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A Note on a Gamma Distribution Computer Program and Graph Paper

HAROLD L. CRUTCHER
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WASHINGTON, D.C.

April 1973

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U.S. NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION

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UDC 519.28:681.326.06:551.5(084.2)

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CONTENTS

A NOTE ON A GAMMA DISTRIBUTION COMPUTER PROGRAM AND GRAPH PAPER

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ABSTRACT. The gamma distribution function may be used as a model for many sets of data. The electronic computer program in the Formula Translator (FORTRAN) IV for this function here provides the analytic solution to a set of data, gives the probabilities of exceeding or not exceeding arbitrary amounts, and indicates the amounts exceeded or not exceeded for arbitrary probabilities.

The developed gamma probability plotting paper serves also for the special cases of the chi-squared, the exponential, and the Poisson distribution functions. Estimates of the scale and shape parameters permit construction of the graph. The graph paper may be used to estimate the scale and shape parameters.

The program, in its general form, permits a maximum of 52 entries, which will suffice for those dealing with weekly data through the year. In addition, in precipitation studies, the user has the option to compute in one pass of the data the two duration and three duration period distributions. These computations are done without program change but by appropriate changes in the control cards. This feature is not limited to the study of precipitation data.

An option permits the computation of the required probabilities and inverses when only the scale and shape parameters are given.

The computer output is designed for easy input to plotter routines.

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I. INTRODUCTION

This paper presents for the gamma distribution function:

1. An electronic computer program in the Formula Translator (FORTRAN) IV to provide an analytic solution for a data set.
2. Probability graph paper that furnishes a best fit straight line to the data.

Pearson and Hartley (1954) and others before them indicate that the chi-square integral, the incomplete gamma function, the type III distribution integral of Pearson (1894), the exponential functions, and the cumulative sum of terms of the Poisson distribution are different forms of the same mathematical function. Rayleigh (Strutt 1919) and the Maxwellian (1859) densities are special cases of the gamma densities. Therefore, the probability plotting paper developed here serves for these distributions as well as the exponential. In queuing theory, the Erlangian distribution is a gamma distribution. (For symbols used in this report, see table 1.)

II. THE GAMMA DISTRIBUTION FUNCTION

Many processes produce data distributions that the gamma distribution model describes well. Naturally, considerable literature exists for this distribution. The model serves for reliability life tests and fatigue problems. It offers advantages in the study of many multiple component systems where time to failure is an important feature. There are many other applications. For example, precipitation is the result of atmospheric processes, and the additive features of the gamma distribution parallel the additive features of atmospheric processes in rainfall production.

Pearson (1916) derives the gamma density function (Pearson's type III) as the solution of a differential equation. The tables edited by Pearson (1922), with subsequent revision through 1957, and those by Pearson and Hartley (1954) permit application of the gamma distribution model to fit and graduate skew data. The above tables permit interpolation for fractional degrees of freedom for the chi-square distribution. Campbell (1923) provides perhaps the first tabulation of the inverse gamma function if only for integer values. These, of course, are equivalent to the chi-square distribution with integer degrees of freedom equal to twice the gamma values. Salvosa (1930) also provides useful tables. Cohen et al. (1969) extend the tables of Salvosa. Birnbaum and Saunders (1958) derive and use the gamma distribution as one of the models for material life length, which may be likened to the life of a storm or the time to failure of precipitation generating processes. Harter (1964, 1969) provides an excellent discussion and extends Pearson's tables. Yet as Harter says, Pearson's work has no serious contender.

Table 1.--Symbols and their meanings

a	Gamma distribution variable dependent on second and third moments
b	Plotting parameter equal to c
c	Plotting parameter with a default option to 0.44
e	Exponential; 2.7183
f	Function
i	(1) sample number
	(2) subscript
j	(1) sample number
	(2) subscript
k	Subscript, such as i or j
n	Number of data
p	(1) probability of nonzero amounts; NX/NNX
	(2) probability level
q	(1 - p), probability of zero amounts (NNX-NX)/NNX
t	Variable
$\bar{t}$	Average t; the overbar indicates an averaging process.
dt	Derivative of t
x	Variable, here generally $y - \alpha$
x'	Transformed x, as $(y - \alpha)/\beta$
$\bar{x}$	Average x; nonzero amounts only
y	Variable
$\bar{y}$	Average y; nonzero amounts only
F	Function
G(x)	Gamma distribution function for a measured set excluding zeros
H(x)	Gamma distribution function for a measured set including zeros
I	Sample number
J	(1) number of duration periods
	(2) number of data combined
K	Kolmogorov (1933)
K-S	Kolmogorov-Smirnov
M	Moment; subscripts indicate type of moment.
ML	Maximum likelihood
NX	Number of data excluding zeros
NNX	Number of data including zeros
S	Smirnov (1936, 1948)
X	Untransformed variable (i.e., an original datum)
Y	Untransformed variable (i.e., an original datum)
α	Alpha; (1) origin
	(2) probability level for rejection
β	Beta; scale parameter
γ	Gamma; shape parameter
$\hat{\beta}$	Beta hat; maximum likelihood estimate of sample scale parameter
$\hat{\gamma}$	Gamma hat; maximum likelihood estimate of sample shape parameter
$\overset{*}{\beta}$	Beta star; Thom's (1958, 1968) estimate of scale parameter
$\overset{*}{\gamma}$	Gamma star; Thom's (1958, 1968) estimate of shape parameter
Γ	Gamma; gamma function
$\int$	Integral
Σ	Sigma; summation
τ	Tau; quantile
$d\tau$	Derivative of τ
χ^2	Chi-square
=	Equal to
>	Greater than
>>	Much greater than
<	Less than
$\leq$	Less than or equal to
$\overline{\quad}$	Overbar; averaging process
∞	Infinity

III. THE GENERAL GAMMA DISTRIBUTION FUNCTION

The general gamma distribution with origin parameter $\alpha(-\infty < \alpha < \infty)$, scale parameter $\beta(\beta > 0)$, and shape parameter $\gamma(\gamma > 0)$ has the probability density function shown in

$$f(y:\alpha,\beta,\gamma) = \beta^{-\gamma}(\Gamma(\gamma))^{-1}(y-\alpha)^{\gamma-1}e^{-(y-\alpha)/\beta}, \quad y > \alpha, \; -\infty < y < +\infty$$

and (1)

$$= 0, \quad y \leq \alpha.$$

The distribution function given in

$$F(y;\alpha,\beta,\gamma) = \int_{\alpha}^{y} f(t;\alpha,\beta,\gamma)dt \qquad (2)$$

is for all $y > \alpha$.

Fisher (1922) first develops the maximum likelihood (ML) equation for the solution for for the incomplete gamma distribution known commonly as the gamma distribution. It is incomplete in the sense that the integral limits of the function do not range from $-\infty$ to $+\infty$ but from some finite point such as α to $+ k$ where k is some real number. If the origin parameter α is zero, this distribution is a special case of the Pearson type III distribution. The solution of the ML equation as developed by Fisher is difficult. Therefore, Thom (1947) develops approximate solutions. Chapman (1956), Greenwood and Durand (1960), Gupta (1960), and Wilk et al. (1962) provide methods to estimate the gamma distribution parameters. Mooley and Crutcher (1968) discuss the variability of the parameter estimates of two gamma distributions. Schickedanz and Krause (1970) present tests for the scale parameters.

Thom's work leads to fruitful use of the gamma distribution in meteorological, climatological, and hydrological applications. Barger and Thom (1949) furnish an evaluation of drought hazard. Friedman and Janes (1957) provide an estimation of rainfall probabilities. Thom (1958) presents a note on the gamma distribution. Barger et al. (1959) give the chances of receiving selected amounts of n-week precipitation in the north-central region of the United States. The last is the model for a number of subsequent publications. Hartley and Lewish (1959) manage the computer hardware and software for the above study. Thom and Vestal (1968) provide a study of monthly rainfall in the conterminous United States.

IV. PARAMETER ESTIMATION

The gamma distribution (Pearson's type III) includes the chi-square and the exponential distributions as special cases. Pearson (1922), Thom (1958), and Hahn and Shapiro (1968) discuss this. Most statistical texts briefly discuss this also. The shape parameter γ is equal to one-half the degrees of freedom for the chi-square distribution and is equal to 1 for the exponential distribution, while the scale parameter β is equal to 1 in the standardized case as well as the last two cases.

Barger et al. (1959) provide plotting paper where the arguments are the mean and the variate. Overlaid straight lines represent probabilities. Each value of the shape parameter γ requires a separate graph. In the same paper, Thom's distribution curves, also prepared from Pearson's tables in 1957, appear. The probability and the variate divided by the scale parameter are the arguments with the shape parameter being overlaid in curved lines over the argument plot.

Wilk et al. (1962) provide techniques to estimate the scale and shape parameters, and they indicate that computer routines are available to provide graphical plots in terms of the quantile probabilities of the distribution and scale units. The theoretical line of best fit is then a straight line. These authors provide a brief set of tables that allows the person with a desk calculator, slide rule, or paper and pencil to interpolate required probability values and scale values and to make a plot of the data against the line obtained from the estimate of the scale and shape parameters. Roy et al. (1971) incorporate the above paper.

Thom (1968) presents direct and inverse tables of the gamma distribution. Thom's tables fill in areas not covered by the Wilk et al. (1962) tables and repeat other portions for verification.

V. ORIGIN

The origin or location parameter α in eq (1) usually is set to zero. However, there are cases where the origin is not zero. Elderton (1953) uses Pearson's moments to locate an origin from which the other parameters of the distribution may be measured. The necessary statements follow:

$$\alpha = \text{origin} = \text{mode} - a,$$

$$a = (2M_2^2/M_3) - (M_3/2M_2),$$

$$\text{mode} = \bar{t} - (M_3/2M_2),$$

and (3)

$$\text{origin} = \bar{t} - ((2M_2^2)/M_3)$$

where M_2 and M_3 are the second and third moments from the mean of the distribution. Barger (1964) discusses this. The expression $\bar{t} - (2M_2^2/M_3)$ does not ensure a positive location estimate even though the observed values are all positive. In some cases, the estimate may be higher than the lowest observed and recorded value in the data set.

Pitman's (1938) estimator for the location (origin) parameter is a minimum variance unbiased estimator if the scale and shape parameters are known. These parameters usually are not known and must be estimated. Pitman's technique is not examined further in this report.

Hastings (1955) provides equations for the estimation of the origin. Greenwood and Durand (1960) also study the estimation of the location parameter. Chapman (1956) provides a tabular aid for iterative procedures to solve for the origin parameter in the untruncated case. He indicates an additional procedure for the truncated case, providing there is sufficient supplementary information. These iterative procedures are not examined in this report.

Blischke (1971 and in prior studies) pursues the solution to the problem. Blischke encountered the same difficulties in the estimation process as is mentioned for the Elderton estimator. This, of course, blocks the calculation of maximum likelihood estimators for the shape and scale parameters or in any estimation process where logarithms are used. Blischke suggests that the lowest value be used as the origin where the estimated origin is above the lowest observed datum. This is the maximum likelihood estimator for the origin. Previously, the present authors found that fit to the gamma distribution may be rejected when this is done, even though a value slightly lower than the minimum datum as the estimator for the origin is used.

The program presents several options for the origin. The default option uses zero as the origin. Such a case would be zero for measured precipitation. If prior experience or theoretical considerations indicate the value(s) of the origin parameter(s), this option is entered in a control card that replaces the default option. A third option uses the lowest value less a small amount to ensure the positive number needed for the logarithms used in the maximum likelihood or Thom's estimators. Additionally, if the lowest value occurs more than once, this value becomes the origin.

The program processes the mixed distribution that consists of two categories, the lower bound and the values above the bound. Categories by nonoccurrence, such as zero precipitation, and the distribution of measurable precipitation above the bound is such a mixed distribution.

Regarding the bias in the estimators of maximum likelihood, it is of interest to refer to Blischke's work and to that of Shenton and Bowman (1970), Fisher (1922), and Thom (1957, 1958). Here we reproduce the comments of Shenton and Bowman:

"In this note we show that Thom's statistics are:

a) slightly biased, no matter how large the sample; however this bias is almost negligible for $\gamma >> 0$, and indeed is only of any real importance if γ is small (say less than 0.1 approximately); the bias in finite samples is about the same as for the maximum likelihood estimators;

b) superior to the maximum likelihood estimators because their variances are less in large sample theory; there is evidence that this property holds in finite samples also;

c) about as near to normality (as measured by skewness and kurtosis) as the maximum likelihood estimators; actually the distribution of $\overset{*}{\beta}$ is generally nearer to the normal form than that of $\hat{\beta}$."

In the above, the $\hat{\beta}$ and $\overset{*}{\beta}$ are respectively the maximum likelihood and Thom estimators.

Removal of bias in the estimators is not attempted in this program and report. Such will be examined later. In view of the large variability of the estimates (Andrews and Barger 1956) and Mooley and Crutcher (1968), removal of the bias may or may not be appropriate.

With α, the origin, obtained, the following expression is pertinent:

$$x = y - \alpha. \tag{4}$$

Then eq (1) becomes

$$f(x;0,\beta,\gamma) = \beta^{-\gamma}(\Gamma(\gamma))^{-1}\, x^{\gamma-1}e^{-x/\beta}, \quad 0 < x < \infty$$

and (5)

$$= 0, \quad x \leq 0.$$

Thom (1968 and in his earlier papers) utilizes this form.

As shown by Thom (1958) and by Wilk et al. (1962), if the variate x assumes a transform by division of the scale parameter β, the distribution function develops as

$$F(x';0,1,\gamma) = (\Gamma(\gamma))^{-1} \int_0^{x'} \tau^{\gamma-1}\, e^{-\tau}\, d\tau, \quad x' > 0$$

and (6)

$$= 0, \quad x' \leq 0.$$

that is a standard form with $\alpha = 0$ and $\beta = 1$ and is positive when $x > 0$.

Figure 1 provides a picture of the effect of the shape parameter and scale parameter on the function curves. Here, the standardized scale (frequency) is plotted against the quantile τ. Curves for shape parameters (A) 0.5, 1.0, 1,5, and 2.0, (B) 1, 2, 3, and 4, and (C) 1, 5, 10, 20, and 30 illustrate the effect. The shape parameter for 1.0 is shown on each subset, but the horizontal scale has been compressed. Hahn and Shapiro (1968) and Falls (1971) provide illustrations for other combinations of the scale and shape parameters. Reference to χ^2 curves also may be made. Where $\gamma = 1$, this is the same as the exponential and the same for χ^2 with two degrees of freedom as well as for a Poisson distribution. Stated somewhat differently, the random variable $(1/2)\chi^2_{2\gamma}$ with 2γ degrees of freedom has the gamma density function with the scale parameter equal to one and the shape parameter equal to γ.

Wilk et al. (1962) and Thom (1968) present the numerical methods to obtain the estimates of the gamma distribution scale and shape parameters β and γ. Masuyama and Kuroiwa (1951) provide a table for the likelihood solution of the gamma distribution. Those papers provide more detail. As Barger et al. (1959) indicate, the estimates of the parameters are subject to rather large variations due to sampling and estimating errors. Mooley and Crutcher (1968) discuss the variance of the probabilities of exceeding stated amounts based on work of Andrews and Barger (1956).

For a particular gamma variate distribution, the product of the shape and scale parameters equals the mean of the nonzero quantities. That is, $\overset{**}{\beta\gamma} = \bar{y}$. If $y_1, y_2, \ldots, y_n$ are independent gamma variates with shape parameters equal to $\gamma_1, \gamma_2, \ldots, \gamma_n$, then $Y = \sum_{i=1}^{n} y_i$ is a gamma variate with a shape parameter equal to $\sum_{i=1}^{n} \gamma_i$ (Kenney and Keeping 1951 and Lancaster 1969). This provides a useful tool for combining parameter estimates, thereby reducing the computation that would be required if the original data sets were combined. The division of the mean of the total set by the new shape parameter estimate provides the new scale parameter estimate.

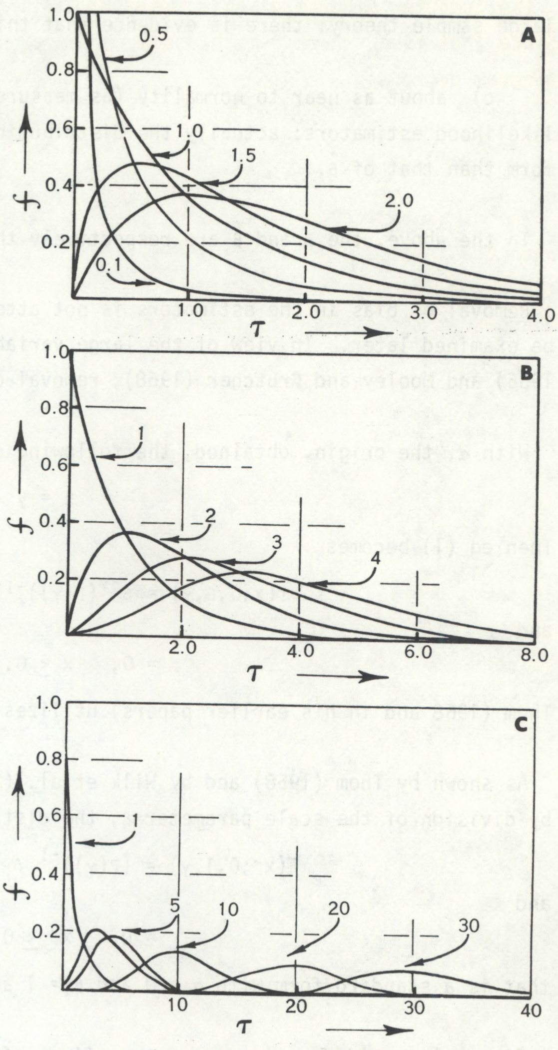


Figure 1.--Selected gamma distribution function curves

An option is available in the computer program discussed below that permits the calculation of probabilities from the input value of the parameter estimates in lieu of entry of original data with subsequent calculation of the estimates.

VI. GAMMA DISTRIBUTION FUNCTION COMPUTER PROGRAM

Elderton (1953) provides the moment estimate procedures for the origin parameter α as indicated previously. Thom (1958, 1968) provides the requisite information and equations to provide the maximum likelihood (ML) and Thom estimates of the scale and shape parameters $\overset{*}{\beta}$ and $\overset{*}{\gamma}$.

The computer program given in the appendix initially follows after Bark and Hofman (1960). Since that time, through much usage, discussions, and changes, resemblance to the original program decreases.

The present program may provide inadequate approximation for values of the probabilities when the shape parameter γ is less than 0.50. In this region, the asymptotic portion of the gamma function distribution, the slope of the curve, is almost indeterminant. Small changes in the shape parameter cause extreme changes in the function.

Pearson (1922) discusses this problem. Where computers of extremely large capacity are available, the approximations may succeed at low gamma and low probability values, though numbers as small as 10^{-35} are reached before failure. For most purposes when dealing with real data, such low gammas and low probabilities are not of too great importance. However, in terms of reliability problems, these may be important. Therefore, further work will be done on this problem in the development of approximation algorithms or techniques. Caution is needed when using this program for shape parameter values < 0.50.

The electronic computer program that forms the appendix, with comments for the FORTRAN IV user, supplies the necessary details. This particular program employs the Univac Series 70/45 Computer. Use with any other computer may require a few changes, but these will be minimal. Other options may be inserted, and changes may be made by the user to satisfy his particular requirements.

Figure 2 illustrates in tabular form output the application of the gamma model to the weekly rainfall distribution at Albany, Ga. The 11th week of the climatological year, May 10-16, for 39 yr with measured precipitation in 29 of the years constitutes the data set. Figure 3 depicts in tabular form the application of the gamma model to maximum rainfall in the Appalachian Mountains (1900-1969) from hurricanes or remnants thereof passing over the mountains or the centers touching the 1,000-ft contour (Haggard et al. 1971). Figure 2 shows output for 20 arbitrarily selected levels, and figure 3 shows output for 52 selected levels, which is the maximum for this program. Fifty-two is also the maximum data set input. This latter restriction, of course, may be bypassed if the option starting with known estimates of the scale and shape parameters is used. Parts A and B in figures 2 and 3 and in data output divide the tabulations into two sets of columns.

PRECIPITATION PROBABILITIES RUN DATE 11/17/71

A	1	2	3	4	5	6	7	8	9	10	11	12
	STATION	I	J	NX	NNX	XBAR	ALPHA	BETA	GAMMA	X2	PROB	K-S
	90140	11	1	29	39	1.030	0.000	1.295	0.796	15.483	0.970	0.086

B 1 SEQ	2 ENTRY DATA	3 ORDER DATA	4 DATA /BETA	5 EMP PROB	6 EMP PROB QUANTILE B=1	7 EMP PROB QUANTILE B=BETA	8 SELECTED PROB VALUES	9 SELECTED QUANTILE B=1	10 SELECTED QUANTILE B=BETA	11 GRAPH PROB (X>0)	12 SELECTED QUANTITY LEVELS	13 EXC PRB FOR PCP LVL
1	0.52	0.00	0.000	0.000	0.000	0.000	0.050	0.000	0.000	0.135	0.100	0.643
2	0.02	0.00	0.000	0.000	0.000	0.000	0.100	0.000	0.000	0.428	0.500	0.426
3	0.00	0.00	0.000	0.000	0.000	0.000	0.150	0.000	0.000	0.567	0.800	0.322
4	0.02	0.00	0.000	0.000	0.000	0.000	0.200	0.000	0.000	0.621	0.950	0.282
5	1.71	0.00	0.000	0.000	0.000	0.000	0.250	0.000	0.000	0.638	1.000	0.269
6	0.51	0.00	0.000	0.000	0.000	0.000	0.300	0.026	0.034	0.766	1.500	0.174
7	0.00	0.00	0.000	0.000	0.000	0.000	0.350	0.070	0.091	0.847	2.000	0.114
8	0.57	0.00	0.000	0.000	0.000	0.000	0.400	0.124	0.160	0.899	2.500	0.075
9	0.05	0.00	0.000	0.000	0.000	0.000	0.450	0.186	0.241	0.933	3.000	0.050
10	2.46	0.00	0.000	0.000	0.000	0.000	0.500	0.258	0.335	0.956	3.500	0.033
11	0.15	0.02	0.015	0.019	0.000	0.000	0.550	0.341	0.442	0.971	4.000	0.022
12	0.00	0.03	0.015	0.054	0.030	0.030	0.600	0.437	0.566	0.980	4.500	0.015
13	2.33	0.03	0.023	0.088	0.044	0.057	0.650	0.548	0.709	0.987	5.000	0.010
14	2.48	0.03	0.023	0.122	0.068	0.087	0.700	0.679	0.879	0.991	5.500	0.007
15	0.00	0.05	0.039	0.157	0.094	0.121	0.750	0.837	1.084	0.994	6.000	0.004
16	2.80	0.15	0.116	0.191	0.122	0.158	0.800	1.034	1.339	0.996	6.500	0.003
17	1.84	0.20	0.154	0.225	0.153	0.198	0.850	1.293	1.674	0.997	7.000	0.002
18	0.00	0.21	0.162	0.260	0.186	0.240	0.900	1.663	2.154	0.998	7.500	0.001
19	0.03	0.41	0.317	0.294	0.221	0.286	0.950	2.308	2.988	0.999	8.000	0.001
20	1.25	0.45	0.348	0.328	0.259	0.336	0.990	3.835	4.966	0.999	8.500	0.001
21	3.95	0.48	0.371	0.363	0.300	0.389						
22	0.59	0.49	0.378	0.397	0.344	0.446						
23	1.32	0.51	0.394	0.431	0.391	0.507						
24	0.00	0.52	0.402	0.466	0.442	0.573						
25	1.21	0.56	0.432	0.500	0.497	0.644						
26	0.56	0.57	0.440	0.534	0.557	0.721						
27	0.00	0.59	0.456	0.569	0.622	0.805						
28	1.61	1.21	0.934	0.603	0.693	0.897						
29	0.20	1.25	0.965	0.637	0.771	0.999						
30	1.63	1.32	1.019	0.672	0.858	1.111						
31	0.41	1.61	1.243	0.706	0.955	1.237						
32	0.49	1.63	1.259	0.740	1.066	1.380						
33	0.00	1.71	1.321	0.775	1.193	1.545						
34	0.00	1.84	1.421	0.809	1.343	1.739						
35	0.48	2.33	1.799	0.843	1.524	1.973						
36	0.00	2.46	1.900	0.878	1.751	2.268						
37	0.03	2.48	1.915	0.912	2.057	2.664						
38	0.00	2.80	2.162	0.946	2.521	3.265						
39	0.21	3.95	3.050	0.981	3.493	4.523						

Figure 2.--Precipitation probabilities for Albany, Ga., during the 11th climatological week of the year, May 10-16. The first week is March 1-7. The gamma model is used. The period of record is 1930-1968.

MAX 24 HOUR TOTAL PRECIP

A	1	2	3	4	5	6	7	8	9	10	11	12
	STATION	I	J	NX	NNX	XBAR	ALPHA	BETA	GAMMA	X2	PROB	K-S
	11	1	1	35	35	6.610	C.000	2.447	2.702	7.571	0.628	0.086

B 1 SEQ	2 ENTRY DATA	3 ORDER DATA	4 DATA /BETA	5 EMP PROB	6 EMP PROB QUANTILE B=1	7 EMP PROB QUANTILE B=BETA	8 SELECTED PROB VALUES	9 SELECTED QUANTILE B=1	10 SELECTED QUANTILE B=BETA	11 GRAPH PROB (X>0)	12 SELECTED QUANTITY LEVELS	13 EXC PRB FOR PCP LVL
1	2.82	0.67	0.274	0.016	0.409	1.001	0.001	0.137	0.334	0.000	0.005	1.000
2	3.98	0.80	0.327	0.044	0.634	1.552	0.003	0.209	0.512	0.000	0.010	1.000
3	4.02	2.82	1.153	0.073	0.794	1.943	0.005	0.256	0.626	0.000	0.015	1.000
4	9.50	3.10	1.267	0.101	0.928	2.272	0.006	0.275	0.673	0.000	0.020	1.000
5	4.50	3.42	1.398	0.130	1.049	2.567	0.007	0.293	0.716	0.000	0.025	1.000
6	11.40	3.50	1.430	0.158	1.161	2.842	0.008	0.309	0.755	0.000	0.030	1.000
7	10.71	3.69	1.508	0.187	1.268	3.103	0.009	0.324	0.792	0.000	0.035	1.000
8	6.31	3.73	1.524	0.215	1.371	3.354	0.010	0.338	0.827	0.000	0.040	1.000
9	4.95	3.98	1.627	0.244	1.471	3.600	0.015	0.399	0.976	0.000	0.045	1.000
10	5.64	4.02	1.643	0.272	1.570	3.841	0.020	0.450	1.101	0.000	0.050	1.000
11	5.51	4.21	1.721	0.301	1.668	4.081	0.025	0.494	1.209	0.000	0.060	1.000
12	13.40	4.46	1.823	0.329	1.766	4.320	0.030	0.534	1.308	0.000	0.070	1.000
13	9.72	4.50	1.839	0.358	1.864	4.560	0.035	0.571	1.398	0.000	0.080	1.000
14	6.47	4.58	1.872	0.386	1.963	4.802	0.040	0.606	1.482	0.000	0.090	1.000
15	10.16	4.75	1.941	0.415	2.063	5.046	0.045	0.638	1.561	0.000	0.100	1.000
16	4.21	4.95	2.023	0.443	2.165	5.297	0.050	0.669	1.636	0.000	0.150	1.000
17	11.60	5.00	2.044	0.472	2.269	5.553	0.055	0.698	1.708	0.000	0.200	1.000
18	4.75	5.51	2.252	0.500	2.377	5.815	0.060	0.726	1.776	0.000	0.250	1.000
19	6.85	5.64	2.305	0.528	2.487	6.085	0.065	0.753	1.843	0.001	0.300	0.999
20	6.25	6.20	2.534	0.557	2.602	6.365	0.070	0.779	1.907	0.001	0.350	0.999
21	3.42	6.25	2.554	0.585	2.721	6.657	0.075	0.805	1.969	0.002	0.400	0.998
22	11.80	6.31	2.579	0.614	2.845	6.962	0.080	0.829	2.029	0.002	0.450	0.998
23	0.80	6.47	2.644	0.642	2.977	7.284	0.085	0.853	2.088	0.003	0.500	0.997
24	3.69	6.85	2.800	0.671	3.116	7.624	0.090	0.877	2.145	0.004	0.550	0.996
25	3.10	7.43	3.037	0.699	3.265	7.988	0.095	0.900	2.202	0.004	0.600	0.996
26	22.22	8.00	3.270	0.728	3.425	8.379	0.100	0.922	2.257	0.005	0.650	0.995
27	7.43	9.50	3.883	0.756	3.599	8.805	0.150	1.129	2.763	0.007	0.700	0.993
28	5.00	9.72	3.973	0.785	3.790	9.274	0.200	1.316	3.220	0.008	0.750	0.992
29	4.58	10.16	4.152	0.813	4.005	9.798	0.250	1.493	3.653	0.009	0.800	0.991
30	4.46	10.71	4.377	0.842	4.249	10.397	0.300	1.666	4.075	0.011	0.850	0.989
31	8.00	11.40	4.659	0.870	4.536	11.098	0.350	1.837	4.496	0.012	0.900	0.988
32	3.73	11.60	4.741	0.899	4.886	11.954	0.400	2.012	4.922	0.014	0.950	0.986
33	3.50	11.80	4.823	0.927	5.340	13.064	0.450	2.190	5.359	0.016	1.000	0.984
34	6.20	13.40	5.477	0.956	6.001	14.683	0.500	2.377	5.815	0.041	1.500	0.959
35	0.67	22.22	9.081	0.984	7.315	17.898	0.550	2.573	6.296	0.078	2.000	0.922
36							0.600	2.784	6.811	0.123	2.500	0.877
37							0.631	2.924	7.153	0.175	3.000	0.825
38							0.650	3.013	7.373	0.232	3.500	0.768
39	THE FOLLOWING CONTROL PARAMETERS HAVE BEEN READ						0.700	3.268	7.997	0.291	4.000	0.709
40							0.750	3.559	8.708	0.350	4.500	0.650
41							0.800	3.902	9.547	0.409	5.000	0.591
42	BEGINNING PERIOD NUMBER IS					1	0.850	4.328	10.589	0.466	5.500	0.534
43	ENDING PERIOD NUMBER IS					1	0.900	4.905	12.000	0.520	6.000	0.480
44	NUMBER OF PRECIP AND/OR PROB LEVELS IS					52	0.910	5.051	12.358	0.617	7.000	0.383
45	OPTION IS					1	0.930	5.394	13.199	0.700	8.000	0.300
46	TWO PERIOD TOTALS REQUIRED, YES IF 2					0	0.950	5.845	14.301	0.768	9.000	0.232
47	THREE PERIOD TOTALS REQUIRED, YES IF 3					0	0.970	6.512	15.933	0.823	10.000	0.177
48	USE DEFINED TABLES OF PRECIP OR PROB, YES IF 0					0	0.980	7.030	17.199	0.960	15.000	0.040
49	NUMBER OF YEARS USED IS					35	0.990	7.895	19.318	0.992	20.000	0.008
50	C VALUE USED IS					0.44	0.995	8.742	21.389	0.999	25.000	0.001
51	ALPHA VALUE TO BE COMPUTED IF IA=2,					1	0.997	9.356	22.892	1.000	30.000	0.000
52	VALUE TO CONTROL ITERATION LIMIT =					10	0.999	10.654	26.068	1.000	35.000	0.000

Figure 3.--Precipitation probabilities for the Appalachian Mountains rainfall from tropical cyclones or remnants thereof crossing the mountains during 1900-1969 (Haggard et al. 1971). The gamma model is used.

Part A, shown on the first line, provides the following.

1	Station	Identification
2	I	Sample number
3	J	Number of duration periods in sample
4	NX	Number of data excluding zeros
5	NNX	Number of data including zeros
6	XBAR	Arithmetic average of data excluding zeros
7	ALPHA	Origin value
8	BETA	Scale parameter estimate, BETA STAR
9	GAMMA	Shape parameter estimate, GAMMA STAR
10	X2	X^2 for chi-square test
11	PROB	Probability of a chi-square value equal to X^2 above
12	K-S	The largest difference in probability between the theoretical and empirical distribution curves. This is the Kolmogorov-Smirnov test statistic (Smirnov 1948).

Part B comprises 13 columns of output information that provide the following.

1 Sequential guidance

2 Data in order of observation or record. These are x or y or transforms of y such as $(y-\alpha)$ or $(y-\alpha)/\beta$.

3 Ordered data of column 2

4 Ordered data of column 2 divided by the scale parameter β of column B-2 data. If the transform $((y-\alpha)/\beta)$ is used, columns B-2 and B-4 ought to be identical except for rounding error.

5 Empirical probability of the ordered data. The expression $(n-c)/(n-c+1)$ provides the probabilities where n is equivalent to NX of part A and c = 0.44 (Gringorten 1963). NX is the number of nonzero data. A program option permits a change in the value of c.

6 Variate quantile associated with the empirical probability of column 5 with the scale parameter β set equal to unity

7 Variate quantile associated with the empirical probability of column 5 with the sample scale parameter $\overset{*}{\beta}$ shown in part A

8 Fifty-two or less arbitrarily selected cumulative theoretical probability values for which columns B-9 and B-10 respectively show corresponding cumulative quantiles and amounts. A program option permits change in these, but allows for no more than 52.

9 Cumulative quantile values of the distribution corresponding respectively to the cumulative probability values of column B-8

10 Cumulative values of the distribution corresponding respectively to the cumulative probability values of column B-8. Multiplication of values in column B-9 by the sample $\overset{*}{\beta}$ value of part A provides column B-10 data.

11 Consider the base. The base is only the distribution of nonzero amounts shown in columns B-2 and B-3. The number of data is the NX of column A-4. Column B-11 then gives the probabilities of occurrence of amounts equal to or less than selected nonzero amounts shown in column B-12. This column is labeled "GRAPH" to indicate that this may be used to graph the set of nonzero amounts.

12 Arbitrarily selected cumulative amounts. The maximum number of amounts is 52. A program option permits change of amounts and < 52 amounts. The option also provides for the amounts to be scaled in terms of the mean of the nonzero amounts.

13 Probabilities of exceeding the arbitrarily selected cumulative amounts shown in column B-12. This is the mixed distribution.

If NX = NNX in part A, columns A-4 and A-5 (i.e., if the original distribution has no zero amounts), then column B-13 is the complement of column B-11.

The plot of columns B-11, B-12, and B-13 (as one set) and columns B-8 and B-9 (as another set) should plot on the straight line of the graphs shown in this report. The data of column B-12 should be scaled by division by the scale parameter β. The empirical probabilities and empirical amounts shown in columns B-5 and B-7 plotted on the graph will show visually and subjectively how good the line of best fit fits the data.

Wherever the approximation routines fail for a particular quantity or probability level, this will be noted in the output. Usually, enough levels will be available so that the loss of a level or two is not important (i.e., interpolation will suffice). If too many levels are noted, then the program routines generally will be inadequate because of difficulties previously mentioned in the asymptotic portion of the distribution.

For most purposes (in the analytical sense), if the gamma model is accepted without question, columns B-8 and B-10 or columns B-12 and B-13 provide the desired information based on the data sample. One set is the inverse of the other, though different levels may be and generally are used.

VII. MIXED DISTRIBUTIONS

Some data sets form a mixed set of distributions. The simplest mixed set consists of two subsets of data:

1. All data equal to or less than α, the origin.
2. All data greater than α.

Where the origin α is zero, the mixed set consists of:

1. The subset of zeros.
2. The subset of measured quantities.

Thus, after Thom (1951),

$$H(x) = q + p\ G(x) \tag{7}$$

where q is the zero set empirical probability, p is the measured set probability, and G(x) is the gamma distribution function for the measured set. For example, if q = 0.40 and p = 0.60, 40% of the observed values are zero and 60% of the observed values are greater than zero. Then, the cumulative probabilities of amounts greater than zero develop from the solution of G(x). These probabilities then are multiplied by 0.60 and added cumulatively to the initial 0.40 probability for the zero. If α is not zero, then the q = 0.40 would apply to values $\leq \alpha$. Also, p refers to values $> \alpha$.

The above procedure, utilizing the simplest mixed distribution, is part of the present computer program. Neither the model nor the program considers or allows for mixtures within the set of measurable quantities.

VIII. PREPARATION OF GAMMA DISTRIBUTION FUNCTION PROBABILITY PLOTTING PAPER

The investigator who relies too much on numbers and the electronic computer to process a data set bypasses the plotting step in many investigations. He expects the computer to do the thinking and the interpretation. This places too much reliance on a computer, which may have internal programming procedures unknown to the investigator. Data plots are many times a last step, if at all, in the reporting step in an investigation.

Plots of data form a first step in the study of any set of experimental or engineering data. The eye is usually a good integrator of the information display. Graphical analysis sometimes replaces numerical analysis. Sometimes, there is no other recourse because the complexity of distributions and their interrelationships defies the ingenuity of the analyst, the programmer, and often, even today, overloads the capacity and capabilities of the electronic computers. Intractable problems from the numerical or even analog point of view sometimes become tractable by means of graphical analysis. Graphical plots in terms of probabilities or of hazards provide ideas, concepts, and answers that numerical procedures cannot provide. Linsley et al. (1949) provide some good examples of graphical correlation procedures.

In a sense, the preplotting of the data permits a quality review of the data. This preplotting may even take the form of simple arrays such as scattergrams, histograms, or isopleth analyses of data arrays, whether in original first differences or transformation. By such means, outlier or questionable data examination is possible prior to inclusion or exclusion in the processing of the data. The inclusion of extremely bad data destroys the validity of any statistical analysis.

The following (Kimball 1960, p. 549) is most appropriate for this discussion, though it pertains in general to the normal distribution.

"Before proceeding further it is to be noted that the simplification afforded by the use of probability-scale graph paper is a <u>visual</u> simplification. The probability paper transforms a curvilinear distribution into a straight line. If the approach is to be purely analytical, there is no point in using the special scale paper.

"It then becomes important to have in mind the purpose served by plotting the observed data on the special scale graph paper. In general there are three rather different kinds of purposes that might be served. These are:

(1) A test as to whether or not the sample data indicate that the universe is of the prescribed type. One argues that the universe is of the prescribed type only if the plotted points tend to lie along a straight line.

(2) The graphical method may be used as a shortcut in estimating the standard deviation of the distribution, which in turn is directly determined from the slope of the fitted straight line.

(3) Graphical extrapolation at one of the extremes. This is the purpose most commonly served in plotting data from an extreme-value universe. Data are often plotted on extreme-value graph paper when it is known that on the lower range of maximum values the extreme-value distribution of Type I does not apply (9, p. 767). If in the upper range there is likely to be good conformity with the Type I distribution, a straight line fitted on the upper range is used as a basis for extrapolation of large extremes beyond the range of plotted points.

"These objectives can overlap. When accenting (2), one may well have a weather eye on Objective (1). For example, in examining a batch of samples taken from different populations one may have inferred that the universes are normal and so accent Objective (2). However, some errant populations may deviate considerably from the normal and so one may also give some weight to the graphical test of normality. Similarly in accenting Objective (3), Objective (2) is important. Furthermore, as noted above, there are situations where the data over the lower range are known not to follow the prescribed distribution; in which case data on the upper range are given greater weight in fitting a straight line. Thus Objective (1) is involved in indicating what part of the range conforms to the prescribed universe."

Wilk et al. (1962) and Thom (1968) provide, for a gamma distributed variable, the necessary information for construction of a probability plot whereby the fitted theoretical line is a straight line.

Plotting of the data often allows the estimation of parameter values. Chernoff and Lieberman (1956) indicate that the optimal construction of a graph paper depends upon the use to which the graph will be put. For example, Nelson and Hendrickson (1969) discuss a computer program for probability plotting and analysis of data, while Shapiro (1969) discusses probability plotting in general. Wilk et al. (1962) discuss this for the gamma distribution, and Nelson and Thompson (1971) discuss this for the Weibull distribution.

This report builds upon the work of all the cited authors back to the work edited by Pearson (1922). Simply, graphical form gives the inverse gamma probability values so that, with an estimate of the shape parameter γ, the appropriate probability grid lines of the graphical plot easily can be drawn. With an estimate of the scale parameter β, the appropriate scale is determined graphically and used as the second argument of the plot or the horizontal grid. Thus, the only requirement for use of the plotting paper illustrated in this report is that the estimates of the scale and shape parameters β and γ must be known or calculated before use of the paper. On the other hand, the graphical techniques provide approximation of the parameters β and γ. Usually, three approximations narrow the estimates sufficiently. Suggestion for an approximation technique is given after the discussion on graph papers.

Hahn and Shapiro (1968) discuss at some length the subject of probability plotting and listing of distribution of assumptions. They indicate a χ^2 distribution probability plotting paper is available from Technical and Engineering Arts for Management, 104 Belrose Avenue, Lowell, Mass. The shape parameter γ is equivalent to one-half the degrees of freedom of the χ^2 distribution. The χ^2 probability plotting paper used for 1, 2, 3, 4,... degrees of freedom can be used for gamma probability plotting paper of 0.5, 1, 1.5, 2,... for gamma. Available χ^2 probability plotting paper is restrictive for the gamma probability plotting paper, but gamma probability paper in the present report is not restrictive for the χ^2 distribution.

When the shape parameter γ is equal to 1.0, then the techniques shown here provide the basis for the exponential distribution function plotting paper.

A. Type A Plotting Paper

The lower bound of the data is zero, and no zeros exist. Type A plotting paper is any rectangular coordinate plotting paper or simply a piece of paper on which unit measurements exist as square blocks of equal size. That is, the units are the same on both the abscissa and the ordinate. Choose the horizontal as the abscissa and the vertical as the ordinate.

Figure 2 illustrates the computer output of a mixed distribution $H(x) = q + p\ G(x)$ where $q = 10/39$ and $p = 29/39$ from columns A-4 and A-5. Columns B-8, B-9, B-10, B-12, and B-13 refer to the mixed distribution; and columns B-11 and B-12 refer to the nonzero portion or p portion. Column B-11 provides G(x). Note that column B-12, selected arbitrary amounts, serves twice.

Figures 4A, 4B, and 4C illustrate the preparation and use of type A plotting paper. The basic chart 4A design is based on Wilk et al. (1962). Figures 4A and 4B refer to the mixed distribution; figure 4C shows both the mixed and the nonzero portions of the distribution.

As a preliminary basic chart, 4A is prepared. Essentially, it consists of a rectangular coordinate graph paper with units and divisions thereof marked as quantiles. A line of slope 1 serves as the theoretical line of best fit. The bound of zero is placed in the lower left-hand corner.

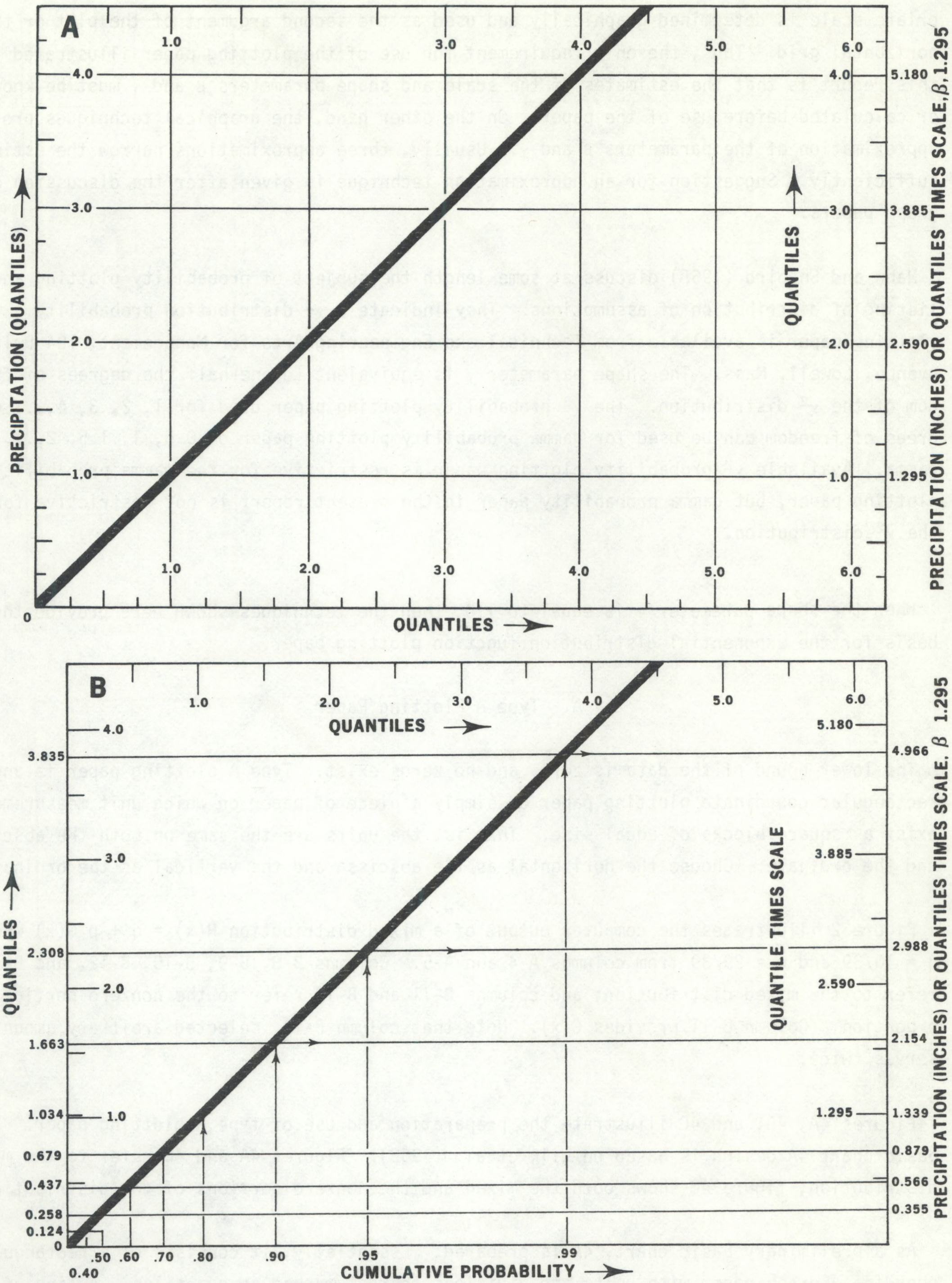


Figure 4.--Albany, Ga., May 10-16 precipitation: (A) quantiles and line of best fit, (B) probability quantiles, (C) probability of exceeding stated amounts in (I) G(X) and (II) H(X), and (D) data plot on the line of best fit

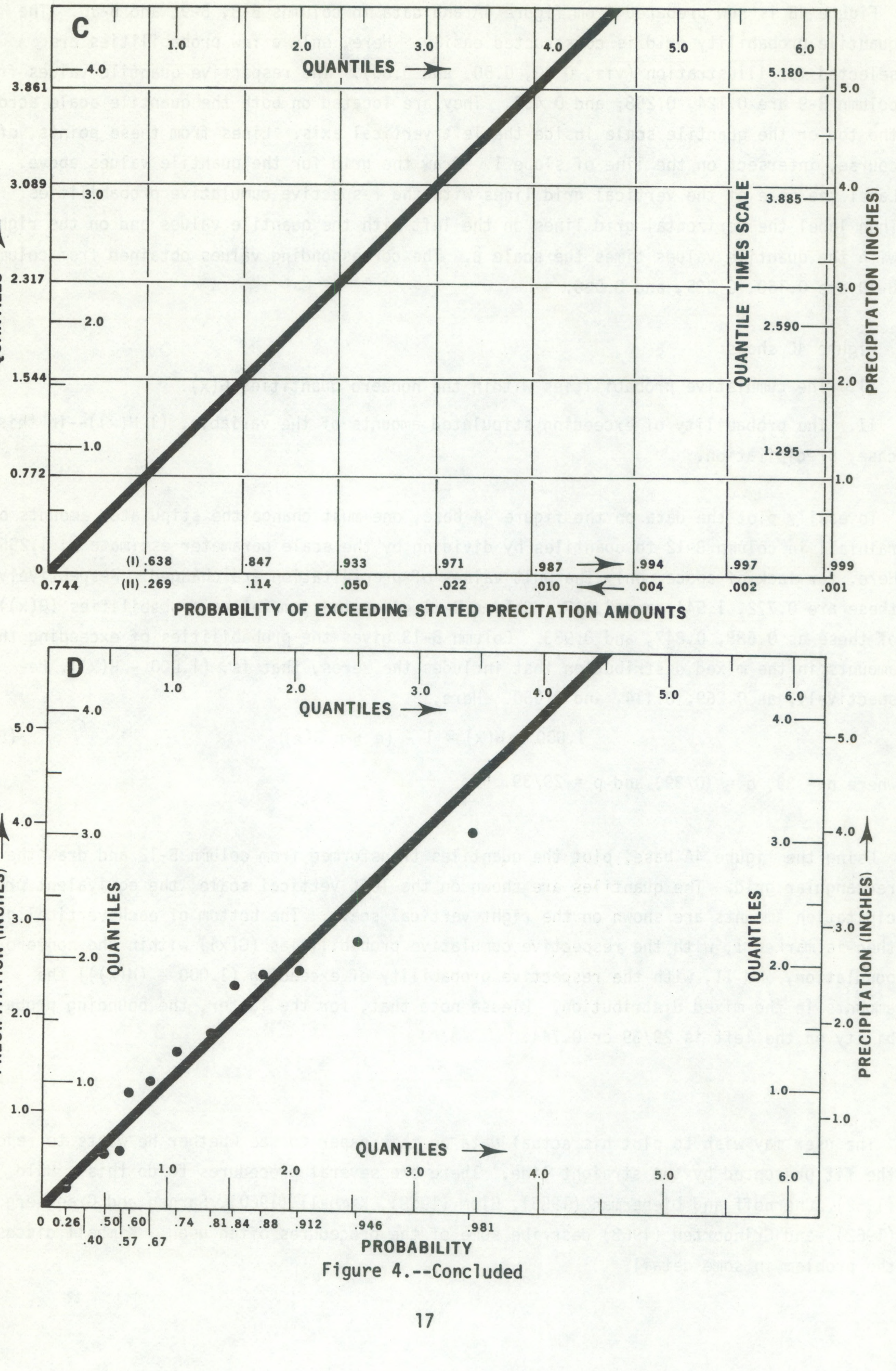


Figure 4.--Concluded

Figure 4B is now prepared from figure 4A and data in columns B-8, B-9, and B-10. The quantile probability grid is constructed easily. Here, only a few probabilities are selected for illustration (viz, 0.40, 0.50, and 0.60). The respective quantile values from column B-9 are 0.124, 0.258, and 0.437. They are located on both the quantile scale across the top or the quantile scale inside the left vertical axis. Lines from these points, of course, intersect on the line of slope 1. Draw the grid for the quantile values above. Label the base of the vertical grid lines with the respective cumulative probabilities. Then label the horizontal grid lines on the left with the quantile values and on the right with the quantile values times the scale $\hat{\beta}$. The corresponding values obtained from column B-10 are 0.160, 0.335, and 0.566.

