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Allocation of Inertial Surveying System Model Parameters

National Geodetic Survey
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ABSTRACT

Mathematical modeling for post-mission adjustments of inertial surveying system observations is examined to determine a parameter allocation scheme which produces an improvement in the final coordinate accuracy. Parameter allocation schemes are systematically varied and the results of the various least squares adjustments are evaluated and compared. A preferred allocation scheme is selected for the given mathematical model and recommendations made for further model research.

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1. INTRODUCTION

Inertial surveying has appeared recently as another technology for the measurement of land. The new inertial surveying equipment is based on hardware originally developed for missile guidance systems. The inertial surveying systems are currently being examined in detail to determine if they indeed offer a promise of quick, reliable survey coordinates.

The data obtained with inertial surveying systems have been shown to contain systematic errors. In an effort to identify and compensate for these errors, the inertial surveying data are subjected to post-mission processing procedures to minimize errors in the final coordinates and thus improve the accuracy and usefulness of these systems.

This report is concerned with a set of mathematical models that have been used to capture the systematic effects in inertial surveying systems. The mathematical models contain model parameters which are part of a post-mission least squares adjustment program. This study explores varying the allocations of the model parameters to determine the most effective scope of each parameter. It is the intention of this study to determine the best allocation of model parameters, using the given mathematical model, to eliminate the systematic errors and provide accurate final survey coordinates.

2. INERTIAL SURVEYING SYSTEMS

Inertial surveying systems are based on maintaining measurement equipment in a particular orientation and then transporting this equipment while it measures changes in position in all three dimensions. Many recent publications have documented the various measurement systems now in use. Only a basic explanation of these systems will be given here.

2.1 Concepts

Two main elements of the inertial surveying system are the gyroscopes and the accelerometers. Ideally, the spinning gyroscopes provide a constant orientation reference to which the measuring equipment can be aligned. The accelerometers are devices which measure accelerations imparted to the equipment in each of the three dimensions.

The gyroscope is made up of several parts. First is the rotor, the part which spins and has most of its mass concentrated at its outer rim. Second, the axis about which the rotor spins is connected to the rest of the equipment by some low-friction bearings. Next, gimbals may be used to permit the gyroscope axis to maintain its orientation while the remaining equipment is forced to move. Unless acted upon by external forces, the ideal gyroscope would maintain its orientation with respect to inertial space.

The gyroscopes, however, do not function in a totally ideal manner or environment. While a gyroscope should maintain its orientation with respect to a point in space, a large number of factors affect its performance and use. One of the most obvious effects is "apparent precession" which is caused by the rotation of the earth itself. This change in apparent tilt of the gyroscope axis is predictable and compensated for. Other effects which may be random and contribute to the drift, or precession of the axis, include gyroscope imbalance, bearing friction, and gimbal inertia. These effects must also be compensated for in some manner.

Accelerometers are devices containing known masses which measure the forces acting upon these known masses. The accelerometers sense the combined effects of the interactions with the gravitational fields and the accelerations related to movement between one point in space and another. In order to separate the gravitational elements from the measured accelerations, an on-board computer contains a model for the expected gravitation effects. The resulting accelerations are then integrated twice with respect to time, also by the computer, in order to obtain the corresponding linear distances.

The accelerometer measurements and integrations are done very rapidly and very often. For the equipment in this study, these processes are repeated every 16-17 milliseconds. At each desired point, the distances in each dimension are then transformed by the computer, using its pre-programmed model for the computational ellipsoid, to obtain differences in latitude, longitude, and elevation. These differences are then applied to the previously inputed or computed latitude, longitude, and elevation to obtain the updated coordinates.

2.2 Sources of Error

This study is concerned with the performance of inertial surveying systems. As with any measurement system, there are systematic errors present which affect the results and which must be handled by a combination of instrumental and computational techniques. Before examining the required techniques, an identification of the primary systematic error sources is necessary.

2.2.1 Scale Errors

As stated earlier, the function of the accelerometers is to quantify the accelerations imparted to the system. The devices contain quantizers which have some imperfections in their ability to accurately measure the accelerations. A complete description of the effects which result is given by Hannah (1982). The primary effects of these imperfections are the accelerometer scale factor errors.

An accelerometer scale factor error results in an along-track error in proportion to the component of distance traveled in the direction of the accelerometer's sensitive axis. If, for example, a scale factor exists in the east accelerometer, traveling in a due east direction will result in a continuously increasing error in longitude. For the equipment in this study, there are two accelerometers in the horizontal channels and one in the vertical channel.

2.2.2 Misalignment Errors

A second category of primary systematic error sources is that which results from some misalignment. A system azimuth misalignment occurs when the reference north axis of the inertial platform is not perfectly aligned with astronomic north. This alignment is supposed to be established prior to the start of the mission when the platform is also leveled with respect to the local vertical. The gyroscope and gimbal system are then used to maintain this orientation throughout the mission.

From the standpoint of coordinate determinations, the effect of a system azimuth misalignment is to produce cross-track errors which are in proportion to the components of distance traveled in directions perpendicular to the accelerometer's sensitive axis. According to Hannah (1982) the misalignment of the platform due to the inertial gyrocompassing may at times be greater than 60 arcseconds. The inertial platform, however, is not the only component of the system which must be properly aligned and compensated.

The accelerometer itself may not be correctly aligned even if the inertial platform is. The accelerometer misalignment cannot be separated from the platform misalignment. As for the platform, if there is some error in the alignment of a sensitive axis of an accelerometer with respect to its corresponding geodetic coordinate axis, then the accelerometer will detect components of acceleration perpendicular to the intended direction of the axis.

The misalignment of a particular accelerometer could be due to some manufacturing imperfection such that the accelerometer axes are not all mutually orthogonal. The accelerometer axis may also not be perfectly aligned with the reference axis of the inertial platform. These imperfections cannot be totally separated from one another but all contribute to the misalignment effects producing the off-track errors.

2.2.3 Drift Errors

The drift errors are those errors due to changes in the scale or orientation of the system that occur with the passage of time. As mentioned earlier, a gyroscope does not operate ideally. Due to its physical limitations and the external forces acting upon it, the axis changes direction or precesses. This precession is a direct cause of drift in system orientation.

Other elements of the inertial surveying system are also subject to changes with respect to time. Some of these changes are predictable and due to normal system performance while others are more complex and due to random effects. These drift errors are differentiated from the other scale and misalignment errors in that they do not result in errors in coordinate determinations which are linear with distance traveled. Most of the modeling for these errors, as will be done in this study, is based on time squared terms. Operational techniques are employed which specifically seek to minimize the effects of these drift errors.

2.2.4 Filtering

Filtering the data as it is observed, is an attempt to minimize the errors due to systematic effects. Inertial surveying systems have been developed with built-in data filtering routines because of the desire for "instant" results. These routines attempt to predict how the systematic effects will behave during the run and then modify the data based on some internal algorithm. These routines contain feedback mechanisms which verify system performance during the mission and update the algorithm as the mission proceeds.

Filtering is given here under error sources because of the unknown nature of the filtering algorithms. In order to protect their proprietary interests, manufacturers of inertial surveying systems have been reluctant to disclose the exact characteristics of the algorithms. To complicate the problem, the observed data are not available on some systems until after they have been modified by this unknown filter.

Thus in their attempt to compensate for systematic errors, some manufacturers have created another error source. The performance of the filter alone cannot be verified. The filter adjusts itself and the data in an unknown way so as to cloud the identification of the systematic behavior of the observing hardware.

2.3 Operations

The alignment of the gyro axes and of the platform holding the accelerometers is then of critical importance in determining coordinated differences with the inertial surveying system. Those factors which degrade this alignment may be minimized by the instrumentation and procedures of inertial surveying. The various operational systems are differentiated by the manner in which they maintain the necessary reference frame.

2.3.1 System Characteristics

One class of inertial surveying systems which does not physically instrument the reference frame is referred to as analytic. Such systems have the inertial platform rigidly attached to the carrying vehicle. The analytic systems use a triad of pulse-rebalanced rate integrating gyros to compute the position of the platform relative to its starting position. Such systems are not yet appropriate for geodetic purposes because of their traditionally lower accuracies.

A second class of inertial surveying systems is referred to as semi-analytic. These are the geodetic quality instruments, which use gyroscopes and gimbals to instrument the reference frame. The systems within this class are further differentiated by the orientation they seek to maintain.

Those systems that are known as space stabilized inertial systems attempt to maintain the platform in a constant orientation without any releveling or realignments to north. These systems sense changes in system orientation during the mission. The platform leveling and gyrocompassing are done mathematically only. The Honeywell Geo-Spin is one of these systems.

Other types of semi-analytic systems are known as local level inertial systems. During the mission, the platform's gyroscopes are torqued to maintain orientation after the system computes the changes necessary because of the motion of the carrying vehicle and the rotation of the earth. This system attempts to maintain the accelerometer sensitive axes in the instantaneous local level reference frame. This type of system includes the Litton inertial surveying system from which the data come for this study.

2.3.2 Observing Procedures

The observing procedures for inertial systems attempt to lessen some of the errors due to the known systematic effects. The procedures dictate establishing an initial known orientation and then maintaining the orientation with respect to the proper reference frame throughout the mission. Because this study deals with data from a Litton Autosurveyor, the procedures discussed are for maintaining the system in the local level orientation.

The operation of the system begins with an initial pre-mission calibration and orientation session. During this time, the gyroscopes stabilize at their operating speeds. The system orients itself to north through its gyrocompassing capability and levels the platform using the direction of local gravity. Approximate coordinates of the system's position are also entered to initialize the filtering algorithm.

The inertial traverse run begins at a point of known coordinates. The system is placed at this point and the exact coordinates are entered into the system keyboard. The system is then moved to its next point. Currently, both ground vehicles and helicopters are used to transport inertial surveying systems.

In order to minimize the problems of drift and to allow the system to stabilize, periodically the system will be held stationary for approximately 20 seconds at what is called a zero velocity update (ZUPT). During the ZUPT, the accelerometers will continue to detect accelerations which the computer will use to update the internal filtering since it "knows" the system is at rest. Also at the ZUPT, the platform of the Litton Autosurveyor is releveled.

It is important that the ZUPTs be done regularly, commonly every three to five minutes. This short interval allows for better prediction of the drift effects which becomes more complex with longer ZUPT intervals. This short interval also reduces the problem of the anomalous gravity field which affects the system since it is releveled at each ZUPT.

At each point where coordinates are desired, coordinates are computed, displayed on the system, and written to magnetic tape. These are referred to as MARK observations.

At each point with known coordinates, exact coordinates are entered into the system keyboard. These coordinates are also used by the computer to improve the filtering information. These observations are known as UPDATEs. It is also important not to go too long between UPDATEs so as to further minimize the problems of drift. Typically the UPDATE interval is no longer than two hours.

2.3.3 Output Data

The data which are given as output from the inertial surveying system consists of the latitude, longitude, and elevation resulting from each UPDATE and MARK observation. Also included with these coordinates is the time that the observation was made. These data elements are written onto magnetic tape cassettes which can then be processed later in an office environment. For this study, these cassettes were transcribed to nine track tapes for processing on larger computers.

This process of ZUPTs, MARKs, and UPDATEs is referred to as a traverse run because of its similarity to a conventional surveying traverse which measures an angle and distance to each new point. The inertial traverses, however, have generally proceeded in as straight a line as possible with almost constant velocity. These restrictions have been employed because of the presence of systematic errors. To eliminate these restrictions and improve the usefulness of inertial surveying systems will require improvements in the modeling of systematic errors in the post-mission processing phase.

3. POST-MISSION PROCESSING

The presence of systematic errors in the inertial surveying data after the real-time data filtering requires some form of post-mission processing. The more complete techniques involve attempts to model the remaining systematic effects with a mathematical model, the processing of the observations through a least squares adjustment, and a statistical analysis of the results.

3.1 Mathematical Models

The first step in adopting a mathematical model is the recognition of the observable quantities. The inertial surveying systems generate coordinate differences which represent differences observed in latitude, longitude, elevation, and time. However, before these observations are available for post-mission processing, they are subjected to the filtering operation contained in the IPS software. The observed quantities themselves are not obtainable. The mathematical models are therefore attempting to capture the systematic effects of the total inertial surveying system including the behavior of the filtering algorithms.

Most determinations of the mathematical models are based on some knowledge of the physical behavior of the inertial surveying hardware, and the error sources, together with empirical verification of the suspected models. An extensive development of models is found in Hannah (1982) and it is his models upon which this study is based.

The models for errors in latitude, longitude, and elevations according to Hannah:

$$\text{Error } (\phi) = S_\phi(\phi_i - \phi_1) + \theta_N(\lambda_i - \lambda_1) + k_1(\lambda_i - \lambda_1)^2$$

$$\text{Error } (\lambda) = S_\lambda(\lambda_i - \lambda_1) + \theta_E(\phi_i - \phi_1) + k_2(t_i - t_1)^2 \qquad (3.1)$$

$$\text{Error } (h) = \theta_{Z,E}(\phi_i - \phi_1) + \theta_{Z,N}(\lambda_i - \lambda_1) + k_3(t_i - t_1)$$

where all coordinate differences are referred to the initial point of the traverse.

This study makes use of these models for error but with a couple of changes. First, the longitude difference squared in the latitude equation will be replaced by time difference squared for consistency with the rest of the equations. This change also recognizes that the term is intended to capture the gyro drift effects which might well behave as time squared. It is also possible that this change would lessen the heading sensitivity problem referred to by Schwarz and Gonthier (1981).

Secondly, Hannah's models have been rewritten in terms of coordinate difference observations. Note that the time of the start of the traverse must still be retained in the time squared terms. When these changes are made and the nine system error parameters are now designated as S_1 through S_9, the models are as follows:

$$\Delta\phi_{ij} = (1 + S_1)(\phi_j - \phi_i) + S_2(\lambda_j - \lambda_i) + S_3\{(t_j - t_o)^2 - (t_i - t_o)^2\}$$

$$\Delta\lambda_{ij} = (1 + S_4)(\lambda_j - \lambda_i) + S_5(\phi_j - \phi_i) + S_6\{(t_j - t_o)^2 - (t_i - t_o)^2\} \quad (3.2)$$

$$\Delta h_{ij} = (h_j - h_i) + S_7(\phi_j - \phi_i) + S_8(\lambda_j - \lambda_i) + S_9(t_j - t_i)$$

between points i and j with t_o = time at start of traverse run.

The units of latitude and longitude in these models are considered to be in radians and the elevations are in meters. Time references are in seconds. Thus the parameters S_1, S_2, S_4, S_5, S_7, and S_8 must be considered to be unitless. The parameters S_3 and S_6 are in radians/second^2 and S_9 is in radians/second.

In these adopted mathematical models, the S_1 and S_4 parameters would capture the scale effects, S_2, S_5, S_7, and S_8 would capture the off-track or misalignment effects, and S_3, S_6, and S_9 would correspond to those biases which behave as linear with time or time squared. Of prime concern in this study is the stability of these effects and the scope of the corresponding parameters.

With respect to the system scale effects, Ball (1978) notes that the scale factors are "quite stable" during a traverse or series of traverses provided the general direction of the traverse heading is not reversed. Todd (1979) also believed initially that the scale factors were very stable, but according to Hannah and Pavlis (1980) more recently has detected significant variations. Schwarz and Gonthier (1981) separately applied models for forward and backward traverses in their investigation. Tindall (1982) has provided scale parameters for each cardinal direction to model accelerometer scale error asymmetry. Others such as Hannah (1982) and Milbert (1982) have applied models where a single scale parameter is applied to both directions of a double run traverse.

With respect to the modeling of the off-track or misalignment effects, it must be recognized that the platform azimuth errors and the accelerometer sensitive axis misalignments are inseparable. That portion of the misalignment which is due to the non-orthogonality of the accelerometer axes is thought to be quite stable by Huddle and Maughmer (1972). The overall misalignment effect for each horizontal axis remaining after the filtering process is usually modeled with linear terms dependent on the coordinate differences in the other axis, and separate drift terms which are time dependent. These terms have been considered stable for the individual traverse run. Recent modeling by Tindall (1982), Hannah (1982), and Milbert (1982) have applied models where single misalignment or drift parameters are used for both directions of a double run traverse.

For this study, the above models will be used but with varying parameter allocations -- six different possible allocations from that of a single direction of the traverse to both directions for the entire set of traverse runs.

3.2 Adjustment Model

The adjustment model to be used in this study is that referred to by Uotila (1967) as the Method of Observation Equations. The following notation is used:

$$L_a = F(X_a) \qquad (3.3)$$

in which L_a represents the observations as functions of the coordinate and model parameters, X_a. The functions are then linearized with a Taylor Series expansion and a design matrix, A evaluated at a particular set of parameter values X_o:

$$A = \left.\frac{\partial F}{\partial X_a}\right|_{X_a = X_o} \tag{3.4}$$

With $L_o = F(X_o)$ and $L = L_o - L_b$, where L_b is a vector of actual observations,

$$A\,X = L + V \tag{3.5}$$

where V is a vector of residuals and $X = X_a - X_o$. With the variance-covariance matrix of the observations given by Σ_{L_b} and σ_o^2 as the a priori variance of unit weight, the weight matrix for the adjustment if given by:

$$P = \sigma_o^2\, \Sigma_{L_b}^{-1} \tag{3.6}$$

The vector of parameter improvements X which is then applied to the vector of initial parameter values X_o is determined by:

$$X = -\,(A'\,P\,A)^{-1}A'\,P\,L \tag{3.7}$$

Due to the non-linearity of the functions in (3.3), this process must be iterated with the updated parameter estimates until the parameter improvements are negligible. Adjusted observations are then computed from the final parameter values and the residual vector is given by:

$$V = L_a - L_b \tag{3.8}$$

The a posteriori variance of unit weight is given by:

$$\hat{\sigma}_o^2 = \frac{V'PV}{n - u} \tag{3.9}$$

where n is the number of observations and u is the number of parameters. The variance-covariance matrix of the adjusted parameters is given by:

$$\Sigma_{X_a} = \sigma_o^2 \, (A' \; P \; A)^{-1} \tag{3.10}$$

The variance-covariance matrix can be determined for other quantities which are functions of these adjusted parameters. This process is known as linear error propagation and is given by Uotila (1978) and Hamilton (1964) as follows:

> Let X_a be the vector of adjusted parameters, Y a vector of quantities derived from X_a and matrix G such that $Y = G \; X_a$. The variance-covariance matrix of Y is given by $\Sigma_Y = G \, \Sigma_{X_a} G'$.

3.3 Analysis Techniques

The purpose of this study is to examine the behavior of system parameters so as to draw some conclusions about the systematic error effects these parameters seek to model. Using the aforementioned mathematical models and adjustment model, a series of minimally constrained least squares adjustments will be run and the results analyzed to study the parameter behavior. Several statistical tests will be applied as well as graphical analysis to evaluate parameter behavior.

3.3.1 F-Tests

The major aspect of this study is the analysis of behavior of system parameters. In order to analyze behavior, the system parameters will be allocated in differing ways in different adjustments and the results compared to determine if the results are significantly different. Valid statistical tests can be applied to compare parameter allocations whenever one allocation scheme represents a constraint on another allocation scheme. That is, when by constraining the behavior of certain system parameters, one may derive one allocation scheme from another. For instance, if a particular scale parameter is allocated one parameter per run where there are three runs on a day, the results of that least squares adjustment can be compared to one in which there is only one scale parameter allocated for the entire day. The scale parameters for each of the three runs are, in effect, constrained to be equal.

The results from a pair of adjustments can be compared using the F-distribution if one of the adjustments represents a constraint of the other. A quantity is derived from the sums of the squares of the weighted residuals (V'PV) and the degrees of freedom for each adjustment. This quantity is then compared with the tabular values of the F-distribution. The comparison will indicate whether the two different allocations of a system parameter produce different enough results to dictate that the parameters must be considered as distinct.

For instance, for some particular system parameter, Sn, the results of two different adjustments can be compared as per Hamilton (1964):

Adjustment 1: $V'PV_1$, DF_1

Adjustment 2: $V'PV_2$, DF_2 with the constraint $Sn_1 = Sn_2$

Hypothesis $H_o : Sn_1 = Sn_2$

$H_1 : Sn_1 \neq Sn_2$

$$\text{If} \quad \frac{\dfrac{V'PV_2 - V'PV_1}{DF_2 - DF_1}}{\dfrac{V'PV_1}{DF_1}} > F_{(DF_2-DF_1),DF_1,\alpha}$$

at some significance level α,

then, reject H_o and conclude that the constraint is not valid and Sn_1 and Sn_2 must be distinct parameters

else, cannot reject H_o and can conclude that Sn_1 and Sn_2 could be one and the same parameter.

The results of these F-tests would then allow for some direct conclusions about the scope of the tested parameters.

3.3.2 Graphical Analysis

In addition to the statistical tests, the various adjustment results will be used to generate graphs depicting individual system parameter behavior and overall model performance. These graphs can then be examined to detect patterns which may not be evident solely from the statistical tests. Such patterns may give clues as to the "best" allocation of model parameters.

Graphs will also be used to compare the results of inertial mathematical models with results obtained from other techniques to evaluate model performance. Graphical comparisons will be made between coordinates derived from adjustments of the inertial observations and coordinates obtained for the same points using conventional surveying techniques. Additionally, the results of adjustments using the described models will be compared with those using the Gregerson models described by Milbert (1982).

3.3.3 Length Relative Accuracies

Horizontal geodetic surveys are classified according to Length Relative Accuracies which result from a minimally constrained least squares adjustment of the survey data. These Length Relative Accuracies express a length discrepancy as the proportional part of the length of the line between two directly connected, adjacent points. The Length Relative Accuracy may be determined by two different methods -- from linear error propagation or from length shifts.

Linear error propagation, as described in Section 3.2, requires the variance-covariance matrix of the adjusted parameters together with the functional relationship of the desired quantity and the adjusted parameters. The variance and standard deviation of the adjusted length between two points is obtainable directly, but is dependent on the a priori estimate for the variance of unit weight and the relative weighting scheme. The standard deviation of the adjusted length divided by the adjusted length gives a value for Length Relative Accuracy. Use of this Length Relative Accuracy implies some confidence in the weighting scheme used for the adjustment.

A Length Relative Accuracy may also be computed directly from the length shift obtained by differencing the adjusted length and the length based on highly precise preliminary coordinates. This length shift divided by the adjusted length gives a value for Length Relative Accuracy. These Length Relative Accuracies require acceptance of the established accuracy of the preliminary coordinates.

