

A new way to visualize and report structural and data uncertainty in stock assessments

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Abstract: We present a visual and tabular representation of fisheries stock assessment model outputs to rapidly examine and effectively communicate sensitivity analysis results from numerous alternative model comparisons. This approach uses multiple output metrics to identify which alternative stock assessment model configurations relative to the reference model deserve further attention when quantifying intermodel uncertainty. An accompanying table of likelihood components, parameters, and model-derived quantities highlights where major changes exist compared with the reference model. The general method is applicable to any stock assessment and should aid in model behavior diagnosis and communicating uncertainty to managers. Specific examples and code are provided for the Stock Synthesis modelling framework.

Résumé : Nous présentons une représentation visuelle et tabulaire de données de sortie de modèles d'évaluation de stocks halieutiques pour l'examen rapide et la communication efficace de résultats d'analyses de sensibilité issus de comparaisons de multiples modèles. Cette approche utilise différents paramètres de sortie pour établir quelles configurations de modèles d'évaluation de stock, comparativement au modèle de référence, méritent plus d'attention pour la quantification de l'incertitude entre modèles. Un tableau associé des composantes de vraisemblance, paramètres et quantités dérivées des modèles fait ressortir les grandes différences par rapport au modèle de référence. La méthode générale peut s'appliquer à toute évaluation de stock et devrait faciliter le diagnostic du comportement de modèles et la communication de l'incertitude aux gestionnaires. Des exemples et du code sont fournis pour le cadre de modélisation Stock Synthesis. [Traduit par la Rédaction]

Introduction

Fisheries stock assessment takes a prominent role in managing marine fisheries to ensure long-term sustainability of marine resources (Hilborn and Walters 1992; Walters and Martell 2004). Stock assessments use mathematical and statistical methods to estimate current status of the stock, reconstruct its past dynamics, and forecast population dynamics into the future under alternative management strategies (Quinn and Deriso 1999; Haddon 2001).

Any model is a mathematical approximation of reality. The goal of a stock assessment scientist is to balance realism and parsimony to describe with measured certainty the general trends in fish populations, derived from a specific model structure and data inputs. It is important, therefore, to not only produce point estimates of management quantities from stock assessments but also describe uncertainty around those estimates and communicate this uncertainty to decision makers to help them choose appropriate management alternatives (Magnusson et al. 2013). Quantifying and communicating uncertainty are two of the most important aspects of conducting a stock assessment.

In a stock assessment framework, such as Stock Synthesis (Methot and Wetzel 2013), a portion of uncertainty is accounted for within the assessment model, and the asymptotic confidence intervals are calculated around the point estimates. These intervals reflect the uncertainty in the model fit to the data included in the assessment, but do not account for model specification uncertainty (e.g., fixed parameters, alternative model configurations). Therefore, stock assessments routinely rely on analysis of model sensitivity to model assumptions to describe a greater degree of

uncertainty than that estimated within a reference model. Stock assessment scientists often need to apply multiple sensitivity tests, resulting in a large number of model comparisons. It is then challenging to effectively present and communicate results of numerous sensitivity analyses to colleagues and decision makers (Levontin et al. 2017, Miller et al. 2019).

In this paper, we describe a method to quickly and transparently identify which aspects of the model contribute to additional uncertainty and deserve further attention when quantifying uncertainty in model outputs and management quantities. The method includes a figure that compares how important stock status metrics from alternative model configurations relate to the uncertainty estimated within the reference model. The figure is accompanied with a detailed table that summarizes likelihood components, parameters, and management quantities for each alternative model configuration. This table aims to easily identify which data and (or) parameters contribute to significant differences beyond the uncertainty captured in the reference model. We illustrate the application of this method to the stock assessment of yelloweye rockfish (*Sebastes ruberrimus*) in the Northeast Pacific Ocean (Gertseva and Cope 2018).

Methods

Visual method description

The proposed method uses outputs from the stock assessment reference model (i.e., the best-fit model based on maximum likelihood used for management) and associated uncertainty estimates to establish a relative comparison among the sensitivity tests. This approach is widely applicable since sensitivity analyses

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should be an integral part of fisheries stock assessment protocol. In this example we develop the approach around the Stock Synthesis modelling framework and provide code (<https://github.com/shcaba/Stock-Assessment-Sensitivity-Plots>) to facilitate creating the figures, table, and output object in any Stock Synthesis-based assessment. We accompany the plotting and table generating code with the sensitivity runs and run code, figures, and complete tables for the example described in this paper (https://github.com/shcaba/Stock-Assessment-Sensitivity-Plots/tree/master/Cope_Gertseva/YEYE_Sensi_models/). The repository will also have code and examples to apply it to the most recent version of Stock Synthesis. The function is also being added to the r4ss package (<https://github.com/r4ss/r4ss>) to facilitate easier use.

