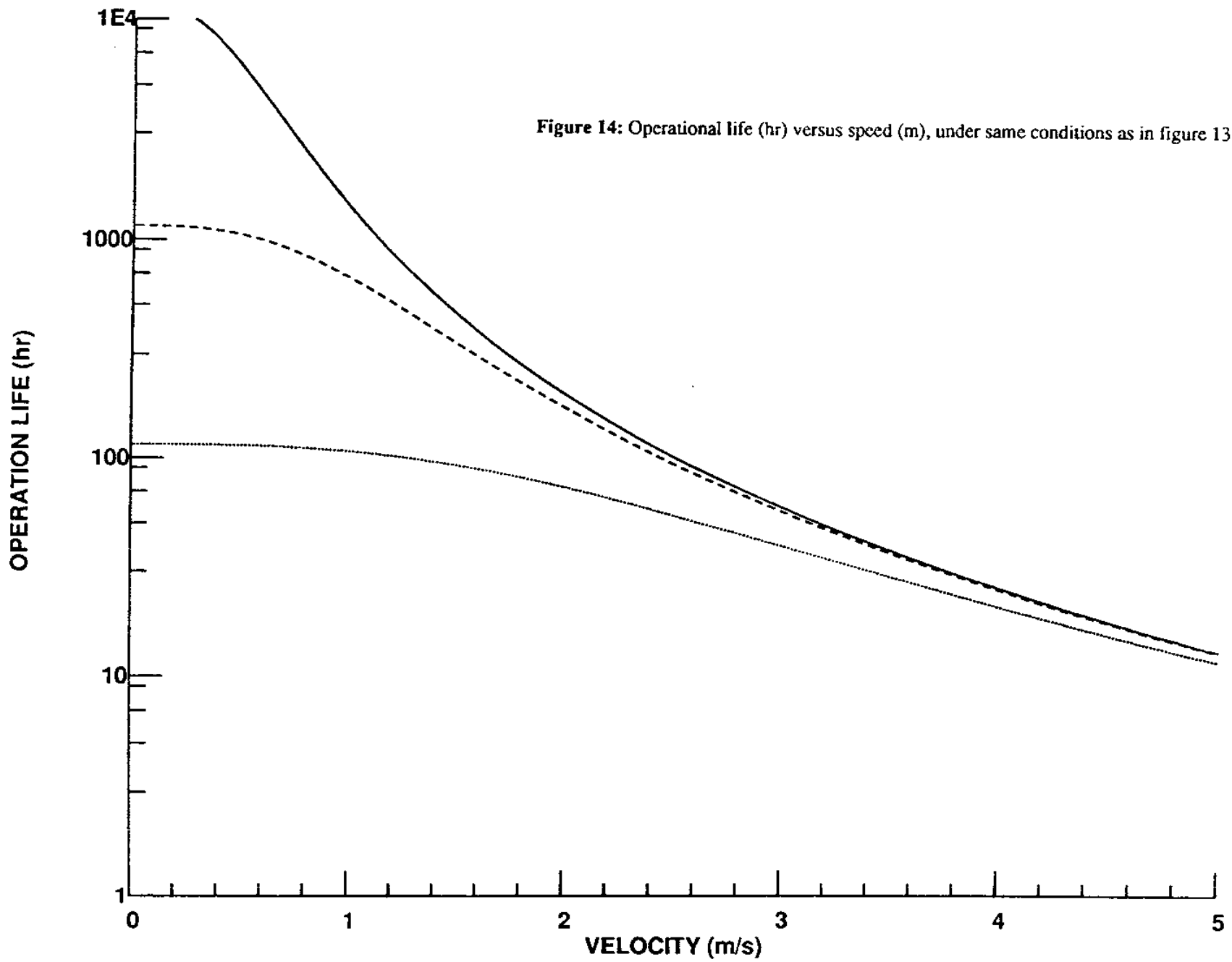
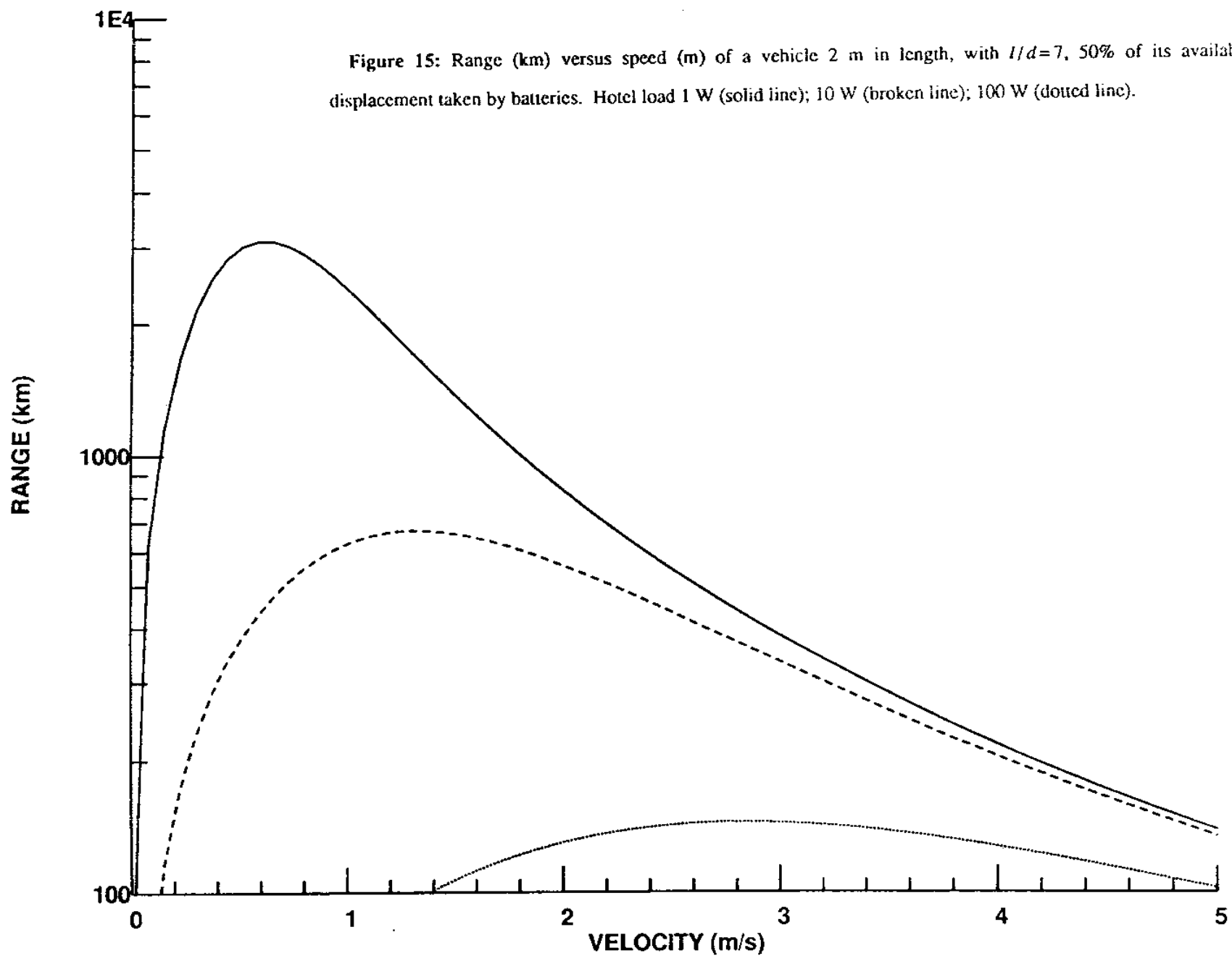
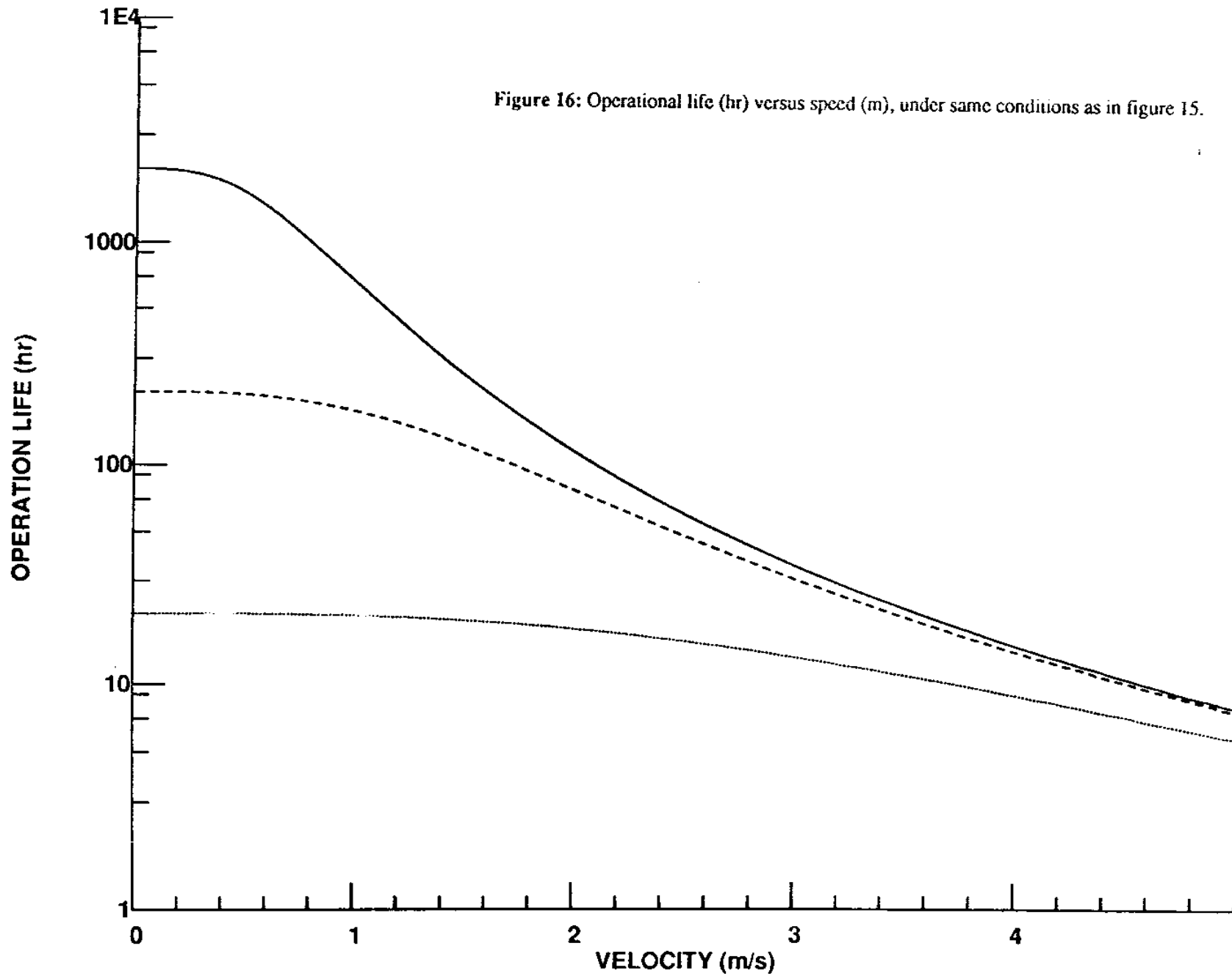


