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College Park, MD 20740

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Extending the Simplified Arakawa-Schubert scheme for meso-scale model applications

Hua-Lu Pan, Qingfu Liu, Jongil Han and Ruiyu Sun

NOAA/NCEP Environmental Modeling Center

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1 Introduction

The Arakawa-Schubert (Arakawa and Schubert, 1974; hereafter referred to as AS) type mass-flux based cumulus parameterization schemes are now used in many global forecast models and General Circulation Models (GCMs). One of the basic assumptions of the AS scheme is that the updraft occurs in a very small area of the grid box. This then led to the famous subsidence warming feature of the mass-flux schemes. The updraft condensation heating is exactly matched by the adiabatic cooling so the updraft stays in a moist-adiabatic profile. It is the compensating subsidence that does the warming and drying of the grid box. At NCEP, the Simplified Arakawa-Schubert (SAS) scheme (Pan and Wu, 1995) has been used in the Global Forecast System (GFS) model since 1994. The SAS scheme utilizes the same small updraft area assumption but is simplified from the original AS scheme in several ways; the use of one cloud type at a time instead of an ensemble of clouds and the addition of a parcel based cumulas convection trigger are two of the more significant ones. This assumption of the small updraft area becomes a problem as we look into applying the scheme to models with grid sizes smaller than five kilometers. As the grid size decreases, more and more grid boxes may contain a large area of updraft. The assumption of small updraft area breaks down more and more often as the model grid sizes get smaller and smaller. At the same time, there are studies (Hong and Dudhia, 2012; Hong, 2013) of model runs without a parameterized convection package in the sub-5km world that show that there is an evidence of over-development of precipitating systems. This is due to the so-called convective feedback mechanism. For example, in a grid column that is conditionally unstable and becoming saturated or super-saturated with lifting, the column becomes absolutely unstable resulting in large vertical motions, low surface pressure, and heavy precipitations. These storms generate precipitations much larger than the actual observed amount and lead to large over-precipitation bias. Tropical storm simulations with explicit schemes tend to result in storms that are too low in surface pressure and too small in size.

So the challenge is to find a way to extend the cumulus parameterization to the smaller sized grid or to go the super-parameterization (Randall et al., 2003) route. The current and planned computer resources for operational forecasts (medium-range to seasonal) make the latter route too expensive. At the same time, the physics that was represented in the AS type schemes are becoming less valid as the grid sizes become smaller and smaller.

Arakawa has been working on a unified parameterization scheme (Arakawa et al., 2011; hereafter referred to as AA2011) and has shown that the fundamental assumptions of the AS scheme will lead to heating and drying profiles that are incompatible with the actual profiles in convection resolving models. In addition, AA2011 laid out formulations of the sub-grid scale fluxes which would still be valid when the cloud area approaches the size of the entire grid area. The increased uncertainty in the estimated environmental properties, when the cloud area becomes the size of the entire grid box, led AA2011 to formulate the fluxes as a function of the updraft property and the grid mean property only.

In this note, we are going to re-derive the sub-grid scale transport term that is at the heart of the AS cumulus scheme removing the assumption that the updraft area is small and we will show that it is possible to formulate a scheme that behaves like the AS scheme when the grid sizes are large (10-km or larger) and, at the same time, can transition to an updraft-dominated scheme when the updraft area becomes large. Our approaches are different from AA2011 in that we retained the environmental property in the equations. This makes the formulation closer to the original scheme at the expense of computational complexity as the updraft area becomes one. Our concern is with the more immediate problem of representing the effect of convection in the 1-5 km models when the updraft area is likely to be greater than .1 but still not close to one. In the following sections, we will perform the derivation of the sub-grid-scale transport and the relevant large-scale prediction equations and discuss how such a scheme can provide a smooth transition and can be implemented as an upgrade to the SAS scheme in models where the updraft area is significant to void the original AS scheme but is still in need of parameterized convection.