The classification of the horizontal geodetic survey is made using the minimum value for Length Relative Accuracy obtained in the adjustment. In the study, these Length Relative Accuracies will be used to compare the distortions remaining in the network after adjustments using the various allocations of parameters. It should be mentioned that there is a lack of confidence in many aspects of the weighting of inertial surveying data. Thus, Length Relative Accuracies based on linear error propagation cannot be relied upon. In this test, however, there will be a priori knowledge of the preliminary coordinates. This supports the use of Length Relative Accuracies based on length shifts to compare the adjustment results.

3.3.4 Chi-Square Tests

The chi-square test can be applied to the adjustment results as explained by Uotila (1975). An H_o hypothesis is made that the system is modeled and functioning correctly. The sum of the squares of weighted residuals (V'PV), the a priori variance of unit weight (σ_o^2) and the degrees of freedom (DF) are used in the one-tailed chi-square test such that if:

$$\frac{V'PV}{\sigma_o^2} > \chi^2_{DF,\alpha}$$

then the H_o hypothesis is rejected at the selected significance level, α.

Such a test shows whether or not V'PV is too large, an important consideration in this study. Possible causes for too large a V'PV include problems with the mathematical models, i.e. incomplete modeling such that significant systematic errors remain. While the intent of this study is to examine the behavior of system parameters in a particular model, the possibility of model deficiencies must not be overlooked.

4. SOFTWARE AND TEST DATA

The adjustment program used in this study is based on INERT1, a program described by Milbert (1982). Substantial changes were made in the models to accommodate parameter flexibility. A series of adjustments was run, all on the same set of test data in order to compare results of varying parameter scope.

4.1 Adjustment Program

Program INERT1 was created at NOAA's National Geodetic Survey for research into the post-mission adjustment of inertial observations. The program provides for a rigorous, simultaneous, least squares adjustment of multiple inertial traverse runs. Program INERT1 served as a basis for program INERTC which was created as part of this study. The input formats for inertial observations are identical.

The adjustment program is modular in construction with each specific task assigned to a different subroutine. The main driver program controls all functions including the reading of data, allocating resources, the forming and solving of normal equations, and the useful presentation of results. Several external subroutine packages are called upon to supply specialized functions.

The least squares adjustment of inertial observations is similar to that of other horizontal geodetic observations in that large sparce matrices must be manipulated. Of major concern is the amount of core storage required in the computer. One of the earlier calls in the program is to a subroutine from Snay (1976) which uses the "Banker's Algorithm" to reorder the unknowns and thereby reduce core requirements. Other routines for handling these sparce matrices are described by Dillinger (1981) and make use of a scratch array for computing phases plus the possibility of additional interim storage not in main memory.

The modular structure of INERT1 provided for ease in changing basic mathematical models. Program INERT1 utilized the Gregerson twelve parameter model whereas for this study, INERTC uses the nine parameter mathematical model referred to earlier in Section 3.1. Changing the mathematical model equations involved changes to only two subroutines. However, to accommodate the different parameter allocations required changes to many subroutines. The primary concern of this study is the comparison of varying parameter allocations and this required significant restructuring of those subroutines responsible for controlling parameter allocation and use.

The first step in the restructuring of parameter allocation was the development of an additional data input record which, by its placement within the input deck, controls the allocation of the nine different categories of system parameters. Such a scheme not only dictates when a new parameter is to be allocated, but also must allow for the reallocation of a parameter used earlier in the adjustment. These parameter allocation records then control the several subroutines which keep track of parameter indices.

Changes were then necessary to those items within the program which dictate the total number of unknowns and control the storage arrays used by the many different subroutines. To determine the total number of unknowns, a preliminary pass through the entire input data deck was instituted which counts the requests for parameter allocation. This approach increased the flexibility and usefulness in this particular study. Once the total number of unknowns is determined, sufficient storage space is set aside for the unknowns and the program is then able to process the observations.

4.2 Description of Test Data

Data used in this study are from the joint effort in March 1981 by the National Geodetic Survey (NGS) and Span International, Inc. of Scottsdale, Arizona. This field test of an inertial surveying system was conducted in southwestern Arizona along the Transcontinental Traverse (TCT) and is described in detail by Leigh (1981).

The site for the test was selected because of the winter season, the proximity to Scottsdale, Arizona, the quality of existing control, and the ability to land a helicopter at the control stations. Of particular concern was the use of the TCT stations in that the TCT has been shown to be accurate in scale to one part in one million (Gergen 1979). Also, the particular portion of the TCT selected for the test provided 80 kilometers along each leg of an L-shaped traverse oriented approximately north-south and east-west (See figure 1). The 80 kilometer distance was determined to be typical of the spacing between arcs of first-order control in the United States.

Originally, a grid pattern had been planned for the test but the L-shaped configuration was adopted due to problems with the test

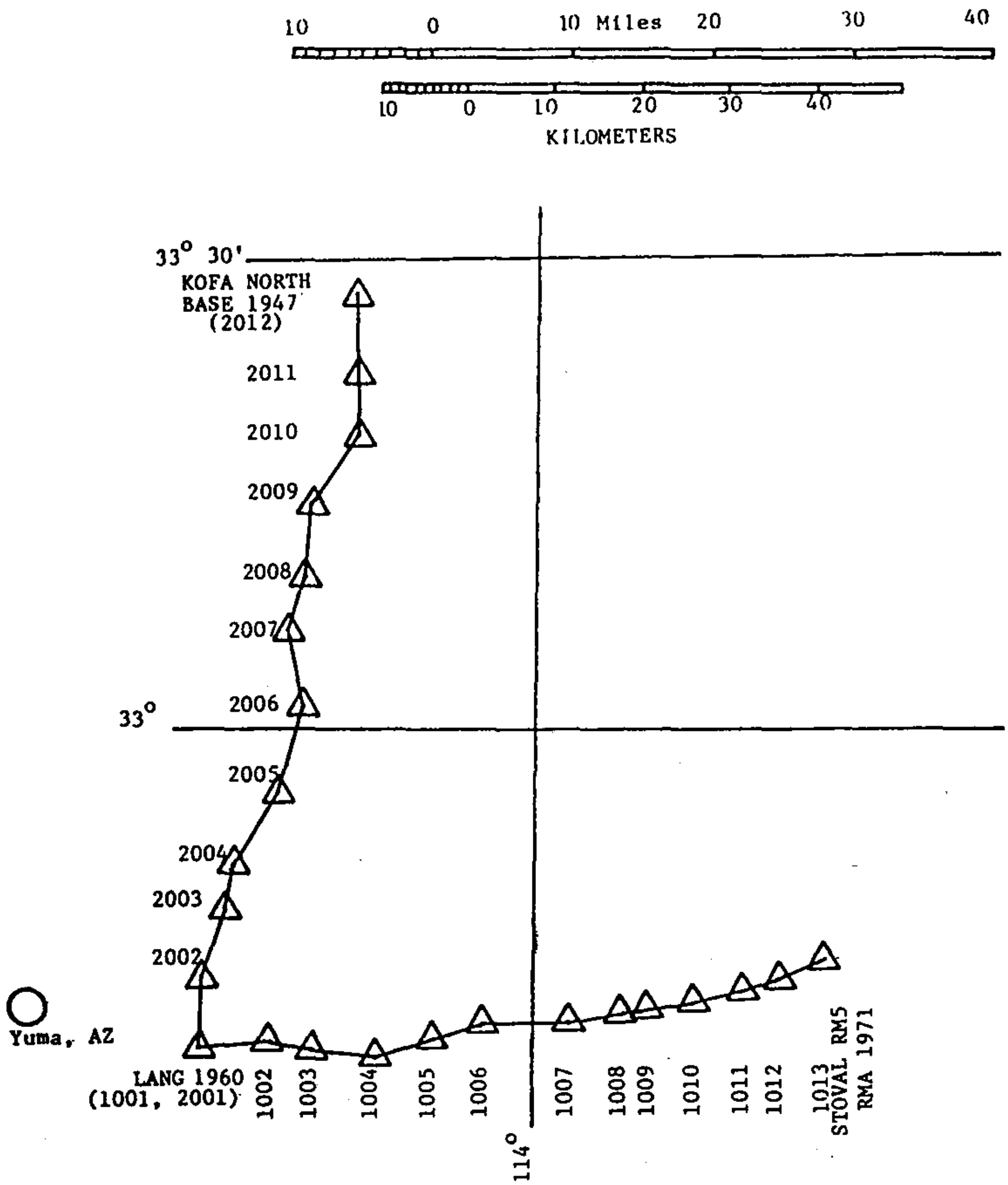


Figure 1. ISS Test Course - SW Arizona

location. Other research in post-mission least squares adjustment of inertial data has resulted in calls for an "area survey" using inertial surveying systems. Hannah and Mueller (1981) noted that such criss-crossing surveys with common points would provide the constraints to increase the degrees of freedom and the use of more comprehensive error models. Schwarz (1981) also calls for the use of cross-over points to provide the possibility of eliminating remaining systematic errors. However, the use of the highly accurate TCT stations with many repeated traverses over the same points should provide sufficient degrees of freedom to allow for all system parameter determinations and the statistical analysis of the results.

To insure the consistency of the geodetic control and to minimize the influence of major network distortion which should be removed with the new adjustment of the North American Datum, all conventional horizontal and astronomic observations in the area of the test site were adjusted in simultaneous, minimally constrained, least squares adjustments. The final adjustment which covered the entire test area contained 829 stations and 6,521 observations and resulted in a variance of unit weight of 1.594 with 3,137 degrees of freedom. The adjusted coordinates from that adjustment are used for the control stations in this study and are given in Table 1.

The inertial observation runs were made during the time period March 18 to April 1, 1981 using a SPANMARK Inertial System (Litton Autosurveyor). The system was transported in a helicopter with each forward run immediately followed by a reverse run. The runs were either north-south or east-west and on some of the days, multiple runs were

Table 1. Adjusted Coordinates - Conventional Observations

Station Name	No.	North Latitude			West Longitude			Elev.
		°	'	"	°	'	"	m
LANG 1960 RM 4	1001	32	40	7.39636	114	24	29.94778	104.
BEACON 2 1971 AZ MK 2	1002	32	40	42.73378	114	18	30.19185	95.1
OVERPASS 1934	1003	32	40	9.24811	114	15	52.09266	88.43
ADONDE 1934	1004	32	39	28.93223	114	11	29.28629	84.3
PASSO 1960 RM 2	1005	32	40	19.77996	114	6	58.91308	87.9
NAVI 1960	1006	32	40	48.51619	114	3	3.21521	102.62
GAEL 1934 RM 5	1007	32	41	46.57782	113	57	18.62695	106.
COLFRED USGS 2 1971	1008	32	42	22.71959	113	53	33.41455	99.9
PEMB 1960	1009	32	42	40.46306	113	51	25.24326	103.49
OWL 1934 RM 4	1010	32	43	6.70717	113	48	10.69975	130.
AWK 1960 RM 4	1011	32	43	41.28930	113	45	26.24630	166.
KIM 1960 RM 3	1012	32	44	18.00110	113	41	46.09492	117.
STOVAL RM 5 RM A 1971	1013	32	45	27.59854	113	38	19.83475	117.48
QUARRY	2002	32	44	38.91312	114	25	13.65760	76.7
BENCH MARK USBR 1934 RM 4	2003	32	48	39.30649	114	22	33.54397	86.
COUNTRY WELL RM 2 RESET	2004	32	51	31.24118	114	21	32.34947	116.
TT 6 USE 1956	2005	32	55	20.80390	114	18	49.69741	208.0
PELIGRO 1949	2006	33	1	19.93113	114	16	53.67928	273.2
HILL TOP 1949 RM 3	2007	33	6	15.20785	114	17	55.48380	342.
INDIAN 1949 RM 2	2008	33	10	25.50005	114	16	34.20444	364.
PGT NO 3 AMS 1971 RM 3	2009	33	14	22.15943	114	15	28.51893	542.
CHOCO 1949 RM 2	2010	33	18	42.14580	114	12	56.57168	448.
KOFA SOUTH BASE 1949 RM 2	2011	33	22	37.06334	114	12	59.96262	417.3
KOFA NORTH BASE 1947	2012	33	27	49.48687	114	12	59.86782	374.6

Note: Standard deviations used in fixing above coordinates

Latitude	0.001 meter
Longitude	0.001 meter
Elevations	
if given to nearest meter	1.0 meter
if given to 0.1 meter	0.1 meter
if given to 0.01 meter	0.01 meter

successfully completed. A summary of the usable runs is given in Table 2. Additional inertial observations were made but not included in this study. Some of these were eliminated because gaps were found in the data received by NGS. Others were not used because the forward run did not have a corresponding complete reverse run. Still others were not used because they involved some experimental variations in observing procedures. The resulting usable data consisted of 18 complete forward and reverse traverse runs taken on nine different days. All adjustments in this study will use this same input data set.

4.3 Various Adjustments

As stated earlier, the purpose of this study is the examination of the behavior of the model parameters. There are nine of these model parameters, S_1 through S_9, in the basic equations of the mathematical models. The various adjustments which will be analyzed and compared will involve different allocations of these nine model parameters.

Appendix 1 gives the 96 test adjustments with the degrees of freedom (DF) and a posteriori variance of unit weight ($\hat{\sigma}_o^2$). These results are grouped according to the allocation of the model parameters. Before examining the tables of adjustment results, the following discussion is necessary to understand the adjustment identification notation.

4.3.1 Model Parameter Allocation

Previous work on post-mission processing of inertial observations has involved several different schemes for allocating the model

Table 2. Traverse Summary

Run Number	Date	Run Directions
1	3/19/81	West - East - West
2	3/21/81	South - North - South
3	3/21/81	South - North - South
4	3/22/81	South - North - South
5	3/22/81	South - North - South
6	3/22/81	South - North - South
7	3/24/81	South - North - South
8	3/26/81	South - North - South
9	3/27/81	West - East - West
10	3/27/81	West - East - West
11	3/27/81	West - East - West
12	3/28/81	West - East - West
13	3/28/81	West - East - West
14	3/28/81	West - East - West
15	3/29/81	West - East - West
16	3/29/81	West - East - West
17	3/29/81	West - East - West
18	3/31/81	South - North - South

parameters. For the sake of completeness, six different allocations will be considered for each model parameter:

A. Parameter allocated per <u>leg</u>.

One parameter for each leg, which is defined to be either a forward or reverse single run traverse.

B. Parameter allocated per <u>run</u>.

One parameter for each run, which is defined to be a double run traverse, forward and reverse.

C. Parameter allocated per <u>direction per day</u>.

One parameter for each cardinal direction for all runs on the same day, e.g. one S_3 for east on March 27th and another S_3 for west on that same day.

D. Parameter allocated per <u>day</u>.

One parameter for each day, whether there is one run or three runs on that day.

E. Parameter allocated per <u>direction for all days</u>.

One parameter for each cardinal direction for the entire test, i.e. one for north, one for south, one for east, one for west.

F. Parameter allocated for <u>all days</u>.

One parameter for the entire test, e.g. one S_5 for all the runs, all of the days.

Thus for each of the nine model parameters, six different allocations can be made for a total of 6^9 or 10,077,696 different possible adjustments. Clearly this would be unreasonable and thus the nine model parameters have been grouped as follows:

Group 1: S_1 S_4 -- "Scale" parameters

Group 2: S_2 S_5 S_7 S_8 -- "Misalignment" parameters

Group 3: S_3 S_6 S_9 -- "Drift" or time-dependent bias parameters

Now if the various adjustments are restricted such that all model parameters in a group are allocated in the same manner, the number of different possible adjustments is 6^3 or 216. To identify the adjustments in this study, the six letters A, B, C, D, E, and F will be used to indicate allocations of model parameter groups. Three of the above letters, in order of appearance, will be used to label the parameter allocations for each adjustment.

Using the allocations as defined on the previous page, for example:

B D A

would indicate an allocation of B (per run) for Group 1 parameters, D (per day) for Group 2 parameters, and A (per leg) for Group 3 parameters.

Thus each adjustment will be labeled by a three letter code which indicates its model parameter allocation: AAA, BAA, DED, etc. To further limit this study, because even 216 different adjustments would be cumbersome and probably of marginal value, adjustments will only be initially run with no more than two different letters in their three letter code. Thus ABA or BAA will be run, but ABC will not. This restriction will still allow for 96 different adjustments involving the nine model parameters and six different allocations.

4.3.2 Minimally Constrained Adjustments

As stated in Section 3.3, the adjustments to be run and analyzed will be minimally constrained. An adjustment will be minimally constrained when the minimum geometric conditions are met for solution of the three-dimensional observation equations.

The mathematical models selected for this study provide for separate scale on each axis and are not reliant on the orthogonality of the axes. Thus, in addition to the three constraints needed for translation in this three-dimensional system, three more constraints are needed to define scale and three more to define orientation of the system.

The total of nine constraints, which are required for the minimally constrained solution, are most simply provided by fixing the coordinates of three points along the inertial traverse. These latitudes, longitudes, and elevations then provide the system definition needed. It can be shown that the selection of the three points to fix along the traverse will not affect the observation residuals and the resulting variance of unit weight.

In the test data set, station LANG 1960 at the junction of the L-shaped traverse was fixed in the minimally constrained adjustments of both the conventional observations (described in Section 4.2) and all adjustments of the inertial data. With the junction point fixed and the recognition that all inertial traverse runs terminated at this junction point, each leg of the "L" can be considered as a separate traverse, each requiring three fixed points. Thus the other endpoints were fixed as was the midpoint of each leg. While the residuals are invariant with respect to the selection of fixed points, the adjusted coordinates are dependent and the midpoints were selected with the intention of minimizing the final overall position shifts.

4.3.3 Weights

The fixing of coordinates explained in the previous section was done by adding direct observations of these coordinates with appropriate observation standard errors. The weights on these coordinates resulted from standard errors of 0.001 meter for latitude and longitude. Weights for elevations were as given in Table 1. The TCT latitudes and longitudes are considered more accurate than the elevations.

The program also has the capability of accepting a priori weights on the model parameters as well. This is useful in validation studies of mathematical models where the possibility of eliminating model parameters is considered. Weights on model parameters were not used or necessary in these test adjustments.

The weights on the inertial coordinate difference observations result from a priori standard errors of 0.1 meter for latitude, longitude, and elevation differences. A diagonal weight matrix is used implying that all observations are uncorrelated. Also, the a priori variance of unit weight was 1.0 in the test adjustments.

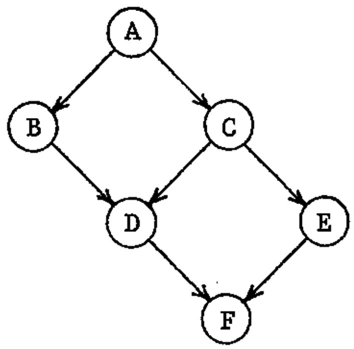
5. COMPARISON OF RESULTS

The results of the test adjustments given in Appendix 1 have been compared as described in Section 3.3. These results demonstrate the behavior of particular model parameters as well as selected combinations of model parameters. Statistical tests and graphical analysis will be used to make the best possible selection for model parameter allocation.

5.1 F-tests on Model Parameters

The grouping of model parameters was done with the assumption that parameters in the same group would behave in a similar manner with respect to scope. The organization of the test adjustments permits an examination of this behavior. Varying the allocation of one group of model parameters at a time while maintaining the same allocation for all other parameters should isolate the significant behavior.

The effect of varying allocations on V'PV can be determined and measured with an F-test. Such an application of the F-test is only valid if one allocation is a constraint of another as explained in Section 3.3.1. Now that the allocations have been labeled with the letters A-F, the valid allocation relationships can be explained with the following diagram:



A : one set per leg

B : one set per run

C : one set per direction per day

D : one set per day

E : one set per direction for all days

F : one set for all days

Figure 2. Allocation Relationships

The valid relationship exists wherever one can proceed from the one letter to the other following the arrows. Thus $A \rightarrow C$, $B \rightarrow F$, $C \rightarrow F$ are valid comparisons whereby C is a constraint of A, F is a constraint of B, and F is a constraint of C. There are twelve such valid comparisons that can be applied.

The 96 test adjustments were run using the same inertial observations but with different allocations of the nine categories of mathematical model parameters. F-tests were used to compare the results of these adjustments and, in more than 94% of the comparisons, it was determined that the different parameter allocations resulted in significant contributions to $V'PV$ as explained in Section 3.3.1. Thus, the different allocations produced different adjustment results.

One objective of this study was to find cases where the different parameter allocation did <u>not</u> produce different adjustment results. Such some other criteria, e.g. to lessen the number of parameters thus increasing the degrees of freedom.

5.1.1 Comparisons on One Parameter Group

The F-test comparisons can be made to examine one group of parameters at a time. The comparisons are to be made to each of the three groups of parameters when the remaining two groups of parameters are allocated in each of six different ways. This scheme therefore results in 12x6x3 or 216 different F-tests.

For each group of parameters, Appendix 2 presents the results of the F-tests in terms of the probabilities of rejecting the hypothesis that the allocations produce similar results. That is, for each comparison, a hypothesis is made that the allocations result in substantially similar values for V'PV. A value is computed from the V'PV and the degrees of freedom for each adjustment, and this value is measured in terms of the F-distribution. Traditionally some significance level, α, is selected which determines the tabular value for the F-distribution. In this study, the resulting value for V'PV and the degrees of freedom were used as input to a computer routine (Amos, 1977) to generate the significance level, or the probability of rejecting the hypothesis. Thus a value of 1.000 indicated a rejection is to be made 100% of the time and a conclusion drawn that the allocations produce significantly different results. The values that are not 1.000 are in the minority and values less that 0.500 are especially rare and therefore noteworthy when they do occur.

The first set of F-tests was applied to those cases in which the allocation is varied for one parameter group at a time. For instance, in Appendix 2, the significance is given for varying only the scale parameters (Group 1 -- S_1, S_4) while maintaining the allocation of the other parameters. Allocating these scale parameters per leg (A) or per direction each day (C) produced the same results in almost all cases, the exception being those in which the remaining parameters are allocated per direction each day (C). This is of interest in that with the allocation of per direction per day (C), the number of scale parameters is substantially reduced as compared with allocation of per leg (A). Such a reduction is a desirable criterion for selection.

The same can be said when comparing allocations of scale parameters per run (B) and per day (D). The allocation per day (D) produced substantially the same results with fewer scale parameters. Neither of these two comparisons justifies the allocations per direction per day (C) or per day (D) for the scale parameters except as a preference over their counterparts (A or B). Further analysis is necessary to make definite statements about the behavior of these scale parameters.

