

An investigation of factors affecting inferences from and reliability of state-space age-structured assessment models: Supplementary Materials

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Referenced Figures

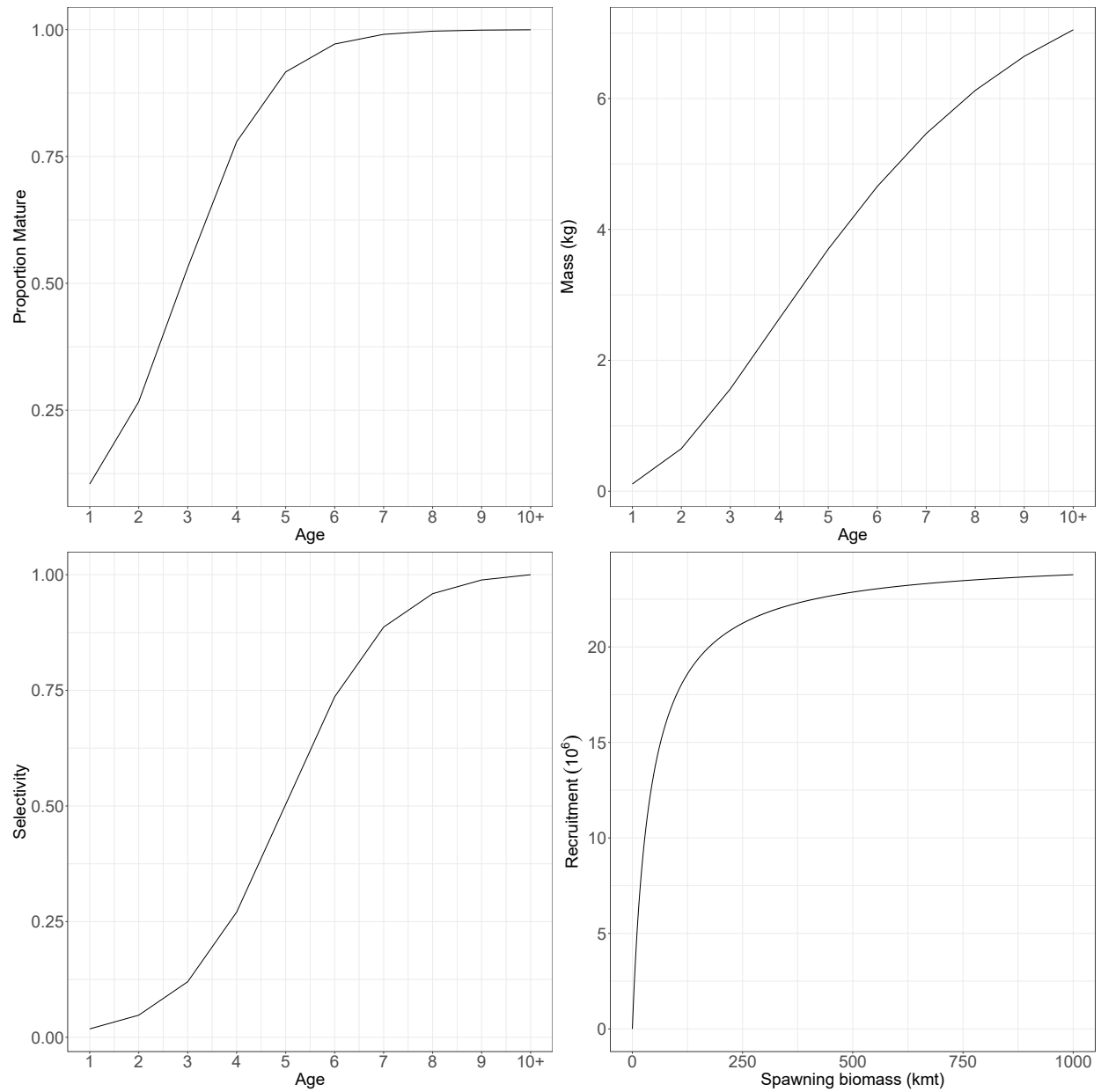


Fig. S1. The proportion mature at age, weight at age, fleet and index selectivity at age, and Beverton-Holt SRR assumed for the population in all OMs. For OMs with random effects on fleet selectivity, this represents the selectivity at the mean of the random effects.

Fig. S3. The maximum of the absolute values of all gradient values for all fits that provided Hessian-based standard errors across all simulated data sets of a given OM configuration (A: R and R+S, B: R+M, C: R+Sel, or D: R+q). Results are conditional on EM fits with alternative process error assumptions (colored points and lines), median natural mortality rate (estimated or known) and recruitment assumptions (Beverton-Holt SRR or not). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

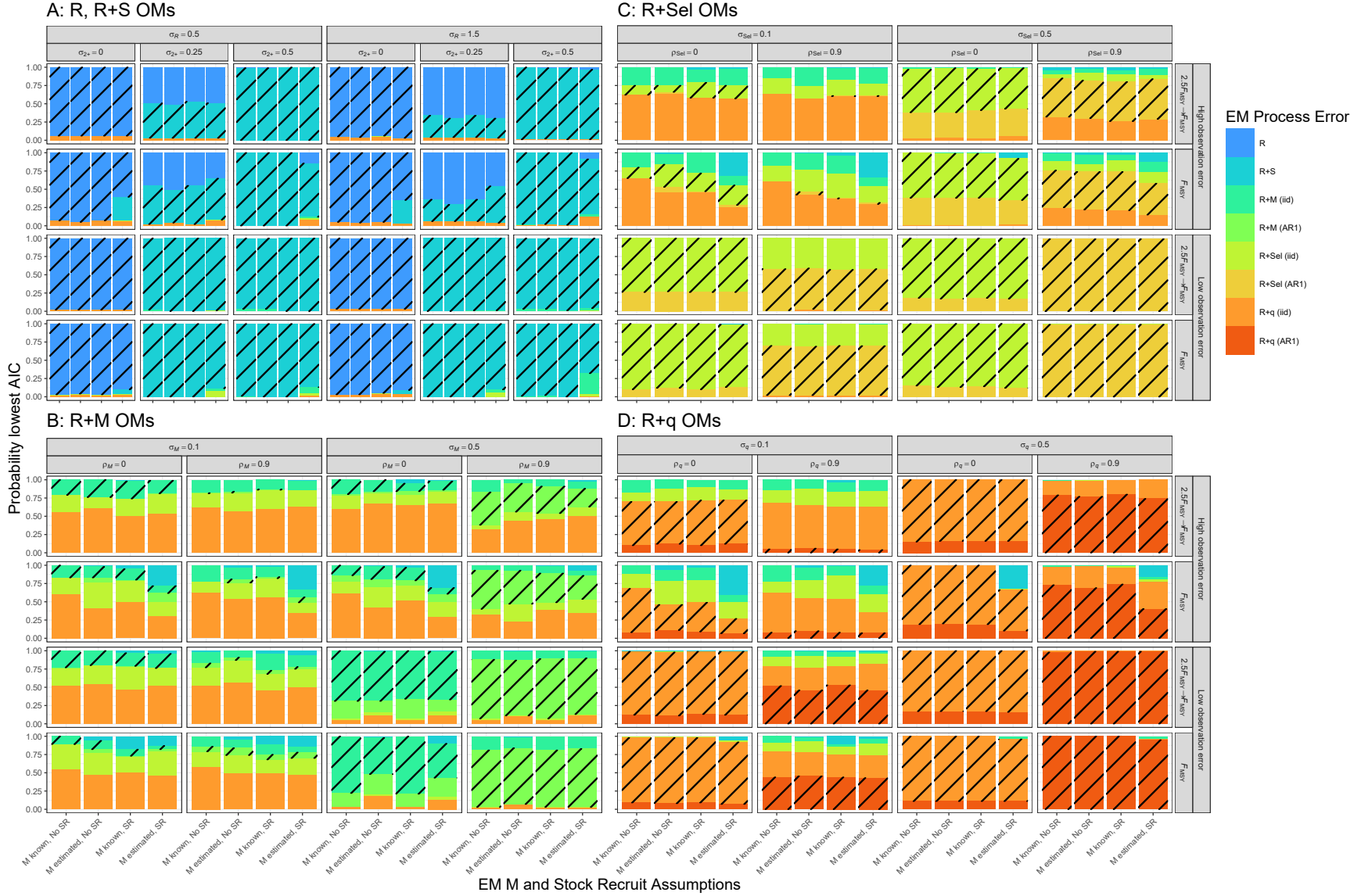


Fig. S4. Estimated probability of lowest AIC for EMs assuming alternative process error assumptions (colored bars) conditional on alternative assumptions for median natural mortality rate (estimated or known) and Beverton-Holt SRR (estimated or not; along x-axis) when fitted to OMs that have R and R+S (A), R+Sel (B), R+M (C), or R+q (D) process error sources. Striped bars indicate results where the EM process error structure matches that of the OM.

