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IS WEATHER CHAOTIC? COEXISTENCE OF CHAOS AND ORDER WITHIN A GENERALIZED LORENZ MODEL

Bo-Wen Shen[1,*], Roger A. Pielke Sr.[2], Xubin Zeng[3], Jong-Jin Baik[4], Sara Faghih-Naini[1,5], Jialin Cui[1,6], and Robert Atlas[7]

ABSTRACT: Since Lorenz's 1963 study and 1972 presentation, the statement "weather is chaotic'' has been well accepted. Such a view turns our attention from regularity associated with Laplace's view of determinism to irregularity associated with chaos. In contrast to single type chaotic solutions, recent studies using a generalized Lorenz model (Shen, 2019a, b; Shen et al., 2019) have focused on the coexistence of chaotic and regular solutions that appear within the same model, using the same modeling configurations but different initial conditions. The results suggest that the entirety of weather possesses a dual nature of chaos and order with distinct predictability. Furthermore, Shen et al. (2021a, b) illustrated the following two mechanisms that may enable or modulate attractor coexistence: (1) the aggregated negative feedback of small-scale convective processes that enable the appearance of stable, steady-state solutions and their coexistence with chaotic or nonlinear limit cycle solutions, referred to as the 1st and 2nd kinds of attractor coexistence; and (2) the modulation of large-scale time varying forcing (heating) that can determine (or modulate) the alternative appearance of two kinds of attractor coexistence.

Recently, the physical relevance of findings within Lorenz models for real world problems has been reiterated by providing mathematical universality between the Lorenz simple weather and Pedlosky simple ocean models, as well as amongst the non-dissipative Lorenz model, and the Duffing, the Nonlinear Schrodinger, and the Korteweg–de Vries equations (Shen, 2020, 2021). We additionally compared the Lorenz 1963 and 1969 models. The former is a limited-scale, nonlinear, chaotic model; while the latter is a closure-based, physically multiscale, mathematically linear model with ill-conditioning. Based on the above findings and results obtained using real-world global models, we then discuss new opportunities and challenges in predictability research, with the aim of improving predictions at extended-range time scales as well as sub-seasonal to seasonal time scales.

Keywords: attractor coexistence, chaos, generalized Lorenz model, Pedlosky model, predictability.

INTRODUCTION

Two studies of Prof. Lorenz (Lorenz, 1963, 1972) laid the foundation of chaos theory that emphasize a Sensitive Dependence of Solutions on Initial Conditions (SDIC). While the concept of SDIC can be found in earlier studies (e.g., Poincare, 1890), the rediscovery of SDIC in Lorenz (1963) changed our view on the predictability of weather and climate, yielding a paradigm shift from Laplace's view of determinism with unlimited predictability to Lorenz's view of deterministic chaos with finite predictability. Based on an insightful analysis of the Lorenz 1963 and 1969 (L63 and L69) models, as well as the recent development of generalized Lorenz models (GLM, Shen, 2014, 2019a, b; Shen et al., 2019), such a conventional view is being revised to emphasize the dual nature of chaos and order in recent studies (Shen et al., 2021a, b). To support and illustrate the revised view, this short report presents the following major features: (1) Continuous vs. Sensitive Dependence on Initial Conditions (CDIC vs. SDIC); (2) single-types of attractors and monostability within the L63 model; (3) coexisting attractors and multistability within the GLM; (4) Skiing vs. Kayaking: an analogy for monostability and multistability; and (5) a list of non-chaotic weather systems.

ANALYSIS AND DISCUSSION

CDIC vs. SDIC

Figure 1 compares the time evolution of solutions from control and parallel runs that apply the same L63 model and parameters. The only difference in the two runs is that a tiny perturbation with $\epsilon = 10^{-10}$ was added into the initial condition of the parallel run. Both runs initially produce very close results but very different results at a later time. Initial comparable results indicate CDIC, an important feature of dynamic systems. Despite initial tiny differences, large differences in both runs, as indicated by the red and blue curves, appear at a later time. Such features are then referred to as SDIC, suggesting that a

[1*]Department of Mathematics and Statistics, San Diego State University, San Diego, CA, USA, Email: bshen@sdsu.edu.
[2]CIRES, University of Colorado at Boulder, Boulder, CO, USA.
[3]Department of Hydrology and Atmospheric Science, University of Arizona, Tucson, AZ, USA.
[4]School of Earth and Environmental Sciences, Seoul National University, Seoul, South Korea.
[5]University of Bayreuth and Friedrich-Alexander University Erlangen-Nuremberg, Germany.
[6]Department of Computer Sciences, North Carolina State University, Raleigh, NC, USA.
[7]AOML, National Oceanic and Atmospheric Administration, Miami, FL, USA.

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tiny change in an IC will eventually lead to a very different time evolution for a solution.

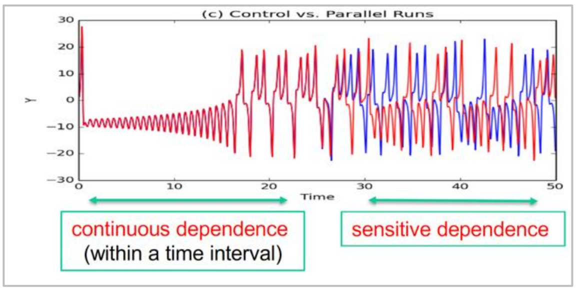

Fig. 1. An illustration of SDIC. Control and parallel runs were performed using the same model and the same model parameters. The only difference is the inclusion of an initial tiny perturbation within the parallel run. Two runs initially produced almost the same result for $\tau \in [0, 25]$, as shown with the red curve. This feature is called CDIC. During longer time integrations, the appearance of two curves (in red and blue) indicates significant differences (i.e., the "rapid divergence") of solutions for the two runs. Such a feature is then called SDIC.