2 Re-formulating the sub-grid scale transport term

The prognostic grid-scale equation for static energy ($s = C_pT + gz$) in a GCM grid can be written as

$$\frac{\partial \overline{s}}{\partial t} = - \bigtriangledown \cdot (\overline{\mathbf{v}}\,\overline{s}) - \frac{1}{\rho}\frac{\partial \rho \overline{w}\,\overline{s}}{\partial z} - \frac{1}{\rho}\frac{\partial \rho \overline{w's'}}{\partial z} + L(c-e), \tag{1}$$

where w' and s' are the perturbations to their grid mean values (represented by $\overline{w}$, $\overline{s}$), c and e are the condensation and evaporation rate, and L is the latent heat of vaporization.

Assuming that a grid can be broken into a convective region (σ)and its environmental region ($1-\sigma$), s_c and w_c represent the mean static energy and vertical velocity in the convective region and $\widetilde{s}$, and $\widetilde{w}$ represent the mean static energy and vertical velocity in the environmental region. The grid-scale mean static energy ($\overline{s}$) and vertical velocity ($\overline{w}$) can be expressed as

$$\overline{s} = \sigma s_c + (1-\sigma)\widetilde{s} \tag{2}$$

$$\overline{w} = \sigma w_c + (1-\sigma)\widetilde{w} \tag{3}$$

When convection occurs, the vertical transport of s on the sub-grid scale is mainly due to convection. Using the above decomposition (Eqns 2 and 3) and based on the Eqn 3 in Tiedtke (1989), $\overline{w's'}$ can be expressed as

$$\begin{aligned}\overline{w's'} &= \sigma_c(w_c - \overline{w})(s_c - \overline{s}) + (1-\sigma_c)(\widetilde{w} - \overline{w})(\widetilde{s} - \overline{s}) \\ &= \sigma_c w_c s_c + \sigma_c \overline{w}\,\overline{s} - \sigma_c w_c \overline{s} - \sigma_c \overline{w} s_c \\ &\quad + (1-\sigma_c)[\widetilde{w}\,\widetilde{s} + \overline{w}\,\overline{s} - \widetilde{w}\,\overline{s} - \overline{w}\,\widetilde{s}] \\ &= \overline{w}\,\overline{s} + \sigma_c w_c(s_c - \overline{s}) - \sigma_c \overline{w}\, s_c \\ &\quad + (1-\sigma_c)\widetilde{w}(\widetilde{s} - \overline{s}) - (1-\sigma_c)\overline{w}\,\widetilde{s} \\ &= \sigma_c w_c(s_c - \overline{s}) + (1-\sigma_c)\widetilde{w}(\widetilde{s} - \overline{s}) \end{aligned} \tag{4}$$

Eqn 4 can be rewritten as

$$\rho\overline{w's'} = M_c(s_c - \overline{s}) + \widetilde{M}(\widetilde{s} - \overline{s}) \tag{5}$$

where $M_c = \rho\sigma_c w_c$, $\widetilde{M} = \rho(1-\sigma_c)\widetilde{w}$, and $\rho\overline{w} = M_c + \widetilde{M}$ have been assumed.

Then, grid-scale dry static energy change due to convection is expressed as

$$\frac{\partial s}{\partial t}_{conv} = -\frac{1}{\rho}\frac{\partial}{\partial z}[M_c(s_c - \overline{s}) + (\rho\overline{w} - M_c)(\widetilde{s} - \overline{s})] + [Lc]_{conv} \tag{6}$$

Similarly, equations for the grid-scale specific humidity and cloud condensate changes due to convection can be derived as

$$\frac{\partial q}{\partial t}_{conv} = -\frac{1}{\rho}\frac{\partial}{\partial z}[M_c(q_c - \overline{q}) + (\rho\overline{w} - M_c)(\widetilde{q} - \overline{q})] - [c]_{conv} \tag{7}$$

$$\frac{\partial q_l}{\partial t}_{conv} = -\frac{1}{\rho}\frac{\partial}{\partial z}[M_c(q_l^c - \overline{q_l}) + (\rho\overline{w} - M_c)(\widetilde{q_l} - \overline{q_l})] + [c - rain]_{conv} \tag{8}$$

where q_l^c and $\widetilde{q_l}$ are the mean cloud condensate over the convection region (σ_i) and its environment ($1-\sigma_i$) respectively.