Figure 4C shows:

I. The cumulative probabilities within the nonzero quantities G(x).

II. The probability of exceeding stipulated amounts of the variable, (1-H(x))--in this case, precipitation.

To easily plot the data on the figure 4A base, one must change the stipulated amounts of rainfall in column B-12 to quantiles by dividing by the scale parameter estimate $\hat{\beta}$, 1.295. Here, for lack of space, only the unit values of precipitation are changed. Respectively, these are 0.772, 1.544, and 2.317. Column B-11 gives the cumulative probabilities (G(x)) of these as 0.638, 0.847, and 0.933. Column B-13 gives the probabilities of exceeding these amounts in the mixed distribution that includes the zeros, that is, (1.000 - H(x)), respectively, as 0.269, 0.114, and 0.050. Here,

$$1.000 - H(x) = 1 - (q + p\ G(x)) \qquad (8)$$

where n = 39, q = 10/39, and p = 29/39.

Using the figure 4A base, plot the quantiles transformed from column B-12 and draw the rectangular grid. The quantiles are shown on the left vertical scale; the equivalent precipitation amounts are shown on the right vertical scale. The bottom of each vertical grid then is marked I, with the respective cumulative probabilities (G(x)) within the nonzero population, and II, with the respective probability of exceeding (1.000 - (H(x))) the amounts in the mixed distribution. Please note that, for the latter, the bounding probability on the left is 29/39 or 0.744.

The user may wish to plot his actual data on this paper to see whether he wants to reject the fit presented by the straight line. There are several procedures to do this. Hald (1952), Chernoff and Lieberman (1956), Blom (1958), Kimball (1960), Sarhan and Greenberg (1962), and Gringorten (1963) describe some of the procedures often used. Kimball discusses the problem in some detail.

The steps are:

1. Order the data from low to high, such as $x_1, x_2, x_3, \ldots, x_n$. See column B-3 of the computer output.

2. Compute the empirical probabilities by means of

$$P_i = (i-c)/(n-c-b+1)$$

where P_i is the empirical probability of i and i is the ith ordered data, b and c are parameters, and n is the number of data. As an approximation, assuming some measure of symmetry whenever appropriate, especially when γ is large, set b = c = 1/2. Blom (1958) suggests the use of 3/8 rather than 1/2. Gringorten (1963), working with extreme values, suggests 0.44. Here, c is set to 0.44. The right-hand member of the equation reduces to (i-0.44)/(n+0.12). Sometimes, when n is small, c is set equal to zero. See computer output column B-5.

The present computer program contains a default option to 0.44. (See Wilk et al. 1962.) The user may decide to use some value for c other than 0.44. Experience dictates the value. Blom (1958), Kimball (1960), Hahn and Shapiro (1968), and Gupta and Groll (1961) present important, pertinent, and interesting reading on the selection of an appropriate value for c. As indicated by Kimball, the various methods put forward to determine plotting positions create confusion of thought in judging what plotting convention is optimum. There still is confusion, but only because the user usually does not realize that the value of c used depends upon a certain feature or certain features of the distribution that are being examined.

3. Plot the data on a base illustrated in figure 4A at the ordered points of (Y_i, p_i). The corresponding quantiles for plotting for p_i are given in columns B-6 and B-7. The user subjectively will decide whether the fit of the straight line to the data is to be rejected or not rejected.

Figure 4A illustrates the basic quantile background for all plotting, though this background usually is not shown. The theoretical cumulative quantiles are shown with the 45° line of best fit (i.e., the scale parameter is 1). On the right, the quantiles for the scale parameter have been multiplied by a scale parameter 1.295. This refers to the data of figure 2, Albany, Ga., precipitation data, 11th climatological week, May 10-16, 1930-1968. Quantiles and quantities associated with arbitrary selected probabilities now are shown in figure 4B where the unit quantile background grid has been dropped. These are taken from figure 2, columns B-8, B-9, and B-10. For example, at the probability level of 0.90, go to the right a quantile value of 1.663; rise vertically on this probability grid value to the 45° sloping line and then to the left and right. Here, the left-hand ordinate has been marked also, but with the value of 1.663; on the right, the quantity level 2.154 (1.663 x 1.295) is shown.

Figure 4C has been prepared from the data provided by columns B-11, B-12, and B-13 of figure 2. The background grid has been prepared by scaling the data in column B-12 by division by the scale parameter 1.295. These then become the units of this particular grid. Here, units of (1.000/1.295) or 0.772 are used. These then are marked, quantiles on the left and precipitation quantities on the right.

Plotting is as follows. Correct the quantities of column B-12 to quantiles by division by the scale parameter, in this case, 1.295. Proceed to the right for a selected quantile, say 0.772, then upward the same distance to the 45° line, then to the left and to the right. Now mark the horizontal end points with the appropriate respective quantile and quantity values, 0.772 and 1.0. Now mark the base of the verticals with the appropriate probability values for the nonzero and the mixed data, G(x) and H(x), respectively. For 1.0-in. precipitation for this set of data from figure 2, the probabilities are respectively 0.638 and 0.269.

Note that in the H(x) data, line (II), the left probability bound is the empirical probability of zeros. In this case, the probability of exceeding zero is 29/39 or 0.744. The lower bounding probability line for the mixed distribution (II) H(x) is labeled 0.744; for the (I) G(x) line within only the measurable precipitation data, the probability of getting less than a measurable amount is zero.

Figure 4D illustrates an overplot of selected data of figure 2 on the graph showing the line of best fit. These are plotted in quantile values from the data in columns B-4 and B-6. These are labeled respectively with corresponding data from columns B-3 and B-5.

Figure 4D shows the theoretical line of best fit of figure 4A with the overplot of empirical data taken from columns B-4, B-5, and B-6 of figure 2. The background grids are not shown here in detail. These are for the nonzero data. The fit of the model to the data may be judged subjectively by eye. Here, the fit could be better. Substantiating this, figure 2 provides on line A three values, one each in columns A-10, A-11, and A-12. The χ^2 value of 15.483 (col. A-10) will be exceeded with a probability of only 0.030 (col. A-11) which indicates that this data set is not too well represented by the gamma model. Here, the distribution was divided into 10 equiprobability class intervals. The value of the K-S test statistic provides, in column A-11, 0.086, a measure of the largest class interval difference between the theoretical and empirical probabilities. The probability of exceeding the value may be obtained by reference to Lilliefors (1967, 1969, 1972) for the normal, exponential, and gamma distributions. Here the gamma is the most appropriate for general use; though if the shape parameter is very large or equal to one, the tables for the normal and exponential respectively can be used. In Lilliefors (1972), for the gamma model, for a sample of 30 (29 were used here), for an α level of 0.005, and for a shape parameter of 1.0 (here, 0.796), the K-S statistic is 0.1863. As the K-S statistic 0.086 does not exceed 0.1863, the model is not rejected. The K-S test is judged to be more powerful than the chi-square test; therefore, though the chi-square test suggests rejection, the model is not rejected.

Here is another example -- figure 3 gives the computer tabular output. The data are 35 cases of maximum rainfall from tropical cyclones or remnants thereof that crossed the Appalachian Mountains during the period 1900-1969 (Haggard et al. 1971).

Figure 3 differs from figure 2 in that there are no zero amounts involved. Thus, paired values of columns B-11 and B-13 add to 1.000. In other words, the column B-11 values are complementary to the column B-13 values. Only one graph is required for the theoretical values. Column B-8 provides arbitrarily selected probability levels for which the theoretical quantiles and amounts are given in columns B-9 and B-10. Four probability levels are selected for illustration, namely, 0.100, 0.500, 0.900, and 0.950. Column B-12 provides arbitrarily selected precipitation amounts for which theoretical probability levels, cumulative and exceeding, are shown respectively in columns B-11 and B-13. As the values in column B-13 are complementary to the column B-11 values, only the column B-11 cumulative probabilities are shown. Because of space limitations, the 1.000-, 5.000-, 10.000-, and 15.000-in. levels with the corresponding probabilities are shown. As in construction of figure 4C, it is necessary to change the column B-12 amounts to quantiles by dividing by the scale parameter 2.447.

Construction of figure 5 from figure 3 information follows that for figure 4C. Figure 5, therefore, shows the cumulative probability for the Appalachian Mountains maximum measured rainfall from cyclones or remnants thereof crossing the mountains during 1900-1969 (after Haggard et al. 1971). The data are taken from figure 3. The line of best fit is the 45° line. The plotting quantile grid used is indicated on the left; the amounts (quantiles times scale parameter) are shown on the right.

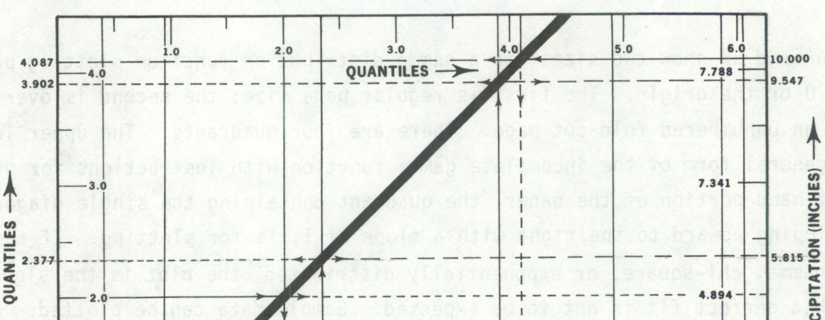


Figure 5.--Maximum measured rainfall from tropical cyclones crossing the Appalachian Mountains during 1900-1969

Figure 5 from figure 3 shows the amounts corresponding to selected probabilities of 0.100, 0.500, and 0.800 from column B-8 against the quantity values from column B-10, 2.257, 5.815, and 9.547, respectively. Probabilities of 0.016 and 0.823 corresponding to selected amounts from column B-11 are 1 and 10 in., respectively. In addition, it further illustrates both the probability level 0.409 and quantity level 4.894 for the quantile value of 2.0.

More complete plotting of the information in figure 3 will permit ready graphical interpolation of information. No overplot of empirical data has been prepared. The chi-square and K-S test value information imply that the data are well fit by the gamma model. The line A, column A-11, chi-square datum of 0.628 indicates that the probability of exceeding the value of 7.571 will be 0.372.

The K-S test value is 0.086. For a sample size of 30, with a shape parameter of 3.0, the probabilities of exceeding values of 0.151 and 0.164 are 0.10 and 0.05, respectively (Lilliefors 1972). The probability of a number larger than 0.086 by chance then is rather large. Please note that, by coincidence, the K-S test statistic for both examples to three decimal places is the same (viz, 0.086) though the shape parameter and data samples are quite different.

Another procedure is to label the ordinate in terms of the quantiles shown in column B-9. Then, construct a line with a slope equal to β and read directly from this slope the probabilities equal to or less than or the probabilities greater than selected amounts.

B. Type B.1 Plotting Paper

Figures 6A and 6B show two sizes for a gamma distribution function plotting paper type B.1 for data > 0 or the origin. The first is regular page size; the second is oversize and appears as an unnumbered fold-out page. There are four quadrants. The upper left quadrant shows the general form of the incomplete gamma function with instructions for use. The lower right-hand portion of the paper, the quadrant containing the single diagonal line of best fit sloping upward to the right with a slope of 1, is for plotting. If the data are precisely gamma, chi-square, or exponentially distributed, the plot is the sloping straight line. Such a perfect fit is not to be expected. Sample data can be plotted. Departures of sample data from the straight line permit subjective decision as to whether the fit is or is not good.

Tables 2 and 3 give recorded maximum 24-hr precipitation from tropical cyclones or remnants thereof that passed over the Appalachian Mountains during 1900-1969. Table 2 provides a data set for the total storm period. Table 3 presents the maximum likelihood estimators of the scale and shape parameters of the gamma distribution for the data of table 2.

Figure 7 illustrates the use of the gamma plotting paper type B.1 and depicts the distribution for the data of table 3. A few values of table 2 are overplotted on the figures to illustrate the fit of the line of quantile slope 1 to the data. The inference is made that the fit is adequate. The size of the graph illustrated precludes easy plotting of all data pairs (Y_i, p_i).

Probability plots of data by computer may be made as indicated by Wilk et al. (1962) who indicate that such a program is available. Here, this graph paper has been developed for use by people to whom a computer is not immediately available or who wish to make graphical estimates of the gamma parameters.

Plot probabilities as the abscissa beginning at zero at the left-hand bound of the plotting area. Plot values of the variate vertically against a scale that may be marked on the left-hand vertical bound of the plotting areas. A scale is available in the appropriate units when the scale parameter β is equal to 1.0.

Specified gamma quantile probabilities appear as curved lines in the upper right-hand quadrant. Tables given by Wilk et al. (1962) and Thom (1968) form the basis of these curved lines. The sloping straight lines shown in the lower left-hand quadrant furnish the appropriate scales for the variate. These are a multiplication artifice.

The user obtains the percentile marks in the fashion shown in the inset of figures 4 and 7. Suppose that the shape parameter is 2.0 and the scale parameter is 3.0. The steps follow.

1. Draw a horizontal line at γ = 2.0 in the upper right-hand quadrant. Note that the line crosses all the gamma probability curves shown for the gamma quantiles.

2. From the intersection of the horizontal line drawn in step 1 and the probability curves, drop perpendicular lines vertically through the lower right-hand quadrant and label each with its appropriate probability value. Labels of the complements provide probabilities exceeding specified values.

3. There are several options to produce scale values. One follows. At the vertical line separating the lower quadrants, proceed upward to the value of the scale parameter 3.0. From this point, draw a line to the extreme lower left at the convergence point of all scale lines. Note the intersection of this line with the horizontal line 1.0 on the vertical separation. This appears as a heavy line on the plotting paper.

4. From the point of intersection found in step 3, draw a perpendicular line. The intersections of the sloping scale and this vertical line provide appropriate scaling units. Draw horizontal lines passing through these points of intersection across the lower right quadrant. In the vertical space along the left-hand side of the plotting quadrant, mark the ends of the horizontal lines with the appropriate scale values.

5. The probabilities equal to or less than or the probabilities greater than selected amounts can be read directly from the heavy line sloping upward to the right.

6. Plot the empirical sample data on the graph to permit subjective evaluation of the fit of the model to the sample data.

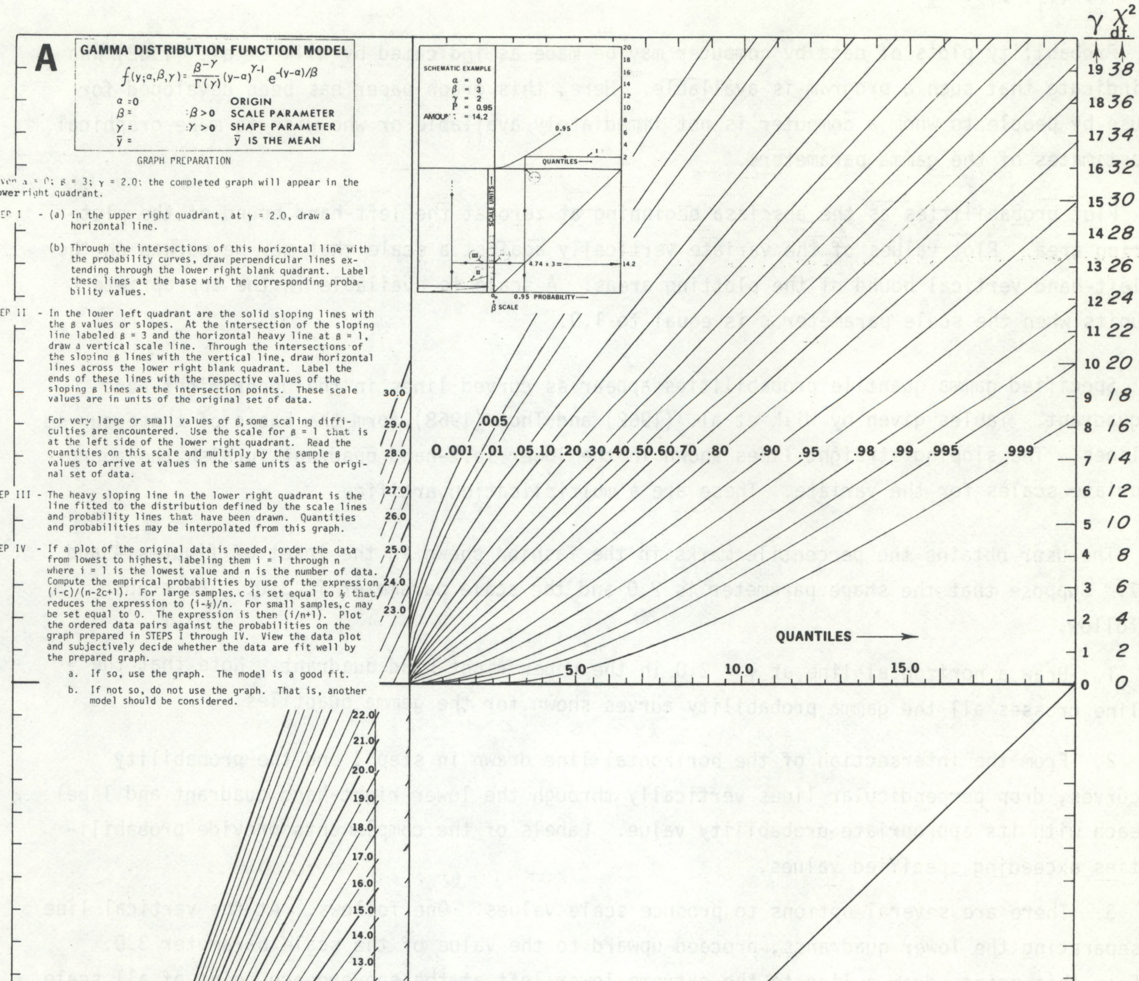


Figure 6.--Gamma distribution function plotting paper type B.1

GAMMA DISTRIBUTION FUNCTION MODEL

$$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$$

$\alpha = 0$		ORIGIN
$\beta =$	$:\beta > 0$	SCALE PARAMETER
$\gamma =$	$:\gamma > 0$	SHAPE PARAMETER
$\bar{y} =$		$\bar{y}$ IS THE MEAN

GRAPH PREPARATION

Given $\alpha = 0$; $\beta = 3$; $\gamma = 2.0$; the completed graph will appear in the lower right quadrant.

STEP I - (a) In the upper right quadrant, at $\gamma = 2.0$, draw a horizontal line.

(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower right blank quadrant. Label these lines at the base with the corresponding probability values.

STEP II - In the lower left quadrant are the solid sloping lines with the β values or slopes. At the intersection of the sloping line labeled $\beta = 3$ and the horizontal heavy line at $\beta = 1$, draw a vertical scale line. Through the intersections of the sloping β lines with the vertical line, draw horizontal lines across the lower right blank quadrant. Label the ends of these lines with the respective values of the sloping β lines at the intersection points. These scale values are in units of the original set of data.

For very large or small values of β, some scaling difficulties are encountered. Use the scale for $\beta = 1$ that is at the left side of the lower right quadrant. Read the quantities on this scale and multiply by the sample β values to arrive at values in the same units as the original set of data.

STEP III - The heavy sloping line in the lower right quadrant is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.

STEP IV - If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression $(i-c)/(n-2c+1)$. For large samples, c is set equal to ½ that reduces the expression to $(i-½)/n$. For small samples, c may be set equal to 0. The expression is then $(i/n+1)$. Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.

a. If so, use the graph. The model is a good fit.

b. If not so, do not use the graph. That is, another model should be considered.

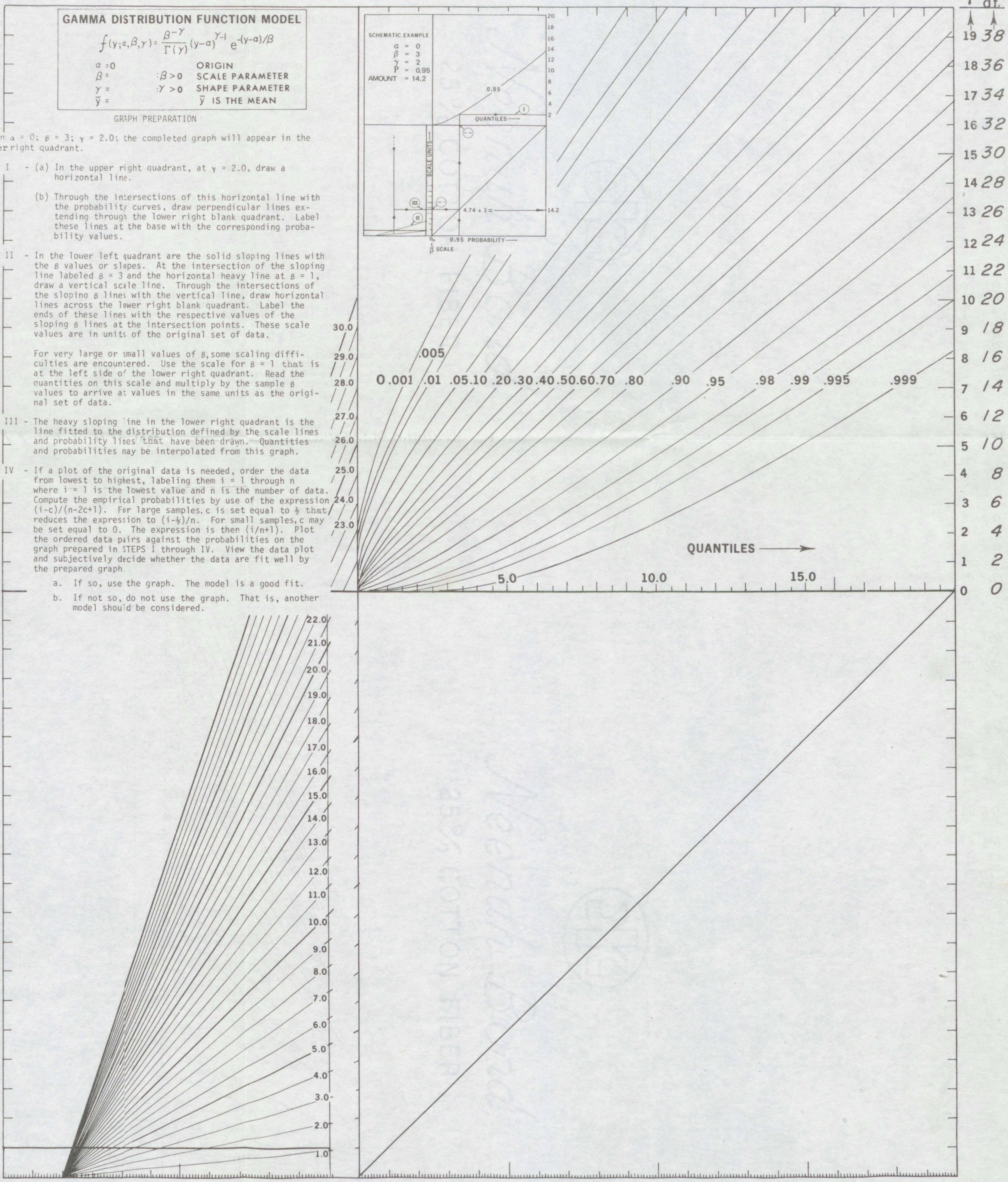


Figure 6.--Concluded

Table 2.--Ordered maximum recorded amounts of Appalachian Mountains precipitation in inches produced by tropical cyclones that passed over the mountains during 1900-1969. A is sequential order; B, amount; C, empirical probability $(1.000 - p_i)$ where $p_i = (i - c)/n - c + 1$ where $c = 0.44$.

A	B	C	A	B	C
1	0.80	0.016	19	6.31	0.514
2	.81	.043	20	6.44	.542 *
3	3.10	.071	21	8.00	.569
4	3.50	.099	22	9.30	.597 *
5	3.74	.126	23	10.84	.625
6	3.79	.154	24	11.00	.652
7	4.02	.182	25	11.07	.680
8	4.46	.209	26	11.22	.708 *
9	4.49	.237	27	13.47	.735
10	4.50	.265	28	15.15	.763
11	4.58	.292	29	15.60	.791
12	5.04	.320	30	16.00	.818
13	5.08	.348	31	16.36	.846
14	5.27	.375	32	16.64	.874
15	5.94	.403 *	33	18.69	.901
16	6.14	.431	34	18.93	.929 *
17	6.18	.458	35	23.73	.957 *
18	6.28	.486	36	27.00	.984 *

* These values are selected arbitrarily to illustrate plotting. The reader is invited to plot other points so as to induce a better understanding of the plotting procedure.

Table 3.--Estimates of the gamma distribution parameters of the maximum recorded Appalachian Mountains precipitation amounts in inches produced by tropical cyclones that passed over the mountains during 1900-1969. Table 2 provides the data.

Number of data	36	
Mean	9.263	
Origin ($\hat{\alpha}$)	0.000	
Beta ($\hat{\beta}$)	4.551	Scale parameter
Gamma ($\hat{\gamma}$)	2.035	Shape parameter
χ^2	10.667	
χ^2 (prob.)	0.846	χ^2 will exceed 10.667 with a probability of (1.000 - 0.846) or 0.154.

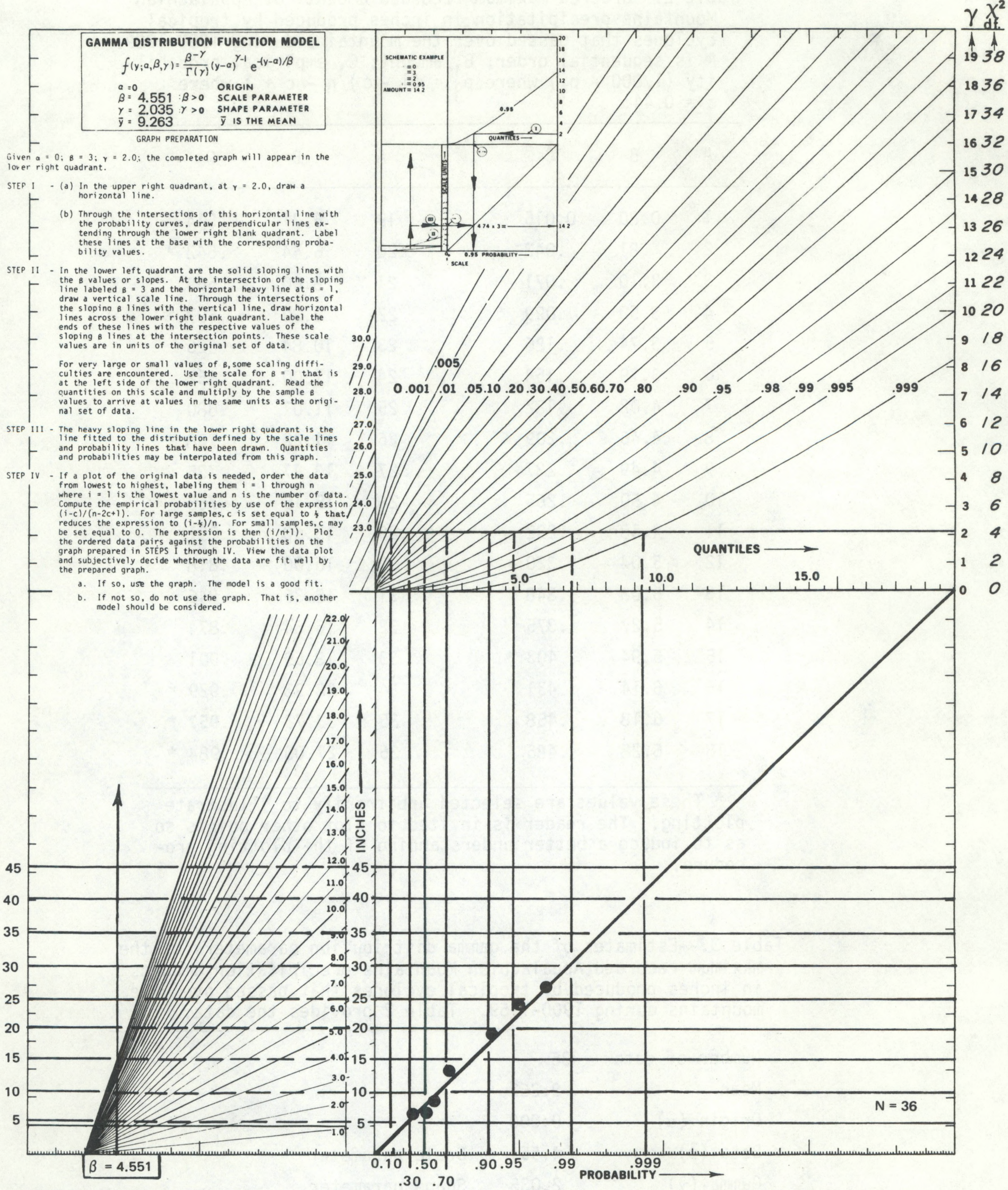


Figure 7.--Probability grid and partial plot of probability of maximum recorded Appalachian Mountains precipitation from tropical cyclones that passed over the mountains during 1900-1968. N = 36.

C. Type C.1 Plotting Paper

Type C.1 plotting paper intrinsically is the same as type B.1. The probability curves and scale curves appear in overlay patterns. The field is larger than in the type B.1 so as to permit easier drafting of the scale and probability grids. Figures 8A and 8B show two sizes for a gamma distribution function plotting paper type C.1 when data are > 0 or the origin:

1. The probability curves run upward toward the right.

2. The scale slopes run upward to the left so as to permit easier reading against the probability curves. These are marked in terms of β, the slope or scale parameter.

3. With a given scale parameter β, select the appropriate slope line. Proceed downward to the right to the intersection with the first horizontal unit line, $\beta=1$. Draw a vertical line and mark this line with the value of the slope lines at the points of intersection. These are the units of the scale of the original data. This scale then forms the ordinates of the graph. The ordinate values then can be drawn horizontally across the grid from the left-hand side of the graph paper (i.e., where probability values are zero). Label these.

4. With a given shape parameter γ, draw a horizontal line through the appropriate γ line. Mark the intersection of the curving probability lines with this horizontal line and respectively label them. Now, draw a vertical line through these points. Label them. These, with borders and the sloping heavy line, complete the grid.

5. The straight line sloping upward at 45° is the theoretical line of best fit as in the types A and B plotting papers.

6. The observed data with their probability levels dictated by $(i-c)/(n-2c+1)$ now may be plotted. See column B-4 of figure 3. View the data plot to determine subjectively whether the straight line fits the data.

Figure 9 illustrates the use of gamma probability plotting paper type C.1 for the parameters given in table 3. Again, a few data pairs of table 2 are overplotted to illustrate the fit of the line to the quantile slope 1 to the data. The inference is, of course, made again that the line is a good fit to the data on the grid shown.

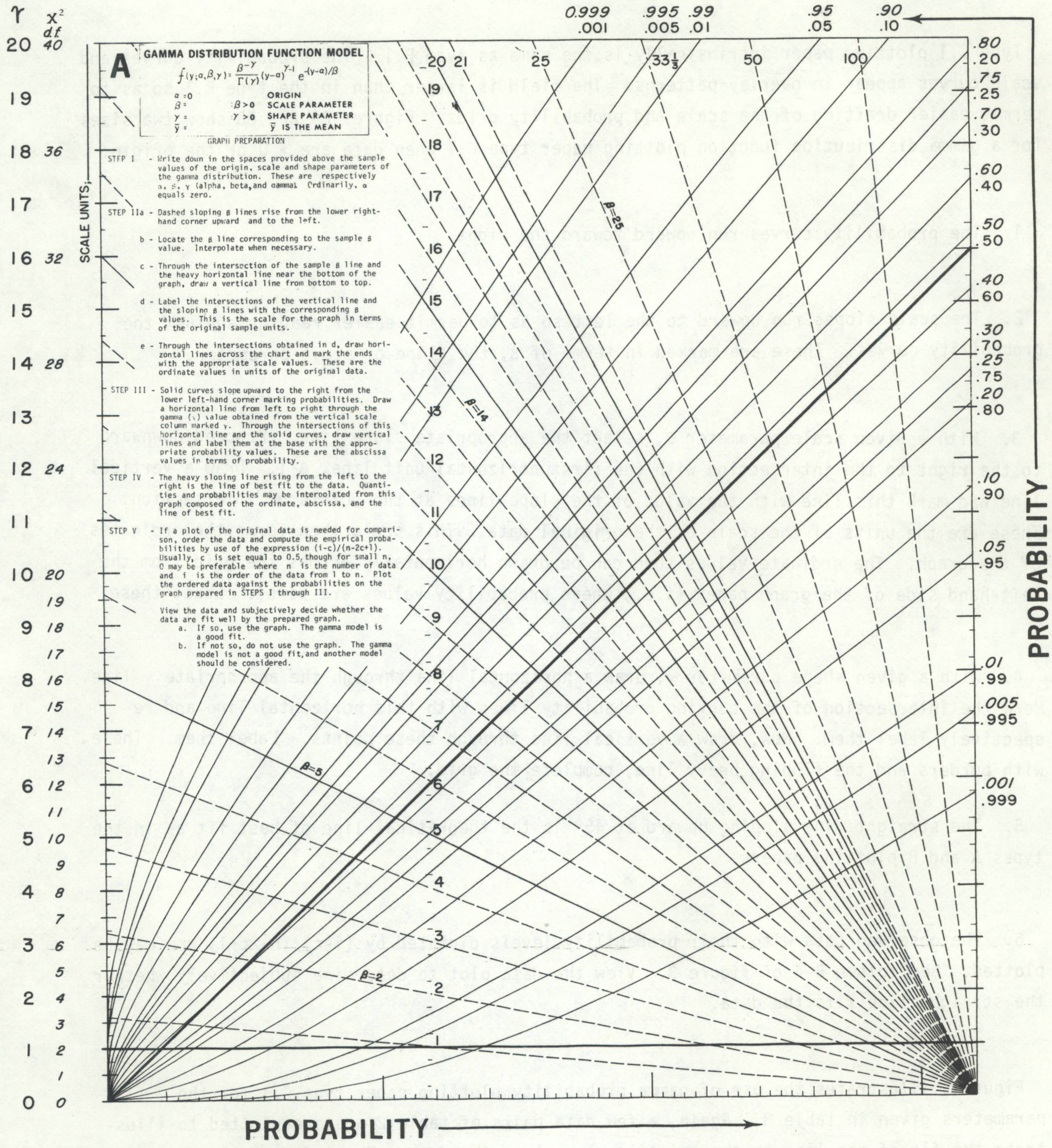


Figure 8.--Gamma distribution function plotting paper type C.1

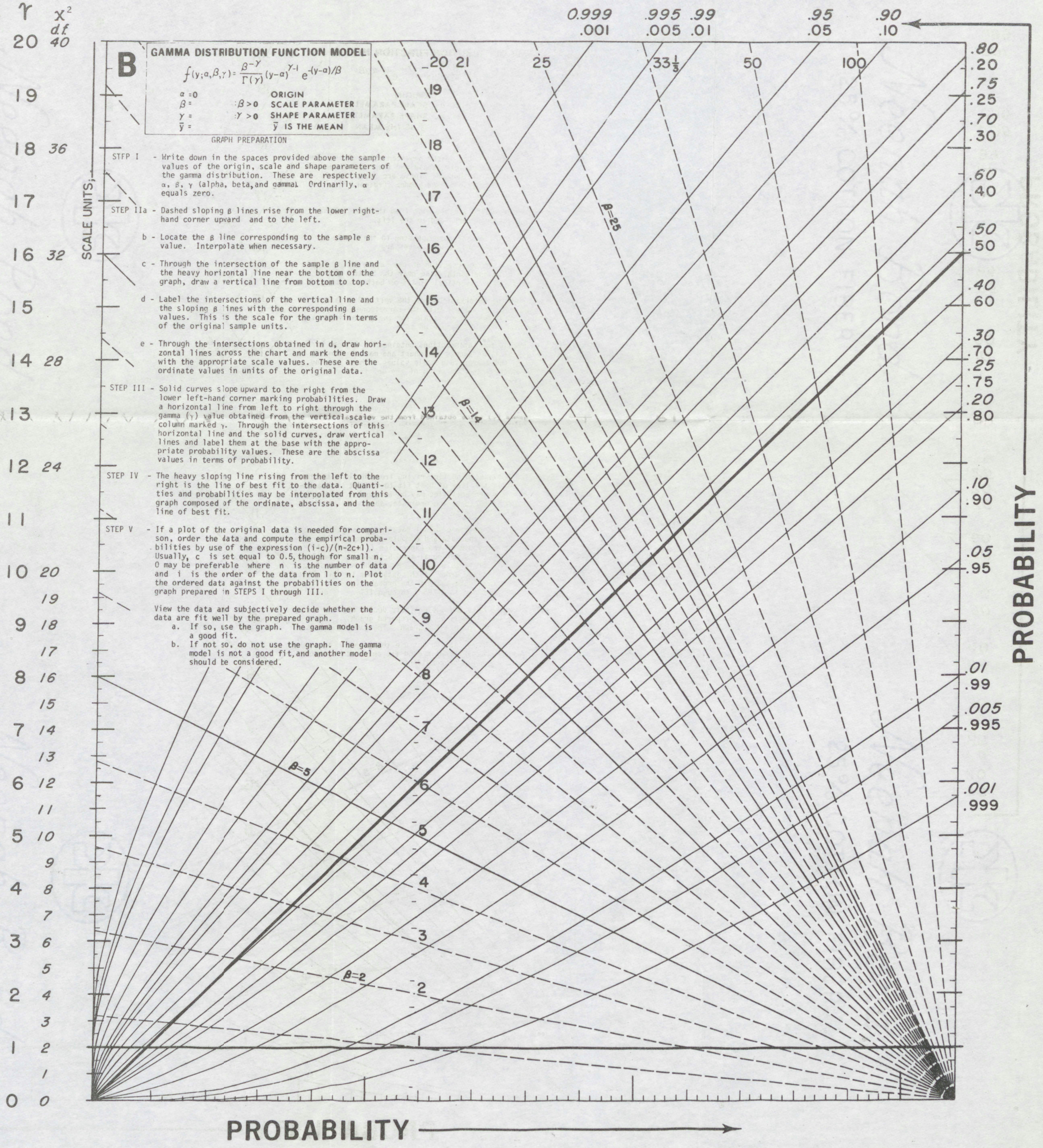


Figure 8.--Concluded

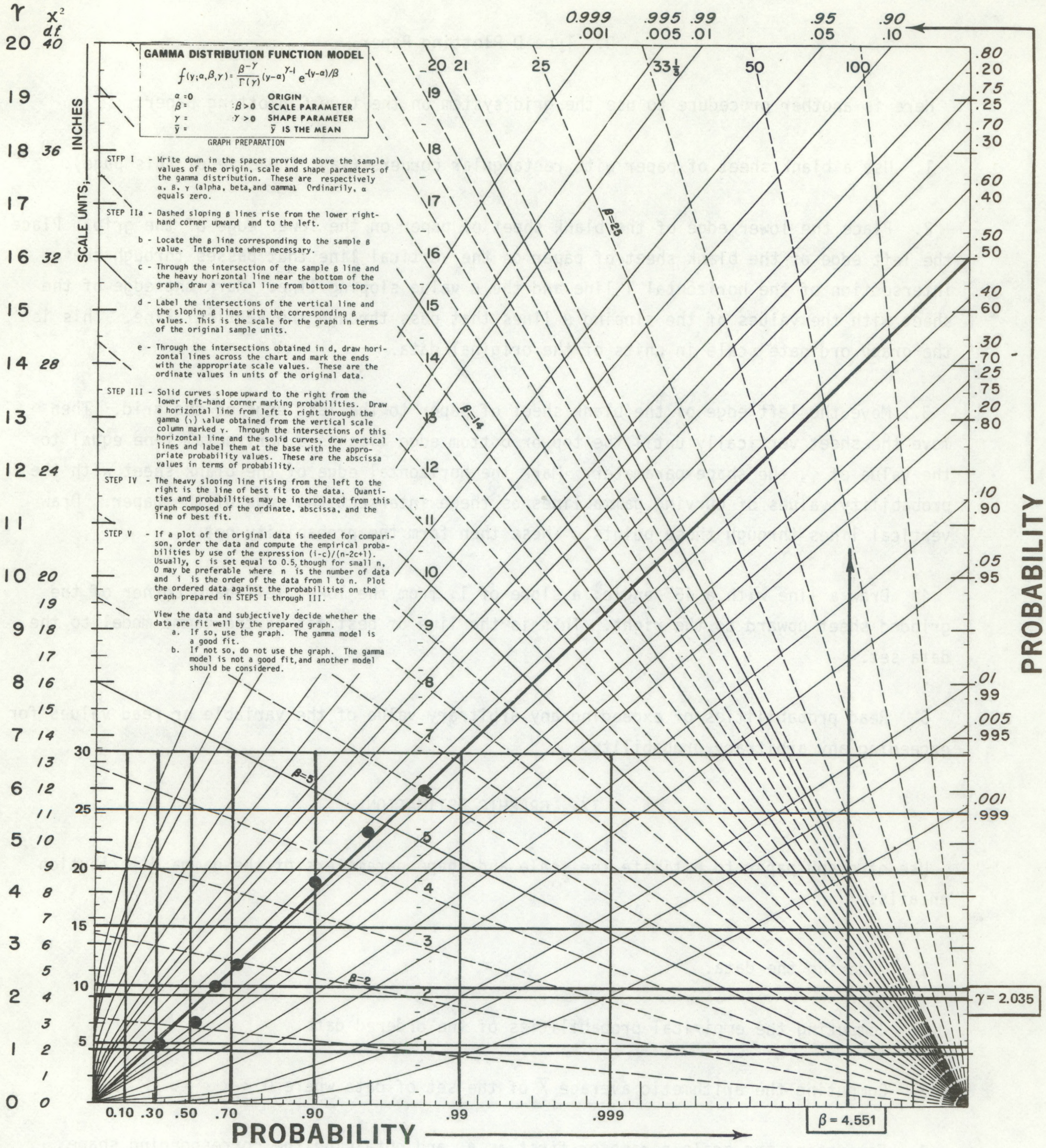


Figure 9.--Probability grid and partial plot of probability of maximum recorded Appalachian Mountains precipitation from tropical cyclones that passed over the mountains during 1900-1968. N = 36.

D. Type D Plotting Paper

Here is another procedure to use the grid system on the type C plotting paper:

1. Use a blank sheet of paper with rectangular corners (say, the size of this page).

2. Place the lower edge of the blank sheet of paper on the lower edge of the grid. Place the left edge of the blank sheet of paper on the vertical line that passes through the intersection of the horizontal 1 line and the β value sloping line. Mark the edge of the sheet with the values of the sloping β lines that pass through the vertical line. This is the graph ordinate scale in units of the original data.

3. Move the left edge of the blank sheet of paper to the left edge of the grid. Then move the sheet vertically until the top or bottom edge lies on a horizontal line equal to the value of γ, the shape parameter. Mark the horizontal edge of the blank sheet with the probability values of curving gamma lines as these intersect the edge of the paper. Draw vertical lines through these points. These then form the probability net.

4. Draw a line with a 45° angle, a slope of 1, from the lower left-hand corner of the gridded sheet upward to the right. This is the line of best fit of the gamma model to the data set.

5. Read probabilities of exceeding any arbitrary value of the variable or read values for exceeding any arbitrary probability.

IX. GRAPHIC ESTIMATION

Use of graph paper to estimate the scale and shape parameters of the gamma distribution entails:

1. Ordering the data.

2. Computing the empirical probabilities of the ordered data.

3. Computing the arithmetic average $\overline{X}$ of the set of data where $0 < x < \infty$.

4. Estimating the scale parameter first as β_1 and obtaining the corresponding shape parameter γ_1 from the expression $\overline{X} = \beta_1\gamma_1$.

5. Alternatively, the first estimate of β, β_1 may be obtained by first estimating the shape parameter γ as γ_1. This may be done by plotting the histogram of the data set and then by looking at the histogram shape. Reference may be made to figure 1 showing various

shape parameters with the corresponding distribution curve shape. Remember:

a. The mean or average is equal to $\beta\gamma$.

b. The mode is one less than γ_1 (i.e., γ-1).

c. If the shape is exponential, γ=1.

d. If the shape is normal, γ is large; estimate 30. Having estimated γ as γ_1, then obtain β_1 from $\overline{X} = \gamma_1\beta_1$.

6. Scaling the lower right diagram (compute ordinate values) using $\beta_1 = \overline{X}/\gamma_1$. This applies to figures 6A, 6B, and 7.

7. Drawing a horizontal line on the upper right diagram corresponding to γ.

8. Drawing vertical probability lines on the lower right diagram corresponding to γ and the empirical probabilities computed in (2).

9. Plotting the data according to points defined by the abscissa and ordinate values determined in (5) and (7).

10. Drawing a line of best fit (by eye) through the plotted data, using all data.

11. Translating the line of best fit so that it intersects the lower left corner of the lower right diagram.

12. For a convenient probability (preferably the highest) take the ratio of the value of the translated line of best fit to the value of the theoretical printed line, which is the 45° line.

13. Multiplying the square or cube of ratio obtained in the latter part of (12) by γ_1 to obtain a second approximation γ_2 that is a closer approximation to the sample shape parameter γ.

