NOAA Technical Memorandum NOS NGS-12
 ADJUSTMENT PROGRAM

Rockville, Md. April 1978

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Specifications to Support Classification, Standards of Accuracy, and General Specifications of Geodetic Control Surveys. Federal Geodetic Control Committee, John 0. Phillips (Chairman), Department of Commerce, NOAA, NOS, 1975, reprinted 1976, 30 p. (PB261037). This publication provides the rationale behind the original publications, "Classification, Standards of Accuracy, ...".
(Continued at end of publication)

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TRAV10 HORIZONTAL NETWORK
ADJUSTMENT PROGRAM

Charles R. Schwarz

National Geodetic Survey
Rockville; Md.
April 1978

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## PREFACE

The TRAV10 program is the result of contributions from many individuals within the National Geodetic Survey (NGS). John G. Gergen laid out the groundwork by designing and coding the first eight programs in the TRAV series. TRAV10 uses many of his routines without change. Robert H. Hanson programmed the HERESI routine for solving the normal equations. The storage structure used in this routine dictates the logic for the rest of the program. David E. Alger wrote the preprocessor and Anna-Mary B. Miller wrote the postprocessor. Richard A. Snay contributed the algorithm and program for the reordering of the unknowns. Primary credit for the program belongs to John F. Isner, who acted as lead programmer and analyst, wrote the main processor, and integrated all the parts. This memorandum was prepared by Charles R. Schwarz, who also converted the program from the CDC 6600 computer to an IBM 360 version.

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#### Abstract

The TRAV10 adjustment program is the major tool for the adjustment of horizontal survey networks at the National Geodetic Survey. It performs a two-dimensional adjustment on the ellipsoid. Many features are similar to those of other programs used by other agencies. The handling of the normal equations, especially for large networks, is the most important design criterion. The TRAV10 program uses the Cholesky solution method with a variable band storage scheme. The normal equations are partitioned into variable sized blocks, stored on random access secondary storage, and paged into main memory as needed. A reordering of the unknowns is used to reduce both the required storage and the number of arithmetic operations.


## 1. INTRODUCTION

The TRAV10 adjustment program is the major tool for the adjustment of horizontal survey networks at the National Geodetic Survey. It has been implemented both on the NOAA CDC 6600 running under SCOPE 3.3, and on the NOAA IBM 360/195, running under OS/MVT. With only very minor exceptions, the two implemented versions of the program are identical.

TRAV10 has grown out of an evolving series of computer programs used at NGS to adjust horizontal survey networks since 1972. Each version has been named after the first major application--the adjustment of the transcontinental traverse of the United States.

The TRAV programs are similar in that they all use observation equations, perform a least-squares adjustment, and iterate the solution to convergence. In purpose they are similar to the GALS program of the Geodetic Survey of Canada, the HAVOC program of the Geodetic Survey Squadron of the U.S. Defense

Mapping Agency, and others. These are all single-pass horizontal adjustment programs, designed to accomplish a complete adjustment from the editing of input observations to the computation of residuals and statistics.

The NGS TRAV programs have differed from each other primarily in the methods they use to form and solve the normal equations. TRAV05, the first version to be put into large-scale operation, used a banded matrix structure completely contained in core memory. It was implemented in three versions, the only difference being the size of the network that could be solved. The controlling factor was generally the amount of central memory allocated to the storage of the normal equations. The small version was configured so that its use of central memory allowed it to run at the highest priority used by the computer center in normal operations. The medium and large versions were configured to use more memory, solve large networks, and run at correspondingly lower priorities.

TRAV06, the second operational program, was used for about one year. It was designed around a variable band storage structure for the normal equations, which were completely contained in central memory. TRAV08 was a larger version of TRAV06.

TRAV07 was a first attempt at partitioning the normal equations and storing the partitions on secondary storage. The partitioning was such that each row of the normal equations was a separate block. Although this scheme allowed the adjustment of much larger networks in a limited area of central memory, the program incurred abnormally high input/output charges and often ran in an I/O bound mode on NOAA's CDC 6600. It was never made operational.

All TRAV programs through TRAV08 were almost independent of the number of observations. Although there were a few fixed size arrays used to index the observations, the number of observations in a network was seldom the limiting factor. The controlling consideration was the limited number of unknowns for which the program could solve.

The operational programs TRAV05, TRAV06, and TRAV08 could handle the majority of projects processed by NGS. However, the few very large projects that exceeded the limitations, and the need for operational efficiency, necessitated a new TRAV program.

## 2. DESIGN CRITERIA

### 2.1 Limitations

The first and most important design criterion for TRAV10 was that the program should not place limits on the number of stations or observations that could be processed, or on the size of
the normal equation matrix. Specifically, a partitioning scheme was needed to avoid limitations of in-core solutions experienced in TRAV05, TRAV06, and TRAV08. At the same time, the partitioning scheme had to be more efficient than the experimental TRAV07.

It was recognized that the computer on which the program runs will eventually place a hardware limit on the size of the network that can be handled. There could always be a network so large that all the disk storage space on the machine would not be sufficient to hold the normal equation partitions. Similarly, there could be so many stations or partitions that even the necessary indices would not fit in the central memory. Thus the objective was that even though the hardware resources placed a limitation, the program itself should not. TRAV10, therefore, has no fixed size in terms of observations, stations, normal equation elements, or partitions. If hardware resources are increased, the size of the network that can be adjusted will increase correspondingly and without limit.

The limitations imposed by finite hardware resources are at least an order of magnitude larger than those which would apply to in-core solution schemes. The hardware limitations are almost never the operative consideration because other factors are of primary consideration.

The first consideration is that the facility operation procedures practiced by the computer center usually define the largest region available under multiprogramming operations. The size of this region is usually smaller than the total multiprogramming area. Of course, it is possible to run in a single thread mode using the whole multiprogramming area, and even to enlarge the multiprogramming area by reconfiguring the operating system. However, this would require that special arrangements be made with the computer center, that the run be made only after all other work of the computer center is completed, and that the special arrangements are valid only on a "one-time" basis. Such special arrangements are seldom worth the effort if the problem can be solved otherwise.

A second consideration concerns human engineering: there is a point at which the output of an adjustment is both physically and conceptually too big to be handled by a human being. When this point is passed, people tend to become cavalier in their analysis of the output, rejecting observations without proper consideration and failing to notice important weaknesses in the network to be adjusted.

A third consideration is the risk that an entire run could be lost if the computer system fails near completion of a run. In general, the longest time a program should ever run without checkpoint safeguards is about 10-20 minutes CPU (about one hour wall clock) time.

All of these considerations point to the same practical limit: about l,000-2,000 stations. This range was selected as the design objective for TRAVl0.

To handle even larger networks, NGS has also been developing a series of programs using the Helmert block technique to partition the normal equation system. This partitioning technique affords a natural checkpoint/restart system. The original concept was that the Helmert block scheme would be used only for adjustments that exceeded the $1,000-2,000$ station practical limit of TRAVIO, and that the size of each Helmert block would be about the same as the largest network handled by TRAV10. Recently it has been suggested that the Helmert block scheme may be used advantageously for networks as small as several hundred stations.

### 2.2 Specification of Parameters

The user should be required to specify as few parameters as possible to the program. For instance, the program should relieve the user of the responsibility for counting the number of stations and the number of observations. Redundant specification of parameters should be avoided. In TRAV10, this criterion is met by requiring the user to specify only a single parameter in the control cards: the size of the region in which the program is to run. (This is done with the REGION parameter on the IBM 360 and the Request Field Length (RFL) statement on the CDC 6600.) The program reads the input and decides how best to use the available core area. The core area is normally divided among the various arrays in the program in such a way that no core space is wasted. Since FORTRAN programs are normally fixed in size, special assembly language interfaces have been used in both the CDC 6600 and the IBM 360 versions to enable the program to access all the central memory in the region in which it runs. In the 6600 version, unused core (should there be any) is returned to the operating system. In the IBM 360 version, the user also preallocates secondary (disk) storage space by estimating the number of stations, observations, and normal equation elements. These estimates may be very approximate and have no effect on the program's execution priority.

### 2.3 Efficiency

The program should be efficient for small and large networks. It should be possible to run small problems with smaller amounts of computer resources. For NGS, as well as for most program users, the efficiency of a program can only be judged with reference to the scheduling algorithm implemented by the computer center. Fewer resource demands by the program means higher priority processing, faster turnaround, more throughput, and higher productivity. TRAV10 achieves this kind of efficiency by attempting to use all the core space available to it and by avoiding time-consuming algorithms that are applicable to only
large networks. For a given network, TRAV10 allows a trade-off to be made between core size and time. To run the program in a small core size, the user pays in terms of the time spent to partition the normal equations and to transfer the partitions to and from secondary storage. The user is advised to make this trade-off so as to place his/her run in a job class of as high priority as possible. Exactly how this is done depends on the scheduling algorithm of the computer center.

### 2.4 Transparency

Details of the data and file structures used by the program and techniques used to handle the normal equations should be transparent to the user. In TRAV10, the user is largely unaware of the reordering and partitioning of the unknowns. The program takes care of these matters automatically so that the user is left free to concentrate on the geodetic aspects of the problem.

### 2.5 Abnormal Terminations

Data errors should not cause the program to terminate abnormally without producing a message to the user in geodetic terms. For TRAVl0, this is accomplished by a program that performs a complete edit of the input data before numerical processing begins.

### 2.6 Program Modularity

The program should be modular so that functions can be clearly separated and modifications easily made. For this reason, TRAVIO is actually a process or sequence of programs rather than a single program. It consists of a choice of two preprocessors, a main processor (also called TRAV10), and a postprocessor. In the IBM 360 version, the main processor and postprocessor are combined into a single program.

All editing of the input data is performed by a preprocessor. This allows for a thorough editing of all data fields. Serious data errors can be trapped at an early stage. When fatal errors are found, all numerical processing is suppressed, although the edit process is carried to conclusion.

The main processor incorporates all the numerical functions concerned with the observation and normal equations. It is the only processor that is cognizant of the method used to partition, store, and solve the normal equations.

The postprocessor reports the residuals, computes their statistics, produces other information to be used in analyzing the results, and writes a file of card images with the adjusted geodetic coordinates.

### 2.7. User's Options

The program should be designed as a production tool for processing large amounts of data in a stable environment. As such a tool, NGS management uses TRAVl0 to specify how computations will be performed. Practices that are considered the prerogatives of management are compiled into the program and cannot be changed by the individual user. Such practices include the editing checks applied by the preprocessor, the default weighting scheme, the numerical values of datum parameters, and the methods used to reorder the unknowns and control the number of iterations. The user is given some options, but most of these are used for controlling printed output and preparing the specification of output reports. In no case can the user's options affect the adjustment model or the numerical results of the adjustment.

## 3. HANDLING OF NORMAL EQUATIONS

The most important consideration in a geodetic least-squares adjustment program is the set of algorithms used to accumulate, store, and solve the normal equations. These algorithms often dictate the logic and structure of the other parts of the program. They determine the program limitations in terms of the number of observations or parameters, and usually whether it is efficient enough to be used for large problems or in a production environment. Designing a good least-squares adjustment program requires some knowledge of the problem to be solved; general purpose programs designed to handle any or all least-squares adjustment problems are not desirable. A distinguishing feature of many adjustment problems in geodesy is that the normal equations are sparse (i.e., there are many more zero than nonzero elements), and algorithms are often designed to take advantage of the a priori knowledge of the location of the zero elements. Normal equation systems arising in horizontal adjustments are sparse, since an off-diagonal element is nonzero only if the two stations to which it corresponds are related by an observation. Furthermore, the percentage of nonzero elements decreases as the size of the network increases.

In TRAV10, the normal equations are solved by subroutine HERESI, which is based on a routine described by Poder and Tscherning (1973).

### 3.1 Solution Algorithm

HERESI implements the Cholesky algorithm (Schmid 1973). The normal equations are decomposed into the product of an upper triangular matrix and its transpose in the form

$$
\mathrm{N}=\mathrm{C}^{\mathrm{T}} \mathrm{C}
$$

The first stage, the triangular factorization or forward solution process, transforms the normal equation system

$$
N X=U
$$

into the system

$$
c X=\left(C^{T}\right)^{-1} U
$$

The back solution process solves the triangular system for the solution vector

$$
X=c^{-1}\left(c^{T}\right)^{-1} U
$$

The algorithm can also be easily extended to yield an inverse of the original matrix, although this is not used in TRAV10.

The basic equations for the forward solution are

$$
c_{i j}=\left\{\begin{array}{c}
\left(n_{i j}-\sum_{k=1}^{i-1} c_{k i} c_{k j}\right) / c_{i i}, i<j  \tag{1}\\
\left(n_{i j}-\sum_{k=1}^{i-1} c_{k i} c_{k j}\right)^{\frac{1}{2}}, i=j \\
0
\end{array}, i>j\right.
$$

Examination of these equations discloses the following properties used by HERESI:

1. Once $n_{i j}$ is used to develop $c_{i j}$, it is no J.onger needed. Thus the matrix $C$ can be developed in the space occupied by matrix N. Since there is no need to store the lower part of $C$, only the upper triangular part of the symmetric matrix $N$ is stored.
2. The triangular matrix $C$ can be developed either row by row or column by column. The HERESI algorithm develops one column at a time, from the first element in the column to the diagonal element.
3. The solution vector can be developed in the same space occupied by the right-hand side of the original equations. Thus the storage requirements are determined only by the size of the original equations; no new storage locations are needed for the solution processes.

