Vortex Visualization of Tropical Cyclones by Liutex

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Abstract: Analysis of the flow field of tropical cyclones in the mature stage by the Liutex methods, such as the Omega Liutex and the Liutex Core Line, is reported. This is the first known study analyzing tropical cyclones with the Liutex method. Liutex has the potential to enable greater understanding of tropical cyclone dynamics and to improve forecasts of their track, intensity, and structure.

Key words: Vortex, tropical cyclone, hurricane, Liutex

1. Introduction

Tropical cyclones are warm-core, non-frontal, synoptic-scale[*], low-pressure systems that are fueled by the heat in large, tropical or subtropical bodies of water and have a closed surface circulation. They cause significant destruction and loss of life due to storm surge, flooding, wind, and wave actions even thousands of km inland from coastal regions and outside of tropical or subtropical regions. Though significant improvements in forecasts have been made, especially in forecasts of where tropical cyclones will go[1], large errors in forecasts of intensity, size, and rainfall continue to occur, and these may be at least partially due to the inability to observe, understand, and model features within the large-scale circulation. For example, numerous structures are frequently seen in the cores and rainbands of tropical cyclones in satellite and radar reflectivity imagery (Fig. 1) that are frequently described as mesovortices or misovortices. This suggests changes to the wind field in and around these features; however, despite frequent observations by aircraft flying through these features and from Doppler radars, no direct evidence of vortices have been seen. As a result, it remains unclear what these features are and what their impact on intensity and structure may be.

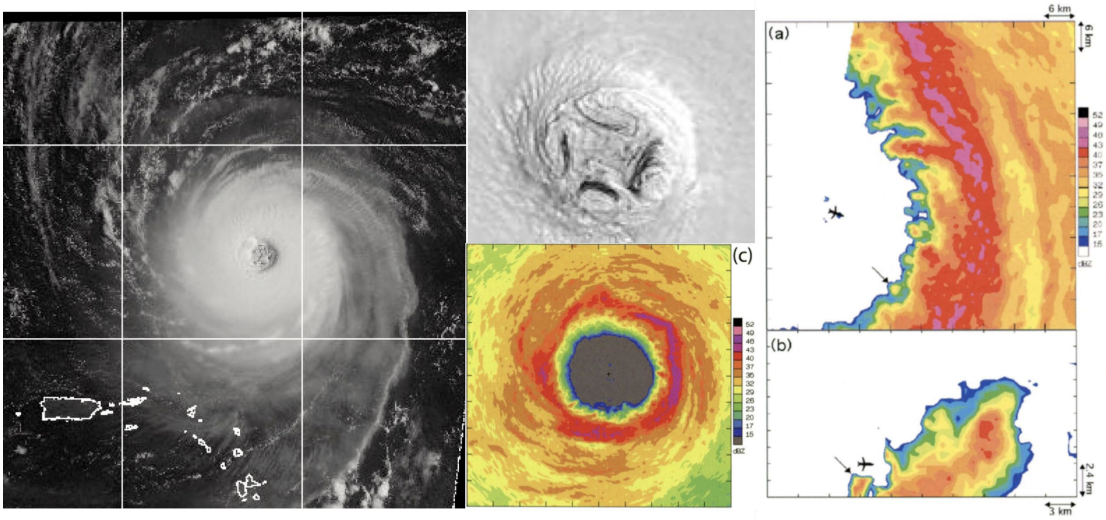

Figure 1. (Left) Visible Geostationary satellite imagery of Hurricane Isabel on 13 September 2003 1800 UTC. (Top middle) Visible Geostationary satellite imagery of

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*: Synoptic scale refers to horizontal scales of motion of many hundreds of km; mesoscale refers to horizontal scales ranging from a few to several hundred km; misoscale refers to horizontal scales up to a few km.

the core of Hurricane Isabel at the same time. (Bottom middle) Reflectivity from a single scan of the Lower Fuselage Radar aboard the NOAA P-3 at the same time. (Top right) Close-up reflectivity of the scan in the bottom middle showing the location of the aircraft and of a small-scale, possibly vortical feature of the type we wish to understand. (Bottom right) Close up reflectivity of a single scan of the Tail Doppler Radar aboard the NOAA P-3 at the same time, showing a vertical cross-section of the feature and of the eyewall.

In Atmospheric Science, a vortex is defined as compact flow that circulates around an axis, characterized by a local extremum in vorticity. Helmholtz proposed a vorticity-based definition,[2] but this definition is subjective and fails in many applications.[3] Vorticity, the curl of the motion, has been commonly used to find vortical regions in a flow field for centuries. However, vorticity and all methods derived from it are contaminated with shear.[4] For example, vorticity can be found in the near-wall boundary layer of a channel flow, where the whole flow field is parallel straight lines, and thus rotation cannot exist. In this case, the vorticity is only induced by shear. Liutex, on the other hand, describes the rigid rotation of a fluid without contamination from shear. Liutex extracts out the pure rotational motion of a fluid.[5–7] Since a vortex is described by rigid body rotation of fluid particle, Liutex should be the most appropriate way to describe the vortex structure of a flow field. In this study, we will present a new theoretical Liutex-based vortex structure on tropical cyclones, based on the observational data in hurricanes Authur (2014) and Fabian (2003). The paper is organized in the following way: In Section 2, we introduce the definition of Liutex, resistance and several other Liutex-based methods; Section 3 discusses the data in the two hurricanes used for the analysis; Section 4 provides tropical cyclone visualizations of vortex structure by Liutex; Some conclusions are given in Section 5.

2. Liutex, resistance, Liutex-Omega, and Liutex core line

2.1 Liutex and Resistance:

The velocity gradient tensor ∇v can be classified into 3 cases based on its discriminant: $\Delta_3 < 0$, $\Delta_3 > 0$, and $\Delta_3 = 0$. When $\Delta_3 < 0$, it has only one real eigenvalue λ_r, and a pair of complex conjugate eigenvalues $\lambda_{cr} \pm i\lambda_{ci}$; when $\Delta_3 > 0$, it has 3 distinct real eigenvalues $\lambda_1, \lambda_2, \lambda_3$; and when $\Delta_3 = 0$, it has repeated real eigenvalues λ_1, λ_2 or just λ. The first case, when $\Delta_3 < 0$, is also referred to as rotational points, whereas the second and third cases, when $\Delta_3 \geq 0$, are referred to as non-rotational points.