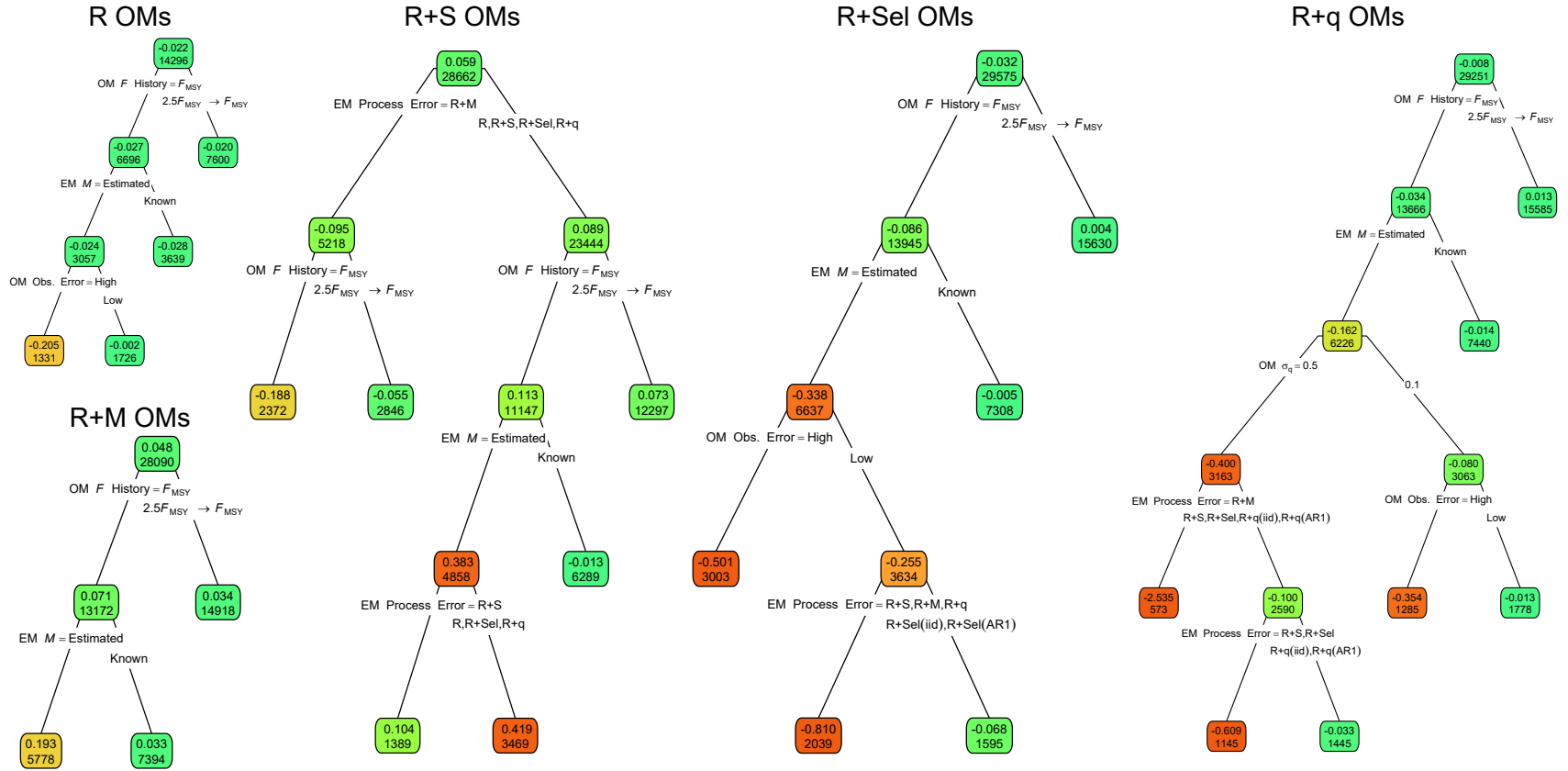


Fig. S5. Regression trees indicating primary factors determining reductions in sums of squares of errors in estimation measured by Eq. 3 for terminal year fully-selected fishing mortality for R+S, R+M, R+Sel and R+q OM. Each node shows the median error (top) and number of observations (bottom) for the corresponding subset. Branch labels indicate the levels of OM and EM factors distinguishing each subset and left branch labels are positioned higher and note the factor. Median errors closer to or further from zero are indicated by more green or red polygons, respectively.

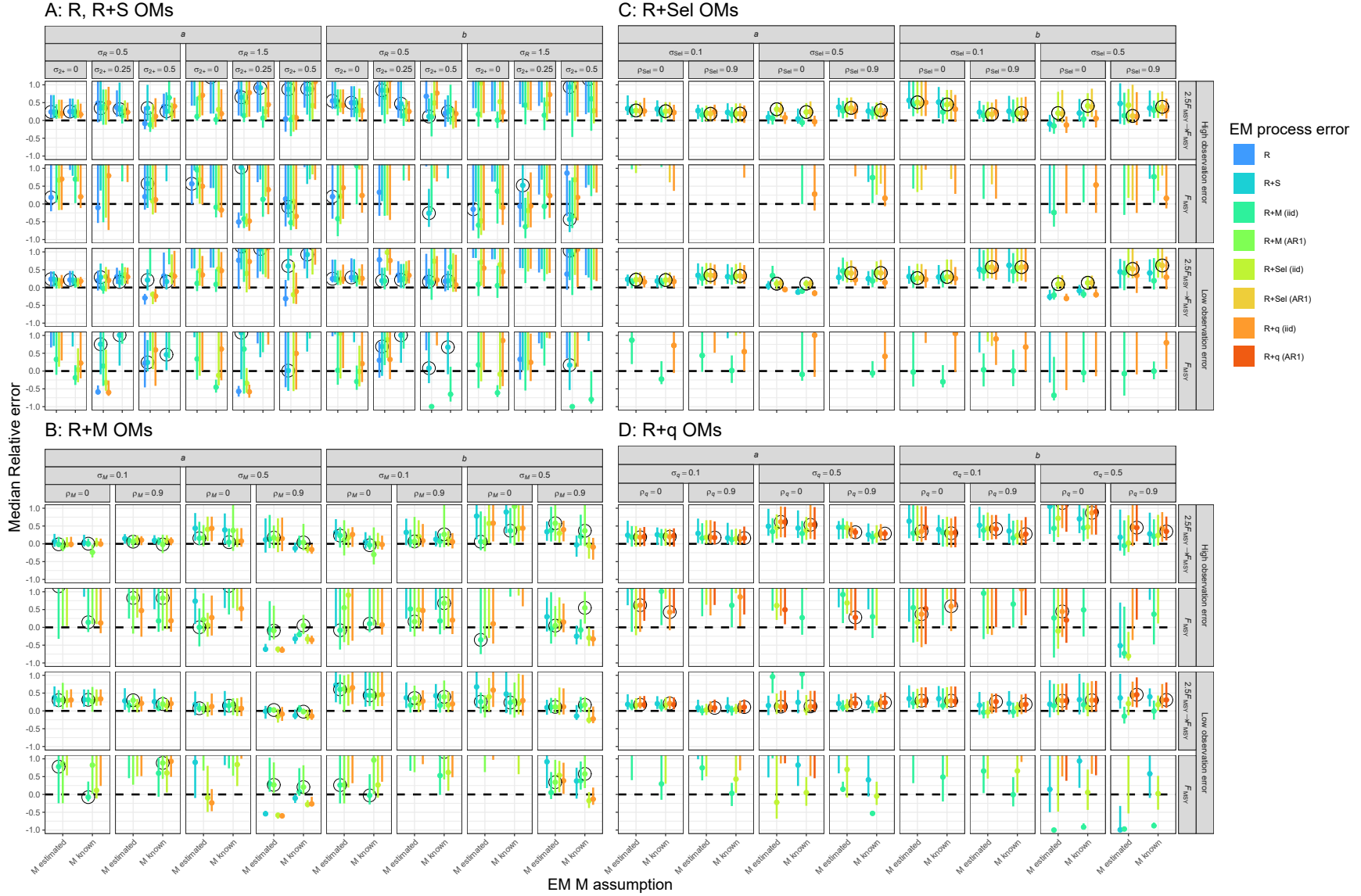


Fig. S7. Median relative error of Beverton-Holt SRR parameters ($a = \alpha$ and $b = \beta$) for EMs fitted to data sets simulated with alternative process error structures: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