14. Computing a new approximation to the correct sample β from $\beta_2 = \overline{X}/\gamma_2$.

15. Repeating this procedure until an estimated γ_k and β_k permit the line of best fit to coincide with the theoretical printed line.

16. The multipliers in (13) may be higher roots or powers for other types of graph paper.

This procedure should provide acceptable estimates with the second approximation, though three approximations may be needed. A bad fitting of the data points by eye will call for the next approximation.

X. PROBABILITY PLOTTING FOR OTHER DISTRIBUTIONS

A. Exponential Distribution

The user is urged always to make a scattergram or some type of plot for the data set. If the data set is bounded on one side and unbounded above, a first guess model is a gamma model. If the shape parameter is near 1, a second guess is that the distribution may be more specific (i.e., exponential). See figure 1. Rarely will the number 1 be obtained precisely because of sampling error. Therefore, with a shape parameter near 1, the exponential distribution is a good first guess. Use exponential distribution plotting paper or the chi-square plotting paper for two degrees of freedom. If these are not available, use the type B or type C probability plotting paper described in this report. The probability grid is determined from gamma equal to one line ($\gamma = 1$).

B. Chi-square Distribution

As the degrees of freedom for the chi-square distribution are integer values equal to twice the value of gamma, then the chi-square distribution for 1, 2, 3, 4, 5,...,n degrees of freedom are plotted on gamma plotting paper for 0.5, 1, 1.5, 2.0, 2.5,..., values for gamma. The graph paper illustrated has the χ^2 degrees of freedom placed parallel to the gamma values along the ordinate on the left.

C. Poisson Distribution

The Poisson distribution holds for even values of the chi-square distribution. Therefore, gamma distribution grids prepared for gamma equal to integer values of gamma may be used for the Poisson distribution.

Please note that the gamma, exponential, chi-square, and Poisson distributions are related. The gamma distribution model is the general model. The shape parameter's range is $0 \leq \gamma < \infty$. The chi-square distribution shape parameters in terms of γ are restricted (i.e., these are 0.5, 1.0, 1.5,..., $< \infty$ gamma values respectively equivalent to chi-square with 1, 2, 3,...,degrees of freedom). The Poisson distribution shape parameters in terms of γ are even more restrictive; i.e., these exist for chi-square degrees of freedom 2, 4, 6,..., $< \infty$, respectively, for gamma shape parameters are 1, 2, 3,.... The exponential distribution has one and only one shape parameter in terms of γ; its value is 1. In other words, for the last distribution, the exponential distribution is equivalent to a gamma distribution with a shape parameter of 1 and a chi-square distribution with two degrees of freedom. A small tabular illustration follows where ... or a value indicates existence and dashes or lack of a mark indicates nonexistence.

Some care will be required if chi-squared tables are used for gamma or vice-versa. The chi-square values must be halved, or the gamma values must be doubled for any cumulative level. For example, at the 0.95 cumulative probability level the chi-square for 2 d.f. is 5.991 while the gamma tabular value at a shape parameter of 1.000 is 2.9957. For 3 d.f., the 0.95 cumulative probability level is 7.815 while the gamma tabular value is 3.9074.

Gamma (Shape Parameter)	Chi-square (Degrees of Freedom (d.f.))	Poisson (d.f.)	Exponential (d.f.)
...	-	-	-
...	-	-	-
...	-	-	-
...	-	-	-
0.5000	1	-	1
...	-	-	
...	-	-	
...	-	-	
...	-	-	
1.0000	2	2	
...	-	-	
...	-	-	
...	-	-	
...	-	-	
1.5000	3	-	
...	-	-	
...	-	-	
...	-	-	
...	-	-	
2.0000	4	4	
.	.	.	
.	.	.	
.	.	.	

XI. SPECIALIZED GAMMA GRAPH PAPER

The foregoing discussion of the gamma probability graph paper treats as large a range of scale and shape parameters as possible. For quality assurance, control and reliability purposes, the user may be interested in the region of the gamma distribution near zero and in the tail. Therefore, five more graphs, types 3 through 7, have been prepared, though it is realized that these may not meet all of the user's needs. These are shown as figures 10, 11, 12, 13, and 14. Readers are invited to correspond with the authors if other specialized forms are needed.

Table 4 provides for the seven graph papers,

1. The ranges of the probabilities, the quantiles, the shape parameters, and the scale parameters, in addition to

2. The slope ratio treatment for the parameter approximating steps.

No examples of use with actual data are provided here.

For figures 12, 13, and 14 the scaling can be $\beta/100$. Figure 11 differs from the others in that the right half is an extension of the left half. Here, the line of best fit may be extended upward toward the right for use with the left half. For the right half a line of best fit may be drawn from its lower left-hand corner and an appropriate shift of scale made from the left half.

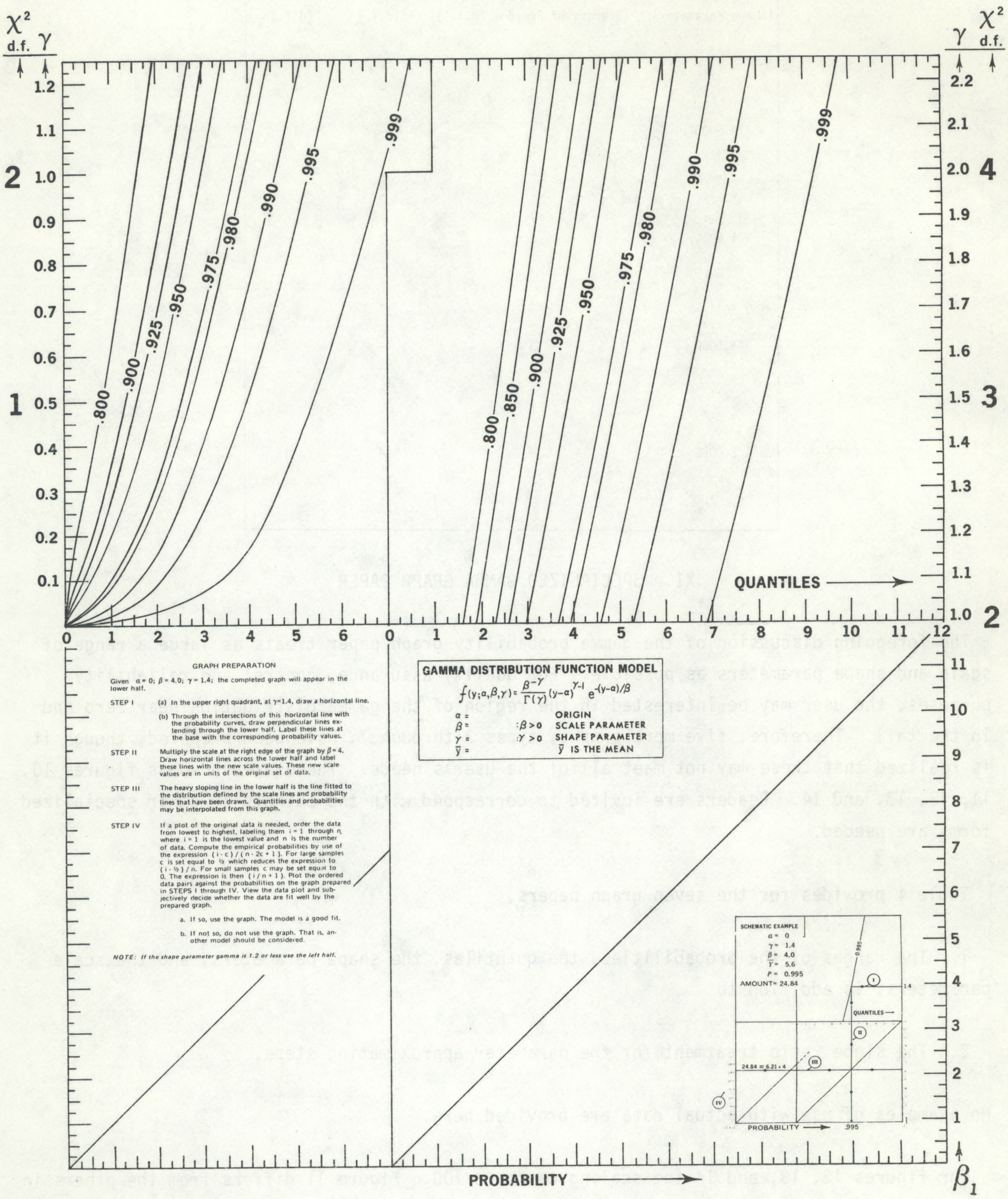
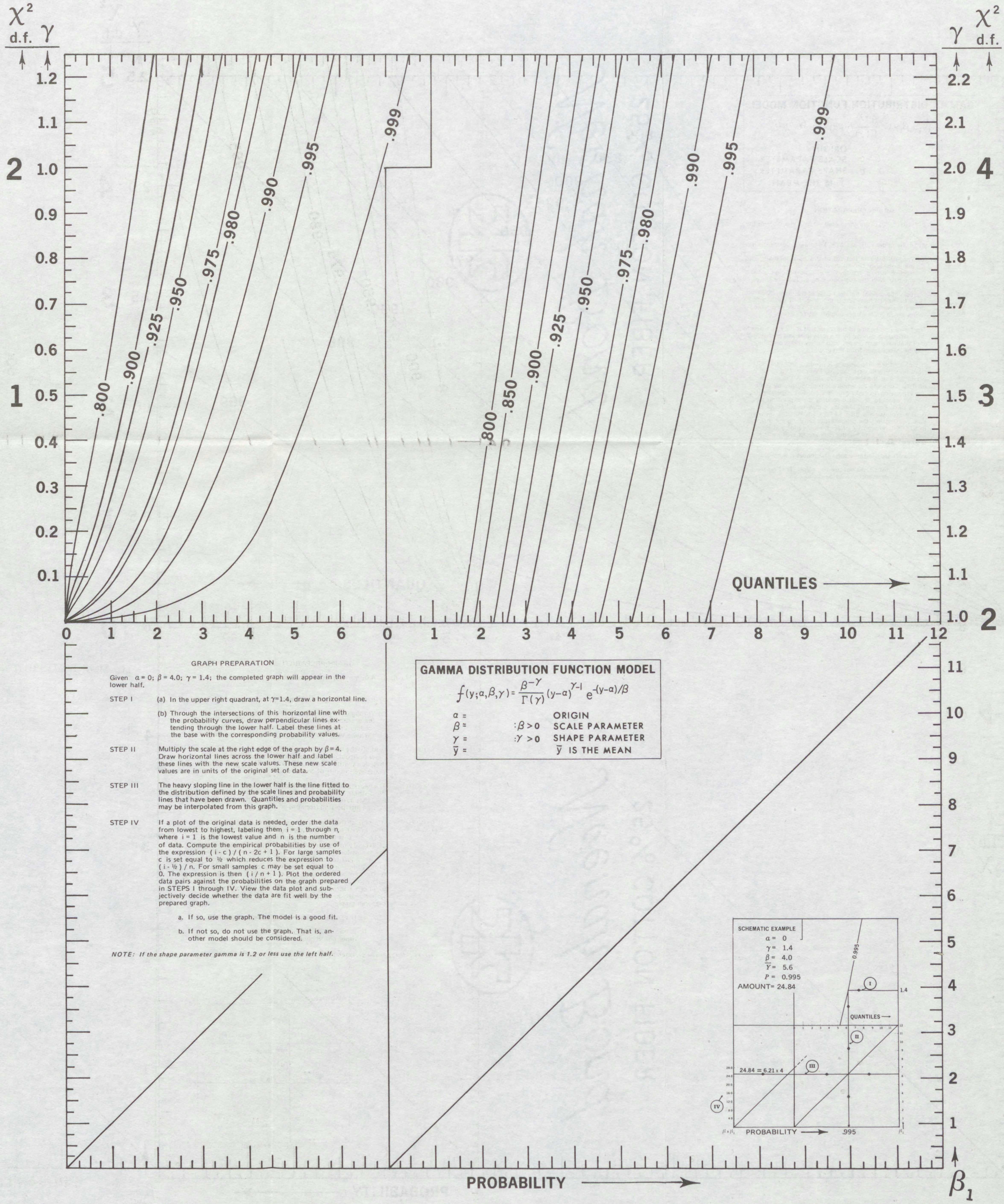


Figure 10.--Gamma distribution function plotting paper type 3



GRAPH PREPARATION

Given $\alpha = 0$; $\beta = 4.0$; $\gamma = 1.4$; the completed graph will appear in the lower half.

STEP I (a) In the upper right quadrant, at γ=1.4, draw a horizontal line.

(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower half. Label these lines at the base with the corresponding probability values.

STEP II Multiply the scale at the right edge of the graph by $\beta = 4$. Draw horizontal lines across the lower half and label these lines with the new scale values. These new scale values are in units of the original set of data.

STEP III The heavy sloping line in the lower half is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.

STEP IV If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i - c) / (n - 2c + 1). For large samples c is set equal to ½ which reduces the expression to (i - ½) / n. For small samples c may be set equal to 0. The expression is then (i / n + 1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.

a. If so, use the graph. The model is a good fit.

b. If not so, do not use the graph. That is, another model should be considered.

NOTE: If the shape parameter gamma is 1.2 or less use the left half.

GAMMA DISTRIBUTION FUNCTION MODEL

$$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$$

α =		ORIGIN
β =	: $\beta > 0$	SCALE PARAMETER
γ =	: $\gamma > 0$	SHAPE PARAMETER
$\overline{y}$ =		$\overline{y}$ IS THE MEAN

Figure 10.--Concluded

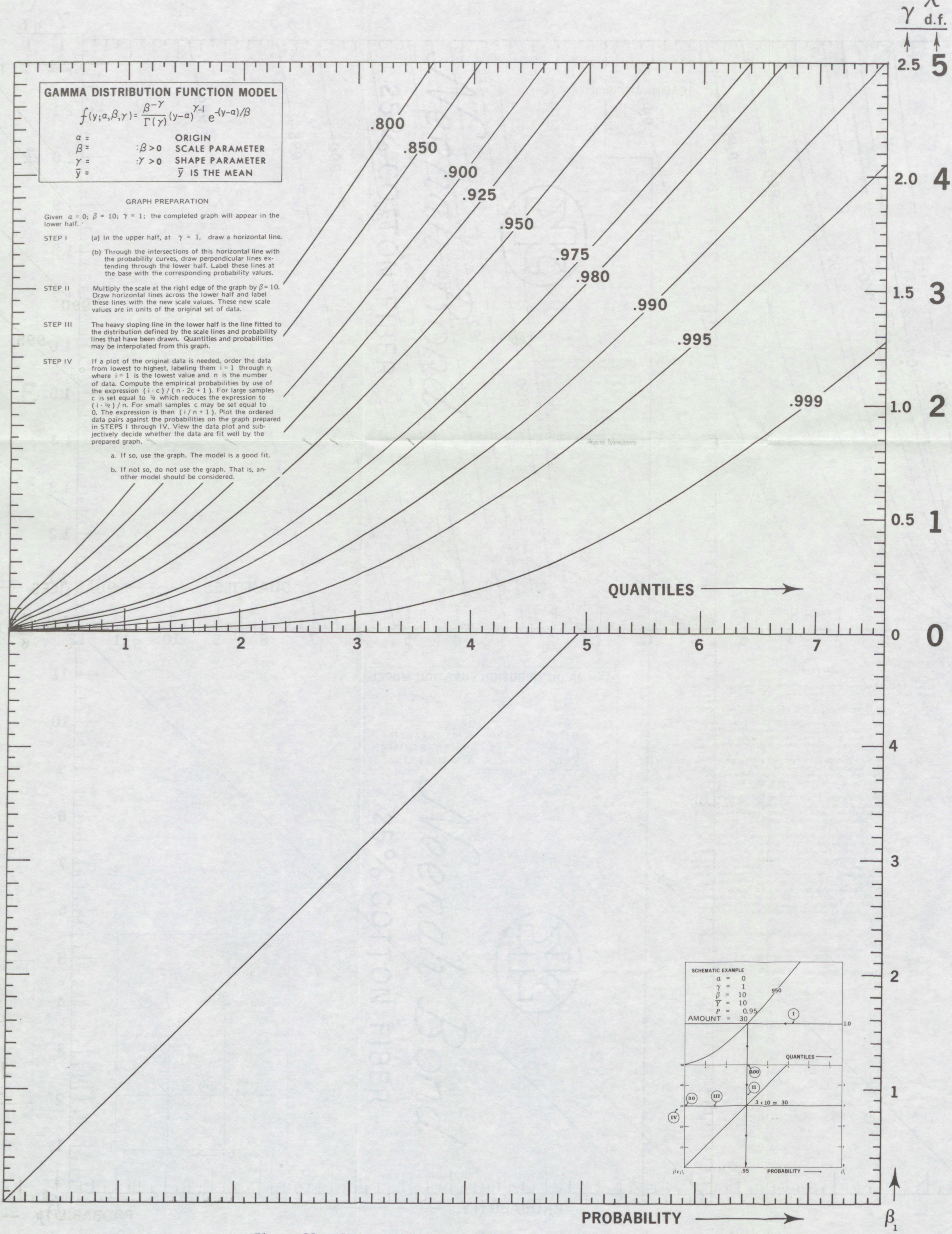


Figure 11.--Gamma distribution function plotting paper type 4

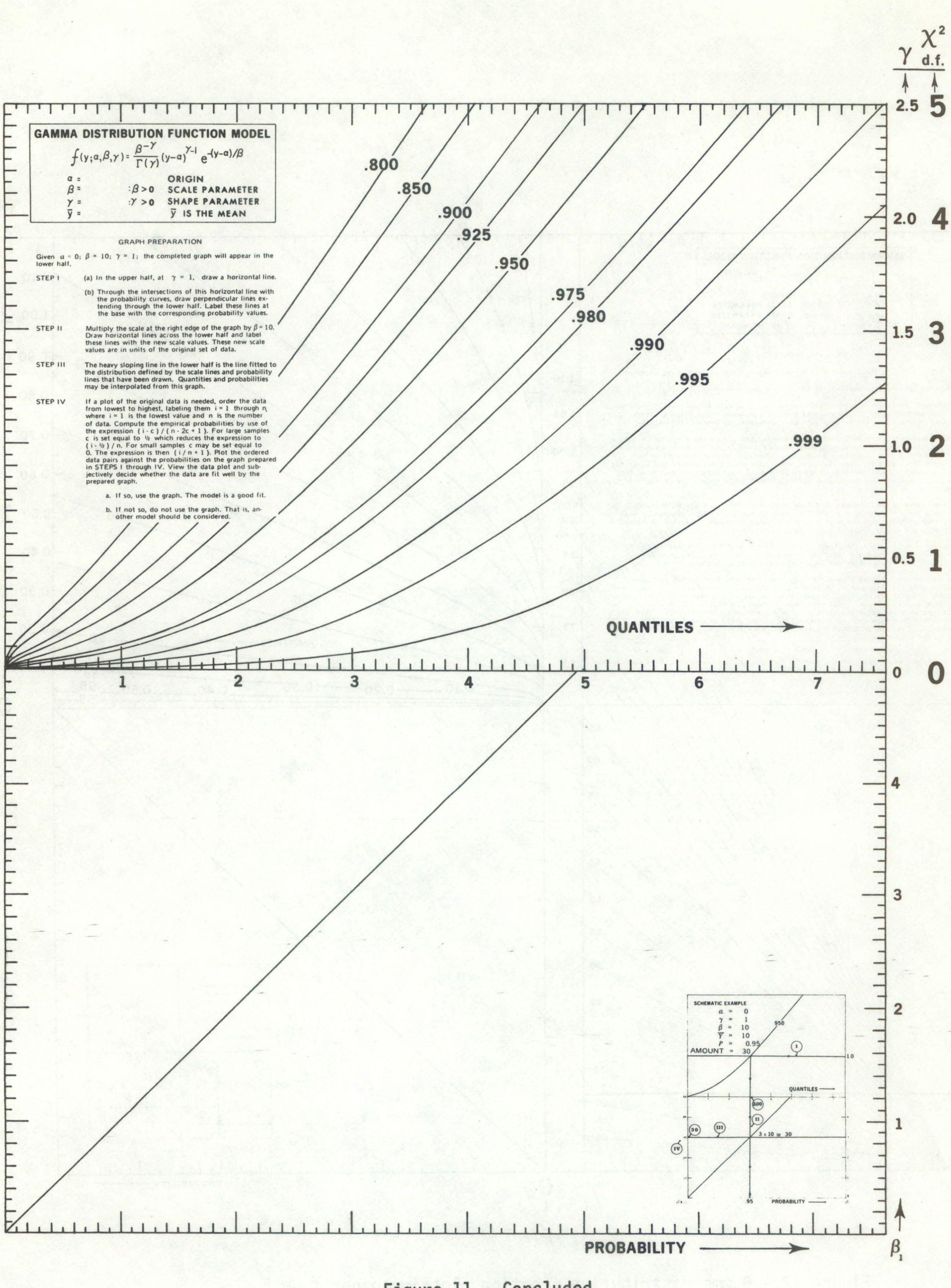


Figure 11.--Concluded

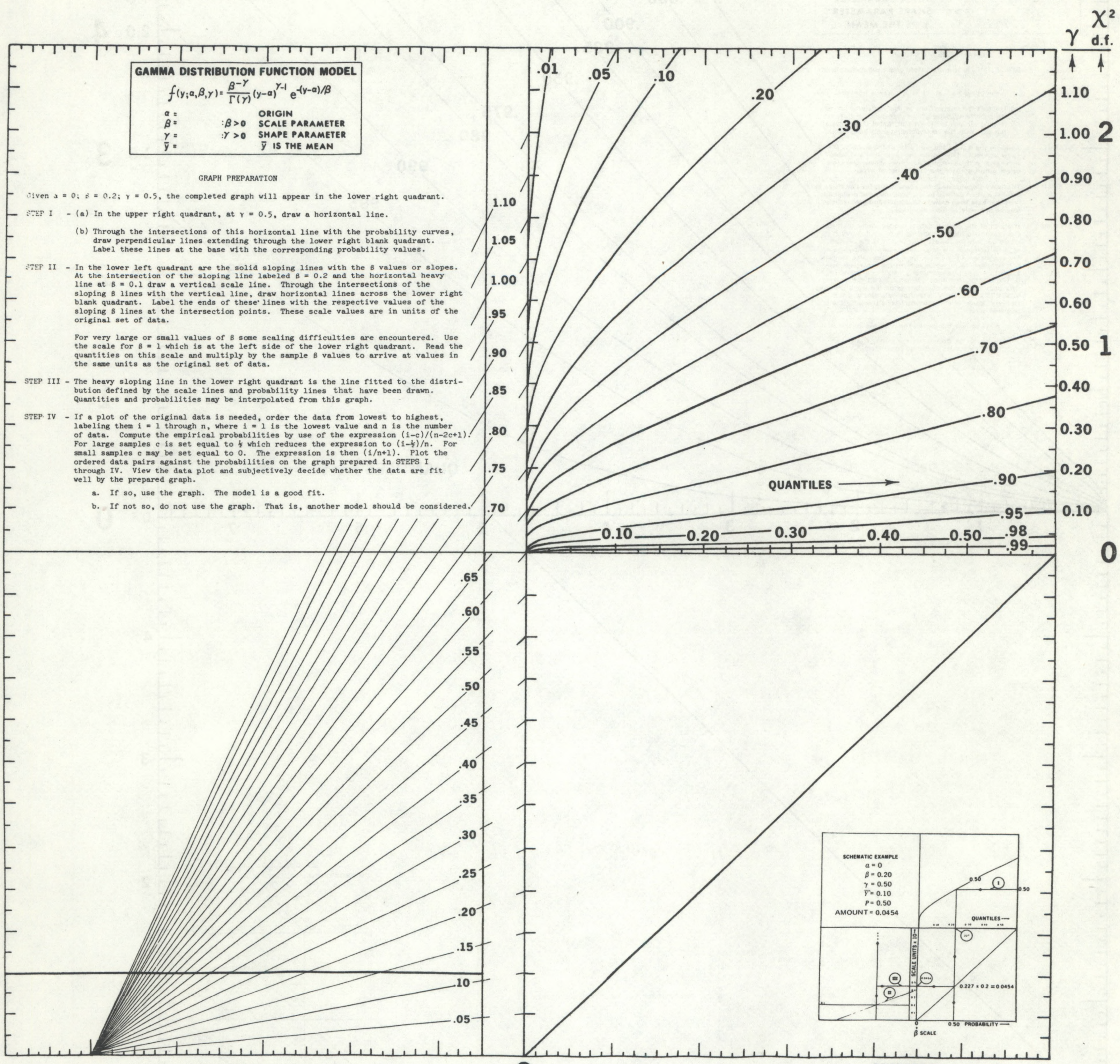


Figure 12.--Gamma distribution function plotting paper type 5

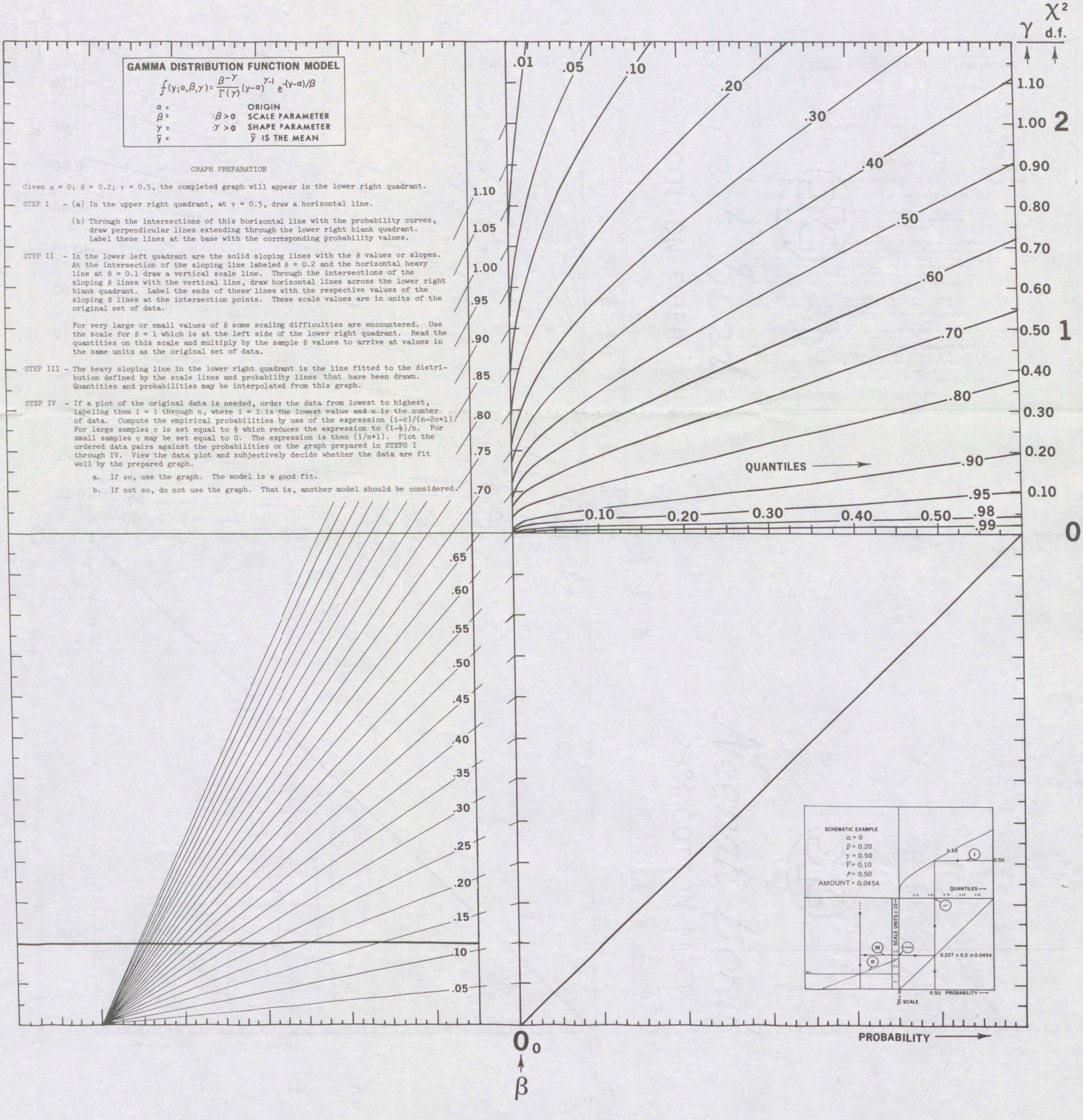


Figure 12.--Concluded

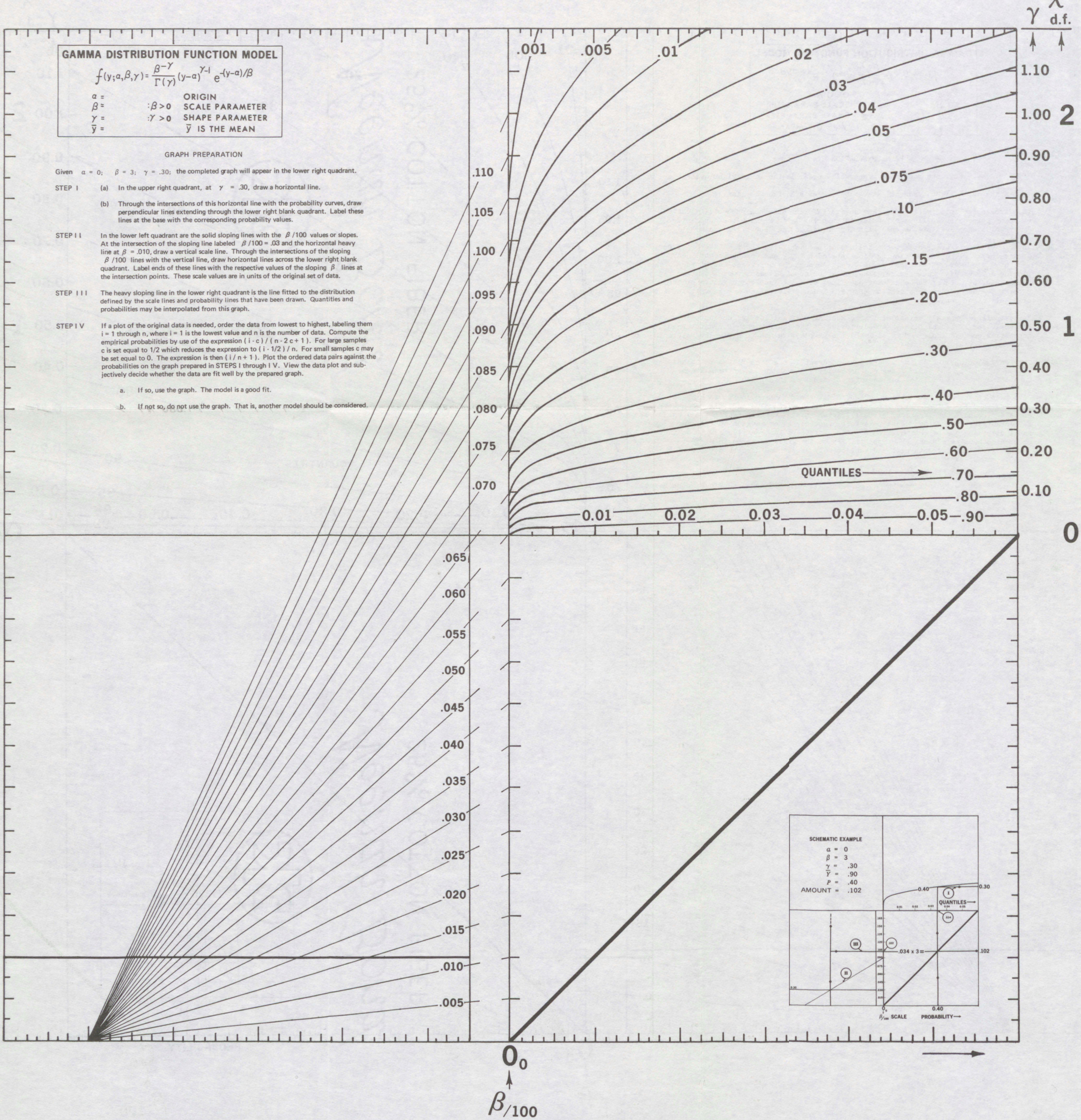


Figure 13.--Gamma distribution function plotting paper type 6

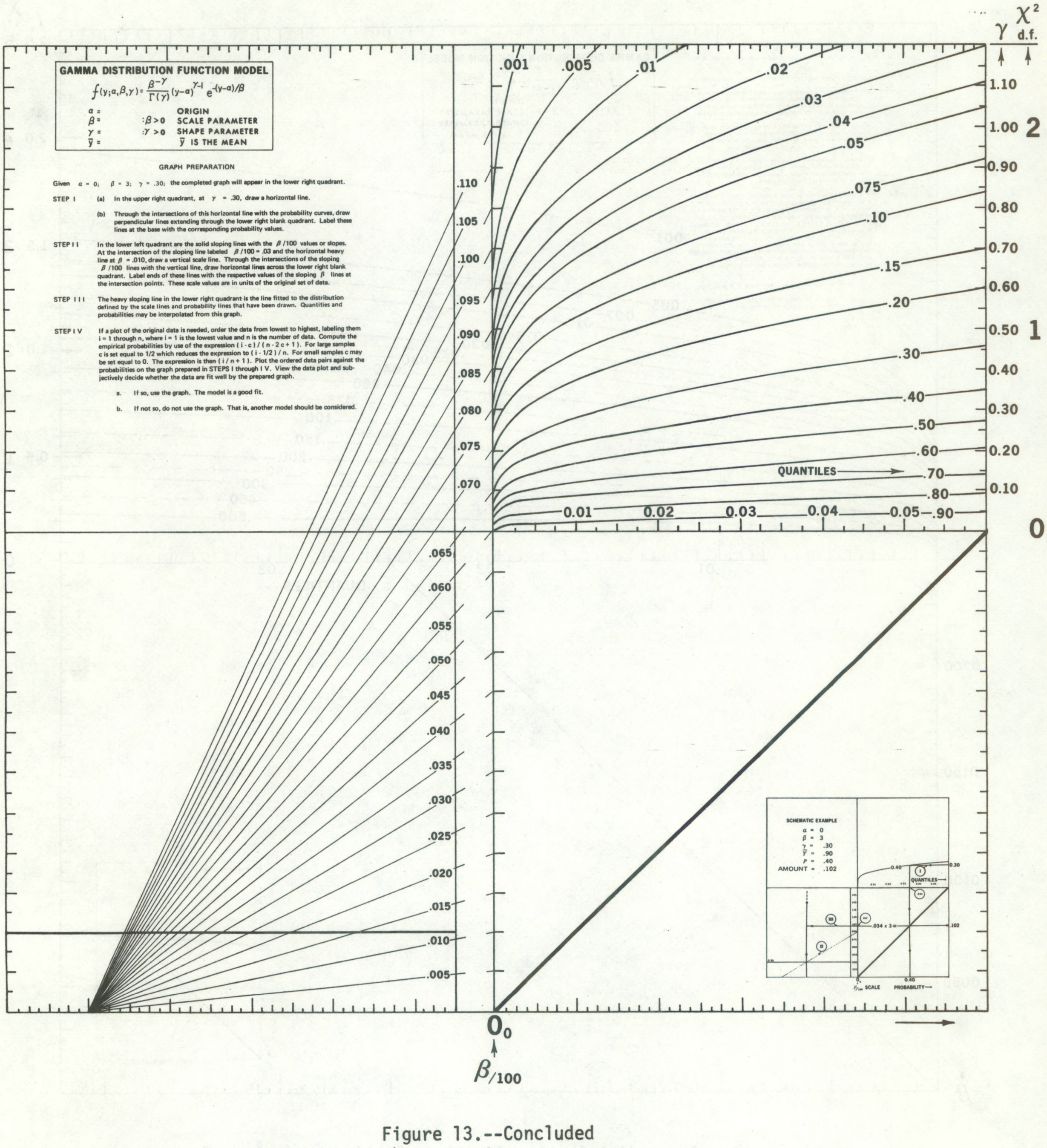


Figure 13.--Concluded

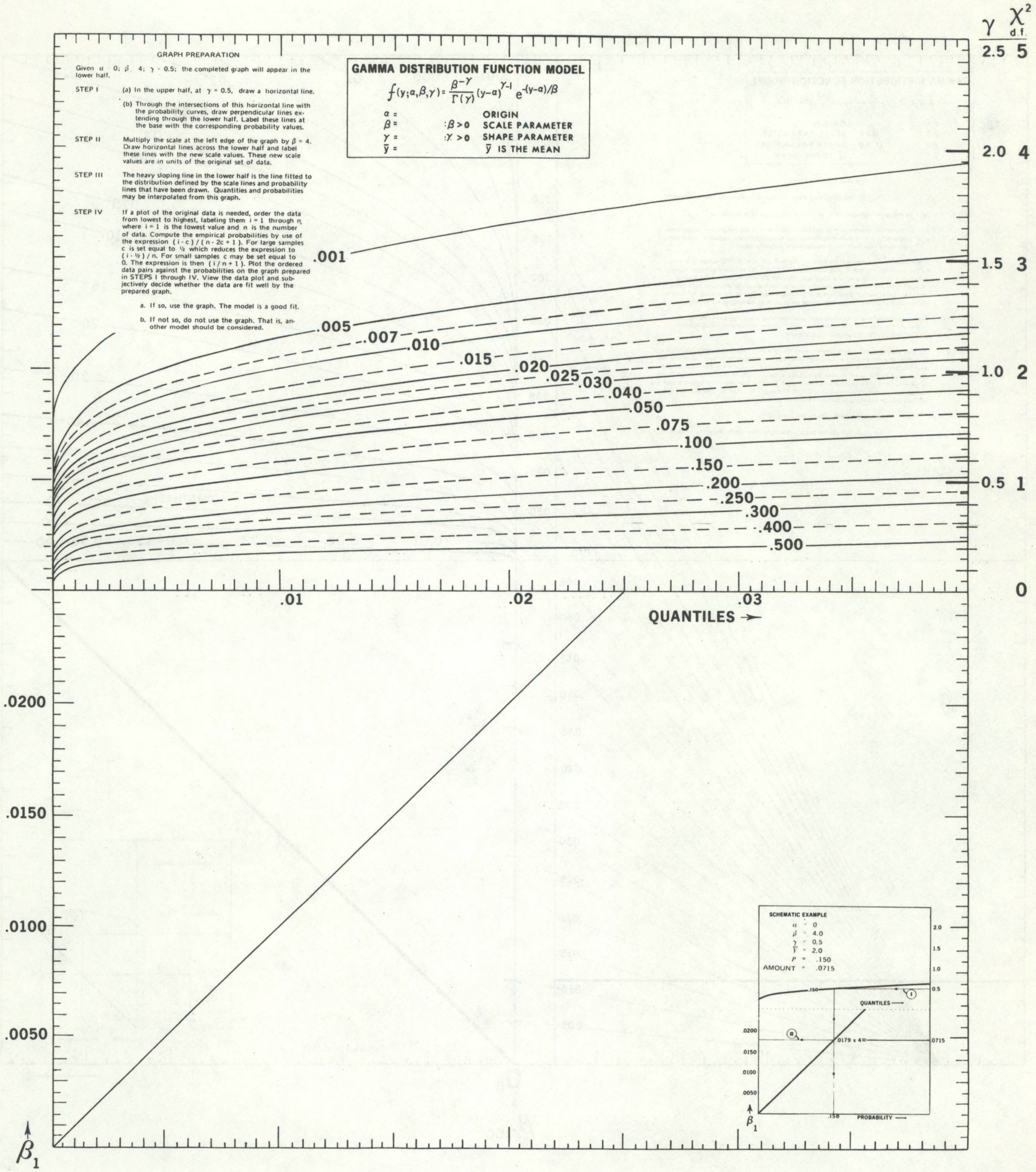


Figure 14.--Gamma distribution function plotting paper type 7

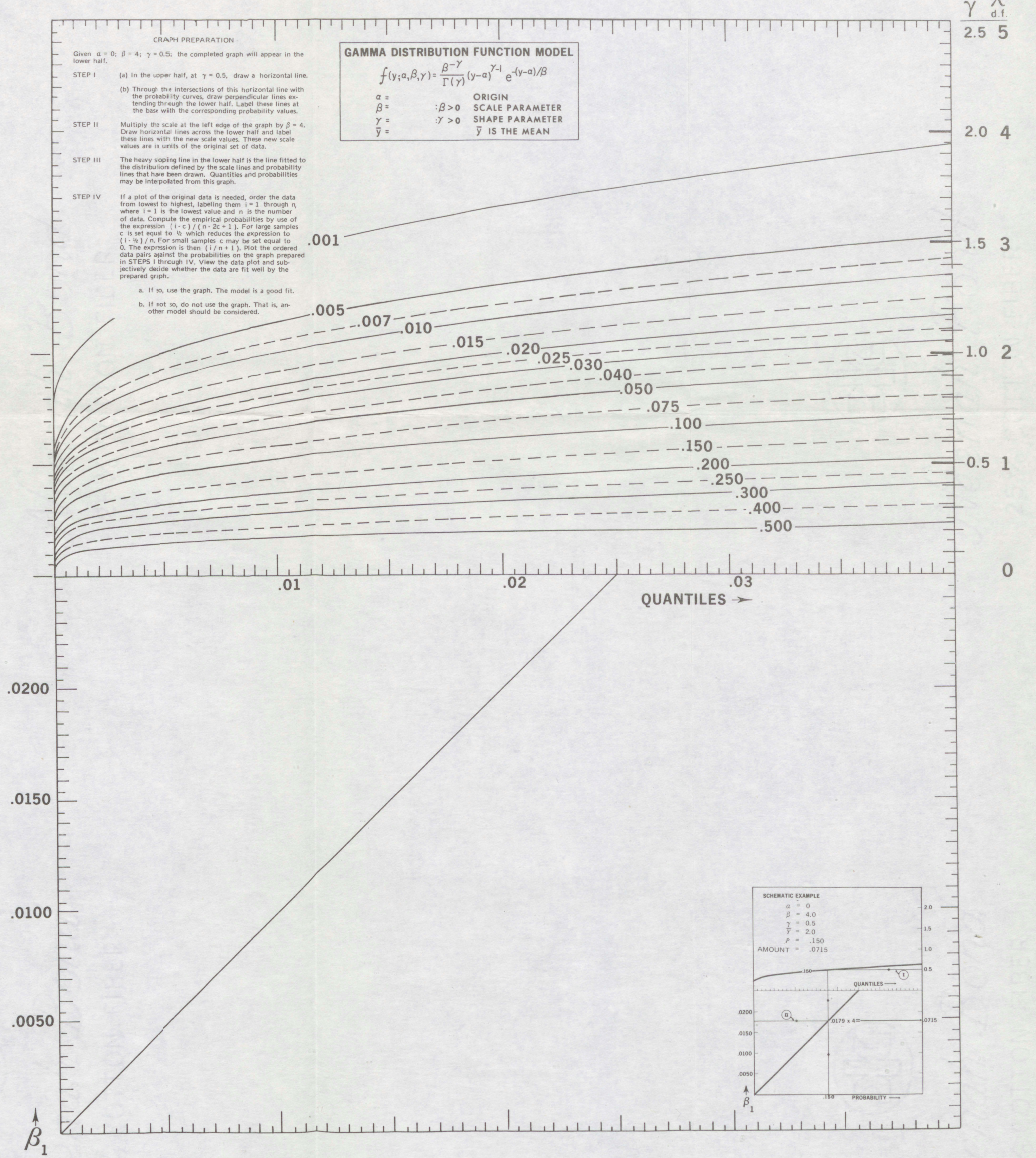


Figure 14.-- Concluded

Table 4.--Ranges for the seven graph papers of the probabilities, the quantiles, the shape and scale parameters

Type	Figure	Probability p	Quantiles	Shape Parameter	Scale Parameter	Slope Ratio r
1*	6	.001 to .999	>0.00 to 20.00	>0.00 to 20.00	>0.00 to 30.00	r^2
2*	8	.001 to .999	>0.00 to 16.00	>0.00 to 20.00	>0.00 to 100.00	r^2
3	10	.800 to .999	>0.00 to 12.00	>0.00 to 2.25	>0.00 † to 12.00	r^2
4	11	.800 to .999	>0.00 to 7.60	>0.00 to 2.50	>0.00 † to 5.00	r^2
5	12	.010 to .990	>0.00 to 0.60	>0.00 to 1.20	>0.00 to 1.10	$r^{1/2}$
6	13	.001 to .900	>0.00 to 0.06	>0.00 to 1.20	>0.00 to 11.00	$r^{1/2}$
7	14	.001 to .500	>0.00 to 0.04	>0.00 to 2.50	>0.00 † to 0.025	$r^{1/4}$

*Type 1 here is type B.1 in the text, and type 2 is type C.1.
†refers to β_1, the multiplier of β.

XII. FUTURE MODIFICATIONS TO THE PROGRAM

The following are five expected modifications planned for the computer program and subroutines given.

1. A subroutine for the determination of an acceptable location (origin) parameter.

2. A possible subroutine for the debiasing of the maximum likelihood and Thom (1958) shape and scale estimators.

3. Modification of routines to permit calculation of probabilities for shape parameter values, plots, and other x-y type plotters using linear scale plotting.

4. A subroutine for cathode ray computer output plots.

5. A separate program designed for low values of the shape parameter (i.e., $\gamma \leq 1.000$).

ACKNOWLEDGMENTS

Acknowledgment is made to Mr. H. C. S. Thom for his discussions and material used from his many papers and to Mr. Danny Fulbright for his help in checking the procedures.

Appreciation is expressed to Dr. M. B. Wilk of the American Telephone and Telegraph Company and to Dr. R. Gnanadesikan of Bell Laboratories for correspondence and discussions. Acknowledgment is made to Dr. James D. McQuigg, Research Meteorologist, Environmental Data Service, NOAA, and to Dr. Sharon LeDuc, Atmospheric Science Department, University of Missouri, for discussions.

Acknowledgment is made to Bradford F. Kimball and to L. R. Shenton and K. O. Bowman for permission to quote material given on pages 13-14 and pages 6-7, respectively. Acknowledgment is made also to the Journal of the American Statistical Association and to NOAA to quote the above material.

Appreciation is tendered to Mr. Warren Buck and to Mr. Bob Ford for the drafting of the figures and graphs, to Dr. Nathaniel Guttman for a review of the report, and to Mrs. Margaret Larabee for final typing of the manuscript.

EDITOR'S NOTE

Under section XII, "Future Modifications to the Program," number 5, techniques were to be developed to permit better calculations when the shape parameter was less than 1. These techniques already have been developed but too late to include here. A modification to the program given in this paper will be issued in the near future. The technique will be extended to the use of higher shape parameters, say 4, as on the average the computing time is halved.

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APPENDIX:
FORTRAN IV ELECTRONIC COMPUTER PROGRAM
FOR APPLICATION OF THE GAMMA DISTRIBUTION FUNCTION TO DATA SETS;
AND
WORK GRAPHS (GAMMA DISTRIBUTION FUNCTION MODEL PLOTTING PAPER)
PERFORATED FOR EASY REMOVAL AT THE END OF THE REPORT

The following are comments for use of the gamma distribution program (FORTRAN IV). The user may go directly to the program for implementation. The program does contain comment cards where deemed appropriate.

The immediately following comments are provided for those who wish to have a more general understanding of just what is required for program initiation. These will be helpful as references if difficulties are encountered either during the initiation or the running of the program.

There are four types of header (control) cards associated with any request. Two of these are required; the remaining two are required only when the user chooses to define his own table of quantiles and probability levels.