2.1.1 Case 1:

For cases when $\Delta_3 < 0$, Liutex was developed to represent the rigid body rotation for rotational points, which can be obtained from the principal velocity gradient tensor $\nabla \boldsymbol{V}$ under a principal coordinate. $\nabla \boldsymbol{V}$ can be obtained from transformation of $\nabla \boldsymbol{v}$ with a rotation matrix $\boldsymbol{U}$:

$$\nabla \boldsymbol{V} = \boldsymbol{U}^T (\nabla v) \boldsymbol{U} = \begin{bmatrix} \lambda_{cr} & -\frac{R}{2} & 0 \\ \varepsilon + \frac{R}{2} & \lambda_{cr} & 0 \\ \xi & \eta & \lambda_r \end{bmatrix}$$

$$\boldsymbol{U} = [\boldsymbol{p} \quad \boldsymbol{q} \quad \boldsymbol{r}] = \begin{bmatrix} p_x & q_x & r_x \\ p_y & q_y & r_y \\ p_z & q_z & r_z \end{bmatrix} \in SO(3) \tag{1}$$

$$\nabla v(\boldsymbol{r}) = \lambda_r \boldsymbol{r} \,;\, \boldsymbol{\omega} \cdot \boldsymbol{r} > 0$$

Here, $\boldsymbol{U}$ is a 3×3 special orthogonal matrix $(SO(3))$, $U^T = U^{-1}$ for being orthogonal, and $|\boldsymbol{U}| = 1$ for being normalized. The 3 column vectors, $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$, are orthonormal, and r is the eigenvector of ∇v with respect to real eigenvalue λ_r, whose dot product with the vorticity ω is positive. R stands for the Liutex magnitude, which can be calculated as follows[8,9], and the Liutex vector $\boldsymbol{R}$ has the same direction as eigenvector $\boldsymbol{r}$ and the magnitude R.

$$\boldsymbol{R} = \left((\boldsymbol{\omega} \cdot \boldsymbol{r}) - \sqrt{(\boldsymbol{\omega} \cdot \boldsymbol{r})^2 - 4\lambda_{ci}^2} \right) \boldsymbol{r} \,;\, \boldsymbol{R} = R\boldsymbol{r} \tag{2}$$

2.1.2 Case 2:

However, Liutex is not defined for non-rotational points, and therefore Resistance L was introduced as being a continuous expansion for it. It is defined as the minimum amount of vorticity required to make the non-rotational point start to rotate.[10] For non-rotational points when $\Delta_3 > 0$, the principal matrix looks as follows:

$$\nabla \boldsymbol{V} = \boldsymbol{U}^T (\nabla v) \boldsymbol{U} = \begin{bmatrix} \frac{\lambda_1 + \lambda_2}{2} & -\frac{L}{2} & 0 \\ \varepsilon + \frac{L}{2} & \frac{\lambda_1 + \lambda_2}{2} & 0 \\ \xi & \eta & \lambda_3 \end{bmatrix} \tag{3}$$

$$U = [\boldsymbol{p} \quad \boldsymbol{q} \quad \boldsymbol{r}] = \begin{bmatrix} p_x & q_x & r_x \\ p_y & q_y & r_y \\ p_z & q_z & r_z \end{bmatrix} \in SO(3)$$

$$\nabla v(\boldsymbol{r}) = \lambda_3 \boldsymbol{r} \,;\, \boldsymbol{\omega} \cdot \boldsymbol{r} > 0 \tag{4}$$

Similarly, $\boldsymbol{U}$ is a 3×3 special orthogonal matrix ($SO(3)$) with 3 orthonormal column vectors, $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$. Here $\boldsymbol{r}$ is the eigenvector with respect to eigenvalue λ_3, $\boldsymbol{p}$, $\boldsymbol{q}$ are orthonormal with $\boldsymbol{r}$, but not necessarily an eigenvector, and the other 2 eigenvalues are λ_1 and λ_2. Hence, we can find the resistance at this direction, and with different choices of λ_3 and $\boldsymbol{r}$, we will have 3 different resistances, thus the overall resistance will be the minimum of them.

$$L = \left\{ \sqrt{(\boldsymbol{\omega} \cdot \boldsymbol{r})^2 + (\lambda_1 - \lambda_2)^2} - \boldsymbol{\omega} \cdot \boldsymbol{r} \right\} \tag{5}$$

2.1.3 Case 3:

For the last case, when $\Delta_3 = 0$ it can be viewed as a middle case in between. The principal matrix looks as follows:

$$\nabla \boldsymbol{V} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ \varepsilon & \lambda_1 & 0 \\ \xi & \eta & \lambda_2 \end{bmatrix} or \begin{bmatrix} \lambda & 0 & 0 \\ \varepsilon & \lambda & 0 \\ \xi & \eta & \lambda \end{bmatrix} \tag{6}$$

Here, λ_1 is the repeated eigenvalue if it has 2 distinct eigenvalues. We can compare this case with case 1, where $\lambda_1 = \lambda_1 \pm 0i$, and $\boldsymbol{R} = \left((\boldsymbol{\omega} \cdot \boldsymbol{r}) - \sqrt{(\boldsymbol{\omega} \cdot \boldsymbol{r})^2 - 4\lambda_{ci}^2} \right) \boldsymbol{r} = 0$. Similarly, we can compare it with case 2 where $\lambda_1 = \lambda_2$, and therefore, we have $L = \sqrt{(\boldsymbol{\omega} \cdot \boldsymbol{r})^2 + (\lambda_1 - \lambda_2)^2} - \boldsymbol{\omega} \cdot \boldsymbol{r} = 0$. If there is only 1 distinct eigenvalue, then it will be a special case where $\lambda_1 = \lambda_2 = \lambda$.

2.1.4 Liutex-Resistance system in 3D and 2D:

We can put Liutex magnitude R and negative resistance $-L$ together as a uniform parameter as Liutex-Resistance system, denoted LR. When $LR > 0$, it means this point is a rotational point, and LR is the Liutex magnitude; When $LR < 0$, it means this point is a non-rotational point, and LR is the negative resistance; When $LR = 0$, it means this point is in case 3 above with neither Liutex nor resistance. Qualitatively, resistance represents the minimum vorticity that required to make the non-rotational point to start rotate, and thus it can be put as 'negative rotate'. Whereas quantitatively, if we consider $\lambda_{1,2} = \lambda_{cr} + i\lambda_{ci}$, then

$$(\lambda_1 - \lambda_2)^2 = (2i\lambda_{ci})^2 = -4\lambda_{ci}^2 \tag{7}$$

$$-L = \boldsymbol{\omega} \cdot \boldsymbol{r} - \sqrt{(\boldsymbol{\omega} \cdot \boldsymbol{r})^2 + (\lambda_1 - \lambda_2)^2} = R \tag{8}$$

In 2-D cases, Liutex and resistance have a closer connection. In 2D cases, the principal matrix can be expressed as $\begin{bmatrix} \lambda_{cr} & -\frac{R}{2} \\ \varepsilon + \frac{R}{2} & \lambda_{cr} \end{bmatrix}$ for rotational points, and $\begin{bmatrix} \frac{\lambda_1 + \lambda_2}{2} & \frac{L}{2} \\ \varepsilon + \frac{L}{2} & \frac{\lambda_1 + \lambda_2}{2} \end{bmatrix}$ for non-rotational points. Since the trace, magnitude of vorticity, and determinate are preserved under special orthogonal transformation. Thus, for rotational points:

$$\begin{cases} \lambda_{cr} = \frac{1}{2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\varepsilon + \frac{R}{2} = \left|\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right| - \frac{R}{2}. \end{cases} \tag{9}$$

And for non- rotational points:

$$\begin{cases} \frac{\lambda_1+\lambda_2}{2} = \frac{1}{2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\varepsilon + \frac{L}{2} = \left|\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right| + \frac{R}{2}. \end{cases} \tag{10}$$

If we use LR for both $-L$ and R, then the principal matrix for both cases can be represented as follows, and LR can be solved through $|\nabla \boldsymbol{V}| = |\nabla \boldsymbol{v}|$.

$$|\nabla \boldsymbol{V}| = \begin{bmatrix} \frac{1}{2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) - \frac{LR}{2}\left|\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right| - \frac{LR}{2}\frac{1}{2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) \end{bmatrix} \tag{11}$$

$$\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} = \frac{1}{4}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2 + \frac{LR}{2}\left(\left|\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right| - \frac{LR}{2}\right) \tag{12}$$

$$LR = \left|\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right| - \sqrt{\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)^2}$$

Therefore, in 2D cases, the Liutex magnitude R and negative resistance $-L$ can both be found through the same explicit formula with respect to velocity gradient only as above.