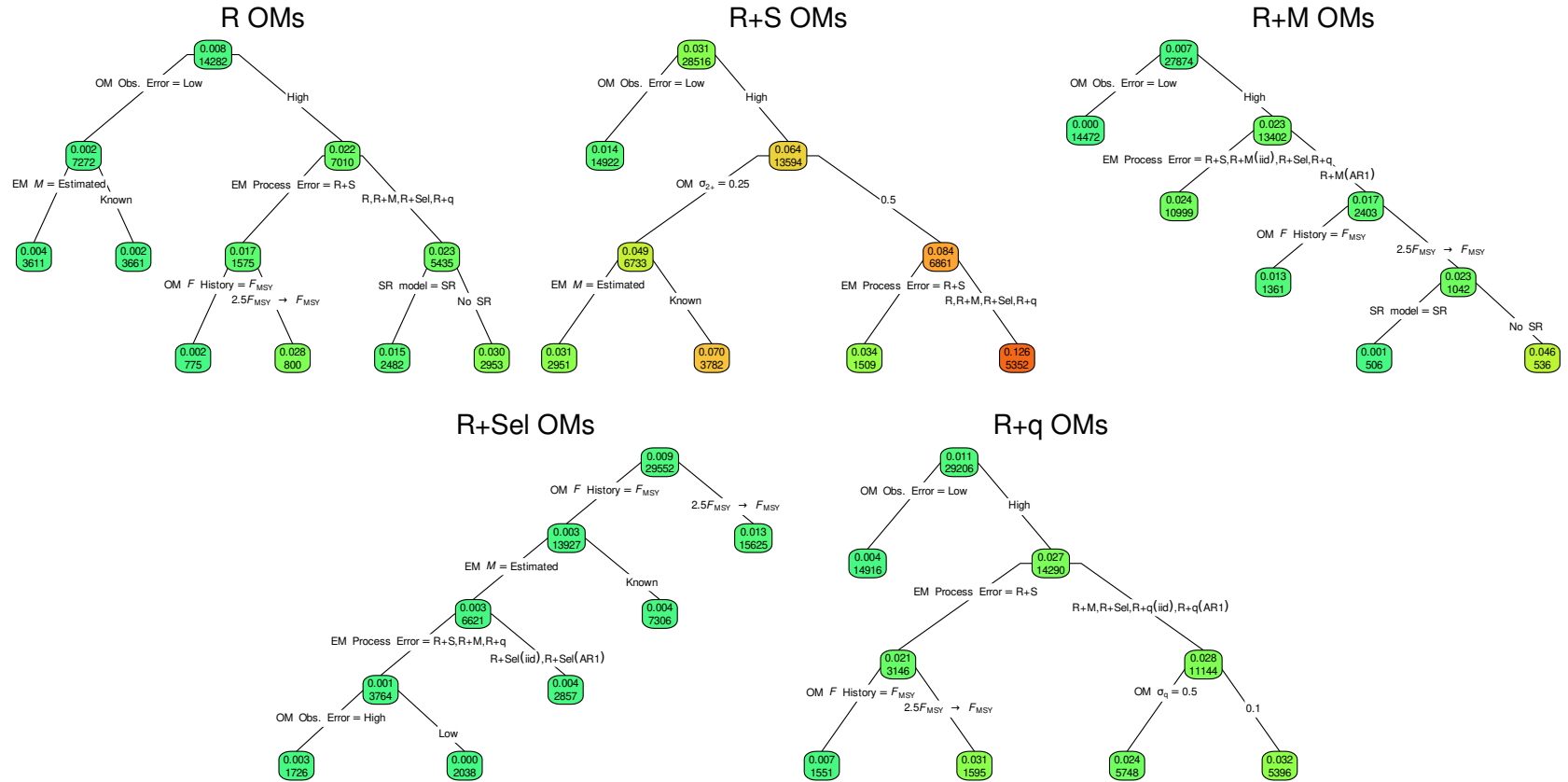


Fig. S8. Regression trees indicating primary factors determining reductions in sums of squares of errors in transformed Mohn's ρ (Eq. 3) for fishing mortality averaged over all age classes for R+S, R+M, R+Sel and R+q OMs. Each node shows the median Mohn's ρ (top) and number of observations (bottom) for the corresponding subset. Branch labels indicate the levels of OM and EM factors distinguishing each subset and left branch labels are positioned higher and note the factor. Median Mohn's ρ closer to or further from zero are indicated by more green or red polygons, respectively.

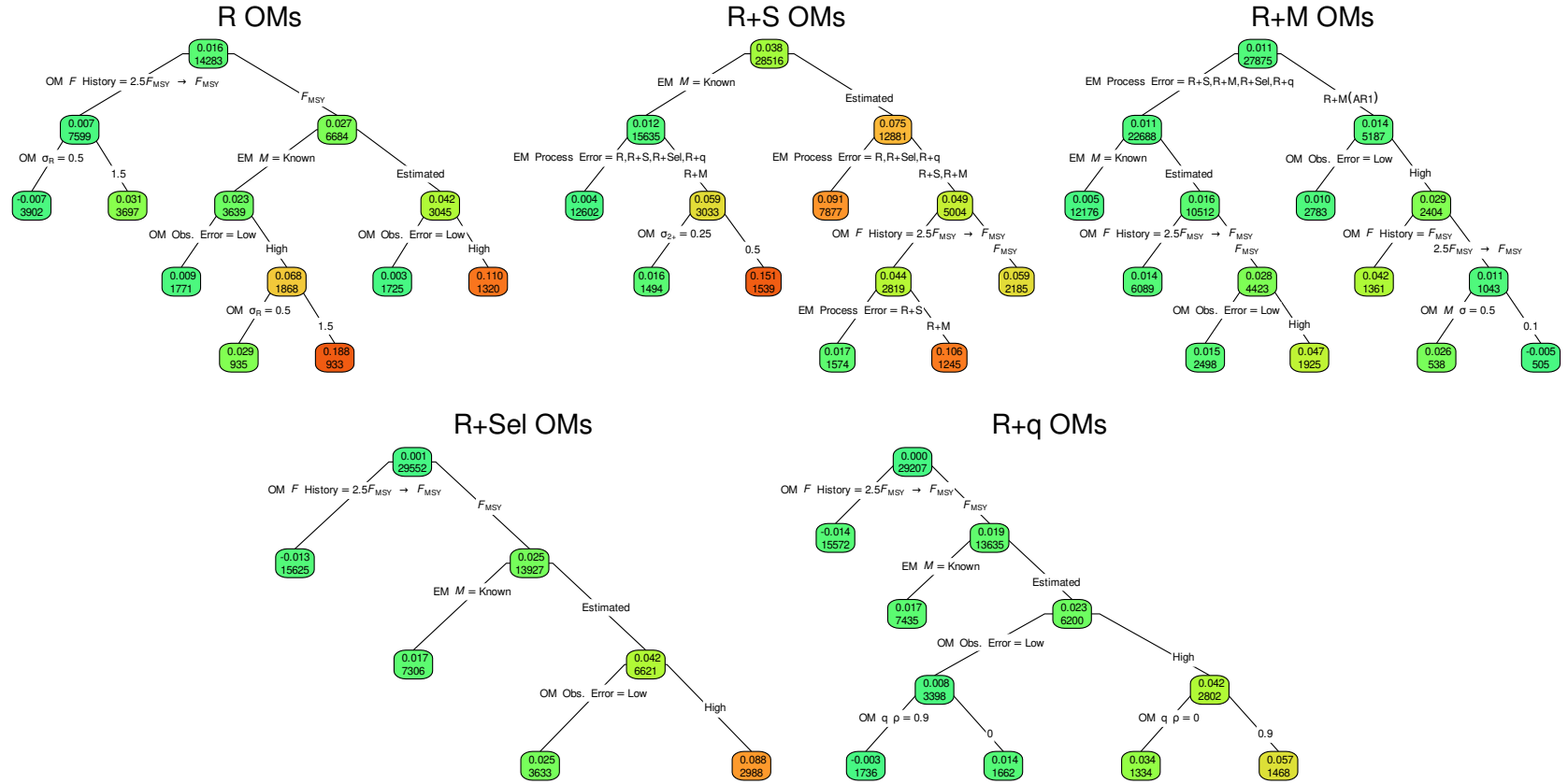


Fig. S9. Regression trees indicating primary factors determining reductions in sums of squares of errors in transformed Mohn's ρ (Eq. 3) for recruitment for R+S, R+M, R+Sel and R+q OMs. Each node shows the median Mohn's ρ (top) and number of observations (bottom) for the corresponding subset. Branch labels indicate the levels of OM and EM factors distinguishing each subset and left branch labels are positioned higher and note the factor. Median Mohn's ρ closer to or further from zero are indicated by more green or red polygons, respectively.

Referenced Tables

Table S1. Distinguishing characteristics of the OMs with random effects on recruitment and apparent survival (R, R+S). When observation uncertainty is low, standard deviations for log-normal distributed indices and logistic normal distributed age composition observations are 0.1 and 0.3, respectively, and when it is high, standard deviations are 0.4 and 1.5, respectively. Fishing mortality either changes from $2.5F_{\text{MSY}}$ to F_{MSY} after year 20 (of 40) or is constant at F_{MSY} over all years.

Model	σ_R	σ_{2+}	Fishing History	Observation Uncertainty
1	0.5		$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
2	1.5		$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
3	0.5	0.25	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
4	1.5	0.25	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
5	0.5	0.50	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
6	1.5	0.50	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
7	0.5		F_{MSY}	Low
8	1.5		F_{MSY}	Low
9	0.5	0.25	F_{MSY}	Low
10	1.5	0.25	F_{MSY}	Low
11	0.5	0.50	F_{MSY}	Low
12	1.5	0.50	F_{MSY}	Low
13	0.5		$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
14	1.5		$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
15	0.5	0.25	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
16	1.5	0.25	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
17	0.5	0.50	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
18	1.5	0.50	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
19	0.5		F_{MSY}	High
20	1.5		F_{MSY}	High
21	0.5	0.25	F_{MSY}	High
22	1.5	0.25	F_{MSY}	High
23	0.5	0.50	F_{MSY}	High
24	1.5	0.50	F_{MSY}	High

Table S2. Distinguishing characteristics of the OMs with random effects on recruitment and natural mortality rate (R+M). When observation uncertainty is low, standard deviations for log-normal distributed indices and logistic normal distributed age composition observations are 0.1 and 0.3, respectively, and when it is high, standard deviations are 0.4 and 1.5, respectively. Fishing mortality either changes from $2.5F_{\text{MSY}}$ to F_{MSY} after year 20 (of 40) or is constant at F_{MSY} over all years. For AR1 process errors, σ_M is defined for the marginal distribution of the processes.