We used five model outputs for comparison purposes: (i) the estimate of the spawning stock biomass or output (SO) in its initial (e.g., unfished) state (SO_0), (ii) the estimate of the SO in the terminal year of assessment (SO_t), (iii) relative stock status, which is the ratio of the spawning stock biomass in the terminal year relative to initial state (SO_t/SO_0), (iv) the estimate of maximum sustainable yield (MSY; or MSY proxy), and (v) the maximum sustainable fishing rate (F_{MSY} or proxy). The first two outputs are used to determine the absolute abundance of the stock. The third output is a relative measure of stock reduction from initial conditions (e.g., 20% means that stock is at 20% of its initial state) and is typically used to determine stock status relative to management reference points. The fourth output serves as measure of both absolute abundance and productivity (i.e., the stock capacity to produce new biomass relative to what is lost due to death) and indicates the absolute amount of fishery removals that the stock can sustain in the long term. The fifth output, F_{MSY} , is a measure of stock productivity relative to fishery selectivity. The general method of examining model sensitivity is not constrained to these five derived quantities, as any derived values can be substituted or added to this list to best match management objectives. All values of MSY and F_{MSY} use the spawning potential ratio (SPR) proxy values as described in Gertseva and Cope (2018).

We define a sensitivity run as an alternative configuration relative to the reference model. The difference in each model output between the alternative run and the reference model was calculated as follows:

$$(1) \quad RC = \frac{X_{alt} - X_{ref}}{X_{ref}}$$

where RC is relative change, X_{alt} is a value for a given model output (e.g., SO_0 , SO_t , SO_t/SO_0 , MSY_{SPR} , or F_{SPR}) estimated by the alternative model, and X_{ref} is a value for a given model output estimated by the reference model. This approach uses the reference model as the point of comparison by which all other models are compared, but does not propose any given reference model is the “correct” model, as this is not a simulation test, but an evaluation of alternative model configurations compared with the reference model. In addition, log relative change is also calculated as

$$(2) \quad \log RC = \ln \frac{X_{alt}}{X_{ref}}$$

This metric addresses the asymmetric relationship between RC above (technically infinite) and below (i.e., when X_{alt} approaches 0, RC approaches -1 , but $\log RC$ approaches negative infinity) the reference value, though its units may offer less intuitive interpretation to stakeholders.

We also translate the reference model 95% asymptotic confidence intervals into each RC measure for each of the derived model outputs. Confidence intervals expressed as an RC range indicate uncertainty within the reference model; thus, alternative

model configuration values outside these intervals highlight metric values with notable difference from the reference model. Such model sensitivity runs would thus be capturing uncertainty beyond the asymptotically derived estimates of the reference model and be of particular interest when capturing uncertainty beyond the bounds of the reference model.

In addition, the figure can include target (TRP) and limit (LRP) stock status reference points (or any other reference points of interest) to identify how results of various sensitivity runs translate to management reference points and which alternative assumptions (either in term of data or parameters) would lead to a notable change in stock status relative to those reference points. Summary plots are provided to compare the pattern among all metrics and sensitivity scenarios, as well as metric-specific plots. An R object of relative change is also produced containing all values and models presented in the plots.

The table that accompanies the figure provides detailed model outputs for the reference model and alternative configurations, as well as likelihood component contributions and parameter values (whether estimated or fixed), so the details of each sensitivity run can be used to trace the source of the significant differences from the reference model seen in the figure.

Application of the visual method to assessment of yelloweye rockfish

We applied the visual method described to stock assessment of the yelloweye rockfish (Gertseva and Cope 2018). The assessment was adopted as the basis for management advice on the US West Coast, the area managed by the Pacific Fishery Management Council (PFMC).