The grid-scale moist static energy ($h = s + Lq$) change due to convection can be derived by combining Eqns 6 and 7

$$\frac{\partial h}{\partial t}_{conv} = -\frac{1}{\rho}\frac{\partial}{\partial z}[M_c(h_c - \overline{h}) + (\rho\overline{w} - M_c)(\widetilde{h} - \overline{h})] \tag{9}$$

Eqns 9, 7, 8 can be equivalently rewritten as

$$\frac{\partial h}{\partial t}_{conv} = -\frac{1}{\rho}\frac{\partial}{\partial z}[M_c(h_c - \widetilde{h}) + \rho\overline{w}(\widetilde{h} - \overline{h})] \tag{10}$$

$$\frac{\partial q}{\partial t}_{conv} = -\frac{1}{\rho}\frac{\partial}{\partial z}[M_c(q_c - \widetilde{q}) + \rho\overline{w}(\widetilde{q} - \overline{q})] - [c]_{conv} \tag{11}$$

$$\frac{\partial q_l}{\partial t}_{conv} = -\frac{1}{\rho}\frac{\partial}{\partial z}[M_c(q_l^c - \widetilde{q_l}) + \rho\overline{w}(\widetilde{q_l} - \overline{q_l})] + [c - rain]_{conv} \tag{12}$$

3 Cloud model

If we can assume the cloud portion of the grid is in a steady state, the cloud budget equations can be written as

$$-\frac{\partial M_c}{\partial z} + E - D = 0 \tag{13}$$

$$-\frac{\partial}{\partial z}[M_c s_c] + E\widetilde{s} - Ds_c + Lc = 0 \tag{14}$$

$$-\frac{\partial}{\partial z}[M_c q_c] + E\widetilde{q} - Dq_c - c = 0 \tag{15}$$

$$-\frac{\partial}{\partial z}[M_c q_l^c] + E\widetilde{q_l} - Dq_l^c + c - rain = 0 \tag{16}$$

where D and E are detrainment and entrainment rate respectively. Eqns 14 and 15 can be combined to give an equation for h.

$$-\frac{\partial}{\partial z}[M_c h_c] + E\widetilde{h} - Dh_c = 0 \tag{17}$$

4 Final equations

Substitute Eqns 17, 15, and 16 into Eqns 10, 11, and 12 we have a set of equations for the contribution of sub-grid scale convection as follows.

$$\frac{\partial \overline{h}}{\partial t}_{conv} = -\frac{1}{\rho}(E\widetilde{h} - Dh_c) + \frac{1}{\rho}\frac{\partial}{\partial z}[M_c\widetilde{h} + \rho\overline{w}(\overline{h} - \widetilde{h})] \tag{18}$$

$$\frac{\partial \overline{q}}{\partial t}_{conv} = -\frac{1}{\rho}(E\widetilde{q} - Dq_c) + \frac{1}{\rho}\frac{\partial}{\partial z}(M_c\widetilde{q} + \rho\overline{w}(\overline{q} - \widetilde{q})] \tag{19}$$

$$\frac{\partial \overline{q_l}}{\partial t}_{conv} = -\frac{1}{\rho}(E\widetilde{q_l} - Dq_l^c) + \frac{1}{\rho}\frac{\partial}{\partial z}(M_c\widetilde{q_l} + \rho\overline{w}(\overline{q_l} - \widetilde{q_l})] \tag{20}$$