Model	σ_R	σ_M	ρ_M	Fishing History	Observation Uncertainty
1	0.5	0.1	0.0	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
2	0.5	0.5	0.0	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
3	0.5	0.1	0.9	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
4	0.5	0.5	0.9	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
5	0.5	0.1	0.0	F_{MSY}	Low
6	0.5	0.5	0.0	F_{MSY}	Low
7	0.5	0.1	0.9	F_{MSY}	Low
8	0.5	0.5	0.9	F_{MSY}	Low
9	0.5	0.1	0.0	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
10	0.5	0.5	0.0	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
11	0.5	0.1	0.9	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
12	0.5	0.5	0.9	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
13	0.5	0.1	0.0	F_{MSY}	High
14	0.5	0.5	0.0	F_{MSY}	High
15	0.5	0.1	0.9	F_{MSY}	High
16	0.5	0.5	0.9	F_{MSY}	High

Table S3. Distinguishing characteristics of the OMs with random effects on recruitment and selectivity (R+Sel). When observation uncertainty is low, standard deviations for log-normal distributed indices and logistic normal distributed age composition observations are 0.1 and 0.3, respectively, and when it is high, standard deviations are 0.4 and 1.5, respectively. Fishing mortality either changes from $2.5F_{\text{MSY}}$ to F_{MSY} after year 20 (of 40) or is constant at F_{MSY} over all years. For AR1 process errors, σ_{Sel} is defined for the marginal distribution of the processes.

Model	σ_R	σ_{Sel}	ρ_{Sel}	Fishing History	Observation Uncertainty
1	0.5	0.1	0.0	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
2	0.5	0.5	0.0	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
3	0.5	0.1	0.9	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
4	0.5	0.5	0.9	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
5	0.5	0.1	0.0	F_{MSY}	Low
6	0.5	0.5	0.0	F_{MSY}	Low
7	0.5	0.1	0.9	F_{MSY}	Low
8	0.5	0.5	0.9	F_{MSY}	Low
9	0.5	0.1	0.0	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
10	0.5	0.5	0.0	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
11	0.5	0.1	0.9	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
12	0.5	0.5	0.9	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
13	0.5	0.1	0.0	F_{MSY}	High
14	0.5	0.5	0.0	F_{MSY}	High
15	0.5	0.1	0.9	F_{MSY}	High
16	0.5	0.5	0.9	F_{MSY}	High

Table S4. Distinguishing characteristics of the OMs with random effects on recruitment and catchability (R+q). When observation uncertainty is low, standard deviations for log-normal distributed indices and logistic normal distributed age composition observations are 0.1 and 0.3, respectively, and when it is high, standard deviations are 0.4 and 1.5, respectively. Fishing mortality either changes from $2.5F_{\text{MSY}}$ to F_{MSY} after year 20 (of 40) or is constant at F_{MSY} over all years. For AR1 process errors, σ_q is defined for the marginal distribution of the processes.

Model	σ_R	σ_q	ρ_q	Fishing History	Observation Uncertainty
1	0.5	0.1	0.0	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
2	0.5	0.5	0.0	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
3	0.5	0.1	0.9	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
4	0.5	0.5	0.9	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	Low
5	0.5	0.1	0.0	F_{MSY}	Low
6	0.5	0.5	0.0	F_{MSY}	Low
7	0.5	0.1	0.9	F_{MSY}	Low
8	0.5	0.5	0.9	F_{MSY}	Low
9	0.5	0.1	0.0	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
10	0.5	0.5	0.0	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
11	0.5	0.1	0.9	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
12	0.5	0.5	0.9	$2.5F_{\text{MSY}} \rightarrow F_{\text{MSY}}$	High
13	0.5	0.1	0.0	F_{MSY}	High
14	0.5	0.5	0.0	F_{MSY}	High
15	0.5	0.1	0.9	F_{MSY}	High
16	0.5	0.5	0.9	F_{MSY}	High

Table S5. Distinguishing characteristics of the EMs and indication (+) of which OM process error sources (R, R+S, R+M, R+Sel, R+q) each EM configuration was fit.

Model	Recruitment model	Median M	Process error	R,R+S OMs	R+M OMs	R+Sel OMs	R+q OMs
1	Mean recruitment	0.2	R ($\sigma_{2+} = 0$)	+			
2	Beverton-Holt	0.2	R ($\sigma_{2+} = 0$)	+			
3	Mean recruitment	Estimated	R ($\sigma_{2+} = 0$)	+			
4	Beverton-Holt	Estimated	R ($\sigma_{2+} = 0$)	+			
5	Mean recruitment	0.2	R+S (σ_{2+} estimated)	+	+	+	+
6	Beverton-Holt	0.2	R+S (σ_{2+} estimated)	+	+	+	+
7	Mean recruitment	Estimated	R+S (σ_{2+} estimated)	+	+	+	+
8	Beverton-Holt	Estimated	R+S (σ_{2+} estimated)	+	+	+	+
9	Mean recruitment	0.2	R+M ($\rho_M = 0$)	+	+	+	+
10	Beverton-Holt	0.2	R+M ($\rho_M = 0$)	+	+	+	+
11	Mean recruitment	Estimated	R+M ($\rho_M = 0$)	+	+	+	+
12	Beverton-Holt	Estimated	R+M ($\rho_M = 0$)	+	+	+	+
13	Mean recruitment	0.2	R+Sel ($\rho_{Sel} = 0$)	+	+	+	+
14	Beverton-Holt	0.2	R+Sel ($\rho_{Sel} = 0$)	+	+	+	+
15	Mean recruitment	Estimated	R+Sel ($\rho_{Sel} = 0$)	+	+	+	+
16	Beverton-Holt	Estimated	R+Sel ($\rho_{Sel} = 0$)	+	+	+	+
17	Mean recruitment	0.2	R+q ($\rho_q = 0$)	+	+	+	+
18	Beverton-Holt	0.2	R+q ($\rho_q = 0$)	+	+	+	+
19	Mean recruitment	Estimated	R+q ($\rho_q = 0$)	+	+	+	+
20	Beverton-Holt	Estimated	R+q ($\rho_q = 0$)	+	+	+	+
21	Mean recruitment	0.2	R+M (ρ_M estimated)		+		
22	Beverton-Holt	0.2	R+M (ρ_M estimated)		+		
23	Mean recruitment	Estimated	R+M (ρ_M estimated)		+		
24	Beverton-Holt	Estimated	R+M (ρ_M estimated)		+		
25	Mean recruitment	0.2	R+Sel (ρ_{Sel} estimated)			+	
26	Beverton-Holt	0.2	R+Sel (ρ_{Sel} estimated)			+	
27	Mean recruitment	Estimated	R+Sel (ρ_{Sel} estimated)			+	
28	Beverton-Holt	Estimated	R+Sel (ρ_{Sel} estimated)			+	
29	Mean recruitment	0.2	R+q (ρ_q estimated)				+
30	Beverton-Holt	0.2	R+q (ρ_q estimated)				+
31	Mean recruitment	Estimated	R+q (ρ_q estimated)				+
32	Beverton-Holt	Estimated	R+q (ρ_q estimated)				+

Table S6. For each OM process error source (columns), percent reduction in deviance for logistic regression models fit to indicators of convergence (maximum absolute gradient $< 10^{-6}$) with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	R	R+S	R+M	R+Sel	R+q
EM Process Error	30.40	0.45	17.57	16.04	24.03
EM M Assumption	2.38	24.11	4.42	1.02	2.66
EM SR Assumption	1.80	0.32	0.96	3.38	2.13
OM Obs. Error	0.12	0.77	0.33	1.76	0.28
OM F History	3.51	6.33	2.36	5.86	5.30
OM σ_R	<0.01	<0.01	—	—	—
OM σ_{2+}	—	<0.01	—	—	—
OM σ_M	—	—	0.39	—	—
OM ρ_M	—	—	0.09	—	—
OM σ_{Sel}	—	—	—	1.08	—
OM ρ_{Sel}	—	—	—	0.01	—
OM σ_q	—	—	—	—	0.06
OM ρ_q	—	—	—	—	<0.01
All factors	43.69	35.72	29.33	34.57	40.69
+ All Two Way	50.53	42.99	43.91	45.93	48.62
+ All Three Way	52.30	48.41	46.81	47.71	50.40

Table S7. For each OM process error source (columns), percent reduction in deviance for linear regression models fit to errors in estimation measured by Eq. 3 for the terminal year fully-selected fishing mortality with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	R	R+S	R+M	R+Sel	R+q
EM M Assumption	2.26	1.33	1.26	2.93	3.26
EM SR assumption	0.11	0.07	0.08	0.07	0.09
EM Process Error	0.46	4.18	0.38	0.13	1.02
OM Obs. Error	1.61	0.06	0.86	0.41	<0.01
OM F History	2.49	3.23	1.42	3.22	4.55
OM σ_R	0.02	0.02	—	—	—
OM σ_{2+}	—	0.87	—	—	—
OM σ_M	—	—	0.16	—	—
OM ρ_M	—	—	0.01	—	—
OM σ_{Sel}	—	—	—	0.24	—
OM ρ_{Sel}	—	—	—	0.05	—
OM σ_q	—	—	—	—	1.03
OM ρ_q	—	—	—	—	0.05
All factors	7.42	9.96	4.37	7.26	10.43
+ All Two Way	17.63	25.76	10.94	13.88	22.07
+ All Three Way	22.97	37.03	14.74	17.32	30.74