### 3.2 Variable Bandwidth

Further examination of eq. (l) shows that if element $n_{m j}$ is the highest nonzero element in column $j$ of $N$, then $C_{m j}$ is the highest nonzero element in column $j$ of $C$. No new nonzero elements are generated above position $m$ in column $j$.

The column profile of a matrix is a graphical display of the position of the highest nonzero element in each column. Figure 1 shows the profile of the upper triangular part of a typical normal equation matrix. The bandwidth of an individual column is the distance of the highest nonzero element from the diagonal. The matrix bandwidth is the largest of the individual column bandwidths. The number of elements within the matrix profile (also called the profile) is obviously the sum of the individual bandwidths.

For many algorithms designed to be operated on banded matrices, the critical measure is the bandwidth. The algorithm in HERESI considers the bandwidth of each column separately. Only elements within the variable band (or profile) are stored, accumulated, and operated upon. Elements outside the profile are known to be zero, and no nonzero elements are generated outside the profile during the Cholesky decomposition. The critical measure, determining the number of locations required for storage and the number of arithmetic operations required for decomposition, is the profile. The processing of each column starts with the first nonzero element, since all those above it remain zero.

The variable bandwidth scheme of matrix storage obviously requires an additional index giving the individual bandwidth of each column. This extra effort is worth the potentially large saving of storage.

### 3.3 The Parititioning Scheme

In TRAV10 the normal equation matrix is divided into partitions or blocks. Each partition consists of some number of pairs of columns of the normal equation matrix. Pairs are used so that the columns corresponding to the latitute and longitude unknowns of a given station are always in the same block. The right-hand side of the normal equations is always a block by itself. Each partition, together with its column index, is stored as a record on random access secondary storage (usually disk) and brought into main memory as needed.

The size of the individual partitions depends on the amount of real memory workspace available to the program. The workspace is divided into two frames. The program automatically partitions the matrix by putting as many pairs of columns into a block. as


Figure l.--Typical matrix profile structures showing the ordering of unknowns by (a) a profile minimization scheme and (b) a bandwidth minimization scheme.
it can without exceeding the size of a frame. The minimum frame size with which the program can work is $4 n+3$ locations, where n is the number of stations. This minimum guarantees that the right-hand side vector (used in computing accuracies) can be held in a frame.

### 3.4 Reordering of Unknowns

The computational and storage savings to be gained from the variable bandwidth approach depend on the size of the matrix profile. In TRAVlo, the unknowns are reordered in such a way as to reduce the profile of the matrix, using the algorithm described by Richard Snay (1976). In practice, the profile of the normal equation matrix requires significantly less storage than a fixed size band, and far less than the upper triangle. For networks comprised of between 30 to 100 stations, the profile is almost always less than $15 \%$ of the elements of the upper triangle. For larger networks, the savings are even more dramatic, since the profile tends to grow only linearly with the number of stations.

The reordering algorithm operates on a machine representation of the network graph. In TRAV10 this graph is represented by a neighbor list for each station. Each neighbor list consists of a variable length sequence of connection records, each of which both identifies the connected station and also indicates the type of observations causing the connection (fig. 2). Since the neighbor lists must be accessed randomly by the reorder routines, they are stored (one physical record per station) on direct access secondary storage and brought into main memory as needed by various routines. Connection records are formed only for observations that are valid in the adjustment. Those formed by observations that will be deleted (i.e., single direction lists) are ignored. When the neighbor lists are formed, all connection records for a given pair of stations are merged together. Connections arising from the elimination of orientation unknowns are also represented, so that the merged neighbor lists provide a representation of the internal structure of the normal equations.

### 3.5 Formation of Normal Equations

The normal equations are accumulated by considering the observation equations one at a time. Rounds of directions (abstracts) are considered as single entities; otherwise, the ordering of the observation equations is immaterial.

NEIGHBOR LIST RECORD FOR STATION K


ID station number. A connection of some sort exists between station $K$ and station ID.

ICODE
bit flags indicating type of connection
rightmost bit - a direction from $K$ to ID
next bit to left - a direction from ID to $K$
next bit to left - an azimuth between $K$ and ID
next bit to left - a distance between $K$ and ID
next bit to left - latitude constraint (ID=K)
next bit to left - longitude constraint (ID=K) NOTE: no bit flags set indicates that the connection arises indirectly from $z$ elimination.

LINK pointer to next connection record in the list. The pointers allow the list to be accessed by ascending order of station number rather than by sequential location. The beginning-of-list pointer is the fourth word of the header.

Figure 2.--Structure of connection records and neighbor list.

### 3.5.1 Elimination of Orientation Unknowns

The orientation unknowns (z's), which arise from each round of directions, are eliminated by the method attributed to Schreiber (Jordan-Eggert 1935, sections 100 and 110). The relationship of this scheme to elimination by matrix partitioning is discussed in section 4. It affords an easy, automatic way of eliminating the orientation unknowns at the earliest opportunity.

Because of the elimination of $z$ 's, only the latitudes and longitudes are left as unknown parameters. The size of the normal equations is reduced, but the meaning of "connection" is changed. Two stations are now connected (i.e., there are nonzero elements in the corresponding rows and columns of the normal equations) whenever there is a direct observation between the two stations, or a third station observes both of them in a single round of directions.

### 3.5.2 Accumulation of Partial Normal Equations

The criterion governing the design of the normal equation partitioning scheme is that two partition frames fit the program's real memory workspace. This is a requirement of the Cholesky factorization routine HERESI.

During the accumulation of partial normal equations, one half of the available memory serves as a frame for transient partitions while the other half is used as a staging area for partial normal equation terms computed when the appropriate partition is not available.

The staging area must be structured such that each partial normal equation term is tagged with its destination. A further requirement is that terms destined for the same location must be ordered on a first-in, first-out basis.

To satisfy the above requirements, the staging area is allocated among as many queues as there are partitions. All space initially "belongs" to an availability list, and all queues are empty. Figure 3 shows this condition for a 3-partition system.

Each list element is large enough to hold the coefficient value, a row and column number, and a pointer to the next element (indicated by an arrow in the figure).

Suppose that partition two currently occupies the paging area. All terms which belong in partition two will be immediately accumulated as they arise. Those belonging to any of the other partitions will be saved in the staging area, linked to the queue corresponding to the partition to which they belong. Figure 4 illustrates the situation after several "normalizations."

If the partition experiencing the greatest "demand" is in the paging area, data movement is minimized and the efficiency of normal equations formation is improved. This is insured if the observations are sorted into the order of elimination of the "from" station, since stations connected by observations are generally close in order of elimination (banding effect). Sorting was judged uneconomical for networks containing more than a few hundred observations, because the sorting expense becomes greater than the cost of doing the solution itself.


PAGING AREA

Figure 3.--Allocation of memory for normal equation accumulation before processing.


PAGING AREA

Figure 4.--Allocation of memory for normal equation accumulation after processing.

Localization of demand is observed even when sorting is not performed. This phenomenon is attributed to rounds of directions, which are entered as input in an unbroken sequence of direction observations. Localization is enhanced by the concatenation of rounds observed from the same station.

Over any small time interval, localization of demand has the net result that either the partition in the paging area is experiencing heavy demand, or one of the queues in the staging area is lengthening rapidly.

The process described above may be interrupted in two possible ways:
a. The observations are depleted.
b. The staging area runs out of free space (all space being taken up by the collection of queues).

In the first case, each partition for which there is a non-empty queue element in the staging area must be recalled to the paging area so that the contents of the respective queue may be "flushed" into the partition. This operation consists of adding each queued partial normal equation term into the proper location in the normal equations.

The second case is referred to as an "overflow." Overflows require immediate remedial action before any processing can continue. The partition currently in the paging area is rewritten on the disk, and the partition corresponding to the longest queue is fetched in its place. The longest queue is then flushed into the new partition and the liberated queue elements are returned to the availability list. The process is then allowed to resume at the point where the overflow condition occurred.

When the system of equations fits into a single partition, the staging area is unused, since all partial coefficients accumulate in their final places. In this case, no wasted movement of data into and out of the staging area occurs, and no input/output of the partition is required. Data movement and input/output will both increase as the number of partitions increases. For a given network, the number of partitions depends on the available memory space, which is determined by the user's field length or REGION parameter. This allows the user to make the trade-off between time, input/output, and core, in order to maximize the job priority under a given scheduling algorithm.
4. MATHEMATICAL SPECIFICATIONS
4.1 Notation

The following notations and adopted values are used in this section:
a $\quad=$ equatorial radius
$\mathrm{f} \quad=$ flattening
$c \quad=a /(l-f)$
$e^{\prime 2}=f(2-f) /(1-f)^{2} \quad$ second eccentricity
$\rho \quad=180 * 3600 / \pi \quad 206264.8062471$
$M_{i} \quad=c /\left(1+e^{2} \cos ^{2} \phi_{i}\right)^{3 / 2} \quad$ radius of curvature in the meridian at point i
$N_{i} \quad=\quad c /\left(1+e^{12} \cos ^{2} \phi_{i}\right)^{\frac{1}{2}}$
radius of curvature in prime vertical at point i
$\alpha_{i j} \quad=\quad \underset{\text { from } i}{ }$ to $j$
$R_{i j} \quad=\frac{M_{i} N_{i}}{N_{i} \cos ^{2} \alpha_{i j}+M_{i} \sin ^{2} \alpha_{i j}} \quad \begin{array}{r}\text { radius of curvature in } \\ \text { azimuth } \alpha_{i j}\end{array}$
s or $s_{i j}=\underset{\text { from } i}{\text { geodetic }} \underset{j}{ }$ distance
$\phi, \lambda \quad=\quad$ geodetic latitude and longitude

| $\Phi r \Lambda$ | ```= astronomic latitude and longitude``` |
| :---: | :---: |
| $\xi$ | $=\Phi-\phi$ component of the deflection of the vertical in the meridian |
| $\eta$ | $=(\Lambda-\lambda) \cos \phi$ component of the deflection in the prime vertical |
| R | $\begin{gathered} =6,371 \text { kilometers mean } \\ \text { earth radius } \end{gathered}$ |
| $\mathrm{N}_{\mathrm{i}}$ | $=$ geoid height at point i (distinguished by radius of curvature in prime vertical by context) |
| H | $=$ orthometric height |
| D | $=$ observed distance |
| k | $=a$ factor used to determine the algebraic sign of certain quantities. $\mathrm{k}=+1$ is used for the North American Datum, with longitude measured positive west and azimuth measured clockwise from South. $k=-1$ is used with other datums, with longitude measured positive to the east and azimuths measured clockwise from North |

### 4.2 Observations and Reductions

TRAV10 accepts direction, azimuth, and distance observations, as well as position (latitude, longitude) constraints. Most observation types are interpreted as being performed by an instrument at a point on the Earth's surface, leveled in the real gravity field, to another point on the Earth's surface. For the purposes of computation, these observations are reduced to the corresponding inferred observations on the ellipsoid. Tables 1, 2, and 3 indicate which corrections are applied. In general, they are applied prior to adjustment and not changed. An exception is the Laplace correction to astronomic azimuths, which is updated after each iteration to take into account the most recent estimate of the geodetic longitude.

Table 1.--Direction observations

|  |  |  | Corrections |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Code | Definition | Weight factor ${ }^{1}$ | Geodesic $^{2}$ | Skew <br> normal |  |
| 1 | First-order | 0.6 | Deflection ${ }^{4}$ |  |  |
| 2 | Second-order | 0.7 | x | x | x |
| 3 | Third-order | 1.2 | x | x | x |
| 4 | Fourth-order | 3.0 | X | x | x |
| R | Direction to <br> reference mark | 3.0 | X | x | x |
| Z | Direction to <br> azimuth mark | 3.0 | x | x | x |

${ }^{1}$ The default standard error of a direction observation is computed from the formula

$$
\sigma^{2}=s_{D}^{2}+2 *(\rho * 0.001 / D)^{2},
$$

where $D$ is the approximate distance between the points and $S_{D}$ is the weight factor from the table. The second term in the formula accounts for a miscentering error of 1 mm of both theodolite and target.

2 All directions receive the geodesic correction.
${ }^{3}$ Directions to stations with an orthometric height receive the skew normal correction. If the geoid height is missing, the orthometric height is used as an approximate height above the ellipsoid.

4 Directions from the stations with both astronomic coordinates and both orthometric and geoid height, to stations with both orthometric and geoid height, receive the deflection correction.

Table 2.--Azimuth observations

| Code | Definition | Default std.error ${ }^{1}$ | Corrections |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Geodesic | Laplace | Deflection ${ }^{2}$ |
| A | First-order astro(NGS) | (1) | X | X | X |
| B | Lower-order astro | 2:0 | X | X | X |
| J | Geodetic azimuth | 2:0 |  |  |  |

${ }^{1}$ The default standard error of a first-order astronomic azimuth is computed from

$$
\sigma^{2}=(.45)^{2}+(.80)^{2}+(\tan \phi / .80)^{2}+(.40 * \sin \phi)^{2},
$$

where $\phi$ is the latitude.
${ }^{2}$ The deflection correction is applied only if the occupied station has both orthometric and geoid heights given.