Control card 1

Card col.	Name in program	Meaning
1-2	II	This is the beginning period number of data set. Usually, this would be 01 if the first period is desired. Care should be taken when working with multiple data points per input record if one chooses to start with other than the first period on each data set. Positional association is used in this program. For example, suppose one has 20 yr of weekly rainfall data in cards with 13 weeks of data contained on 1 card--therefore 4 cards per year comprising a data set. If the user chooses to define II = 12, he should make certain that fields 1-11 do not contain invalid punches (blanks are permissible). The data for this year should then fall into the 12th field of the input card. If one chooses to start with 14, the first card for weeks 1-13 will not be required, however the card number must be 2 since data storage is computed by index = (card no. - 1) * No. pts/card + pt # in this card $01 \leq II \leq 52$.
3-4	JJ	Ending period number $II \leq JJ \leq 52$
5-6	N	Number of quantile and probability levels to compute. If the standard set is chosen, N = 52, otherwise N is specified by the user. Note if N < 52, the user should define his own set since the first N values of the defined set would be used. $01 \leq N \leq 52$
7	ICOD	Code definition required by the program. If period totals of a quantity (i.e., weekly rainfall) are the input data, ICOD = 1. If parameter data are input (i.e., γ, β, $\overline{X}$), ICOD = 2. $1 \leq ICOD \leq 2$

Control card 1 concluded

8	I2	Coded as 1 if 2 period totals are required, otherwise blank or zero. $0 \leq I2 \leq 1$
9	I3	Coded as 1 if 3 period totals are required, otherwise blank or zero. $0 \leq I3 \leq 1$
10	ITAB	If the defined set of quantiles and probability levels are used, ITAB = 0; if the user specified the tables, ITAB = 1. If ITAB = 1, the tables are read under format specifications of F4.2. If ITAB = 2, the user may specify the tables and these will be read under F4.0. $0 \leq ITAB \leq 2$
11-13	K1	The number of years in the data sample. This number is checked by the program and, if incorrect, an appropriate error message is printed. $001 \leq K1 \leq 999$
14-77	ASTN	64 character heading of the user's choice to appear at the top of each output page.
78	IFACT	IFACT = 1 if the user wishes to compute the quantiles by $\overline{X}/N$ where $\overline{X}$ is the gamma distribution mean for an individual period and N is defined in col. 5-6 above. Note that the user may or may not use the defined tables as provided by the program. If he does, then ITAB = 0 (col. 10) and IFACT = 1. If the user wants to use less than 52 levels, he must include a card for the quantity levels even though they will be overlaid by this option. $0 \leq IFACT \leq 1$
79	IA	If IA = 0, alpha (origin) is assumed to be zero. If IA = 1, alpha is defined by control card 4, col. 9-16. If IA = 2, alpha is computed by the program.
80	ICN	Coded for card recognition.

Control card 2

Card col.	Name in program	Meaning
1-4	P(1)	Quantile levels in format of F4.2 if ITAB = 1 or are in F4.0 if ITAB = 2. Note this card is not required if ITAB = 0.
5-8	P(2)	
.	.	
.	.	
.	.	
77-80	P(K)	
1-4	P(K=1)	If 20 < N ≤ 40, then a second card is needed. If 40 < N ≤ 52, a third card is required.
.	.	
.	.	
.	.	
	P(N)	

Control card 3

Card col.	Name in program	Meaning
1-4	PL(1)	Probability levels in format of F4.2. Note this card is not required if ITAB = 0. The same conditions hold for the number of cards or in the quantile definition above.
5-8	PL(2)	
.	.	
.	.	
.	.	
77-80	PL(K)	
1-4	PL(K+1)	

Control card 4

Card col.	Name in program	Meaning
1-2	IP	IP = number of data points contained on each individual card.
3-4	INT	INT = number of levels of chi-square grouping. If blank or zero, a default of 10 is used.
5-8	C	C = constant for computation of empirical probabilities. Default of 0.44 is used if C = blank or zero.
9-16	ALPHA	Origin definition if IA in card 1 is set to 1. If IA = 0, leave ALPHA blank or zero.
17-24	AJJ	AJJ is the largest value in the data set entry that is used by the program to detect missing data. For example, in the case of precipitation, if 1 year-week were missing and the user had coded the missing value 99.99, then AJJ should be coded 99.99. AJJ is read under format specification F8.2. It should be noted that, if the user requested 2 or 3 period totals and the case of missing data were encountered with 99.99 defined for missing, the output would show an entry in the affected period that is greater than the 99.99; however, this would be omitted by the test of >99.99.
25-26	LIMIT	LIMIT is the controlling iteration value. If blank or zero, the default value of 10 is chosen. LIMIT is not used in the current version of the program.
27-73	AFMT	AFMT is the user defined data format. Example of period total, 13 values/card and ICOD = 1. (I5, I2, I1, 13F4.2) I5 - STN or data set identifier in col. 1-5. I2 - Year of sequence number in col. 6-7. I1 - Card number within sequence # in col. 8. 13F4.2 - Thirteen fields of data with each field 4 cols. in width and an assumed decimal for data recorded to the nearest 0.01. Example for ICOD = 2. (I5, I2, I2, I3, I3, F6.2, F6.2, F6.2) I5 - Data set identifier in col. 1-5. I2 - Period number I col. 6-7. I2 - No. of weeks in period J in col. 8-9. I3 - NX = No. of years of nonzero entries, col. 10-12. I3 - NNX = No. of total years, col. 13-15. F6.2 - XBAR = gamma distribution mean, col. 16-21. F6.2 - GAMMA = shape parameter, col. 22-27. F6.2 - BETA = scale parameter, col. 28-33
74	MILL	Input data are not in inches but millimeters (mm) and user wants quantiles converted to mm, code MILL = 1, otherwise 0.
75-80	BLANK	Not used.

If the user has multiple data sets to run with uniform characteristics (i.e., all options identical), only one control set of cards is required. Two blank cards terminate the run.

If the data sets are different, for example, the number of years in one set is different from the remainder and requires its own set of control cards, then one blank card should be used to separate stations run under different controls.

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FORTRAN IVL27 SOURCE PROGRAM                                          05/26/72    PAGE 0001

      PROGRAM POA04
C
C     POA04 IS A FORTRAN IV COMPUTER PROGRAM WRITTEN AT THE NATIONAL CLIMATIC
C     CENTER TO COMPUTE PRECIPITATION AMOUNTS AND/OR PROBABILITIES FROM PERIOD
C     TOTALS.
C     THE PROGRAM IS WRITTEN TO ALLOW AS MUCH LATITUDE AS POSSIBLE FOR THE USER
C     BY PERMITTING BY CONTROL CARD SPECIFICATION THE FOLLOWING:
C        1 DEFINITION OF BEGINNING AND ENDING PERIOD
C        2 SELECTION OF THE NUMBER OF LEVELS TO COMPUTE
C        3 ALLOWING INPUT TO BE FROM PRE-COMPUTED PARAMETERS
C        4 ALLOWING OPTION OF COMPUTING TWO PERIOD OR THREE PERIOD STATISTICS
C        5 ALLOWING THE USER TO SPECIFY LEVELS OF QUANTILES AND PROBABILITIES OR
C          TO USE A PRE-DEFINED SET
C        6 ALLOW THE USER TO DEFINE THE HEADER LINE APPEARING ON THE OUTPUT
C        7 ALLOW THE USER TO SPECIFY THE ORIGIN OR PERMIT ITS COMPUTATION IF UNKNOWN
C        8 ALLOW THE USER TO SPECIFY THE NUMBER OF INPUT DATA POINTS PER RECORD
C          AND THEIR FORMAT AT RUN TIME
C        9 ALLOW THE USER TO SPECIFY THE NUMBER OF INTERVALS OF DATA GROUPING
C          FOR COMPUTATION OF CHI-SQUARE
C       10 ALLOW THE USER TO SPECIFY THE CONSTANT TO BE USED IN COMPUTING THE
C          EMPIRICAL PROBABILITIES
C
C     THE REQUIRED AND OPTIONAL CONTROL CARDS ARE AS FOLLOWS
C     FIRST CONTROL CARD
C     POSITION   NAME   DEFINITION
C     1-2        II     BEGINNING PERIOD NUMBER
C     3-4        JJ     ENDING PERIOD NUMBER
C     5-6        N      NUMBER OF QUANTILE AND PROBABILITY LEVELS TO COMPUT
C     7          ICOD   CODED 1 IF PERIOD TOTALS TO BE USED. CODED 2 FOR PARAMETERS
C     8          I2     CODED 1 IF TWO PERIOD TOTALS ARE REQUIRED OTHERWISE BLANK
C     9          I3     CODED 1 IF 3 PERIOD TOTALS ARE REQUIRED OTHERWISE BLANK
C     10         ITAB   CODED 0 FOR DEFINED TABLES, 1 IF YOU SPECIFY TABLES
C     CODE 2 IF DATA ARE IN MM AND QUANTILES ARE IN MM
C     11-13      K1     NUMBER OF YEARS IN DATA SAMPLE
C     14-77      ASTN   64 CHARACTER HEADING LINE
C     78         IFACT  CODE 1 IF PRECIP LEVELS ARE TO BE DIFINED BY XBAR/N
C     79         IA     USER WILL SUPPLY ALPHA IF CODED 1, 0 MEANS ALPHA=0,
C                       2 REQUIRES COMPUTATION OF ALPHA
C     80         ICN    CODED 1 FOR CARD RECOGNITION
C
C     IF  ITAB=1 ON THE PRECEEDING CARD THEN N VALUES OF QUANTILES AND
C     PROBABILITIES MUST BE READ. THESE ARE READ UNDER FORMAT CONTROL OF
C     20F4.2  QUANTILES ARE READ FIRST WITH AS MANY CARDS USED AS ARE
C     REQUIRED TO CONTAIN N VALUES. PROBABILITIES ARE THEN READ IN THE SAME
C     FASHION. (NOTE THESE ARE A SEPARATE SET. UNUSED PORTIONS OF THE QUANTILE
C     CARD CAN NOT BE USED TO DEFINE PROBABILITIES)
C
C     THE NEXT HEADER CARD IS AS FOLLOWS:
C     POSITION   NAME   DEFINITION
C     1-2        IP     NUMBER OF DATA POINTS PER RECORD
```

```
C 3-4         INT    NUMBER OF INTERVALS FOR GROUPS (CHI-SQ.) DEFAULT=10 IF BLK
C 5-8         C      CONSTANT FOR EMPIRICAL PROB  DEFAULT=.44 IF BLANK
C 9-16        ALPHA  ORIGIN IF KNOWN AND IF IA ON PREVIOUS RECORD IS 1
C 17-24       AJJ    LARGEST VALUE TO BE ASSIGNED FOR MISSING DATA   F8.2
C 25-26       LIMIT  CONTROLLING ITERATION FACTOR  I2
C 27-73       AFMT   USER DEFINED DATA FORMAT
C 74          MILL   INPUT VALUES ARE IN MM AND QUANTILES SHOULD BE CONVERTED TO
C                    MM EQUIVALENT  I1 1=YES BLANK OR ZERO =NO
C
C  INPUT EQUIPMENT CONSISTS OF A CARD READER  DEVICE 5
C  OUTPUT IS THE LINE PRINTER  DEVICE 6
C
C  STORAGE ALLOCATION IS AS FOLLOWS:
C   X(75)   TEMPORARY READ IN AREA
C   X1(3900)  INPUT DATA BY WEEK (OR MONTH) FOR 1 PERIOD TOTALS
C   X2(3900)  INPUT DATA BY WEEK (OR MONTH) FOR 2 PERIOD TOTALS
C   X3(3900)  INPUT DATA BY WEEK (OR MONTH) FOR 3 PERIOD TOTALS
C   O(75)     ORDERED DATA SET FOR CURRENT PERIOD
C   EMP(75)   EMPIRICAL PROB FOR CURRENT PERIOD
C   P1(52)    QUANTILES FOR EMPIRICAL PROB WITH BETA=1
C   P2(52)    QUANTILES FOR EMPIRICAL PROB WITH BETA=BETA
C   P3(52)    QUANTILES FOR SELECTED PROB LEVELS WITH BETA=1
C   P4(52)    QUANTILES FOR SELECTED PROB LEVELS WITH BETA=BETA
C   P5(52)    PROBABILITIES FOR SELECTED QUANTILES
C   P6(52)    GRAPH PROB FOR SELECTED QUANTILES (FOR X>0)
C   P(52)     SELECTED QUANTILE LEVELS
C   PL(52)    SELECTED PROBABILITY LEVELS
C   ATAB1(52) PRE-DEFINED QUANTILE TABLE
C   ATAB2(52) PRE-DEFINED PROB TABLE
C
      COMMON SX,SLX,NX,NNX,NUM,XBAR,GAMMA,BETA,GAM,PEA,N,II,JJ,QQ,FLAG,V
      COMMON X(75),X1(3900),X2(3900),X3(3900),O(75),EMP(75),AJJ
      COMMON P1(52 ),P2(52 ),P3(52 ),P4(52 ),P5(52),P6(52),P(52),PL(52)
      COMMON ASTN(8),H(20),CHI(20),INT,K1,SKTEST,PROB,ID1,ALPHA,IA,C
      COMMON LIMIT,IFACT
      DIMENSION TEM(15),ATAB1(52),ATAB2(52),AFMT(6)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*4 X,X1,X2,X3,O,AJJ
      DATA ATAB1/.005,.010,.015,.020,.025,.030,.035,.040,.045,.050,.060
     1,.070,.080,.090,.100,.15,.20,.25,.30,.35,.40,.45,.50,.55,.60,.65,
     2.70,.75,.80,.85,.90,.95,1.0,1.5,2.0,2.5,3.0,3.5,4.0,4.5,5.0,5.5,
     36.0,7.0,8.0,9.0,10.0,15.0,20.0,25.0,30.0,35.0/
      DATA ATAB2/.001,.003,.005,.006,.007,.008,.009,.010,.015,.020,.025,
     1           .030,.035,.040,.045,.050,.055,.060,.065,.070,.075,.080,
     2           .085,.090,.095,.100,.150,.200,.250,.300,.350,.400,.450,
     3           .500,.550,.600,.631,.650,.700,.750,.800,.850,.900,.910,
     4           .930,.950,.970,.980,.990,.995,.997,.999/
C  READ FIRST CONTROL CARD
15    READ (5,1) II,JJ,N,ICOD,I2,I3,ITAB,K1,ASTN,IFACT,IA,ICN
1     FORMAT  (3I2,4I1,I3,8A8,3I1)
```



```
C CLEAR PRECIP AND PROB TABLES TO ZERO.
      DO 14 M=1,52
      P1(M)=0.
      P2(M)=0.
      P3(M)=0.
      P4(M)=0.
      P5(M)=0.
      P6(M)=0.
14    CONTINUE
      ALPHA=0.
C  IS THIS THE FIRST CONTROL CARD OF A GROUP? IF NOT PRINT AND STOP
      IF (ICN.EQ.1) GO TO 16
      WRITE (6,13) II,JJ,N,ICOD,I2,I3,ITAB,K1,ASTN,IFACT,IA,ICN
13    FORMAT (1X,3I2,4I1,I3,8A8,3I1)
      WRITE  (6,2)
2     FORMAT (1H1,' GAMMA PROGRAM IS FINISHED'////)
      STOP 1111
16    IF (ITAB.EQ.0) GO TO 18
C ITAB=0 IMPLIES THE USE OF DEFINED VALUES OF PRECIP AND PROBABILITIES.
      IF (ITAB.EQ.2) GO TO 17
      READ (5,3) (P(I),I=1,N)
3     FORMAT (20F4.2)
      READ (5,3) (PL(I),I=1,N)
      GO TO  20
C IF ITAB=2, DATA AND QUANTILES ARE CODED IN MM
6     FORMAT (20F4.0)
17    READ (5,6) (P(I),I=1,N)
      READ (5,3) (PL(I),I=1,N)
      GO TO 20
18    DO 19 I=1,52
      P(I)=ATAB1(I)
19    PL(I)=ATAB2(I)
C  READ NEXT CONTROL CARD
20    READ (5,4)  IP,INT,C,ALPHA,AJJ,LIMIT,AFMT,MILL
4     FORMAT (2I2,F4.2,F8.5,F8.2,I2,6A8,I1)
C  AFMT IS THE USER DEFINED OBJECT TIME FORMAT
C   SET DEFAULT VALUE IF UNDEFINED FOR C
      IF (C.EQ.0) C=.44
C   SET DEFAULT VALUE IF UNDEFINED FOR INT
      IF (INT.EQ.0) INT=10
C SET DEFAULT VALUE IF UNDEFINED FOR ALPHA
      IF (IA.EQ.0) ALPHA=0.
C SET DEFAULT VALUE IF LIMIT IS ZERO OR BLANK
      IF (LIMIT.EQ.0) LIMIT=10
      WRITE (6,5) II,JJ,N,ICOD,I2,I3,ITAB,K1,C,IA,LIMIT
5     FORMAT (1H1,' THE FOLLOWING CONTROL PARAMETERS HAVE BEEN READ'///
     1' BEGINNING PERIOD NUMBER IS ',24X,I2/
     2' ENDING PERIOD NUMBER IS ',27X,I2/
     3' NUMBER OF PRECIP AND/OR PROB LEVELS IS ',12X,I2/
     4' OPTION IS ',41X,I2/
```

```
     5' TWO PERIOD TOTALS REQUIRED, YES IF 2 ',14X,I2/
     6' THREE PERIOD TOTALS REQUIRED, YES IF 3 ',12X,I2/
     7' USE DEFINED TABLES OF PRECIP OR PROB, YES IF 0 ',4X,I2/
     8' NUMBER OF YEARS USED IS ',26X,I3/
     9' C VALUE USED IS ',33X,F4.2/
     9' ALPHA VALUE TO BE COMPUTED IF IA=2,           ,10X,I2/
     9' VALUE TO CONTROL ITERATION LIMIT = ',16X,I2)
C COMPUTE INTERVAL VALUES FOR CHI SQUARE COMPUTATIONS  (K INTERVALS)
      L=INT-1
      V=INT
      DO 35 I=1,L
      H(I)= I/V
35    CONTINUE
C SHOULD THE QUANTILE TABLE BE CONVERTED FROM INCHES TO MM
      IF (MILL.EQ.0) GO TO 37
      DO 36 I=1,N
 36   P(I)=P(I)*25.4
C  ICOD=2 IMPLIES USE OF PRE-COMPUTED PARAMETER VALUES.
 37   IF (ICOD.EQ.2) GO TO 55
C WE WILL READ ALL INPUT DATA AND STORE AWAY INTO 1, 2 AND 3 PERIOD TOTALS
C  SET NUM=0 FOR CHECKING THE ACTUAL NUMBER OF YEARS OF DATA TO BE READ
      NUM=0
      READ (5,AFMT) ID1,IY,IC,(TEM(J),J=1,IP)
C  ID1= IDENTIFICATION OF THIS SAMPLE
C  IY = YEAR NUMBER
C  IC = CARD NUMBER WITHIN YEAR
C  TEM= DATA POINTS  (IP OF THEM)
C
44    NUM=NUM+1
45    DO  46 I=1,IP
C COMPUTE POINT NUMBER FOR DATA ARRAY
      J=(IC-1)*IP+I
46    X(J)=TEM(I)
      READ (5,AFMT) ID2,IY1,IC,(TEM(J),J=1,IP)
      IF (IY.EQ.IY1) GO TO 45
C AT THIS POINT WE HAVE READ A COMPLETE DATA YEAR AND HAVE STORED VALUES IN X(I)
C
C K1 EQUALS TOTAL NUMBER OF YEARS TO PROCESS
      DO 50 J=II,JJ
      I=(J-1)*K1+NUM
C  DATA ARE STORED ALL YEARS BY WEEK (OR MONTH) NUMBER FOR DURATION PERIOD 1
      X1(I)=X(J)
      IF (I2.EQ.0.AND.I3.EQ.0) GO TO 50
      K=J+1
      L=J+2
      IF (J.EQ.JJ) GO TO 47
C  DATA ARE STORED ALL YEARS BY WEEK (OR MONTH) NUMBER FOR DURATION PERIOD 2
      X2(I)=X(J)+X(K)
      IF (J.EQ.(JJ-1)) GO TO 48
C  DATA ARE STORED ALL YEARS BY WEEK (OR MONTH) NUMBER FOR DURATION PERIOD 3
```



```
      X3(I)=X(J)+X(K)+X(L)
      GO TO 50
47    IF (ID1.NE.ID2) GO TO 50
      X2(I)=X(J)+TEM(1)
      X3(I)=X(J)+TEM(1)+TEM(2)
      GO TO 50
48    IF (ID1.NE.ID2) GO TO 50
      X3(I)=X(J)+X(K)+TEM(1)
50    CONTINUE
C
C WE NOW HAVE DATA STORED YEARS IN SUCCESSION BY WEEK NO. FOR  1, 2 AND  3 DUR
C PERIODS
      IY=IY1
      IF  (ID2.EQ.ID1) GO TO 44
C ALL  DATA FOR THIS STATION HAVE NOW BEEN READ.
      IF (NUM.EQ.K1) GO TO 60
C  SOMETHING WRONG LETS TAKE A LOOK.
      WRITE (6,51)  K1,NUM
 51   FORMAT (1H1,' YOU INDICATED ',I5,' WERE TO BE USED, HOWEVER OUR CO
     2UNT OF YEARS READ IS',I5///)
      STOP 1111
C
C
C
C  DATA TO BE READ FOR CONDITION ICOD=2, EACH CARD CONSTITUTES 1 DATA SET
55    READ (5,AFMT) ID1,I,J,NX,NNX,XBAR,GAMMA,BETA
      IF (ID1.EQ.0) GO TO 15
C  COMPUTE PROB FOR SELECTED QUANTILES
      CALL  COMPUT (ICOD,P,P5,N)
      CALL COMPUT (99,P,P6,N)
      CALL  PRINT  (X1,I,J,ICOD)
      GO TO 55
C
60    DO 100  KOUNT=1,3
      GO TO (58,56,57),KOUNT
C  CHECK FOR NEED OF 2 DURATION PERIODS
56    IF (I2-0) 58,100,58
C  CHECK FOR NEED OF 3 DURATION PERIODS
57    IF (I3-0) 58,100,58
C  COMPUTE PARAMETERS FOR 1 DURATION PERIOD
58    DO  75   I=II,JJ
      GO TO (61,62,63),KOUNT
61    CALL SUM (X1,I,0,CHI,ICOD)
      CALL COMPUT (ICOD,P,P5,N)
      CALL COMPUT (99,P,P6,N)
      CALL PRINT (X1,I,1,ICOD)
      GO TO 75
C  COMPUTE PARAMETERS FOR 2 DURATION PERIODS
62    CALL SUM (X2,I,0,CHI,ICOD)
      CALL COMPUT (ICOD,P,P5,N)
```

```
      CALL COMPUT (99,P,P6,N)
      CALL PRINT  (X2,I,2,ICOD)
      GO TO 75
C  COMPUTE PARAMETERS FOR 3 DURATION PERIODS
63    CALL SUM (X3,I,O,CHI,ICOD)
      CALL COMPUT (ICOD,P,P5,N)
      CALL COMPUT (99,P,P6,N)
      CALL PRINT (X3,I,3,ICOD)
75    CONTINUE
100   CONTINUE
      ID1=ID2
      IY=IY1
      NUM=1
      IF (ID2.NE.0) GO TO 45
      GO TO 15
      END
```

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```
      SUBROUTINE SUM (Y,I,O,CHI,ICOD)
C
C  THE SUM SUBROUTINE DOES THE FOLLOWING
C    1 SELECTS THE ITH WEEK FROM THE APPROPRIATE DURATION PERIOD
C    2 SORTS SELECTED DATA IN ASCENDING ORDER OF MAGNITUDE
C    3 IF REQUIRED COMPUTES ORIGIN ALPHA
C    4 COMPUTES SUMS AND LOGS FOR COMPUTATION OF PARAMETERS
C
      COMMON SX,SLX,NX,NNX,NUM,XBAR,GAMMA,BETA,GAM,PEA,N,II,JJ,QQ,FLAG,V
      COMMON X(75),X1(3900),X2(3900),X3(3900),O(75),EMP(75),AJJ
      COMMON P1(52 ),P2(52 ),P3(52 ),P4(52 ),P5(52),P6(52),P(52),PL(52)
      COMMON ASTN(8),H(20),CHI(20),INT,K1,SKTEST,PROB,ID1,ALPHA,IA,C
      COMMON LIMIT,IFACT
      DIMENSION Y(1),TEM(50),T(50)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*4 X,X1,X2,X3,O,AJJ,Y
C  IF ICOD=2 GO TO RETURN SINCE PARAMETERS ARE ALREADY AVAILABLE
      IF (ICOD.EQ.2) GO TO 70
      SX=0.
      SLX=0.
      NX=0
      NNX=0
      DO 7  J=1,NUM
 7    O(J)=AJJ
C  COMPUTE BEGIN AND END OF STORAGE FOR DATA SELECTION
      M=I*NUM-(NUM-1)
      NN=M+NUM-1
      V=0.
      K=1
C  MOVE SELECTED DATA INTO O
      DO 8 J=M,NN
      IF (Y(J).GE.AJJ) GO TO 8
      O(K)=Y(J)
      K=K+1
      IF (Y(J).GT.0.) V=V+1.
8     CONTINUE
      IF (K.EQ.1) GO TO 70
      K=K-1
C  SORT DATA IN O INTO ASCENDING ORDER
      DO 25 L1=1,K
      DO 25 L2=L1,K
      IF (O(L2).GT.O(L1))  GO TO 25
      QQ=O(L2)
      O(L2)=O(L1)
      O(L1)=QQ
25    CONTINUE
C  IF IA=1, ALPHA WAS SPECIFIED IN THE HEADER CARD
      IF (IA.LT.2) GO TO 9
C  IF THE LEAST VALUE OF O IS ZERO, THE ORIGIN IS DEFINED
      IF (O(1).EQ.0.) GO TO 6
```

```
C  IF AT LEAST 2 VALUES (SMALLEST) ARE EQUAL ALPHA IS MADE THIS VALUE
      IF (D(1).EQ.D(2)) GO TO 6
C  ALPHA IS MADE SLIGHTLY SMALLER THAN THE SMALLEST ENTRY
      ALPHA=D(1)-D(1)*.00001
      GO TO 9
6     ALPHA=D(1)
9     NX=0
      SX=0.
      SLX=0.
C  COMPUTE SUMS AND LOGS FOR THE NON-ZERO ENTRIES
      DO 20 J=M,NN
      IF (Y(J).GE.AJJ ) GO TO 20
      IF ((Y(J)-ALPHA).LE.0.) GO TO 5
      SX=SX+Y(J)-ALPHA
      SLX=SLX+DLOG(Y(J)-ALPHA)
      NX=NX+1
5     NNX=NNX+1
20    CONTINUE
      IF (NX.LT.6) GO TO 70
C  COMPUTE CHI-SQUARE
      L=INT-1
      XBAR=SX/NX
      D=4.*(DLOG(XBAR)-SLX/NX)
      GAMMA=(1.+SQRT(1.+D/3.))/D
      BETA=XBAR/GAMMA
      CALL GAMIT(1)
      CALL INVGAM (BETA,H,CHI,L,NX,NX)
      DO 26 J=1,50
      TEM(J)=0.
      T(J)=0.
26    CONTINUE
      M=0
      K=1
      L=0
      DO 40 J=1,NNX
      IF (D(J).EQ.0.) GO TO 40
27    IF (D(J).LE.CHI(K)) GO TO 30
      T(K)=M
      TEM(K)=L
      K=K+1
      L=0
      IF (K-INT) 27,75,80
30    M=M+1
      L=L+1
40    CONTINUE
45    DEN=NX/FLOAT(INT)
      SS =0.
      DO 50 L=1,INT
      SS =SS + (TEM(L)-DEN)*(TEM(L)-DEN)
50    CONTINUE
```

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```
      QQ=SS /DEN*.5
      GAMMA= (INT-3)*.5
      BETA=1.
      IF (IFACT.EQ.0) GO TO 51
      P(1)=10.*XBAR/N
      PART=P(1)
      DO 52 L=2,N
      P(L)=P(L-1)+PART
52    CONTINUE
C  COMPUTE PROBABILITY OF CHI-SQUARE
51    CALL COMPUT (99,QQ,PROB,1)
C  COMPUTE S-K
      QQ=QQ+QQ
      DIFF=-9999.
      SS =0.
      DO 65 L=1,K
      SS=SS+DEN
      CHECK=ABS(T(L)-SS)
      IF (CHECK.GT.DIFF) DIFF=CHECK
65    CONTINUE
      SKTEST=DIFF/NX
70    RETURN
75    J=K-1
      TEM(K)=NX-T(J)
      T(K)=T(J)+TEM(K)
      GO TO 45
80    WRITE (6,81) (H(J),CHI(J),TEM(J),T(J),J=1,INT)
81    FORMAT (3X,4F18.10/)
      STOP 1111
      END
```

```
      SUBROUTINE COMPUT (ICOD,W,Y,NZ)
C
C  COMPUTE SUBROUTINE PROVIDES COMPUTATION OF PROBABILITIES FOR SELECTED
C  QUANTILE VALUES
C
      COMMON SX,SLX,NX,NNX,NUM,XBAR,GAMMA,BETA,GAM,PEA,N,II,JJ,QQ,FLAG,V
      COMMON X(75),X1(3900),X2(3900),X3(3900),D(75),EMP(75),AJJ
      COMMON P1(52 ),P2(52 ),P3(52 ),P4(52 ),P5(52),P6(52),P(52),PL(52)
      COMMON ASTN(8),H(20),CHI(20),INT,K1,SKTEST,PROB,ID1,ALPHA,IA,C
      COMMON LIMIT,IFACT
      DIMENSION UU(100),W(1),Y(1)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*4 X,X1,X2,X3,D,AJJ
      IF (NX.LT.6) GO TO 200
      EN=NX
      ENN=NNX
      TRACE= (ENN-EN)/ENN
C  IF  ICOD=2 OR ICOD=99 NOT COMPUTE PARAMETERS
      IF  (ICOD.EQ.2.OR.ICOD.EQ.99) GO TO 68
      IF (NX.EQ.0) GO TO 200
      XBAR=SX/EN
      D=4.*(DLOG(XBAR)-SLX/EN)
      GAMMA=(1.+SQRT(1.+D/3.))/D
      BETA=XBAR/GAMMA
C COMPUTE  GAMMA OF GAMMA
68    CALL GAMIT(2)
      SEE=SQRT(GAMMA)
      YOU=1./(BETA*SEE)
      DO 70 K=1,NZ
70    UU(K)=W(K)*YOU
      DO 90 M=1,NZ
C IF PROB EXCEEDS ALLOWABLE RANGE OF THE MEAN TERMINATE COMPUTATION FOR THIS LEV
      IF (W(M).GT.(LIMIT*XBAR)) GO TO 300
      SEA=UU(M)*SEE
      Z=2.
      TERM=SEA/(PEA+Z)
      SERIES=TERM+1.
      DO 75  L=1,5000
      Z=Z+1.
      TERM=(TERM*SEA)/(PEA+Z)
      SERIES=SERIES+TERM
      IF (TERM.LT..1E-7) GO TO 80
75    CONTINUE
C  IF FLAG=1 GAMMA>50 AND GAMMA OF GAMMA (GAM) IS COMPUTED AS LOG(GAMMA)
80    IF  (FLAG.EQ.1.)  GO TO 81
      EYE= ((SEA**GAMMA)*EXP(-SEA)*SERIES)/GAM
      GO TO 82
81    EYE=  EXP(LOG(SEA)*GAMMA-SEA+LOG(SERIES)-GAM)
82    Y(M)=1.-(((1.-TRACE)*EYE)+TRACE)
      IF ( Y(M).LE.1.) GO TO 85
```



```
      Y(M)=1.
85    IF ( Y(M).LT..00004)  Y(M)=0.
      IF (ICOD.EQ.99) Y(M)=EYE
90    CONTINUE
      CALL GAMIT(1)
      IF (ICOD.EQ.99) GO TO 200
C  COMPUTE AND ACCUMULATE EMPIRICAL PROBABILITIES
      L=1
      DEN= V -2.*C+1.
      DO 95 I=1,K1
      IF (D(I)-0.) 96,96,97
96    EMP(I)=0.
      P1(I)=0.
      P2(I)=0.
      GO TO 95
97    IF (D(I).GE.AJJ) GO TO 96
      EMP(I)=(L-C)/DEN
      L=L+1
95    CONTINUE
C COMPUTE PRECIP AMOUNTS FOR COMPUTED EMPIRICAL PROBALITIES WITH BETA=1.
C
      CALL INVGAM (1.,EMP,P1,NNX,K1,K1)
C COMPUTE PRECIP AMOUNTS FOR SELECTED PROBABILITY LEVELS WITH BETA=1.
      CALL INVGAM (1.,PL,P3,N,NX,NNX)
      DO 100 I=1,NZ
      P4(I)=P3(I)*BETA
100   CONTINUE
      DO 101 I=1,NNX
      P2(I)=P1(I)*BETA
101   CONTINUE
200   RETURN
300   Y(M)=0.
      GO TO 90
      END
```

```
      SUBROUTINE INVGAM(BET,PP,PK,NN,NY,NNY)
C
C  INVGAM SUBROUTINE COMPUTES QUANTILES FOR SELECTED PROBABILITIES
C
      COMMON SX,SLX,NX,NNX,NUM,XBAR,GAMMA,BETA,GAM,PEA,N,II,JJ,QQ,FLAG,V
      COMMON X(75),X1(3900),X2(3900),X3(3900),O(75),EMP(75),AJJ
      COMMON P1(52 ),P2(52 ),P3(52 ),P4(52 ),P5(52),P6(52),P(52),PL(52)
      COMMON ASTN(8),H(20),CHI(20),INT,K1,SKTEST,PROB,ID1,ALPHA,IA,C
      COMMON LIMIT,IFACT
      DIMENSION PP(1),PK(1)
      DIMENSION AA(11),BB(11),CC(11),AAA(11),BBB(11),CCC(11)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*4 X,X1,X2,X3,O,AJJ
      DATA AA/.935,.952,.969,.967,.950,.986,.994,.971,.957,.937,.937/
      DATA BB/-1.2,-.8,-.5,-.4,.0,.2,.4,1.2,1.8,2.2,2.2/
      DATA CC/.05,.1,.2,.25,.4,.5,.6,.8,.9,.95,.99/
      DATA AAA/-.0032,-.0045,-.0125,-.0158,-.0249,-.0262,-.0307,.0408,
     1.1942,.4,.7/
      DATA BBB/.0625,.0950,.1608,.1907,.2540,.2551,.2578,-.45,-1.8592,
     14.,6./
      DATA CCC/-.03,-.068,-.0995,-.0965,.0519,.2615,.583,3.0662,7.0493,
     110.,14./
      XO=0.
      IJJ=0
      SG=SQRT(GAMMA)
      IF (PEA.GE..1) SP=SQRT(PEA)
      EN=NY
      ENN=NNY
      O= (ENN-EN)/ENN
      DO  55  I=1,NN
      IJ=0
      PK(I)=0.
      DXO=10.
      CHECK=1.E-6
      IF (PP(I).GT..9.OR.PP(I).LT..5) CHECK=1.E-8
C  DL IS THE ADJUSTED PROB LEVEL TAKING INTO ACCOUNT THE ZERO PORTION OF THE
C  DISTRIBUTION
      DL=(PP(I)-Q)/(1.-Q)
      IF (DL.LT..0001) GO TO 55
C  FIND THE NEAREST LEVEL TO THE ADJUSTED PROB LEVEL IN ORDER TO CHOOSE
C  CONSTANTS FOR MAKING A FIRST APPROXIMATION
      DO 20 K=1,11
      M=K
      IF (CC(K)-DL) 20,20,25
   20 CONTINUE
   25 M=M-1
      IF (M.EQ.0) M=1
      IF (GAMMA.LT.1.2) GO TO 60
C MAKE FIRST GUESS APPROXIMATION FOR LARGE GAMMA (GAMMA>3)
C
```



```
   26 XO=ABS((AA(M)*SP+BB(M))*SG)
   27 Z=2.
      T=XO/(PEA+Z)
      S=T+1.
      DO 30 L=1,5000
      Z=Z+1.
      T=(T*XO)/(PEA+Z)
      S=S+T
      IF (T.LT.CHECK) GO TO 35
   30 CONTINUE
      WRITE (6,31) GAMMA,XBAR,BETA,SG,SP,S,TEM,XO,XX,PP(I),DL,TT,SUM,
     1DX,DXO
   31 FORMAT ('  GAMMA= ',E15.8,'  XBAR= ',F5.3,'  BETA= ',F5.3,'  SQRT
     1GAMMA= ',E15.8,'  SQRT (G-1)= ',E15.8/'  SERIES SUM= ',E15.8,'  SE
     2RIES TERM= ',E15.8,'  XO= ',E15.8,'  OLD XO= ',E15.8/'  PROB LEVEL
     3= 'F5.3,'  ADJUSTED PROB LEVEL= ',E15.8,'  TT= ',E15.8,'  SUM= ',
     4E15.8/' DX = ',E15.8,' DXO = ',E15.8//)
      STOP 1111
   35 IF (FLAG.EQ.1.) GO TO 38
      XX= XO - (XO/GAMMA)* S + (GAM*DL*EXP(XO)) / (EXP(PEA* LOG(XO)))
      GO TO 39
   38 XX= XO - (XO/GAMMA)* S + (EXP(GAM+ LOG(DL)+ XO - PEA* LOG(XO)))
   39 DX=XX-XO
      XO=XX
      IF (XO.LE.0.) GO TO 70
      IF (ABS(DX).GT.ABS(DXO))GO TO 70
      DXO=DX
      IF (ABS(DX).GT.CHECK) GO TO 27
      PK(I)= XX*BET
   55 CONTINUE
      RETURN
C  MAKE APPROXIMATION FOR SMALL GAMMA
   60 XO= AAA(M)*(GAMMA**3)+ BBB(M)*(GAMMA**2)+ CCC(M)*GAMMA
      IF (XO.LE.0.)  XO= ABS(XO)+.00001
      XO=XO*SG
      GO TO 27
  100 IF (GAMMA.LT..05) GO TO 105
      XO=.0001
      GO TO 27
  105 XO=.000001
      GO TO 27
   70 IF (IJ.GT.5) GO TO 95
      IJ=IJ+1
      IF (GAMMA.GT.1.1) GO TO 60
      SP=.001
      GO TO 26
   95 IF (IJJ.NE.0) GO TO 97
      WRITE (6,96) PP(I)
   96 FORMAT (1H1,' UNABLE TO CONTINUE WITH THIS LEVEL ',F10.4////)
      IJJ=1
```

```
      GO TO 55
97    WRITE (6,98) PP(I)
98    FORMAT (1X ,' UNABLE TO CONTINUE WITH THIS LEVEL ',F10.4////)
      GO TO 55
200   XO=XX
      GO TO 27
      END
```



```
      SUBROUTINE PRINT (Y,I,M,ICOD)
      COMMON SX,SLX,NX,NNX,NUM,XBAR,GAMMA,BETA,GAM,PEA,N,II,JJ,QQ,FLAG,V
      COMMON X(75),X1(3900),X2(3900),X3(3900),O(75),EMP(75),AJJ
      COMMON P1(52 ),P2(52 ),P3(52 ),P4(52 ),P5(52),P6(52),P(52),PL(52)
      COMMON ASTN(8),H(20),CHI(20),INT,K1,SKTEST,PROB,ID1,ALPHA,IA,C
      COMMON LIMIT,IFACT
      DIMENSION Y(1)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*4 X,X1,X2,X3,O,AJJ,Y,Z(75)
      IF (NX.LT.6) GO TO 26
      LINE=45
      IF (ICOD.EQ.2)  GO TO  50
      V=0.
      L=1
      K=I*NUM-(NUM-1)
      NN=K+NUM-1
      IF  (N.GT.NUM) NN=NN+N-NUM
      DO 10 J=1,NUM
      Z(J)=0.
      IF (O(J).GE.AJJ) GO TO 10
      Z(J)=O(J)/BETA
10    CONTINUE
      DO 25 J=K,NN
      IF (LINE.LT.45) GO TO 6
      LINE=1
      WRITE (6,1) ASTN
1     FORMAT (1H1,33X,8A8//)
      WRITE (6,2)
2     FORMAT (7X,'STATION       I       J       NX       NNX        XBAR         A
     1LPHA        BETA        GAMMA           X2           PROB         K-S')
      WRITE (6,3) ID1,I,M,NX,NNX,XBAR,ALPHA,BETA,GAMMA,QQ,PROB,SKTEST
3     FORMAT (6X,3I7,2I8,5F11.3,2F10.3//)
      WRITE (6,4)
4     FORMAT (53X,'EMP PROB  EMP PROB  SELECTED  SELECTED  SELECTED
     1GRAPH   SELECTED   EXC PRB'/16X,'ENTRY           ORDER        DATA        E
     2MP  QUANTILE QUANTILE         PROB  QUANTILE  QUANTILE       PROB    QU
     3ANTITY       FOR'/ 8X,  'SEO        DATA         DATA        /BETA        PRO
     4B          B=1     B=BETA    VALUES           B=1     B=BETA       (X>0)
     5LEVELS   PCP LVL'/)
6     IF (LINE.GT.N.OR.V.EQ.1.) GO TO 30
      IF  (LINE.GT.NUM.OR.V.EQ.2.) GO TO 40
      WRITE (6,5) L,Y(J),O(L),Z(L),EMP(L),P1(L),P2(L),PL(L),P3(L),P4(L),
     1P6(L),P(L),P5(L)
5     FORMAT (I11,2F10.2,2F10.3,2F10.3,F10.3,3F10.3,F11.3,F10.3)
20    LINE=LINE+1
      L=L+1
25    CONTINUE
26    RETURN
30    WRITE (6,31) L,Y(J),O(L),Z(L),EMP(L),P1(L),P2(L)
31    FORMAT (I11,2F10.2,2F10.3,2F10.3)
```



```
      V=1.
      GO TO 20
40    WRITE (6,41) L,PL(L),P3(L),P4(L),P6(L),P(L),P5(L)
41    FORMAT (I11,60X,F10.3,3F10.3,F11.3,F10.3)
      V=2.
      GO TO 20
50    ALPHA=0.
      DO  60 J=1,N
      IF (LINE.LT.45) GO TO 59
      LINE=1
      WRITE (6,1) ASTN
      WRITE (6,2)
      WRITE (6,51) ID1,I,M,NX,NNX,XBAR,ALPHA,BETA,GAMMA
51    FORMAT (6X,3I7,2I8,4F11.3)
      WRITE (6,4)
59    WRITE (6,52) J,EMP(J),P1(J),P2(J),PL(J),P3(J),P4(J),P6(J),P(J),
     1P5(J)
      LINE=LINE+1
60    CONTINUE
52    FORMAT (I11,30X, F10.3,2F10.3,F10.3,3F10.3,F11.3,F10.3)
      GO TO 26
      END
```

```
      SUBROUTINE GAMIT (I)
      COMMON SX,SLX,NX,NNX,NUM,XBAR,GAMMA,BETA,GAM,PEA,N,II,JJ,QQ,FLAG
      DIMENSION U(200)
      IMPLICIT REAL*8 (A-H,O-Z)
      A=.035868343
      V= -.193527818
      C= .482199394
      D= -.756704078
      E= .918206857
      F= -.897056937
      G= .988205891
      B= -.577191652
      FLAG=0.
      IF (GAMMA.EQ.1.) GO TO 55
      PEA=GAMMA-1.
      L=PEA+I
      IF (L.LT.1) GO TO 25
      U(1)=PEA+I
      DO 10 K=1,L
      KK=K+1
10    U(KK)=U(K)-1.
      PD=1.
      IF (GAMMA.GT.50) GO TO 45
      IF (L.LT.2) GO TO 16
      DO 15 K=2,L
      PD=PD*U(K)
15    CONTINUE
16    Y= U(L)-1
      GO TO 30
25    Y=GAMMA-L
      PD=1./Y
30    GAM=PD*((((((((A*Y+V)*Y+C)*Y+D)*Y+E)*Y+F)*Y+G)*Y+B)* Y+1.)
40    RETURN
55    GAM=1.
      GO TO 40
45    FLAG=1.
      PD=0.
      L=L-1
      DO 50 K=2,L
      PD= PD+ LOG(U(K))
50    CONTINUE
      Y=U(L+1)-1
      GAM=    ((((((((A*Y+V)*Y+C)*Y+D)*Y+E)*Y+F)*Y+G)*Y+B)* Y+1.)
      GAM=PD + LOG(GAM)
      GO TO 40
      END
```

FORTRAN IVL27 OBJECT SUMMARY POA04 PROGRAM 05/26/72 PAGE 0018

```
OBJECT DECK ORIGIN  FDEDA0        FIRST CARD SEQ NO  0001
INSTRUCTIONS                           00B80
ARRAYS                                 003E8
FORMATS, NAMELISTS, LITCONS            002F8
VARIABLES, CONSTANTS, TEMPORARIES      00460
                   POA04     SD  01  016C0     ITF#MPI   ER  02     ITF#MPS   LD   01260
                             CM  04  0CAB8
                   PRINT     EV  05            COMPUT    EV  06     SUM       EV  07
                   ITF#OR    EV  08            ITF#IR    EV  09     ITF#OX    EV  0A
                   ITF#OI    EV  0B            ITF#OH    EV  0C     ITF#OF    EV  0D
                   ITF#OA    EV  0E            ITF#IZ    EV  0F     ITF#IX    EV  10
                   ITF#IS    EV  11            ITF#IL    EV  12     ITF#II    EV  13
                   ITF#IH    EV  14            ITF#IG    EV  15     ITF#IF    EV  16
                   ITF#IE    EV  17            ITF#ID    EV  18     ITF#IA    EV  19
                   ITF#CG    EV  1A            ITF#IC    EV  1B     ITF#ST    EV  1C
```

FORTRAN IVL27 STORAGE MAP POA04 PROGRAM 05/26/72 PAGE 0019

SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE
AFMT	(R8)	01	00EC0	ITAB	I4	01	012FC	P3	(R8)	04	0BF38	17	STMT	01	001FC
AJJ	R4	04	0BBF0	IY	I4	01	01318	P4	(R8)	04	0C0D8	18	STMT	01	00294
ALPHA	R8	04	0CA98	IY1	I4	01	01328	P5	(R8)	04	0C278	19	STMT	01	002BC
ASTN	(R8)	04	0C8F8	I2	I4	01	012F4	P6	(R8)	04	0C418	20	STMT	01	002EC
ATAB1	(R8)	01	00B80	I3	I4	01	012F8	QQ	R8	04	00058	35	STMT	01	00416
ATAB2	(R8)	01	00D20	J	I4	01	01320	SKTEST	R8	04	0CA80	36	STMT	01	0046A
BETA	R8	04	00030	JJ	I4	04	00050	SLX	R8	04	00008	37	STMT	01	0049E
C	R8	04	0CAA8	K	I4	01	0132C	SX	R8	04	00000	44	STMT	01	00516
CHI	(R8)	04	0C9D8	KOUNT	I4	01	01330	TEM	(R8)	01	00EF0	45	STMT	01	0052C
EMP	(R8)	04	0B998	K1	I4	04	0CA7C	V	R8	04	00068	46	STMT	01	00564
FLAG	R8	04	00060	L	I4	01	01314	X	(R4)	04	00070	47	STMT	01	00738
GAM	R8	04	00038	LIMIT	I4	04	0CAB0	XBAR	R8	04	00020	48	STMT	01	0079C
GAMMA	R8	04	00028	M	I4	01	01304	X1	(R4)	04	0019C	50	STMT	01	007E6
H	(R8)	04	0C938	MILL	I4	01	01310	X2	(R4)	04	03E8C	51	FORMT	01	01206
I	I4	01	01308	N	I4	04	00048	X3	(R4)	04	07B7C	55	STMT	01	008A6
IA	I4	04	0CAA0	NNX	I4	04	00014	1	FORMT	01	00F68	56	STMT	01	00958
IC	I4	01	0131C	NUM	I4	04	00018	2	FORMT	01	00F93	57	STMT	01	00974
ICN	I4	01	01300	NX	I4	04	00010	3	FORMT	01	00FBA	58	STMT	01	00990
ICOD	I4	01	012F0	O	(R4)	04	0B86C	4	FORMT	01	00FCA	60	STMT	01	00926
ID1	I4	04	0CA90	P	(R8)	04	0C5B8	5	FORMT	01	00FE8	61	STMT	01	009C2
ID2	I4	01	01324	PEA	R8	04	00040	6	FORMT	01	00FC2	62	STMT	01	00A1A
IFACT	I4	04	0CAB4	PL	(R8)	04	0C758	13	FORMT	01	00F7C	63	STMT	01	00A72
II	I4	04	0004C	PROB	R8	04	0CA88	14	STMT	01	0009A	75	STMT	01	00ABC
INT	I4	04	0CA78	P1	(R8)	04	0BBF8	15	STMT	01	00006	100	STMT	01	00AF0
IP	I4	01	0130C	P2	(R8)	04	0BD98	16	STMT	01	0012A				

FORTRAN IVL27 STORAGE MAP POA04 PROGRAM 05/26/72 PAGE 0020

SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE
15	STMT	01	00006	100	STMT	01	00AF0	J	I4	01	01320	D	(R4)	04	0B86C
14	STMT	01	0009A	ATAB1	(R8)	01	00B80	ID2	I4	01	01324	EMP	(R8)	04	0B998
16	STMT	01	0012A	ATAB2	(R8)	01	00D20	IY1	I4	01	01328	AJJ	R4	04	0BBF0
17	STMT	01	001FC	AFMT	(R8)	01	00EC0	K	I4	01	0132C	P1	(R8)	04	0BBF8
18	STMT	01	00294	TEM	(R8)	01	00EF0	KOUNT	I4	01	01330	P2	(R8)	04	0BD98
19	STMT	01	0028C	1	FORMT	01	00F68	SX	R8	04	00000	P3	(R8)	04	0BF38
20	STMT	01	002EC	13	FORMT	01	00F7C	SLX	R8	04	00008	P4	(R8)	04	0C0D8
35	STMT	01	00416	2	FORMT	01	00F93	NX	I4	04	00010	P5	(R8)	04	0C278
36	STMT	01	0046A	3	FORMT	01	00FBA	NNX	I4	04	00014	P6	(R8)	04	0C418
37	STMT	01	0049E	6	FORMT	01	00FC2	NUM	I4	04	00018	P	(R8)	04	0C5B8
44	STMT	01	00516	4	FORMT	01	00FCA	XBAR	R8	04	00020	PL	(R8)	04	0C758
45	STMT	01	0052C	5	FORMT	01	00FE8	GAMMA	R8	04	00028	ASTN	(R8)	04	0C8F8
46	STMT	01	00564	51	FORMT	01	01206	BETA	R8	04	00030	H	(R8)	04	0C938
47	STMT	01	00738	ICOD	I4	01	012F0	GAM	R8	04	00038	CHI	(R8)	04	0C9D8
48	STMT	01	0079C	I2	I4	01	012F4	PEA	R8	04	00040	INT	I4	04	0CA78
50	STMT	01	007E6	I3	I4	01	012F8	N	I4	04	00048	K1	I4	04	0CA7C
55	STMT	01	008A6	ITAB	I4	01	012FC	II	I4	04	0004C	SKTEST	R8	04	0CA80
60	STMT	01	00926	ICN	I4	01	01300	JJ	I4	04	00050	PROB	R8	04	0CA88
56	STMT	01	00958	M	I4	01	01304	QQ	R8	04	00058	ID1	I4	04	0CA90
57	STMT	01	00974	I	I4	01	01308	FLAG	R8	04	00060	ALPHA	R8	04	0CA98
58	STMT	01	00990	IP	I4	01	0130C	V	R8	04	00068	IA	I4	04	0CAA0
61	STMT	01	009C2	MILL	I4	01	01310	X	(R4)	04	00070	C	R8	04	0CAA8
62	STMT	01	00A1A	L	I4	01	01314	X1	(R4)	04	0019C	LIMIT	I4	04	0CAB0
63	STMT	01	00A72	IY	I4	01	01318	X2	(R4)	04	03E8C	IFACT	I4	04	0CAB4
75	STMT	01	00ABC	IC	I4	01	0131C	X3	(R4)	04	07B7C				

FORTRAN IVL27 OBJECT SUMMARY SUM SUBROUTINE 05/26/72 PAGE 0021