Before continuing with the scale parameters, though, the F-test comparisons in the remainder of Appendix 2 can be examined in a similar manner for the other two groups of model parameters. For the misalignment parameters, given together in Group 2, the significant entries indicate that, in most cases, similar results are obtained for allocations per run (B) and per day (D). Thus, the allocation per day would likewise be preferred with fewer misalignment parameters, therefore fewer unknowns to solve for and greater degrees of freedom.

For those parameters given together in Group 3 to capture that behavior which is time dependent, the F-test comparisons do not justify one allocation scheme over another. All entries related to Group 3 would dictate rejection of the hypothesis of similar parameter behavior. Thus no statement can yet be made supporting one particular allocation over another for these time dependent parameters.

5.1.2 Comparisons on Two Parameter Groups

In order to investigate the behavior of these model parameters further, allocations can also be compared where two of the groups of parameters are allocated alike. In a similar manner as is used in the previous section, the two groups can be allocated in each of the six different ways. Such comparisons were possible out of the same 96 adjustments and may lead to further refinements of the parameter allocations.

Appendix 3 gives the results of 216 additional F-tests stated, as earlier, as the probability of rejecting the hypothesis that model parameters are equal and giving similar results in the adjustments. Here again, in most cases, changing the model parameter allocations gives significantly different results and the constraints would not be acceptable. Those values which are considerably smaller than 1.000 are very significant and permit some conclusions to be drawn.

The first table in Appendix 3 gives the comparisons for varying the scale and misalignment parameters together. The comparisons clearly indicate that allocations per run (B) and allocations per day (D) model the system behavior in a similar manner and give similar adjustment results. This combination behavior is consistent with that noted separately for these two parameter groups in the previous section. Therefore, of the two allocations, the allocations per day (D) for both scale and misalignment parameters would be preferred.

The other two sets of F-tests given in the remainder of Appendix 3 involve comparisons where allocations of Group 3 are paired with Group 1 and with Group 2. All of these comparisons do not lead to a preference of one allocation scheme over another as all F-tests would dictate rejection of the similar parameter hypothesis.

5.2 System Parameter Behavior

The statistical tests of the preceding sections were found to be inconclusive in making the selection of the proper allocation of system parameters. It is therefore desirable to determine the behavior of each system parameter in its least constrained situation with the intent of determining its proper allocation. This behavior can be discerned by examining graphs based on adjustments with system parameters allocated one set for each leg (AAA). Each parameter group can then be analyzed to look for patterns in its behavior.

For each system parameter, S_1 through S_9, the behavior is shown initially for a "free" adjustment as well as a "calibration" adjustment. The "free" adjustment is one with the minimum number of constraints, with the three constrained stations for each leg of the L-shaped traverse. The "calibration" adjustment is one in which all of the TCT derived preliminary coordinates are held fixed, thus giving an immediate indication of the appropriateness of the model.

The graphs of system parameter behavior presented in the following figures plot the system parameters against time. The parameter values have been "normalized" by dividing by the resulting standard deviation. The value plotted then indicates the significance of the parameter. All graphs represent data taken from minimally constrained or "free" adjustments except for those explicitly labeled as "calibration". The time scale is not linear. The values are plotted for each traverse run, 1 to 18. As given in Table 2, there were several days with more than one traverse run.

5.2.1 Scale Parameters

Figures 3-6 illustrate the behavior of S_1 and S_4, the scale parameters. There appears to be regular patterns on the graphs, particularly between runs 14 and 18 for S_4. The regular patterns prompted the generation of the additional graphs with separate lines for each direction of the traverse run. Thus on Figure 5 and Figure 7, the behavior of the scale parameters is shown separately for observations proceeding in the forward and reverse directions.

Examination of Figure 5 reveals some separation between the forward and reverse direction but this separation is not consistent. There does, however, appear to be some possible consistency between runs taken on the same day. Figure 7 contains a similar pattern with more separation but notable extremes in runs 1, 9, 12 and 16. Once again, there appears to be some possible daily pattern though it is a little difficult to discern at this point.

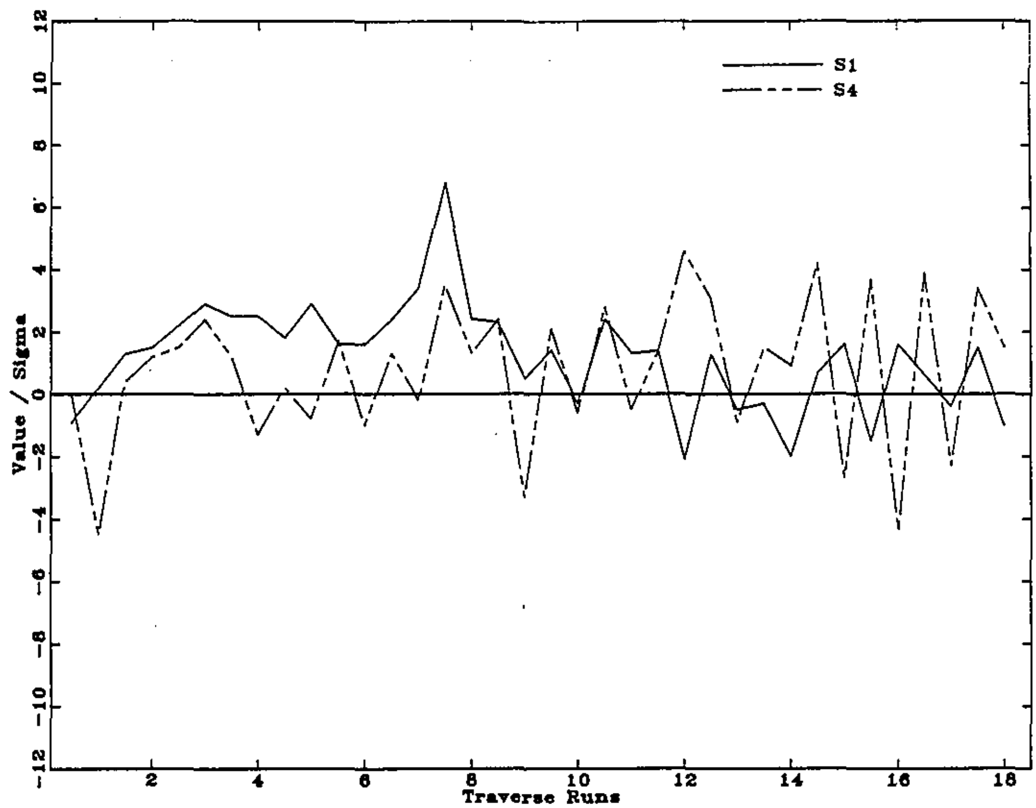


Figure 3. S1,S4, scale parameters.

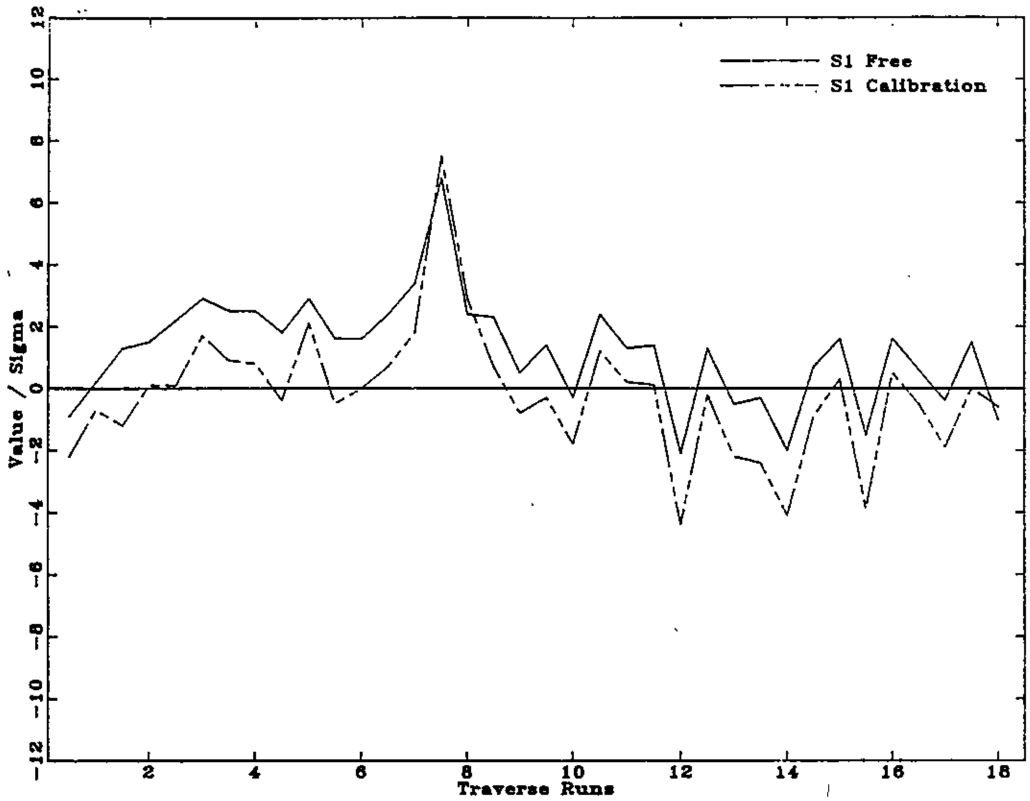


Figure 4. S1, scale parameters.

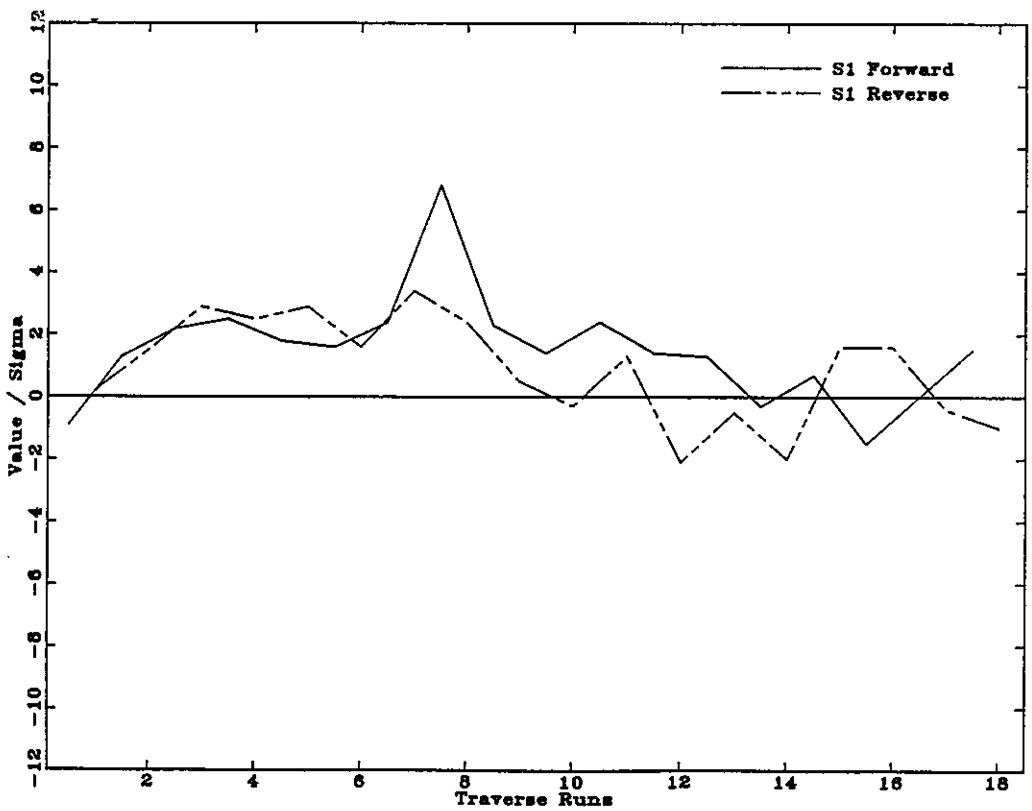


Figure 5. S1, scale parameters.

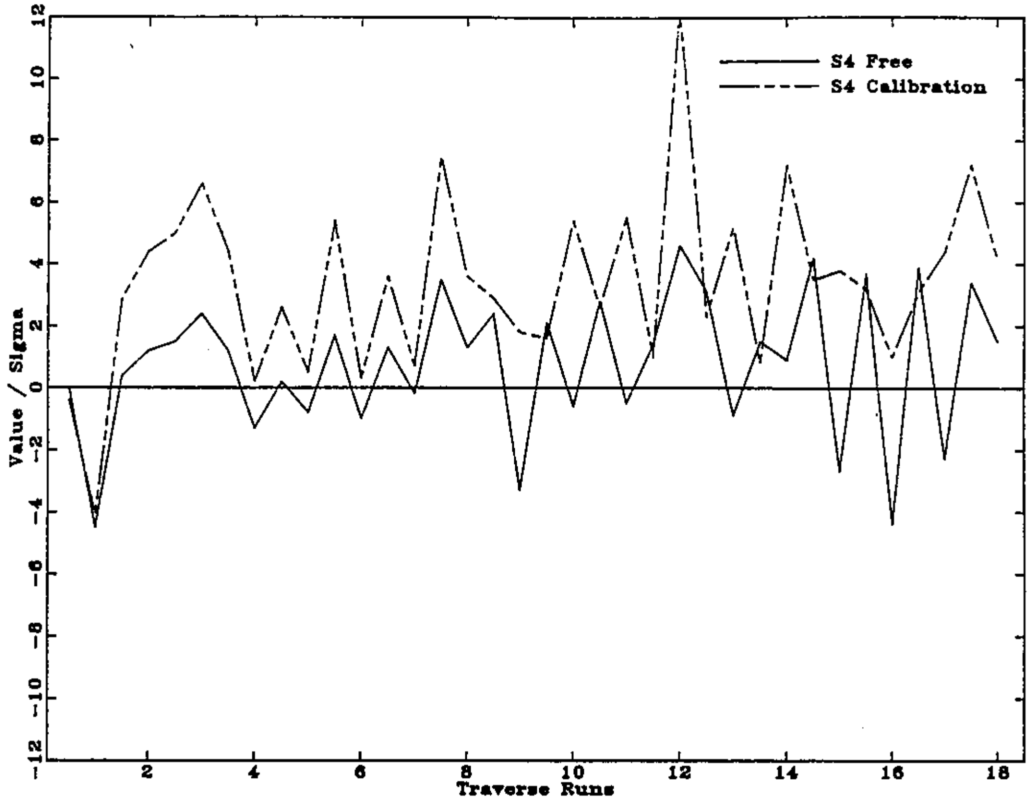


Figure 6. S4, scale parameters.

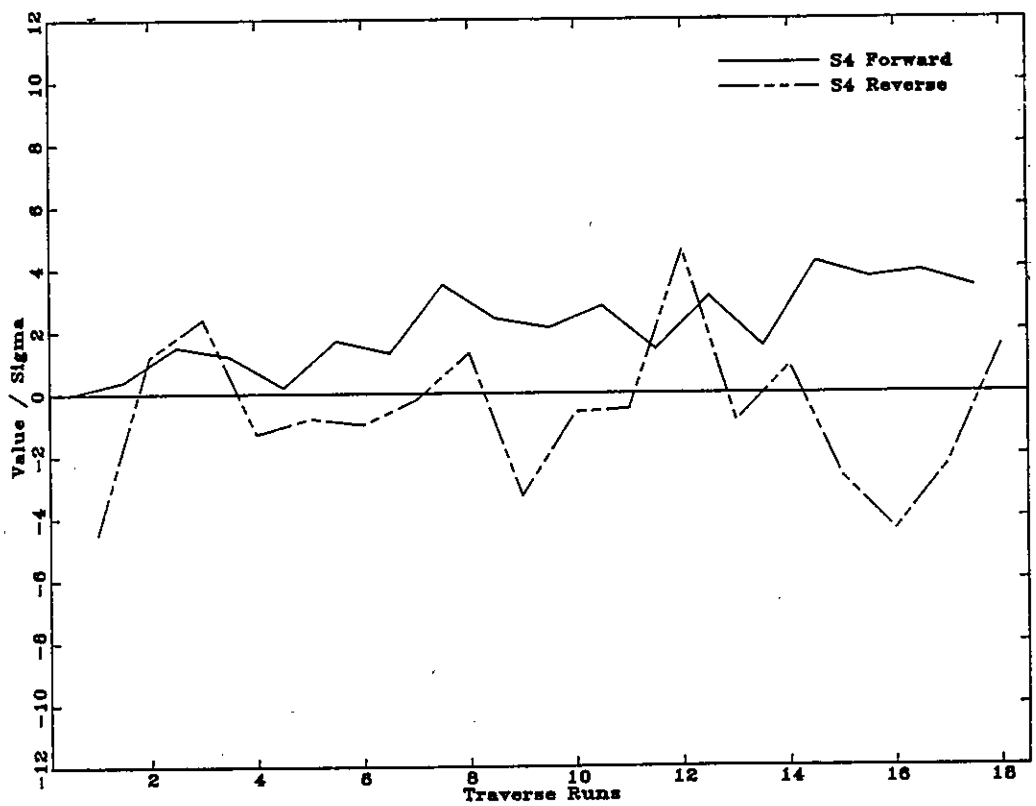


Figure 7. S4, scale parameters.

5.2.2 Misalignment Parameters

The graph of the behavior of the misalignment parameters contains several more definitive saw tooth patterns that indicate allocations which distinguish between forward and reverse directions. This behavior would dictate an allocation of A, C, or E for the misalignment parameters. Furthermore, there is a definite shift of the patterns between run 8 and run 9, precisely where the runs change direction between north-south and east-west. This direction shift pattern leads to an allocation of C or E.

The more detailed examination of each misalignment parameter provided by Figures 9-16 also exhibit the significant shift where the direction of traverse changes. However, the variability between runs 9 through 18 for S_2 (on Figures 9-10) and between runs 1 through 8 for S_5 (on Figures 11-12) implies an allocation of C, each direction for each day. There is not enough consistency between days for these runs to consider an allocation of E, each direction for all days, to represent misalignment parameter behavior.

Parameters S_7 and S_8 also exhibit behavior similar to that of S_2 and S_5. The shift with direction change is there but the plots of these parameters reflect the substantially higher standard deviations. Thus these parameters do not appear to be as significant as they are plotted nearer the zero axis. However, allocations of one set per direction per day (C) appear to most closely reflect the behavior of these parameters as well.

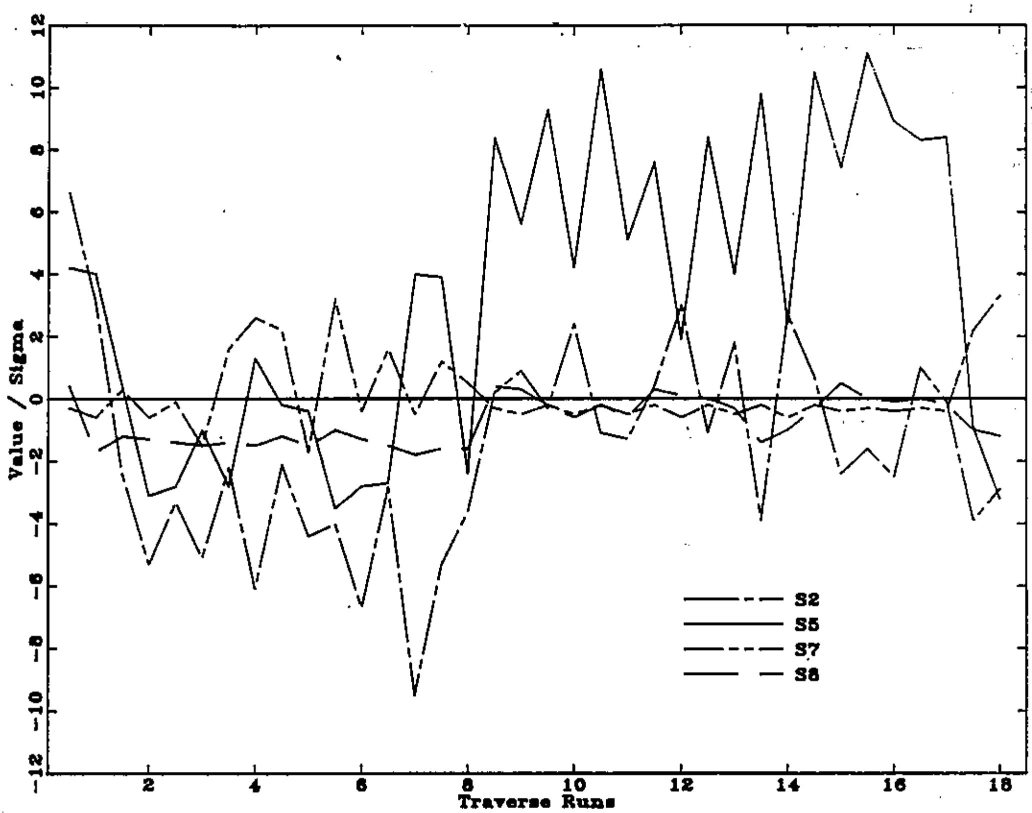


Figure 8. S2, S5, S7, S8, misalignment parameters.

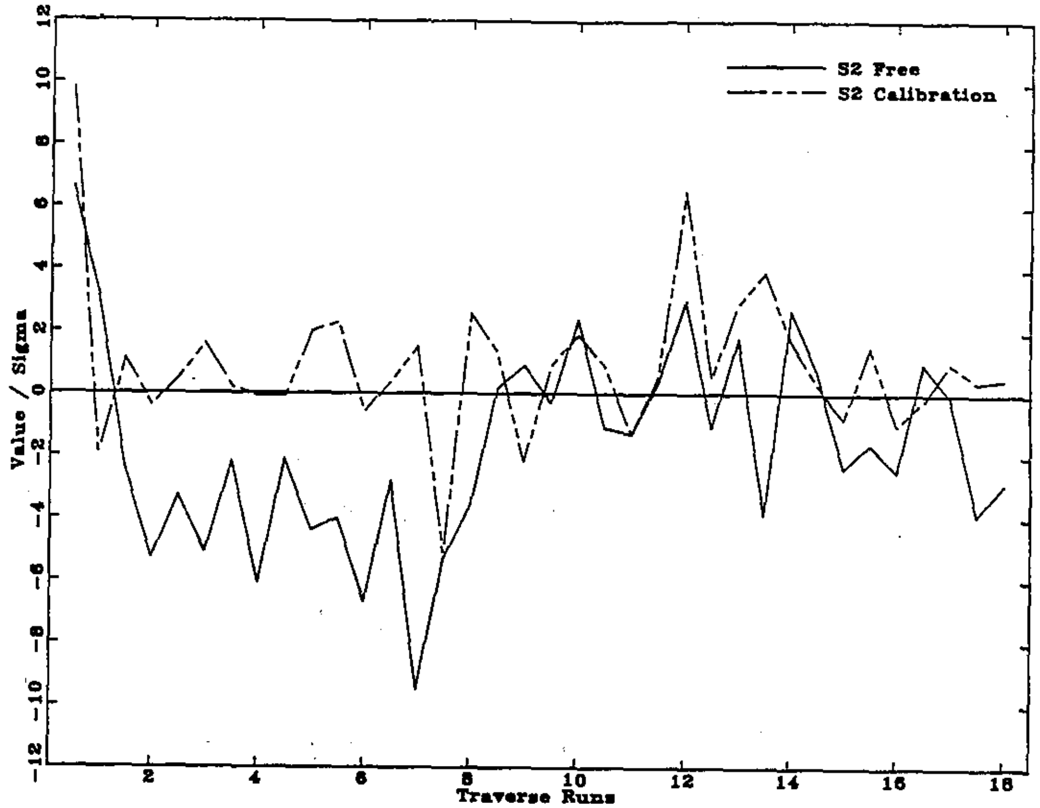


Figure 9. S2, misalignment parameters.