Table 3.--Distance observations

| Code | D Definition | Corrections |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Weight $\mathrm{S}_{1}$ | $\begin{aligned} & \text { factor }^{1} \\ & \mathrm{~S}_{2} \end{aligned}$ | $\begin{aligned} & \text { Sea } \\ & \text { level }{ }^{2} \end{aligned}$ | Arc | $\begin{aligned} & \text { Geoi } \\ & \text { heig } \end{aligned}$ | id2nd <br> ght ${ }^{3}$ velocity. |
| C | Electro-optical infrared | 15 mm | 1.0 ppm |  |  | X |  |
| G | Electro-optical infrared | 15 | 1.0 |  |  | x | X |
| X | Electro-optical mark to mark | 15 | 1.0 | X | X |  |  |
| F | Reference marks, feet | 10 | 0.5 |  |  | x |  |
| M | ```Reference marks, meters``` | 10 | 0.5 |  |  | X |  |
| T | Taped, sea level | 10 | 0.5 |  |  | X |  |
| U | Taped, mark to mark | 10 | 0.5 | X | X |  |  |
| E | Microwave, sea level | 30 | 3.0 |  |  | x |  |
| Y | Microwave, mark to mark | 30 | 3.0 | X | X |  | - |

${ }^{1}$ The standard error of a distance observation is computed from the weight factors by the formula

$$
\sigma^{2}=S_{1}^{2}+\left(D S_{2}\right)^{2}+\left(0.00005\left(h_{2}-h_{1}\right) / 3\right)^{2}
$$

where $D$ is the distance and $h_{1}, h_{2}$ are the heights.
${ }^{2}$ Mark to mark distances must have both orthometric and geoid heights at both ends of the line; otherwise, they are rejected.
${ }^{3}$ Geoid height corrections are made only if geoid heights are available for both ends of the line.

The corrections to directions and azimuths are taken from Bomford (1971, pp. 121-122). As applied in TRAV10, these are:
a. Geodesic correction

$$
-\frac{\rho}{12} \frac{e^{i^{2}} s^{2}}{N^{2}} \cos ^{2} \phi \sin 2 \alpha
$$

b. Skew normal correction

$$
\frac{\rho h_{2}}{2 N} e^{t 2} \cos ^{2} \phi \sin 2 \alpha
$$

c. Deflection correction

$$
\frac{\mathrm{h}_{2}-\mathrm{h}_{1}}{\mathrm{D}}
$$

The Laplace correction, which transforms astronomic azimuths to geodetic azimuths, is computed in the form
$\eta \tan \phi$.
When the adjustment is performed on the North American Datum, the observatory correction of 0.51 is added to all astronomic (west) longitudes before the computation of the deflection in the prime vertical. This correction is applicable to astronomic longitudes referred to the U. S. Naval Observatory and observed on or after January 1, 1962. The effect of the correction is to make all astronomic longitudes consistent with the adopted origin of the North American Datum, which is based on the adopted longitude of the U. S. Naval Observatory prior to 1962 .

The corrections for distance observations are taken from Meade (1972). As applied in TRAV10, these are
a. Sea level correction

$$
\left(\frac{D^{2}-\left(h_{2}-h_{1}^{2}\right)}{\left(1+\frac{h_{1}}{R_{12}}\right)\left(1+\frac{h_{2}}{R_{21}}\right)}\right)^{\frac{1}{2}}-D
$$

b. Arc correction

c. Geoid height correction

d. Second velocity correction

$$
\frac{C_{r}\left(C_{r}-2\right) D^{3}}{24 R}
$$

Position observations (constraints) require no corrections. The standard deviations of latitude and the standard deviations of longitude may be specified at the discretion of the user. Default values are $\sigma_{\phi}=\sigma_{\lambda}=10^{-10}$ second of arc. The default values are intended to serve as a means of effectively fixing a station's coordinates.

### 4.3 Observation equation coefficients

An observation equation is formed for each observed quantity. No observation equation involves more than five unknown parameters, so that the matrix of observation equations is sparse. Symbolically, each observation equation is written:

$$
a_{1} \delta \phi_{1}+a_{2} \delta \lambda_{1}+a_{3} \delta \phi_{2}+a_{4} \delta \lambda_{2}+a_{5} \delta z=\ell+v
$$

The units of the coordinate corrections $\delta \phi, \delta \lambda$ are seconds of arc. The units of angular and position observations are seconds of arc, and distance observations are in meters.

The observation equation coefficients are based on the forms given by Bomford (1971, p. 145).

For direction observations, the coefficients are

$$
\begin{gathered}
a_{1}=-k \frac{M_{1}}{S} \sin \alpha_{12} \quad a_{3}=-k \frac{M_{2}}{S} \sin \alpha_{21} \\
a_{2}=\frac{N_{2}}{S} \cos \phi_{2} \cos \alpha_{21} \quad a_{4}=-a_{3} \\
a_{5}=1 .
\end{gathered}
$$

For astronomic azimuth observations, the coefficients are

$$
\begin{gathered}
a_{1}=-\frac{M_{1}}{S} \sin \alpha_{12} \quad a_{3}=-\frac{M_{2}}{S} \sin \alpha_{21} \\
a_{2}=\frac{N_{2}}{S} \cos \phi_{2} \cos \alpha_{21}+\sin \phi_{1} \quad a_{4}=\frac{N_{2}}{S} \cos \phi_{2} \cos \alpha_{21} \\
a_{5}=0
\end{gathered}
$$

For distance observations, the coefficients are

$$
\begin{gathered}
a_{1}=\frac{M_{1}}{\rho} \cos \alpha_{12}
\end{gathered} \quad a_{3}=k \frac{M_{2}}{\rho} \cos \alpha_{21}, ~ a_{4}=-a_{2} .
$$

For a direct observation of latitude, $a_{1}=1$ and $a_{2}=a_{3}=a_{4}=a_{5}=0$. For a direct observation of longitude, $a_{2}=l^{3}$ and $a_{1}=a_{3}=a_{4}=a_{5}=0$.

The coefficient $a_{5}$ is never formed explicitly in the program nor is space ever allocated for it.

The right-hand side in the equation, $\ell$, is taken in the sense "observed minus computed" where the "observed" value is the input value plus the correction discussed in section 4.2. The "computed" values of the geodetic azimuth and distance are obtained from the geodetic inverse problem

$$
\left(\begin{array}{c}
\alpha_{12} \\
\alpha_{21} \\
S_{12}
\end{array}\right)=f\left(\phi_{1}, \lambda_{1}, \phi_{2}, \lambda_{2}\right),
$$

where the values for the latitudes and longitudes are either the input values or the values from the most recent iteration. The Helmert iterative method with the computational arrangement presented by Vincenty $(1975,1976)$ is used to solve the geodetic inverse problem.

For direction observations, the constant term is computed as $d^{b}-\alpha^{c}-z^{0}$, where $\alpha^{c}$ is the "computed" azimuth, $d^{b}$ is the "observed"direction, and $z^{\ominus}$ is an approximation to the orientation unknown for the round of directions. The approximation $z^{0}$ is obtained from the equation $z^{0}=d^{b}-\alpha c$ using the direction and azimuth for the first direction in the round. This causes the constant term in the first observation equation of each round of directions to be zero, and the constant term in the other equations to be generally small. This is done as a convenience to the geodesist who is interested in treating a large misclosure as an indicator of a large error or blunder. Otherwise, since the observation equation is linear in this unknown, we could just as easily use $z^{0}=0$.

### 4.4 Weights

Weights are always computed as the inverse of the square of the standard deviation of the observation. The standard deviation may be specified together with the observation. A single standard deviation is given for angular and position observations. For distance observations, both a constant part and a part proportional to the distance must be specified. If no standard deviation is given, the default observational standard deviation shown in tables $1,2,3$, and section 4.2 are used. The observational standard deviations are in the same units as the corresponding observations, except for positional constraints when the standard deviations of latitude and longitude are specified in meters.

### 4.5 Rejections

The following observations are rejected by the program:

1. Any observations for which the stations at both ends of the line are not in the input list of geodetic positions. These observations cannot be processed, since approximate values are not available for all the unknowns involved.
2. Any single direction list. These observations can add nothing to the adjustment.
3. Any astronomic azimuth with a Laplace correction in excess of 10 minutes of arc.
4. Any mark to mark distance for which the difference in endpoint elevations is greater than the distance itself.
5. Any mark to mark distance for which both the orthometric height and the geoid height are not available for both ends of the line.

### 4.6 Normal Equations

Each azimuth, distance, and position observation generates an observation equation that can be written

$$
A_{k} X=L_{k}+V_{k}
$$

where X is a vector containing corrections to all latitudes and longitudes. $A_{k}$ is a row matrix, all of whose elements except four will always vanish. $\mathrm{I}_{\mathrm{k}}$ and $\mathrm{V}_{\mathrm{k}}$ are single elements. If $P_{k}$ is the weight of the observation, the corresponding partial normal equation is

$$
\mathrm{N}_{\mathrm{k}} \mathrm{X}=\mathrm{U}_{\mathrm{k}},
$$

where

$$
N_{k}=A_{k}^{T} P_{k} A_{k}
$$

and

$$
U_{k}=A_{k}^{T} P_{k} L_{k}
$$

Direction observations require special consideration because of the presence of the orientation unknowns. The method of Schreiber (Jordan-Eggert, sections 100 and 110) is used. Let the group of observation equations generated by the kth abstract be written as

$$
A_{k} X+E_{k} \delta Z_{k}=I_{k}+V_{k}
$$

The matrix $A_{k}$ and the vectors $L_{k}$ and $V_{k}$ now have as many rows as there are directions in the abstract; $E_{k}$ is a vector of ones. If $P_{k}$ is the weight matrix for this group of observations, the following partial normal equations are generated:

$$
\left(\begin{array}{llll}
A_{k}^{T} P_{k} & A_{k} & A_{k}^{T} & P_{k} \\
E_{k} \\
E_{k}^{T} & P_{k} & A_{k} & E_{k}^{T} \\
P_{k} & E_{k}
\end{array}\right)\left(\begin{array}{r}
x \\
\\
\delta z_{k}
\end{array}\right)=\left(\begin{array}{lll}
A_{k}^{T} & P_{k} & I_{k} \\
E_{k}^{T} & P_{k} & L_{k}
\end{array}\right)
$$

Since all observations involving the orientation unknown $\delta \mathrm{z}_{\mathrm{k}}$ have been processed, it can be eliminated at the partial normal equation staqe. This leads to the following reduced partial normal equation:

$$
\begin{align*}
\left(A_{k}^{T} P_{k} \cdot A_{k}\right. & \left.-A_{k}^{T} P_{k} E_{k}\left(E_{k}^{T} P_{k} E_{k}\right)^{-1} E_{k}^{T} P_{k} A_{k}\right) X  \tag{2}\\
& =A_{k}^{T} P_{k} L_{k}-A_{k}^{T} P_{k} E_{k}\left(E_{k}^{T} P_{k} E_{k}\right)^{-1} E_{k}^{T} P_{k} L_{k}
\end{align*}
$$

which is similarly written

$$
\mathrm{N}_{\mathrm{k}} \mathrm{X}=\mathrm{U}_{\mathrm{k}}
$$

The column matrix $\mathrm{E}_{\mathrm{k}}$ and the terms containing it are never generated explicitly. Instead, the direction observation equations are processed while ignoring the orieptation unknown, generating the terms $A_{k}^{T} P_{k} A_{k}$ on the left and $A_{k} P_{k} L_{k}$ on the right. As the observations are processed, the "Schreiber equation" is formed. This is written

$$
E_{k}^{T} P_{k} A_{k} X=E_{k}^{T} P_{k} L_{k}, \quad \text { weight }=-\left(E_{k}^{T} P_{k} E_{k}\right)^{-1}
$$

or in a more intuitive form,

$$
\left.\underset{i}{(\Sigma} \quad P_{k i} \quad A_{k i}\right) X=\underset{i}{\Sigma} P_{k i} \quad I_{k i}, \quad \text { weight }=-\left(\underset{i}{\Sigma} P_{k i}\right)^{-l},
$$

where the sum is taken over all directions in the abstract.
After all directions in the round are processed, a partial normal equation is formed from the Schreiber equation as if it were an actual observation equation. This is added to the contributions from the actual direction observations, giving rise to the second term on each side of the reduced partial normal equation for the abstract (eq. 2).

All contributions to the normal equations are accumulated as partial normal equations are generated. After all observations are processed, the final normal equations take the form

$$
\begin{aligned}
\mathrm{NX} & =\mathrm{U}, \\
\text { where } \mathrm{N} & =\underset{\mathrm{k}}{\Sigma} \mathrm{~N}_{\mathrm{k}} \\
\text { and } \mathrm{U} & =\sum_{\mathrm{k}} \mathrm{U}_{\mathrm{k}} .
\end{aligned}
$$

### 4.7 Iterations

The normal equations are solved for the corrections X to the latitude and longitude unknowns. After this, the entire process of forming observation and normal equations is repeated with the updated approximations to the unknowns. The iterative process is terminated when any of the following conditions exists:

1. Satisfactory convergence is achieved. This occurs whenever the root mean square (rms) corrections to both latitude and longitude are less than 0.0001, i.e.,

$$
\left(\frac{\Sigma(\delta \phi)^{2}}{\mathrm{n}}\right)^{\frac{1}{2}} \leq 0.0001 \text { and }\left(\frac{\Sigma(\delta \lambda)^{2}}{\mathrm{n}}\right)^{\frac{1}{2}} \leq 0.0001 .
$$

2. The number of iterations exceeds 4 (the first solution is counted as iteration zero).
3. The solution diverges on two iterations. Divergence is detected when the rms residual increases between two successive solutions. This is allowed to happen once, but iterations are terminated if it occurs a second time.