```
OBJECT DECK ORIGIN  FDF148      FIRST CARD SEQ NO  0200
INSTRUCTIONS                          00B88
ARRAYS                                00320
FORMATS, NAMELISTS, LITCONS           00010
VARIABLES, CONSTANTS, TEMPORARIES     002C0
                  SUM       SD  01    01178        ITF#MPI    ER  02            I$F#BUG   EV  03
                            CM  04    0CAB8
                  INVGAM    EV  05                 GAMIT      EV  06            COMPUT    EV  07
                  ITF#OR    EV  08                 ITF#OX     EV  09            ITF#OF    EV  0A
                  ITF#IC    EV  0B                 ITF#ST     EV  0C            ITF#05    EV  0D
                  ITF#31    EV  0E
```

FORTRAN IVL27 STORAGE MAP SUM SUBROUTINE 05/26/72 PAGE 0022

SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE
AJJ	R4	04	0BBF0	INT	I4	04	0CA78	P1	(R8)	04	0BBF8	5	STMT	01	0042C
ALPHA	R8	04	0CA98	J	I4	01	00F6C	P2	(R8)	04	0BD98	6	STMT	01	0032A
ASTN	(R8)	04	0C8F8	JJ	I4	04	00050	P3	(R8)	04	0BF38	7	STMT	01	00086
BETA	R8	04	00030	K	I4	01	00F7C	P4	(R8)	04	0C0D8	8	STMT	01	0018E
C	R8	04	0CAA8	K1	I4	04	0CA7C	P5	(R8)	04	0C278	9	STMT	01	0033E
CHECK	R8	01	00FB8	L	I4	01	00F88	P6	(R8)	04	0C418	20	STMT	01	00440
CHI	¤(R8)			LIMIT	I4	04	0CAB0	QQ	R8	04	00058	25	STMT	01	0026C
D	R8	01	00F90	L1	I4	01	00F80	SKTEST	R8	04	0CA80	26	STMT	01	00596
DEN	R8	01	00F98	L2	I4	01	00F84	SLX	R8	04	00008	27	STMT	01	0061A
DIFF	R8	01	00FB0	M	I4	01	00F70	SS	R8	01	00FA0	30	STMT	01	006CE
EMP	(R8)	04	0B998	N	I4	04	00048	SX	R8	04	00000	40	STMT	01	006F6
FLAG	R8	04	00060	NN	I4	01	00F78	T	(R8)	01	00D18	45	STMT	01	0071A
GAM	R8	04	00038	NNX	I4	04	00014	TEM	(R8)	01	00B88	50	STMT	01	007A4
GAMMA	R8	04	00028	NUM	I4	04	00018	V	R8	04	00068	51	STMT	01	008C2
H	(R8)	04	0C938	NX	I4	04	00010	X	(R4)	04	00070	52	STMT	01	0089E
I	¤I4	01	00F74	D	¤(R4)			XBAR	R8	04	00020	65	STMT	01	00962
IA	I4	04	0CAA0	P	(R8)	04	0C5B8	X1	(R4)	04	0019C	70	STMT	01	009AC
ICOD	¤I4	01	00F68	PART	R8	01	00FA8	X2	(R4)	04	03E8C	75	STMT	01	009BC
ID1	I4	04	0CA90	PEA	R8	04	00040	X3	(R4)	04	07B7C	80	STMT	01	00A2C
IFACT	I4	04	0CAB4	PL	(R8)	04	0C758	Y	¤(R4)			81	FORMT	01	00EA8
II	I4	04	0004C	PROB	R8	04	0CA88								

FORTRAN IVL27 STORAGE MAP SUM SUBROUTINE 05/26/72 PAGE 0023

SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE
CHI	¤(R8)			80	STMT	01	00A2C	NX	I4	04	00010	P1	(R8)	04	0BBF8
Y	¤(R4)			TEM	(R8)	01	00B88	NNX	I4	04	00014	P2	(R8)	04	0BD98
D	¤(R4)			T	(R8)	01	00D18	NUM	I4	04	00018	P3	(R8)	04	0BF38
7	STMT	01	00086	81	FORMT	01	00EA8	XBAR	R8	04	00020	P4	(R8)	04	0C0D8
8	STMT	01	0018E	ICOD	¤I4	01	00F68	GAMMA	R8	04	00028	P5	(R8)	04	0C278
25	STMT	01	0026C	J	I4	01	00F6C	BETA	R8	04	00030	P6	(R8)	04	0C418
6	STMT	01	0032A	M	I4	01	00F70	GAM	R8	04	00038	P	(R8)	04	0C5B8
9	STMT	01	0033E	I	¤I4	01	00F74	PEA	R8	04	00040	PL	(R8)	04	0C758
5	STMT	01	0042C	NN	I4	01	00F78	N	I4	04	00048	ASTN	(R8)	04	0C8F8
20	STMT	01	00440	K	I4	01	00F7C	II	I4	04	0004C	H	(R8)	04	0C938
26	STMT	01	00596	L1	I4	01	00F80	JJ	I4	04	00050	INT	I4	04	0CA78
27	STMT	01	0061A	L2	I4	01	00F84	QQ	R8	04	00058	K1	I4	04	0CA7C
30	STMT	01	006CE	L	I4	01	00F88	FLAG	R8	04	00060	SKTEST	R8	04	0CA80
40	STMT	01	006F6	D	R8	01	00F90	V	R8	04	00068	PROB	R8	04	0CA88
45	STMT	01	0071A	DEN	R8	01	00F98	X	(R4)	04	00070	ID1	I4	04	0CA90
50	STMT	01	007A4	SS	R8	01	00FA0	X1	(R4)	04	0019C	ALPHA	R8	04	0CA98
52	STMT	01	0089E	PART	R8	01	00FA8	X2	(R4)	04	03E8C	IA	I4	04	0CAA0
51	STMT	01	008C2	DIFF	R8	01	00FB0	X3	(R4)	04	07B7C	C	R8	04	0CAA8
65	STMT	01	00962	CHECK	R8	01	00FB8	EMP	(R8)	04	0B998	LIMIT	I4	04	0CAB0
70	STMT	01	009AC	SX	R8	04	00000	AJJ	R4	04	0BBF0	IFACT	I4	04	0CAB4
75	STMT	01	009BC	SLX	R8	04	00008								

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FORTRAN IVL27 OBJECT SUMMARY  COMPUT   SUBROUTINE                    05/26/72    PAGE 0024

          OBJECT DECK ORIGIN  FDF560     FIRST CARD SEQ NO  0345
          INSTRUCTIONS                          00780
          ARRAYS                                00320
          FORMATS, NAMELISTS, LITCONS           00000
          VARIABLES, CONSTANTS, TEMPORARIES     002C0
                    COMPUT    SD  01  00D60     ITF#MPI    ER  02           I$F#BUG   EV  03
                              CM  04  0CAB8
                    INVGAM    EV  05            GAMIT      EV  06           ITF#X4    EV  07
                    ITF#01    EV  08            ITF#05     EV  09           ITF#05    EV  0A
                    ITF#31    EV  0B
```

FORTRAN IVL27 STORAGE MAP   COMPUT   SUBROUTINE   05/26/72   PAGE 0025

SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE
AJJ	R4	04	0BBF0	IFACT	I4	04	0CAB4	P2	(R8)	04	0BD98	X2	(R4)	04	03E8C
ALPHA	R8	04	0CA98	II	I4	04	0004C	P3	(R8)	04	0BF38	X3	(R4)	04	07B7C
ASTN	(R8)	04	0C8F8	INT	I4	04	0CA78	P4	(R8)	04	0C0D8	Y	¤(R8)		
BETA	R8	04	00030	JJ	I4	04	00050	P5	(R8)	04	0C278	YOU	R8	01	00B70
C	R8	04	0CAA8	K	I4	01	00B5C	P6	(R8)	04	0C418	Z	R8	01	00B88
CHI	(R8)	04	0C9D8	K1	I4	04	0CA7C	QQ	R8	04	00058	68	STMT	01	00130
D	R8	01	00B60	L	I4	01	00BA0	SEA	R8	01	00B80	70	STMT	01	00188
DEN	R8	01	00BB0	LIMIT	I4	04	0CAB0	SEE	R8	01	00B68	75	STMT	01	002B4
EMP	(R8)	04	0B998	M	I4	01	00B7C	SERIES	R8	01	00B98	80	STMT	01	002D0
EN	R8	01	00B40	N	I4	04	00048	SKTEST	R8	04	0CA80	81	STMT	01	00332
ENN	R8	01	00B48	NNX	I4	04	00014	SLX	R8	04	00008	82	STMT	01	00374
EYE	R8	01	00BA8	NUM	I4	04	00018	SX	R8	04	00000	85	STMT	01	003D0
FLAG	R8	04	00060	NX	I4	04	00010	TERM	R8	01	00B90	90	STMT	01	00418
GAM	R8	04	00038	NZ	¤I4	01	00B78	TRACE	R8	01	00B50	95	STMT	01	0058E
GAMMA	R8	04	00028	O	(R4)	04	0B86C	UU	(R8)	01	00780	96	STMT	01	004F2
H	(R8)	04	0C938	P	(R8)	04	0C5B8	V	R8	04	00068	97	STMT	01	00532
I	I4	01	00BA4	PEA	R8	04	00040	W	¤(R8)			100	STMT	01	0060C
IA	I4	04	0CAA0	PL	(R8)	04	0C758	X	(R4)	04	00070	101	STMT	01	0065C
ICOD	¤I4	01	00B58	PROB	R8	04	0CA88	XBAR	R8	04	00020	200	STMT	01	00680
ID1	I4	04	0CA90	P1	(R8)	04	0BBF8	X1	(R4)	04	0019C	300	STMT	01	00690

FORTRAN IVL27 STORAGE MAP   COMPUT   SUBROUTINE   05/26/72   PAGE 0026

SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE
W	¤(R8)			TRACE	R8	01	00B50	NUM	I4	04	00018	P2	(R8)	04	0BD98
Y	¤(R8)			ICOD	¤I4	01	00B58	XBAR	R8	04	00020	P3	(R8)	04	0BF38
68	STMT	01	00130	K	I4	01	00B5C	GAMMA	R8	04	00028	P4	(R8)	04	0C0D8
70	STMT	01	00188	D	R8	01	00B60	BETA	R8	04	00030	P5	(R8)	04	0C278
75	STMT	01	002B4	SEE	R8	01	00B68	GAM	R8	04	00038	P6	(R8)	04	0C418
80	STMT	01	002D0	YOU	R8	01	00B70	PEA	R8	04	00040	P	(R8)	04	0C5B8
81	STMT	01	00332	NZ	¤I4	01	00B78	N	I4	04	00048	PL	(R8)	04	0C758
82	STMT	01	00374	M	I4	01	00B7C	II	I4	04	0004C	ASTN	(R8)	04	0C8F8
85	STMT	01	003D0	SEA	R8	01	00B80	JJ	I4	04	00050	H	(R8)	04	0C938
90	STMT	01	00418	Z	R8	01	00B88	QQ	R8	04	00058	CHI	(R8)	04	0C9D8
96	STMT	01	004F2	TERM	R8	01	00B90	FLAG	R8	04	00060	INT	I4	04	0CA78
97	STMT	01	00532	SERIES	R8	01	00B98	V	R8	04	00068	K1	I4	04	0CA7C
95	STMT	01	0058E	L	I4	01	00BA0	X	(R4)	04	00070	SKTEST	R8	04	0CA80
100	STMT	01	0060C	I	I4	01	00BA4	X1	(R4)	04	0019C	PROB	R8	04	0CA88
101	STMT	01	0065C	EYE	R8	01	00BA8	X2	(R4)	04	03E8C	ID1	I4	04	0CA90
200	STMT	01	00680	DEN	R8	01	00BB0	X3	(R4)	04	07B7C	ALPHA	R8	04	0CA98
300	STMT	01	00690	SX	R8	04	00000	O	(R4)	04	0B86C	IA	I4	04	0CAA0
UU	(R8)	01	00780	SLX	R8	04	00008	EMP	(R8)	04	0B998	C	R8	04	0CAA8
EN	R8	01	00B40	NX	I4	04	00010	AJJ	R4	04	0BBF0	LIMIT	I4	04	0CAB0
ENN	R8	01	00B48	NNX	I4	04	00014	P1	(R8)	04	0BBF8	IFACT	I4	04	0CAB4

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FORTRAN IVL27 OBJECT SUMMARY  INVGAM   SUBROUTINE                    05/26/72    PAGE 0027

          OBJECT DECK ORIGIN  FDF418     FIRST CARD SEQ NO  0457
          INSTRUCTIONS                          00838
          ARRAYS                                00210
          FORMATS, NAMELISTS, LITCONS           001A0
          VARIABLES, CONSTANTS, TEMPORARIES     00354
                    INVGAM    SD  01  00F3C     ITF#MPI    ER  02           I$F#BUG   EV  03
                              CM  04  0CAB8
                    ITF#OR    EV  05            ITF#OX     EV  06           ITF#OH    EV  07
                    ITF#OF    EV  08            ITF#OE     EV  09           ITF#X2    EV  0A
                    ITF#IC    EV  0B            ITF#ST     EV  0C           ITF#01    EV  0D
                    ITF#05    EV  0E            ITF#31     EV  0F
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FORTRAN IVL27 STORAGE MAP   INVGAM   SUBROUTINE   05/26/72   PAGE 0028

SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE
AA	(R8)	01	00838	IA	I4	04	0CAA0	PP	¤(R8)			XX	R8	01	00D20
AAA	(R8)	01	00940	ID1	I4	04	0CA90	PROB	R8	04	0CA88	X1	(R4)	04	0019C
AJJ	R4	04	0BBF0	IFACT	I4	04	0CAB4	P1	(R8)	04	0BBF8	X2	(R4)	04	03E8C
ALPHA	R8	04	0CA98	II	I4	04	0004C	P2	(R8)	04	0BD98	X3	(R4)	04	07B7C
ASTN	(R8)	04	0C8F8	IJ	I4	01	00CD4	P3	(R8)	04	0BF38	Z	R8	01	00CF8
BB	(R8)	01	00890	IJJ	I4	01	00C98	P4	(R8)	04	0C0D8	20	STMT	01	001C6
BBB	(R8)	01	00998	INT	I4	04	0CA78	P5	(R8)	04	0C278	25	STMT	01	001EA
BET	¤R8	01	00D40	JJ	I4	04	00050	P6	(R8)	04	0C418	26	STMT	01	00236
BETA	R8	04	00030	K	I4	01	00CF0	Q	R8	01	00CC8	27	STMT	01	0025C
C	R8	04	0CAA8	K1	I4	04	0CA7C	QQ	R8	04	00058	30	STMT	01	002FA
CC	(R8)	01	008E8	L	I4	01	00D10	S	R8	01	00D08	31	FORMT	01	00A48
CCC	(R8)	01	009F0	LIMIT	I4	04	0CAB0	SG	R8	01	00CA0	35	STMT	01	0035A
CHECK	R8	01	00CE0	M	I4	01	00CF4	SKTEST	R8	04	0CA80	38	STMT	01	003E8
CHI	(R8)	04	0C9D8	N	I4	04	00048	SLX	R8	04	00008	39	STMT	01	00446
DL	R8	01	00CE8	NN	¤I4	01	00CD0	SP	R8	01	00CA8	55	STMT	01	004F4
DX	R8	01	00D38	NNX	I4	04	00014	SUM	R8	01	00D30	60	STMT	01	0054C
DXO	R8	01	00CD8	NNY	¤I4	01	00CC0	SX	R8	04	00000	70	STMT	01	00632
EMP	(R8)	04	0B998	NUM	I4	04	00018	T	R8	01	00D00	95	STMT	01	0069A
EN	R8	01	00CB0	NX	I4	04	00010	TEM	R8	01	00D18	96	FORMT	01	00B79
ENN	R8	01	00CB8	NY	¤I4	01	00C9C	TT	R8	01	00D28	97	STMT	01	006F8
FLAG	R8	04	00060	O	(R4)	04	0B86C	V	R8	04	00068	98	FORMT	01	00BB0
GAM	R8	04	00038	P	(R8)	04	0C5B8	X	(R4)	04	00070	100	STMT	01	005D6
GAMMA	R8	04	00028	PEA	R8	04	00040	XBAR	R8	04	00020	105	STMT	01	00612
H	(R8)	04	0C938	PK	¤(R8)			XO	R8	01	00C90	200	STMT	01	00734
I	I4	01	00CC4	PL	(R8)	04	0C758								

FORTRAN IVL27 STORAGE MAP   INVGAM   SUBROUTINE   05/26/72   PAGE 0029

SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE
PP	¤(R8)			96	FORMT	01	00B79	TT	R8	01	00D28	O	(R4)	04	0B86C
PK	¤(R8)			98	FORMT	01	00BB0	SUM	R8	01	00D30	EMP	(R8)	04	0B998
20	STMT	01	001C6	XO	R8	01	00C90	DX	R8	01	00D38	AJJ	R4	04	0BBF0
25	STMT	01	001EA	IJJ	I4	01	00C98	BET	¤R8	01	00D40	P1	(R8)	04	0BBF8
26	STMT	01	00236	NY	¤I4	01	00C9C	SX	R8	04	00000	P2	(R8)	04	0BD98
27	STMT	01	0025C	SG	R8	01	00CA0	SLX	R8	04	00008	P3	(R8)	04	0BF38
30	STMT	01	002FA	SP	R8	01	00CA8	NX	I4	04	00010	P4	(R8)	04	0C0D8
35	STMT	01	0035A	EN	R8	01	00CB0	NNX	I4	04	00014	P5	(R8)	04	0C278
38	STMT	01	003E8	ENN	R8	01	00CB8	NUM	I4	04	00018	P6	(R8)	04	0C418
39	STMT	01	00446	NNY	¤I4	01	00CC0	XBAR	R8	04	00020	P	(R8)	04	0C5B8
55	STMT	01	004F4	I	I4	01	00CC4	GAMMA	R8	04	00028	PL	(R8)	04	0C758
60	STMT	01	0054C	Q	R8	01	00CC8	BETA	R8	04	00030	ASTN	(R8)	04	0C8F8
100	STMT	01	005D6	NN	¤I4	01	00CD0	GAM	R8	04	00038	H	(R8)	04	0C938
105	STMT	01	00612	IJ	I4	01	00CD4	PEA	R8	04	00040	CHI	(R8)	04	0C9D8
70	STMT	01	00632	DXO	R8	01	00CD8	N	I4	04	00048	INT	I4	04	0CA78
95	STMT	01	0069A	CHECK	R8	01	00CE0	II	I4	04	0004C	K1	I4	04	0CA7C
97	STMT	01	006F8	DL	R8	01	00CE8	JJ	I4	04	00050	SKTEST	R8	04	0CA80
200	STMT	01	00734	K	I4	01	00CF0	QQ	R8	04	00058	PROB	R8	04	0CA88
AA	(R8)	01	00838	M	I4	01	00CF4	FLAG	R8	04	00060	ID1	I4	04	0CA90
BB	(R8)	01	00890	Z	R8	01	00CF8	V	R8	04	00068	ALPHA	R8	04	0CA98
CC	(R8)	01	008E8	T	R8	01	00D00	X	(R4)	04	00070	IA	I4	04	0CAA0
AAA	(R8)	01	00940	S	R8	01	00D08	X1	(R4)	04	0019C	C	R8	04	0CAA8
BBB	(R8)	01	00998	L	I4	01	00D10	X2	(R4)	04	03E8C	LIMIT	I4	04	0CAB0
CCC	(R8)	01	009F0	TEM	R8	01	00D18	X3	(R4)	04	07B7C	IFACT	I4	04	0CAB4
31	FORMT	01	00A48	XX	R8	01	00D20								

FORTRAN IVL27 OBJECT SUMMARY   PRINT   SUBROUTINE   05/26/72   PAGE 0030

OBJECT DECK ORIGIN FDF4C8   FIRST CARD SEQ NO 0614

INSTRUCTIONS	00758
ARRAYS	00130
FORMATS, NAMELISTS, LITCONS	002B0
VARIABLES, CONSTANTS, TEMPORARIES	00260

PRINT	SD	01	00D98	ITF#MPI	ER	02		I$F#BUG	EV	03	
	CM	04	0CAB8								
ITF#OR	EV	05		ITF#OX	EV	06		ITF#OI	EV	07	
ITF#OH	EV	08		ITF#OF	EV	09		ITF#OA	EV	0A	
ITF#IC	EV	0B									

FORTRAN IVL27 STORAGE MAP   PRINT   SUBROUTINE   05/26/72   PAGE 0031

SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE
AJJ	R4	04	0BBF0	JJ	I4	04	00050	P2	(R8)	04	0BD98	2	FORMT	01	00897
ALPHA	R8	04	0CA98	K	I4	01	00BE4	P3	(R8)	04	0BF38	3	FORMT	01	0090F
ASTN	(R8)	04	0C8F8	K1	I4	04	0CA7C	P4	(R8)	04	0C0D8	4	FORMT	01	0092B
BETA	R8	04	00030	L	I4	01	00BE0	P5	(R8)	04	0C278	5	FORMT	01	00A83
C	R8	04	0CAA8	LIMIT	I4	04	0CAB0	P6	(R8)	04	0C418	6	STMT	01	001F6
CHI	(R8)	04	0C9D8	LINE	I4	01	00BD8	QQ	R8	04	00058	10	STMT	01	00128
EMP	(R8)	04	0B998	M	¤I4	01	00BF4	SKTEST	R8	04	0CA80	20	STMT	01	00336
FLAG	R8	04	00060	N	I4	04	00048	SLX	R8	04	00008	25	STMT	01	0035E
GAM	R8	04	00038	NN	I4	01	00BEC	SX	R8	04	00000	26	STMT	01	00382
GAMMA	R8	04	00028	NNX	I4	04	00014	V	R8	04	00068	30	STMT	01	00392
H	(R8)	04	0C938	NUM	I4	04	00018	X	(R4)	04	00070	31	FORMT	01	00AB6
I	¤I4	01	00BE8	NX	I4	04	00010	XBAR	R8	04	00020	40	STMT	01	00444
IA	I4	04	0CAA0	O	(R4)	04	0B86C	X1	(R4)	04	0019C	41	FORMT	01	00AD0
ICOD	¤I4	01	00BDC	P	(R8)	04	0C5B8	X2	(R4)	04	03E8C	50	STMT	01	004E8
ID1	I4	04	0CA90	PEA	R8	04	00040	X3	(R4)	04	07B7C	51	FORMT	01	00AF2
IFACT	I4	04	0CAB4	PL	(R8)	04	0C758	Y	¤(R4)			52	FORMT	01	00B05
II	I4	04	0004C	PROB	R8	04	0CA88	Z	(R4)	01	00758	59	STMT	01	0059C
INT	I4	04	0CA78	P1	(R8)	04	0BBF8	1	FORMT	01	00888	60	STMT	01	0065C
J	I4	01	00BF0												

FORTRAN IVL27 STORAGE MAP PRINT SUBROUTINE 05/26/72 PAGE 0032

SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE
Y	¤(R4)			51	FORMT	01	00AF2	GAM	R8	04	00038	P4	(R8)	04	0C0D8
10	STMT	01	00128	52	FORMT	01	00B05	PEA	R8	04	00040	P5	(R8)	04	0C278
6	STMT	01	001F6	LINE	I4	01	00BD8	N	I4	04	00048	P6	(R8)	04	0C418
20	STMT	01	00336	ICOD	¤I4	01	00BDC	II	I4	04	0004C	P	(R8)	04	0C5B8
25	STMT	01	0035E	L	I4	01	00BE0	JJ	I4	04	00050	PL	(R8)	04	0C758
26	STMT	01	00382	K	I4	01	00BE4	QQ	R8	04	00058	ASTN	(R8)	04	0C8F8
30	STMT	01	00392	I	¤I4	01	00BE8	FLAG	R8	04	00060	H	(R8)	04	0C938
40	STMT	01	00444	NN	I4	01	00BEC	V	R8	04	00068	CHI	(R8)	04	0C9D8
50	STMT	01	004E8	J	I4	01	00BF0	X	(R4)	04	00070	INT	I4	04	0CA78
59	STMT	01	0059C	M	¤I4	01	00BF4	X1	(R4)	04	0019C	K1	I4	04	0CA7C
60	STMT	01	0065C	SX	R8	04	00000	X2	(R4)	04	03E8C	SKTEST	R8	04	0CA80
Z	(R4)	01	00758	SLX	R8	04	00008	X3	(R4)	04	07B7C	PROB	R8	04	0CA88
1	FORMT	01	00888	NX	I4	04	00010	O	(R4)	04	0B86C	ID1	I4	04	0CA90
2	FORMT	01	00897	NNX	I4	04	00014	EMP	(R8)	04	0B998	ALPHA	R8	04	0CA98
3	FORMT	01	0090F	NUM	I4	04	00018	AJJ	R4	04	0BBF0	IA	I4	04	0CAA0
4	FORMT	01	0092B	XBAR	R8	04	00020	P1	(R8)	04	0BBF8	C	R8	04	0CAA8
5	FORMT	01	00A83	GAMMA	R8	04	00028	P2	(R8)	04	0BD98	LIMIT	I4	04	0CAB0
31	FORMT	01	00AB6	BETA	R8	04	00030	P3	(R8)	04	0BF38	IFACT	I4	04	0CAB4
41	FORMT	01	00AD0												

FORTRAN IVL27 OBJECT SUMMARY GAMIT SUBROUTINE 05/26/72 PAGE 0033

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OBJECT DECK ORIGIN  FDF4E8         FIRST CARD SEQ NO  0730
INSTRUCTIONS                          004D8
ARRAYS                                00640
FORMATS, NAMELISTS, LITCONS           00000
VARIABLES, CONSTANTS, TEMPORARIES     00208
        GAMIT    SD  01   00D20       ITF#MPI   ER  02       I$F#BUG   EV  03
                 CM  04   00068
        ITF#05   EV  05
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FORTRAN IVL27 STORAGE MAP GAMIT SUBROUTINE 05/26/72 PAGE 0034

SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE
A	R8	01	00BA0	GAMMA	R8	04	00028	NX	I4	04	00010	10	STMT	01	0015C
B	R8	01	00BD8	I	¤I4	01	00BE4	PD	R8	01	00BF0	15	STMT	01	0021C
BETA	R8	04	00030	II	I4	04	0004C	PEA	R8	04	00040	16	STMT	01	00240
C	R8	01	00BB0	JJ	I4	04	00050	QQ	R8	04	00058	25	STMT	01	00268
D	R8	01	00BB8	K	I4	01	00BE8	SLX	R8	04	00008	30	STMT	01	0029E
E	R8	01	00BC0	KK	I4	01	00BEC	SX	R8	04	00000	40	STMT	01	002F2
F	R8	01	00BC8	L	I4	01	00BE0	U	(R8)	01	004D8	45	STMT	01	0031E
FLAG	R8	04	00060	N	I4	04	00048	V	R8	01	00BA8	50	STMT	01	00392
G	R8	01	00BD0	NNX	I4	04	00014	XBAR	R8	04	00020	55	STMT	01	00302
GAM	R8	04	00038	NUM	I4	04	00018	Y	R8	01	00BF8				

FORTRAN IVL27 STORAGE MAP GAMIT SUBROUTINE 05/26/72 PAGE 0035

SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE	SYMBOL	TYPE	ESID	VALUE
10	STMT	01	0015C	A	R8	01	00BA0	K	I4	01	00BE8	GAMMA	R8	04	00028
15	STMT	01	0021C	V	R8	01	00BA8	KK	I4	01	00BEC	BETA	R8	04	00030
16	STMT	01	00240	C	R8	01	00BB0	PD	R8	01	00BF0	GAM	R8	04	00038
25	STMT	01	00268	D	R8	01	00BB8	Y	R8	01	00BF8	PEA	R8	04	00040
30	STMT	01	0029E	E	R8	01	00BC0	SX	R8	04	00000	N	I4	04	00048
40	STMT	01	002F2	F	R8	01	00BC8	SLX	R8	04	00008	II	I4	04	0004C
55	STMT	01	00302	G	R8	01	00BD0	NX	I4	04	00010	JJ	I4	04	00050
45	STMT	01	0031E	B	R8	01	00BD8	NNX	I4	04	00014	QQ	R8	04	00058
50	STMT	01	00392	L	I4	01	00BE0	NUM	I4	04	00018	FLAG	R8	04	00060
U	(R8)	01	004D8	I	¤I4	01	00BE4	XBAR	R8	04	00020				

LINKAGE EDITOR --- PARAMETERS AND DIAGNOSTICS 05/26/72 PAGE 1

(P) ERREXIT A=000001

LINKAGE EDITOR --- PROGRAM MAP 05/26/72 PAGE 2

PROGRAM

NAME OF PROGRAM	POA04	COMPUTED LENGTH	00095984	MAXIMUM LENGTH	00095984
		NUMBER OF REGIONS	001	NUMBER OF OVERLAY POINTS	000
		NUMBER OF SEGMENTS	001	NUMBER OF ENTRY POINTS	00089
		NUMBER OF MODULES	035	STARTING EXECUTION ADDR.	014D20
		BLANK COMMON LENGTH	00051896	BLANK COMMON LOAD ADDR.	000000

SEGMENT

NAME OF SEGMENT	(ROOT)	NUMBER 001	SEGMENT LENGTH	00095984	STARTING ADDRESS	000000
			SYMBOLIC OVERLAY POINT	(ROOT)	REGION NUMBER	001
			NEXT SEGMENT IN PATH	(ROOT)	NUMBER OF MODULES IN SEGMENT	035

MODULES	NAME OF MODULE	LOAD ADDRESS	MODULE LENGTH	NUMBER OF ENTRYS	METHOD USED TO BIND MODULES
	POA04	00CAB8	00005824	00002	EXPLICIT
	SUM	00E178	00004472	00001	EXPLICIT
	COMPUT	00F2F0	00003424	00001	EXPLICIT
	INVGAM	010050	00003904	00001	EXPLICIT
	PRINT	010F90	00003480	00001	EXPLICIT
	GAMIT	011D28	00003360	00001	EXPLICIT
	ITF#AG	012A48	00000064	00003	IMPLICIT
	ITF#IA	012A88	00000104	00002	IMPLICIT
	ITF#IC	012AF0	00003312	00007	IMPLICIT
	ITF#IC2	0137E0	00000832	00009	IMPLICIT
	ITF#ID	013B20	00000768	00006	IMPLICIT
	ITF#IFMT	013E20	00002176	00003	IMPLICIT
	ITF#IG	0146A0	00000048	00002	IMPLICIT
	ITF#IH	0146D0	00000048	00002	IMPLICIT
	ITF#II	014700	00000416	00002	IMPLICIT

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ITF#IL	0148A0	00000096	00002	IMPLICIT
ITF#ILR	014900	00000520	00004	IMPLICIT
ITF#IS	014B08	00000216	00002	IMPLICIT
ITF#IX	014BE0	00000016	00002	IMPLICIT
ITF#IZ	014BF0	00000304	00002	IMPLICIT
ITF#MPI	014D20	00000904	00005	IMPLICIT
ITF#OA	0150A8	00000104	00002	IMPLICIT
ITF#OH	015110	00000048	00002	IMPLICIT
ITF#OI	015140	00000128	00002	IMPLICIT
ITF#OLR	0151C0	00000432	00003	IMPLICIT
ITF#OX	015370	00000048	00002	IMPLICIT
ITF#PA	0153A0	00000704	00005	IMPLICIT
ITF#X2	015660	00000200	00001	IMPLICIT
ITF#X4	015728	00000280	00001	IMPLICIT
ITF#01	015840	00000400	00001	IMPLICIT
ITF#09	0159D0	00000456	00002	IMPLICIT
ITF#31	015B98	00000184	00001	IMPLICIT
ITFDATAD	015C50	00002512	00003	IMPLICIT
ITF#BUG	016620	00004176	00003	IMPLICIT
ITF#IC1	017670	00000128	00001	IMPLICIT

***END LNKEDT

A

GAMMA DISTRIBUTION FUNCTION MODEL

$$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$$

$\alpha = 0$		ORIGIN
$\beta =$	$: \beta > 0$	SCALE PARAMETER
$\gamma =$	$: \gamma > 0$	SHAPE PARAMETER
$\bar{y} =$		$\bar{y}$ IS THE MEAN

GRAPH PREPARATION

Given $\alpha = 0$; $\beta = 3$; $\gamma = 2.0$; the completed graph will appear in the lower right quadrant.

STEP I - (a) In the upper right quadrant, at $\gamma = 2.0$, draw a horizontal line.

(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower right blank quadrant. Label these lines at the base with the corresponding probability values.

STEP II - In the lower left quadrant are the solid sloping lines with the β values or slopes. At the intersection of the sloping line labeled $\beta = 3$ and the horizontal heavy line at $\beta = 1$, draw a vertical scale line. Through the intersections of the sloping β lines with the vertical line, draw horizontal lines across the lower right blank quadrant. Label the ends of these lines with the respective values of the sloping β lines at the intersection points. These scale values are in units of the original set of data.

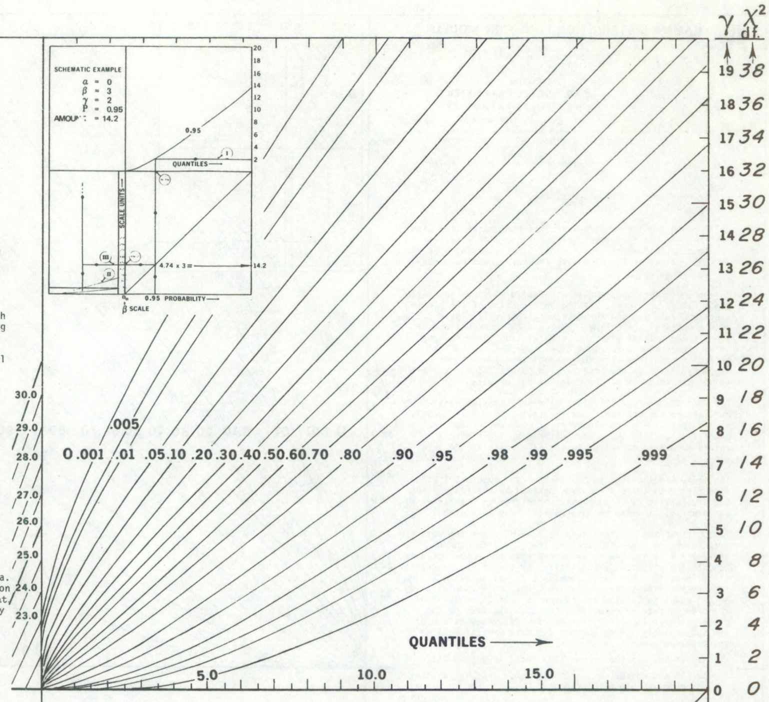
For very large or small values of β, some scaling difficulties are encountered. Use the scale for $\beta = 1$ that is at the left side of the lower right quadrant. Read the quantities on this scale and multiply by the sample β values to arrive at values in the same units as the original set of data.

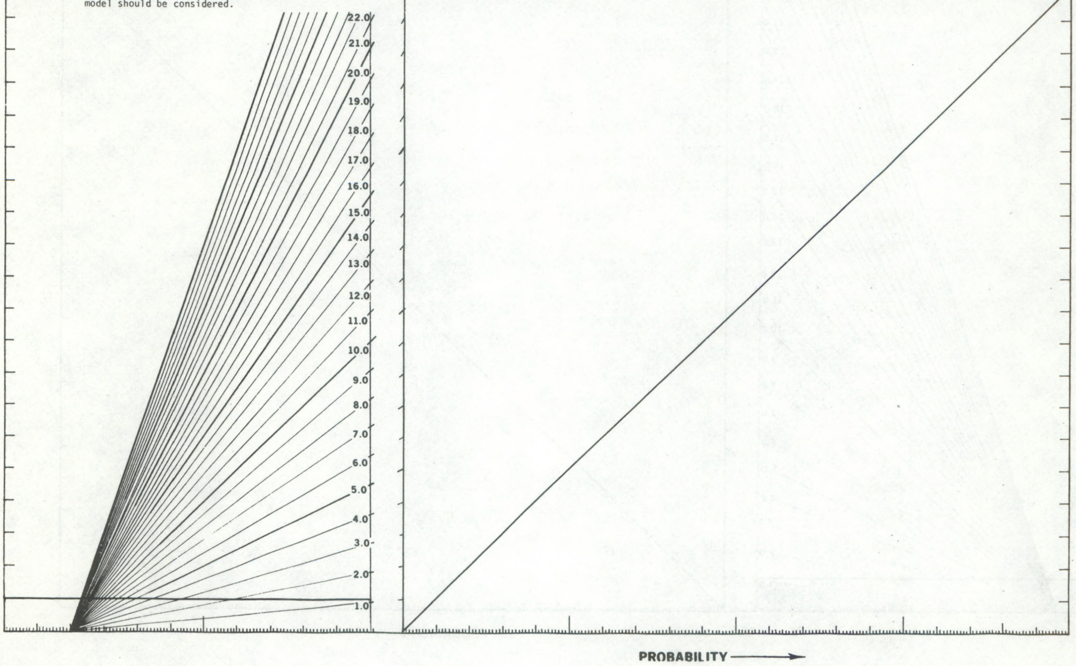
STEP III - The heavy sloping line in the lower right quadrant is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.

STEP IV - If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression $(i-c)/(n-2c+1)$. For large samples, c is set equal to ½ that reduces the expression to $(i-½)/n$. For small samples, c may be set equal to 0. The expression is then $(i/n+1)$. Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.

a. If so, use the graph. The model is a good fit.

b. If not so, do not use the graph. That is, another model should be considered.