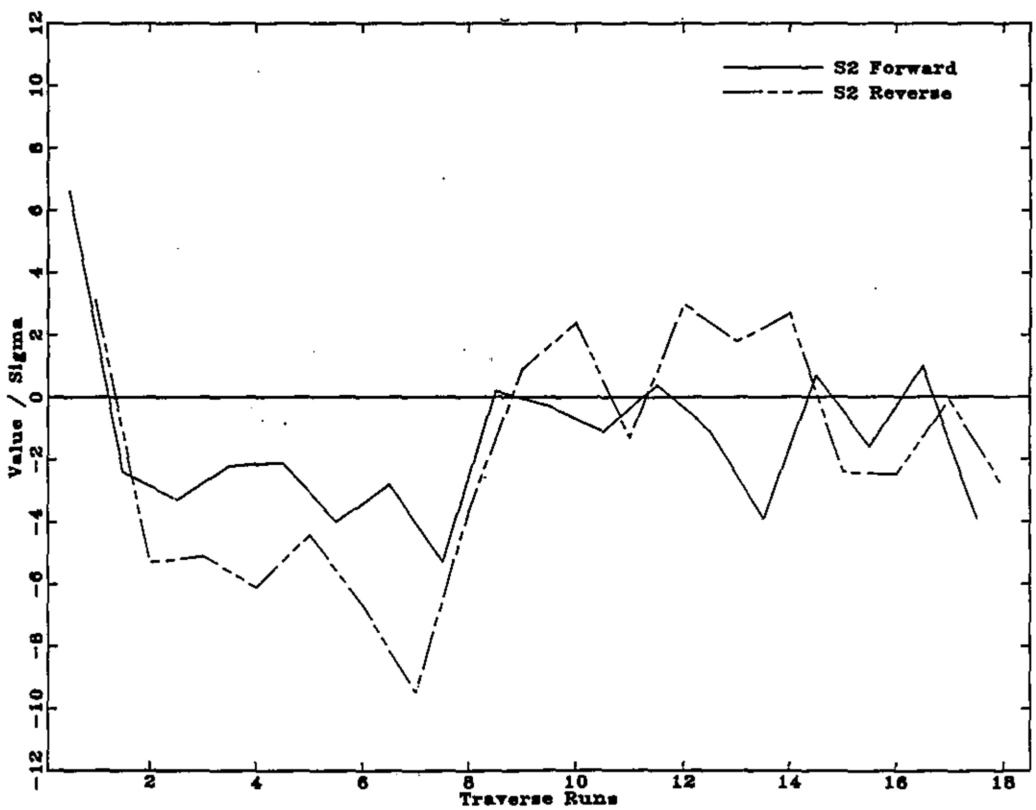


Figure 10. S2, misalignment parameters.

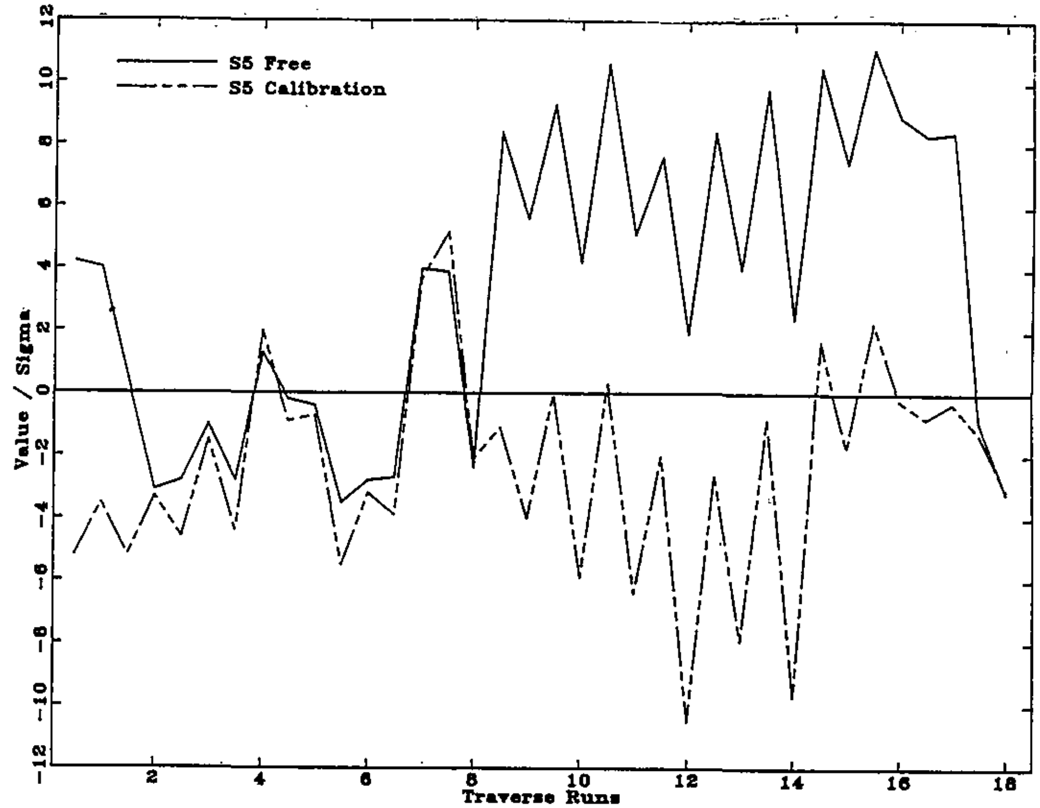


Figure 11. S5, misalignment parameters.

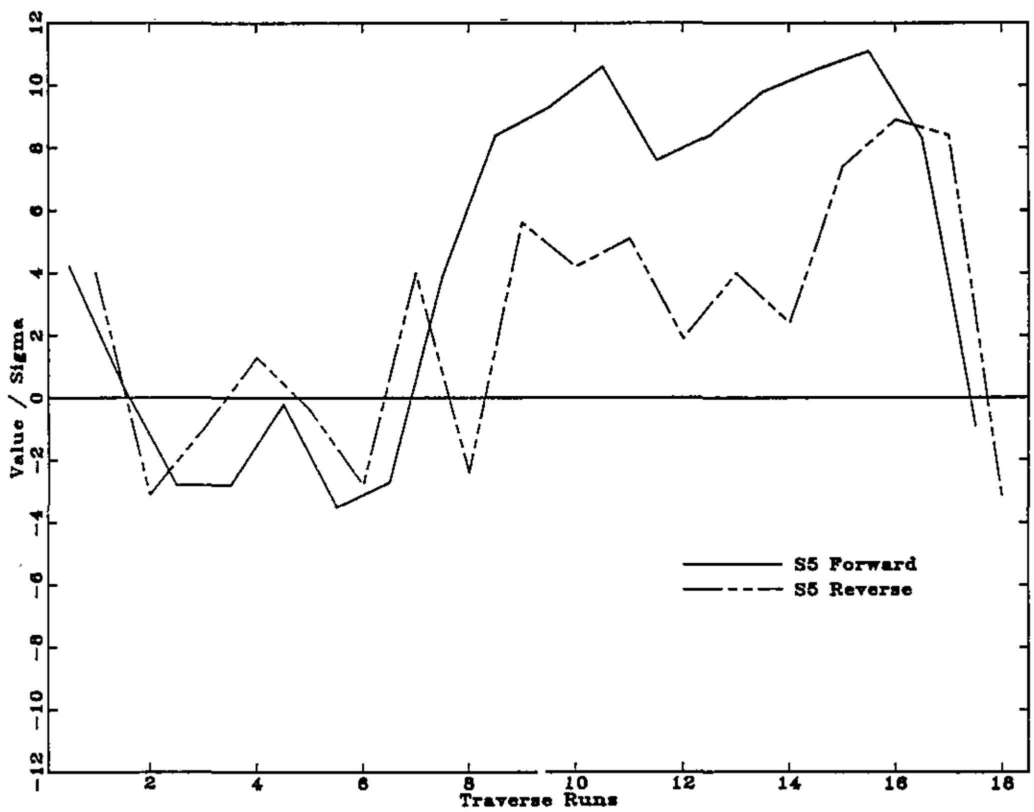


Figure 12. S5, misalignment parameters.

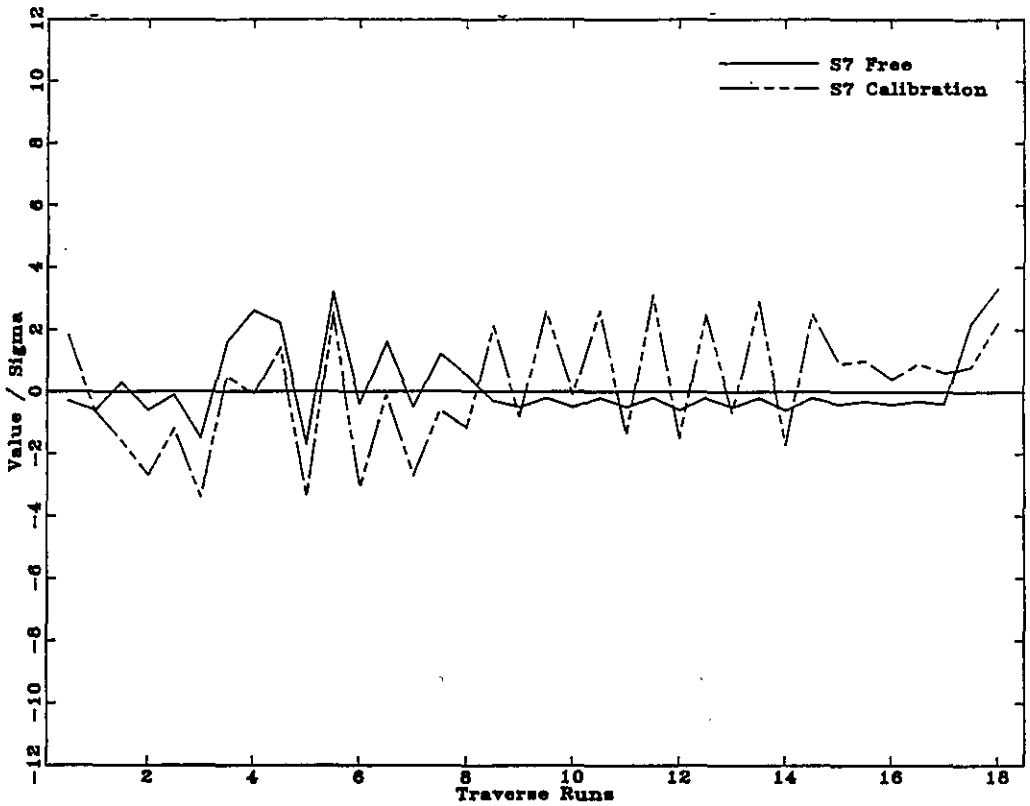


Figure 13. S7, misalignment parameters.

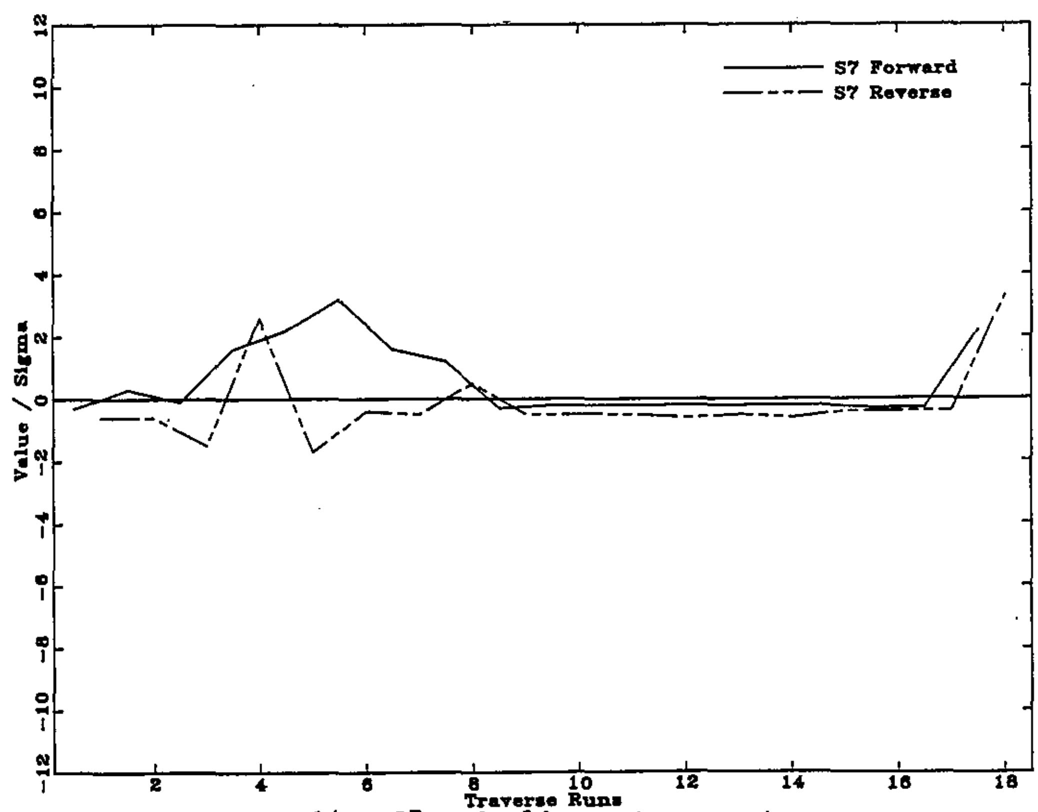


Figure 14. S7, misalignment parameters.

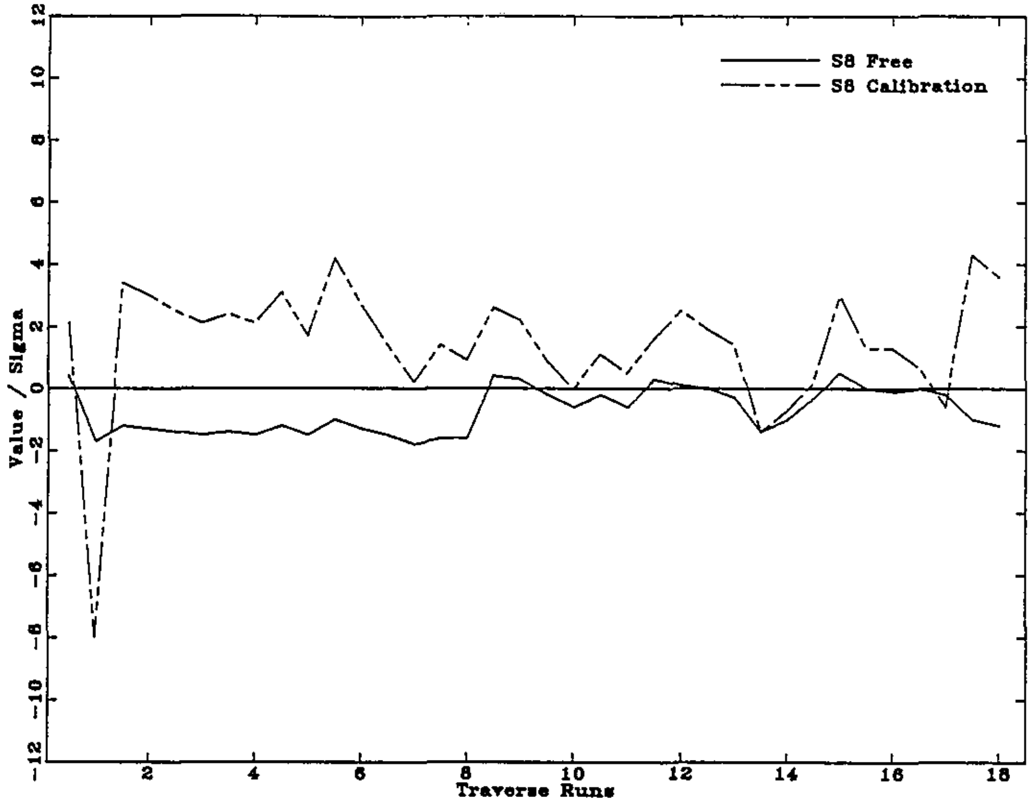


Figure 15. S8, misalignment parameters.

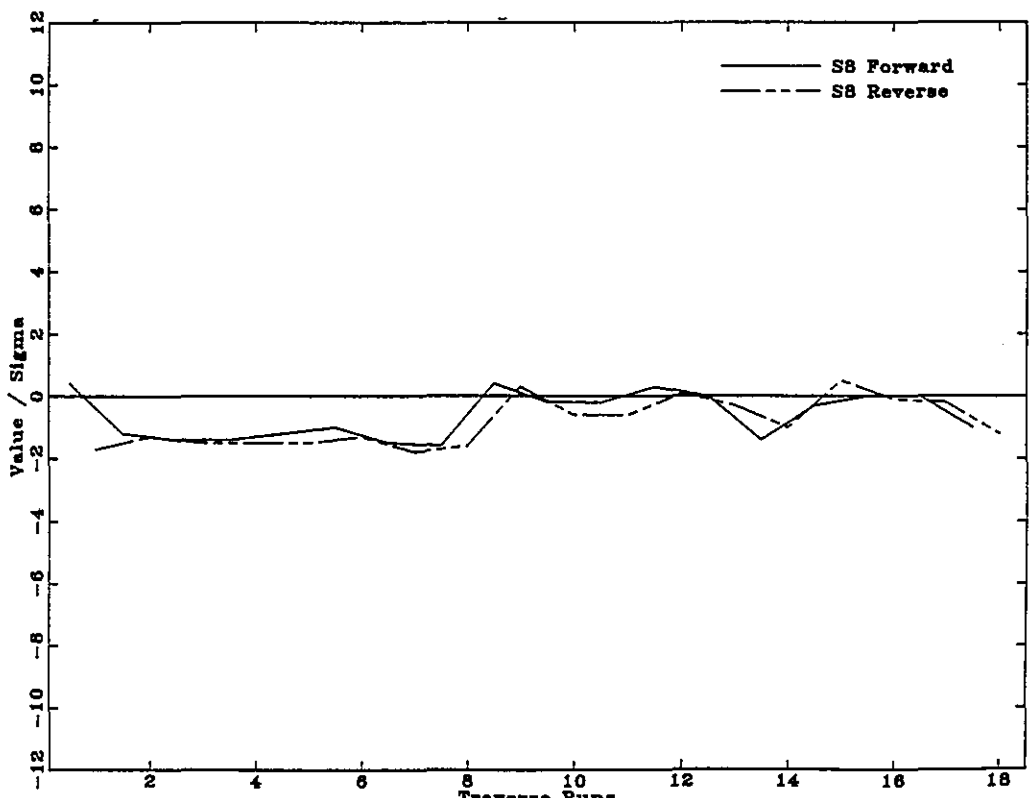


Figure 16. S8, misalignment parameters.

5.2.3 Drift Parameters

The behavior of the drift parameters appears more erratic than that of the scale or misalignment parameters. The predominant saw tooth patterns on Figure 17 clearly dictate allocations based on separating forward and reverse runs. This is particularly evident for S_6 and S_9. The variability of S_3 per run on Figure 19 would imply an allocation of per day (A) or per run (B). Even S_6 and S_9 display significant variations between each run. The lack of pattern therefore dictates an allocation of per leg (A) for the drift parameters.

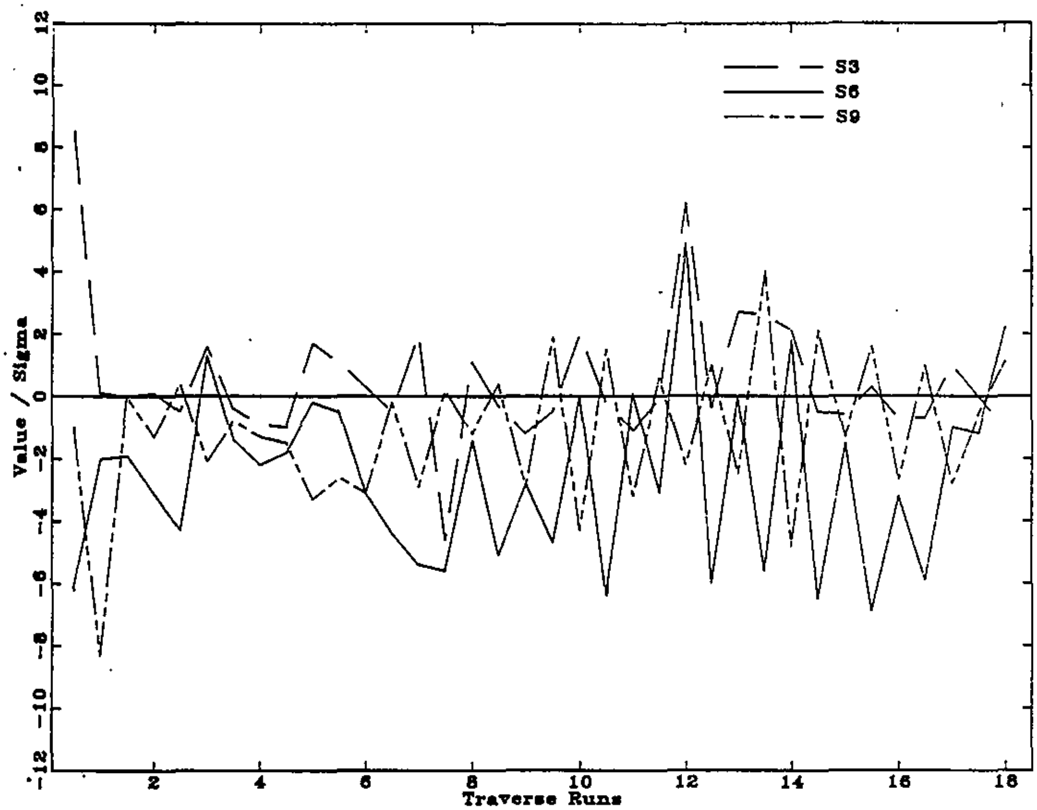


Figure 17. S3, S6, S9, drift parameters.