2.2 Liutex-Omega and Liutex core line methods

2.2.1 Liutex-Omega

If we only use Liutex method, and if the flow field contains both strong and weak vortices, then sometimes, the weak vortices will be neglected if the difference in Liutex magnitude is too large between strong and weak vortices. Thus, Liutex-Omega[11] method is developed for this situation and defined as follows:

$$\Omega_R = \frac{\boldsymbol{\omega}\cdot\boldsymbol{r}}{2\left[(\boldsymbol{\omega}\cdot\boldsymbol{r})^2 - 2\lambda_{ci}^{\ 2} + 2\lambda_{cr}^{\ 2} + \lambda_r^{\ 2}\right] + \varepsilon} \tag{13}$$

Here, $\boldsymbol{\omega}$ is the vorticity; $\boldsymbol{r}$ is the eigenvector of λ_r with $\boldsymbol{\omega}\cdot\boldsymbol{r} > 0$; λ_r is still the real eigenvalue; λ_{cr} and λ_{ci} are the real and imaginary part of the complex eigenvalues; and ε is a small number to prevent the denominator to be 0.

2.2.2 Liutex core line[12]

Although Liutex magnitude is used a lot in vortex visualization, it is originally defined as a vector. Therefore, we can further explore it into a Liutex tube to present the vortex structure, and thus we can find the core line of this Liutex tube as the vortex core line. We can pick a small region and find a point with a local maximum of Liutex magnitude inside this region. The local maximum of Liutex magnitude can be defined as points with $\nabla \boldsymbol{R} \times R = 0$, and it can be found numerically as $|\nabla \boldsymbol{R} \times R|$ is less than some small threshold. The Liutex core line can be plotted as the streamline of Liutex vector, passing through this point. The advantage of the Liutex core line is its feature of threshold-free and uniqueness. Liutex core lines are similar to streamlines of a velocity filed which does not need threshold but has capability to show the vortex/Liutex strength with different colors.

3. Tropical cyclone visualizations of vortex structure by Liutex

High-resolution analyses of tropical cyclones are created using the Hurricane Ensemble Data Assimilation System (HEDAS), an Ensemble Kalman Filter-based data assimilation system designed to ingest high-resolution TC observations collected by Hurricane Hunter aircraft, though it can assimilate any observation for which operators exists[13]. Three-dimensional analyses in General Regularly distributed Information in Binary form (GRIB) format were post-processed by calculating Liutex. The post-processed data containing Liutex was visualized using Tecplot 360 software.

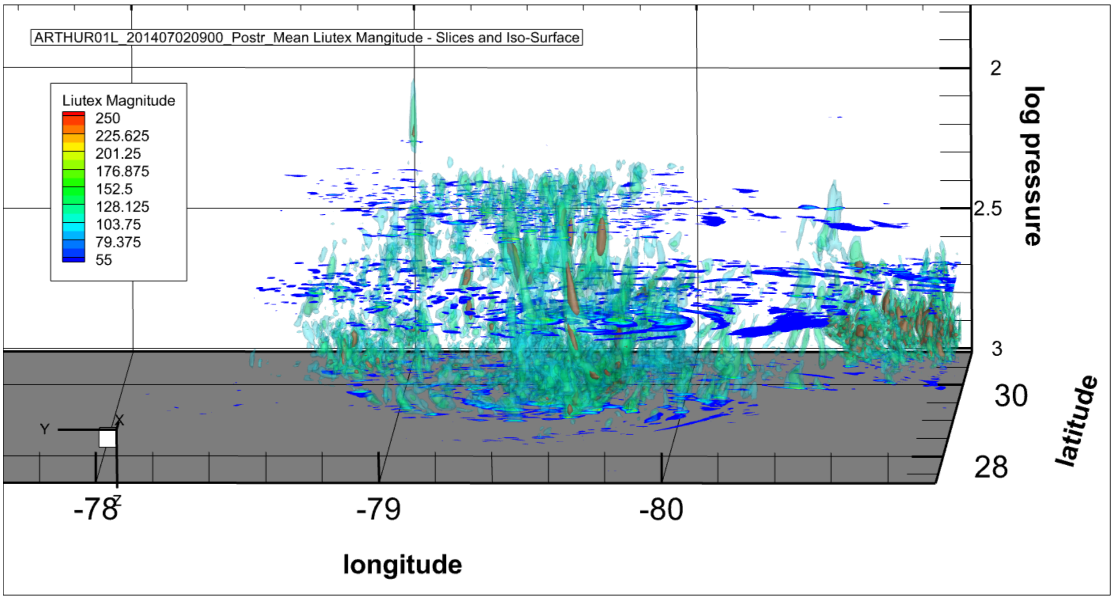

Figure 2. Three-dimensional analysis of Hurricane Arthur on 2 July 2014 0900 UTC visualized using Liutex magnitude iso-surfaces as well as horizontal slices. You can clearly see where all vortices are and their magnitudes based on the color.

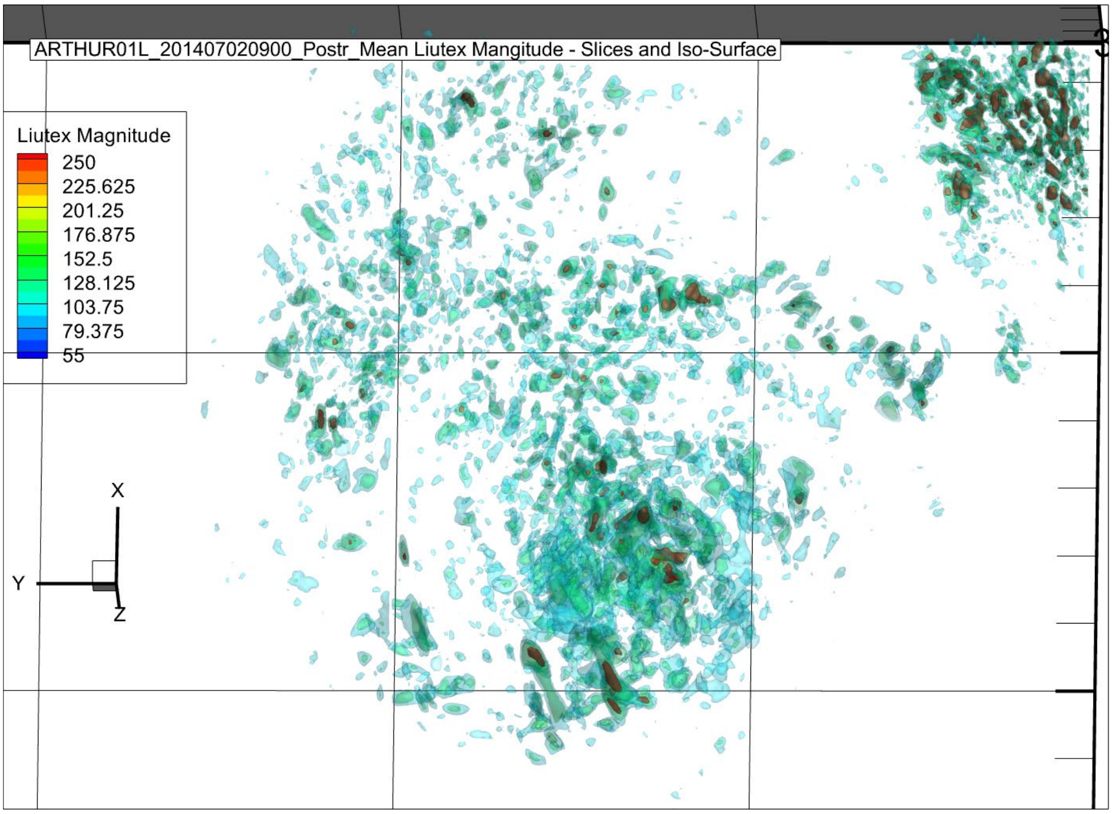

Figure 3. As in Fig. 2 but viewed from the top using Liutex magnitude iso-surfaces. From this angle, we can see that Arthur consists of numerous, individual small-scale vortices.