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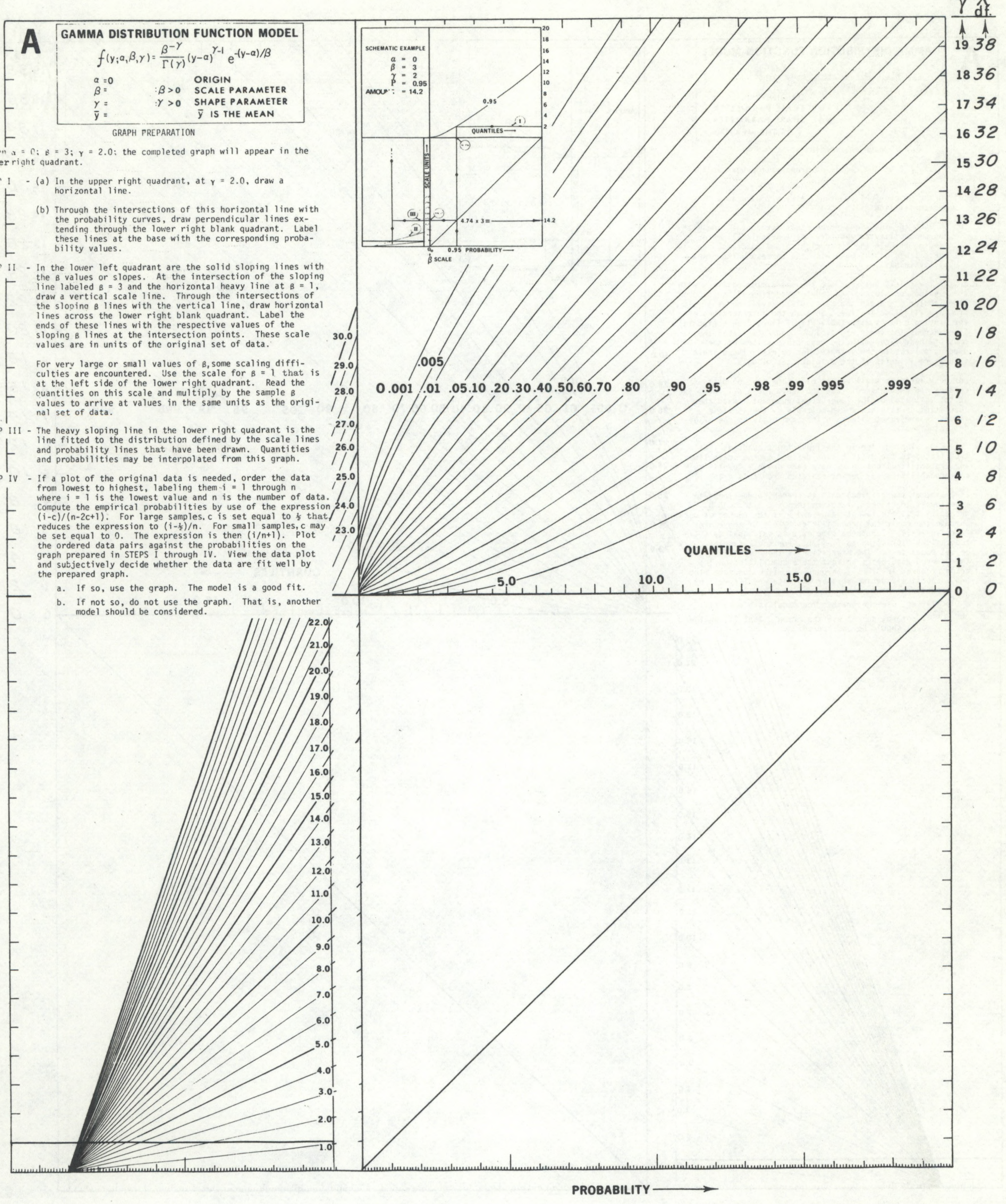

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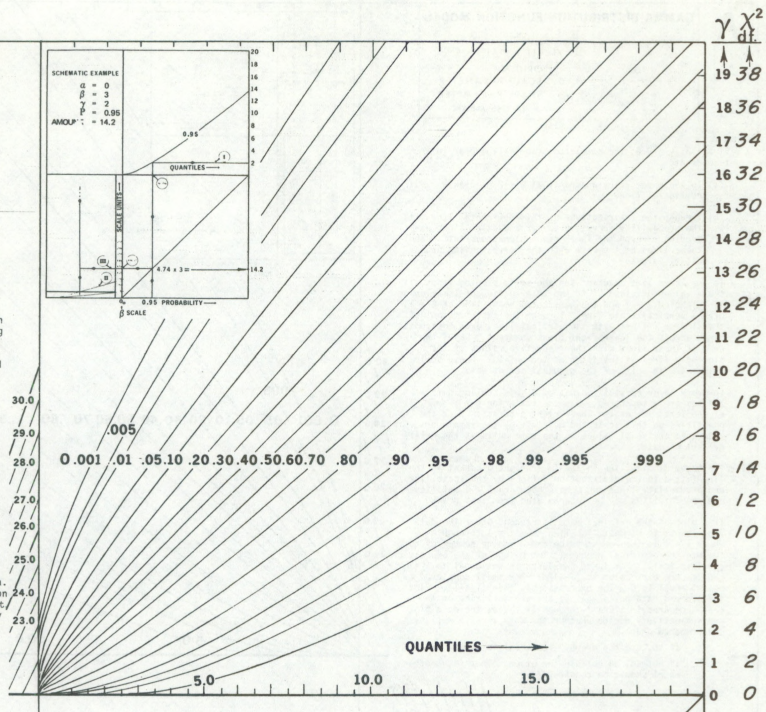

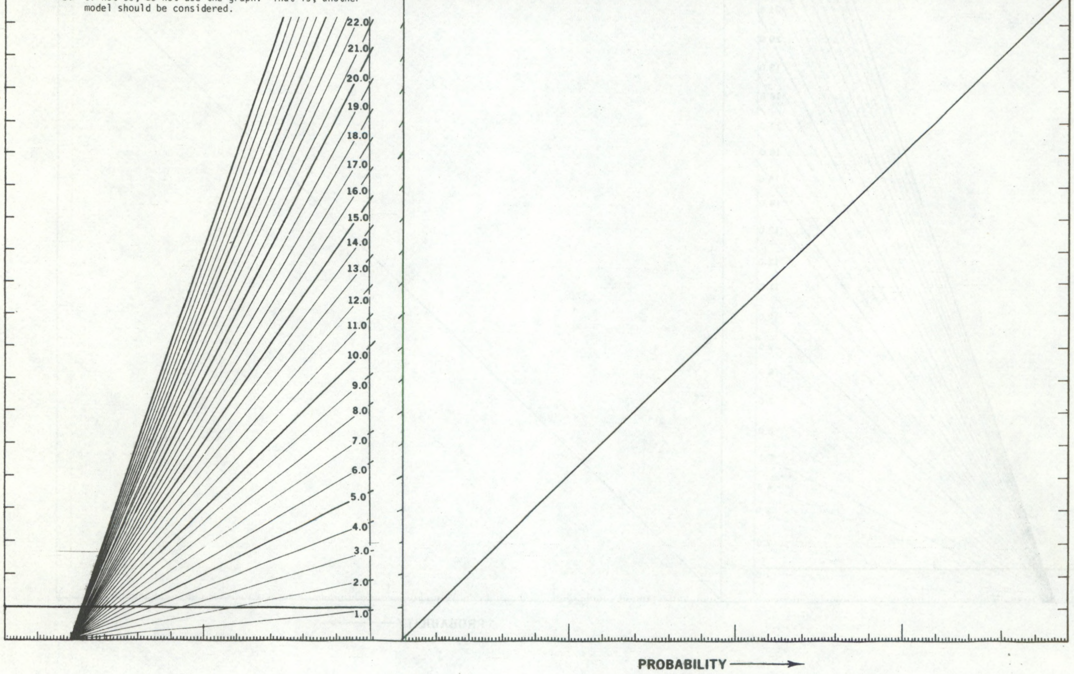

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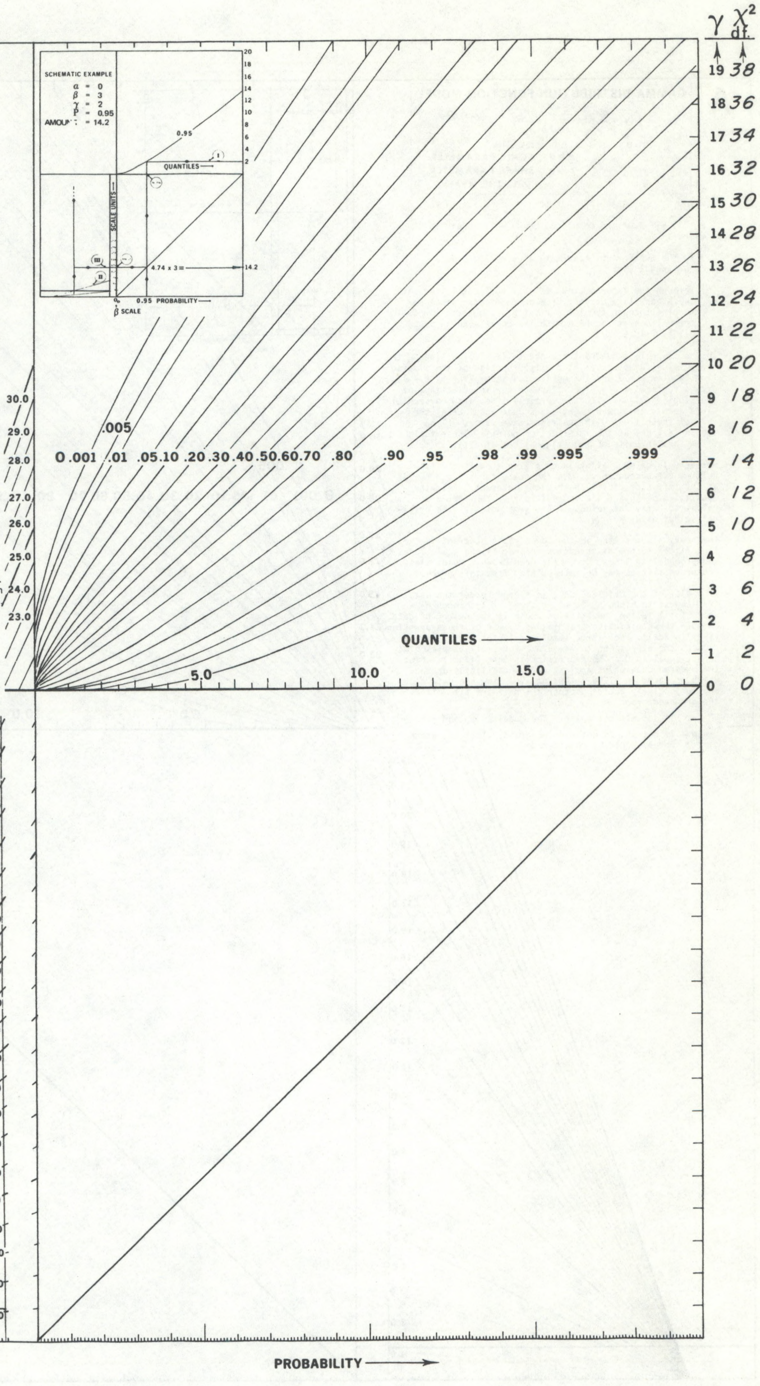

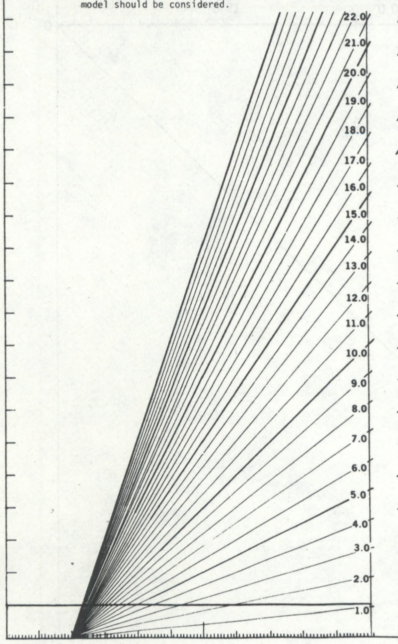

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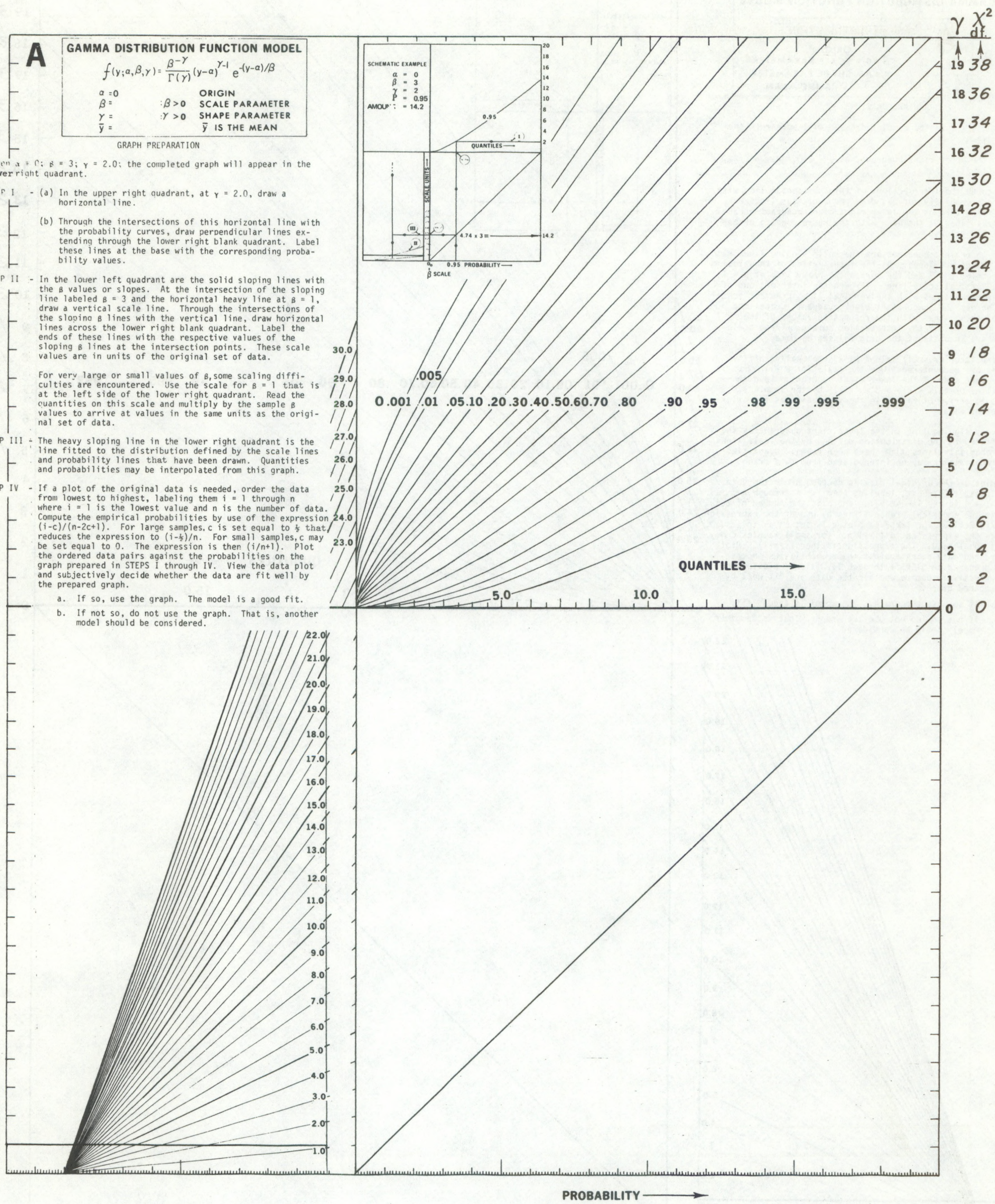

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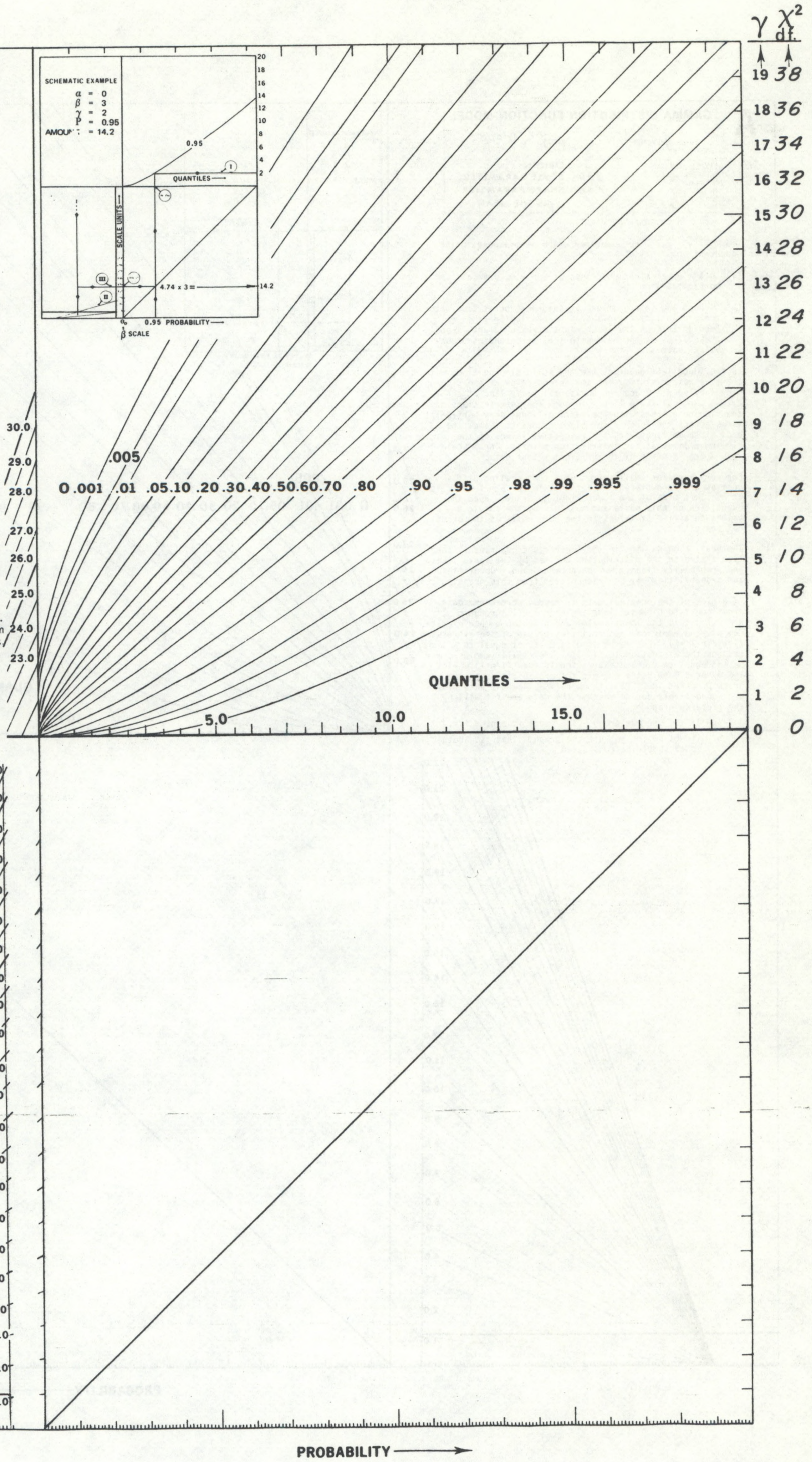

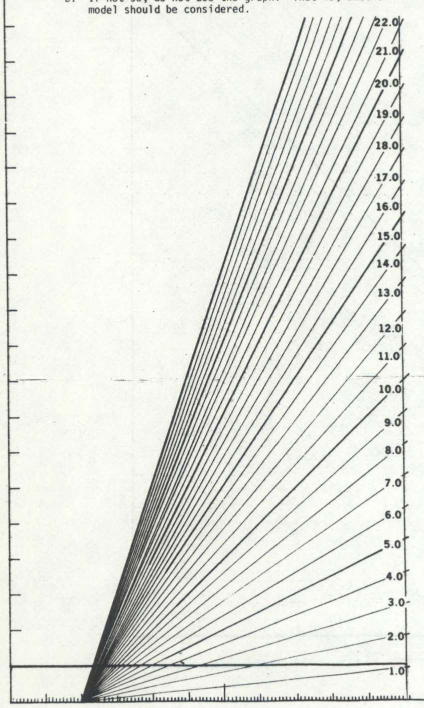

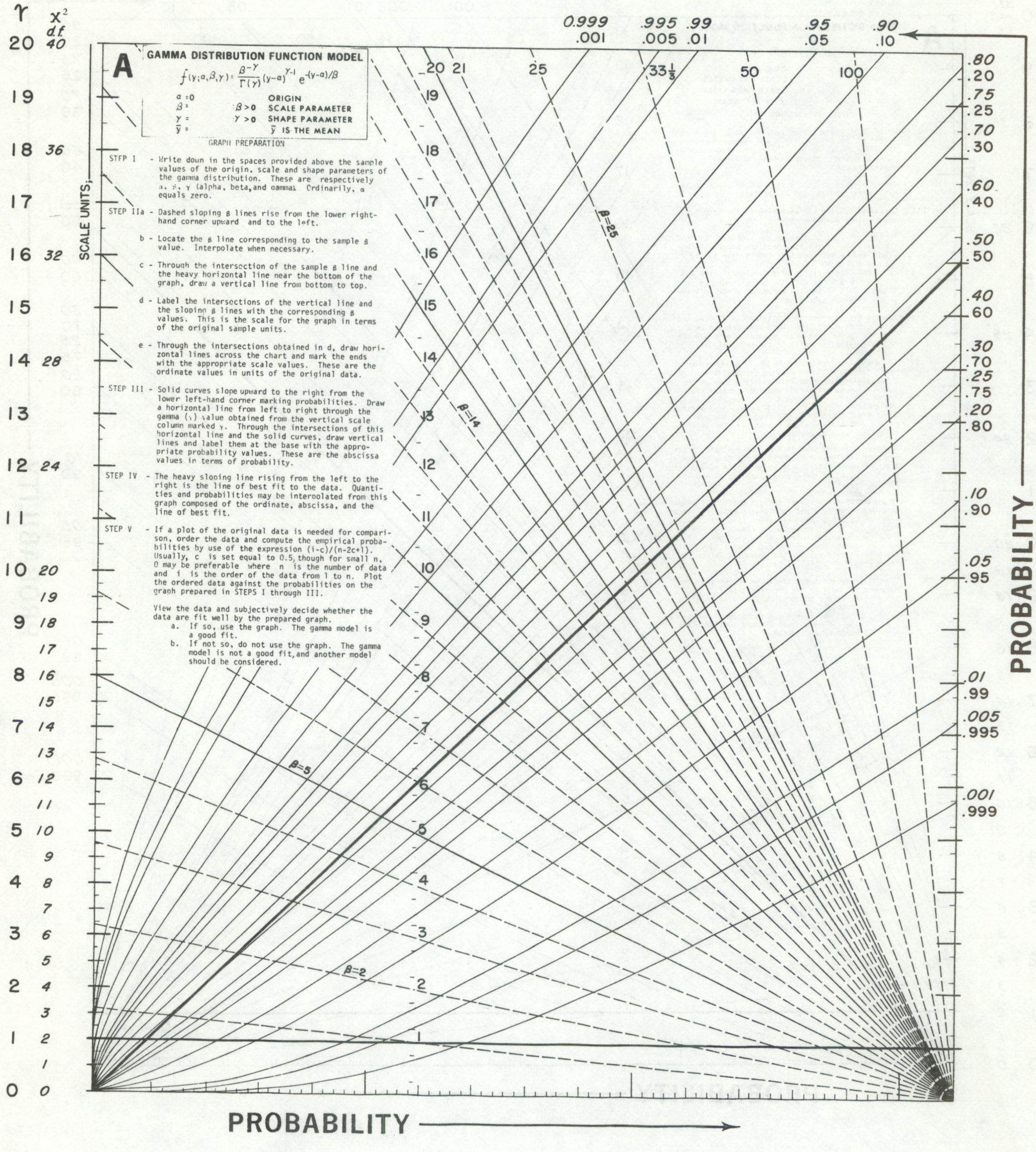
A
GAMMA DISTRIBUTION FUNCTION MODEL
$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$
α =0 ORIGIN
β = :β > 0 SCALE PARAMETER
γ = :γ > 0 SHAPE PARAMETER
ȳ = ȳ IS THE MEAN
GRAPH PREPARATION
STEP I - Write down in the spaces provided above the sample values of the origin, scale and shape parameters of the gamma distribution. These are respectively α, β, γ (alpha, beta, and gamma). Ordinarily, α equals zero.
STEP IIa - Dashed sloping β lines rise from the lower right-hand corner upward and to the left.
b - Locate the β line corresponding to the sample β value. Interpolate when necessary.
c - Through the intersection of the sample β line and the heavy horizontal line near the bottom of the graph, draw a vertical line from bottom to top.
d - Label the intersections of the vertical line and the sloping β lines with the corresponding β values. This is the scale for the graph in terms of the original sample units.
e - Through the intersections obtained in d, draw horizontal lines across the chart and mark the ends with the appropriate scale values. These are the ordinate values in units of the original data.
STEP III - Solid curves slope upward to the right from the lower left-hand corner marking probabilities. Draw a horizontal line from left to right through the gamma (γ) value obtained from the vertical scale column marked γ. Through the intersections of this horizontal line and the solid curves, draw vertical lines and label them at the base with the appropriate probability values. These are the abscissa values in terms of probability.
STEP IV - The heavy sloping line rising from the left to the right is the line of best fit to the data. Quantities and probabilities may be interpolated from this graph composed of the ordinate, abscissa, and the line of best fit.
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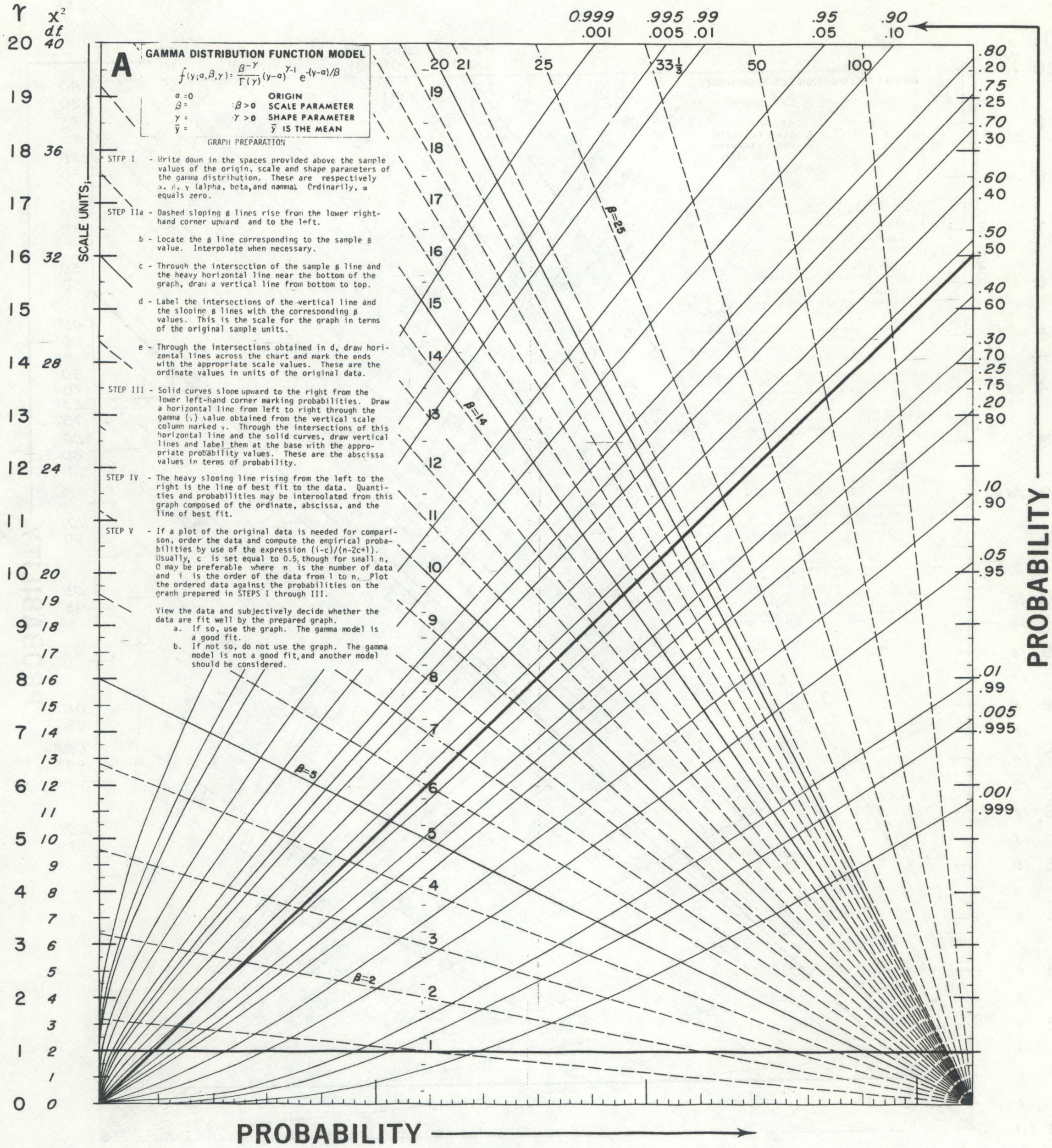
γ
χ²
d.f.
SCALE UNITS;
β=25
β=14
β=5
β=2
PROBABILITY
PROBABILITY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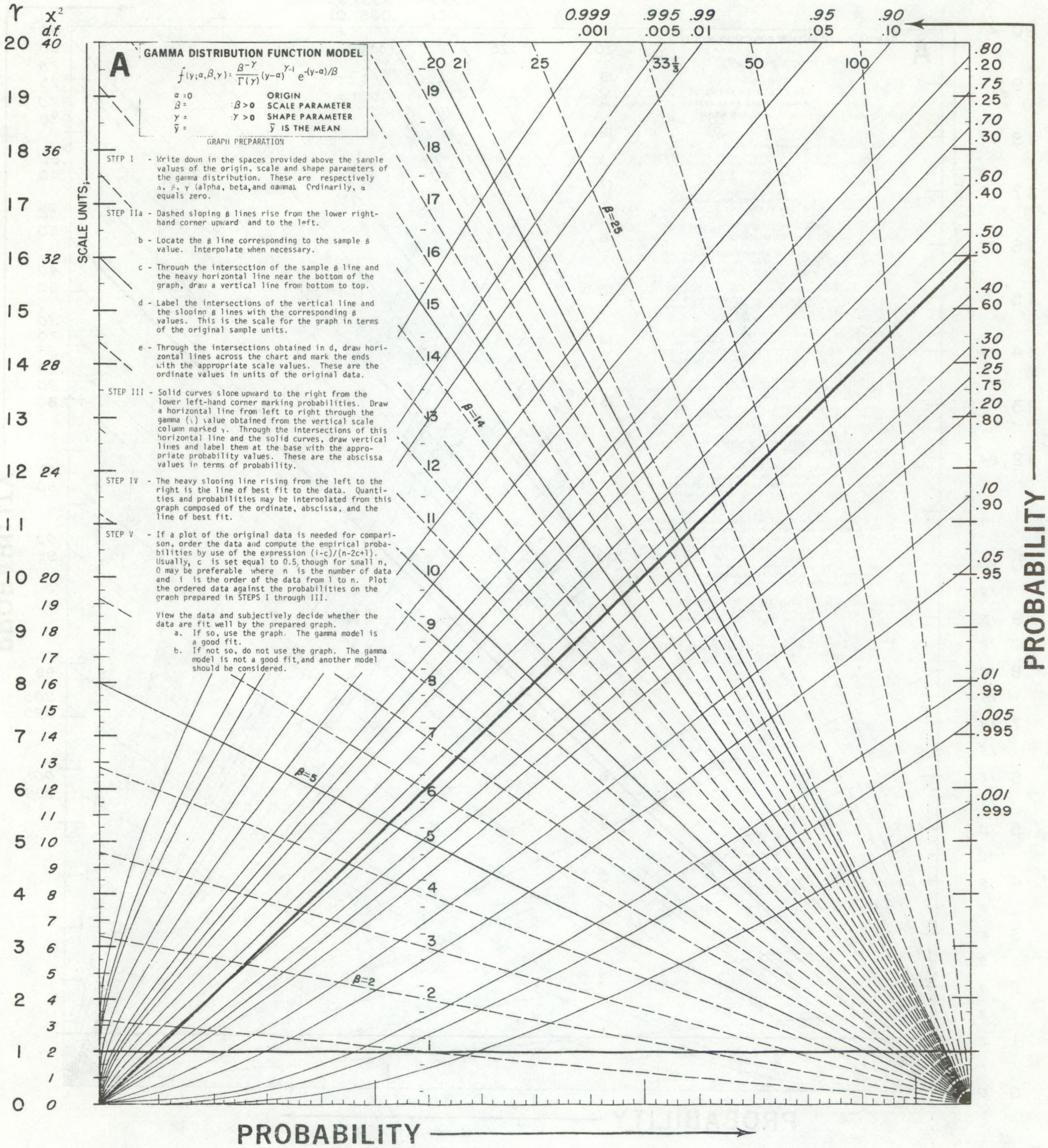
γ
χ^2 df
SCALE UNITS;
β=25
β=14
β=5
β=2
20 21 25 33⅓ 50 100
0.999 .001
.995 .005
.99 .01
.95 .05
.90 .10
.80 .20
.75 .25
.70 .30
.60 .40
.50 .50
.40 .60
.30 .70
.25 .75
.20 .80
.10 .90
.05 .95
.01 .99
.005 .995
.001 .999
PROBABILITY
PROBABILITY



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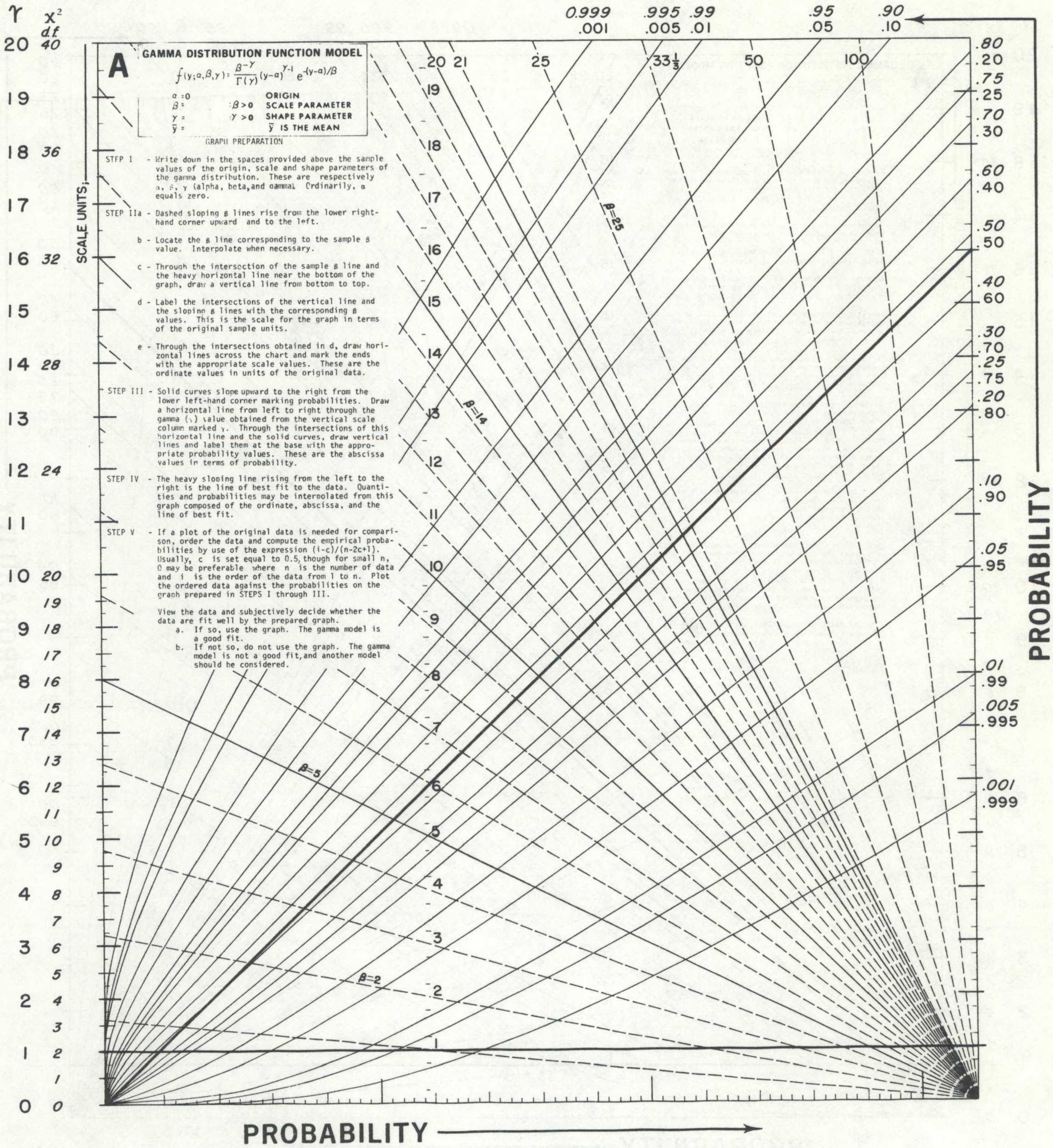
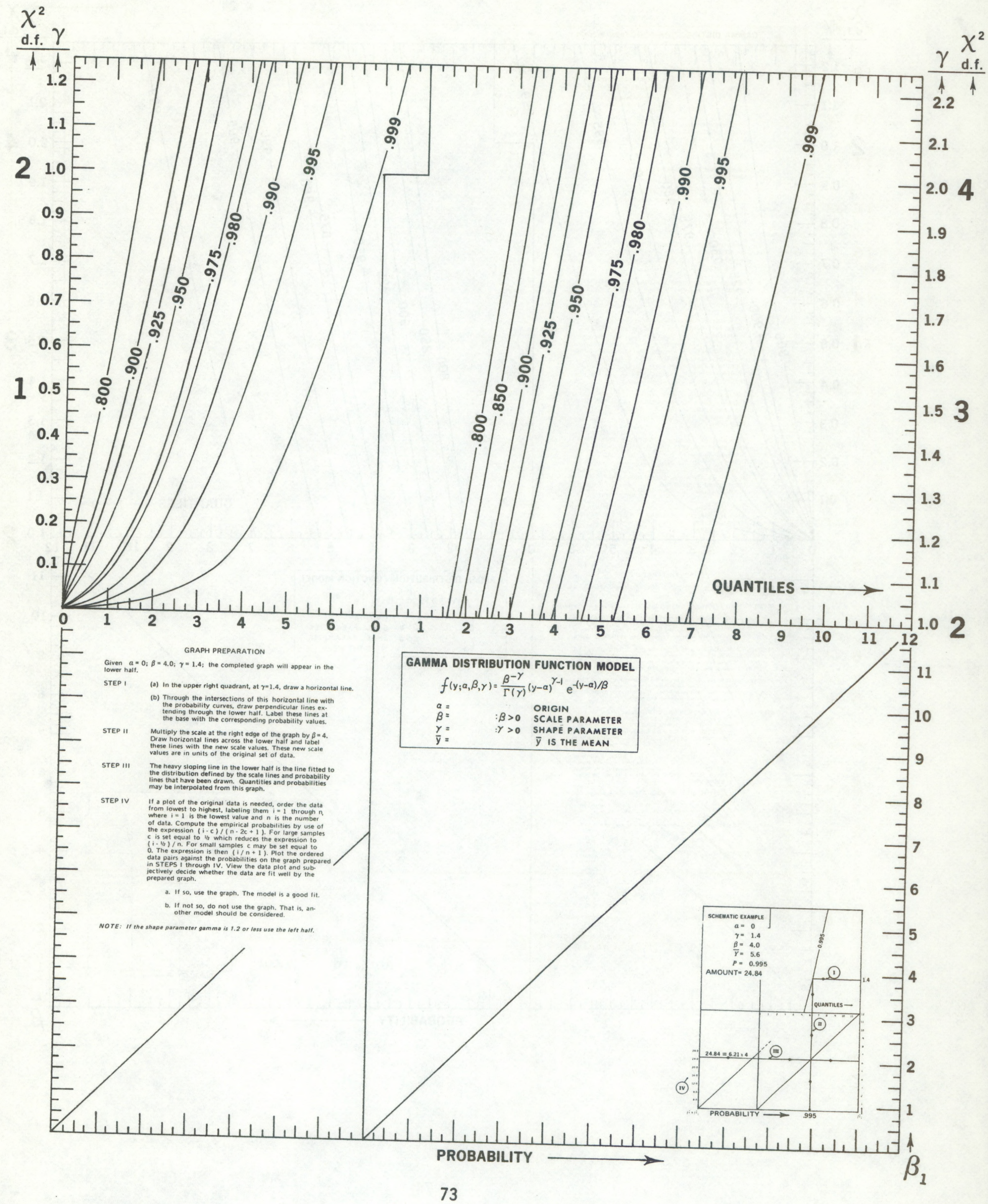
d - Label the intersections of the vertical line and the sloping β lines with the corresponding β values. This is the scale for the graph in terms of the original sample units.
e - Through the intersections obtained in d, draw horizontal lines across the chart and mark the ends with the appropriate scale values. These are the ordinate values in units of the original data.
STEP III - Solid curves slope upward to the right from the lower left-hand corner marking probabilities. Draw a horizontal line from left to right through the gamma (γ) value obtained from the vertical scale column marked γ. Through the intersections of this horizontal line and the solid curves, draw vertical lines and label them at the base with the appropriate probability values. These are the abscissa values in terms of probability.
STEP IV - The heavy sloping line rising from the left to the right is the line of best fit to the data. Quantities and probabilities may be interpolated from this graph composed of the ordinate, abscissa, and the line of best fit.
STEP V - If a plot of the original data is needed for comparison, order the data and compute the empirical probabilities by use of the expression (i-c)/(n-2c+1). Usually, c is set equal to 0.5, though for small n, 0 may be preferable where n is the number of data and i is the order of the data from 1 to n. Plot the ordered data against the probabilities on the graph prepared in STEPS I through III.
View the data and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The gamma model is a good fit.
b. If not so, do not use the graph. The gamma model is not a good fit, and another model should be considered.
γ
χ² d.f.
SCALE UNITS;
β=25
β=14
β=5
β=2
0.999 .001
.995 .005
.99 .01
.95 .05
.90 .10
20 21 25 33⅓ 50 100
PROBABILITY
PROBABILITY



A
GAMMA DISTRIBUTION FUNCTION MODEL
$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$
$\alpha = 0$ ORIGIN
$\beta =$: $\beta > 0$ SCALE PARAMETER
$\gamma =$: $\gamma > 0$ SHAPE PARAMETER
$\bar{y} =$ $\bar{y}$ IS THE MEAN
GRAPH PREPARATION
STEP I - Write down in the spaces provided above the sample values of the origin, scale and shape parameters of the gamma distribution. These are respectively α, β, γ (alpha, beta, and gamma). Ordinarily, α equals zero.
STEP IIa - Dashed sloping β lines rise from the lower right-hand corner upward and to the left.
b - Locate the β line corresponding to the sample β value. Interpolate when necessary.
c - Through the intersection of the sample β line and the heavy horizontal line near the bottom of the graph, draw a vertical line from bottom to top.
d - Label the intersections of the vertical line and the sloping β lines with the corresponding β values. This is the scale for the graph in terms of the original sample units.
e - Through the intersections obtained in d, draw horizontal lines across the chart and mark the ends with the appropriate scale values. These are the ordinate values in units of the original data.
STEP III - Solid curves slope upward to the right from the lower left-hand corner marking probabilities. Draw a horizontal line from left to right through the gamma (γ) value obtained from the vertical scale column marked γ. Through the intersections of this horizontal line and the solid curves, draw vertical lines and label them at the base with the appropriate probability values. These are the abscissa values in terms of probability.
STEP IV - The heavy sloping line rising from the left to the right is the line of best fit to the data. Quantities and probabilities may be interpolated from this graph composed of the ordinate, abscissa, and the line of best fit.
STEP V - If a plot of the original data is needed for comparison, order the data and compute the empirical probabilities by use of the expression (i-c)/(n-2c+1). Usually, c is set equal to 0.5, though for small n, 0 may be preferable where n is the number of data and i is the order of the data from 1 to n. Plot the ordered data against the probabilities on the graph prepared in STEPS I through III.
View the data and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The gamma model is a good fit.
b. If not so, do not use the graph. The gamma model is not a good fit, and another model should be considered.
γ
χ^2 d.f.
SCALE UNITS;
β=25
β=14
β=5
β=2
0.999 .001
.995 .005
.99 .01
.95 .05
.90 .10
PROBABILITY
PROBABILITY



GRAPH PREPARATION

Given $\alpha = 0$; $\beta = 4.0$; $\gamma = 1.4$; the completed graph will appear in the lower half.

STEP I

(a) In the upper right quadrant, at $\gamma = 1.4$, draw a horizontal line.

(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower half. Label these lines at the base with the corresponding probability values.

STEP II

Multiply the scale at the right edge of the graph by $\beta = 4$. Draw horizontal lines across the lower half and label these lines with the new scale values. These new scale values are in units of the original set of data.

STEP III

The heavy sloping line in the lower half is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.

STEP IV

If a plot of the original data is needed, order the data from lowest to highest, labeling them $i = 1$ through n, where $i = 1$ is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression $(i - c)/(n - 2c + 1)$. For large samples c is set equal to ½ which reduces the expression to $(i - ½)/n$. For small samples c may be set equal to 0. The expression is then $(i/n + 1)$. Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.

a. If so, use the graph. The model is a good fit.

b. If not so, do not use the graph. That is, another model should be considered.

***NOTE:** If the shape parameter gamma is 1.2 or less use the left half.*

GAMMA DISTRIBUTION FUNCTION MODEL

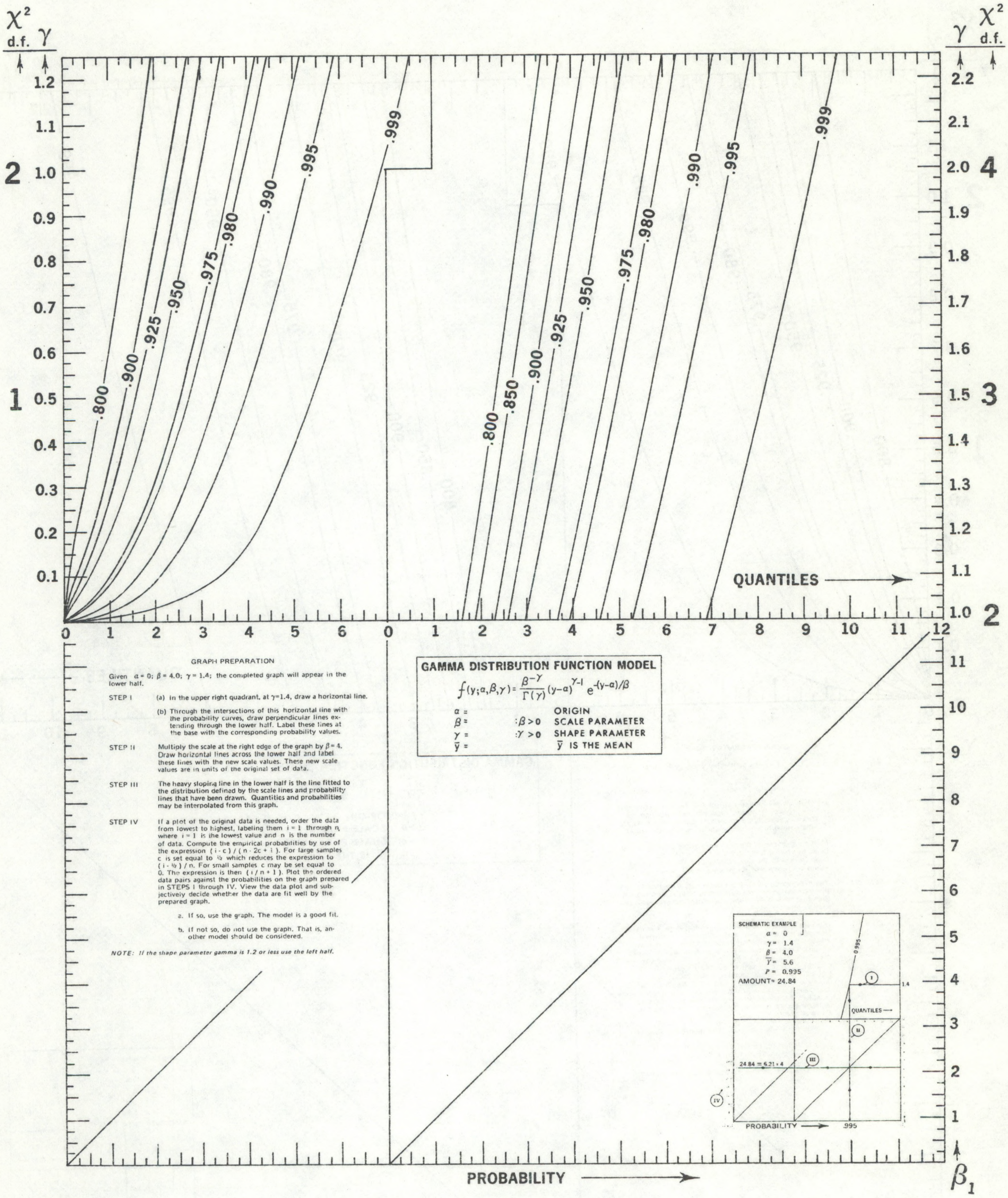
$$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1}e^{-(y-\alpha)/\beta}$$