Table S8. For each OM process error source (columns), percent reduction in deviance for linear regression models fit to errors in estimation measured by Eq. 3 for the terminal year recruitment with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	R	R+S	R+M	R+Sel	R+q
EM M Assumption	1.96	0.40	0.69	3.52	3.03
EM SR assumption	0.06	0.02	0.05	0.02	0.05
EM Process Error	0.39	4.74	0.41	0.12	1.16
OM Obs. Error	1.47	0.08	0.64	0.18	<0.01
OM F History	2.54	2.66	1.11	4.18	5.06
OM σ_R	0.03	0.01	—	—	—
OM σ_{2+}	—	1.05	—	—	—
OM σ_M	—	—	0.36	—	—
OM ρ_M	—	—	0.02	—	—
OM σ_{Sel}	—	—	—	0.23	—
OM ρ_{Sel}	—	—	—	0.06	—
OM σ_q	—	—	—	—	1.09
OM ρ_q	—	—	—	—	0.06
All factors	6.90	9.01	3.43	8.58	10.90
+ All Two Way	16.48	24.64	9.73	15.76	22.75
+ All Three Way	21.46	35.60	13.56	19.07	31.15

Further Results

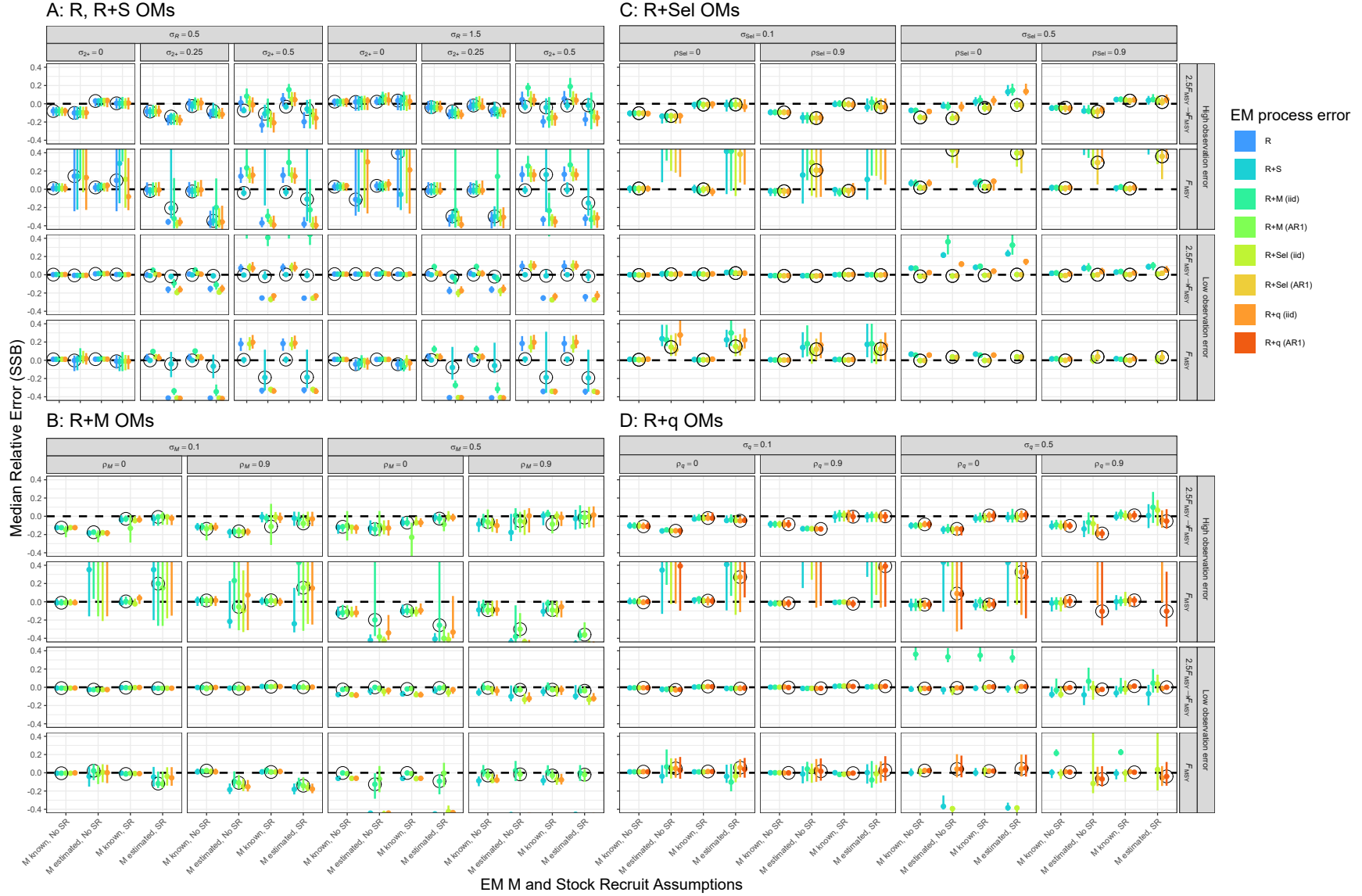


Fig. S10. Median relative error of terminal year SSB for EMs fitted to data sets simulated with alternative process error sources: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.



Fig. S11. Median relative error of terminal year fully-selected fishing mortality for EMs fitted to data sets simulated with alternative process error sources: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

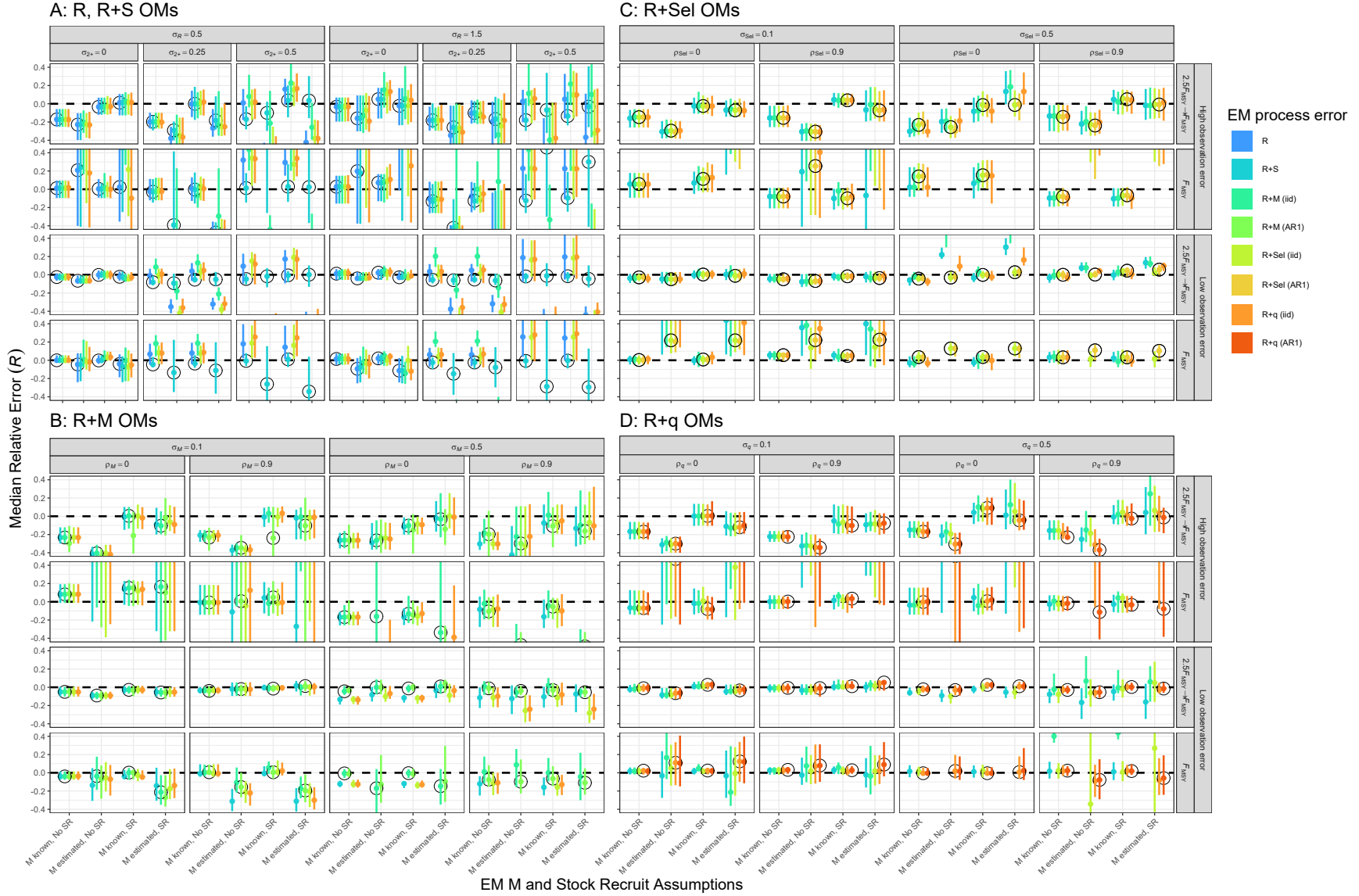


Fig. S12. Median relative error of terminal year recruitment for EMs fitted to data sets simulated with alternative process error sources: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