Yelloweye rockfish assessment overview

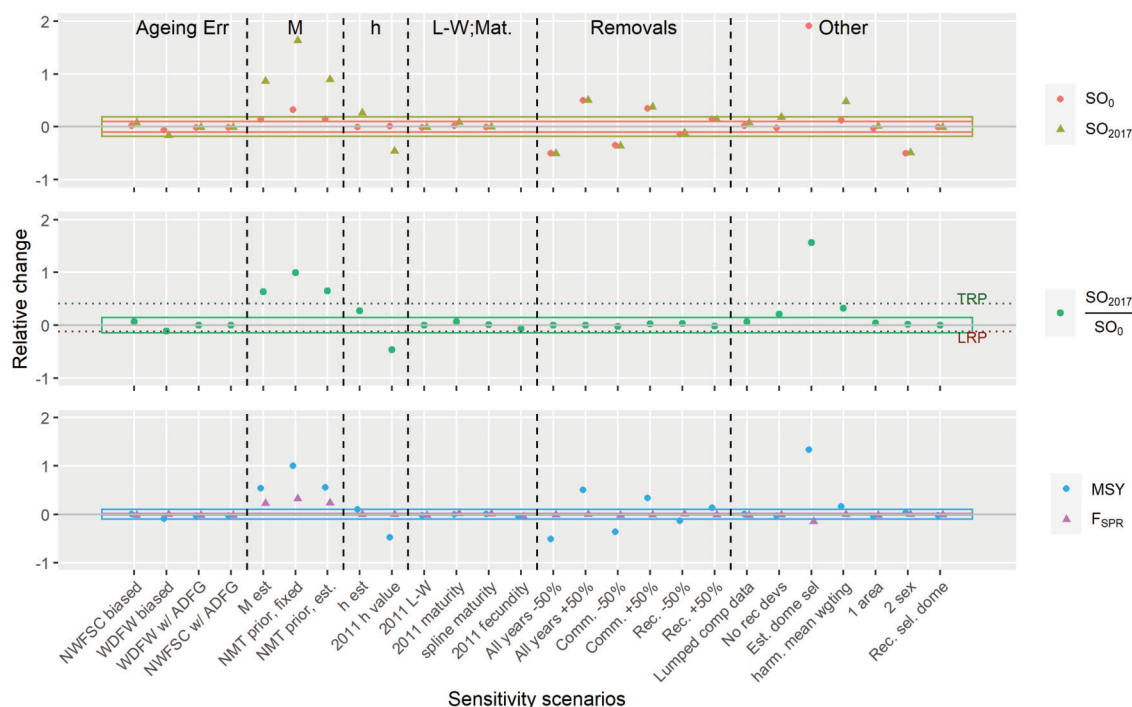
Yelloweye rockfish is a long-lived (>100 years old) US West Coast groundfish that gained management attention when it was declared overfished in 2002. Catches of yelloweye rockfish have been highly constrained and have had a profound impact on several fishery sectors. The assessment took a fresh look at all aspects of data treatment and assessment model structure and assumptions to arrive at a reference model. This reference model passed review and was considered the best available scientific information for management.

The yelloweye rockfish assessment used the Stock Synthesis framework (Methot and Wetzel 2013) and is relatively data-limited compared with many other West Coast groundfishes. The modeling period goes back to 1889, when the yelloweye rockfish stock was assumed to be in an unfished state. The stock was modelled with two explicit spatial areas: waters off California and waters off Oregon and Washington, which are linked by a common stock-recruit relationship. Each area has its own unique catch history and a number of fishing fleets specific to each area. Females and males in the assessment are combined, since estimates of growth parameters did not differ between sexes. Several life history parameters, essential for understanding the dynamics of the stock, such as natural mortality and stock-recruit steepness, were internally inestimable and needed to be fixed at values estimated outside the model. The above reference model decisions led to plausible alternative model configurations that we used to explore assumptions in data and parameters in the reference model.

Model configurations explored in the assessment

Within the assessment we explored model sensitivity to different data sources that contribute to likelihood estimates within the model and to a broad spectrum of alternative model configurations. To demonstrate the use of our visual method, we focus on analysis related to the alternative model configurations. Application of this method to analysis of the likelihood components is available in Gertseva and Cope (2018). Several additional treatments of model parameters were presented in the assessment as

Fig. 1. Sensitivity to alternative model specification of the yelloweye rockfish assessment model described in the text in terms of relative change ($RC = (X_{alt} - X_{ref})/X_{ref}$) across five different derived model metrics. Boxes correspond to the 95% confidence interval of a derived quantity (indicated by colour) in the reference model. Values outside the box would indicate significant departure from the uncertainty provided in the reference model. Abbreviations in the groupings are as follows: Ageing Err = ageing error assumptions; *M* = natural mortality; *h* = spawner–recruit steepness; and L-W;Mat. = length–weight and maturity. Abbreviations in the specifications are as follows: NWFS = Northwest Fisheries Science Center; WDFW = Washington Department of Fish and Wildlife; ADFG = Alaska Department of Fisheries and Game; NMT = Natural Mortality Tool; Comm. = commercial; Rec. = recreational; comp = composition; est = estimate; sel. = selectivity. In the middle panel showing stock status, TRP = target reference point and LRP = limit reference point. [Colour online.]



likelihood profiles, but are not presented here, as this paper is only using a subset of sensitivities to demonstrate this new way of presenting and interpreting a suite of sensitivity analyses, not attempting to be a complete description of all analyses done in the yelloweye rockfish stock assessment.