α = ORIGIN

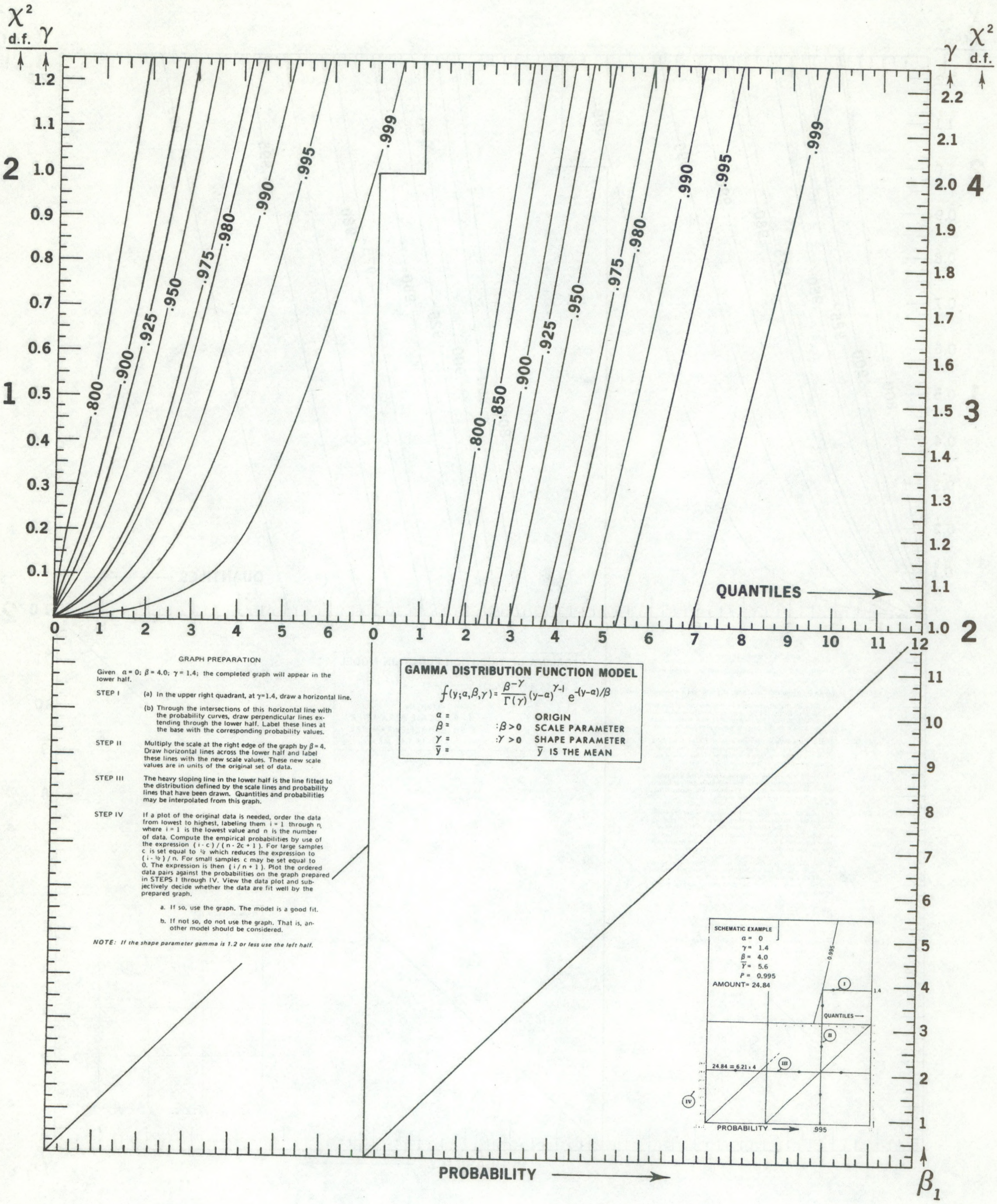
β = $:\beta > 0$ SCALE PARAMETER

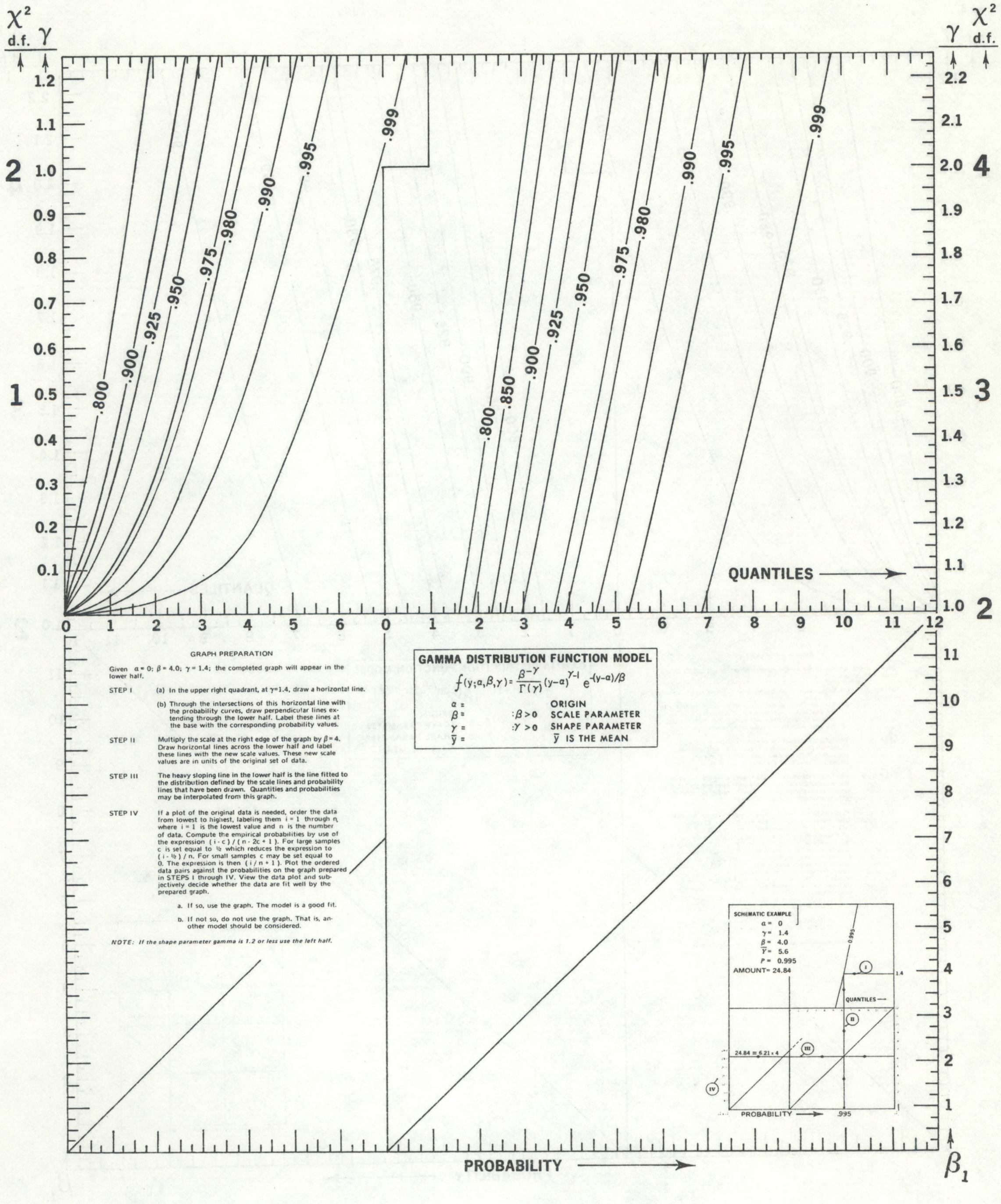
γ = $:\gamma > 0$ SHAPE PARAMETER

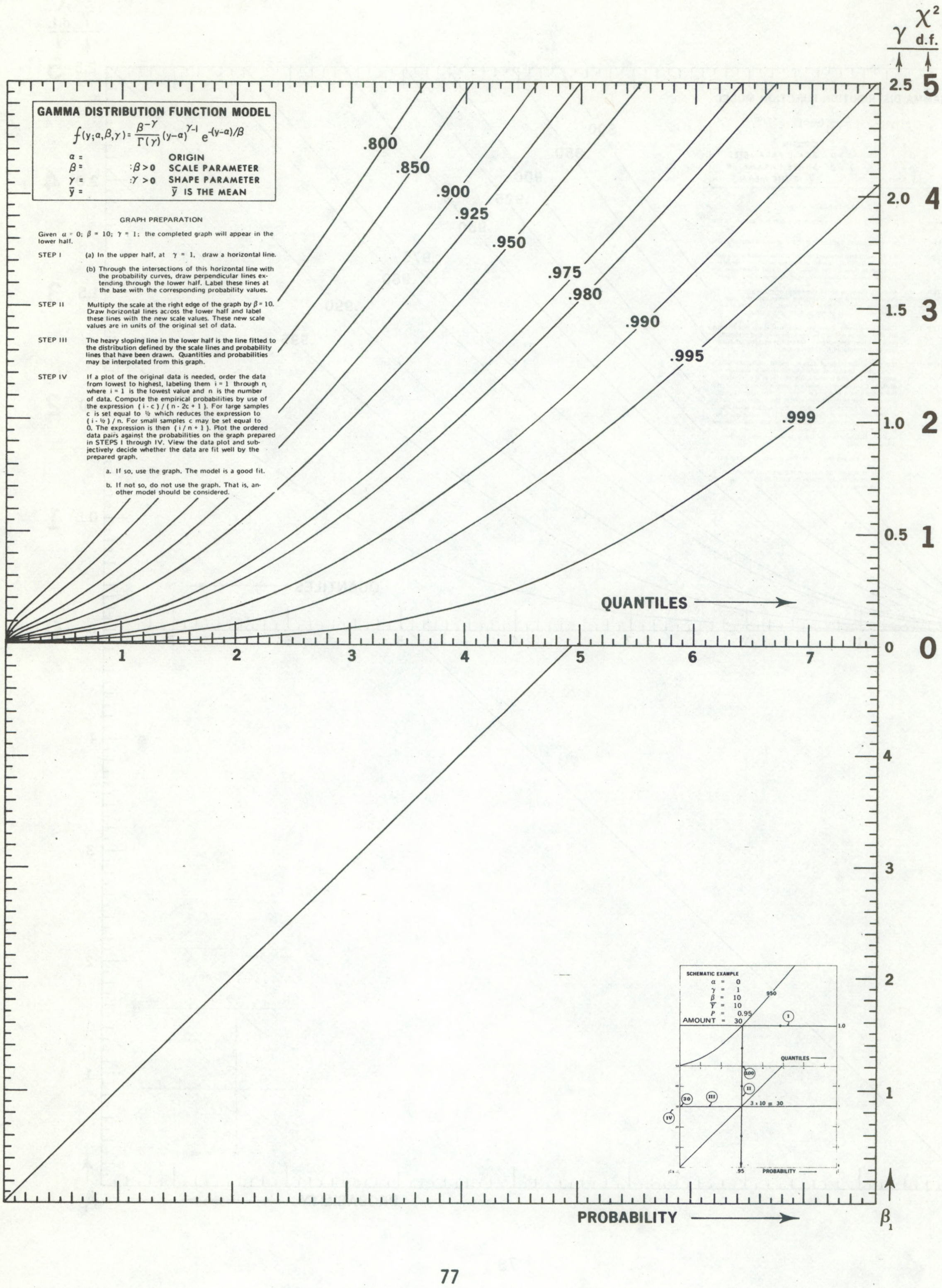
$\bar{y}$ = $\bar{y}$ IS THE MEAN

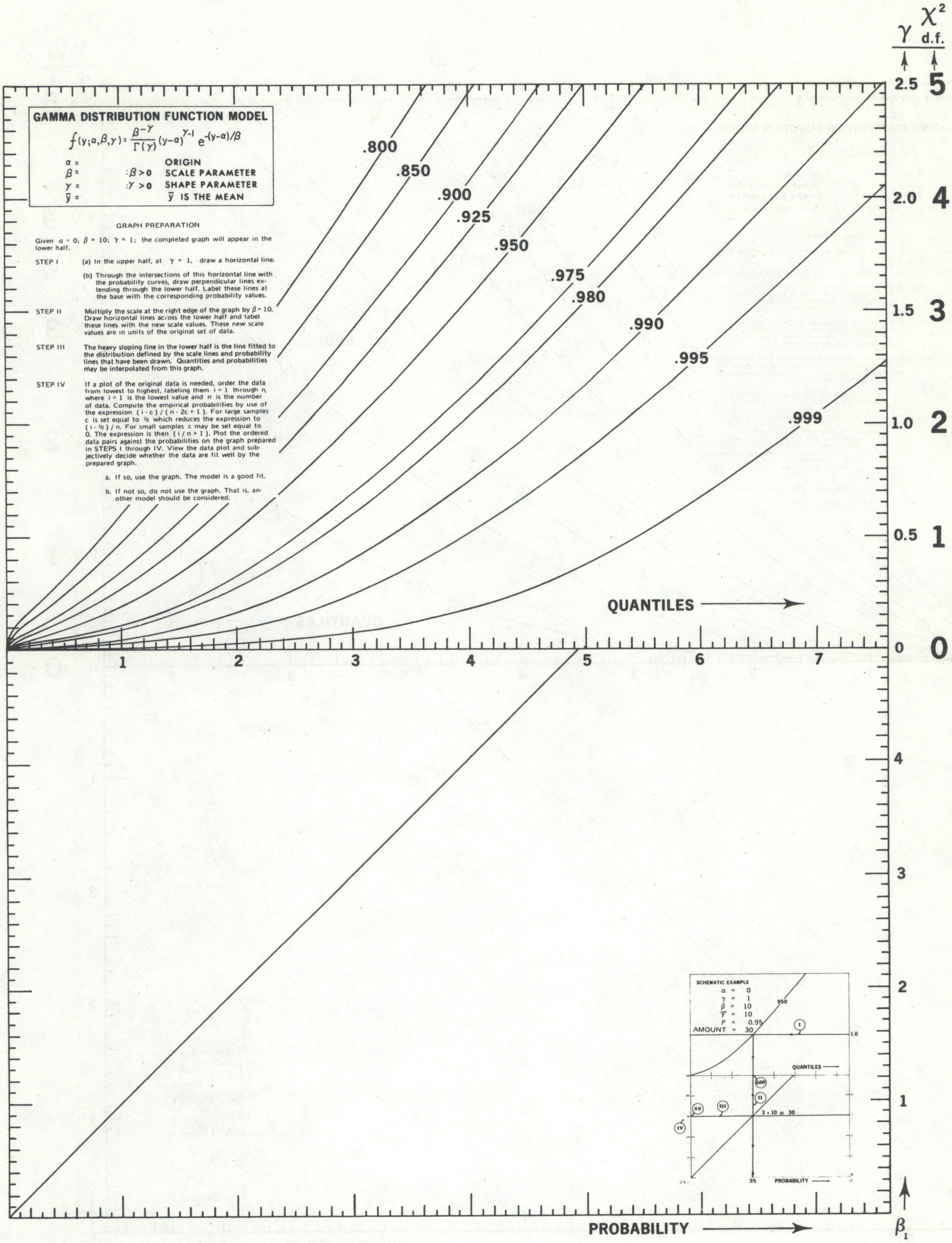


χ^2 d.f.
γ
1.2
1.1
2 1.0
0.9
0.8
0.7
0.6
1 0.5
0.4
0.3
0.2
0.1
.800
.900
.925
.950
.975
.980
.990
.995
.999
.850
2.2
2.1
2.0 4
1.9
1.8
1.7
1.6
1.5 3
1.4
1.3
1.2
1.1
1.0 2
QUANTILES →
0 1 2 3 4 5 6
0 1 2 3 4 5 6 7 8 9 10 11 12
11
10
9
8
7
6
5
4
3
2
1
β_1
PROBABILITY →
GAMMA DISTRIBUTION FUNCTION MODEL
$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1}\,e^{-(y-\alpha)/\beta}$
α = ORIGIN
β = : $\beta > 0$ SCALE PARAMETER
γ = : $\gamma > 0$ SHAPE PARAMETER
$\bar{y}$ = $\bar{y}$ IS THE MEAN
GRAPH PREPARATION
Given $\alpha = 0$; $\beta = 4.0$; $\gamma = 1.4$; the completed graph will appear in the lower half.
STEP I (a) In the upper right quadrant, at γ=1.4, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower half. Label these lines at the base with the corresponding probability values.
STEP II Multiply the scale at the right edge of the graph by $\beta = 4$. Draw horizontal lines across the lower half and label these lines with the new scale values. These new scale values are in units of the original set of data.
STEP III The heavy sloping line in the lower half is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
STEP IV If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i - c) / (n - 2c + 1). For large samples c is set equal to ½ which reduces the expression to (i - ½) / n. For small samples c may be set equal to 0. The expression is then (i / n + 1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The model is a good fit.
b. If not so, do not use the graph. That is, another model should be considered.
NOTE: If the shape parameter gamma is 1.2 or less use the left half.
SCHEMATIC EXAMPLE
α = 0
γ = 1.4
β = 4.0
$\bar{Y}$ = 5.6
P = 0.995
AMOUNT= 24.84
0.995
I
1.4
QUANTILES →
II
III
24.84 = 6.21 × 4
IV
PROBABILITY →
.995

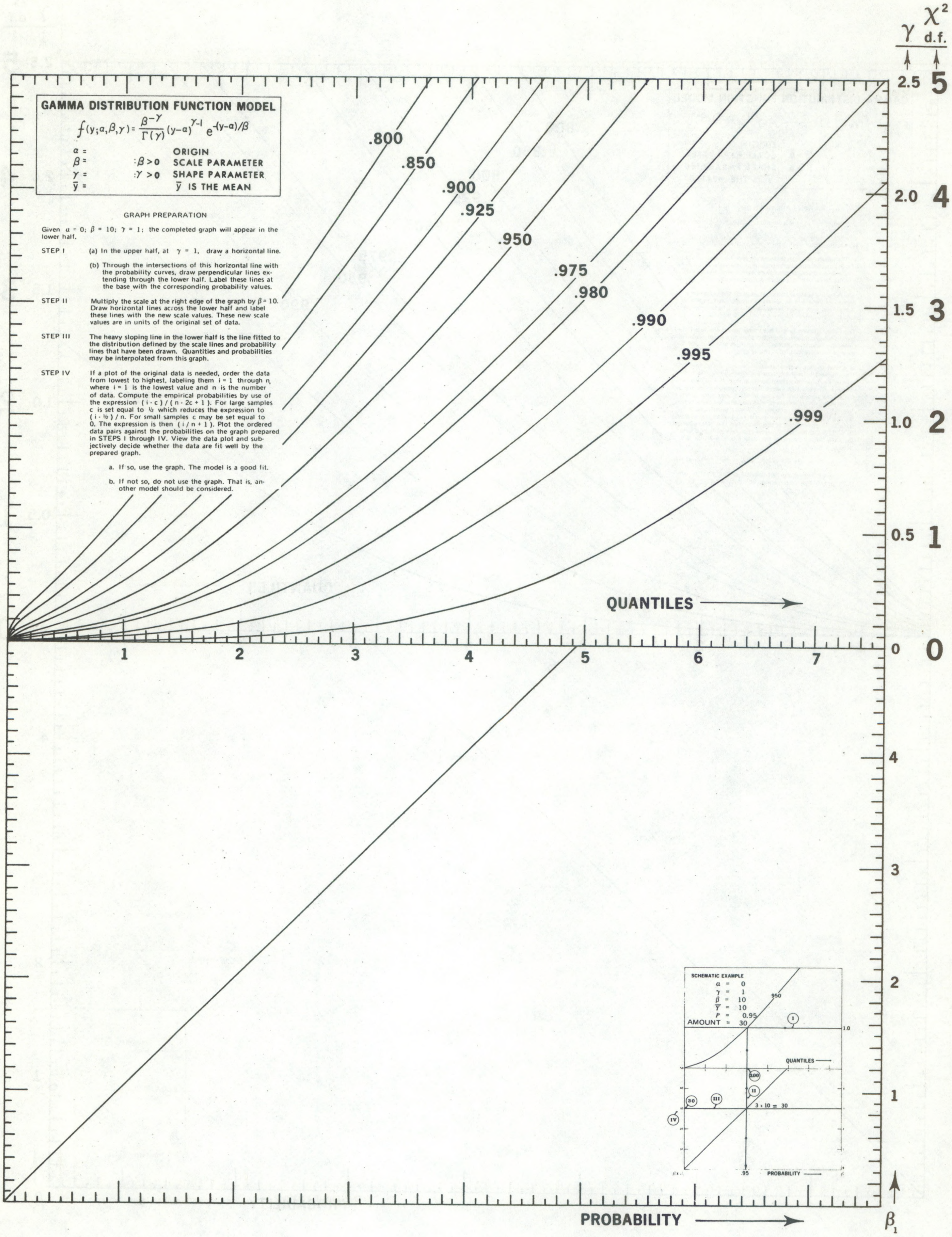

χ²
d.f.
γ
1.2
1.1
2 1.0
0.9
0.8
0.7
0.6
1 0.5
0.4
0.3
0.2
0.1
.800
.900
.925
.950
.975
.980
.990
.995
.999
.800
.850
.900
.925
.950
.975
.980
.990
.995
.999
γ
χ²
d.f.
2.2
2.1
2.0 4
1.9
1.8
1.7
1.6
1.5 3
1.4
1.3
1.2
1.1
1.0 2
QUANTILES
0 1 2 3 4 5 6
0 1 2 3 4 5 6 7 8 9 10 11 12
11
10
9
8
7
6
5
4
3
2
1
β1
PROBABILITY
GRAPH PREPARATION
Given α = 0; β = 4.0; γ = 1.4; the completed graph will appear in the lower half.
STEP I (a) In the upper right quadrant, at γ=1.4, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower half. Label these lines at the base with the corresponding probability values.
STEP II Multiply the scale at the right edge of the graph by β = 4. Draw horizontal lines across the lower half and label these lines with the new scale values. These new scale values are in units of the original set of data.
STEP III The heavy sloping line in the lower half is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
STEP IV If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i - c) / (n - 2c + 1). For large samples c is set equal to ½ which reduces the expression to (i - ½) / n. For small samples c may be set equal to 0. The expression is then (i / n + 1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The model is a good fit.
b. If not so, do not use the graph. That is, another model should be considered.
NOTE: If the shape parameter gamma is 1.2 or less use the left half.
GAMMA DISTRIBUTION FUNCTION MODEL
$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$
α = ORIGIN
β = :β > 0 SCALE PARAMETER
γ = :γ > 0 SHAPE PARAMETER
ȳ = ȳ IS THE MEAN
SCHEMATIC EXAMPLE
α = 0
γ = 1.4
β = 4.0
Ȳ = 5.6
P = 0.995
AMOUNT= 24.84
0.995
I
1.4
QUANTILES
II
III
24.84 = 6.21 x 4
IV
PROBABILITY
.995

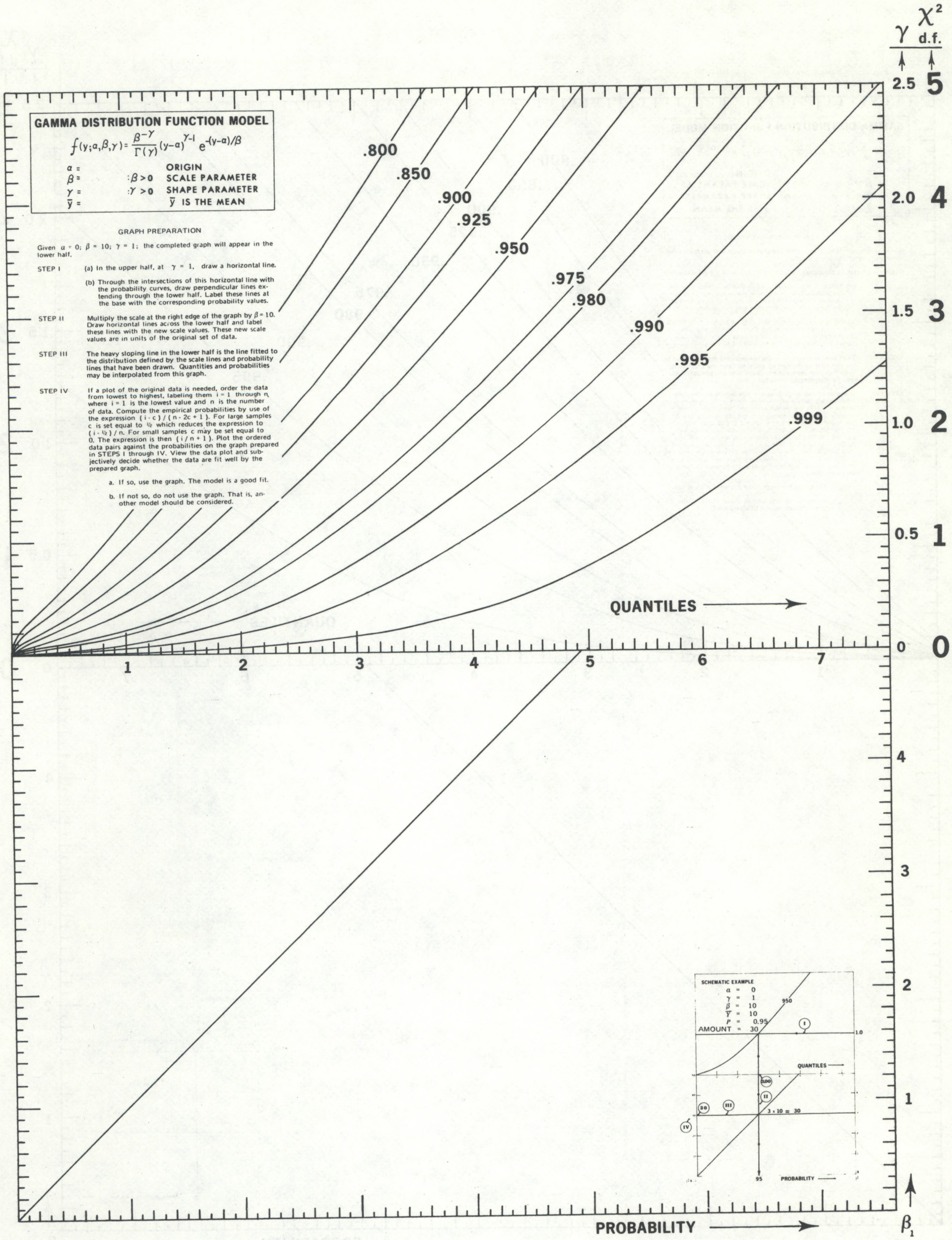

χ^2
d.f. γ
1.2
1.1
2 1.0
0.9
0.8
0.7
0.6
1 0.5
0.4
0.3
0.2
0.1
0
.800
.900
.925
.950
.975
.980
.990
.995
.999
0 1 2 3 4 5 6
γ χ^2
d.f.
2.2
2.1
2.0 4
1.9
1.8
1.7
1.6
1.5 3
1.4
1.3
1.2
1.1
1.0 2
QUANTILES
.800
.850
.900
.925
.950
.975
.980
.990
.995
.999
0 1 2 3 4 5 6 7 8 9 10 11 12
GRAPH PREPARATION
Given $\alpha = 0$; $\beta = 4.0$; $\gamma = 1.4$; the completed graph will appear in the lower half.
STEP I (a) In the upper right quadrant, at $\gamma = 1.4$, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower half. Label these lines at the base with the corresponding probability values.
STEP II Multiply the scale at the right edge of the graph by $\beta = 4$. Draw horizontal lines across the lower half and label these lines with the new scale values. These new scale values are in units of the original set of data.
STEP III The heavy sloping line in the lower half is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
STEP IV If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i - c) / (n - 2c + 1). For large samples c is set equal to ½ which reduces the expression to (i - ½) / n. For small samples c may be set equal to 0. The expression is then (i / n + 1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The model is a good fit.
b. If not so, do not use the graph. That is, another model should be considered.
NOTE: If the shape parameter gamma is 1.2 or less use the left half.
GAMMA DISTRIBUTION FUNCTION MODEL
$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$
α = ORIGIN
β = : $\beta > 0$ SCALE PARAMETER
γ = : $\gamma > 0$ SHAPE PARAMETER
$\bar{y}$ = $\bar{y}$ IS THE MEAN
11
10
9
8
7
6
5
4
3
2
1
SCHEMATIC EXAMPLE
α = 0
γ = 1.4
β = 4.0
$\bar{Y}$ = 5.6
P = 0.995
AMOUNT= 24.84
0.995
I
1.4
QUANTILES
II
III
24.84 = 6.21 x 4
IV
PROBABILITY
.995
PROBABILITY
β_1

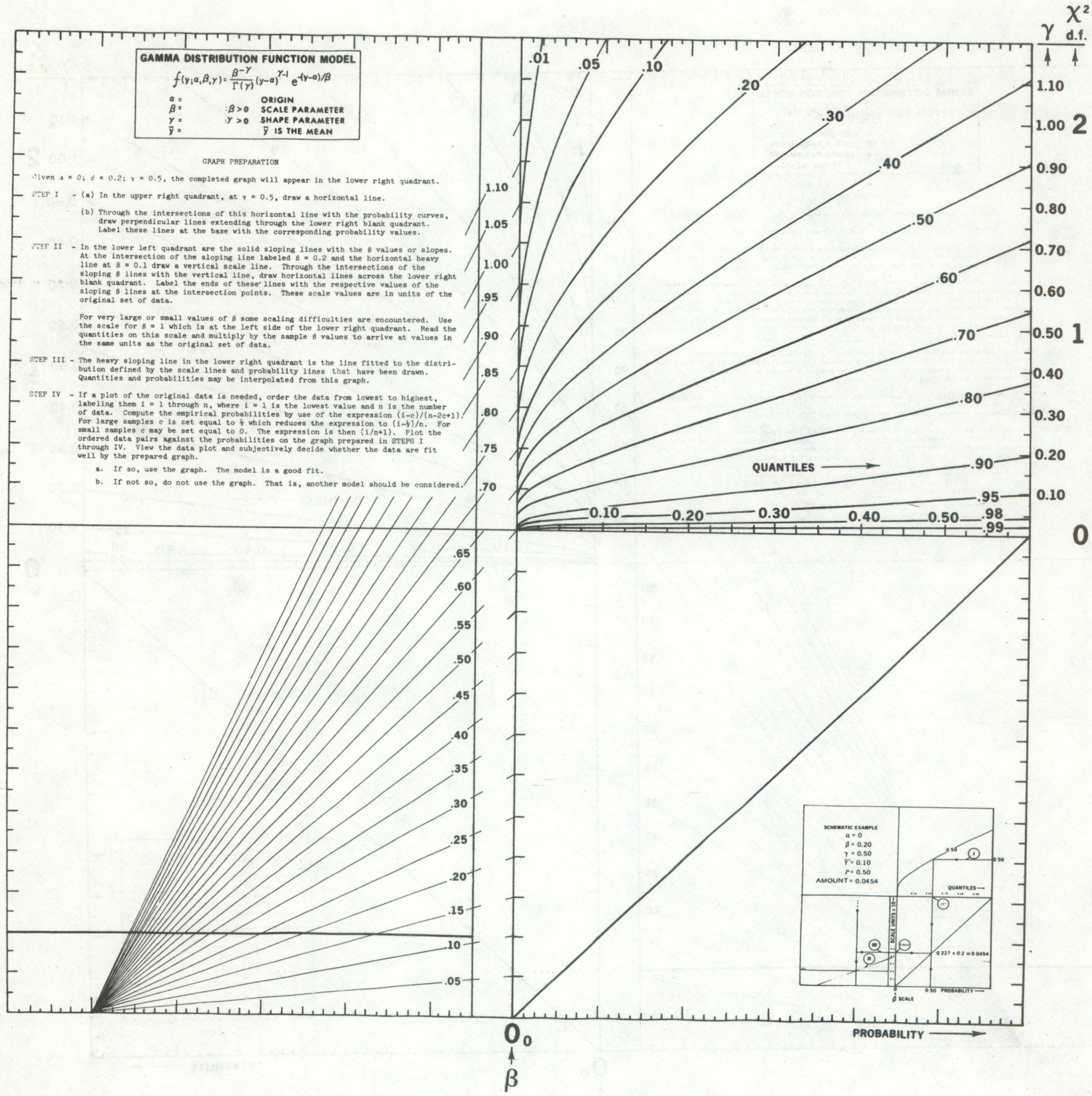

GAMMA DISTRIBUTION FUNCTION MODEL
$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$
α = ORIGIN
β = : $\beta > 0$ SCALE PARAMETER
γ = : $\gamma > 0$ SHAPE PARAMETER
$\bar{y}$ = $\bar{y}$ IS THE MEAN
GRAPH PREPARATION
Given $\alpha = 0$; $\beta = 10$; $\gamma = 1$: the completed graph will appear in the lower half.
STEP I (a) In the upper half, at $\gamma = 1$, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower half. Label these lines at the base with the corresponding probability values.
STEP II Multiply the scale at the right edge of the graph by $\beta = 10$. Draw horizontal lines across the lower half and label these lines with the new scale values. These new scale values are in units of the original set of data.
STEP III The heavy sloping line in the lower half is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
STEP IV If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i - c) / (n - 2c + 1). For large samples c is set equal to ½ which reduces the expression to (i - ½) / n. For small samples c may be set equal to 0. The expression is then (i / n + 1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The model is a good fit.
b. If not so, do not use the graph. That is, another model should be considered.
.800
.850
.900
.925
.950
.975
.980
.990
.995
.999
γ
χ^2
d.f.
2.5 5
2.0 4
1.5 3
1.0 2
0.5 1
0 0
QUANTILES
1
2
3
4
5
6
7
4
3
2
1
SCHEMATIC EXAMPLE
α = 0
γ = 1
β = 10
$\bar{Y}$ = 10
P = 0.95
AMOUNT = 30
950
I
1.0
QUANTILES
3.00
II
30
III
3 x 10 = 30
IV
.95
PROBABILITY
PROBABILITY
β_1



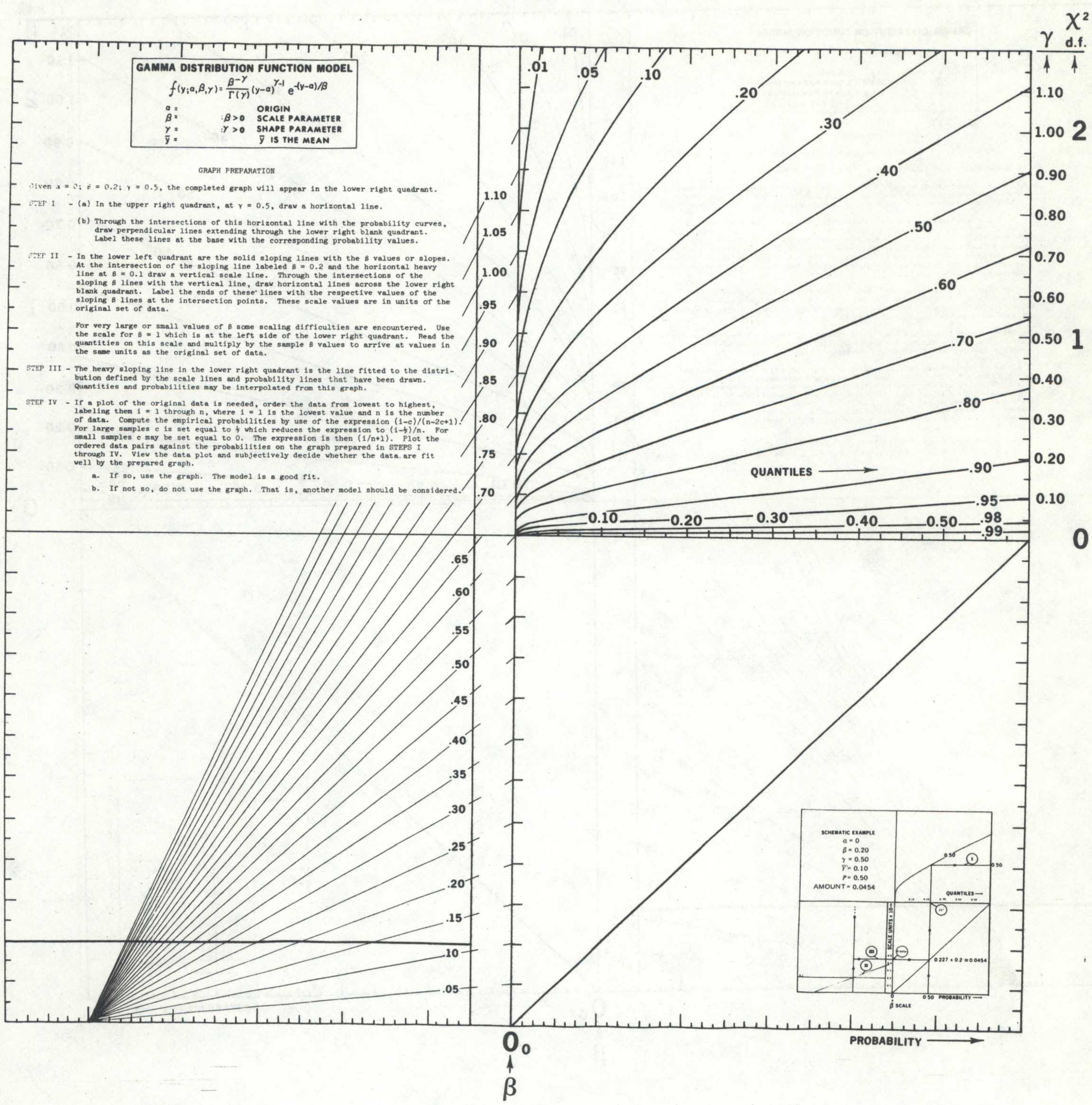
GAMMA DISTRIBUTION FUNCTION MODEL
$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$
α = ORIGIN
β = :β > 0 SCALE PARAMETER
γ = :γ > 0 SHAPE PARAMETER
ȳ = ȳ IS THE MEAN
GRAPH PREPARATION
Given α = 0; β = 10; γ = 1; the completed graph will appear in the lower half.
STEP I (a) In the upper half, at γ = 1, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower half. Label these lines at the base with the corresponding probability values.
STEP II Multiply the scale at the right edge of the graph by β = 10. Draw horizontal lines across the lower half and label these lines with the new scale values. These new scale values are in units of the original set of data.
STEP III The heavy sloping line in the lower half is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
STEP IV If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i - c) / (n - 2c + 1). For large samples c is set equal to ½ which reduces the expression to (i - ½) / n. For small samples c may be set equal to 0. The expression is then (i / n + 1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The model is a good fit.
b. If not so, do not use the graph. That is, another model should be considered.
.800
.850
.900
.925
.950
.975
.980
.990
.995
.999
γ
χ² d.f.
2.5 5
2.0 4
1.5 3
1.0 2
0.5 1
0 0
QUANTILES
1
2
3
4
5
6
7
4
3
2
1
PROBABILITY
β₁
SCHEMATIC EXAMPLE
α = 0
γ = 1
β = 10
ȳ = 10
P = 0.95
AMOUNT = 30
QUANTILES
3 x 10 = 30
PROBABILITY

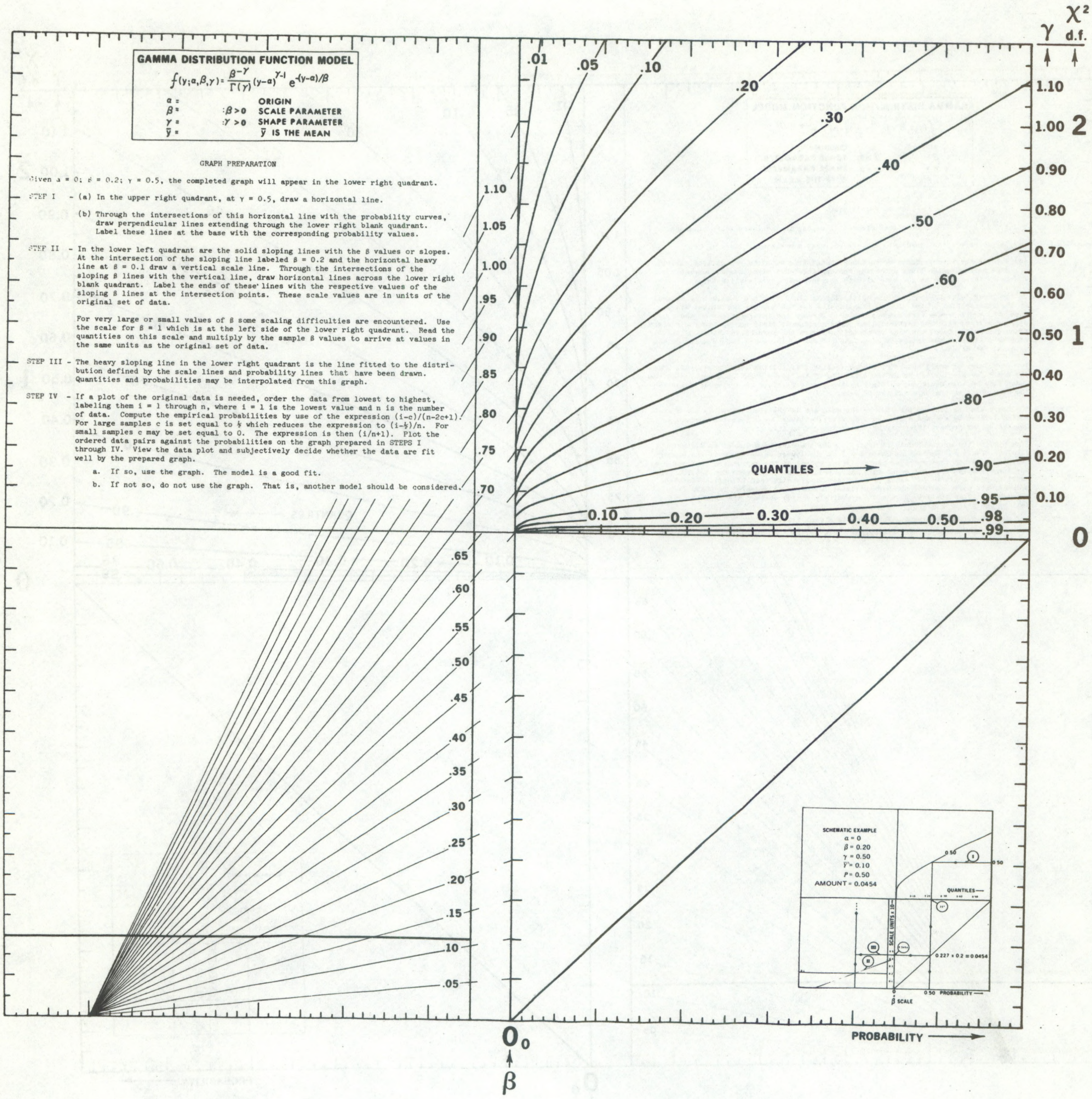

GAMMA DISTRIBUTION FUNCTION MODEL
$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$
α = ORIGIN
β = : $\beta > 0$ SCALE PARAMETER
γ = : $\gamma > 0$ SHAPE PARAMETER
$\bar{y}$ = $\bar{y}$ IS THE MEAN
GRAPH PREPARATION
Given $\alpha = 0$; $\beta = 10$; $\gamma = 1$; the completed graph will appear in the lower half.
STEP I (a) In the upper half, at $\gamma = 1$, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower half. Label these lines at the base with the corresponding probability values.
STEP II Multiply the scale at the right edge of the graph by $\beta = 10$. Draw horizontal lines across the lower half and label these lines with the new scale values. These new scale values are in units of the original set of data.
STEP III The heavy sloping line in the lower half is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
STEP IV If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i - c) / (n - 2c + 1). For large samples c is set equal to ½ which reduces the expression to (i - ½) / n. For small samples c may be set equal to 0. The expression is then (i / n + 1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The model is a good fit.
b. If not so, do not use the graph. That is, another model should be considered.
.800
.850
.900
.925
.950
.975
.980
.990
.995
.999
γ
χ^2
d.f.
2.5 5
2.0 4
1.5 3
1.0 2
0.5 1
0 0
QUANTILES
1
2
3
4
5
6
7
4
3
2
1
PROBABILITY
β_1
SCHEMATIC EXAMPLE
α = 0
γ = 1
β = 10
$\bar{Y}$ = 10
P = 0.95
AMOUNT = 30
.950
QUANTILES
3 x 10 = 30
.95
PROBABILITY


GAMMA DISTRIBUTION FUNCTION MODEL
f(y;α,β,γ) = β^−γ / Γ(γ) (y−α)^γ−1 e^−(y−α)/β
α = ORIGIN
β = :β > 0 SCALE PARAMETER
γ = :γ > 0 SHAPE PARAMETER
ȳ = ȳ IS THE MEAN
GRAPH PREPARATION
Given α = 0; β = 10; γ = 1; the completed graph will appear in the lower half.
STEP I (a) In the upper half, at γ = 1, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower half. Label these lines at the base with the corresponding probability values.
STEP II Multiply the scale at the right edge of the graph by β = 10. Draw horizontal lines across the lower half and label these lines with the new scale values. These new scale values are in units of the original set of data.
STEP III The heavy sloping line in the lower half is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
STEP IV If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i - c) / (n - 2c + 1). For large samples c is set equal to ½ which reduces the expression to (i - ½) / n. For small samples c may be set equal to 0. The expression is then (i / n + 1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The model is a good fit.
b. If not so, do not use the graph. That is, another model should be considered.
.800
.850
.900
.925
.950
.975
.980
.990
.995
.999
γ
χ² d.f.
2.5 5
2.0 4
1.5 3
1.0 2
0.5 1
0 0
QUANTILES
1
2
3
4
5
6
7
4
3
2
1
SCHEMATIC EXAMPLE
α = 0
γ = 1
β = 10
ȳ = 10
P = 0.95
AMOUNT = 30
3 · 10 = 30
PROBABILITY
β₁

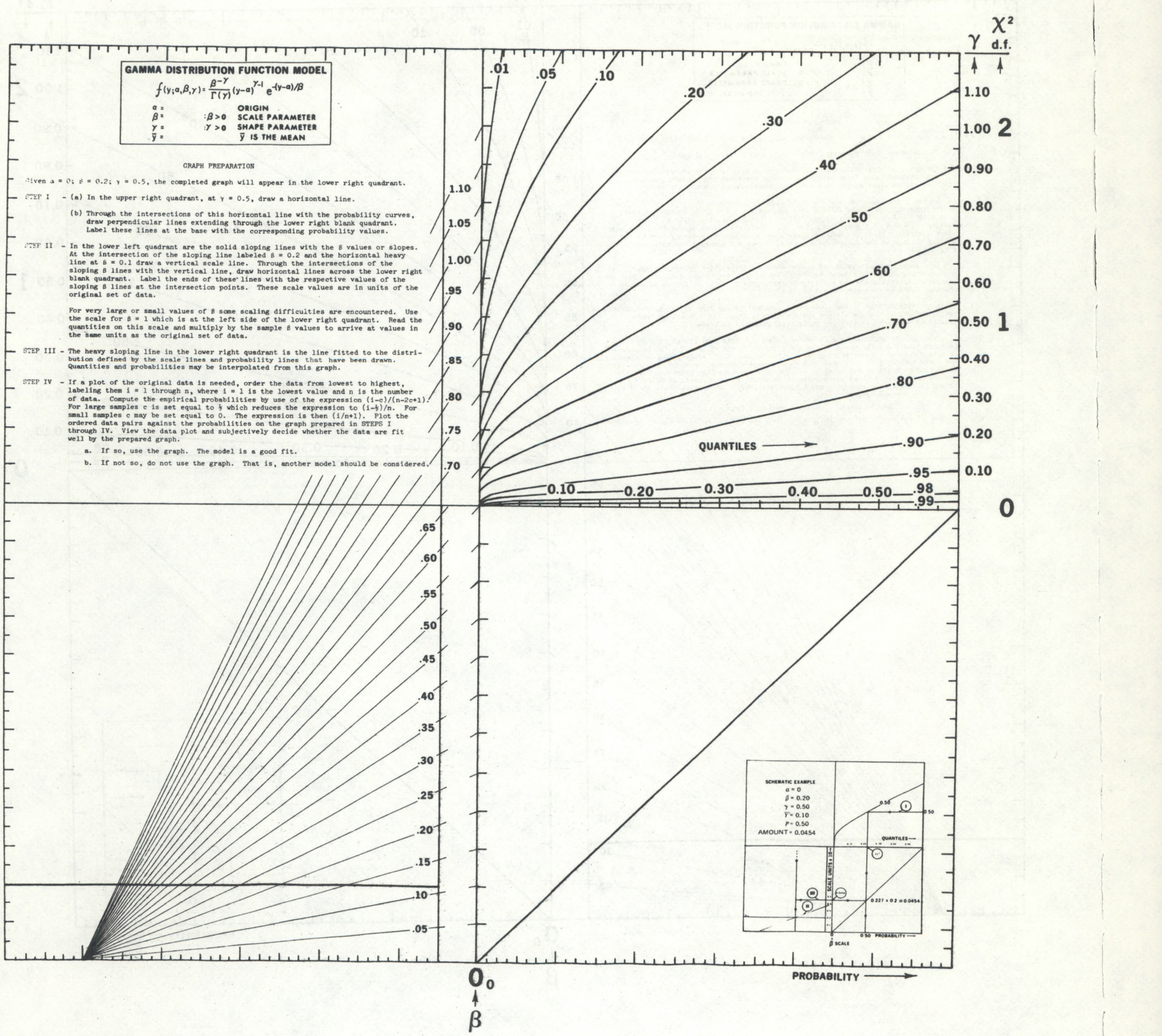


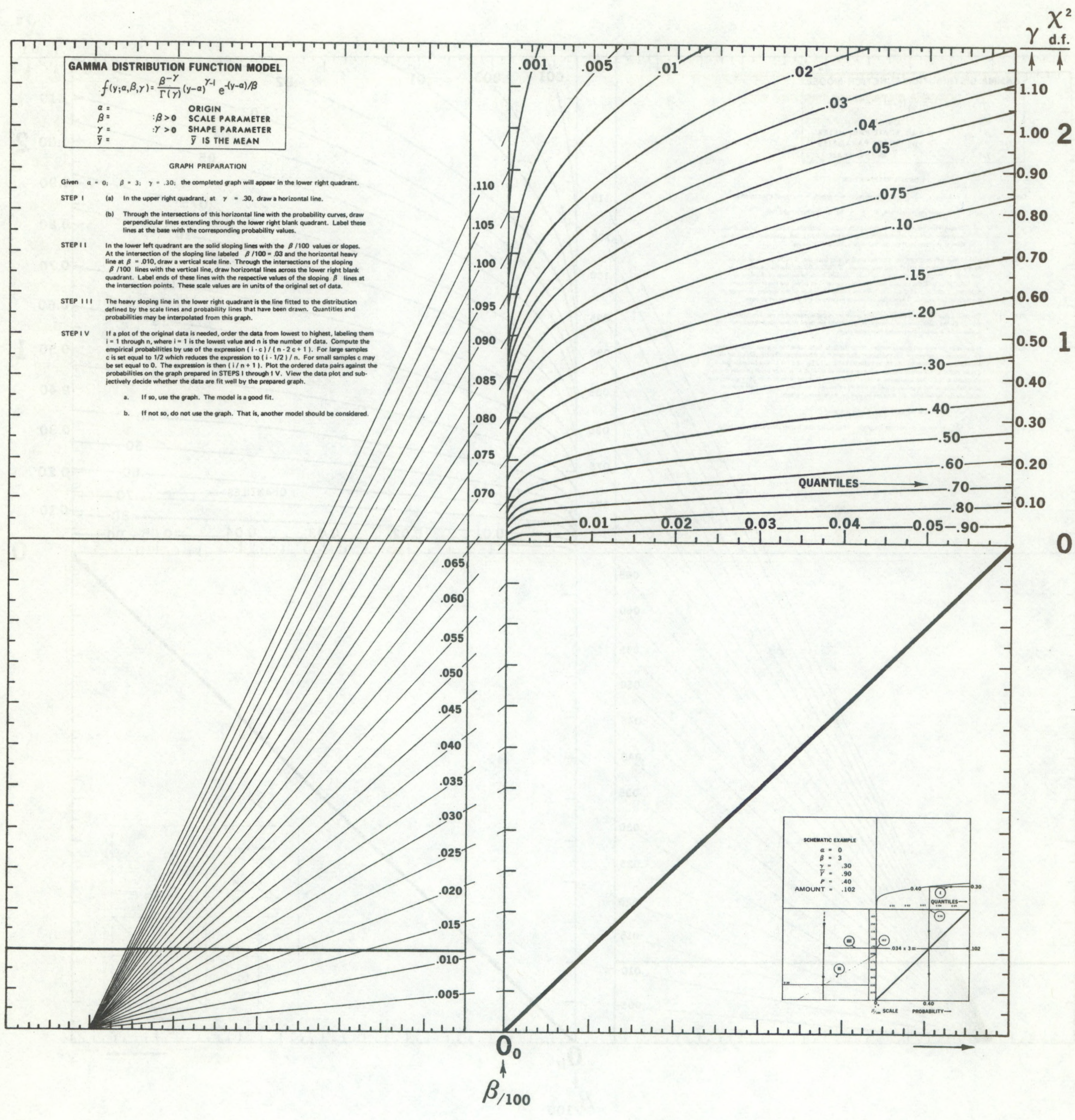
GAMMA DISTRIBUTION FUNCTION MODEL
$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$
α = ORIGIN
β = : $\beta > 0$ SCALE PARAMETER
γ = : $\gamma > 0$ SHAPE PARAMETER
$\bar{y}$ = $\bar{y}$ IS THE MEAN
GRAPH PREPARATION
Given $\alpha = 0$; $\beta = 0.2$; $\gamma = 0.5$, the completed graph will appear in the lower right quadrant.
STEP I - (a) In the upper right quadrant, at $\gamma = 0.5$, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower right blank quadrant. Label these lines at the base with the corresponding probability values.
STEP II - In the lower left quadrant are the solid sloping lines with the β values or slopes. At the intersection of the sloping line labeled $\beta = 0.2$ and the horizontal heavy line at $\beta = 0.1$ draw a vertical scale line. Through the intersections of the sloping β lines with the vertical line, draw horizontal lines across the lower right blank quadrant. Label the ends of these lines with the respective values of the sloping β lines at the intersection points. These scale values are in units of the original set of data.
For very large or small values of β some scaling difficulties are encountered. Use the scale for $\beta = 1$ which is at the left side of the lower right quadrant. Read the quantities on this scale and multiply by the sample β values to arrive at values in the same units as the original set of data.
STEP III - The heavy sloping line in the lower right quadrant is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
STEP IV - If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i-c)/(n-2c+1). For large samples c is set equal to ½ which reduces the expression to (i-½)/n. For small samples c may be set equal to 0. The expression is then (i/n+1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The model is a good fit.
b. If not so, do not use the graph. That is, another model should be considered.
.01 .05 .10 .20 .30 .40 .50 .60 .70 .80 .90 .95 .98 .99
QUANTILES
0.10 0.20 0.30 0.40 0.50
γ
χ^2 d.f.
1.10
1.00 2
0.90
0.80
0.70
0.60
0.50 1
0.40
0.30
0.20
0.10
0
1.10 1.05 1.00 .95 .90 .85 .80 .75 .70 .65 .60 .55 .50 .45 .40 .35 .30 .25 .20 .15 .10 .05
0_0
β
PROBABILITY
SCHEMATIC EXAMPLE
α = 0
β = 0.20
γ = 0.50
$\bar{Y}$ = 0.10
P = 0.50
AMOUNT = 0.0454
QUANTILES
0.227 × 0.2 = 0.0454
β SCALE
0.50 PROBABILITY