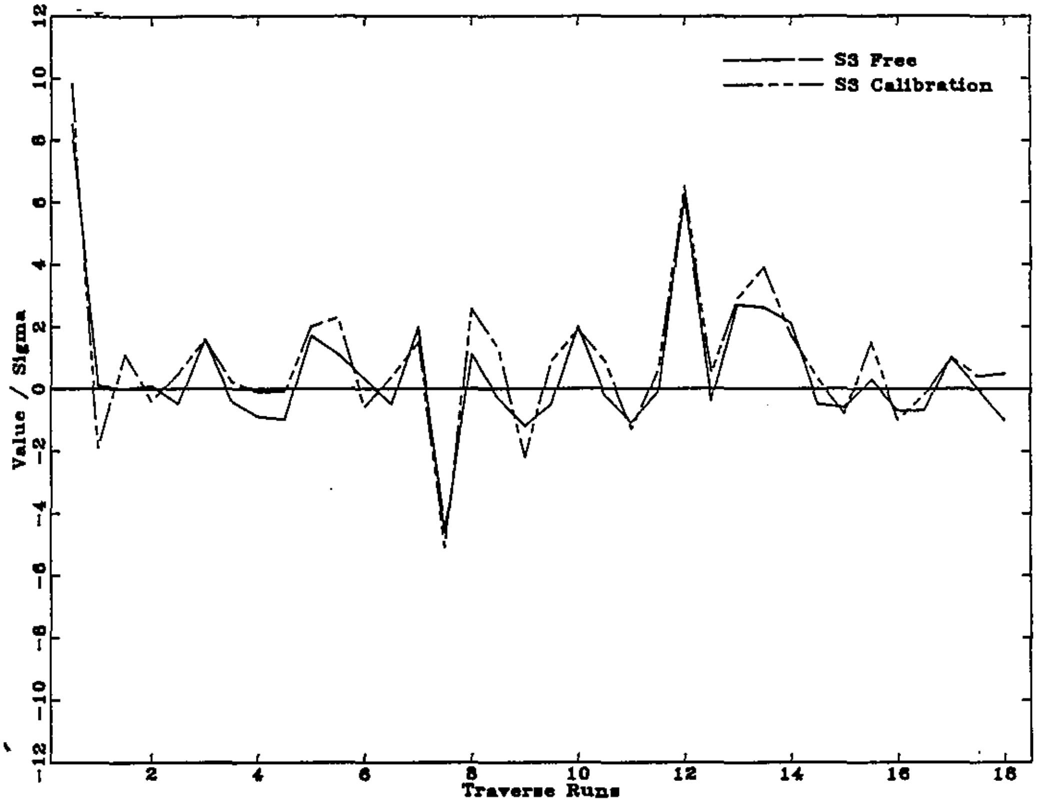


Figure 18. S3, drift parameters.

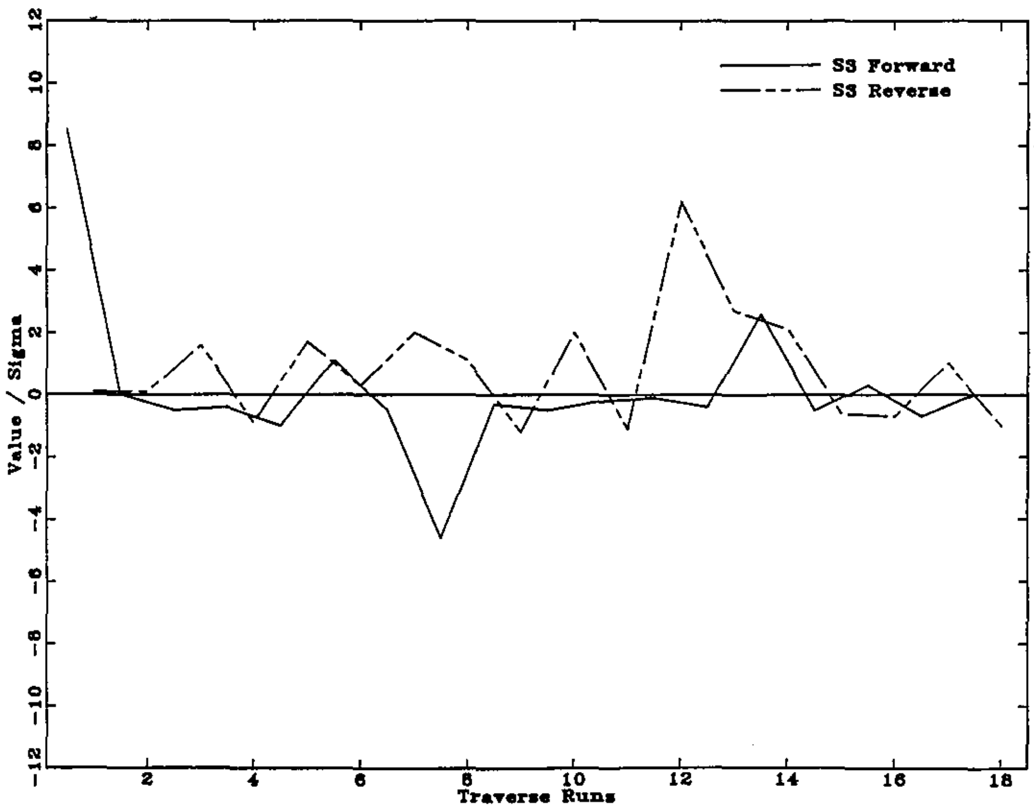


Figure 19. S3, drift parameters.

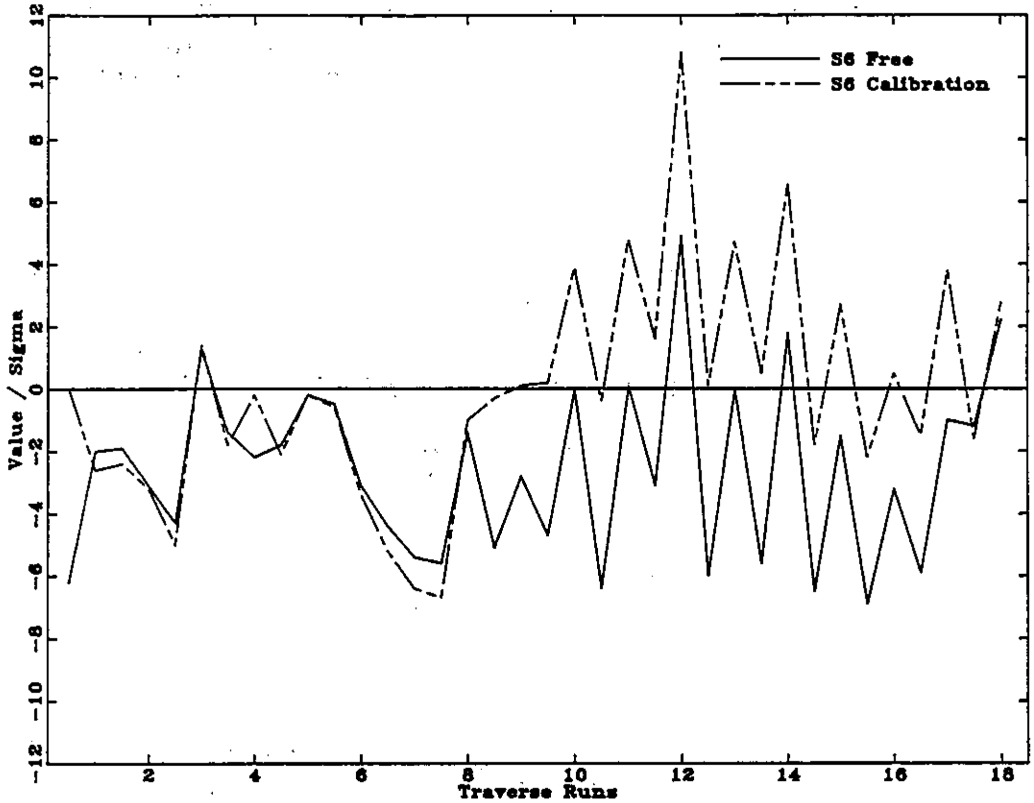


Figure 20. S6, drift parameters.

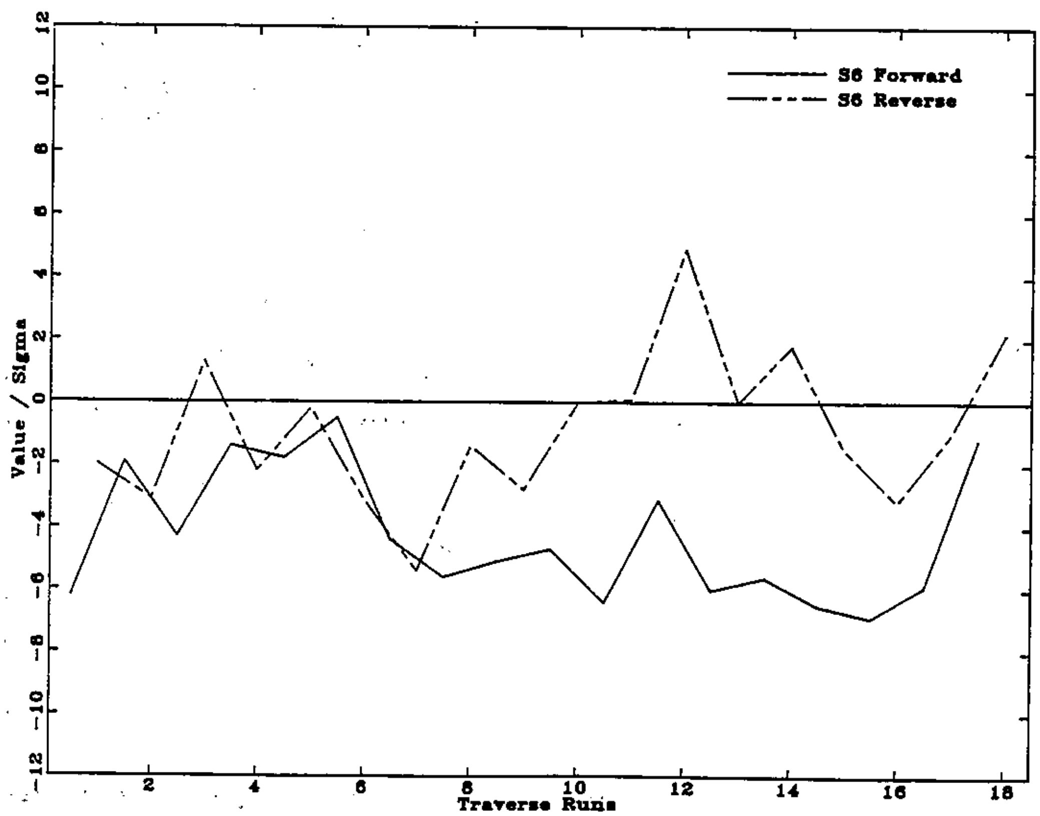


Figure 21. S6, drift parameters.

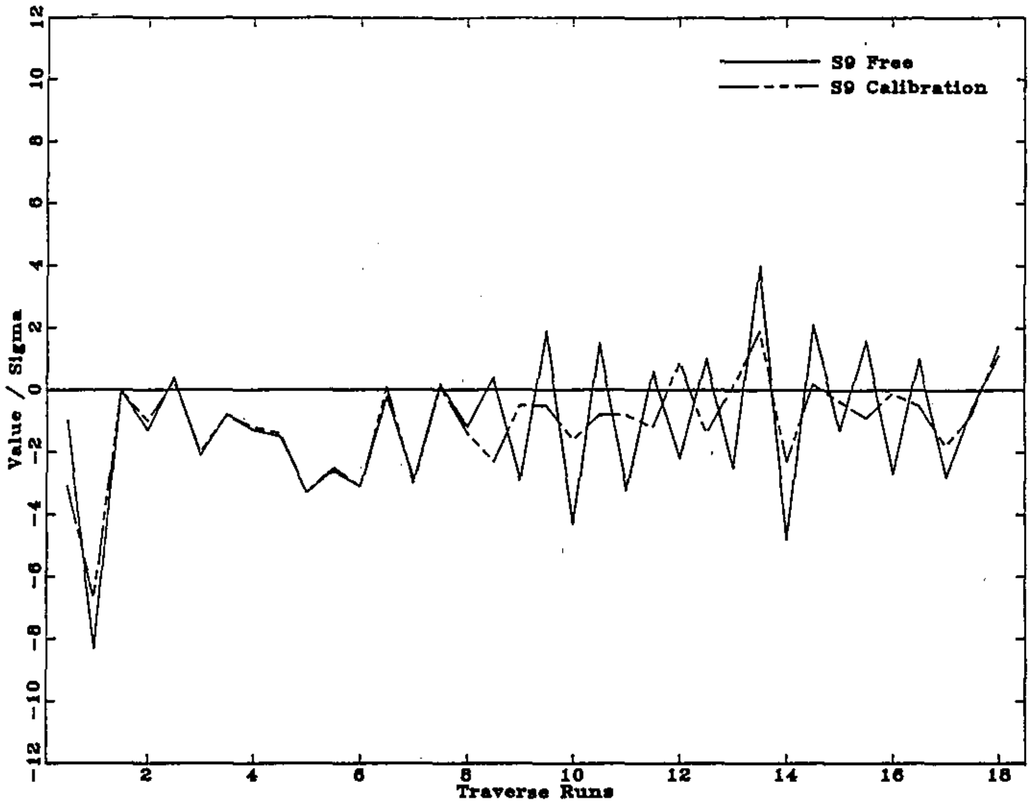


Figure 22. S9, drift parameters.

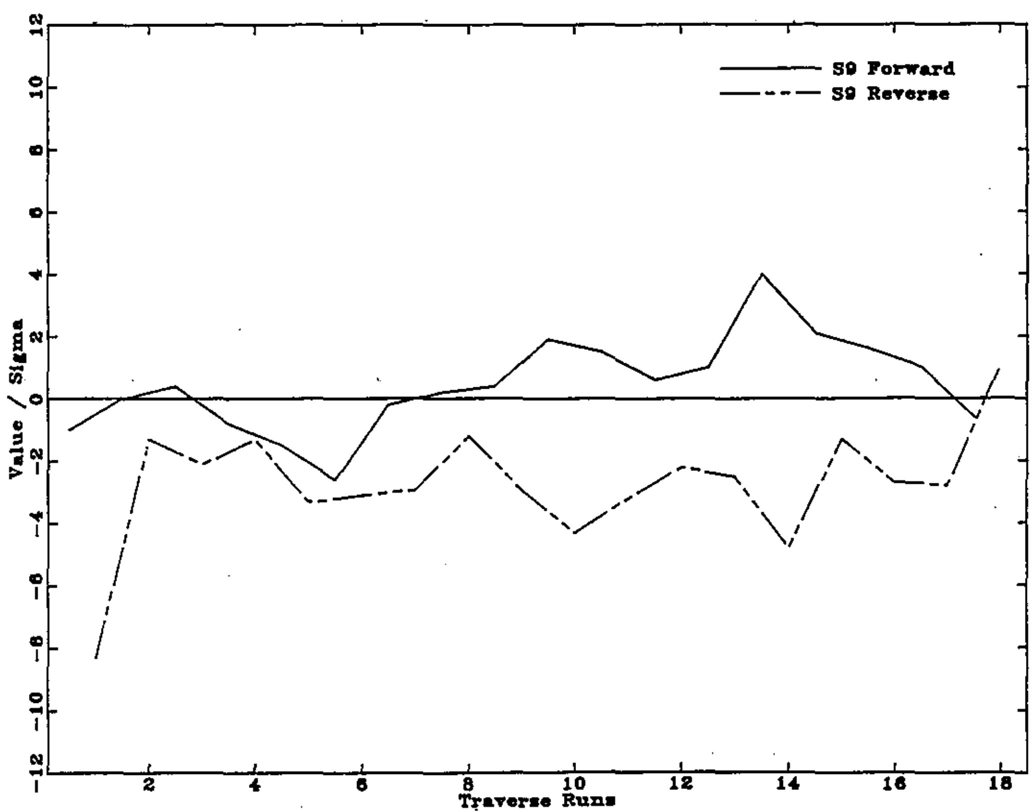


Figure 23. S9, drift parameters.

5.3 Length Relative Accuracies

In each of the 96 test adjustments, Length Relative Accuracies were computed both from linear error propagation and from length shifts. These accuracies were obtained for all lines in the adjustment which connected previous control points. In all adjustments, the minimum Length Relative Accuracy computed from error propagation was better than that computed from length shifts, often by an order of magnitude. This is significant for several reasons:

1. The minimum Length Relative Accuracy is the standard by which horizontal survey work is classified in the United States.
2. There is much uncertainty in the weighting schemes used in adjusting inertial observations and therefore in any propagated Length Relative Accuracy.
3. This particular test involved length shifts computed from coordinates of known accuracy.

These minimum Length Relative Accuracies are extremely important as an evaluative tool for any surveying method, particularly those involving new techniques. Many claims have been made about the capability of inertial systems without carefully identifying the basis for the claims. The minimum value obtained for Length Relative Accuracy in a project must be used for proper project classification.

The distinction between quantities derived through error propagation and those derived from comparison with known quantities is extremely important in new technology. Modifying some a priori weights can greatly improve the error propagation results. These results are only as reliable as the initial weight estimates which must be validated against a known standard.

This test used the results from the least squares adjustment including the Transcontinental Traverse observations producing coordinates of known relative accuracy. Thus the Length Relative Accuracies computed from two dimensional length shifts can be used to reliably compare the adjustment results and distortion remaining after adjustment. The values for minimum Length Relative Accuracy from the 96 test adjustments ranged from 1 : 10,070 to 1 : 28,258. Rather than list all 96 values, only the top 15 are given in Table 3. In addition to the relative accuracy, the actual length shift is given in meters. To aid in evaluating the significance of the shift, the a posteriori value for the 3-sigma level is also given. Note that all shifts exceed the 3-sigma level, often by a factor of 2 or more.

The minimum Length Relative Accuracy is the basis for classification of horizontal control work and measures the worst case of accurate system performance. The system performance here refers to the modeling in the post-mission adjustment as well as that of the hardware and filter algorithms. Poor system performance could be attributable to any element of this system. For this segment, proper performance of the earlier elements of the system is assumed and the length relative accuracies are then used to compare the various observation model parameter allocations.

Table 3 lists those adjustments which had the largest minimum Length Relative Accuracy. Several items are evident from the listing.

Table 3. Minimum Length Relative Accuracies Computed From Length Shifts

Adjustment	Relative Accuracy	Shift (meters)	$3\hat{\sigma}$ (meters)
DFD	1 : 28,258	0.265	0.150
CFC	1 : 28,250	0.265	0.135
BFB	1 : 28,243	0.265	0.150
AFA	1 : 28,079	0.267	0.132
FFA	1 : 27,748	0.340	0.141
FFC	1 : 27,717	0.340	0.143
FFB	1 : 27,314	0.345	0.165
FFD	1 : 27,276	0.346	0.167
FFE	1 : 26,474	0.356	0.192
BDB	1 : 26,026	0.363	0.143
DDB	1 : 25,973	0.363	0.143
BDD	1 : 25,972	0.363	0.145
DBB	1 : 25,931	0.364	0.143
BBB	1 : 25,924	0.364	0.144
DDD	1 : 25,894	0.364	0.145

First, an allocation of one set for all days (F) appears for Group 2 in the top nine entries. Thus modeling the misalignment effects with one set of parameters has produced the "best" results by this measurement criteria.

The second obvious pattern in the listing in Table 3 is the similar allocation of Group 1 and Group 3 for the top four entries. Of these top entries, allocations of one set per day for Group 1 and Group 3 lead the list. The allocation of a set per day is very significant in that this indicates the scale parameters and the drift parameters behave best when allocated for the entire day.

One must be careful when viewing this coupling between Group 1 and Group 3 in that the adjustments done were limited to those in which no more than two different allocation schemes were used. There could be improvement when a third allocation is also allowed. Additionally, it must be recognized that Length Relative Accuracies, when applied to the traverse configuration of this test, are really only evaluating scale accuracies. A better understanding of model performance with respect to accuracy can be obtained by examining the coordinate differences themselves.

5.4 Coordinate Accuracy

The test data set in this study provides an excellent opportunity to validate certain aspects of system parameter performance because of the a priori knowledge of the geodetic coordinates. These coordinates permit the evaluation of the mathematical model by its ability to replicate the coordinates. Here again, graphical analysis will be used to discern patterns and make statements concerning model parameter allocations.

In each of the following graphs, the coordinate differences resulting from six representative adjustments have been plotted against changes in latitude or changes in longitude. The six adjustments each involved the same allocations for all nine system parameters, from AAA to FFF. By comparing coordinate differences vs. change in latitude or longitude, conclusions can be drawn about model performance in scale or misalignment.

From a practical standpoint, discerning six different line types on one graph became difficult and so results from two of the parameter allocations are represented by point symbols. Also, to help in deciphering the information displayed, each set of three graphs includes a major graph with the six allocations and two auxiliary graphs each displaying three allocations.

The first set of graphs, Figures 24-26, depicts latitude differences vs. latitude. These graphs indicate model performance with respect to latitude scale. Close comparison will reveal that, in general, allocations of B or D provide latitude differences closer to the zero axis. The same holds true for the next set of graphs that indicates longitude scale. These graphs support the behavior noted earlier for the scale parameters.

The next two sets of graphs give indication of model performance with respect to misalignment effects. The first of these sets, Figures 30-32, indicates similar performance for all six allocations. Figures 33-35, however, indicate the better results are obtained with allocations of A or C. These figures demonstrate that the allocations of E produced significant distortion in the coordinates. Based on this criterion, therefore, an allocation per leg (A) or per direction for each day (C) would be more desirable for the misalignment parameters. The selection of per direction for each day (C) results in fewer parameters and thus is preferred. This conclusion is supported by the parameter behavior noted in the earlier graphical analysis, but is not supported by the examination based on length relative accuracies.