### 4.8 Accuracies

TRAV10 has the capability of computing the relative error between any two specific points. The relative error is expressed as the standard error of the adjusted azimuth $\sigma_{\alpha}$, the standard error of the adjusted distance $\sigma_{d}$, and the covariance between them $\sigma_{\alpha d}$.

Let $\mathrm{X}^{\mathrm{a}}$ denote all the adjusted latitudes and longitudes. Symbolically, we can write the azimuth and distance as

$$
\left(\frac{\alpha}{d}\right)=g\left(x^{a}\right),
$$

even though only two latitudes and two longitudes are acutally involved. We further let

$$
G=\frac{\partial g\left(x^{a}\right)}{\partial x^{a}}=\frac{\partial(\alpha, a)}{\partial X^{a}}
$$

The covariance matrix of the azimuth and distance is symbolically propagated from the covariance matrix of the latitude and longitude unknowns:

$$
\begin{aligned}
\Sigma_{\alpha d} & =G \Sigma_{X} G^{T}=\sigma_{0}^{2} G N^{-1} G^{T}=\sigma_{0}^{2} G\left(C^{T} C\right)^{-1} G^{T} \\
& =\sigma_{0}^{2} G C^{-1}\left(C^{T}\right)^{-1} G^{T} \\
& =\sigma_{0}^{2}\left(\left(C^{T}\right)^{-1} G^{T}\right)^{T}\left(\left(C^{T}\right)^{-1} G^{T}\right) .
\end{aligned}
$$

The product $\left(\left(C^{T}\right)^{-1} G^{T}\right)$ is computed by solving the equation $C^{T} U=G^{T}$ with the HERESI subroutine. The computations are equivalent to performing a forward solution for the last two columns of the original normal equations augmented with the two columns $\mathrm{G}^{\mathrm{T}}$.

TRAV10 does not explicitly compute terms of the inverse of the normal equations. The point uncertainty of the coordinates of any station can be found by computing the accuracy of the desired station relative to any fixed point in the adjustment. This approach avoids superfluous computations that are not of interest to the analyst and allows all computations to be contained within the matrix profile.

## 5. GEODETIC ANALYSIS AIDS

### 5.1 Detection of Blunders

The magnitude of the right-hand side or constant term in the observation equation is often a good indicator of blunders in the data. This is especially true when the input coordinates are very accurate (as is often the case in horizontal surveys) or when there are few blunders and a heavily overdetermined system. The program displays those observations for which the constant term is large. The definition of "large" for each type of observation is given in section 12.10 of the user instructions (appendix).

### 5.2 Solvability Analysis

A logical solvability analysis is performed as part of the reordering process. Based on the observations used, the stations read as part of the input are grouped into components. Each component is an independent network, unconnected to any other component. To be solvable, each component requires a definition of the origin, orientation, and scale of the coordinate system. This is normally supplied by fixing one or more points in each component.

An analysis of the observations at each station is produced. The number of unique independent observations at a station is counted by the formula

$$
L+\operatorname{MAX}(0, N D F R O M-1)+N D T O+N A Z I+N D I S T,
$$

where
NDFROM is the number of directions emanating from the station,

NDTO is the number of directions to the station,

NAZI is the number of azimuths either from or to the station, and

NDIST is the number of distances either from or to the station.

If $L<2$, the station is flagged as undetermined. If $L=2$, it is flagged as a no-check station.

The counts of the number of observations are based on the connection records (fig. 2). Because all connection records for a given pair of stations are merged, repeated observations of the same type over the same line are counted only once. Thus, for a station to be considered possibly determinable, it must be involved in at least two unique observations.

When undetermined stations are detected, the matrix of normal equation coefficients is known to be singular and the program suppresses any attempt to solve the normal equations. In the practical adjustments of horizontal networks at NGS, missing observations and undetermined stations have been found to be a very frequent cause of singular normal equation systems. Thus, the solvability analysis is frequently able to identify the cause of the singularity.

There are, however, certain unusual configurations which can cause the normal equations to be singular even though no undetermined stations are detected. For example, consider an intersection station seen from two other points where all three points lie on a straight line. Only one component of the intersection station's position can be determined even though the number of observations meets the minimum required for determining both components.

### 5.3 Analysis of Residuals

The reporting and analysis of the residuals are performed by the postprocessor phase of the adjustment. This phase is implemented only after the solution phase has iterated to convergence.

Residuals are computed only after the last iteration of the solution process. Both the linear residuals ( $\mathrm{v}_{\mathrm{i}}=\mathrm{A}_{\mathrm{i}} \mathrm{X}-\ell_{\mathrm{i}}$ ) and the normalized residuals ( $\mathrm{V}_{\mathrm{i}} / \mathrm{P}_{\boldsymbol{i}}$ ) are displayed. The user of the program is offered the following tools for analysis:

1. On option, only the observations are listed for which the absolute value of the normalized residual is greater than 1.0. This serves to highlight the potentially troublesome observations.
2. Observations are flagged when the absolute value of the normalized residual is greater than the tau statistic at the 95\% confidence level (5\% probability of type 1 error). The use of the tau statistic for screening residuals from an adjustment is described by Allen Pope (1976).
3. On option, the residuals are sorted by observed station and all the residuals for each intersection station are displayed as a group.
4. The minimum, maximum, and mean absolute value residual is displayed for each of the observation codes. This is done both for the total population of residuals and for the limited population of observations over short lines.
5. The range, minimum, maximum, mean, and average absolute value of the normalized residuals are displayed.
6. The observation sequence numbers for the 20 largest normalized residuals are displayed. This immediately guides the user to the largest residuals.
7. The 95\% confidence interval of the $\chi^{2}$ statistic is displayed for the testing of the estimated variance of unit weight.

### 5.4 Detection of Singularities

If the normal equations are singular, then a zero will be generated by the Cholesky triangular factorization process for some diagonal element. Once this occurs, no more elements in the row corresponding to that element can be processed, since the algorithm requires division by the diagonal element of the row being processed.

In practice, roundoff errors and other effects cause small nonzero numbers to appear on the diagonal during factorization of singular matrices, so that the test for zero must be replaced by a comparison against a tolerance. In the program, each squared diagonal element of the triangular matrix factor is normalized by division by the corresponding element of the normal equation matrix before comparison to the tolerance, i.e.,

$$
g_{i}=\frac{c_{i j}^{2}}{n_{i i}}
$$

This normalization was suggested to NGS by William D. Googe of the Defense Mapping Agency Topographic Center, and is, therefore, called the "Googe number." It allows selection of a tolerance which is independent of network size, observation types, or weights.

In the program, the tolerance is set equal to 0.000001. Whenever a column is reduced and $g_{i}<0.000001$, a message is produced indicating that the solution breaks down at that point. The corresponding row and column of the normal equations (and triangular factor) are set equal to zero and the solution is continued. In effect, this procedure finds the solution that would be obtained if the offending unknown were set equal to its a priori (approximate) value. It allows the geodesist to see all, not only the first, of the unknowns that cannot be determined from the given data.

The Googe number can be given an interesting geometrical interpretation. Let $A_{i-1}$ denote the matrix consisting of the first i-l columns of the matrix of coefficients of the observation equations, and let $a_{i}$ denote the ith column, i.e.,

$$
A_{i}=\left(\begin{array}{ll}
A_{i-l} & a_{i}
\end{array}\right)
$$

and

$$
\mathrm{A}=\mathrm{A}_{\mathrm{u}}
$$

where $u$ is the total number of unknowns.
The portion of the normal equations corresponding to the first i columns is

$$
N_{i}=\left(\begin{array}{cc}
N_{i-i} & \beta_{i} \\
\beta & \\
i & \gamma_{i}
\end{array}\right)
$$

where

$$
\begin{aligned}
& N_{i-1}=A_{i-1}^{T} P A_{i-1} \\
& \beta_{i}=A_{i-1}^{T} P a_{i}
\end{aligned}
$$

and

$$
\gamma_{i}=a_{i}^{T} P a_{i}
$$

After the Cholesky triangular factorization procedure has been applied to the first i-l columns of the normal equations, the space originally occupied by the upper triangular part of $\mathrm{N}_{\mathrm{i}}$ contains
where $C_{i-1}^{T} C_{i-1}=N_{i-1}$.

$$
\left(\begin{array}{cc}
c_{i-1} & \beta_{i} \\
& \gamma_{i}
\end{array}\right)
$$

After reduction of column i (but before taking the square root of the diagonal element), this space contains

$$
c_{i}=\left(\begin{array}{ll}
C_{i-1} & \left(C_{i-1}^{T}\right)^{-1} \beta_{i} \\
& \\
& \gamma_{i}-\beta_{i}^{T} C_{i-1}^{-1}\left(C_{i-1}^{T}\right)^{-1} \beta_{i}
\end{array}\right)
$$

The lower right corner of the matrix is

$$
\begin{aligned}
c_{i i}^{2} & =\gamma_{i}-\beta_{i}^{T} C_{i-1}^{-1}\left(C_{i-1}^{T}\right)^{-1} \beta_{i} \\
& =\gamma_{i}-\beta_{i}^{T} N_{i-1}^{-1} \beta_{i} \\
& =a_{i}^{T} P a_{i}-a_{i}^{T} P A_{i-1}\left(A_{i-1}^{T} P A_{i-1}\right)^{-1} A_{i-1}^{T} P a_{i} \\
& =a_{i}^{T} P^{\frac{1}{2}}\left(I-P^{\frac{1}{2}} A_{i-1}\left[\left(P^{\frac{1}{2} A} A_{i-1}\right)^{T} P^{\frac{1}{2} A_{i-1}}\right]^{-1}\left(P^{\frac{1}{2}} A_{i-1}\right)^{T}\right) P^{\frac{1}{2}} a_{i} \\
& =\bar{a}_{i}^{T}\left(I-\bar{A}_{i-1}\left(\bar{A}_{i-1}^{T} \bar{A}_{i-1}\right)^{-1} \bar{A}_{i-1}^{T}\right) \bar{a}_{i},
\end{aligned}
$$

where $P^{\frac{1}{2}}$ is the square root of the weight matrix and the overbars indicate normalization by $P^{\frac{1}{2}}$.

Let

$$
S_{i-1}=I-\bar{A}_{i-1}\left(\bar{A}_{i-1}^{T} \bar{A}_{i-1}\right)^{-1} \bar{A}_{i-1}^{T}
$$

Then

$$
c_{i i} \quad=\bar{a}_{i}^{T} S_{i-1} \bar{a}_{i}
$$

or, since $S$ is idempotent,

$$
c_{i i}^{2}=\left(S_{i-1} \bar{a}_{i}\right)^{T}\left(S_{i-1} \bar{a}_{i}\right)=\left|S_{i-1} \bar{a}_{i}\right|^{2}
$$

The matrix $S_{i-1}$ is a projector onto the orthogonal complement of the sub-space spanned by the columns of $\overline{\bar{A}}_{i-1}$. It may also be viewed as a projector in the space with metric $P$ onto the orthogonal complement of the sub-space spanned by the columns of $A_{i-1}$. The number $c_{i j}^{2}$ is the square of the length of the projection of $\bar{a}_{i}$ onto this space. We also have

$$
n_{i i}=\gamma_{i}=a_{i}^{T} P a_{i}=\left|\bar{a}_{i}\right|^{2}
$$

The Googe number $g_{i}=c_{i j}^{2} / n_{i i}$ can now be given the following interpretation: When $g_{i}=0$, the $i$ th column of the observation equations is a linear combination of the first i-1 columns; when $g_{i}=1$, the $i$ th column is orthogonal to the first i-s columns. Thus $g_{i}$ is a measure of the independence of the $i$ th column from the first i-1 columns. Geometrically, it may be interpreted as the square of the sine of the angle $\alpha$ in the following drawing:


Since $0 \leq g_{i} \leq 1$, the magnitude of $g_{i}$ is independent of the number of unknowns, number of observations, units used, or weights used. In practice, the test for $g_{i}<0.000001$ has proven to be a reliable indicator of problems in the set of data being adjusted.

## 6. TRAVIO PERFORMANCE

Figure 5 provides a general indication of the performance of TRAVIO on the NOAA CDC 6600. For small networks, about 150-200 stations can be adjusted per minute of central processor time. For larger networks, one can process 100-150 stations per minute. On the NOAA IBM 360/195, roughly two and one-half times as many stations per minute can be processed as on the CDC 6600.