The first two equations are similar to the large-scale prediction equations in the original AS manuscript (eqs.74 and 75)where dry static energy s is used instead of moist static energy h for the convection effect. We retained all the entrainment and detrainment terms as the SAS is a bulk cloud model and not an ensemble cloud model. For the second term on the right-hand side of Eqns 18 and 19, the first portion shows the compensating subsidence effect except the property of the environment is specifically used instead of the grid mean as in AS. The second portion of term 2 represents the additional effect when the updraft area is large. Interpretation of this effect is still needed. The three equations (18, 19, and 20) were derived without the assumption about the size of the updraft. What we now need is a specification of the updraft region fraction (σ) of the model grid. With the knowledge of σ, we can derive the variables for the environmental region used in Eqns 18- 20. Specifying entrainment and detrainment as functions of the mass flux leaves the mass flux (Mc) as the one variable to close the system just like the original AS scheme and the SAS scheme.

Eqn 20 in the current form is quite different from the actual practice in the current SAS. In SAS, the only term retained in the right hand side of Eqn 20 is the detrained cloud water term. In the cloud model of the current SAS, the total cloud condensate is transported from one layer to the layer above instantly. The key assumption made is that the vertical velocity is so large that maintenance of moist adiabat is instantaneous. Entrainment of water vapor from the environment is considered in the transport but entrainment of cloud condensate is not. This assumption is effectively assuming that the environmental cloud condensate is zero (or not interacting with the cumulus). When the transported total cloud water exceeds the local saturation specific humidity, detrainment of cloud water is parameterized as well as conversion to rain water. So Eqn 17 is used to build the cloud model while the only effect retained in Eqn 20 is the detrainment of cloud water. A more elaborate cloud model can be built that will allow more interaction with the micro-physics schemes for meso-scale models in the future.

The SAS closure is a modified form of the original AS closure of quasi-equilibrium (Lord, 1978). We modified the fraction of climatological cloud work function and the relaxation time based on the large-scale vertical motion at cloud base. So the mass flux is still the quantity to be determined at closure. With the present set of equations (18, 19, and 20), it is still possible to use the same closure provided we understand that the closure comes from the portion of the mass flux that is due to the cloud work function (or buoyancy) only. We will discuss this point in the next section after we propose a way to determine the updraft area σ.

5 Determination of the updraft area

Given that the original AS scheme simply assumed that the updraft area is small, the accuracy of the actual specification of this variable may not be crucial. We should make sure that the area (σ) is small (5-10%) for grid sizes over 50 km and is allowed to become large as the grid area becomes smaller. Parameterization, by definition, is a crude simulation of reality. The accuracy of the specification of σ will be one of those tunable variables. We have tentatively decided to use the ratio of the scaled cloud vertical motion ($=\sqrt{\overline{w_b}^2 + 2 * cloudworkfunction}$) and the grid column maximum vertical motion ($\overline{w}$) to determine the fraction ($\sigma = \overline{w} \ / \ w_c$) . This is equivalent to using Equation 3 neglecting the $\widetilde{w}$ term. Physically, updraft speed is usually much larger than the compensating subsidence ($w_c >> \widetilde{w}$) as subsidence occurs over a much larger area. So when the grid area is large, the grid mean vertical velocity is likely much smaller than the updraft speed. When the updraft area becomes comparable to the size of the model grid, (1-σ) becomes small while $\widetilde{w}$ remains small so the use of w_c and $\overline{w}$ to estimate σ is an even better approximation compared to situations when the grid size is much larger than the updraft area. The value of the updraft area (σ) is less important when the grid sizes are large, as the AS type of mass flux schemes actually make the assumption that sigma is zero except when it is multiplied with the updraft speed to form the mass flux. In the current formulation, we are actually likely making a factor of two error in the estimation of sigma for small sigma while the approximation gets better as σ increases. In order to still reduce the errors for larger size grids, we have adopted a formula for σ assuming that the magnitude of the subsidence is an order of magnitude smaller than that of the updraft. This leads to the formula σ = .91 $\overline{w}$ / w_c + .09.