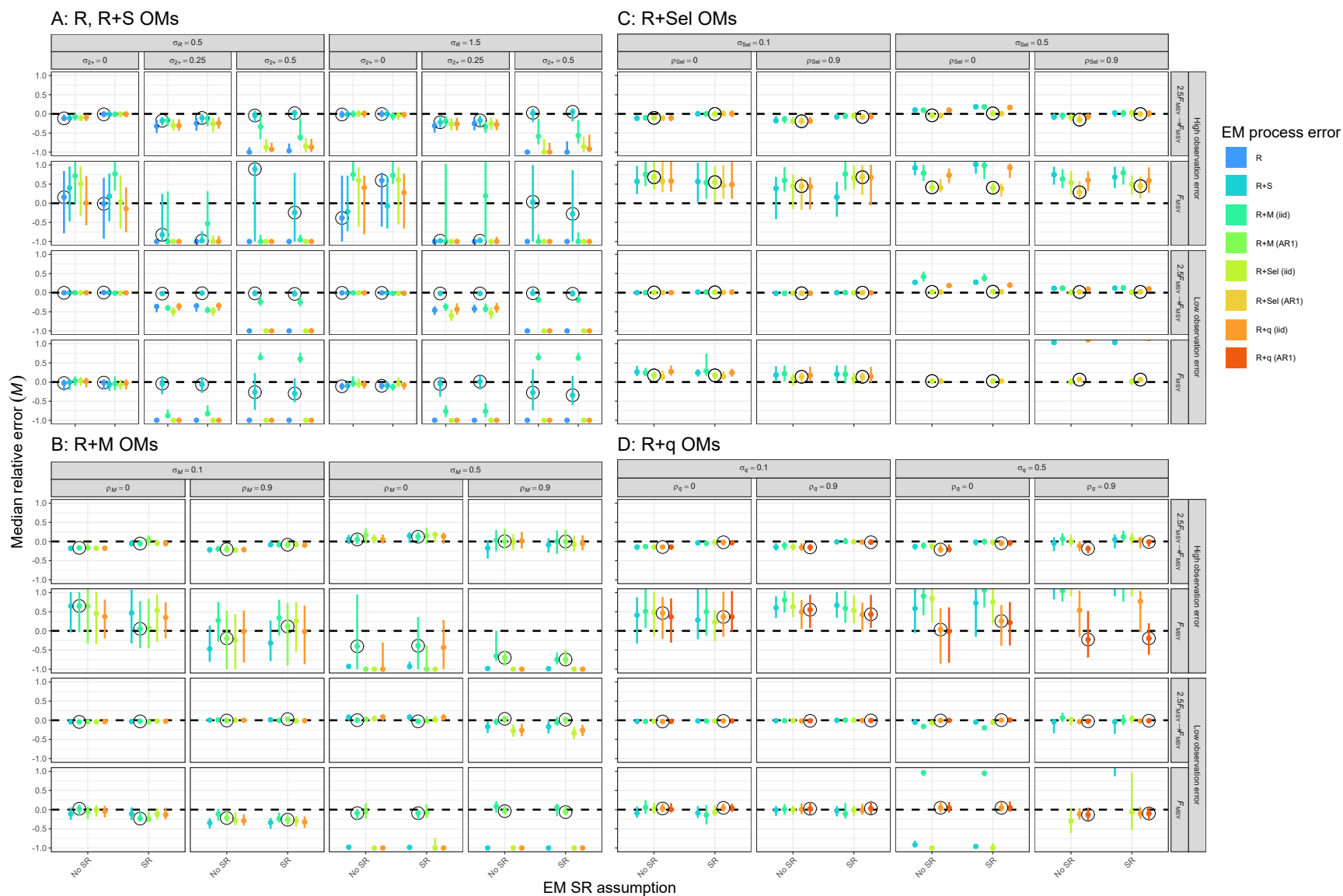


Fig. S13. Median relative error of median natural mortality rate for EMs fitted to data sets simulated with alternative process error sources: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

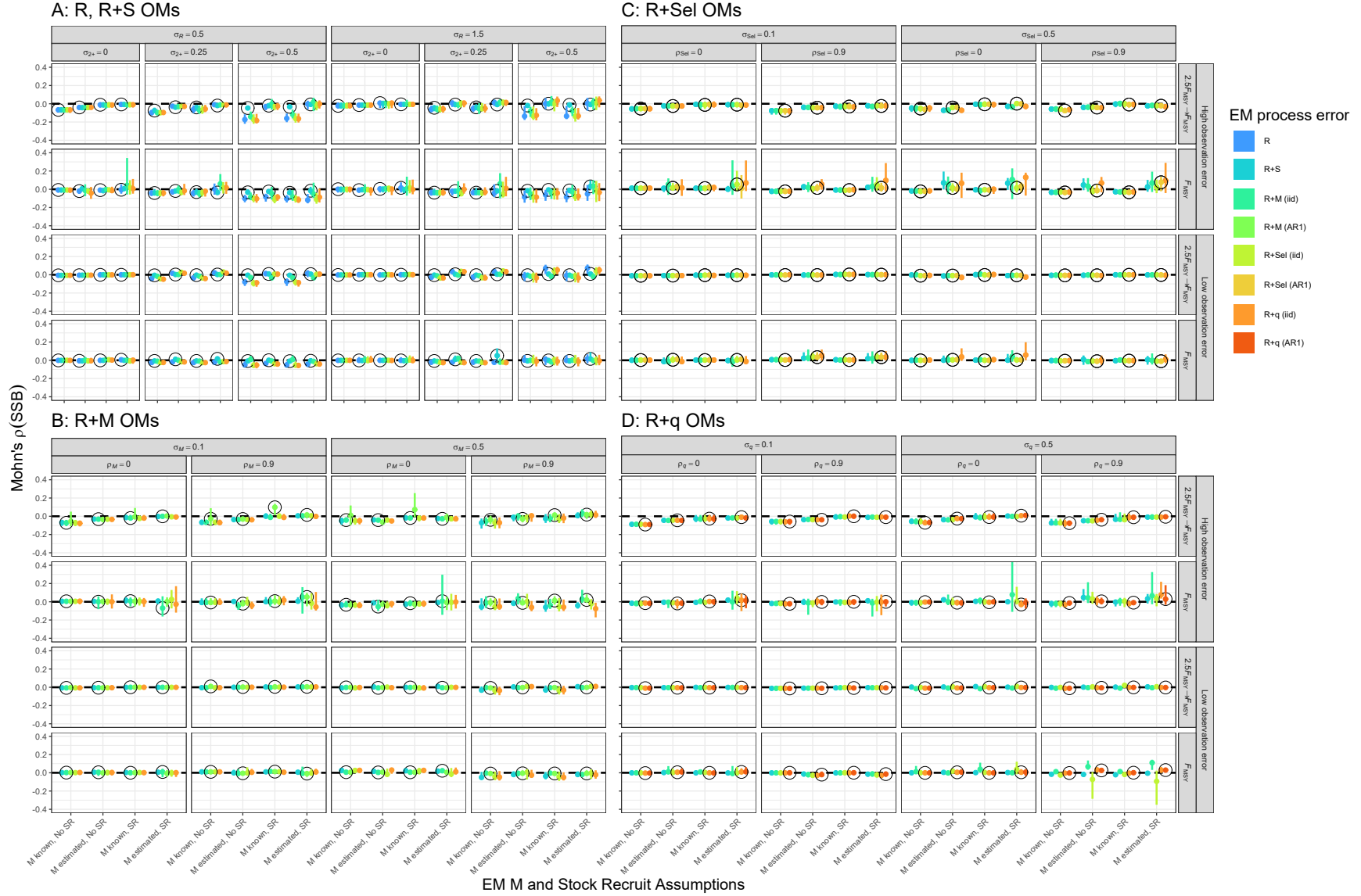


Fig. S14. Median Mohn's ρ for SSB for EMs fitted to data sets simulated with alternative process error sources: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