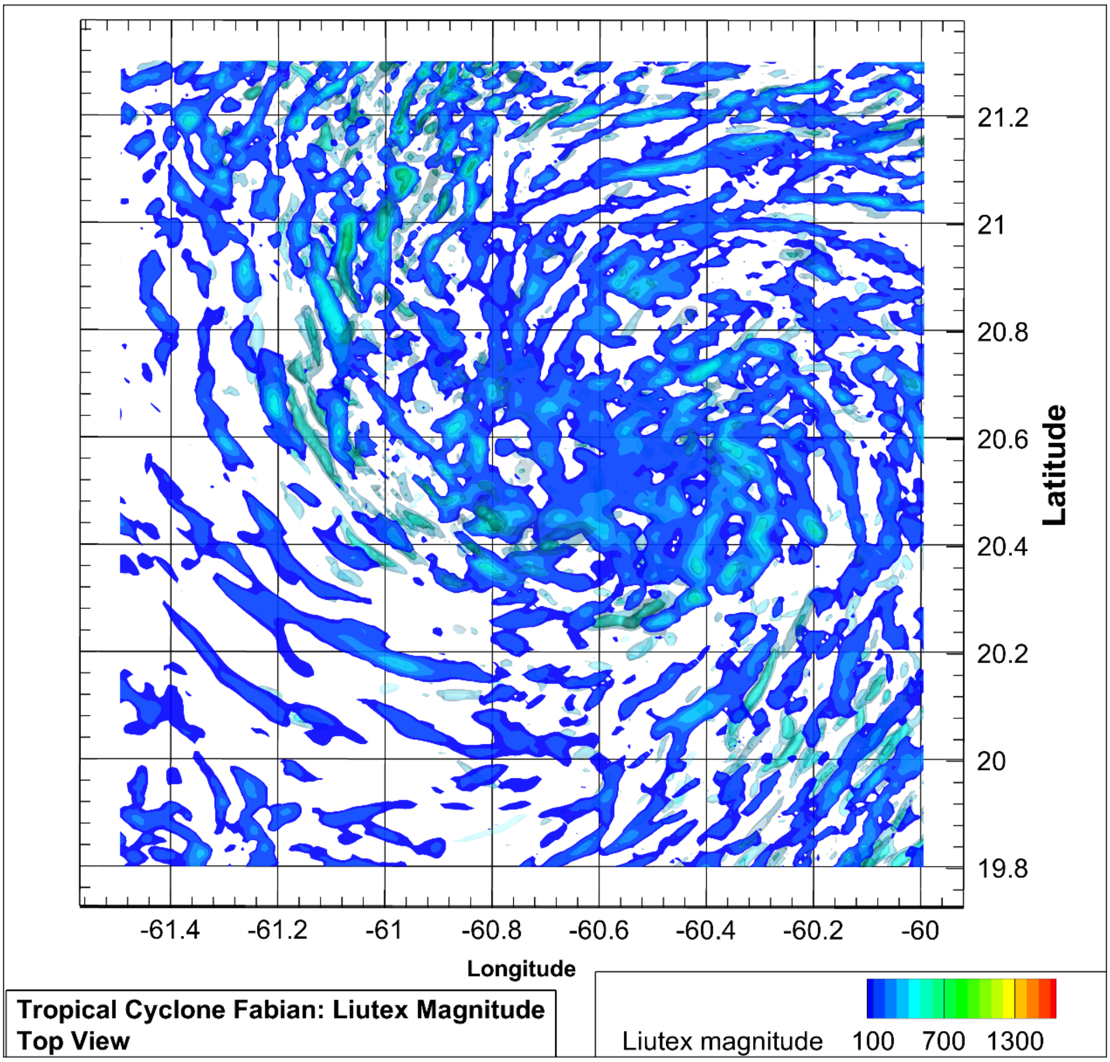

Figure 4. Liutex magnitude slices of Hurricane Fabian on 2 September 2003 1830 UTC.

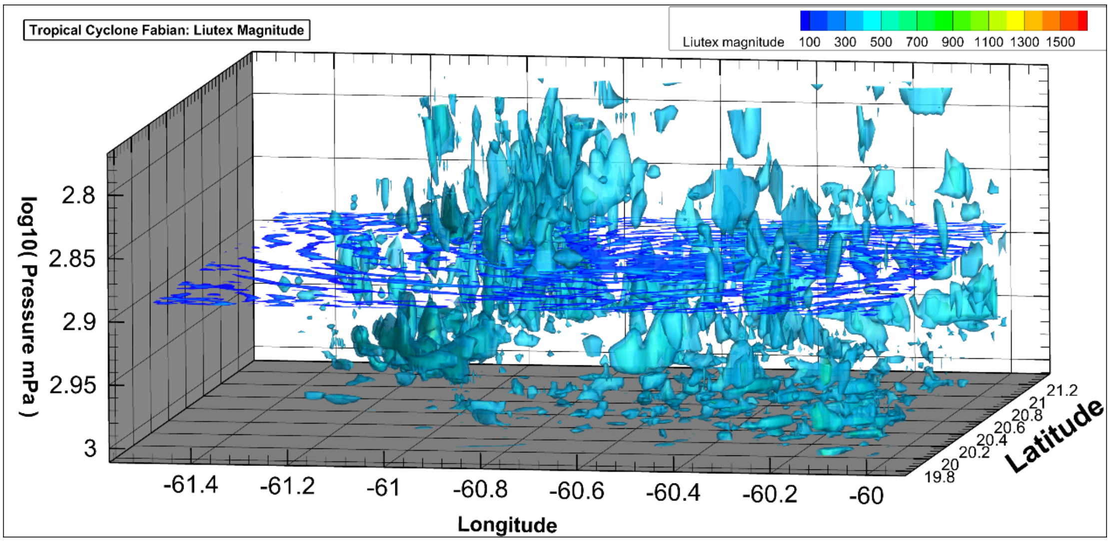

Figure 5. As in Fig. 4, but in three dimensions.