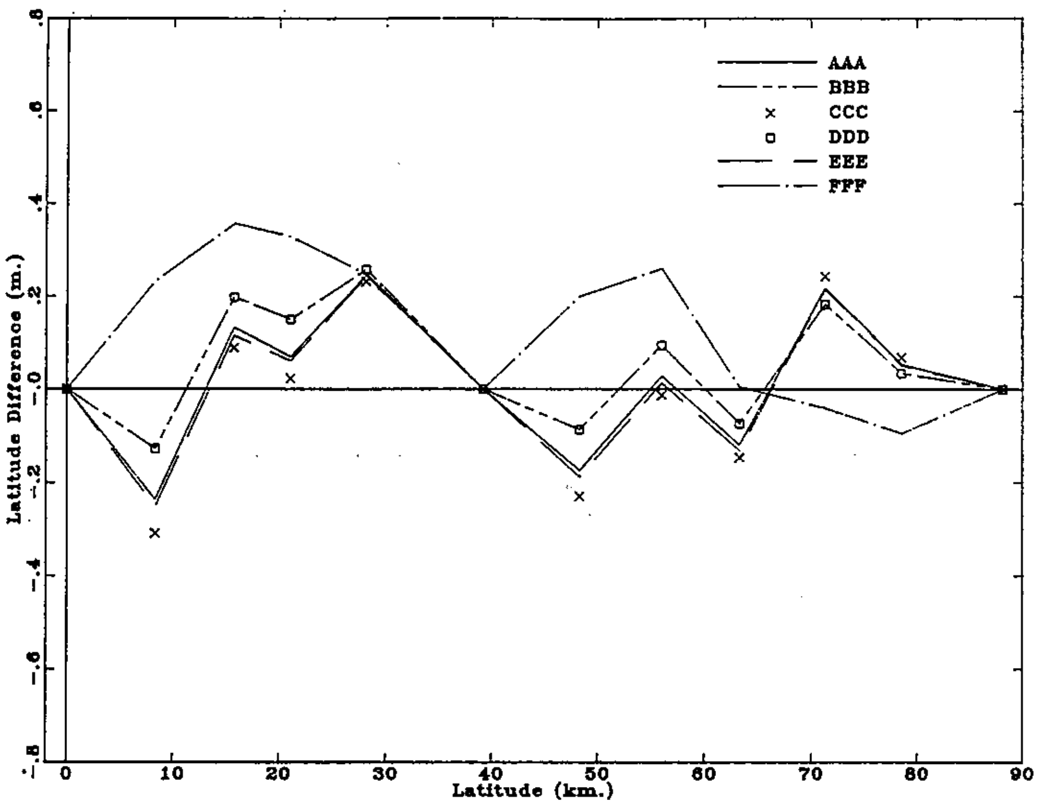


Figure 24. Latitude differences of ISS and TCT, south-north direction.

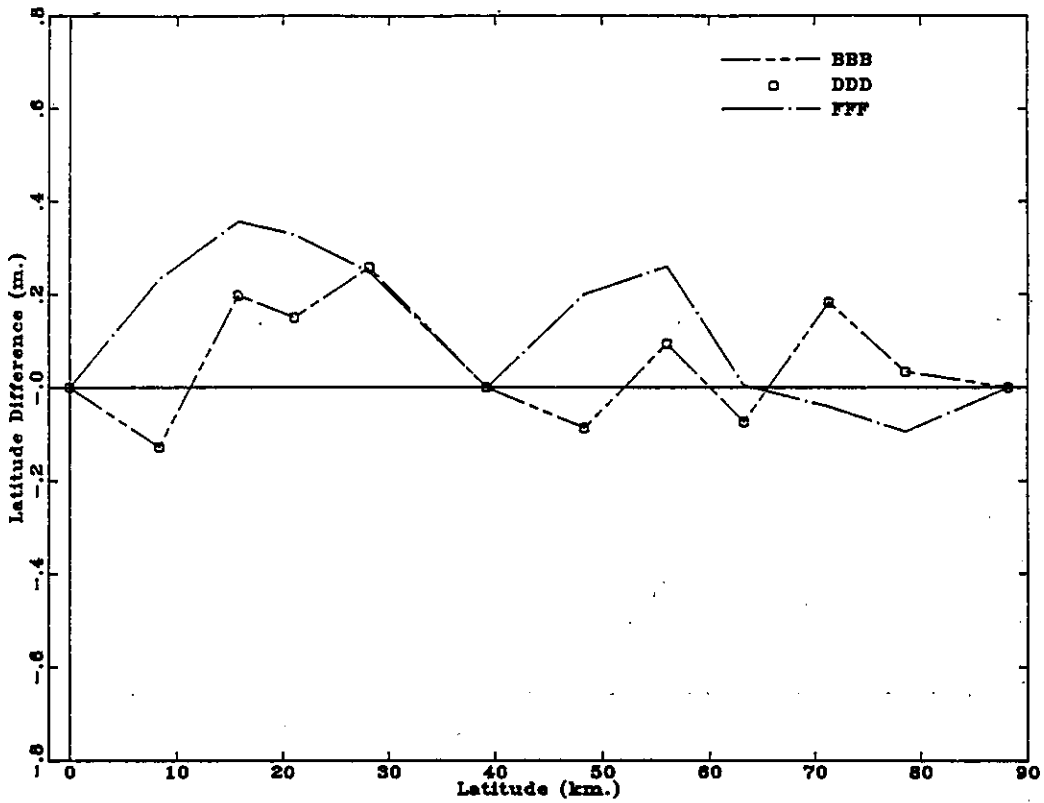


Figure 25. Latitude differences of ISS and TCT, south-north direction.

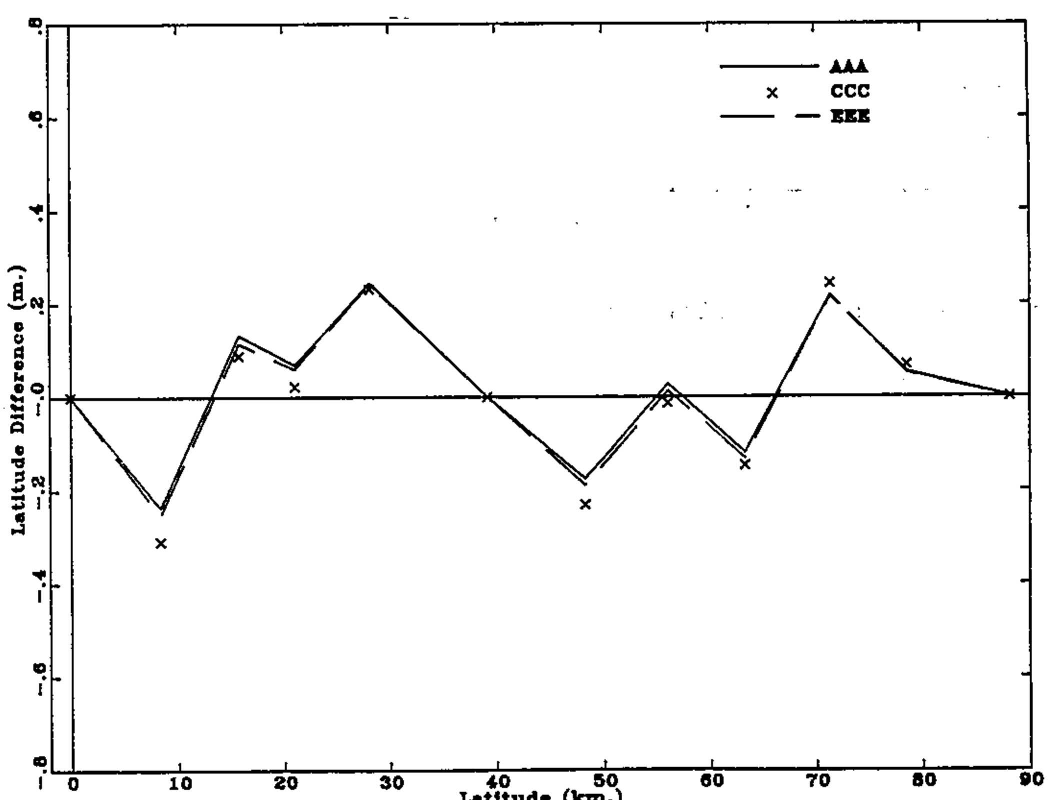


Figure 26. Latitude differences of ISS and TCT, south-north direction.

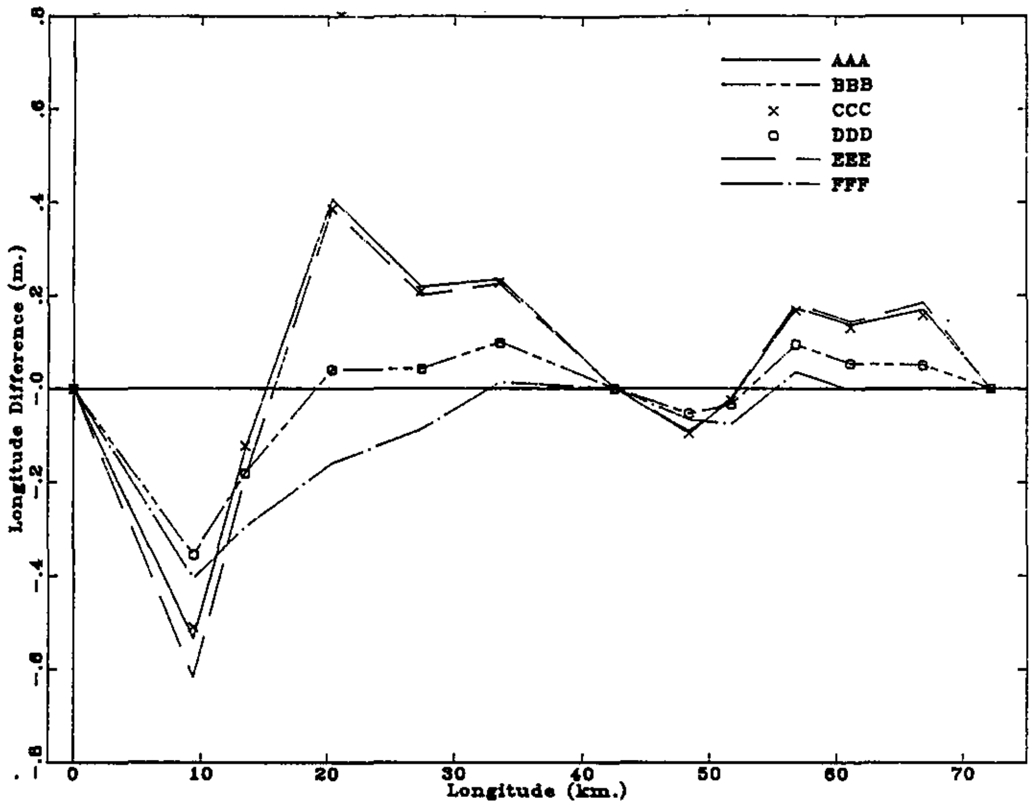


Figure 27. Longitude differences of ISS and TCT, west-east direction.

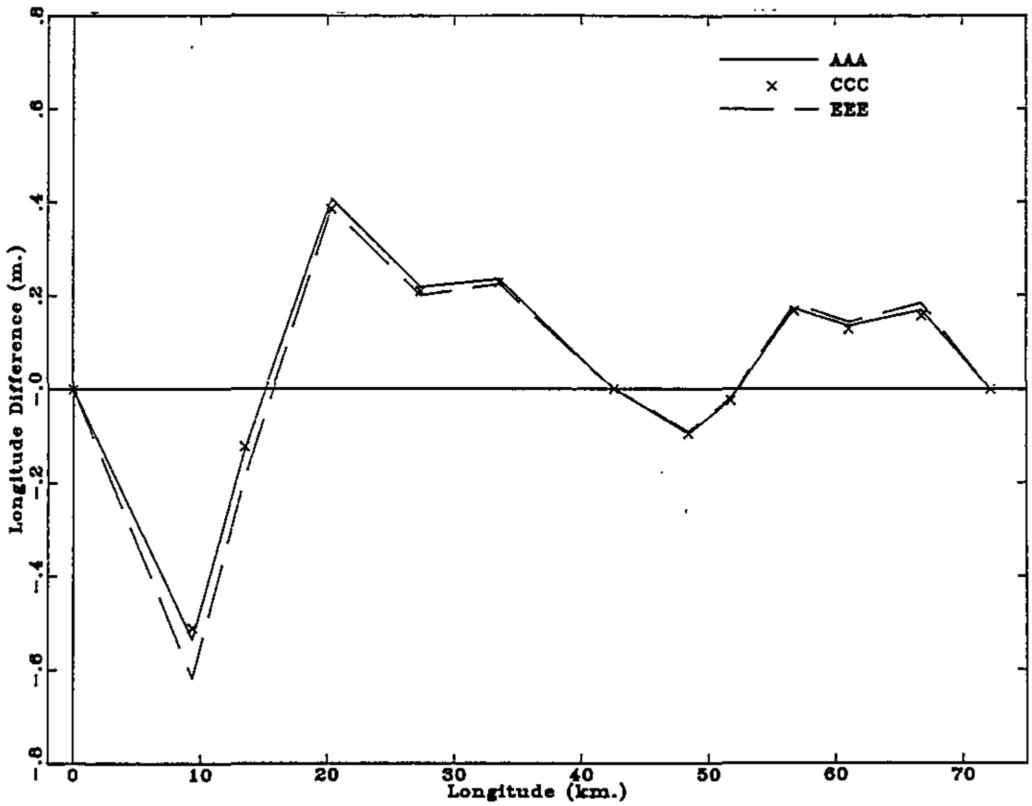


Figure 28. Longitude differences of ISS and TCT, west-east direction.

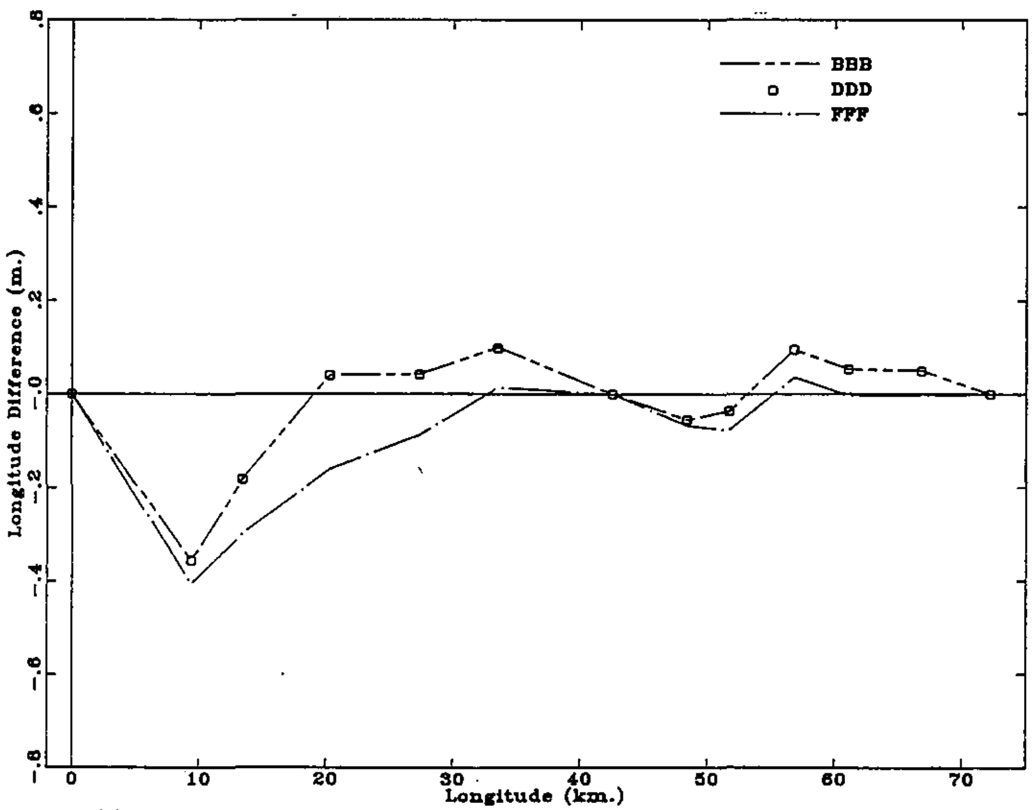


Figure 29. Longitude differences of ISS and TCT, west-east direction.

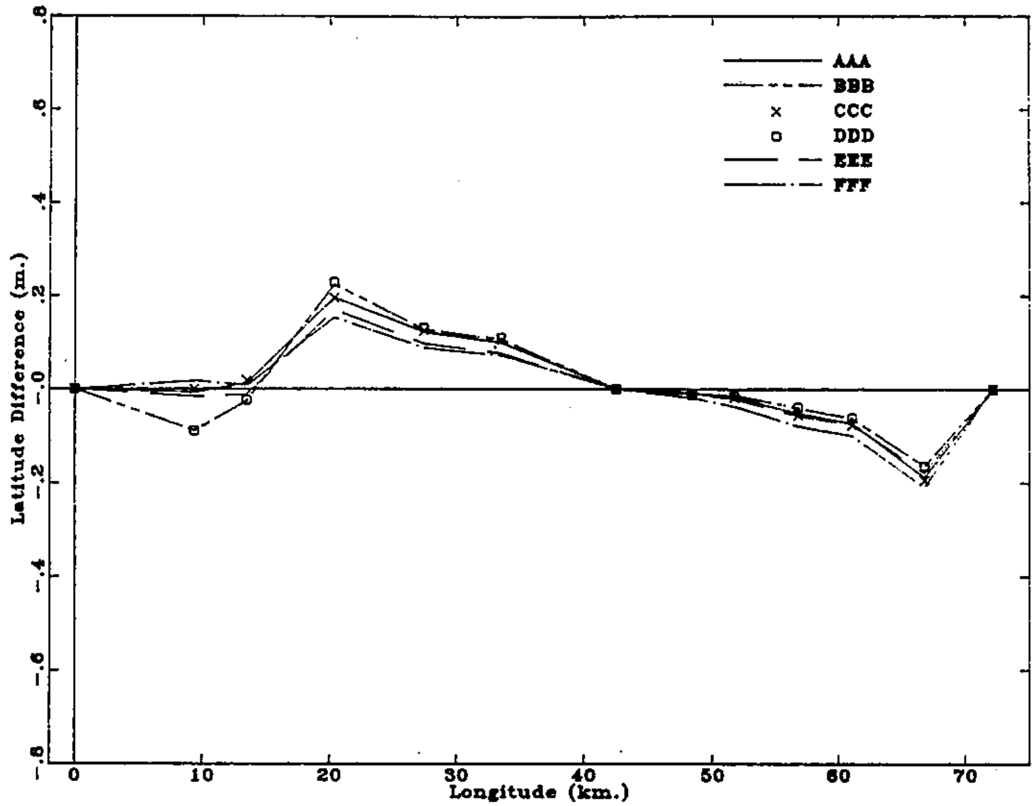


Figure 30. Latitude differences of ISS and TCT, west-east direction.

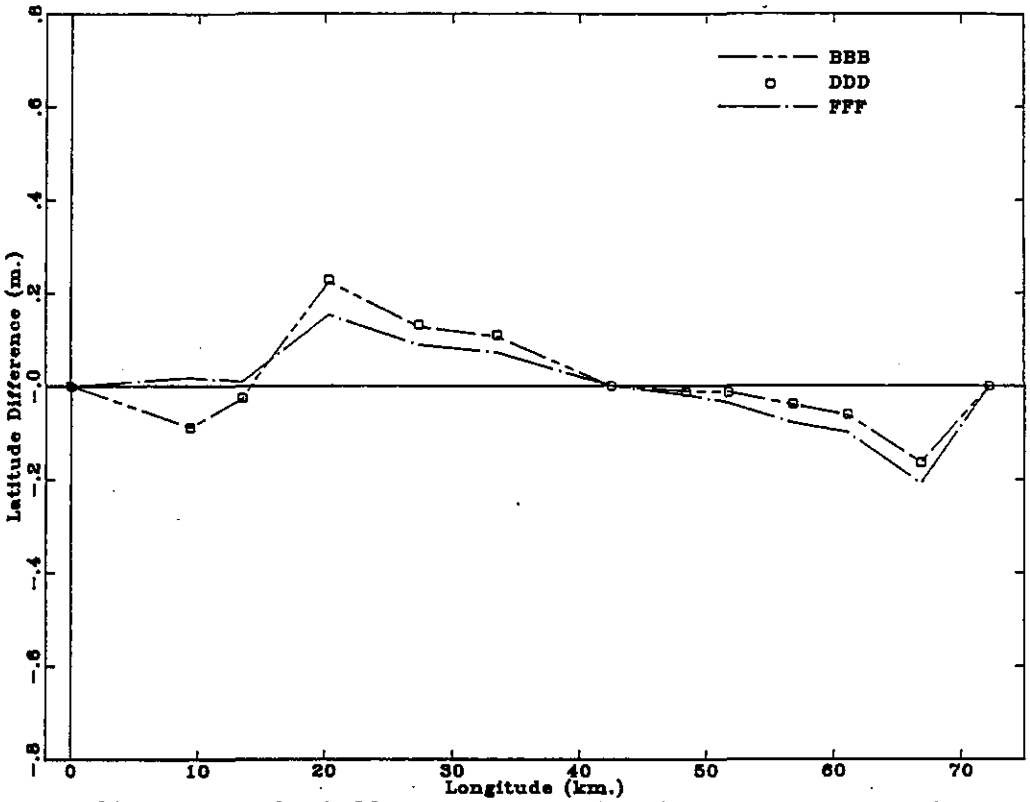


Figure 31. Latitude differences of ISS and TCT, west-east direction.