The time required to adjust any given network is remarkably well approximated by a linear function of the number of stations. Other factors, of course, may be considered. The CPU time required depends very strongly on the number of observations to be processed, but the number of observations tends to be a linear function of the number of stations. The matrix profile tends to grow somewhat faster than a linear function of the number of stations, which accounts for the observation that somewhat fewer stations per minute can be processed for large networks. Other factors, such as the number of normal equation partitions, have very little effect on the CPU time, although they may affect the charges a job incurs for input/output activity.


Figure 5.--TRAVI0 run time as a function of the number of stations.

The following representative sampling of jobs (table 4) was used to construct figure 5 .

Table 4.--TRAV10 performance

| Number <br> stations | Observations/station | Profile/station | CPU seconds |
| :---: | :---: | :---: | :---: |
| 100 | 6 | 50 | 30 |
| 300 | 6 | 80 | 135 |
| 500 | 6 | 200 | 220 |
| 800 | 6 | 220 | 370 |

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## APPENDIX.--USER INSTRUCTIONS

User operating instructions, which are maintained in machinereadable form, appear on the following pages. This sample is for the IBM 360 version of the program. Instructions for the CDC 6600 version differ only in details concerning the use of control cards. In general, a TRAV deck will run the same and produce identical answers on either machine.

|  | 00000010 <br> 00000020 <br> 00000030 <br> 00000040 <br> 00000050 <br> 00000060 <br> 00000070 <br> 00000080 <br> 00000090 <br> 00000100 <br> 00000110 |
| :---: | :---: |
| DATE OF REVISION $\quad$ JANUARY, 1977 | $\begin{aligned} & 00000110 \\ & 00000120 \\ & 00000130 \end{aligned}$ |
| 1. PURPOSE | $00000140$ $00000150$ |
| to adjust a horizontal survey nethork by the method OF OBSERVATION EQUATIONS ON THE ELLIPSOID | $\begin{aligned} & 00000160 \\ & 00000170 \\ & 00000180 \end{aligned}$ |
| 2. FEATURES | $\begin{aligned} & 00000190 \\ & 00000200 \end{aligned}$ |
| TRAV10 IS HRITTEN TO BE THE PRIMARY PRODUCTION TOOL OF | 00000210 |
| THE HORIZONTAL NETHORK BRANCH OF THE MATIONAL GEODETIC | 00000220 |
| SURVEY. IT IS DESIGNED SO THAT BOTH VERY LARGE AND VERY | 00000230 |
| SHALL HETHORKS CAN BE ADJUSTED. HHILE STILL HAIMTAINING | 00000240 |
| EFFICIENT USE OF COMPUTER TIME AND CORE MEHORY. THE USER | 00000250 |
| COMMUNICATES THE SIZE OF THE HETHORK TO BE ADJUSTED TO | 00000260 |
| THE PROGRAM THROUGH A MINIMUH NUMBER OF PARAMETERS. | 00000270 |
| THE EDITING OF INPUT DATA AND ERROR MESSAGES IS DESIGNED | 00000280 |
| TO BE COMPREHENSIVE. | $00000290$ |
| 3. PROGRAM HISTORY | $\begin{aligned} & 00000310 \\ & 00000320 \end{aligned}$ |
| SInCE 1972, THE MATIONAL GEODETIC SURVEY HAS USED A SERIES | 00000330 |
| OF PROGRAHS HAMED TRAVXX ON A CDC6600, VERSION $8 / 76$ | 00000340 |
| OF TRAV10 IS AN IBM 360 VERSIOM OF TRAV10 ON THE | 00000350 |
| CDC6600, CURRENT AS OF FEBRUARY 1977. | 00000360 |
| IT IS SIMILAR TO THE CDC 6600 VERSION OF THE PROGRAN IN | 00000370 |
| MOST RESPECTS. THE PRIMARY difference being that the | 00000380 |
| POST-PROCESSOR HAS BEEN MADE AN INTEGRAL PART OF TRAV10 | 00000390 |
| AND NO LONGER REOUIRES A SEPARATE JOB STEP. | 00000400 |
| 4. PREPROCESSORS | 00000420 |
|  | 00000430 |
| TRAV10 MUST BE RUN IN CONJUNCTION HITH A PREPROCESSOR. | 00000440 |
| WHICH PERFORHS THE NAHE NUMBERING FUNCTION AND PASSES | 00000450 |
| 90 CHARACTER RECORDS TO THE MAIN PROCESSOR. | 00000460 |
| THO PREPROCESSORS ARE PROVIDED FOR THIS PURPOSE | 00000470 |
| PREPROC - THE FULL PREPROCESSOR PERFORMS A COMPLETE | 00000480 |
| EDIT OF THE IMPUT TRAVDECK, CHECKING FOR | 00000490 |
| BOTH VALID FIELD CONTENTS AND VALID DECK | 000005500 |
| STRUCTURE. | $00000510$ |
| OUKPROL NUMBERING FUNCTION ONLY. IT DOES ABSOLUTELY | 00000530 |
| NO CHECKING FOR INVALID FIELDS OR DECK STRUCTURE | 00000540 |
| ERRORS. IT SHOULD BE USED ONLY HHEN THE USER | 00000550 |
| IS ABSOLUTELY CERTAIN THAT HIS TRAVDECK CONTAINS | 00000560 |
| HO ERRORS. | 00000570 |
| 5. PROCEDURES | 00000580 00000590 |
|  | 0000060 |
| THREE PROCEDURES ARE PROVIDED | 00000610 |
| 5.1 CCTRAV10 - EXECUTES THE FULL PREPROCESSOR AND THE MAIN | 00000620 |
| PROGRAH. | 00000630 |
| 5.2 cctravo - EXECUTES THE OUICK PREPROCESSOR AND THE MAIN | 00000640 |
| PROGRAM. | 0000065 |

    5.3 CCTRAVED - EXECUTES THE FULL PREPROCESSOR ONLY. 00000660
        AND IS USED ONLY TO EDIT TRAVDECKS. 00000670
        FOR ALL THREE PROCEDURES, THE PROCEDURE STEPS ARE HAMED AS 00000680
        PREPROC - THE STEP THAT EXECUTES THE FULL PREPROCESSOR 00000700
    QUIKPROC - THE STEP THAT EXECUTES THE QUICK PREPROCESSOR 00000710
    trav10 - the step that executes the main processor travio 00000720
        00000730
        00000740
        00000750
        6. INPUT
        THE INPUT TO ALL THREE PROCEDURES IS THE STANDARD TRAVDECK,
    DESCRIBED IN THE NGS 6600 PROGRAM LIBRARY USER'S WRITE-UPS,
IN THE STANDARD OPERATING PROCEDURES OF THE HORIZONTAL
NETHORK BRANCH OF NGS, AND IN APPENDIX B.
INPUT IS PASSED TO THE PROCEDURES THROUGH ONE OF THO FILES
(BUT NOT BOTH)
- FOR 80 CHARACTER CARD Ihages
UPDATEF - FOR 100 CHARACTER IMAGES (CARD IMAGES IN CC 1-80 00000860
AND RECORD IDENTIFIERS IN EC 81-100). 00000870
00000870
00000880
EXAMPLES
FOR DATA SUBMITTED THROUGH THE RUN STREAM, INCLUDE THE CARD 00000900
//CARDIN DD * 00000910
ORA PREVIOUS 00000920
FOR DATA PREPARED WITH THE SNAPUP UTILITY IN A PREVIOUS 00000930
STEP OF THE SAME JOB, USE 00000940
//CARDIN DD DSN=++IMAGES,DISP=(OLD,DELETE) 00000950
FOR DATA PREPARED HITH R. MILLER'S ROUTINE FOR SIMULATING 00000970
THE CDC 6600 UPDATE SYSTEM ([CRJMUPD), USE 00000980
//UPDATEF DD DSN=++COMPILE,DISP=(OLD,DELETE) 00000990
7. PROGRAM SIZE - NUMBER OF STATIONS. 00001010
7.1 THE FULL PREPROCESSOR IS FIXED IN SIZE AT 1500 STATIONS. 00001020
IF IT IS NECESSARY TO PROCESS MORE STATIONS, SEE THE 00001040
PROGRAMHING STAFF TO HAVE A SPECIAL VERSION OF THE 00001050
PREPROCESSOR COMPILED.
00001060
PREPROCESSOR COMPILED. 00001070
7.2 THE OUICK PREPROCESSOR AND THE TRAV10 PROGRAM ARE VARIABLE 00001080
IN SIZE, SO THAT THE NUMBER OF STATIONS WHICH CAN BE PROCESSED00001090
IS LIMITED ONLY BY THE SIZE OF THE REGION IN HHICH THE 00001100
PROGRAH RUNS. 00001110
00001110
00001120
8. CALCULATING REGION SIZE
00001130
00001140
ANY JOB EXECUTING ONE OF THE PROCEDURES SHOULD CONTAIN THE REGION 00001150
PARAMETER ON THE JOB CARD, HHERE THE REGION SIZE FOR EACH 00001160
PROCEDURE IS COMPUTED AS DESCRIBED BELOW 00001170
8.1 CCTRAVED - USE REGION=140K 00001190
8.2 CCTRAVO - USE ONE OF THE TWO METHODS BELOH 00001200
8.2.1 COMPUTE THE REGION SIZE IN UNITS OF K (I:E..UNITS 00001230
OF 1024 BYTES) FROH THE FORHULA 00001240
REGION SIZE $=$ HS +90 K
HHERE 90K IS THE PROGRAM SIZE (INCLUDING ALL CODE, FIXED
0001260
LENGTH ARRAYS, BUFFERS, ACCESS METHOD ROUTINES, LOADER 00001270
ROUTIMES, ETC.) FOR VERSION $8 / 76$ OF PROGRAM TRAV10, 00001280
AMD HS IS THE AMOUNT OF HORK SPACE NEEDED BY TRAV10. 00001290
00001290
00001300
00001310


| 2500 | 392 | 00001980 |
| :--- | :--- | :--- |
| 3000 | 453 | 00001990 |
|  |  | 00002000 |
|  | 00002010 |  |

8.3 CCTRAVIO 00002020

USE THE LARGER OF 140K AND THE REGION SIZE COMPUTED ACCORDING 00002030 TO THE METHODS OF PARAGRAPH 8.2 .2 00002040

00002050
9. PARAMETERS PASSED TO THE PROCEDURES

THE FOLLOHING KEYHORD PARAHETERS CAN BE PASSED TO THE PROCEDURES 00002070.
GPS - THE NUMBER OF GP'S IN THE JOB 00002080
OBS - THE NUMBER OF OBSERVATIONS 00002100
PROFILE - THE PROFILE OF THE NORMAL EOUATION MATRIX 00002110
DEFAULT VALUES ARE GPS $=50,0 B S=300$, PROFILE $=100000002130$
THESE PARAMETERS ARE USED ONLY FOR THE CALCULATIOM OF THE
AUXILIARY (DISK ARERAGE MEEDED HHILE RUNMING THE PROGRAH, 00002160
BUT HAVE NO EFFECT ON THE PROGRAHS THEMSELVES OR THE JOB 00002170
CLASS UNDER HHICH THE PROGRAH HILL RUN. THEREFORE, TO PREVENT 00002180
THE PROGRAMS FROM TERMINATING DUE TO INSUFFICIENT DISK SPACE, 00002190
IT IS HISE TO BE GENEROUS IN ESTIMATING THESE PARAMETERS. 00002200
THE PROFILE PARAMETER IS NOT PERTINENT AMD SHOULD BE OMITTED when 00002210
RUNHING THE PROCEDURE CCTRAVED. 00002220
10. OUTPUT

00002230
00002240
ERROR MESSAGES FROM THE PROGRAMS ARE LISTED IM APPEMDIX C.
00002270
IF REQUESTED BY A ' 1 ' PUNCH IN CC 70 OF THE OPTION CARD, 00002280 TRAV10 PRODUCES A FILE OF GP CARD IHAGES CONTAINING ADJUSTED 00002300 GP'S. THIS FILE IS PASSED TO SUBSEQUENT STEPS IN THE SAHE JOB, 00002310
but is LOSt AT THE END OF THE JOB. TO ACCESS THIS FILE IN 00002320
A SUBSEQUENT JOB STEP, USE
00002330
I/YOURDDMAME DD DSN $=++$ NEWGPS.DISP=0LD 00002340
00002350
00002360
00002370
00002380
11.1 PERFORM AN ADJUSTMENT OF A TRAVDECK WHICH EXISTS ON CARDS, 00002390 HITH 30 GP'S AND 500 OBSERVATIONS. ALREADY EDITED. 00002400 //JOBNAHE JOB ACCOUNTING INFO......REGION=100K,TIME=1 00002410 $1 /$ EXEC CCTRAVO,GPS=30,0BS=500,PROFILE=1000 00002420 I/CARDIN DD * 00002430

TRAVDECK 00002450
00002450
$\begin{array}{ll}1 * & 00002470 \\ 11 & 00002480\end{array}$
00002490
11.2 A TRAVDECK IS IN THE MEMBER MAMED DATA01 OF THE CATALOGUED00002500 DATASET MAMED MOS.MGS.MYDATA. THE JOB HAS 200 GPS, 2000 OBS, 00002510 AND A PROFILE OF 5000 ELEMENTS. CORRECTIONS ARE TO BE HADE 00002520 USING SMAPUP. AND AN ADJUSTHENT DONE. THE MEW GP CARDS ARE 00002530 TO BE HRITTEM ON TAPE. . 00002540 //JOBMAME JOB ACCOUNTIMG IMFO.....REGIOM=200X,TIME=1 00002560 // EXEC SMAPUP,DATASET='MOS.NGS.MYDATA(DATAO1)' 00002570 //SYSIM DD