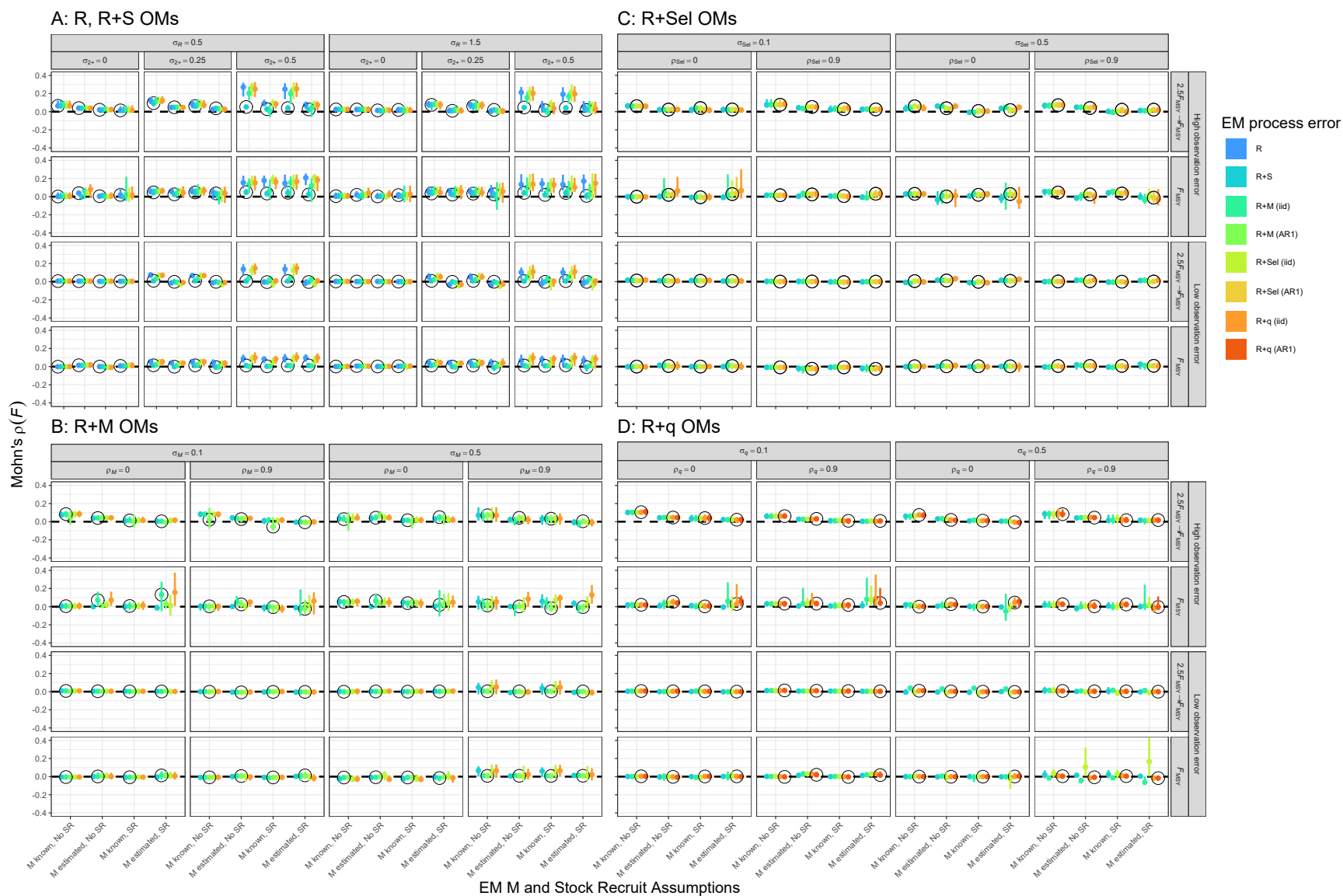


Fig. S15. Median Mohn's ρ of fishing mortality averaged over all age classes for EMs fitted to data sets simulated with alternative process error sources: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.



Fig. S16. Median Mohn's ρ of recruitment for EMs fitted to data sets simulated with alternative process error sources: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

Fig. S17. Probability of EMs providing Hessian-based standard errors with alternative process error (colored points and lines), and median natural mortality rate (estimated or known) and Beverton-Holt SRR (estimated or not; along x-axis) assumptions when fitted to OM that have R and R+S (A), R+Sel (B), R+M (C), or R+q (D) process error sources. Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

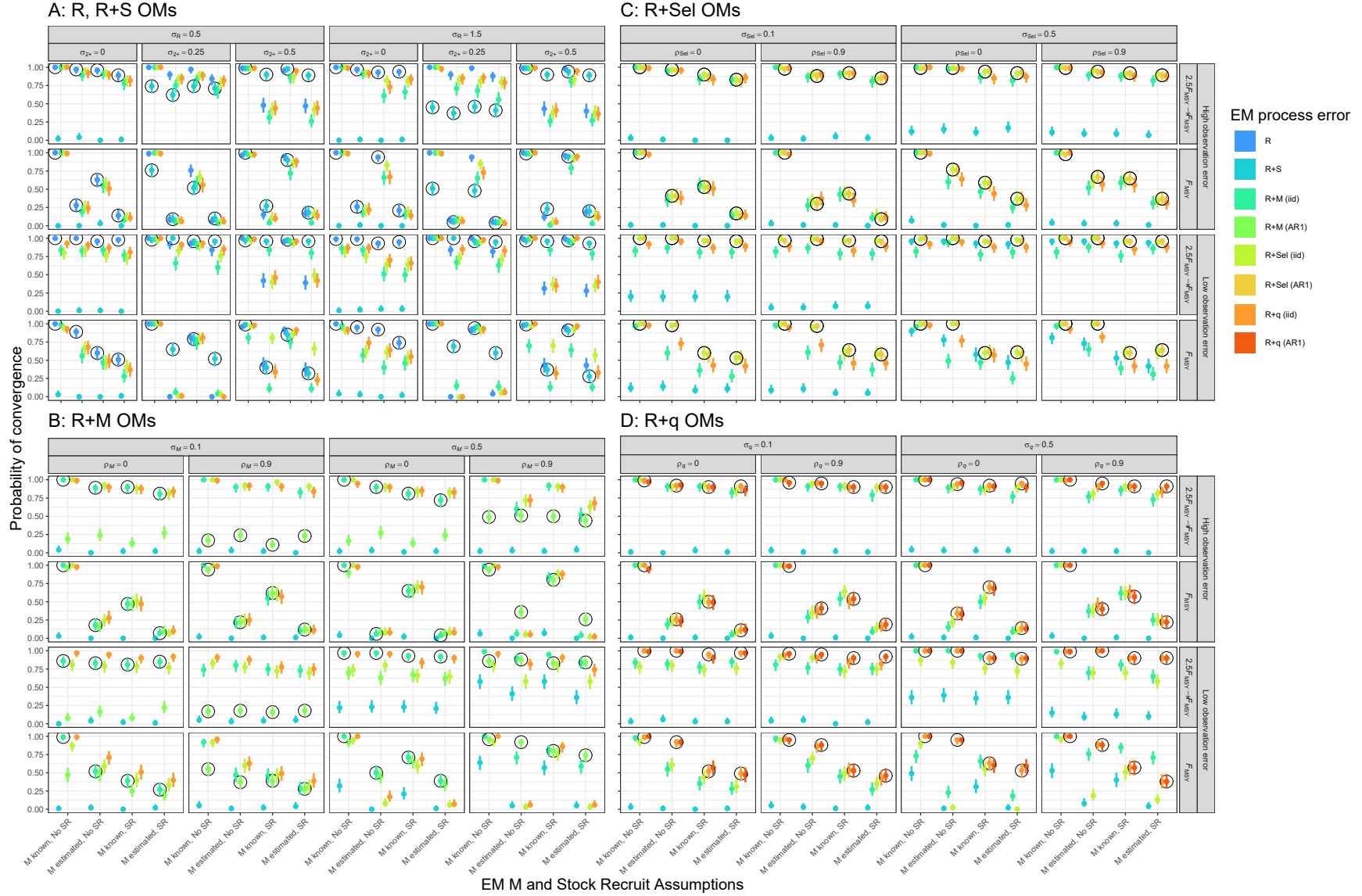


Fig. S18. Probability of EMs providing maximum absolute values of gradients less than 10^{-6} with alternative process error (colored points and lines), and median natural mortality rate (estimated or known) and Beverton-Holt SRR (estimated or not; along x-axis) assumptions when fitted to OMs that have R and R+S (A), R+Sel (B), R+M (C), or R+q (D) process error sources. Circled values indicate results where the EM process error structure matches that of the OM and vertical lines represent 95% confidence intervals.