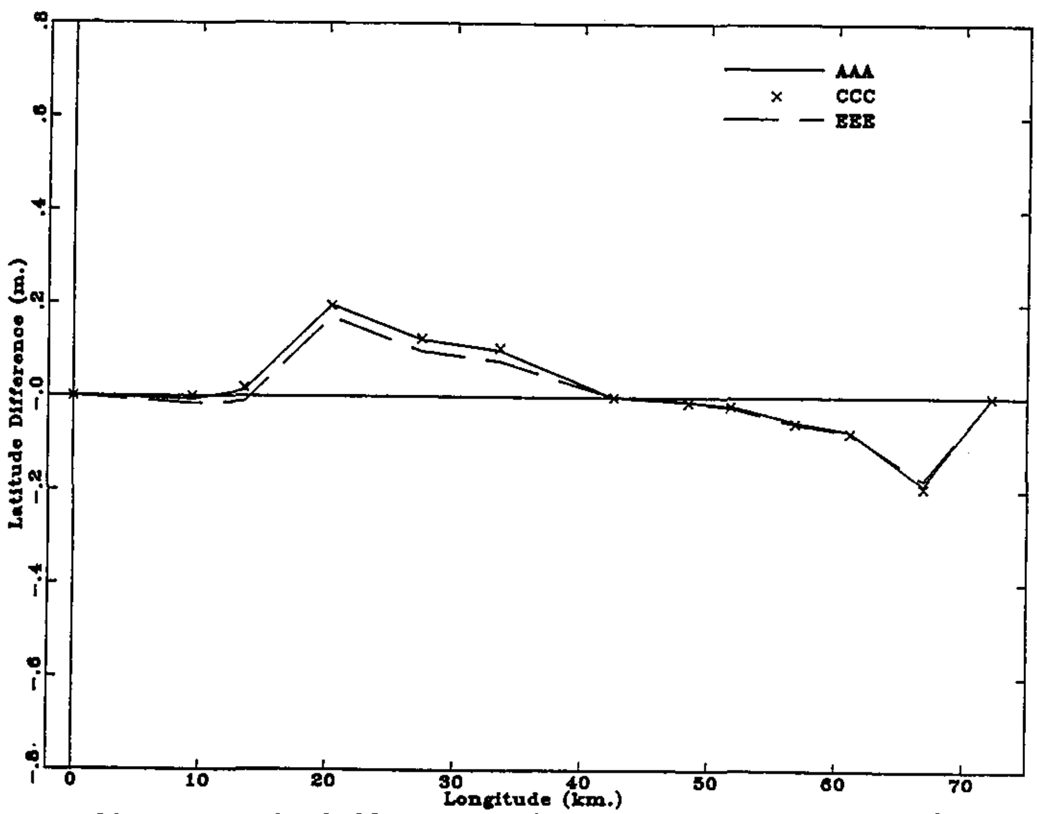


Figure 32. Latitude differences of ISS and TCT, west-east direction.

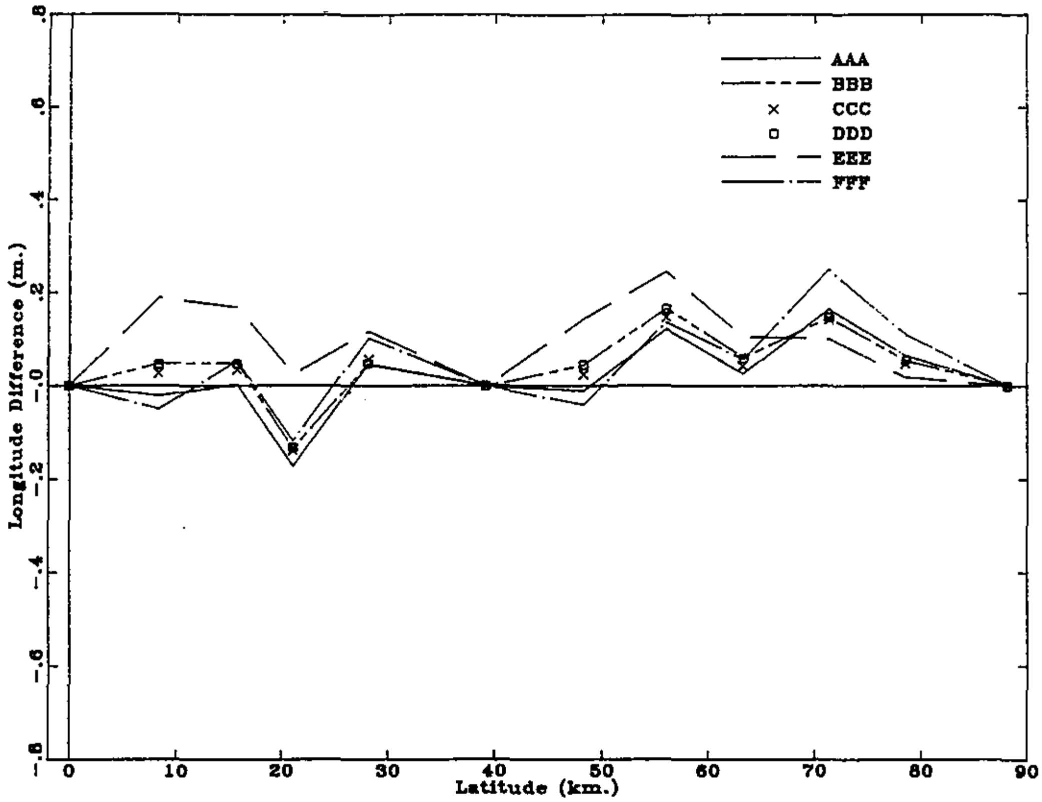


Figure 33. Longitude differences of ISS and TCT, south-north direction.

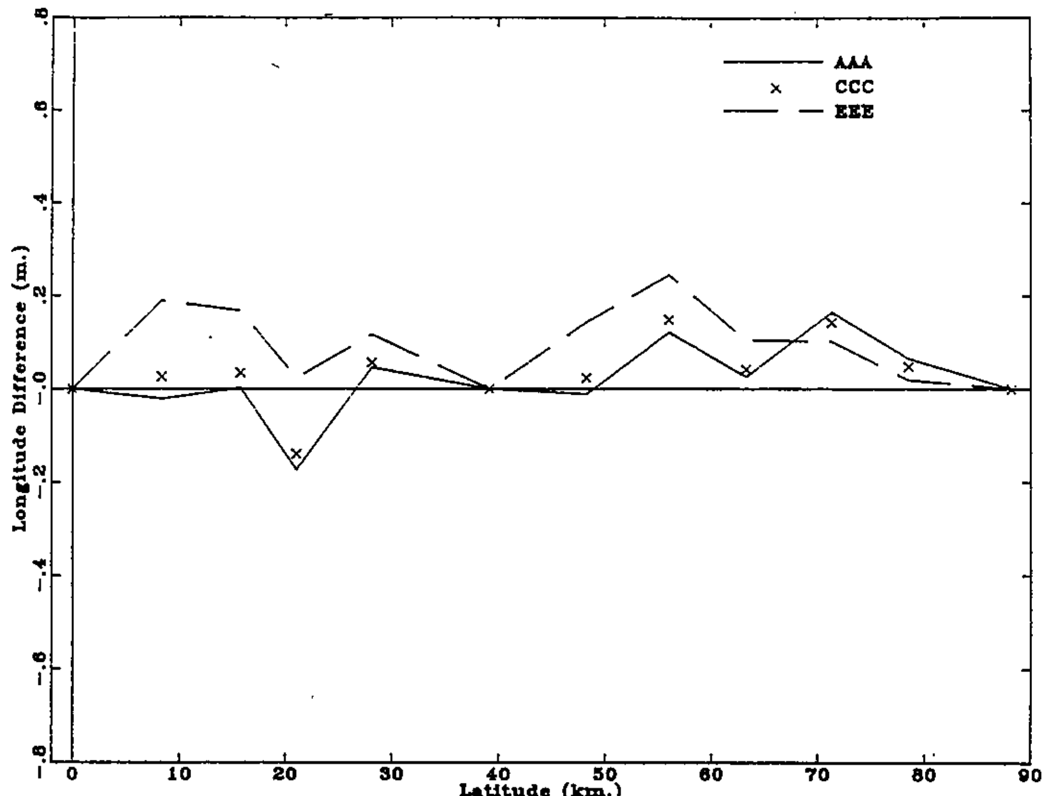


Figure 34. Longitude differences of ISS and TCT, south-north direction.

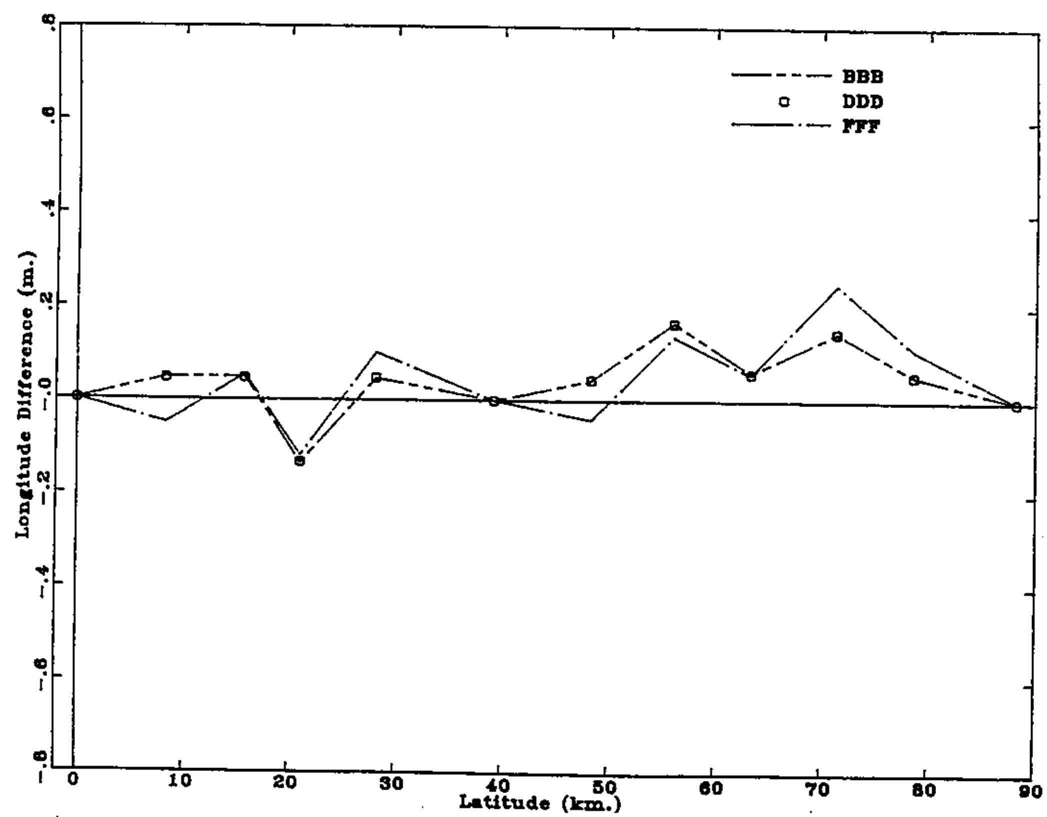


Figure 35. Longitude differences of ISS and TCT, south-north direction.

6. CONCLUSIONS AND RECOMMENDATIONS

In this study, an initial mathematical model was adopted which attempted to model the behavior of a Litton Autosurveyor inertial surveying system. This modeling was intended to capture the systematic elements which remain after the application of the on-board filter and therefore affect the accuracy of the results. The mathematical model selected was based on that used by Hannah (1982) but was modified to allow for examination of behavior of the model parameters. The results of test adjustments with varying parameter allocations have been analyzed and permit some conclusions to be drawn regarding model parameter behavior. Additionally, the approach of this study is applicable to continuing research into inertial surveying models and therefore recommendations for future study are appropriate as well.

6.1 Mathematical Model

The mathematical model parameters were examined to determine how they should be applied to best capture the systematic effects present in the inertial surveying data. This examination involved several different analysis techniques which could be applied because of the unique situation provided by the test site location. In addition to analysis based on the internal precision of the test adjustments, the highly accurate TCT coordinates for the test stations permitted analysis based on the ability of the model to replicate these "known" coordinates. The many test adjustments have been evaluated and some conclusions can now be made concerning the usefulness of the resulting model.

6.1.1 Model Parameter Allocation

The model parameters were grouped in this study by the general category of errors which they were intended to model. Of the nine system parameters, there were two for scale errors, four for misalignment errors, and three for drift errors. The test adjustments varied the allocations of these groups of system parameters and the results were analyzed.

First, the analysis for the scale parameters can be reviewed. The F-tests on the parameter constraints demonstrated that allocations per run (B) and allocations per day (D) are identical as were allocations per leg (A) and allocations per direction for each day (C). Next the graphs of system parameter behavior indicated a slight preference for allocations per day (D). An examination of the Length Relative Accuracies also indicated a slight preference for allocations per day (D), significant in that Length Relative Accuracies for such a traverse configuration primarily evaluate scale. Finally, the graphical analysis of the coordinate differences from the test adjustments also demonstrated a preference for allocations per day (D). Thus, for the scale parameters, it is reasonable to select an allocation per day (D) as preferred.

The F-tests on the misalignment parameters also indicated that allocations per run (B) and allocations per day (D) were roughly identical. In the graphical analysis of system parameter behavior, the patterns indicated significant changes with change of direction, particularly with the S_5 parameter. There were clear indications that the parameters behaved differently on the forward and reverse runs as well. This variability lead to a rejection of the support for allocating the misalignment parameters for all days (F) as was indicated by the Length Relative Accuracies. Again, it must be emphasized that the Length Relative Accuracies are more reflective of scale problems with this traverse configuration. The analysis of misalignment effects based on coordinate differences further refined the selection when it was noted that better results were obtained with the misalignment parameters allocated per direction per day (C) and therefore is most reasonable.

The drift parameters exhibited the most erratic behavior with distinct differences between forward and reverse runs. The primary bases for assignment of drift parameters are the graphs of parameter behavior, Figures 17-23. Due to the lack of information from the F-tests or any other conclusive graph pattern, an allocation of per leg (A) seems most appropriate for the drift parameters.

Thus to summarize the selections made:

Scale	S_1 S_4	D	Allocation per day
Misalignment	S_2 S_5 S_7 S_8	C	Allocation per direction per day
Drift	S_3 S_6 S_9	A	Allocation per leg

Using this selection of parameter allocations, additional adjustments were made with the same test data set. From the adjustment in which all known preliminary coordinates were held fixed, representative values can be obtained for the nine system parameters. Some values and their respective standard deviations are as follows:

Sn	Value	$\hat{\sigma}$	
1	-4.72×10^{-5}	2.36×10^{-5}	
2	4.31×10^{-5}	6.46×10^{-6}	
3	9.96×10^{-14}	9.84×10^{-15}	radians/sec^2
4	-1.04×10^{-5}	6.16×10^{-6}	
5	-1.81×10^{-4}	3.61×10^{-5}	
6	6.93×10^{-15}	1.05×10^{-14}	radians/sec^2
7	$3.94 \times 10^{+2}$	$2.13 \times 10^{+2}$	
8	$3.20 \times 10^{+2}$	$1.52 \times 10^{+2}$	
9	-2.27×10^{-3}	7.21×10^{-4}	radians/sec

Further studies may also find the system parameter correlations useful. Though beyond the scope of this study, the correlation coefficients of the adjusted parameters are given in Appendix 4.

Finally, as an evaluation of the selected model, adjustments were also made of the test data set using another model, the Gregerson twelve parameter model. This model is the basis of another adjustment program which was developed at NOAA's National Geodetic Survey. The coordinate differences produced in minimally constrained adjustments using both models are shown in Figures 36-39. These graphs indicate that the selected model (DCA) performs in a similar manner for latitude scale, and somewhat worse for longitude scale. However, both the final two

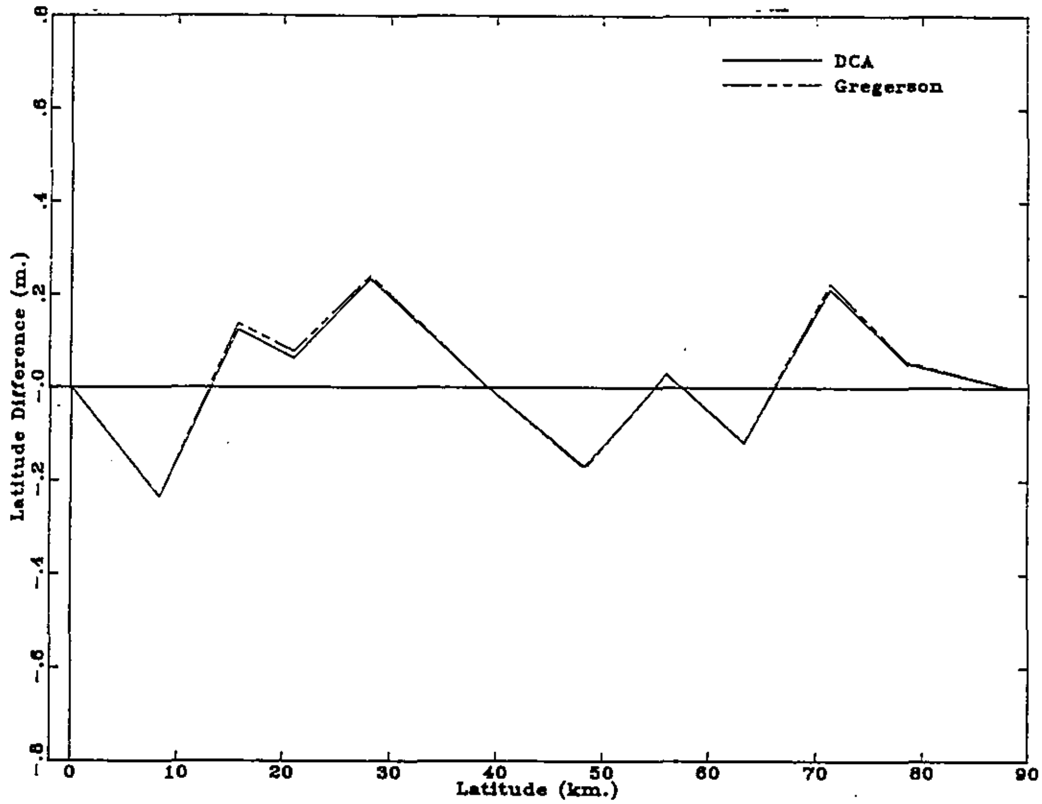


Figure 36. Latitude differences of ISS and TCT, south-north direction.

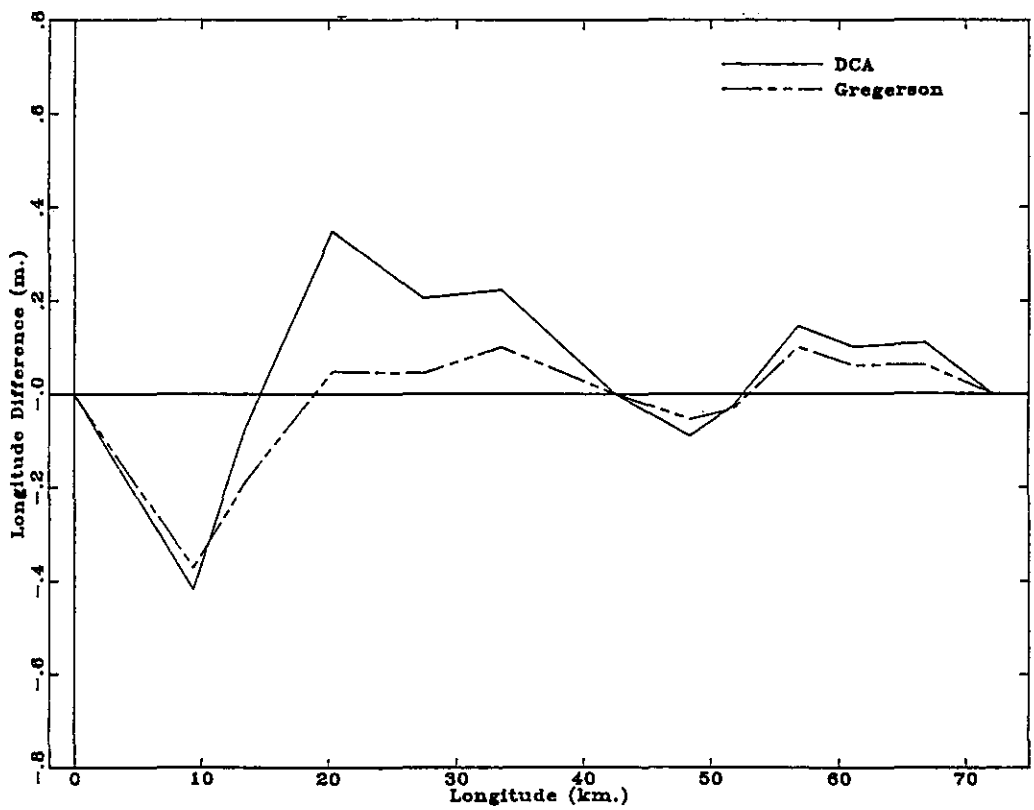


Figure 37. Longitude differences of ISS and TCT, west-east direction.

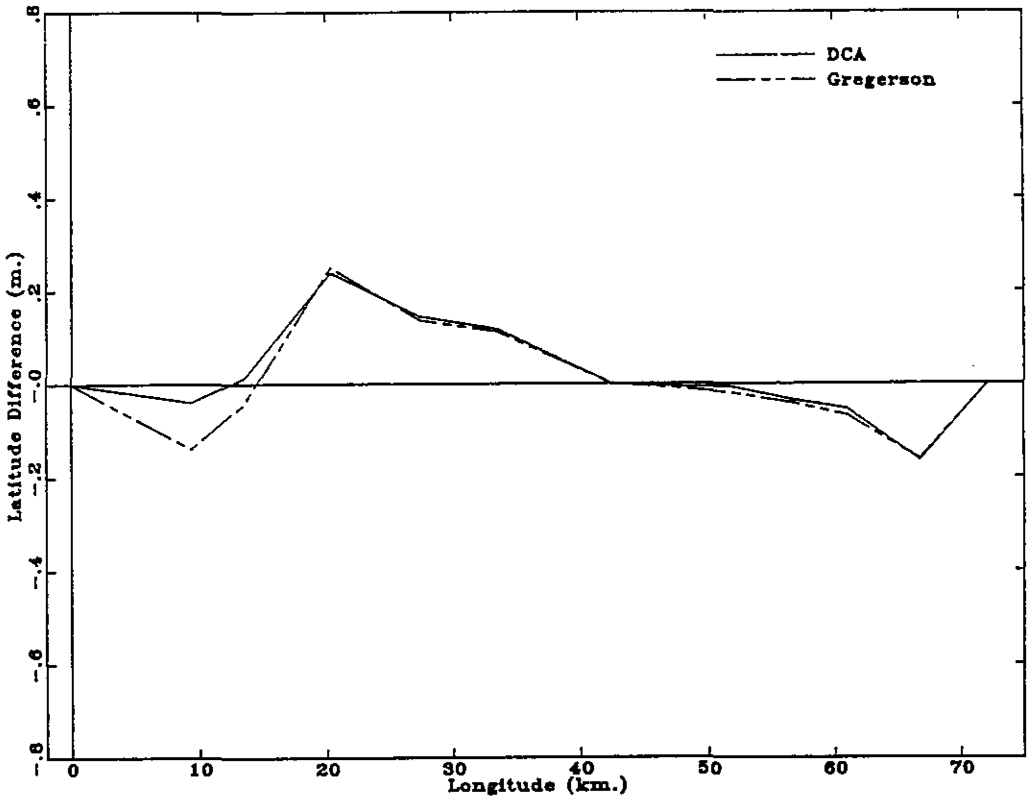


Figure 38. Latitude differences of ISS and TCT, west-east direction.