Single-type of attractors and monostability

Since Lorenz (1963), chaotic solutions have been a focal point for several decades, yielding the statement of "weather is chaotic". In fact, depending on the relative strength of heating, the L63 model also produces non-chaotic solutions such as steady-state and limit cycle solutions (e.g., Figure 1 of Shen et al., 2021b). Given a model configuration, only one-type of solution appears, referred to as monostability, as shown in Fig. 2 (left).

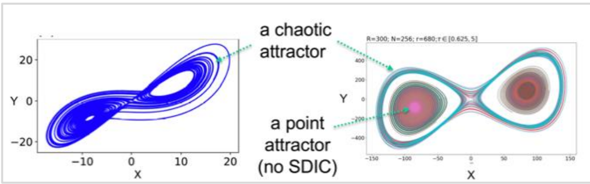

Fig. 2. Monostability illustrated by the chaotic solution of the L63 model (left), and multistability by coexisting chaotic and steady-state solutions of the GLM (right).

Coexisting attractors and multistability

By comparison, as shown in Fig. 2 (right), one of the major features within the GLM is so-called multistability with coexisting attractors. Two kinds of attractor coexistence include the 1st kind that contains coexisting chaotic and steady-state solutions and the 2nd kind that possesses coexisting, limit-cycle, and steady-state solutions. As a result of the multistability, SDIC does not always appear.

Skiing vs. Kayaking: an analogy for monostability and multistability

To illustrate SDIC, Lorenz (1993) applied the activity of skiing (left in Fig. 3) and developed an idealized skiing model for revealing the sensitive dependence of time-varying paths on starting points (middle in Fig. 3). The left panel for skiing may indicate monostability when slopes are steep everywhere. By comparison, the right panel for kayaking is used to illustrate multistability. In the photo, both strong currents and a stagnant area (outlined with a white box) can be identified, suggesting both instability and local stability. As a result, when two kayaks move along strong currents, their paths display SDIC. On the other hand, when two kayaks move into the stagnant area, they become trapped. The features of kayaking reveal the nature of multistability.

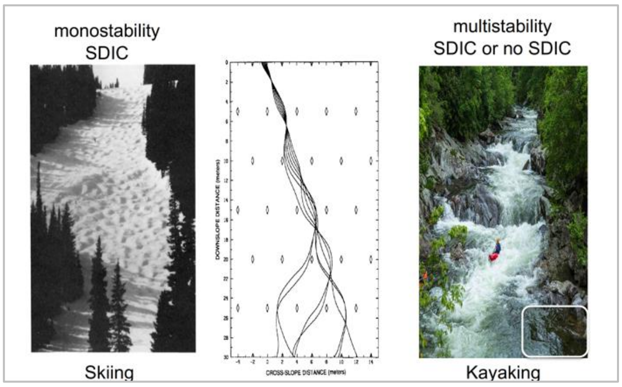

Fig. 3. Skiing as used to reveal SDIC and monostability (left and middle, Lorenz, 1993), and kayaking as used to indicate multistability (right, Copyright: ©Carol-stock.adobe.com).

Non-chaotic weather systems

Although chaotic solutions have received great attention, non-chaotic solutions have also been applied for understanding the dynamics of different weather systems, including steady-state solutions for investigating atmospheric blocking (e.g., Charney and DeVore, 1979; Crommelin et al., 2004), limit cycles for studying Quasi-Biennial Oscillations (e.g., Renaud et al., 2019) and vortex shedding (Ramesh et al., 2015), and nonlinear solitary-pattern solutions for understanding morning glory (i.e., low-level roll clouds, Goler and Reeder, 2004).

CONCLUDING REMARKS

By deploying a generalized Lorenz model (GLM) containing major features of attractor coexistence and multistability, we discussed time varying multistability that is enabled and/or modulated by (1) the aggregated negative feedback of small-scale convective processes and (2) large-scale time varying forcing (heating). Using an insightful analysis of the L63 and L69 models, we

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presented two types of sensitivities. The SDIC of the L63 model was applied to define the butterfly effect. The 2nd type of sensitivity with ill-conditioning was found in the L69 model. As a result of multistability, SDIC does not always appear within the GLM.

Based on the classification of intrinsic and practical predictabilities that display a dependence on flow, and imperfect numerical tools and observations, respectively, we further showed that:

- The L63 nonlinear model is effective for revealing the chaotic nature of weather, suggesting finite intrinsic predictability.
- The GLM with multistability suggests both limited and unlimited intrinsic predictability for chaotic and non-chaotic solutions, respectively.
- Using selected cases within a global model (e.g., Shen, 2019b), a practical predictability of 30 days was previously documented.

The above results suggest a revised view on the dual nature of chaos and order with distinct predictability in weather and climate. The refined view on the dual nature of weather is neither too optimistic nor pessimistic as compared to the Laplacian view of deterministic predictability that is unlimited and the Lorenz view of deterministic chaos with finite predictability. The refined view may unify our theoretical understanding of different predictability and recent global model simulations that display promising results at two to four week time scales.

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