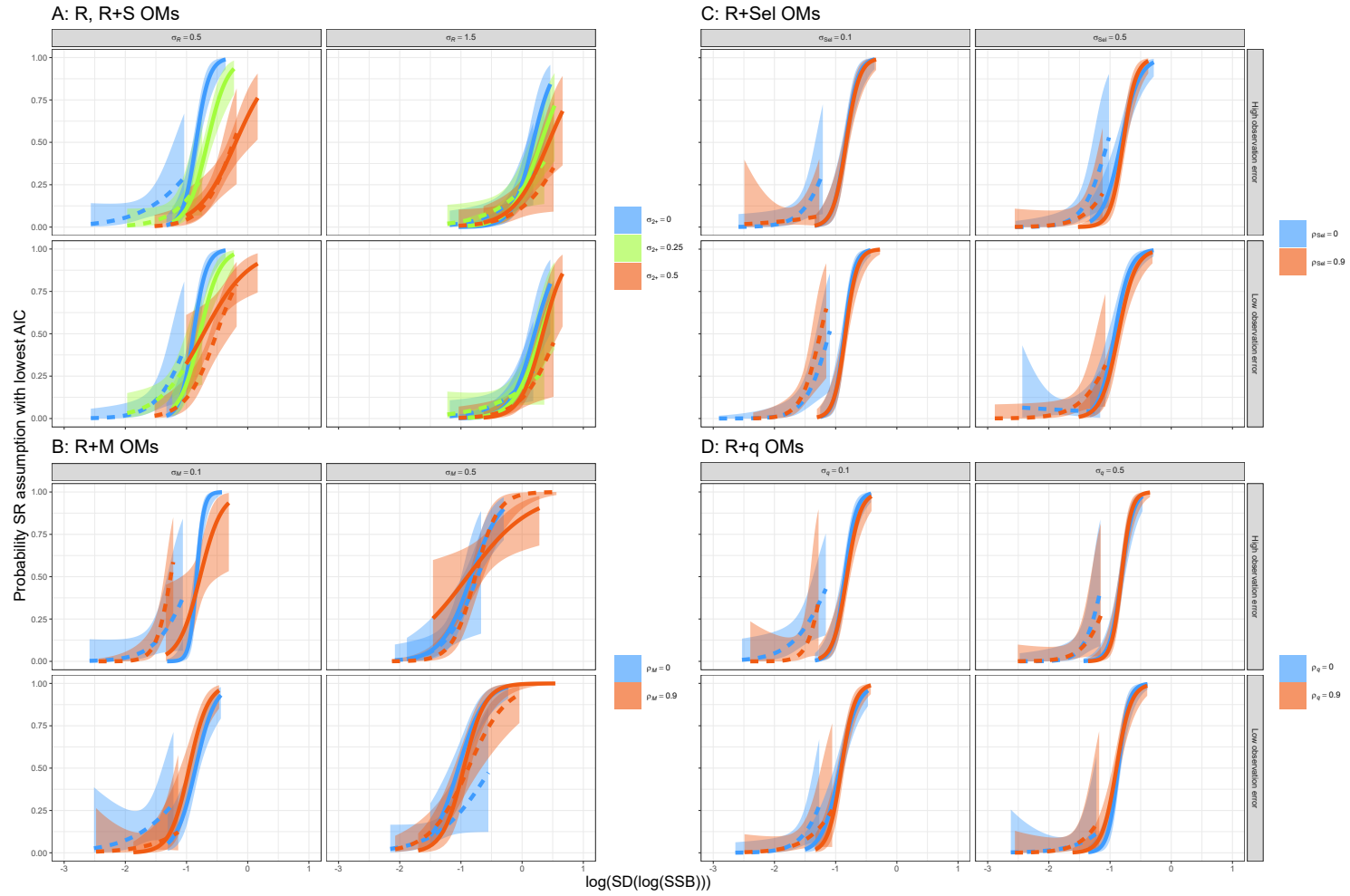


Fig. S19. Probability of lowest AIC from logistic regression on the log-standard deviation of the true log(SSB) in each simulation for EM with Beverton-Holt SRRs, rather than the otherwise equivalent EM without the SRR. Results are conditional on median M is known in the EM and alternative assumptions EMs having the correct process error structure: R and R+S (A), R+Sel (B), R+M (C), or R+q (D), and median M is assumed known in the EM. Solid and dashed lines are for OM with and without temporal contrast in fishing pressure, respectively, and polygons represent 95% confidence intervals. Range of results indicates the range of log-standard deviation of log(SSB) for simulations of the particular OM.

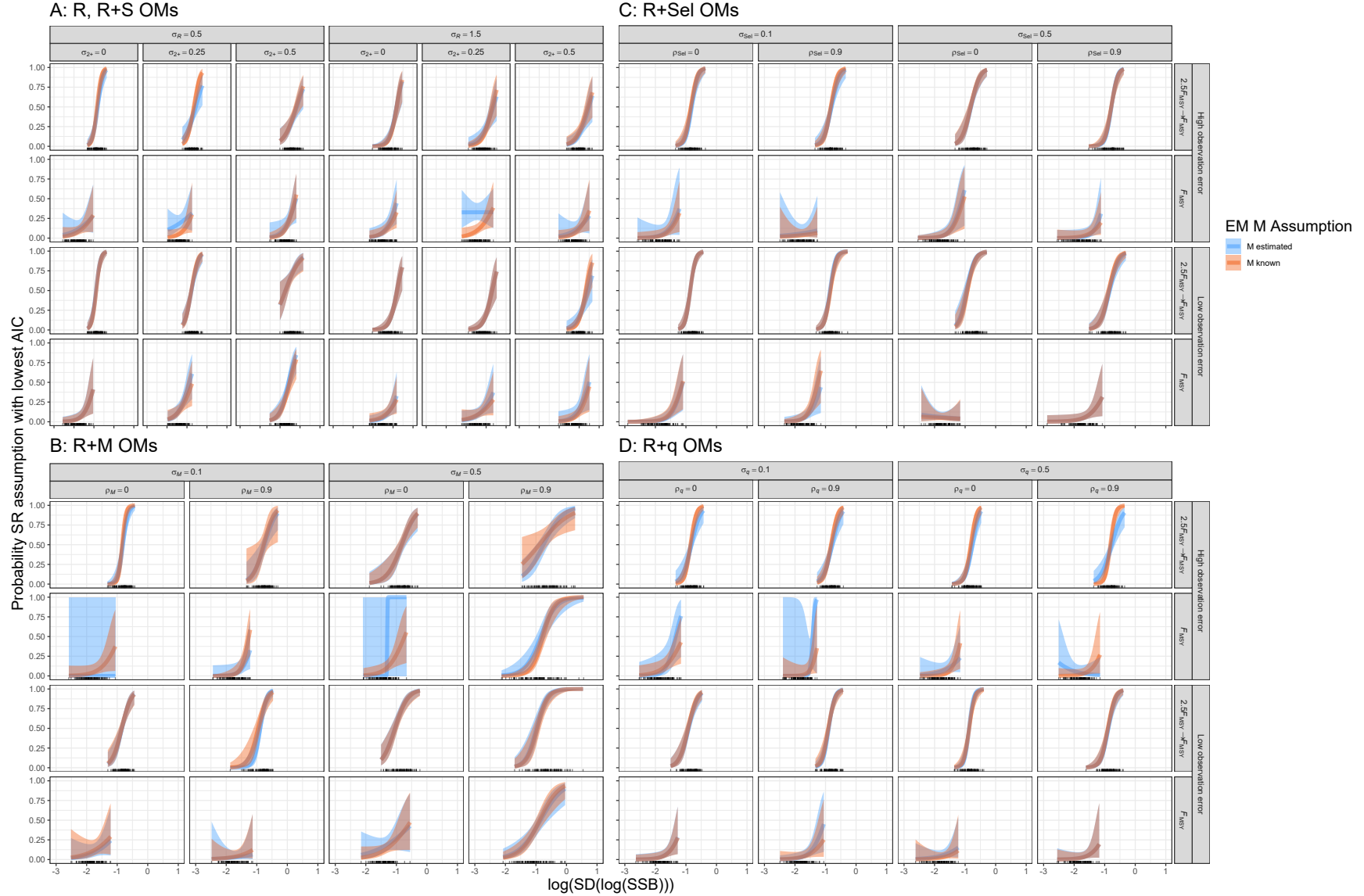


Fig. S20. Estimated probability of lowest AIC from logistic regression on the log-standard deviation of the true $\log(SSB)$ in each simulation for EM with Beverton-Holt SRRs, rather than the otherwise equivalent EM without the SRR. Results are conditional on alternative assumptions for median natural mortality rate (estimated or known) and on EMs having the correct process error structure: R and R+S (A), R+Sel (B), R+M (C), or R+q (D). Rug along x-axis denotes $SD(\log(SSB))$ values for each simulation and polygons represent 95% confidence intervals.

Table S9. For each OM process error source (columns), percent reduction in deviance for linear regression models fit to transformed Mohn’s ρ values for each simulation (Eq. 3) for fishing mortality averaged over all age classes with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	R	R+S	R+M	R+Sel	R+q
EM M Assumption	0.06	0.09	0.01	0.12	0.01
EM SR assumption	0.01	<0.01	0.01	0.02	0.01
EM Process Error	0.03	0.07	0.02	0.06	0.03
OM Obs. Error	0.16	0.10	0.05	0.02	0.07
OM F History	0.07	0.02	0.03	0.24	0.03
OM σ_R	<0.01	0.01	–	–	–
OM σ_{2+}	–	0.09	–	–	–
OM σ_M	–	–	<0.01	–	–
OM ρ_M	–	–	<0.01	–	–
OM σ_{Sel}	–	–	–	0.01	–
OM ρ_{Sel}	–	–	–	<0.01	–
OM σ_q	–	–	–	–	<0.01
OM ρ_q	–	–	–	–	0.01
All factors	0.32	0.38	0.12	0.48	0.15
+ All Two Way	0.65	0.67	0.30	0.95	0.43
+ All Three Way	1.18	1.11	0.63	1.34	0.90

Table S10. For each OM process error source (columns), percent reduction in deviance for linear regression models fit to transformed Mohn’s ρ values for each simulation (Eq. 3) for recruitment with each OM and EM factor (rows) included individually, combined, and with second and third order interactions.

Factor	R	R+S	R+M	R+Sel	R+q
EM M Assumption	0.86	0.56	0.16	1.00	1.27
EM SR assumption	<0.01	0.02	0.01	0.01	0.01
EM Process Error	0.01	0.59	0.18	0.07	0.04
OM Obs. Error	0.34	0.01	0.08	0.24	0.27
OM F History	0.91	0.22	0.06	1.20	1.67
OM σ_R	<0.01	0.14	—	—	—
OM σ_{2+}	—	0.11	—	—	—
OM σ_M	—	—	0.01	—	—
OM ρ_M	—	—	<0.01	—	—
OM σ_{Sel}	—	—	—	0.01	—
OM ρ_{Sel}	—	—	—	0.01	—
OM σ_q	—	—	—	—	0.01
OM ρ_q	—	—	—	—	0.01
All factors	2.28	1.74	0.51	2.66	3.51
+ All Two Way	4.20	2.74	1.08	5.08	6.51
+ All Three Way	4.83	3.79	1.79	6.03	7.82