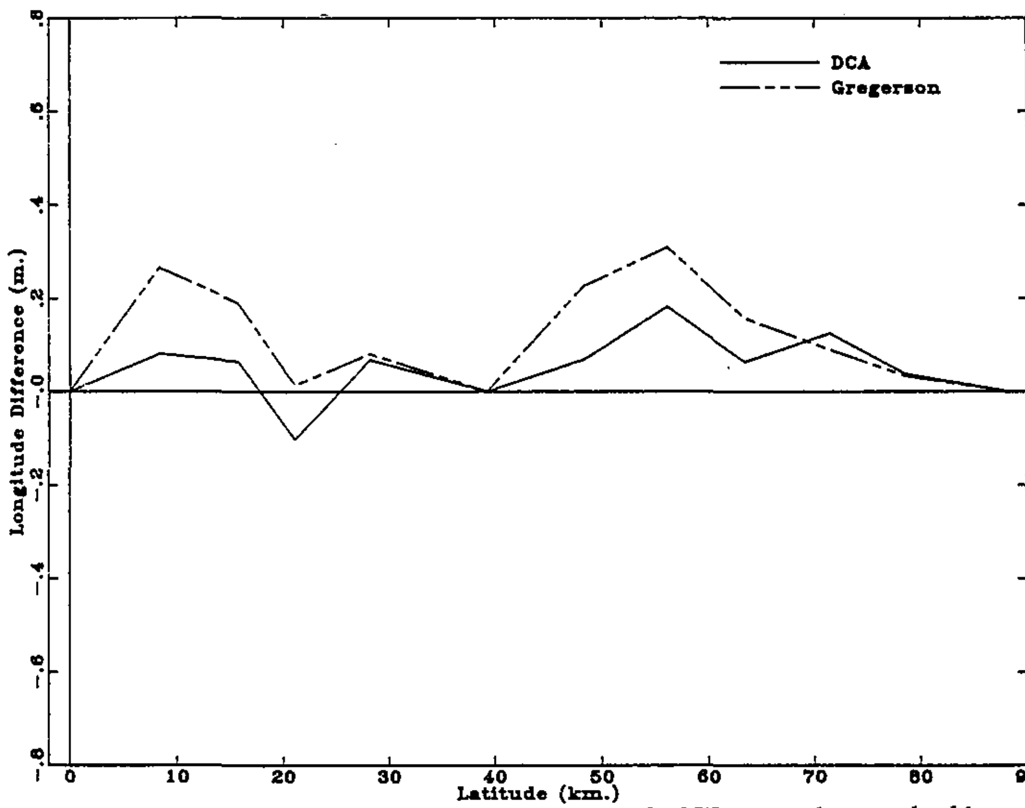


Figure 39. Longitude differences of ISS and TCT, south-north direction.

graphs indicate that the selected model is better than Gregerson's for handling misalignment effects.

6.2 Limitations of the Study

Before accepting the best overall allocation of the mathematical model parameters, it is important to discuss the limitations imposed upon this study; first, the potential problems with the mathematical model itself, second, the unavailability of the raw observed data, and finally, the limitations due to problems with the test data set.

6.2.1 Mathematical Model Problems

When examining the results of the test adjustments and applying statistical tests, an assumption was made that the system was modeled and functioning correctly. According to Uotila (1975) this hypothesis can be tested by applying the chi-square test to the V'PV resulting from the adjustment as explained in Section 3.3.4. The chi-square test establishes an acceptable range for V'PV based on the degrees of freedom and some selected significance level. Since the degrees of freedom were approximately 1,000 on each test adjustment, the acceptable range is not very wide. When the chi-square test was applied to the results of the 96 test adjustments, all failed to fall within the acceptable range at any significance level.

A further look at model adequacy is provided by additional examination of Table 3. Even though the Length Relative Accuracies given were computed directly from length shifts, linear error propagation was used to compute the standard deviations associated with these relative accuracies. The adjustment program generates the standard deviation which was then multiplied by the a posteriori standard deviation of unit weight and given in meters for comparison with the length shift, also in meters. The 3-sigma level is shown and in all cases the length shift exceeds the 3-sigma level. If the system were functioning correctly, statistics would predict that this level should be exceeded in less than 0.3% of the cases.

The chi-square tests and the excessive length shifts both indicate the presence of problems in the mathematical models. Such problems could be with the weighting of the observations or due to the inadequacies in the mathematical models themselves. Further work in model research is necessary to eliminate these problems and to allow more definitive refinements in the model parameter allocations.

6.2.2 Unavailability of Raw Observations

Another major limitation in the inertial systems research today is the unavailability of the raw observed data. Most systems, including the Litton Autosurveyor which produced the data in this study, filter the data before they are available. The filtering algorithms have not been totally disclosed and therefore cannot be undone to get the actual observations. What is left to be modeled then in the post-mission analysis is the performance of the inertial measurement hardware clouded by the on-board filtering software. Errors in the measurement system cannot be separated from the errors in the filtering. The systematic errors of one type, such as scale, are therefore coupled with errors of another type, such as misalignment, in the filtering attempt. The system parameter correlations given in Appendix 4 do not indicate problems in this area though further work is warranted.

6.2.3 Problems with Test Data

The inertial test data set used in this study has characteristics which limit the conclusions about systematic effects. First, the system was run continuously for the entire set, 24 hours a day. This continuous use does not provide for the distinction of systematic effects which may or may not transcend the stopping and restarting of the system.

Secondly, due to problems with observing procedures, intentional introduction of false coordinate updates, and problems with transcribing the data into computer readable form, some of the observed data was not usable. The data that could be used did not include any day's observations which were taken along both the north-south leg and the east-west leg of the L-shaped traverse. Finally, the conventional observations associated with the points in the test area do not include sufficient vertical observations to precisely determine station elevations, thus limiting conclusions based on height difference models.

6.3 Recommendations for Future Study

Clearly, even with these indications of model problems and the limitations imposed by the test data, this study has demonstrated that some of the inertial observation model parameters which capture the systematic effects should be allocated for more than just one traverse run. With the particular observation models selected for this study, most of the results were obtained using only two different allocations of model parameters for scale, misalignment, and drift. Further research with these models may find that additional allocation schemes are desirable for the nine model parameters. Varying the allocation of model parameters should be explored using other observation models as well.

Future tests of inertial surveying equipment should also involve observing routines which result in sufficient degrees of freedom to allow for these detailed studies of the systematic effects. The adjustments in this study had a large number of degrees of freedom due to the many repeated measurements over the same points. Another approach would be to observe in a grid like pattern with many common crossover points to provide the necessary internal constraints. Such an observation plan should include observations in all possible directions on one day.

Finally, a major advantage of this study was the use of points with established two dimensional accuracy. Transcontinental Traverse stations allowed for direct comparisons between "known" coordinates and inertially derived coordinates. This validation of new surveying equipment is vital, no matter what technology is involved. If future inertial observation model research proceeds in this manner, inertial surveying systems can be used with confidence to produce quick and reliable survey coordinates.

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Appendix 1

Test Adjustment Results

Adj #	S_1S_4	$S_2S_5S_7S_8$	$S_3S_6S_9$	DF	$\hat{\sigma}_o^2$
1	A	A	A	954	3.09
2	A	B	B	1080	3.61
3	A	C	C	1080	3.10
4	A	D	D	1143	3.67
5	A	E	E	1178	5.68
6	A	F	F	1199	6.21
7	B	A	B	1044	3.37
8	C	A	C	1044	3.01
9	D	A	D	1089	3.30
10	E	A	E	1114	3.26
11	F	A	F	1129	3.54
12	B	B	A	1062	3.54
13	C	C	A	1062	2.95
14	D	D	A	1116	3.45
15	E	E	A	1146	3.38
16	F	F	A	1164	3.80

A one set per leg

B one set per run

C one set per direction each day

D one set per day

E one set per direction for all days

F one set for all days

Appendix 1

Test Adjustment Results

Adj #	S_1S_4	$S_2S_5S_7S_8$	$S_3S_6S_9$	DF	$\hat{\sigma}_o^2$
17	B	B	B	1116	3.66
18	B	A	A	990	3.18
19	B	C	C	1116	3.21
20	B	D	D	1179	3.73
21	B	E	E	1214	6.12
22	B	F	F	1235	6.82
23	A	B	A	1026	3.48
24	C	B	C	1116	3.52
25	D	B	D	1161	3.71
26	E	B	E	1186	5.11
27	F	B	F	1201	5.51
28	A	A	B	1008	3.31
29	C	C	B	1116	3.19
30	D	D	B	1170	3.60
31	E	E	B	1200	4.97
32	F	F	B	1218	5.25

A one set per leg

B one set per run

C one set per direction each day

D one set per day

E one set per direction for all days

F one set for all days

Appendix 1

Test Adjustment Results

Adj #	S_1S_4	$S_2S_5S_7S_8$	$S_3S_6S_9$	DF	$\hat{\sigma}_o^2$
33	C	C	C	1116	3.13
34	C	A	A	990	3.04
35	C	B	B	1116	3.58
36	C	D	D	1179	3.67
37	C	E	E	1214	5.64
38	C	F	F	1235	6.15
39	A	C	A	1026	3.01
40	B	C	B	1116	3.28
41	D	C	D	1161	3.40
42	E	C	E	1186	3.38
43	F	C	F	1201	3.64
44	A	A	C	1008	3.05
45	B	B	C	1116	3.60
46	D	D	C	1170	3.60
47	E	E	C	1200	3.54
48	F	F	C	1218	3.93

A one set per leg

B one set per run

C one set per direction each day

D one set per day

E one set per direction for all days

F one set for all days

Appendix 1

Test Adjustment Results

Adj #	S_1S_4	$S_2S_5S_7S_8$	$S_3S_6S_9$	DF	$\hat{\sigma}_o^2$
49	D	D	D	1197	3.73
50	D	A	A	1008	3.15
51	D	B	B	1134	3.63
52	D	C	C	1134	3.23
53	D	E	E	1232	6.09
54	D	F	F	1253	6.77
55	A	D	A	1062	3.45
56	B	D	B	1152	3.63
57	C	D	C	1152	3.54
58	E	D	E	1222	5.09
59	F	D	F	1237	5.48
60	A	A	D	1035	3.31
61	B	B	D	1143	3.74
62	C	C	D	1143	3.33
63	E	E	D	1227	5.06
64	F	F	D	1245	5.33

A one set per leg

B one set per run

C one set per direction each day

D one set per day

E one set per direction for all days

F one set for all days

Appendix 1

Test Adjustment Results

Adj #	S_1S_4	$S_2S_5S_7S_8$	$S_3S_6S_9$	DF	$\hat{\sigma}_o^2$
65	E	E	E	1242	6.76
66	E	A	A	1018	3.18
67	E	B	B	1144	3.69
68	E	C	C	1144	3.25
69	E	D	D	1207	3.77
70	E	F	F	1263	7.24
71	A	E	A	1082	3.32
72	B	E	B	1172	4.49
73	C	E	C	1172	3.44
74	D	E	D	1217	4.57
75	F	E	F	1257	6.99
76	A	A	E	1050	3.21
77	B	B	E	1158	5.15
78	C	C	E	1158	3.29
79	D	D	E	1212	5.09
80	F	F	E	1260	7.13

A one set per leg

B one set per run

C one set per direction each day

D one set per day

E one set per direction for all days

F one set for all days

Appendix 1

Test Adjustment Results

Adj #	S_1S_4	$S_2S_5S_7S_8$	$S_3S_6S_9$	DF	$\hat{\sigma}_o^2$
81	F	F	F	1269	7.47
82	F	A	A	1024	3.23
83	F	B	B	1150	3.73
84	F	C	C	1150	3.30
85	F	D	D	1213	3.82
86	F	E	E	1248	6.82
87	A	F	A	1094	3.68
88	B	F	B	1184	4.76
89	C	F	C	1184	3.78
90	D	F	D	1229	4.81
91	E	F	E	1254	7.15
92	A	A	F	1059	3.42
93	B	B	F	1167	5.49
94	C	C	F	1167	3.46
95	D	D	F	1221	5.40
96	E	E	F	1251	6.94

A one set per leg

B one set per run

C one set per direction each day

D one set per day

E one set per direction for all days

F one set for all days

Appendix 2

F-Test Results

S_1 S_4 - Group 1

Hypothesis H_o : $S_{1i} = S_{1j}$

$S_{4i} = S_{4j}$

The probabilities of rejecting H_o :

Comparing Allocations	-AA	-BB	-CC	-DD	-EE	-FF
A-- : B--	0.997	0.951	1.000	0.977	1.000	1.000
A-- : C--	0.015	0.133	0.888	0.530	0.156	0.066
A-- : D--	0.955	0.734	1.000	0.956	1.000	1.000
A-- : E--	0.988	0.976	1.000	0.993	1.000	1.000
A-- : F--	0.999	0.997	1.000	1.000	1.000	1.000
B-- : D--	0.030	0.034	0.874	0.543	0.152	0.037
B-- : F--	0.960	0.988	0.999	0.998	1.000	1.000
C-- : D--	1.000	0.986	1.000	0.995	1.000	1.000
C-- : E--	1.000	1.000	1.000	1.000	1.000	1.000
C-- : F--	1.000	1.000	1.000	1.000	1.000	1.000
D-- : F--	1.000	1.000	0.999	1.000	1.000	1.000
E-- : F--	0.999	0.995	0.999	0.999	0.991	1.000

Note:
A one set per leg
B one set per run
C one set per direction each day
D one set per day
E one set per direction for all days
F one set for all days

Appendix 2

F-Test Results

S_2 S_5 S_7 S_8 - Group 2

Hypothesis $H_o : S_{2i} = S_{2j}$ $S_{7i} = S_{7j}$

$S_{5i} = S_{5j}$ $S_{8i} = S_{8j}$

The probabilities of rejecting H_o :

Comparing Allocations	A-A	B-B	C-C	D-D	E-E	F-F
-A- : -B-	1.000	1.000	1.000	1.000	1.000	1.000
-A- : -C-	0.007	0.002	0.999	0.994	0.999	0.992
-A- : -D-	1.000	1.000	1.000	1.000	1.000	1.000
-A- : -E-	1.000	1.000	1.000	1.000	1.000	1.000
-A- : -F-	1.000	1.000	1.000	1.000	1.000	1.000
-B- : -D-	0.137	0.128	0.785	0.783	0.306	0.223
-B- : -F-	1.000	1.000	1.000	1.000	1.000	1.000
-C- : -D-	1.000	1.000	1.000	1.000	1.000	1.000
-C- : -E-	1.000	1.000	1.000	1.000	1.000	1.000
-C- : -F-	1.000	1.000	1.000	1.000	1.000	1.000
-D- : -F-	1.000	1.000	1.000	1.000	1.000	1.000
-E- : -F-	1.000	1.000	1.000	1.000	1.000	1.000

Note:
A one set per leg
B one set per run
C one set per direction each day
D one set per day
E one set per direction for all days
F one set for all days

Appendix 2

F-Test Results

$S_3\ S_6\ S_9$ - Group 3

Hypothesis $H_o : S_{3i} = S_{3j} \quad S_{9i} = S_{9j}$

$S_{6i} = S_{6j}$

The probabilities of rejecting H_o :

Comparing Allocations	AA-	BB-	CC-	DD-	EE-	FF-
--A : --B	1.000	0.999	1.000	1.000	1.000	1.000
--A : --C	0.100	0.951	1.000	1.000	1.000	0.999
--A : --D	1.000	1.000	1.000	1.000	1.000	1.000
--A : --E	0.994	1.000	1.000	1.000	1.000	1.000
--A : --F	1.000	1.000	1.000	1.000	1.000	1.000
--B : --D	0.535	0.997	1.000	1.000	0.994	0.986
--B : --F	0.998	1.000	1.000	1.000	1.000	1.000
--C : --D	1.000	1.000	1.000	1.000	1.000	1.000
--C : --E	1.000	1.000	1.000	1.000	1.000	1.000
--C : --F	1.000	1.000	1.000	1.000	1.000	1.000
--D : --F	1.000	1.000	1.000	1.000	1.000	1.000
--E : --F	1.000	1.000	1.000	1.000	1.000	1.000

Note:

A one set per leg

B one set per run

C one set per direction each day

D one set per day

E one set per direction for all days

F one set for all days

Appendix 3

F-Test Results

S_1 S_2 S_4 S_5 S_7 S_8 - Group 1 and Group 2

Hypothesis H_o : $S_{1i} = S_{1j}$ $S_{4i} = S_{4j}$ $S_{7i} = S_{7j}$

$S_{2i} = S_{2j}$ $S_{5i} = S_{5j}$ $S_{8i} = S_{8j}$

The probabilities of rejecting H_o :

Comparing Allocations	--A	--B	--C	--D	--E	--F
AA- : BB-	1.000	1.000	1.000	1.000	1.000	1.000
AA- : CC-	0.000	0.001	0.961	0.683	0.960	0.812
AA- : DD-	1.000	1.000	1.000	1.000	1.000	1.000
AA- : EE-	1.000	1.000	1.000	1.000	1.000	1.000
AA- : FF-	1.000	1.000	1.000	1.000	1.000	1.000
BB- : DD-	0.000	0.021	0.524	0.402	0.079	0.016
BB- : FF-	1.000	1.000	1.000	1.000	1.000	1.000
CC- : DD-	1.000	1.000	1.000	1.000	1.000	1.000
CC- : EE-	1.000	1.000	1.000	1.000	1.000	1.000
CC- : FF-	1.000	1.000	1.000	1.000	1.000	1.000
DD- : FF-	1.000	1.000	1.000	1.000	1.000	1.000
EE- : FF-	1.000	1.000	1.000	1.000	1.000	1.000

Note:
- A one set per leg
- B one set per run
- C one set per direction each day
- D one set per day
- E one set per direction for all days
- F one set for all days

Appendix 3

F-Test Results

S_1 S_3 S_4 S_6 S_9 - Group 1 and Group 3

Hypothesis H_o : $S_{1i} = S_{1j}$ $S_{4i} = S_{4j}$ $S_{9i} = S_{9j}$

$S_{3i} = S_{3j}$ $S_{6i} = S_{6j}$

The probabilities of rejecting H_o :

Comparing Allocations	-A-	-B-	-C-	-D-	-E-	-F-
A-A : B-B	1.000	1.000	1.000	1.000	1.000	1.000
A-A : C-C	0.016	0.820	0.997	0.976	0.996	0.982
A-A : D-D	1.000	1.000	1.000	1.000	1.000	1.000
A-A : E-E	0.998	1.000	1.000	1.000	1.000	1.000
A-A : F-F	1.000	1.000	1.000	1.000	1.000	1.000
B-B : D-D	0.002	0.938	1.000	0.998	0.978	0.901
B-B : F-F	1.000	1.000	1.000	1.000	1.000	1.000
C-C : D-D	1.000	1.000	1.000	1.000	1.000	1.000
C-C : E-E	1.000	1.000	1.000	1.000	1.000	1.000
C-C : F-F	1.000	1.000	1.000	1.000	1.000	1.000
D-D : F-F	1.000	1.000	1.000	1.000	1.000	1.000
E-E : F-F	1.000	1.000	1.000	1.000	1.000	1.000

Note:
A one set per leg
B one set per run
C one set per direction each day
D one set per day
E one set per direction for all days
F one set for all days

Appendix 3

F-Test Results

$S_2\ S_3\ S_5\ S_6\ S_7\ S_8$ - Group 2 and Group 3

Hypothesis H_o : $S_{2i} = S_{2j}$ $S_{5i} = S_{5j}$ $S_{7i} = S_{7j}$

$S_{3i} = S_{3j}$ $S_{6i} = S_{6j}$ $S_{8i} = S_{8j}$

The probabilities of rejecting H_o :

Comparing Allocations	A--	B--	C--	D--	E--	F--
-AA : -BB	1.000	1.000	1.000	1.000	1.000	1.000
-AA : -CC	0.594	0.739	0.966	0.947	0.924	0.922
-AA : -DD	1.000	1.000	1.000	1.000	1.000	1.000
-AA : -EE	1.000	1.000	1.000	1.000	1.000	1.000
-AA : -FF	1.000	1.000	1.000	1.000	1.000	1.000
-BB : -DD	0.939	0.964	0.989	0.994	0.980	0.988
-BB : -FF	1.000	1.000	1.000	1.000	1.000	1.000
-CC : -DD	1.000	1.000	1.000	1.000	1.000	1.000
-CC : -EE	1.000	1.000	1.000	1.000	1.000	1.000
-CC : -FF	1.000	1.000	1.000	1.000	1.000	1.000
-DD : -FF	1.000	1.000	1.000	1.000	1.000	1.000
-EE : -FF	1.000	1.000	1.000	1.000	1.000	1.000

Note:

A one set per leg

B one set per run

C one set per direction each day

D one set per day

E one set per direction for all days

F one set for all days

Appendix 4

Adjusted System Parameter Correlation Coefficients

1	2	3	4	5	6	7	8	9
$1.000x10^{+0}$	$-.106x10^{+0}$	$-.432x10^{+0}$	$-.221x10^{-6}$	$.104x10^{-4}$	$-.657x10^{-5}$	$.298x10^{-6}$	$-.194x10^{-6}$	$.133x10^{-8}$
	$1.000x10^{+0}$	$-.669x10^{+0}$	$-.462x10^{-6}$	$-.199x10^{-6}$	$.616x10^{-6}$	$-.323x10^{-7}$	$-.155x10^{-6}$	$.428x10^{-6}$
		$1.000x10^{+0}$	$.556x10^{-6}$	$-.514x10^{-5}$	$.274x10^{-5}$	$-.112x10^{-6}$	$.269x10^{-6}$	$-.475x10^{-6}$
			$1.000x10^{+0}$	$-.168x10^{+0}$	$-.524x10^{+0}$	$-.109x10^{-6}$	$-.816x10^{-7}$	$.386x10^{-6}$
				$1.000x10^{+0}$	$-.589x10^{+0}$	$.626x10^{-6}$	$.683x10^{-6}$	$-.257x10^{-5}$
					$1.000x10^{+0}$	$-.323x10^{-6}$	$-.346x10^{-6}$	$.129x10^{-5}$
						$1.000x10^{+0}$	$-.877x10^{+0}$	$-.182x10^{-1}$
							$1.000x10^{+0}$	$-.418x10^{+0}$
								$1.000x10^{+0}$