Because the cloud vertical velocity is a combination of the cloud base vertical velocity and the buoyancy effect, the normal closure which estimates the change of the air column due to the change of a unit of cloud work function needs to be modified when the mass flux is no longer dominated by the buoyancy effect. Instead, we propose to scale the Eqns 18- 20 by the portion that is due to the buoyancy effect.

6 Convergence

In principle, when the updraft area approaches unity, the updraft properties (s_c and w_c) approach the grid mean values ($\overline{s}$ and $\overline{w}$). The sub-grid scale transport in Eqn 4 approaches zero (in the first term on the right-hand-side because s_c approaches $\overline{s}$ and in the second term because (1-σ) approaches zero). So the effects of the parameterized convection should diminish as the updraft area increases. In our experience, as the environment becomes more moist (such as in the hurricane environment), the cloud work function becomes smaller as the sounding approaches moist adiabatic even with the current version of the SAS. As pointed out by AA2011, the use of the environmental properties in this scheme is troublesome as these properties become undefined when the updraft area is the same as the grid area. We currently terminate the scheme when the updraft area is greater than .9. When the updraft area is that large, the numerical problem due to the grid-scale condensation is likely to be small so turning off the parameterization should be safe. We have achieved the convergence in theory but not completely in practice. The problem will be addressed in future studies.

7 Testing the scheme

The new scheme, now named the meso-SAS, has been tested in the NCEP Hurricane WRF(Weather Research and Forecast) model together with a modification of the initialization procedure (Developmental Testbed Center release v3.5a of the HWRF system, 14, August, 2013). The current HWRF is a tripple nest model with the nest resolutions of 27, 9, and 3 km. The current configuration of the HWRF runs with SAS parameterization only in the outer two nests while the inner nest runs without a parameterized convection package. This is done because it is no longer justified to run the SAS when the grid resolution is less than 5km as the assumption in SAS that the updraft area be small is no longer valid. While preliminary diagnosis shows that the updraft area in most of the HWRF runs are still small (the largest values in the run are in the .1 to .2 range), it does show that there is a need to use a scheme that does not rely on the assumption of small updraft area. The meso-SAS is implemented in all three nests as the scheme is written to run in coarse and fine mesh resolutions.

In Fig. 1, we show the track errors for all forecasts made for 2012 for the Atlantic basin. It can be seen that the modified initial conditions are able to improve the track forecasts at all time ranges while the meso-SAS addition improves upon the modified initial condition in time ranges after 48 hours. In Fig. 2, we show the intensity error of all forecasts made for 2012 for the Atlantic basin. We can conclude that modified initial condition improves the intensity forecast at all time ranges while the meso-SAS provides further improvements in forecast beyond 48 hours. In Fig. 3 and 4, we show the same statistics for the eastern Pacific basin and the same conclusion can be drawn.

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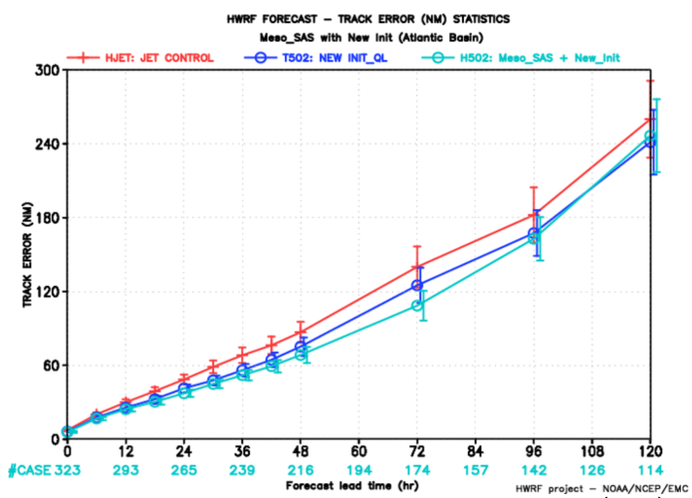