The major structural choices in the assessment included assumptions about (i) spatial structure of the model (two-area versus one-area assessment), (ii) sex structure (one-sex versus two-sex model), (iii) shape of the selectivity curves for fishing fleets (asymptotic versus dome-shaped), and (iv) recruitment deviations (estimated within the assessment versus deterministically taken from the stock–recruit curve). We also explored alternative assumptions about catch history and examined alternative values of life history parameters that were either estimated or fixed in the assessment. These included alternative treatments of (i) natural mortality (*M*), (ii) stock–recruit compensation (i.e., steepness or *h*), (iii) maturity, (iv) fecundity, and the (v) length–weight relationship. We additionally explored model sensitivity to alternative data weighting approaches, including those of Francis (2017) and McAllister and Ianelli (1997). Finally, in the assessment we examined model sensitivity to alternative ageing error assumptions. In total, 26 alternative configurations are presented in this comparison with the reference model.

Results

The RC associated with 130 combinations of the five metrics and 26 alternative model configurations for the yelloweye rockfish assessment model are shown in Fig. 1, with RC on the y axis and specific model configurations on the x axis. Optional user-specified group labels offer a way to recognize patterns within and among model specification groups. The sensitivity groups include (i) ageing

error, (ii) *M*, (iii) *h*, (iv) length–weight and maturity, (v) removal history, and (vi) other, which is catchall for additional sensitivities.

The multipanel summary slide offers a quick way of looking across all metrics and model specification scenarios (Fig. 1), though separate figures for each metric are also produced (available on the GitHub site). Reference model confidence intervals for each metric are shown as rectangular boxes of different colours that correspond to colours of individual metrics. The levels of RC in relative stock status that correspond to TRP (40% relative stock status for the yelloweye rockfish) and LRP (25% relative stock status for the yelloweye rockfish) are also shown. The current reference model estimates that yelloweye rockfish stock is below the TRP but above LRP ($RC = 0$ on middle panel of Fig. 1).

This representation of the sensitivity results in Fig. 1 reveals that the assessment model is very sensitive to assumptions about *M* and *h*. In sensitivity analyses, both *M* and *h* were estimated using Hamel (2015) and Thorson et al. (2019) priors, respectively. We also estimated *M* using the prior produced from the Natural Mortality Tool publicly available at <https://github.com/shcaba/Natural-Mortality-Tool>. All runs with alternative *M* values produced larger estimates of the spawning output and more optimistic estimates of status and productivity than the reference model, with the relative *SO* above the TRP in each alternative model. The reference model was also sensitive to assumptions about stock–recruit steepness. *SO*₀ is generally insensitive to steepness changes, but *SO*_{*t*} and therefore (*SO*_{*t*}/*SO*₀) both show significant differences when *h* is changed, though overall productivity of the stock increased with a significantly higher *MSY* value.

The model was also sensitive to alternative assumptions about catch histories. Even though these model configurations differed in their estimates of *SO*, and thus the *MSY* proxy, estimates of

Table 1. Model selection (AIC, Δ AIC relative to reference), likelihood component (Δ AIC relative to reference), parameter values, and derived model quantities for a subset of model specification sensitivity runs for the yelloweye rockfish stock assessment.