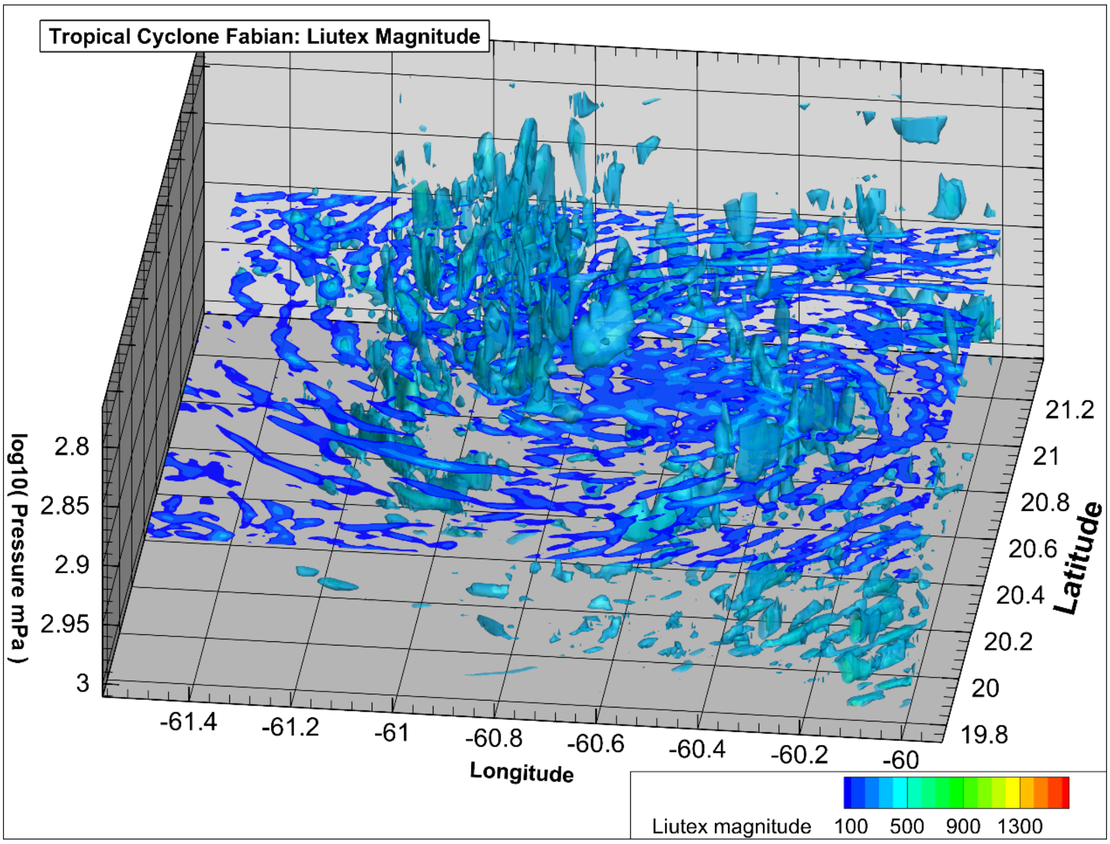

Figure 6. As in Fig. 5, but from a different angle.

Liutex magnitude in sample tropical cyclones are shown in Figs. 2-6. Liutex has made it possible to visualize vortices without being misled by the contamination of shear. The Modified Liutex-Omega method can also be used to visualize vortices (Figs. 7-8). This method can help us observe both strong and weak vortices at the same time using an iso-surface at a threshold of 0.52.

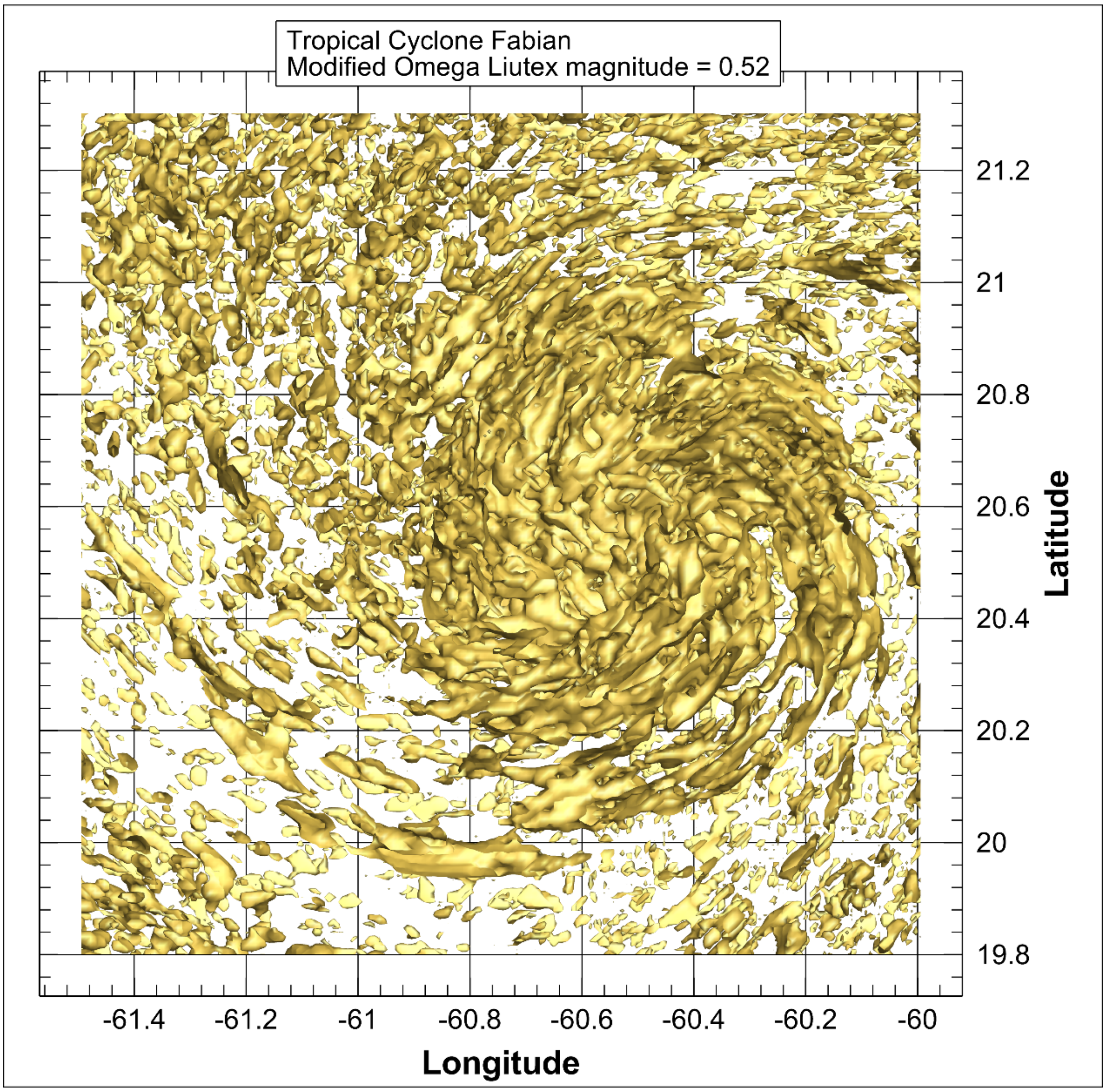

Figure 7. Modified Liutex Omega magnitude iso-surface as viewed from the top from the same hurricane Fabian dataset as in Figs. 4-6. Modified Liutex Omega is a threshold-insensitive vortex identification method where both strong and weak vortices are simultaneously shown. Both the weaker outer vortices and stronger inner vortices merge into one iso-surface allowing us to view vortices at the same time.