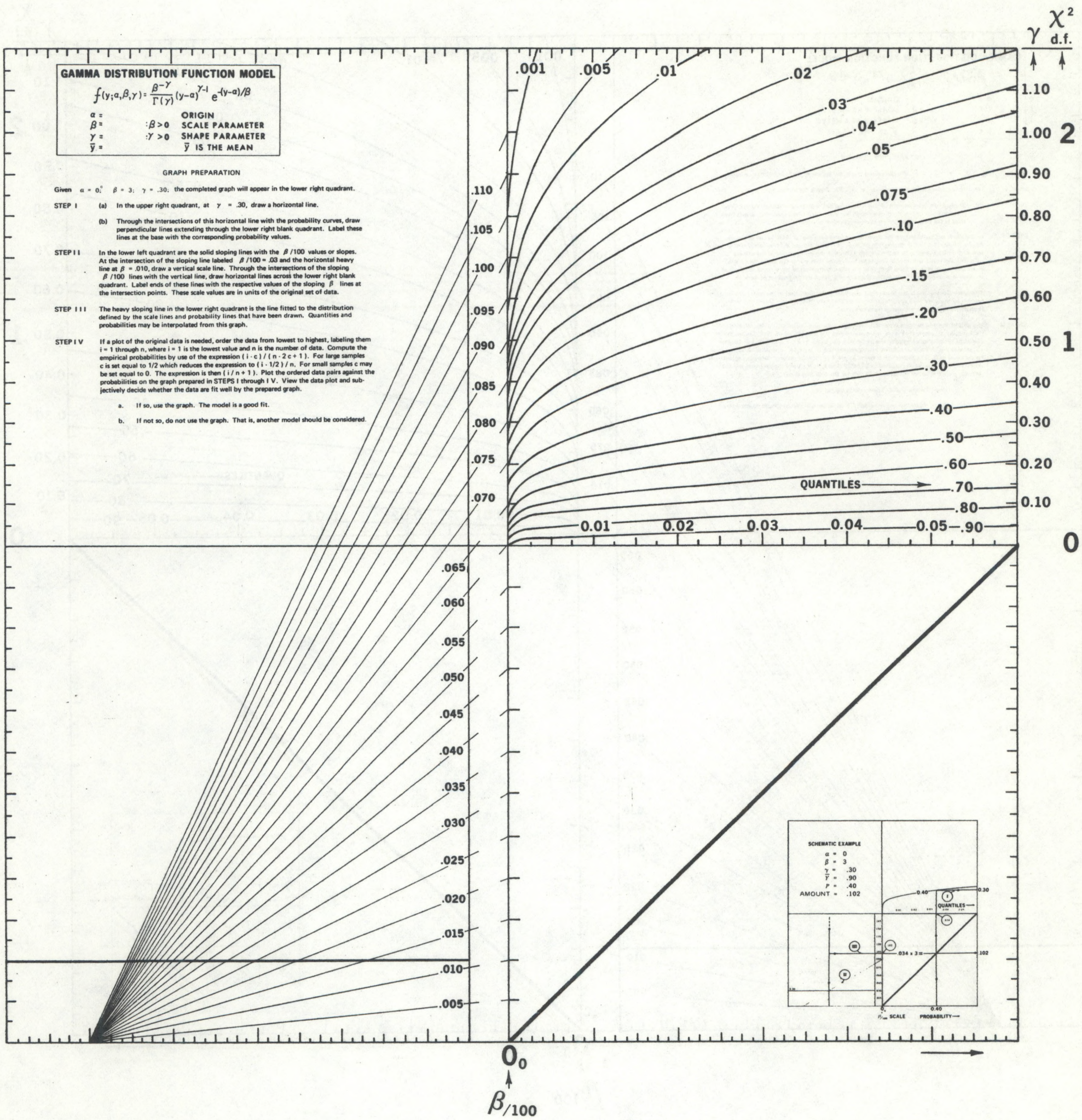

GAMMA DISTRIBUTION FUNCTION MODEL
$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$
α = ORIGIN
β = : β > 0 SCALE PARAMETER
γ = : γ > 0 SHAPE PARAMETER
ȳ = ȳ IS THE MEAN
GRAPH PREPARATION
Given α = 0; β = 0.2; γ = 0.5, the completed graph will appear in the lower right quadrant.
STEP I - (a) In the upper right quadrant, at γ = 0.5, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower right blank quadrant. Label these lines at the base with the corresponding probability values.
STEP II - In the lower left quadrant are the solid sloping lines with the β values or slopes. At the intersection of the sloping line labeled β = 0.2 and the horizontal heavy line at β = 0.1 draw a vertical scale line. Through the intersections of the sloping β lines with the vertical line, draw horizontal lines across the lower right blank quadrant. Label the ends of these lines with the respective values of the sloping β lines at the intersection points. These scale values are in units of the original set of data.
For very large or small values of β some scaling difficulties are encountered. Use the scale for β = 1 which is at the left side of the lower right quadrant. Read the quantities on this scale and multiply by the sample β values to arrive at values in the same units as the original set of data.
STEP III - The heavy sloping line in the lower right quadrant is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
STEP IV - If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i-c)/(n-2c+1). For large samples c is set equal to ½ which reduces the expression to (i-½)/n. For small samples c may be set equal to 0. The expression is then (i/n+1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The model is a good fit.
b. If not so, do not use the graph. That is, another model should be considered.
1.10
1.05
1.00
.95
.90
.85
.80
.75
.70
.65
.60
.55
.50
.45
.40
.35
.30
.25
.20
.15
.10
.05
.01
.05
.10
.20
.30
.40
.50
.60
.70
.80
.90
.95
.98
.99
QUANTILES
0.10
0.20
0.30
0.40
0.50
γ
χ²
d.f.
1.10
1.00 2
0.90
0.80
0.70
0.60
0.50 1
0.40
0.30
0.20
0.10
0
SCHEMATIC EXAMPLE
α = 0
β = 0.20
γ = 0.50
ȳ = 0.10
P = 0.50
AMOUNT = 0.0454
QUANTILES
0.227 × 0.2 = 0.0454
0.50 PROBABILITY
β SCALE
0 0
β
PROBABILITY

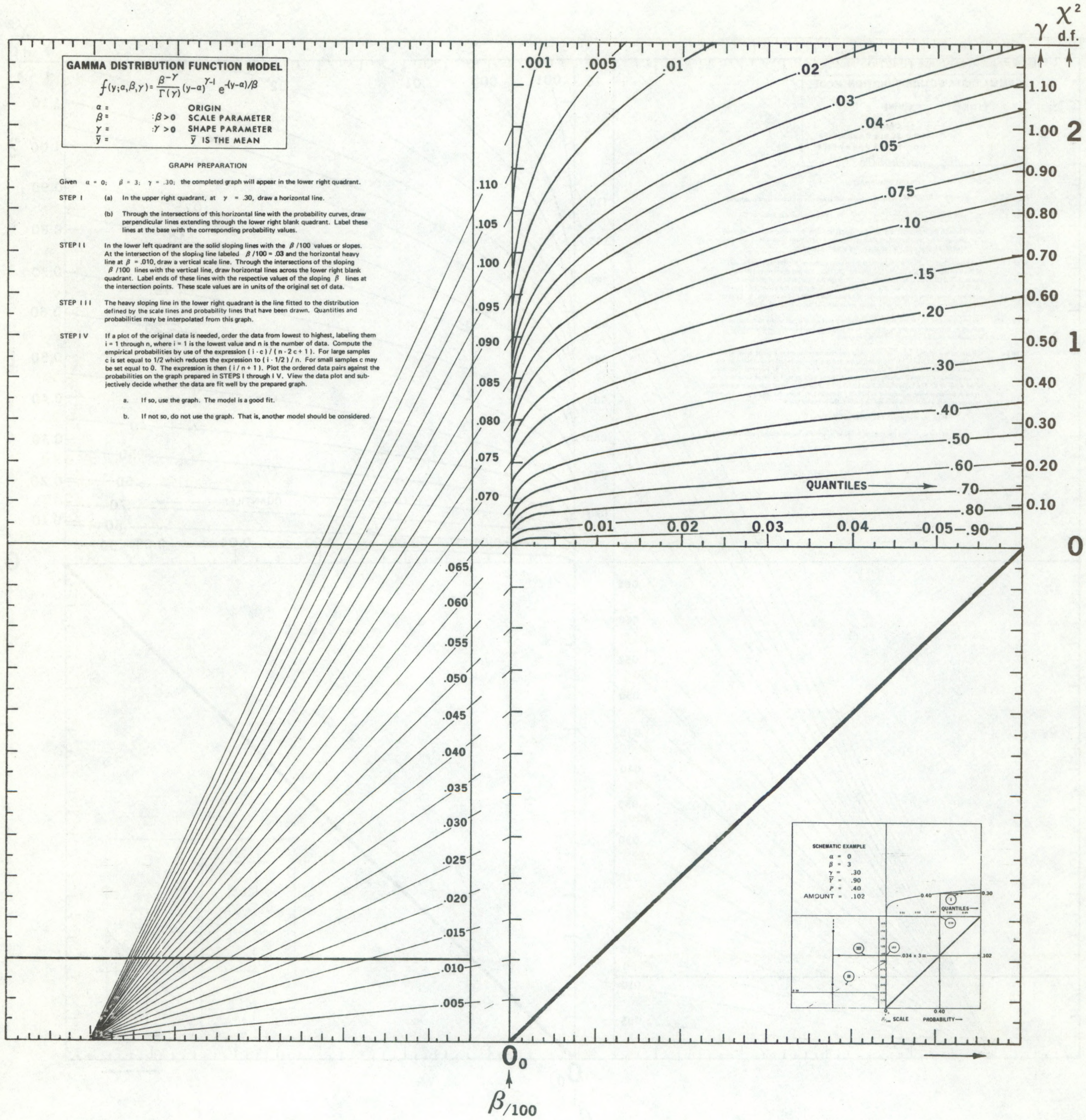


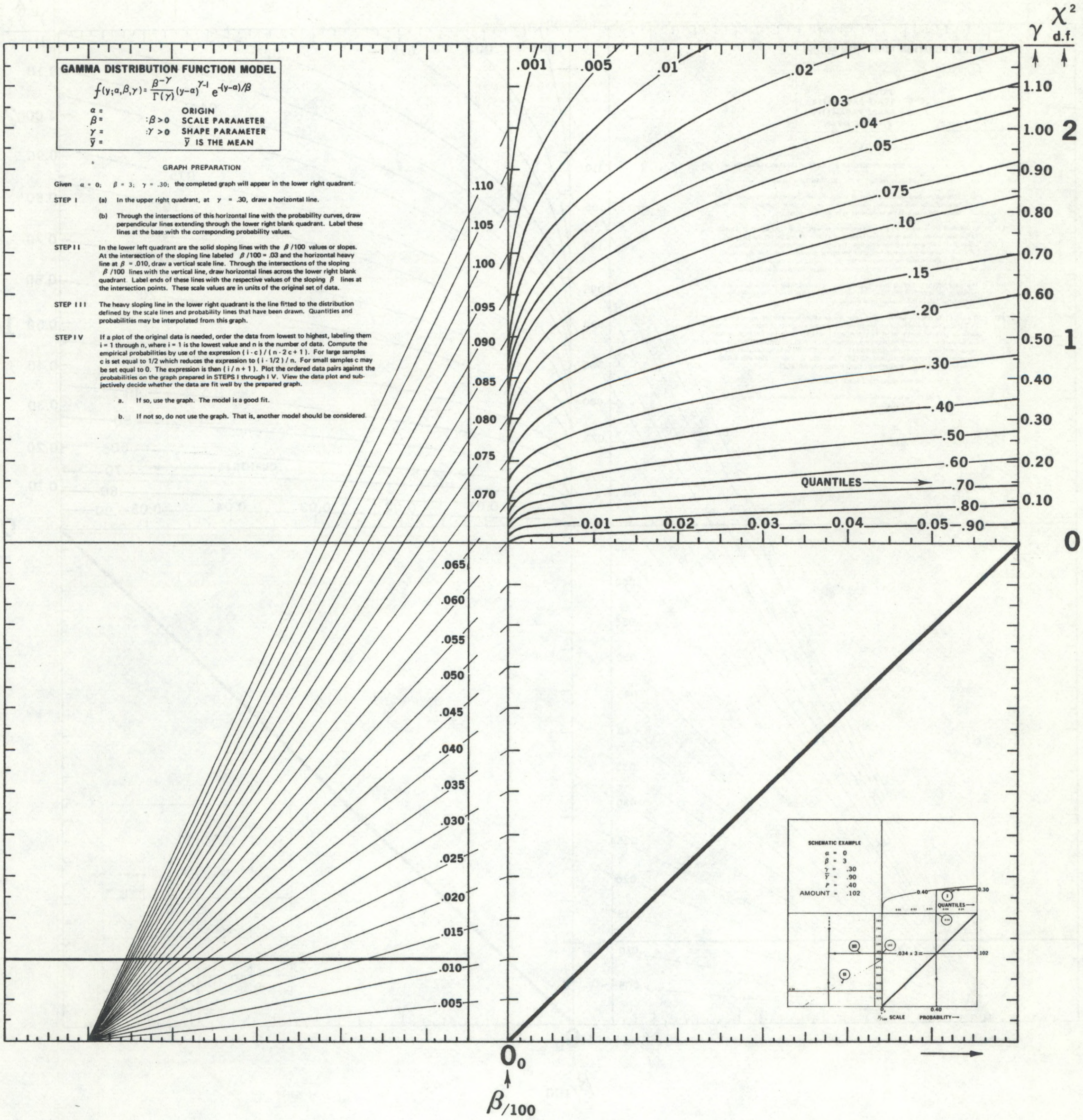
GAMMA DISTRIBUTION FUNCTION MODEL
$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$
α = ORIGIN
β = : $\beta > 0$ SCALE PARAMETER
γ = : $\gamma > 0$ SHAPE PARAMETER
$\bar{y}$ = $\bar{y}$ IS THE MEAN
GRAPH PREPARATION
Given α = 0; β = 0.2; γ = 0.5, the completed graph will appear in the lower right quadrant.
STEP I - (a) In the upper right quadrant, at γ = 0.5, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower right blank quadrant. Label these lines at the base with the corresponding probability values.
STEP II - In the lower left quadrant are the solid sloping lines with the β values or slopes. At the intersection of the sloping line labeled β = 0.2 and the horizontal heavy line at β = 0.1 draw a vertical scale line. Through the intersections of the sloping β lines with the vertical line, draw horizontal lines across the lower right blank quadrant. Label the ends of these lines with the respective values of the sloping β lines at the intersection points. These scale values are in units of the original set of data.
For very large or small values of β some scaling difficulties are encountered. Use the scale for β = 1 which is at the left side of the lower right quadrant. Read the quantities on this scale and multiply by the sample β values to arrive at values in the same units as the original set of data.
STEP III - The heavy sloping line in the lower right quadrant is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
STEP IV - If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i-c)/(n-2c+1). For large samples c is set equal to ½ which reduces the expression to (i-½)/n. For small samples c may be set equal to 0. The expression is then (i/n+1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The model is a good fit.
b. If not so, do not use the graph. That is, another model should be considered.
γ
χ²
d.f.
1.10
1.00
2
0.90
0.80
0.70
0.60
0.50
1
0.40
0.30
0.20
0.10
0
.01
.05
.10
.20
.30
.40
.50
.60
.70
.80
.90
.95
.98
.99
QUANTILES
0.10
0.20
0.30
0.40
0.50
1.10
1.05
1.00
.95
.90
.85
.80
.75
.70
.65
.60
.55
.50
.45
.40
.35
.30
.25
.20
.15
.10
.05
0₀
β
PROBABILITY
SCHEMATIC EXAMPLE
α = 0
β = 0.20
γ = 0.50
ȳ = 0.10
P = 0.50
AMOUNT = 0.0454
QUANTILES
SCALE UNITS x 10
0.227 x 0.2 = 0.0454
β SCALE
0.50
PROBABILITY

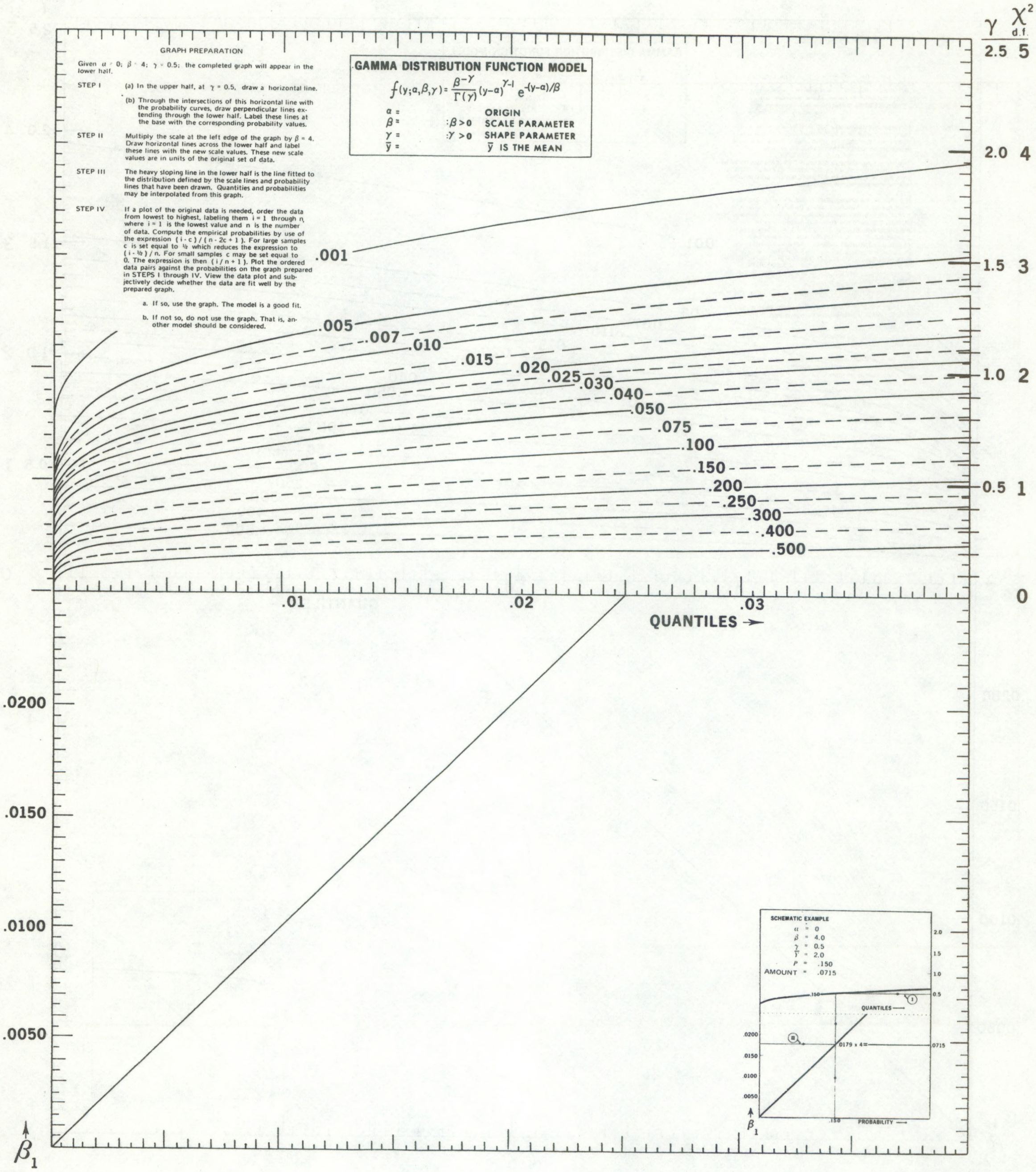

GAMMA DISTRIBUTION FUNCTION MODEL
$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$
α = ORIGIN
β = :β>0 SCALE PARAMETER
γ = :γ>0 SHAPE PARAMETER
ȳ = ȳ IS THE MEAN
GRAPH PREPARATION
Given α = 0; β = 0.2; γ = 0.5, the completed graph will appear in the lower right quadrant.
STEP I - (a) In the upper right quadrant, at γ = 0.5, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower right blank quadrant. Label these lines at the base with the corresponding probability values.
STEP II - In the lower left quadrant are the solid sloping lines with the β values or slopes. At the intersection of the sloping line labeled β = 0.2 and the horizontal heavy line at β = 0.1 draw a vertical scale line. Through the intersections of the sloping β lines with the vertical line, draw horizontal lines across the lower right blank quadrant. Label the ends of these lines with the respective values of the sloping β lines at the intersection points. These scale values are in units of the original set of data.
For very large or small values of β some scaling difficulties are encountered. Use the scale for β = 1 which is at the left side of the lower right quadrant. Read the quantities on this scale and multiply by the sample β values to arrive at values in the same units as the original set of data.
STEP III - The heavy sloping line in the lower right quadrant is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
STEP IV - If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i-c)/(n-2c+1). For large samples c is set equal to ½ which reduces the expression to (i-½)/n. For small samples c may be set equal to 0. The expression is then (i/n+1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The model is a good fit.
b. If not so, do not use the graph. That is, another model should be considered.
.01 .05 .10 .20 .30 .40 .50 .60 .70 .80 .90 .95 .98 .99
QUANTILES
0.10 0.20 0.30 0.40 0.50
γ
χ²
d.f.
1.10
1.00 2
0.90
0.80
0.70
0.60
0.50 1
0.40
0.30
0.20
0.10
0
1.10 1.05 1.00 .95 .90 .85 .80 .75 .70 .65 .60 .55 .50 .45 .40 .35 .30 .25 .20 .15 .10 .05
0
0
β
PROBABILITY
SCHEMATIC EXAMPLE
α = 0
β = 0.20
γ = 0.50
ȳ = 0.10
P = 0.50
AMOUNT = 0.0454
QUANTILES
PROBABILITY
β SCALE

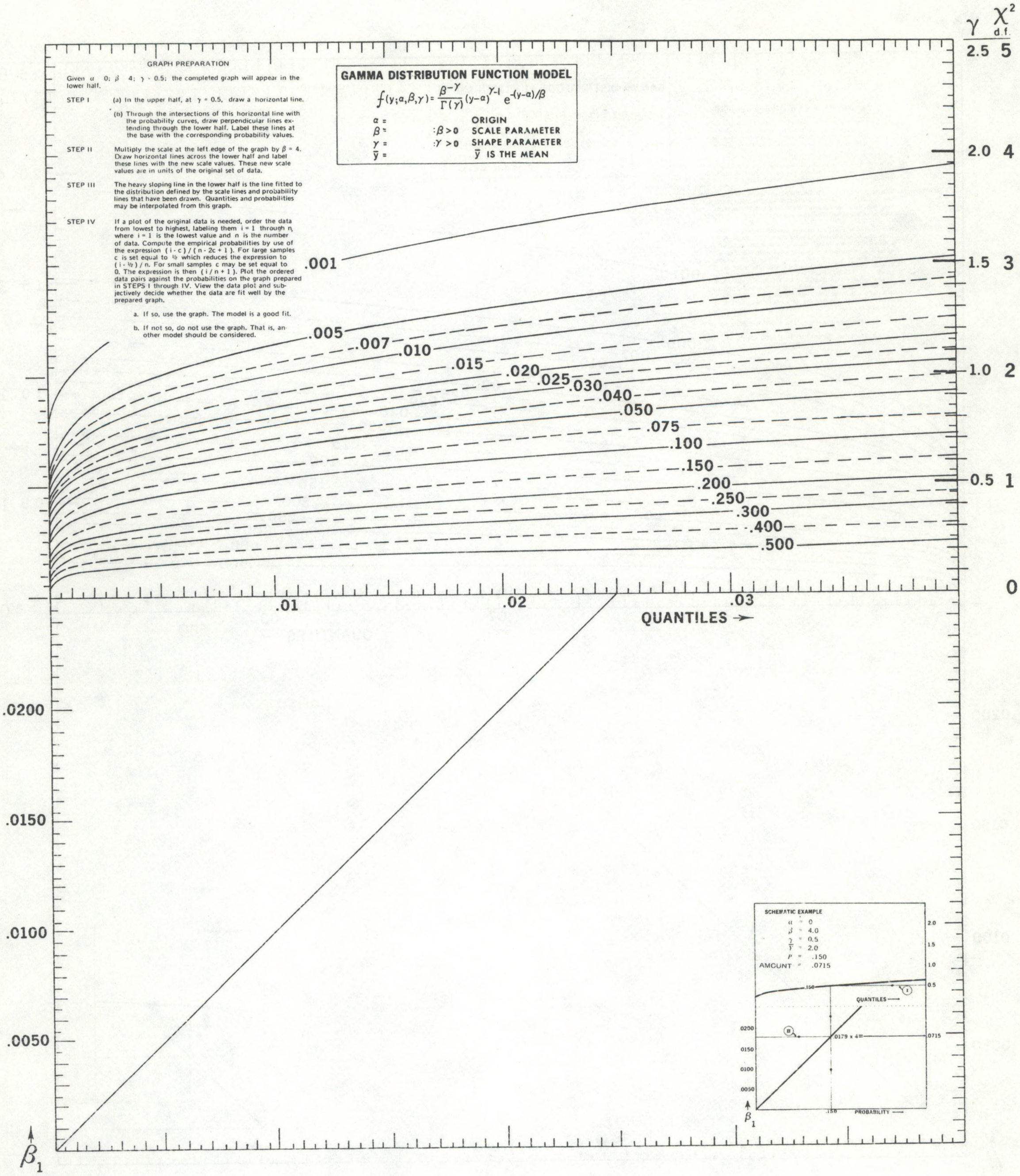

GAMMA DISTRIBUTION FUNCTION MODEL
$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)} (y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$
α = ORIGIN
β = :β > 0 SCALE PARAMETER
γ = :γ > 0 SHAPE PARAMETER
ȳ = ȳ IS THE MEAN
GRAPH PREPARATION
Given α = 0; β = 3; γ = .30; the completed graph will appear in the lower right quadrant.
STEP I (a) In the upper right quadrant, at γ = .30, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower right blank quadrant. Label these lines at the base with the corresponding probability values.
STEP II In the lower left quadrant are the solid sloping lines with the β /100 values or slopes. At the intersection of the sloping line labeled β /100 = .03 and the horizontal heavy line at β = .010, draw a vertical scale line. Through the intersections of the sloping β /100 lines with the vertical line, draw horizontal lines across the lower right blank quadrant. Label ends of these lines with the respective values of the sloping β lines at the intersection points. These scale values are in units of the original set of data.
STEP III The heavy sloping line in the lower right quadrant is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
STEP IV If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i - c) / (n - 2 c + 1). For large samples c is set equal to 1/2 which reduces the expression to (i - 1/2) / n. For small samples c may be set equal to 0. The expression is then (i / n + 1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The model is a good fit.
b. If not so, do not use the graph. That is, another model should be considered.
.001 .005 .01 .02 .03 .04 .05 .075 .10 .15 .20 .30 .40 .50 .60 .70 .80 .90
QUANTILES
0.01 0.02 0.03 0.04 0.05
γ
χ²
d.f.
1.10 1.00 0.90 0.80 0.70 0.60 0.50 0.40 0.30 0.20 0.10
2
1
0
.110 .105 .100 .095 .090 .085 .080 .075 .070 .065 .060 .055 .050 .045 .040 .035 .030 .025 .020 .015 .010 .005
0₀
β/100
SCHEMATIC EXAMPLE
α = 0
β = 3
γ = .30
ȳ = .90
P = .40
AMOUNT = .102
PROBABILITY


GAMMA DISTRIBUTION FUNCTION MODEL
$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)} (y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$
α = ORIGIN
β = : $\beta > 0$ SCALE PARAMETER
γ = : $\gamma > 0$ SHAPE PARAMETER
$\bar{y}$ = $\bar{y}$ IS THE MEAN
GRAPH PREPARATION
Given $\alpha = 0$; $\beta = 3$; $\gamma = .30$; the completed graph will appear in the lower right quadrant.
STEP I (a) In the upper right quadrant, at $\gamma = .30$, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower right blank quadrant. Label these lines at the base with the corresponding probability values.
STEP II In the lower left quadrant are the solid sloping lines with the $\beta/100$ values or slopes. At the intersection of the sloping line labeled $\beta/100 = .03$ and the horizontal heavy line at $\beta = .010$, draw a vertical scale line. Through the intersections of the sloping $\beta/100$ lines with the vertical line, draw horizontal lines across the lower right blank quadrant. Label ends of these lines with the respective values of the sloping β lines at the intersection points. These scale values are in units of the original set of data.
STEP III The heavy sloping line in the lower right quadrant is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
STEP IV If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i - c) / (n - 2 c + 1). For large samples c is set equal to 1/2 which reduces the expression to (i - 1/2) / n. For small samples c may be set equal to 0. The expression is then (i / n + 1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The model is a good fit.
b. If not so, do not use the graph. That is, another model should be considered.
.001 .005 .01 .02 .03 .04 .05 .075 .10 .15 .20 .30 .40 .50 .60 .70 .80 .90
QUANTILES
0.01 0.02 0.03 0.04 0.05
γ $\chi^2_{d.f.}$
1.10 1.00 0.90 0.80 0.70 0.60 0.50 0.40 0.30 0.20 0.10
2 1 0
.110 .105 .100 .095 .090 .085 .080 .075 .070 .065 .060 .055 .050 .045 .040 .035 .030 .025 .020 .015 .010 .005
0_0
$\beta/100$
SCHEMATIC EXAMPLE


GAMMA DISTRIBUTION FUNCTION MODEL
f(y;α,β,γ) = β^−γ / Γ(γ) (y−α)^γ−1 e^−(y−α)/β
α = ORIGIN
β = :β > 0 SCALE PARAMETER
γ = :γ > 0 SHAPE PARAMETER
ȳ = ȳ IS THE MEAN
GRAPH PREPARATION
Given α = 0; β = 3; γ = .30; the completed graph will appear in the lower right quadrant.
STEP I (a) In the upper right quadrant, at γ = .30, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower right blank quadrant. Label these lines at the base with the corresponding probability values.
STEP II In the lower left quadrant are the solid sloping lines with the β/100 values or slopes. At the intersection of the sloping line labeled β/100 = .03 and the horizontal heavy line at β = .010, draw a vertical scale line. Through the intersections of the sloping β/100 lines with the vertical line, draw horizontal lines across the lower right blank quadrant. Label ends of these lines with the respective values of the sloping β lines at the intersection points. These scale values are in units of the original set of data.
STEP III The heavy sloping line in the lower right quadrant is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
STEP IV If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i · c) / (n · 2 c + 1). For large samples c is set equal to 1/2 which reduces the expression to (i · 1/2) / n. For small samples c may be set equal to 0. The expression is then (i / n + 1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The model is a good fit.
b. If not so, do not use the graph. That is, another model should be considered.
γ
χ² d.f.
.001 .005 .01 .02 .03 .04 .05 .075 .10 .15 .20 .30 .40 .50 .60 .70 .80 .90
1.10 1.00 0.90 0.80 0.70 0.60 0.50 0.40 0.30 0.20 0.10 0
2 1 0
QUANTILES
0.01 0.02 0.03 0.04 0.05
.110 .105 .100 .095 .090 .085 .080 .075 .070 .065 .060 .055 .050 .045 .040 .035 .030 .025 .020 .015 .010 .005
0₀
β/100
SCHEMATIC EXAMPLE
α = 0
β = 3
γ = .30
ȳ = .90
P = .40
AMOUNT = .102
QUANTILES
.034 x 3 =
.102
0.40
SCALE
PROBABILITY


GAMMA DISTRIBUTION FUNCTION MODEL
$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$
α = ORIGIN
β = : $\beta > 0$ SCALE PARAMETER
γ = : $\gamma > 0$ SHAPE PARAMETER
$\bar{y}$ = $\bar{y}$ IS THE MEAN
GRAPH PREPARATION
Given $\alpha = 0$; $\beta = 3$; $\gamma = .30$; the completed graph will appear in the lower right quadrant.
STEP I (a) In the upper right quadrant, at $\gamma = .30$, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower right blank quadrant. Label these lines at the base with the corresponding probability values.
STEP II In the lower left quadrant are the solid sloping lines with the $\beta/100$ values or slopes. At the intersection of the sloping line labeled $\beta/100 = .03$ and the horizontal heavy line at $\beta = .010$, draw a vertical scale line. Through the intersections of the sloping $\beta/100$ lines with the vertical line, draw horizontal lines across the lower right blank quadrant. Label ends of these lines with the respective values of the sloping β lines at the intersection points. These scale values are in units of the original set of data.
STEP III The heavy sloping line in the lower right quadrant is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
STEP IV If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i - c) / (n - 2 c + 1). For large samples c is set equal to 1/2 which reduces the expression to (i - 1/2) / n. For small samples c may be set equal to 0. The expression is then (i / n + 1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through I V. View the data plot and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The model is a good fit.
b. If not so, do not use the graph. That is, another model should be considered.
γ
χ^2 d.f.
.001 .005 .01 .02 .03 .04 .05 .075 .10 .15 .20 .30 .40 .50 .60 .70 .80 .90
QUANTILES
0.01 0.02 0.03 0.04 0.05
1.10 1.00 0.90 0.80 0.70 0.60 0.50 0.40 0.30 0.20 0.10
2 1 0
.110 .105 .100 .095 .090 .085 .080 .075 .070 .065 .060 .055 .050 .045 .040 .035 .030 .025 .020 .015 .010 .005
SCHEMATIC EXAMPLE
α = 0
β = 3
γ = .30
$\bar{y}$ = .90
$\hat{\gamma}$ = .40
AMOUNT = .102
QUANTILES
.034 x 3 =
.102
0.40
0.30
SCALE
PROBABILITY
0_0
$\beta/100$


GRAPH PREPARATION
Given α = 0; β = 4; γ = 0.5; the completed graph will appear in the lower half.
STEP I (a) In the upper half, at γ = 0.5, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower half. Label these lines at the base with the corresponding probability values.
STEP II Multiply the scale at the left edge of the graph by β = 4. Draw horizontal lines across the lower half and label these lines with the new scale values. These new scale values are in units of the original set of data.
STEP III The heavy sloping line in the lower half is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
STEP IV If a plot of the original data is needed, order the data from lowest to highest, labeling them i = 1 through n, where i = 1 is the lowest value and n is the number of data. Compute the empirical probabilities by use of the expression (i - c) / (n - 2c + 1). For large samples c is set equal to ½ which reduces the expression to (i - ½) / n. For small samples c may be set equal to 0. The expression is then (i / n + 1). Plot the ordered data pairs against the probabilities on the graph prepared in STEPS I through IV. View the data plot and subjectively decide whether the data are fit well by the prepared graph.
a. If so, use the graph. The model is a good fit.
b. If not so, do not use the graph. That is, another model should be considered.
GAMMA DISTRIBUTION FUNCTION MODEL
$f(y;\alpha,\beta,\gamma) = \frac{\beta^{-\gamma}}{\Gamma(\gamma)}(y-\alpha)^{\gamma-1} e^{-(y-\alpha)/\beta}$
α = ORIGIN
β = :β > 0 SCALE PARAMETER
γ = :γ > 0 SHAPE PARAMETER
ȳ = ȳ IS THE MEAN
γ
$\chi^2_{d.f.}$
2.5 5
2.0 4
1.5 3
1.0 2
0.5 1
0
.001
.005
.007
.010
.015
.020
.025
.030
.040
.050
.075
.100
.150
.200
.250
.300
.400
.500
.01
.02
.03
QUANTILES →
.0200
.0150
.0100
.0050
β_1
SCHEMATIC EXAMPLE
α = 0
β = 4.0
γ = 0.5
ȳ = 2.0
P = .150
AMOUNT = .0715
.150
QUANTILES →
.0179 x 4=
.0715
PROBABILITY →


GRAPH PREPARATION
Given α 0; β 4; γ · 0.5: the completed graph will appear in the lower half.
STEP I (a) In the upper half, at γ = 0.5, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower half. Label these lines at the base with the corresponding probability values.
STEP II Multiply the scale at the left edge of the graph by β = 4. Draw horizontal lines across the lower half and label these lines with the new scale values. These new scale values are in units of the original set of data.
STEP III The heavy sloping line in the lower half is the line fitted to the distribution defined by the scale lines and probability lines that have been drawn. Quantities and probabilities may be interpolated from this graph.
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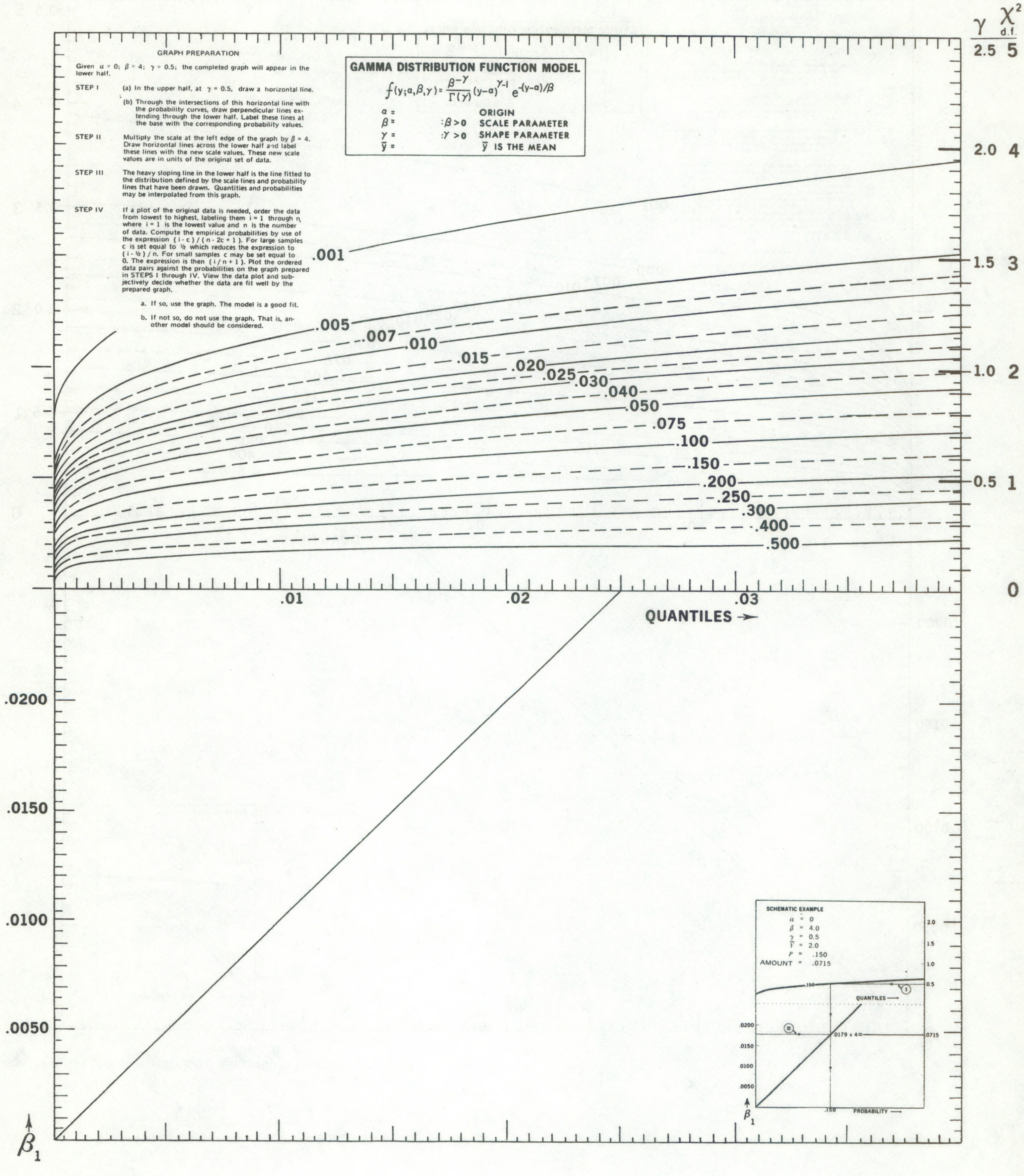
β = :β > 0 SCALE PARAMETER
γ = :γ > 0 SHAPE PARAMETER
ȳ = ȳ IS THE MEAN
.001
.005
.007
.010
.015
.020
.025
.030
.040
.050
.075
.100
.150
.200
.250
.300
.400
.500
γ $\chi^2_{d.f.}$
2.5 5
2.0 4
1.5 3
1.0 2
0.5 1
0
.01
.02
.03
QUANTILES →
.0200
.0150
.0100
.0050
β_1
SCHEMATIC EXAMPLE
α = 0
β = 4.0
γ = 0.5
ȳ = 2.0
P = .150
AMOUNT = .0715
.150
QUANTILES →
.0179 x 4 = .0715
PROBABILITY →


GRAPH PREPARATION
Given α = 0; β = 4; γ = 0.5; the completed graph will appear in the lower half.
STEP I (a) In the upper half, at γ = 0.5, draw a horizontal line.
(b) Through the intersections of this horizontal line with the probability curves, draw perpendicular lines extending through the lower half. Label these lines at the base with the corresponding probability values.
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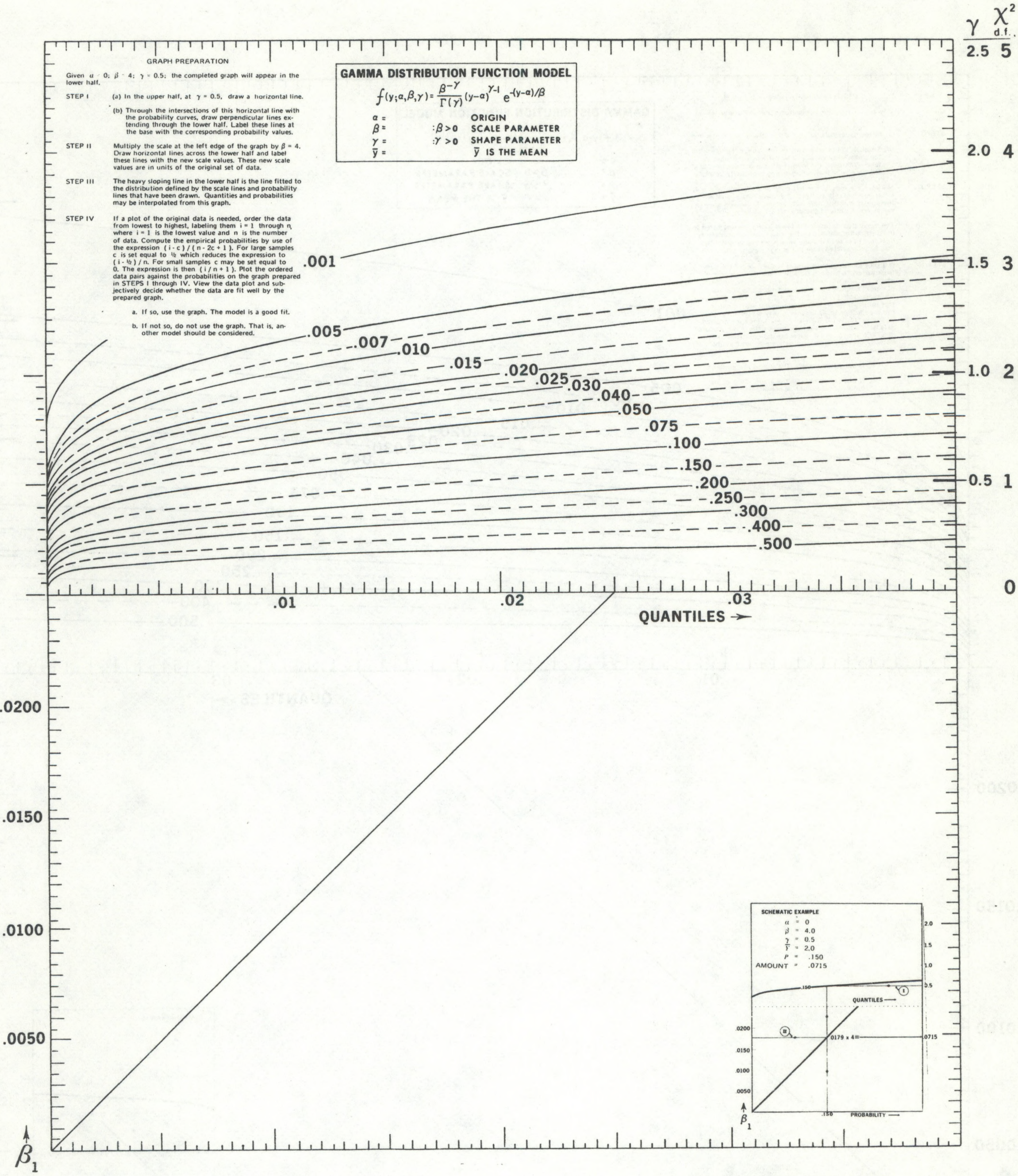
β = :β > 0 SCALE PARAMETER
γ = :γ > 0 SHAPE PARAMETER
ȳ = ȳ IS THE MEAN
γ
χ^2
d.f.
2.5 5
2.0 4
1.5 3
1.0 2
0.5 1
0
.001
.005
.007
.010
.015
.020
.025
.030
.040
.050
.075
.100
.150
.200
.250
.300
.400
.500
.01
.02
.03
QUANTILES →
.0200
.0150
.0100
.0050
β_1
SCHEMATIC EXAMPLE
α = 0
β = 4.0
γ = 0.5
Ȳ = 2.0
P = .150
AMOUNT = .0715
2.0
1.5
1.0
0.5
.150
I
QUANTILES →
II
.0200
.0150
.0100
.0050
.0179 x 4 =
.0715
.150
PROBABILITY →
β_1



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