	Sensitivity scenario									
	Reference model	Ageing error				Natural mortality			Steepness	
		Alt. error matrix 1	Alt. error matrix 2	Alt. error matrix 3	Alt. error matrix 4	Estimated	Fixed to alt. value	Est. with alt. prior	Estimated	Fixed to old value
AIC	11 784	11 856	11 840	11 788	11 811	11 765	11 778	11 764	11 778	11 827
Δ AIC	0	73	56	5	27	-19	-6	-19	-6	43
Index likelihood components (ΔAIC)										
CA dockside recreational	0	0	0	0	0	0	0	0	0	2
OR dockside recreational	0	-1	1	0	0	0	0	0	0	0
WA dockside recreational	0	-1	2	0	0	-7	-11	-7	-2	5
CA CPFV	0	0	-1	0	0	3	4	3	1	-3
OR onboard	0	0	0	0	0	0	0	0	0	0
AFSC triennial	0	0	0	0	0	1	2	1	0	0
NWFSC WCGTS	0	0	0	0	0	0	0	0	0	0
IPHC	0	0	0	0	0	-1	-1	-1	0	-1
Length likelihood components										
CA trawl	0	0	-2	0	0	2	2	2	0	0
CA non-trawl	0	0	0	0	0	0	-1	0	0	0
CA dockside recreational	0	-2	1	0	0	0	0	0	0	0
OR-WA trawl	0	1	2	0	0	-1	-1	-1	0	2
OR-WA non-trawl	0	0	0	0	0	-1	-1	-1	-1	2
OR dockside recreational	0	0	-2	1	1	-5	-6	-5	-2	5
WA dockside recreational	0	4	4	4	11	0	6	0	0	0
CA CPFV	0	-1	3	0	1	-2	-2	-2	0	2
OR onboard	0	0	2	0	0	-2	-2	-2	-1	4
AFSC triennial	0	0	0	0	0	0	1	0	0	0
NWFSC trawl	0	0	0	0	0	0	0	0	0	0
IPHC	0	0	1	0	0	0	0	0	0	0
Age likelihood components										
CA non-trawl	0	0	2	0	0	0	1	0	0	0
CA dockside recreational	0	-12	-8	0	1	0	0	0	0	0
OR-WA trawl	0	3	-1	-1	0	2	3	2	0	0
OR-WA non-trawl	0	2	5	0	-1	-1	-2	-1	0	0
OR dockside recreational	0	6	1	-3	-1	5	7	5	0	-1
WA dockside recreational	0	4	15	0	-2	-5	-7	-5	0	1
NWFSC WCGTS	0	43	17	0	3	0	-1	0	0	-1
IPHC	0	-9	-14	-1	-1	2	2	2	1	-3
Parameters										
Length at A_{\min}	8.61	9.41	7.01	8.60	8.52	8.42	8.18	8.42	8.56	8.88
Length at A_{\max}	63.45	63.19	64.12	63.46	63.40	63.51	63.48	63.51	63.48	63.38
VBGF k	0.06	0.06	0.07	0.06	0.07	0.06	0.06	0.06	0.06	0.07
CV young	0.19	0.18	0.19	0.19	0.18	0.19	0.19	0.19	0.19	0.19
CV old	0.05	0.06	0.06	0.05	0.06	0.05	0.05	0.05	0.05	0.06
Recruitment distribution CA	0.41	0.43	0.35	0.41	0.40	0.60	0.77	0.61	0.43	0.39
$\ln R_0$	5.39	5.43	5.26	5.38	5.38	6.14	6.58	6.17	5.39	5.40
M	0.044	0.044	0.044	0.044	0.044	0.058	0.066	0.059	0.044	0.044
Length at A_{\max}	63.42	63.42	63.42	63.42	63.42	63.42	63.42	63.42	63.42	63.42
VBGF k	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
CV old	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Steepness	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.95	0.42
Derived quantities										
SO_0	1139	1157	1071	1127	1129	1298	1506	1307	1134	1148
SO_{2017}	323	349	270	320	321	601	851	613	409	176
SO_{2017}/SO_0	28%	30%	25%	28%	28%	46%	57%	47%	36%	15%
Yield at $SPR_{50\%}$	105	105	96	102	101	161	210	164	116	55
F at $SPR_{50\%}$	0.022	0.021	0.022	0.022	0.021	0.027	0.029	0.027	0.022	0.022

Note: The gray cells provide an example of identifying notable changes in likelihood components and parameters for an example scenario. Model selection criteria of ± 2 units are considered significantly different. Abbreviations are as follows: CA = California; OR = Oregon; WA = Washington; CPFV = commercial passenger fishing vessel; AFSC = Alaska Fisheries Science Center; NWFSC = Northwest Fisheries Science Center; WCGTS = West Coast Bottom Trawl Survey; IPHC = International Pacific Halibut Commission; A_{\min} = minimum age used to estimate growth curve; A_{\max} = maximum age used to estimate growth curve; VBGF k = von Bertalanffy growth function parameter; CV = coefficient of variation; M = natural mortality; SO = spawning stock output; SPR = spawning potential ratio; F = fishing rate. The full table is available in the example folder posted at https://github.com/shcaba/Stock-Assessment-Sensitivity-Plots/tree/master/Cope_Gertseva.