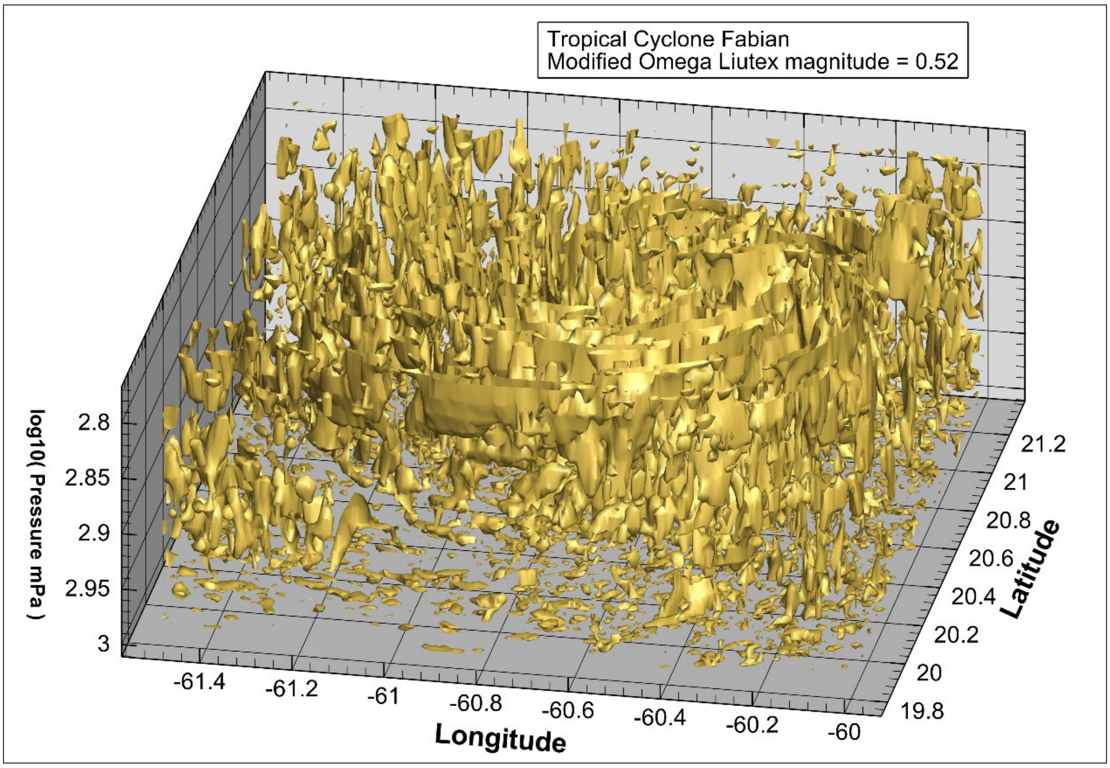

Figure 8. As in Fig. 7, but viewed from the side.

Though modified Liutex-Omega gives a good depiction of all vortices in a region, it cannot provide individual vortex structures. And even though it is suggested to use a threshold value of 0.52 for modified Liutex-Omega, this method is still not threshold - free as you can simply adjust the threshold arbitrarily. So, we need a different method to define the unique structure of a vortex. This is why the Liutex Core Line is introduced. The Liutex Core Line uniquely defines a vortex structure. The Liutex core line is a threshold-free method that defines the center axis of rotation of any vortex. These center axes of rotation define the structure of vortices. Here, we will find the Liutex core line of a single vortex within Hurricane Arthur.

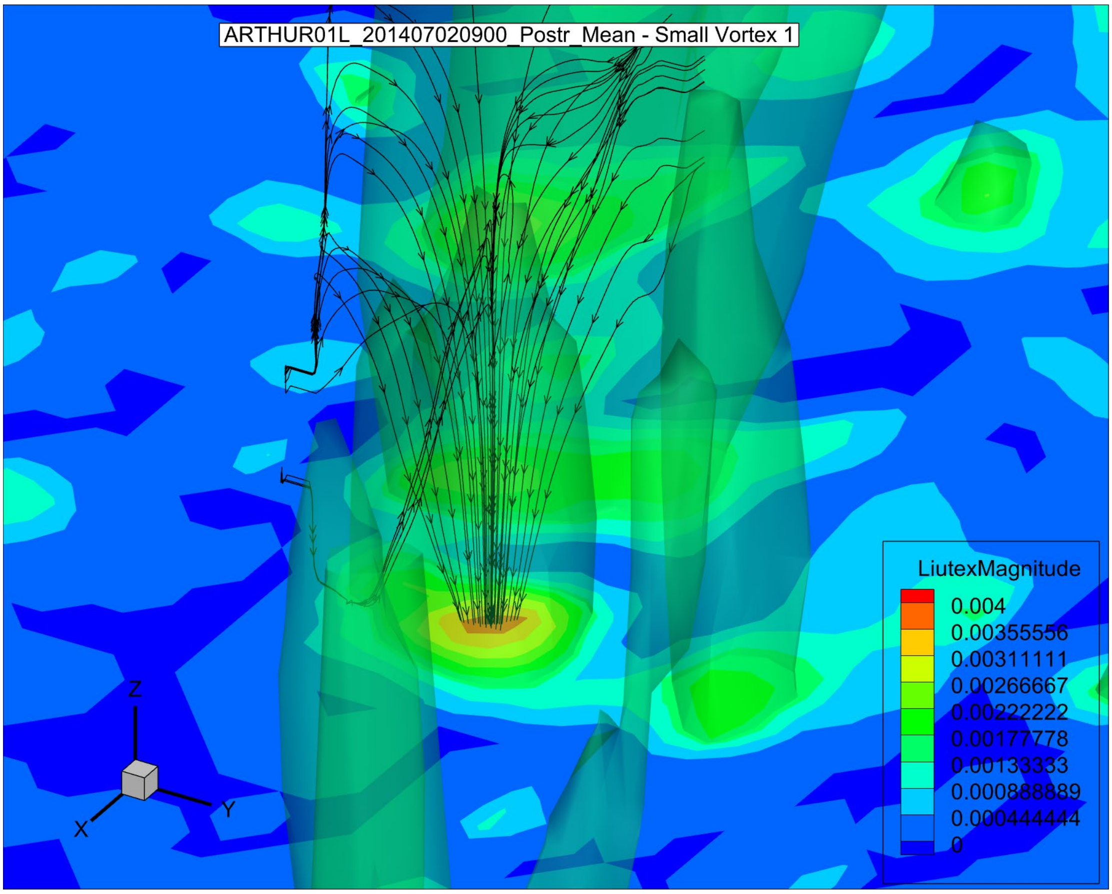

Figure 9. The first step in finding the Liutex core line manually. Liutex magnitude gradient lines are plotted on the iso-surface. These Liutex magnitude gradient lines converge to the center axis of rotation which is near a local maximum. These lines serve as a guide as to where we seed our Liutex core line.