SWAPUP DIRECTIVES
J. 00002600

11 EXEC CCTRAV10,GPS=200.OBS=2000,PROFILE=5000 00002610
//CARDIM DD DSM $=++$ IMAGES.DISP=OLD 00002620
// EXEC PGM=IEBGEMER 00002630

```
//SYSUT1 DO DSN=&+MENGPS,DISP=OLD
//SYSUTZ DO UNIT=TAPE9.VOL=SER=XXXXX,DCB=:.SYSUT1.DISP=NEH
//SYSIN DD DUMNY
//SYSPRIMT DD SYSOUT=A
/1
```

12. special features - - special features of the prograh hith hhich the user should be familiar are discussed below.
12.1 REORDERIMG OF UMKMONHS - - THE PROGRAM DEALS HITH THO differemt orderings of the stations. The first, known as input order, is the order in hhich the stations appear in the gr card portion of the imput travoeck. the second ORDERIMG. THE ORDER OF ELIMINATION, IS DETERMIMED BY THE PROGRAH. FOR THE MOST PART, USERS MEED NOT BE CONCERMED hith the order of elimination or even that a second ordering exists. host messages are keyed to the input order or the stations. only the message *e=singular solution*** **:SOLUTIOM BROKE DOHN AT STATION XXX: :
*******EXECUTION TERHIMATIMG********, USES THE ORDER OF elimimation to identify the station (in the field XXX). the CORRESPONDENCE BETWEEN THE IMPUT ORDER AND THE ORDER OF elimimation is given hith the observatiomal sumaray amd solvability amalysis.
the reordering of the unknohns is perforhed to reduce the mumber of computations involved in solving the mormal equations, and 15 based on the hethod in moan techmical memoramdun mos mgs-4 reducing the profile of sparse symbetric matrices,' by richard a. smay.

00002640 00002650 00002660 00002670 00002680 00002690 00002700 00002710 00002720 00002730 00002740 00002750 00002760 00002770 00002780 00002790 00002800 00002810 00002820 00002830 00002840 00002850 00002860 00002870 00002880 00002890 00002900 00002910 00002920 00002930 00002940 00002950 UMLESS SUPPRESSED BY A ' 11 P PUMCH IN CC 63 OF THE OPTIOM CARD00002960 REORDERING HILL PROCEED AUTOMATICALLY. USE OF THE REORDER 00002970 feature is generally recohmended. The payoff. In terhs of 00002980 REDUCING THE RUMNING TIME FOR SOLVIMG A GIVEM METHORK, IS 00002990 MARGIMAL FOR SHALL METHORKS BUT EXTREMELY SIGMIFICANT FOR 00003000 LARGE METHORKS (UHLESS THE IHPUT ORDER ALREADY REPRESENTS 00003010 an order hhich himimizes the profile of the normal eguationoooe3020 COEFFICIEMT MATRIX). 00003030 00003040
12.2 TABLE OF CONMECTIONS

AS A BY-PRODUCT OF THE REORDERIMG PROCESS, A TABLE OF CONMECTIONS IS BUILT AMD DISPLAYED TO THE USER. THE ITEHS DISPI.AYED FOR EACH STATION ARE 00003050 IHPNT ORDER EACH STATIDI ARE 00003060 IMPUT ORDER 00003070 order of elimimation 00003080
00003100
COMPOMENT TO HHICH THE STATIOM BELONGS 00003110
HUMBER OF UNIOUE DIRECTIONS ORIGIMATIMG FROM THE STATION 00003120 (SIMgle direction lists are hot counted)
MUMBER OF UNIOUE DIRECTIONS HHICH SEE THE STATION 00003140
(SIMGLE DIRECTION LISTS ARE MOT COUNTED)
mubber of umioue azimuths having the station at one end
(ASTRO AZIMUTHS MUST HAVE ASTRO LONGI TUDE)
00003160
mumber of uhioue distances havimg the station at one end. 00003180
SOLVABILITY MOTE (SEE BELOW)
00003170
00003190
the hord unidue above means that observations of the same
kind over the sahe lime are couwted owly once.
am elementary solvability amalysis is performed at each 00003240
STATIOM. THE AMALYSIS IS BASED SOLELY ON THE UNIOUE MUMBER00003250
AMD KIMDS OF OBSERVATIONS INVOLVIMG THE STATION. THE RESULT00003260
OF THE AMALYSIS IS POSTED II THE TABLE OF CONHECTIONS 00003270
WHENEVER OME OF THE FOLLOHING THREE COMDITIONS IS MET 00003280
FIXED STATIOM

NO-CHECK STATION 00003300
UNDETERMINED STATION 00003310
DUE TO THE SIMPLIIITY OF THE AHALYSIS. THERE MAY BE 00003320
STATIONS WHOSE POSITION IS NOT DETERMENED OR IS NOT 00003340
OVERDETERMINED BUT HHICH ARE NOT FLAGGED. 00003350
A COMPONENT IN TRIANGULATION IS A SUBNETWORK WITHIN HHICH 00003380
EVERY POINT HAS A PATH TO EVERY OTHER POINT. NORMALLY. 00003390
ONE ATTEMPTS TO ADJUST A SINGLE NETHORK, OR A SINGLE 00003400
COMPONENT: HOHEVER, BECAUSE OF MISSING OBSERVATIONS, THE 00003410
nETHORK MAY ACTUALLY BREAK DOHN INTO THO SUBNETHORKS. 00003420
WHEN THIS HAPPENS EITHER NEW OBSERVATIONS MUST BE SUPPLIED00003430
OR FIXED CONTROL, SCALE, AND ORIENTATION HUST BE SUPPLIED 00003440
FOR EACH COMPONENET. 00003450
12.4 POSITIOMAL CONSTRAINTS.
POSITIONS ARE CONSTRAINED AT THEIR INPUT GP BY HEIGHTED 00003480
CONSTRAINTS. THE HEIGHTS ARE COMPUTED FROM THE STANDARD 00003490
deviations in latitude and longitude given on the 00003500
CONSTRAINED POSITION CARD. HOHEVER, IF THE STANDARD 00003510
DEVIATION FIELD FOR EITHER LATITUDE OR LONGITUDE IS 00003520
BLANK OR ZERO, A STANDARD DEVIATION OF 0.0000000001 00003530
SECONDS OF ARC HILL BE ASSIGNED. THUS EFFECTIVELY. 00003540
FIXING THE COORDINATE. 00003550
EVERY OBSERVATION TAKES PART In THE ADJUSTHENT AS LONG AS 00003580
BOTH END POINTS ARE IN THE GP SECTION OF THE TRAVDECK AND00003590
WITH THE EXCEPTION OF SINGLE DIRECTION ABSTRACTS AMD 00003600
UNREASONABLE OBSERVATIONS. UNREASOMABLE OBSERVATIONS 00003610
WHICH MAY BE REJECTED INCLUDE THE FOLLOWING 00003620
1. ASTRO AZIMUTHS WITH LAPLACE CORRECTIONS IN EXCESS 00003630
OF 10 MINUTES OF ARC.
2. MARK-TO-MARK DISTANCES HITH ENDPOINT ELEVATION 00003650
DIFFERENCES GREATER THAN OR EOUAL TO THE DISTAMCE00003660
ITSELF. 00003670
3. MARK-TO-MARK DISTANCES FOR WHICH BOTH THE 00003680
ORTHOMETRIC AND GEOID HEIGHT ARE NOT AVAILABLE 00003690
FOR BOTH ENDS OF THE LINE. 00003700
REJECTED OBSERVATIONS ARE FLAGGED BY THE MESSAGE 00003710
DELETED OBSERVATION+++++++++++++++++++ 00003720
00003730
00003740
GP SECTIOM OF THE TRAVDECK. THE OBSERVATION LS FLAGGED BY 00003750
THE MESSAGE 00003760
DELETED OBSERVATION ******************* 00003770
DELETION OF DIRECTIONS OFTEN RESULTS IN ONLY A SINGLE 00003780
ACTIVE DIRECTION REMAINING. HHICH IS THEN REJECTED AS A 00003790
SINGLE DIRECTION LIST. SINGLE DIRECTION LISTS ARE NOT 00003800
FLAGGED EXPLICITLY, BUT THEIR OBSERVATION SEOUEWCE , 00003810
$\begin{array}{lll}\text { FLAGGED EXPLICITLY, BUT THEIR OBSERVATION SEOUENCE } & 00003810 \\ \text { NUMBER IS REASSIGNED TO THE MEXT OBSERVATION }\end{array}$
NUHBER IS REASSIGNED TO THE NEXT OBSERVATION 00003820
12.6 WEIGHTS 00003840
THE HEIGHT ASSOCIATED HITH AN OBSERVATION IS THE IHVERSE 00003850
OF THE SQUARE OF THE STANDARD ERROR OF THE OBSERVATION. 00003860
THE STANDARD ERROR IS EITHER PUNCHED ON THE OBSEVATION 00003870
CARD OR COMES FROM INTERMAL DEFAULTS. THE DEFAULT 00003880
HEIGHTING SCHEME IS DOCUMENTED ON THE FIRST PAGE OF THE 00003890
TRAV10 OUTPUT. 00003900
12.7 ABSTRACTS 00003920
MULTIPLE ABSTRACTS OF DIRECTIONS AT THE SAHE STATION 00003930
MAY BE USED, BUT MUST BE DISTINGUISHED BY VARYING THE 00003940
LIST NUMBER. ALL DIRECTIONS IN AN ABSTRACT MUST BE 00003950

TOGETHER IN THE INPUT, BUT DIFFERENT ABSTRACTS AT THE SAME STATION CAN BE SEPARATED BY OTHER ABSTRACTS.
12.8 ASTRONOMIC LONGITUDES

ALL ADJUSTMENTS ON HAD 1927 USE ASTRONOHIC LONGITUDES
REFERRED TO THE U.S. NAVAL OBSERVATORY. SIMCE ASTRO LONGITUDES OBSERVED AFTER JAN 1, 1962 ARE BASED ON THE 1968 BIH SYSTEH, THE PROGRAM ADDS 0.51 SECONDS to all input longitudes.
12.9 TRIANGLE CLOSURES TRIANGLE CLOSURES ARE NOT COHPUTED
12.10 MISCLOSURES
an attempt has been made to screen out truly TROUBLESOME OBSERVATIONS BY DISPLAYING THOSE FOR WHICH THE 'OBSERVED MINUS COMPUTED' TERM IS LARGE. THE PROGRAM COMPUTES LINEAR MISCLOSURES FOR ALL OBSERVATIONS. FOR ANGULAR OBSERVATIONS, THE LINEAR MISCLOSURE IS GIVEN BY D*TAN(L), HHERE D IS THE LINE LENGTH and L is the angular hisclosure (OBSERVED MIMUS COMPUTED TERH).
the follohing rules govern the printing of MISCLOSURES

FOR ANGULAR OBSERVATIONS, PRINT THOSE FOR WHICH 1. ANGULAR MISCLOSURE IS GREATER THAM 30 SECONDS 00004210 2. LINEAR MISCLOSURE IS GREATER THAN 5 METERS. 00004230 00004230 FOR LINEAR OBSERVATIONS, PRINT THOSE FOR HHICH 00004250 1. THE MISCLOSURE IS GREATER THAN 0.5 METER AND 00004260 THE DISTANCE IS LESS THAH 500 METERS, OR 00004270
2. THE MISCLOSURE IS GREATER THAN 5 METERS AND 00004280 THE DISTANCE IS GREATER THAN 500 HETERS. 00004290
IN ADDITION, ANY MISCLOSURE GREATER THAN 10000 METERS 00004310 (SIGNIFYING GROSS BLUNDERS IN INPUT POSITIONS) 00004320 WILL TERMINATE THE RUN WITH THE MESSAGE 00004330 RUN ABORTED DUE TO EXCESSIVE N-TERHS************* 00004340 00004350 00004360
12.11 ACCURACIES

TRAV10 HILL COMPUTE THE STANDARD DEVIATION OF THE ADJUSTED AZIMUTH AND DISTANCE BETHEEN ANY PAIR OF POINTS, AS REQUESTED II THE ACCURACY REQUEST PURTION OF THE TRAVDECK. IT IS NOT NECESSARY THAT THERE BE ANY ACTUAL OBSERVATIONS BETHEEN THE THO POINTS. Standard deviations of coordinates are mot computed.
12.12 INPUT THE STRUCTURE OF THE INPUT DECK IS DESCRIBED IN APPENDIX B. 00004370 00004380 00004390 00004400 00004410 00004420 00004430 00004440 00004450 00004460 00004470 00004480 00004490 00004500 00004510 00004520 00004530 00004540 00004550 00004560 00004570 00004580 00004590
B. 1 TRAV DECK FORMAT SPECIFICATIONS :\#******** 00004600 00004610