SO_t/SO_0 varied only slightly among the runs. This demonstrates that constant proportional change to catch histories will scale the population up and down, but does not change the relative stock status. Finally, Fig. 1 shows that assumptions of dome-shaped selectivity produced more optimistic estimates of the current stock status than estimated in the reference model. Of the 26 total alternative model configurations, 15 scenarios produced at least one model output outside the reference model confidence intervals (the colored boxes), indicating significant uncertainty not being captured in the reference model, while also providing the directional change (i.e., above or below the reference value) in that uncertainty. Note that some values may be off the limits of any given plot (e.g., Fig. 1, top panel, SO_{2017} for the dome-shaped selectivity scenario). Users should be vigilant about missing values (the generated R object contains RC values for all metrics and scenarios) or initially use broader y limits (a user input option) to see all values before reducing limits to better see patterns.

Likelihood component differences from the reference model, parameter values, and the five derived model outputs for each alternative model configuration are reported in Table 1. Owing to space limitations, Table 1 presents only a shortened version of the full table (available in the GitHub repository) to illustrate its structure. For example, under the scenario “Natural Mortality, Estimated” are the outputs for that particular model where M is estimated using a prior based on maximum age. The figure indicated that it creates significant differences in the derived outputs and brings the population status above the TRP; the table shows those changes are due to different fits to two indices of abundance, one length composition and two age composition data sources (Table 1, gray cells). Some of these data sources fit better (i.e., have lower negative log-likelihood values), while others fit worse (i.e., have higher negative log-likelihood values). Overall, the model selection criteria (Akaike’s information criterion (AIC) and ΔAIC relative to the reference model; Burnham and Anderson 2002) indicate a much improved fit when M is estimated (Table 1). Inclusion of the AIC values gives a quick ability to consider how much better or worse a model fits the data relative to the reference model, though bear in mind this approach is not appropriate when data or data treatment changes (e.g., a change in data-weighting; adding or excluding data sources). In addition to M being quite different from the reference model, this example also shows a large change in the estimate of recruitment distribution across areas. Such exploration of data fits and changes in parameter estimates improves transparency and enhances the ability to communicate differences in alternate model configurations, important considerations when ultimately concluding how intermodel, not just intramodel, variability should be captured.

Discussion

Complexity of integrated statistical stock assessments may lead to shortcuts in communicating results, wherein only summary statistics (e.g., mean or median) are used to represent inherently uncertain model outputs. Communicating how alternative model configurations may change model outputs beyond the reference model uncertainty and relative to management reference points is a critical and challenging task. The method introduced here allows a quick and transparent evaluation of a large number of sensitivity analyses and identifies which model assumptions deserve further attention when quantifying model configuration uncertainty for management consideration.

Within the US West Coast management system, sensitivity analyses help identify “major axes of uncertainty” in the assessment. These major axes of uncertainty are model configurations that cause the greatest departures from the reference model output. The axes of uncertainty often include parameters fixed in the reference model, such as M or h , as well as other model inputs, such as catch time series. The identified axes of uncertainty are

used to construct a decision table (Hilborn and Walters 1992) and define alternative “states of nature” relative to the reference model, when the model is run with “low” and “high” values of assumed parameters under different catch trajectories. The model projections under low and high states of nature are provided to decision makers along with the results of the reference model to communicate a broader range of uncertainty than in the reference model alone. The method presented here offers a systematic way to identify possible states of nature when developing such decision tables.

Another approach to translate structural uncertainty in stock assessments into management decisions is to use an ensemble modeling approach (Stewart and Martell 2015; Stewart and Hicks 2018), when multiple models’ outputs are integrated to develop management advice. Ensemble models account for a range of uncertainty in model structural choices, bypassing the need for one reference model, and are increasingly common in fisheries (Townsend et al. 2014). Our method can be used to highlight and discuss which models should be considered when formulating a model ensemble or when formulating a range of operating models for management strategy evaluation (Punt et al. 2016).

There are a variety of stock assessment frameworks and management reference points in use worldwide. While we highlight the use of Stock Synthesis here, this method of presentation can be adapted to any stock assessment framework, as well as incorporate any derived output deemed useful by fisheries managers, not just the five metrics we present here. Additionally, if available sensitivity analyses also include measures of uncertainty, one could include errors bars representing uncertainty intervals on each point. Finally, the model configurations identified that show the most interesting sensitivities when using this visual tool can then be subjected to a more in-depth time series comparison with the reference model.

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