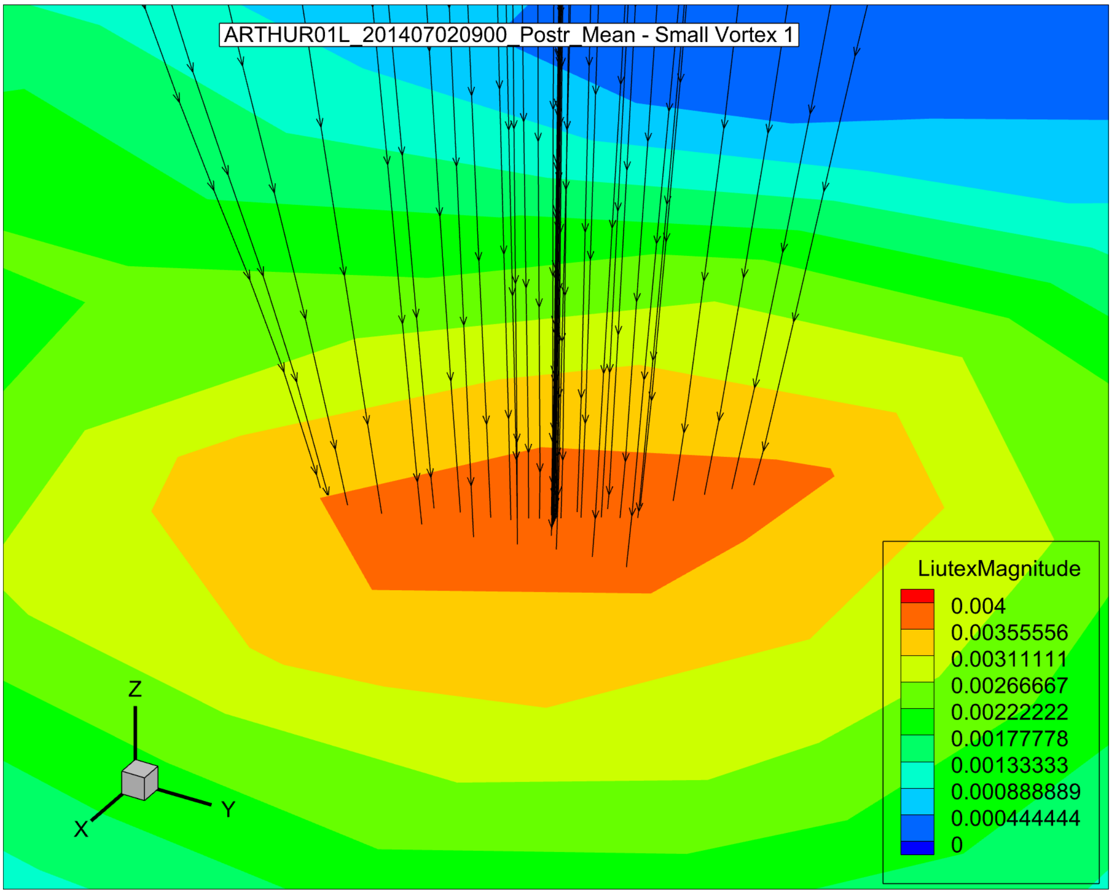

Figure 10. A closer look at the slice intersecting the Liutex magnitude gradient lines as they converge on the local maximum. This allows us to find the exact location of where to draw the Liutex line.

First, we follow the Liutex magnitude gradient lines to the center axis of rotation (Figs. 9 and 10). The Liutex magnitude gradient lines lead us to a local maximum where the center of the vortex is located. At this point, we are located on the Liutex core line, or the center axis of rotation of the vortex. From this point, we draw a Liutex line using the components of the Liutex vector.

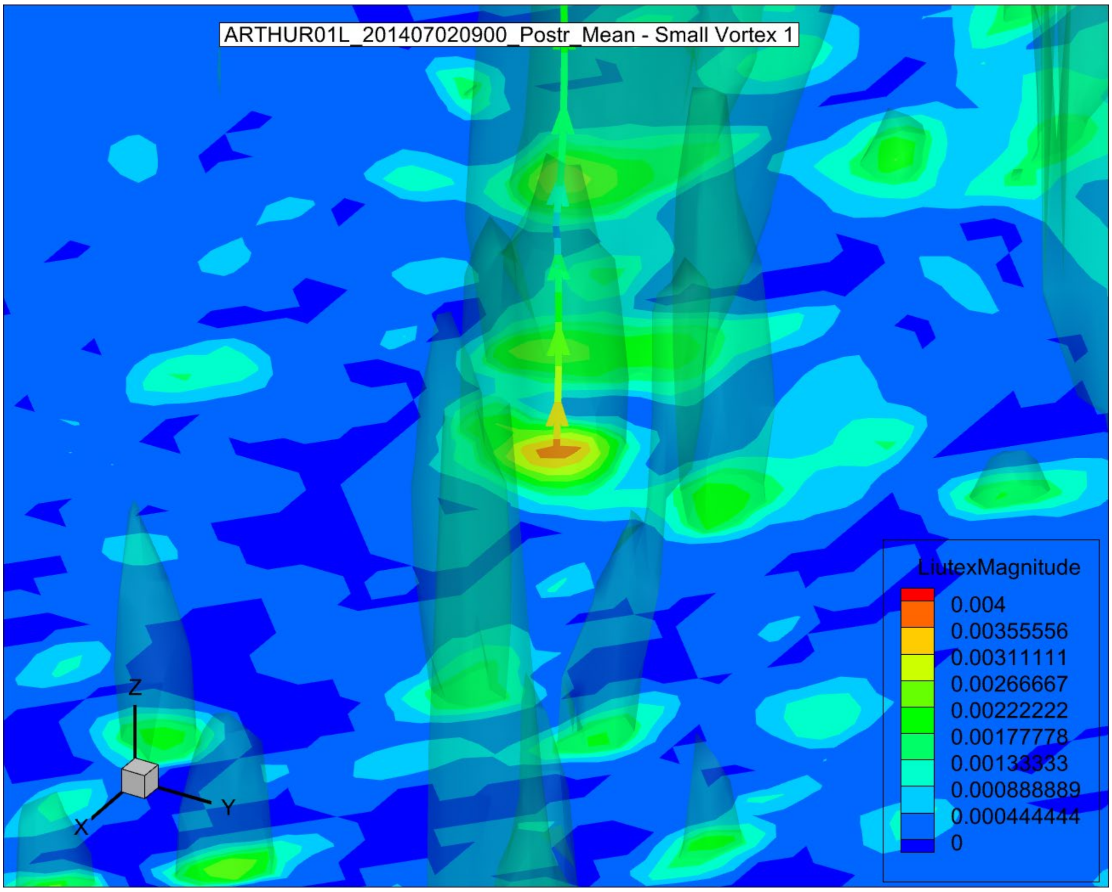

Figure 11. The Liutex Core Line (center axis of rotation) going through the depicted vortex colored by the Liutex magnitude. This core line defines the unique structure of the vortex providing the rotation direction and strength simultaneously.

The contour of the Liutex core line represents the Liutex magnitude, or the angular velocity of rotation at that point in the core line. The Liutex core line passes through several layers (Fig. 12) and Liutex magnitude iso-surface (Fig. 13). The Liutex core line is centered within the iso-surface as it represents the axis of rotation for that particular vortex.