Figure 14: Operational life (hr) versus speed (m), under same conditions as in figure 13.





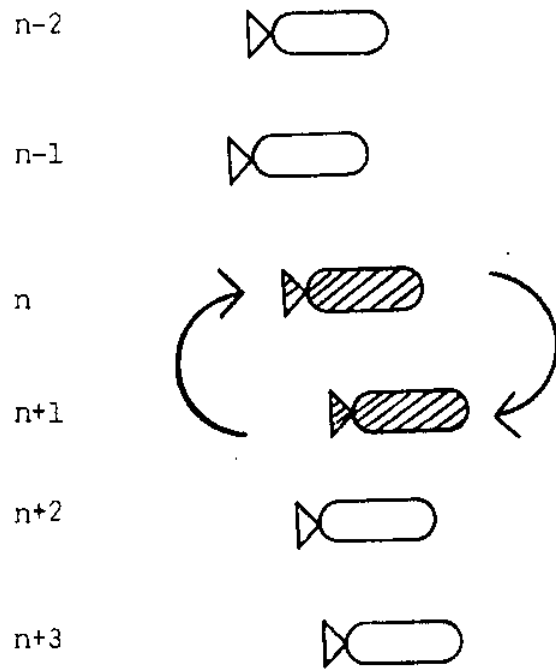


Figure 17: Operation of vehicle interchange.