APPENDIX C - THE FULL PREPROCESSOR - PREPROC
C. 1 GENERAL FLOW
PREPROC IS A THO PASS PROGRAM. ERROR MESSAGES ARE NOTED IM SEC.
2.2 FOR EACH PASS THROUGH THE DATA. ALL MESSAGES NOTED ARE
fatal. pass 1 messages cause immediate termination of editing
at the end of that section. any messages concerwing improper
DECK STRUCTURE (MESSAGES 1 THROUGH 10 OF PASS 1) HILL SHOH THE
FIRST 10 ERRORS. IF THE NUMBER OF ERRORS EXCEED 10 THE OUTPUT I
CANCELLED (THE HESSAGE WILL GIVE TOTAL COUNT OF RECORDS IM
ERROR) ALL OTHER MESSAGES ARE DESCRIBED IN MESSAGES 11 AND 12.00006060
THESE ARE NOTED BY UNDERSCORING BY X FOR FATAL AND H FOR HARNING. 00006070
OURING PASS 2. MESSAGES 1 AND 2 HILL TERHINATE CHECKING AT THE 00006090
END OF THE GEOGRAPHIC POSITIONS. MESSAGES OF TYPE 1 AND 3 00006100
PRODUCED AFTER THE GPS HILL CAUSE TERMINATION AT END OF DATA. 00006110
these messages are noted in sec. 2:3.
PREPROC CAN ACCEPT BOTH CARD DECKS AND UPDATE FORMAT DECKS
ERRORS ARE FLAGGED AND THE CARD INPUT SEQUENCE NUMBER IS GIVEN
OR THE UPDATE SEQUENCE NUMBER IS GIVEN TO ALLOW FOR EASIER
CORRECTION OF ERRORS.
C. 2 ERROR MESSAGES
2.1 SEVERITY OF ERRORS
H - HARNING ISSUED, BUT EXECUTION WILL CONTINUE
F - fatal. deck scanning continues. but no output file
(TRAVIN) HILL BE GENERATED. A DUMP HILL BE CALLED AFTER
SCANHING DECK FOR ADDITIONAL ERRORS.
2.2 MESSAGES - PASS I
1.OPTION CARD IN ERROR
2. HO GPS IH DECK
3.GPS OUT OF ORDER OR ALL (NO. OF GPS) IN ERROR.
4. NO FIXED POSITIONS IM DECK
5. FIXED POSITIONS OUT OF ORDER OR ALL (NO. OF FIXED POSITIONS)00006340
FIXED POSITIONS IN ERROR.
6.ASTRO DATA OUT OF ORDER OR ALL (NO. OF ASTRO POSITIONS) 00006360
ASTRO POSITIONS IN ERROR
7.DIRECTIONS OUT OF ORDER OR ALL (HO. OF DIRECTIONS)
DIRECTIONS IN ERROR.
8. AZIMUTHS OUT OF ORDER OR ALL (NO. OF AZIMUTHS) AZIMUTHS IN
ERROR.
9.DISTANCES OUT OF ORDER OR ALL (NO. OF DISTANCES) DISTANCES
IN ERROR.
10. IMPROPER DECK STRUCTURE PREMATURE END OF DATA.
10. IMPROPER DECK STRUCTURE PREMATURE EMD OF DATA.
11. FATAL ERRORS IN DATA HAVE TERHINATED ANY FURTHER PROCESSING 000006450
OF THIS JOB. PASS I OF PROGRAM HILL LIST THE FATAL ERRORS 00006460
GENERATED BY EACH SECTION OF THE INPUT FILE. THESE ERRORS 00006470
are flagged by underscoring an X in the column in error. 00006480
2.3 MESSAGES - PASS 11 MAME FIELD 00006500
1. ILLEGAL CHARACTER IM FIRST TWO CHARACTERS OF NAME FIELD.
THIS MESSAGE DESCRIBES THE ERROR IH THE NAME FIELD OF THE
PREVIOUSLY LISTED DATA RECORD. THE FIELDS WHERE POSSIBLE
Illegal characters may be are underscored by an X. The job 00006540
IS TERMIMATED IF THE ERROR OCCURS IN THE GP DECK AT THE END 00006550
OF THE GPS.
2. DUP GP IN DECK. FATAL ERROR, STOPS PROCESSING BEFORE 00006570
OBSERVATIOMS ARE NUMBERED.
3.FROM AMD TO STATIOH SAME. ANY OBSERVATION WITH THE MAME 00006580
00006590
OBSERVATIOMS ARE NUMBERED.
3.FROM AMD TO STATIOH SAME. ANY OBSERVATION WITH THE MAME 00006580
00006590

FIELDS EOUAL (EXCEPT BLANK NAMES IN ACCURACY CARDS) HILL 00006600 tERMIHATE ON A FATAL ERROR.

NOTE
ALL DATA RECORDS LISTED AS ERRORS SHOW A SEOUENCE NUMBER OF 20 CHARACTERS ON THE RIGHT.
A.CARD DECK INPUT - SEQ. NO. IS SEQUENTIAL HITHIN INPUT DECK.
b.update file - SEO. NO. ShOHS the deck name and record numbers relative to the input deck.

## C. 3 NAMES

GP CARDS, FIXED POSITION CARDS AND ASTRO POSITION CARDS HAVE ONE HAME ONLY IN CC7-36. OTHER CARD TYPES HAVE THO HAMES, A FROH-STATION-NAME (FSN) IN CC7-30 AND A TO-STATION-NAME (TSN) IN CC37-66. NOTE THAT FSNS ARE ALHAYS 24.OR FEWER CHARACTERS IN LENGTH. IT IS UNDERSTOOD THAT FSNS ARE PADDED WITH SIX BLANKS On the right before comparison hith the table of names.

NAMES MUST BEGIN IN THE PROPER COLUMNS OR F ERRORS WILL OCCUR. NAMES MUST ALSO START SECOMD CHARALSO START WITH A LETTER (A-2) OR NUHBER (O-9) AND THE 00006800 ONE OF THE FOLLOWING SPECIAL CHARACTERS

## BLANK

PERIOD
HYPHEN
or else the message illegal character in name field will be DISPLAYED ALONG WITH A FATAL ERROR FLAG.
C. 4 SPECIFICATIONS FOR PREPROC FIELD CHECKING BY CARD TYPE
4.1 OPTION CARD

| COLUMN ALLOWABLE CONTENTS | SEVERITY |
| :--- | :--- | :--- |
| 07 | BLANK |

### 4.2 GP CARD

COLUMN 01
07-08 LEGAL HAME CHARS
37--38 INTEGER ) $=90$
39--40
41--47
(NTEGER ) $=60$
48--50
51--52
INTEGER $)=360$
INTEGER ) $=60$
F

53--59 INTEGER $)=6000000$
60--63 SIGNED INTEGER (*)
64--65 INTEGER OR SC
66--68 SIGNED INTEGER (*)
69
SIGNED INTEGER (*)
INTEGER
F
70--72 ALLOHABLE PCZ (**)
H
W
OR BLANK
79--80 ALLOHABLE CLASS/
ORDER CODE H
(**) SEE TABLE Of Valid state plane coordinate zones
(:E*)MUST CONTAIN VALUE EXCEPT WHEN CLASSIORDER CODE EOUALS
34. 45. OR 49



## APPENDIX D - TRAV10 OUTPUTS

D. 1 OUTPUT from the input and adjustment phases

1. STANDARD ERRORS AND OPTIONS USED IN THIS RUN
2. INPUT STATION POSITIONS, ELEVATIONS, ETC.
3. FIXED POSITIONS WITH THEIR ASSIGNED SEQUENCE NUMBER
4. LIST OF INPUT ASTRONOMICAL POSITIONS
5. INPUT DIRECTIONS: COMPUTED CORRECTIONS FOR DEFLECTION OF THE VERTICAL, NORMAL SECTION TO GEODESIC CORRECTION. AND SKEH NORMAL ( FOR ELEVATION OF THE FOREPOINT): CORRECTED DIRECTION. DIRECTIONS FOR HHICH BOTH END POINTS ARE NOT IN THE GP LIST ARE DELETED. SINGLE DIRECTION LISTS ARE DELETED.
6. OBSERVED AZIMUTHS: LAPLACE CORRECTION: GEODETIC AZIMUTH. AZIMUTHS FOR HHICH BOTH ENDS OF THE LIME ARE NOT IN THE GP LIST ARE DELETED HITH ASTERIKS IN THE ELLIPSOIDAL AZIMUTH FIELD. AZIMUTHS FOR WHICH THE LAPLACE CORRECTION IS LARGER THAM 600 SECONDS OF ARC (HHICH IS USUALLY AN INDICATION THAT THE ASTRONOHIC LONGITUDE EITHER WAS NOT INPUT OR WAS INPUT INCORRECTLY) ARE DELETED WITH PLUS SIGNS IN THE ELLIPSOIDAL AZIMUTH FIELD.
7. INPUT DISTANCES: CORRECTIONS FOR REFRACTION AND GEOID HEIGHT: ELLIPSOIDAL GEODESIC DISTANCE. IF BOTH END POINTS OF THE LINE ARE NOT IN THE GP LIST, THE DISTANCE IS DELETED HITH ASTERISKS IN THE ELLIPSOIDAL DISTANCE FIELD. FOR MARK TO MARK DISTANCES, IF BOTH STATIONS DO NOT HAVE BOTH ELEVATIONS AND GEOID HEIGHTS, OR IF THE HEIGHT difference is greater than the hark to mark DISTANCE BETHEEN THE POINTS, THE OBSERVATION IS deleted hith plus signs in the ellipsoldal distance FIELD.
8. parameterization of hormal equatiom structure and size. OBSERVATIONAL SUMMARY AND SOLVABILITY AMALYSIS. ORDER OF ELIMIMATION OF EACH STATION AFTER REORDERIMG OF UNKHOWMS.