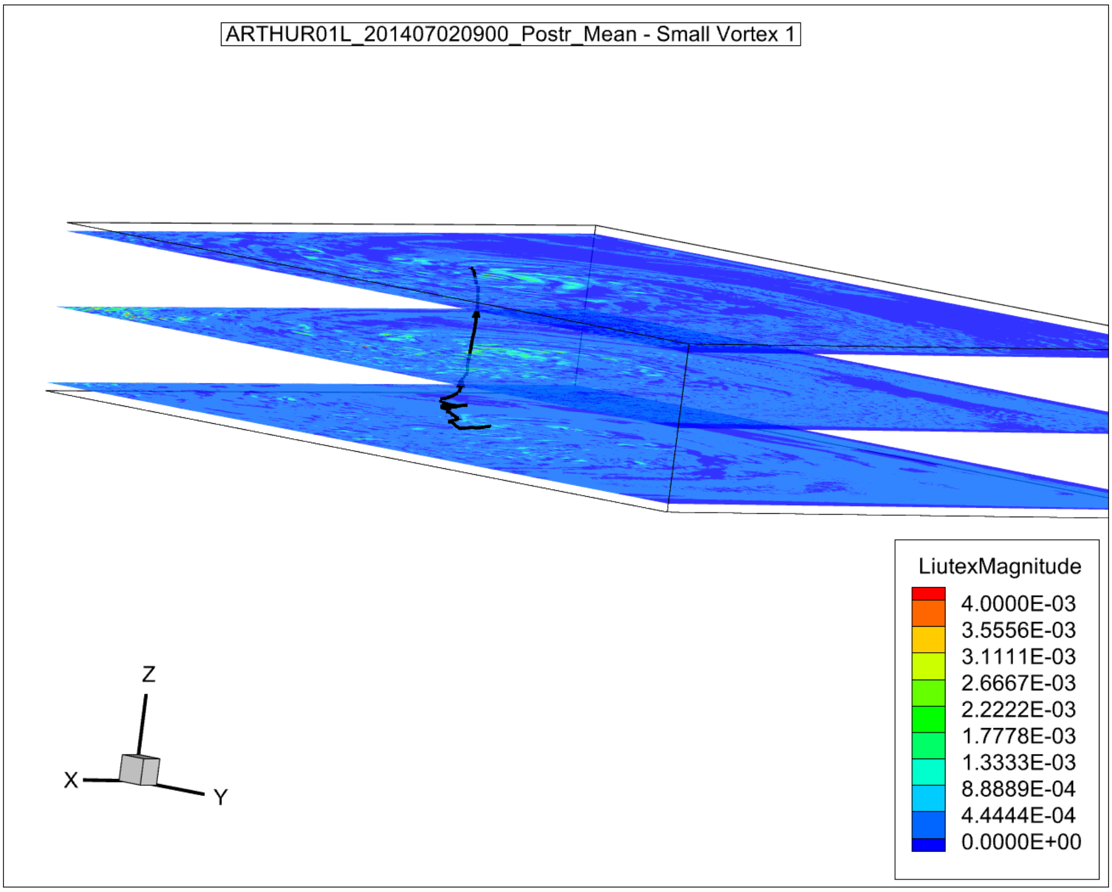

Figure 12. The path of the Liutex core line throughout the entire tropical cyclone showing the vertical extent of the vortex.

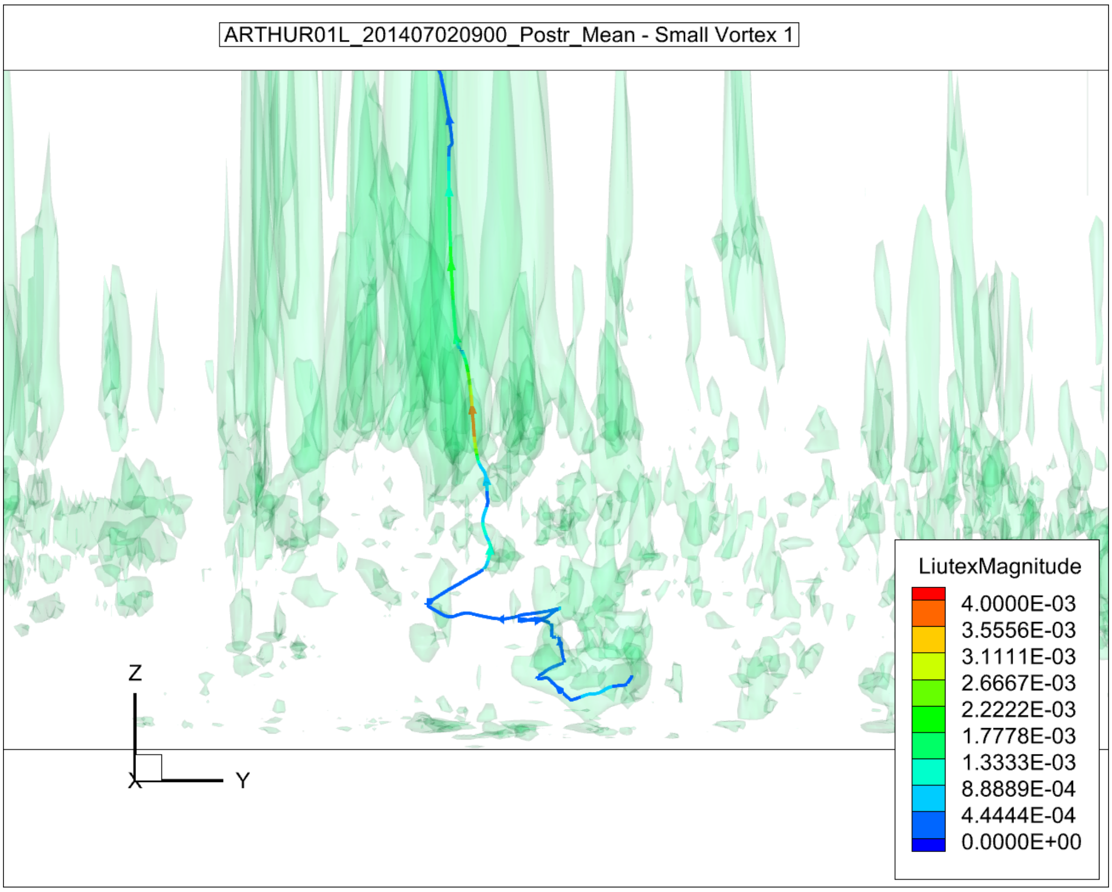

Figure 13. The full Liutex core line representing the vortex of. We can see how our vortex is weaker towards the bottom but becomes stronger with height.

With tools such as the Liutex core line, we can uniquely represent any vortex structure. This is not possible with any other methods before Liutex due to the scalar property and the contamination of shear which skews the results of what we understand a vortex to be rotation.

Figure 14 compares the three-dimensional Liutex magnitude and its two-dimensional version, LR, the Liutex-Resistance at 700 hPa showing the similarity for green with two figures on the left, and red with 2 figures on the right. The similarity suggests that the flow and rotation of a hurricane is mainly in the horizontal.

Figure 14. Figures of three-dimensional Liutex magnitude (two on the left) and two-dimensional Liutex-Resistance (two on the right) at 700 hPa. This upper two cover the whole cyclone, and the lower two focus on the center.

4. Conclusions

Vortices should not be defined by vorticity, since vorticity may be contaminated by shear. Liutex uniquely defines vortices with a direction parallel to the rotation axis with twice the angular speed as its magnitude. A better representation of vortex structures in tropical cyclones may help to better understand features such as those seen Fig. 1, and to better predict tropical cyclone track, intensity, structure, and impacts.

The Liutex iso-surface method is shear-free, but still requires a threshold. The Liutex-Omega method is threshold insensitive and able to capture both strong and weak vortices simultaneously. Finally, the Liutex core line method is threshold-free and is the only method which can provide a unique and accurate vortex structure.

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Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent: Informed consent was obtained from all individual participants included in the study.

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