Figure 1: Track error for the Atlantic basin (nm)

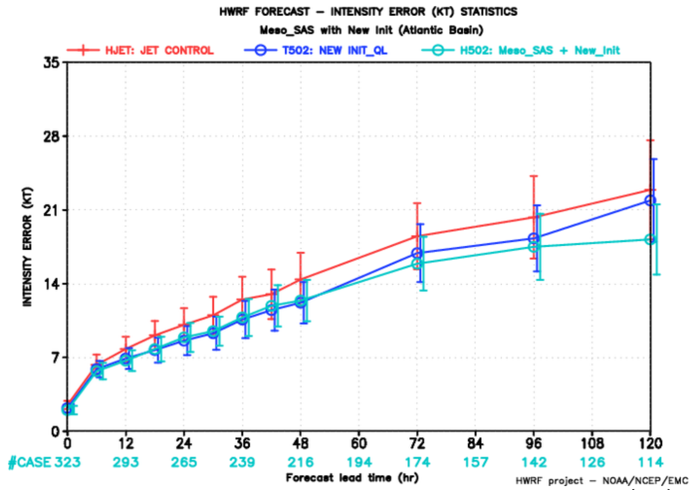

Figure 2: Intensity error for the Atlantic basin (kt)

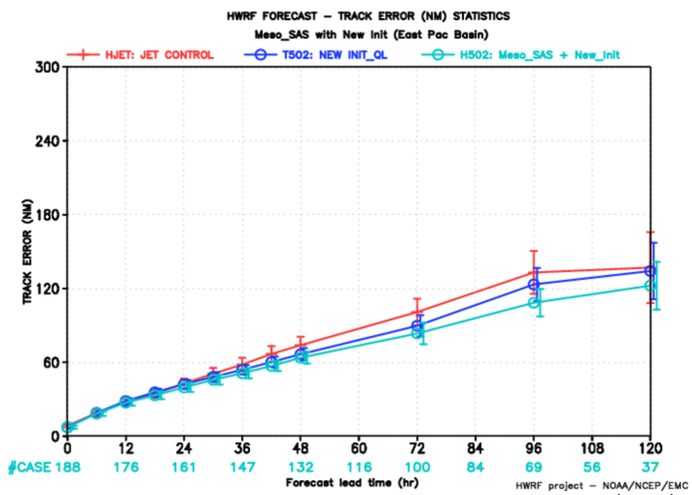

Figure 3: Track error for eastern Pacific basin (nm)

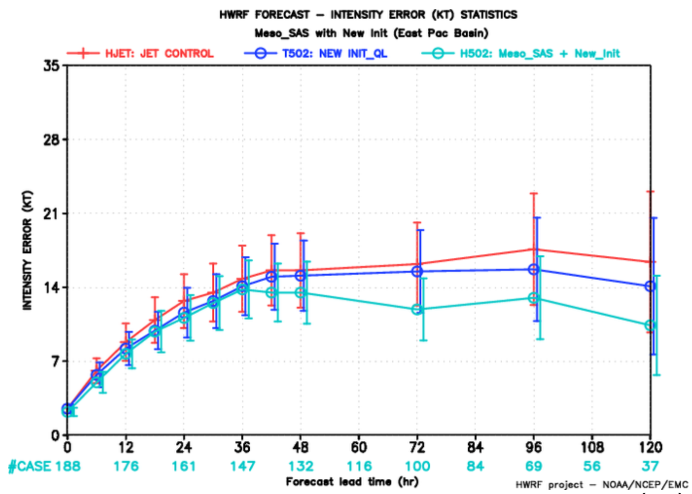

Figure 4: Intensity error for eastern pacific basin (kt)