00007920 00007930 00007940 00007950 00007960 00007970 00007980 00007990 00008000 00008010 00008020 00008030 00008040 00008050 00008060 00008070 00008080 00008090 00008100 00008110 00008120 00008130 00008140 00008150 00008160 00008170 00008180 00008190 00008200 00008210 00008220 00008230 00008240 00008250 00008260 00008270 00008280 00008290 00008300 00008310 00008320 00008330 00008340 00008350 00008360 00008370 00008380 00008390 00008400 00008410 00008420 00008430 00008440 00008450 00008460 00008470 00008480 00008490 00008500 00008510 00008520 00008530 00008540 00008550 00008560 00008570
SUMHARY BY COMPONENT. 00008580
9. FOR EACH ITERATION 00008590
A. OBSERVATIONS FOR WHICH THE VOBSERVED HINUS COMPUTEDV ..... 00008600
TERM IS LARGE. TERMS WHICH HILL BE PRINTED ARE 00008610
00008620DISCUSSED IN SECTION 12.1000008620
B. RHS CORRECTION TO LATITUDE AND LONGITUDE (IN SECONDS ..... 00008630
of arc). Variance of Unit weight and degrees of freedom.10. STANDARD DEVIATIOH OF THE AZIMUTH AND DISTANCE BETHEEN PAIRSOF POINTS. AS REQUESTED IN THE ACCURACY SECTION OF THE 0000866000008650travdeck.
OUTPUT... THE PROGRAM OUTPUTS THE FOLLOHING IMFORMATION IN ALL CASES. 00008710
1. ADJUSTED POSITIONS IN DEGREES, MINUTES, AND SECONDS, 00008720
HITH STATION SEQUENCE NUMBER, G-NUMBER, NAME, ELEVATION, 00008730
GEOID HEIGHT, STATE PLANE COORDINATE ZONE(S). AND ORDERI 00008740
TYPE.
00008750
2. ADJUSTED OBSERVATIONS, WITH SEOUENCE MUHBER, FROH- AND 00008760
TO-STATION NUMBERS AND NAMES, WEIGHT, RESIDUAL, RESIDUAL 00008770
TIMES SQUARE ROOT OF WEIGHT, AND ORIGINAL OBSERVATION 00008780
(SECONDS ONLY, IN THE CASE OF DIRECTIONS AND AZIMUTHS). 00008790
SEE OPTION 11 BELOH
00008800
3. MAXIMUH RESIDUAL, MEAN RESIDUAL, AND MEAN ABSOLUTE VALUE00008810
OF RESIDUAL, FOR EACH CATEGORY OF OBSERVATION REPRESENTED00008820
IN THE ADJUSTHENT, FOR LONG AND SHORT LINES. 00008830
4. THE NUMBER OF NO-CHECK OBSERVATIONS. 00008840
5. NUMBER OF OBSERVATIONS, MAXIMUM AND MINIMUH NORHALIZED 00008850
RESIDUAL AND RANGE, MEAN NORMALIZED RESIDUAL, MEAN ABSO- 00008860
LUTE VALUE OF NORMALIZED RESIDUAL.
00008870
6. SEQUENCE NUMBERS OF OBSERVATIONS HITH NORMALIZED RESI- 00008880
DUALS GREATER THAN 2. IF THERE ARE HORE THAN 20 SUCH 00008890
OBSERVATIOHS, ONLY THE 20 HITH THE GREATEST ABSOLUTE 00008900
VALUE OF NORMALIZED RESIDUAL ARE PRINTED.
7. VALUE OF TAU, USED FOR COMPARISOM HITH NORHALIZED
- Value of tau. used for comparison hith norhalized
RESIDUALS (SEE NOTE ON REJECTS BELOW). 00008930
8. VARIANCE OF THE UNIT WEIGHT, DEGREES OF FREEDOM. AND 00008940
ACCEPTABLE RANGE OF VARIANCE USING A CHI-SQUARE TEST. 00008950
9. STATION HAMES IN ALPHABETICAL ORDER, HITH THEIR SEQUENCE00008960
NUMBERS.
0000897
00008980
... OPTIONAL OUTPUT (ONLY IF SIGNALS IN INPUT ARE ACTIVAIED).
10. OLD FREE STATIONS HAVE G-MUMBER CHANGED.
11. MORMALLY ALL ADJUSTED OBSERVATIONS ARE PRINTED.
00009000
00009000
OPTIOMALLY, ONLY THOSE OBSERVATIONS FOR WHICH THE 00009020
00009010
NORHALIZED RESIDUAL IS GREATER THAN 1 ARE PRINTED. 00009030
12. FOR EVERY OLD FREE STATION. THE DIFFERENCES BETHEEN
INPUT AND ADJUSTED POSITIONS IN LATITUDE, LONGITUDE.
INPISTANCE, AND AZIMUTH, ARE PRINTED.
13. FOR EVERY INTERSECTION STATION. ALL OBSERVATIONS HHICH 00009070
have that station as a to-station are printed. Including 00009080
TYPE OF OBSERVATION. SEQUENCE MUMBER. FROH- AND TO- 00009090
STATION NUMBERS AND NAMES, RESIDUAL, FORHARD AZIMUTH AND 00009100
DISTANCE.
14. FOR EVERY PAIR OF STATIOMS BETHEEN WHI.CH OBSERVATIONS
EXIST, THE FROM AND TO STATION NAHES ARE PRINTED 00009130
00009110
00009120
TOGETHER HITH THE ADJUSTED FORWARD AZIMUTH, BACK 00009140
AZIMUTH, AMD DISTAMCE.
15. ALL IMFORHATION MEMTIOMED IN POINT 1 UNDER BASIC
OUTPUT (EXCEPT SEQUEMCE NUMBER) IS WRITTEN TO DATA SET
MENGPS IN GP-CARD FORMAT.
00009150
00009160
00009170
00009180
00009190
REJECTS... MO DATA IS REJECTED IM POSTPRC. HOHEVER, THE RESIDUALS ARE 00009200
COMPARED AGAIMST A TAU VALUE WHICH IS A FUNCTION OF THE
00009210
VARIAMCE, DEGREES OF FREEDOH, AHD NUMBER OF OBSERVATIONS 00009220
IM THE ADJUSTHEMT (FORMULATION BY A. POPE). IF ANY MORMAL- 00009230

```
I2ED RESIDUAL (RESIDUAL TIMES SQUARE ROOT OF WEIGHT) IS 00009240
GREATER THAN TAU, IT IS MARKED HITH AN ASTERISK NEXT TO 00009250
THE PRINTED NORMALIZED RESIDUAL IN SECTION 2 ABOVE. THIS. }0000926
MEANS THE OBSERVATION SHOULD BE LOOKED AT. THE OBSERVATION 00009270
IS N O T REMOVED FROM THE DATA FILE BY THE PROGRAM. 00009280
    FURTHER PROCESSING OF THE JOB. THE JCL LOG HILL INDICATE A }0000932
    COMPLETION CODE OF USER 240, HHICH IS ISSUED BY THE FORTRAN 00009330
    ERROR MONITOR. THE IHN240I MESSAGE HILL INDICATE THE USER 0}0000934
    COMPLETION CODE ISSUED BY TRAV10. THE MEANINGS OF THESE CODES 00009350
    ARE
    101 THE KORE ROUTINE CANNOT OBTAIN CENTRAL PROCESSOR MEMORY 00009380
    SPACE. THIS MESSAGE SHOULD NEVER BE ISSUED. IF IT OCCURS, 00009390
    SEE THE PROGRAMMING STAFF.
    EQUATION PARTITIONS FROM MASS STORAGE. SEE THE PROGRAMHING 00009430
    STAFF.
    EQUATION PARTITIONS FROH HASS STORAGE. SEE THE PROGRAMMING 00009470
    STAFF. 00009480
    00009490
    00009500
    ON MASS STORAGE. THE ERROR IS PROBABLY DUE TO A HARDWARE (00009510
    PROBLEM ON THE DISK. RESUBHIT THE JOB. IF THE ERROR PERSISTS,00009520
    SEE THE PROGRAMMING STAFF. 000009530
    200 TRAV10 PURPOSELY ABORTS A RUN BECAUSE OF INSUFFICIENT 00009550
    HORK SPACE OR BECAUSE OF A DIVERGING OR SINGULAR SOLUTION. 00009560
    THE MESSAGE vTHE FOLLOWIMG DUMP IS STRICTLY IHTENTIONALV 00009570
    IS PRODUCED ON THE OUTPUT LISTING. THE EXACT REASON FOR THE 00000580
    ABORT IS FOUND PRECEDING THE JOB STATISTICS. }0000959
    IF THE REASON IS INSUFFICIENT SPACE, INCREASE THE REGION 00009600
    PARAMETER ON THE JOB CARD AND RESUBHIT THE JOB. THE OTHER 00009610
    ERROR MESSAGES INDICATE THAT THE PROBLEH LIES HITH THE 00009620
    DATA. SEE SECTION D.4.
    201 THE HERESI ROUTINE DID NOT HAVE ENOUGH SPACE TO GEMERATE
        A BACK SOLUTION. THIS IS A PROGRAM LOGIC ERROR AND SHOULD
        00009680
        301 LOGIC ERROR IN THE REORDER ALGORITHM. SEE THE 00009690
        PROGRAMMING STAFF.
D.4 DATA DEPENDENT MESSAGES
    4.1 TRAV10 DID NOT FIND ANY DATA.
        SELF EXPLANATORY. USUALLY CAUSED BY A JCL ERROR SUCH THAT THE
        INPUT TRAVDECK IS NOT PROPERLY PASSED TO TRAV10.
    4.2 ERROR
    TOO MANY POSITIONS
    THE MAXIMUM NUMBER OF POSITIONS IS XXX
    THE REGION SIZE UNDER WHICH THE PROGRAM IS RUNMING IS TOO
        SHALL TO SUPPORT EVEN THE INPUT OF GP CARDS. RERUN THE JOB
        IN A LARGER REGION. SEE SECTION & FOR A GUIDE TO CALCULATING
        THE PROPER REGION SIZE.
    4.3 ERROR
    SYSTEM LACKS DEGREES OF FREEDOM, ADJUSTMENT IMPOSSIBLE. 00009880
    00009890
```



|  | THEM. | $\begin{aligned} & 00010560 \\ & 00010570 \end{aligned}$ |
| :---: | :---: | :---: |
| 4 | ERROR | 00010580 |
|  | RUM ABORTED DUE TO EXCESSIVE M-TERMS. | 00010590 |
|  |  | 00010600 00010610 |
|  | SEE SECTION 12.10 FOR THE DEFIMITION OF EXCESSIVE N-TERMS | 00010610 00010620 |
|  |  | 00010620 |
| 4.11 | ERROR | 00010640 |
|  | SIMGUlar solution | 00010650 |
|  | SOLUTION BROKE DOWN AT STATION XXXX LATITUDE (OR LONGITUDE) | 00010660 |
|  |  | 00010670 |
|  | THE SOLUTION BREAKS DOWN DURING THE REDUCTION OF THE MORMAL | 00010680 |
|  | EOUATIOMS, Indicating a singular systeh caused by one | 00010690 |
|  | Or more undetermined coordiimates. This condition may arise | 00010700 |
|  | BECAUSE OF A SUBTLE TRUE SINGULARITY OR BECAUSE OF WEAK | 00010710 |
|  | GEOMETRY. FIND THE CAUSE OF THE SIMGULARITY AND REMEDY | 00010720 |
|  | BY ADDIng more data or constraints. The station number | 00010730 |
|  | XXXX REFERS TO THE ORDER OF ELIMINATION OF THE STATIONS. | 00010740 |
|  |  | 00010750 |
| 4.12 | SLOWLy converging or diverging solution. | 00010760 |
|  |  | 00010770 |
|  | CHECK FOR BAD PRELIMINARY POSITIONS. THIS CONDITION MAY | 00010780 |
|  | also be caused by a combination of critical geometry and | 00010790 |
|  | UnREALISTIC HEIGHTS, PARTICULARLY OVER SHORT LINES. | 00010800 |
|  | UnREALISIIC WEIGTS. PARTICJLakly OVER SHORT LINES. | 00010810 |
| ***** | *\#\#*************************さ**************************** | 00010820 |

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## (Continued from inside front cover)

NOAA Technical Memorandums National Ocean Survey National Geodetic Survey subseries

NOS NGS-1 Use of climatological and meteorological data in the planning and execution of National Geodetic Survey field operations. Robert J. Leffler, December 1975, 30 p. (PB249677). Availability, pertinence, uses, and procedures for using climatological and meteorological data are discussed as applicable to NGS field operations.

NOS NGS-2 Final report on responses to geodetic data questionnaire. John F. Spencer, Jr., March 1976, 39 p. (PB254641). Responses (20\%) to a geodetic data questionnaire, malled to $36,000 \mathrm{U}$. S. land surveyors, are analyzed for projecting future geodetic data needs.

NOS NGS-3 Adjustment of geodetic field data using a sequential method. Marvin C. Whiting and Allen J. Pope, March 1976, 11 p. (PB253967). A sequential adjustment is adopted for use by NGS field parties.

NOS NGS-4 Reducing the profile of sparse symmetric matrices. Richard A. Snay, June 1976, 24 p. (PB258476). An algorithm for improving the profile of a sparse symetric matrix is introduced and tested against the widely used reverse Cuthill-McKee algorithm.

NOS NGS-5 National Geodetic Survey data: availability, explanation, and application. Joseph F. Dracup, June 1976, 45 p. (PB258475). This publication summarizes the data and services available from NGS, reviews survey accuracies, and illustrates how to use specific data.

NOS NGS-6 Determination of North American Datum 1983 coordinates of map covers. T. Vincenty, October 1976, 8 p. (PB262442). Predictions of changes in coordinates of map corners are detailed.

NOS NGS-7 Recent elevation change in Southern California. S.R. Holdahl, February 1977, 19 p. (PB265940). Velocities of elevation change have been determined from Southern Calif. leveling data for 1906-62 and 1959-76 epochs.

NOS NGS-8 Establishment of calibration base lines. Joseph F. Dracup, Charles J. Fronczek, and Raymond W. Tomlinson, August 1977, 22 p. (PB277130). Specifications are given for establishing calibration base lines.
(Continued on following page)

NOS NGS-9 National Geodetic Survey Publications on surveying and geodesy 1976. September 1977. 17 p. (PB275181). This compilation lists publications authored by NGS staff in 1976 , sources of availability of out-of-print Coast and Geodetic Survey publications, and information on subscriptions to the Geodetic Control Data Automatic Mailing List.

NOS NGS-10 Use of calibration base lines. Charles J. Fronczek, December 1977, 38 p. A detailed explanation is given for evaluating electronic distance measuring instruments.

NOS NGS-11 Applicability of Array Algebra. Richard A. Snay, February 1978, 22 p. Conditions required for the transformation from matrix equations into computationally more efficient array equations are considered.

NOAA Technical Reports National Ocean Survey National Geodetic Survey Subseries

NOS 65 NGS 1 The statistics of residuals and the detection of outliers. Allen J. Pope, May 1976, 133 p. (PB258428). A criterion for rejection of bad geodetic data is derived on the basis of residuals from a simultaneous least-squares adjustment; subroutine TAURE is included.

NOS 66 NGS 2 Effect of Geoceiver observations upon the classical triangulation network. R. E. Moose, and S. W. Henriksen, June 1976, 65 p. (PB260921). The use of Geoceiver observations is investigated as a means of improving triangulation network adjustment results.

NOS 67 NGS 3 Algorithms for computing the geopotential using a simplelayer density model. Foster Morrison, March 1977, 41 p. (PB265421). Several algorithms are developed for computing the gravitational attraction with high accuracy of a simple-density layer at arbitrary altitudes. Computer program is included.

NOS 68 NGS 4 Test results of first-order class III leveling. Charles T. Whalen and Emery Balazs, November 1976, 30 p. (PB265421). Specifications for releveing the National vertical control net were tested and the results published.

NOS 70 NGS 5 Selenocentric geodetic reference system. Frederick J. Doyle, Atef A. Elassal, and James R. Lucas, February 1977, 53 p. (PB266046). Reference system was established by simultaneous adjustment of 1,244 metric-camera photographs of the lunar surface from which 2,662 terrain points were positioned.
(Continued on inside back cover)

NOS 71 NGS 6 Application of digital filtering to satellite geodesy. C. C. Goad, May 1977, 73 p. (PB270192). Variations in the orbit of GEOS-3 were analyzed for $M_{2}$ tidal harmonic coefficient values which perturb the orbits of artificial satellites and the Moon.

NOS 72 NGS 7 Systems for the determination of polar motion. Soren W. Henriksen, May 1977, 55 p. Methods for determining polar motion are described and their advantages and disadvantages compared.

