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STATISTICAL WAVE FORECAST EQUATIONS FOR A DEEPWATER BUOY LOCATION OFF THE WASHINGTON-OREGON COAST

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1. INTRODUCTION

Since 1971, the National Oceanic and Atmospheric Administration (NOAA) has deployed moored environmental monitoring buoys in the Atlantic, Pacific, Gulf of Mexico, and Great Lakes. Fig. 1 shows the locations of buoys that were measuring hourly wind and wave data in June 1981 and planned sites for future deployments. Data measured at these buoys are used to develop, tune, initialize, and verify meteorological and oceanographical forecast models.

This office note presents the derivation and evaluation of wave height and period forecast equations. These equations were derived and evaluated with measured data at buoy 46005 (called EB-21 prior to September 1976), located approximately 300 n mi off the Washington-Oregon coast (Fig. 2). The equations, which can be used to forecast deepwater waves off the Washington-Oregon coast, may also be helpful in predicting conditions at the Columbia River bar. The experience gained in this study will be useful in developing wave forecast equations at other buoy locations.

2. BUOY DATA

Buoy 46005, of the 12-meter disk type shown in Fig. 3, is equipped to measure wind speed and direction, air temperature, barometric pressure, wave spectra, and sea surface temperature. Table 1 shows some of the features associated with measurement sensors aboard the buoy. Measurements taken by these sensors are transmitted every hour to the National Meteorological Center through the Geostationary Environmental Satellite data relay system. These measurements, which are later saved on magnetic tape, were used to derive wave height and period forecast equations.

3. DERIVATION

I used a multiple regression screening program to correlate measured wave heights and periods (predictands) with measured predictors. This approach, where predictand data are correlated with measured predictors is called "perfect prog" in contrast to the Model Output Statistics (MOS) approach where predictand data are correlated with forecasts from a model. I used approximately 1 year of data, 1333 measurements recorded at 6-h intervals (0000, 0600, 1200, and 1800 GMT) from November 2, 1978 through October 29, 1979, in the derivation of the equations.

A. Predictands

Significant wave height and average period, the predictands, are computed from wave spectra by the NOAA Data Buoy Office (NDBO) and stored on magnetic tape. A brief explanation of wave spectra and how it is used to compute wave height and period follows.

A deformed sea surface is a combination of a large number of sinusoidal waves each having its own amplitude and period. Each wave makes an energy contribution to the deformed sea surface. This contribution is proportional to the square of the wave's surface displacement. The accelerometer aboard buoy 46005 samples the water level displacement every 0.67 seconds for 20 minutes every hour. A spectral analysis of these recorded data gives the distribution of wave energy by wave period. The expression of wave energy as a function of wave frequency (recipocal of wave period), shown in Fig. 4, is called a wave-energy spectrum. This expression is convenient because the area under the spectral energy curve equals the total energy of the wave system. The significant wave height (average height of the one-third highest waves) can be expressed by the term $4(E)^{1/2}$, where E is the area in square meters under the spectral energy curve. This is the wave height I used to derive wave height forecast equations.

The second predictand is a coded average wave period which was converted to seconds. The average period may be derived from a wave record by dividing the record length by the number of up or down crossings.

For the development data, significant wave heights (recorded in half-meters by international agreement) varied from 1 to 16 half-meters, with an average height of approximately 4 half-meters. Wave periods averaged about 8 seconds and varied from less than 5 seconds to 13 seconds. Table 2, number of waves tabulated by categories of height and period, gives a more complete picture of the distribution of the wave height and period measurements for the development data. The majority of wave heights are less that 7 half-meters, while most wave periods are less than 10 seconds. In general and as expected, the larger the wave height the longer the period. The correlation between wave height and period is 0.57.

B. Predictors

Wave height and period are determined by fetch (distance over water that wind has essentially constant direction and speed), wind speed, and duration (length of time the wind has blown over the fetch). Depth of water is <u>not</u> a consideration at buoy 46005 because the buoy is moored in 1500 fathoms of water. In an attempt to capture the effects of fetch and duration, which are not measured directly, I offered the regression screening program the following measured variables at the buoy as predictors:

1. wind speed and direction,
2. air temperature, sea surface temperature, air-sea temperature difference, and
3. wave height and period at earlier times (persistence).

Meteorological variables are measured 10 meters above the water line.

a. Wind speed and direction

The first predictors investigated were wind speed and direction. Wind speed in knots, measured at the same time as the waves, as well as speeds measured 6, 12, 18, 24, and 30 hours before the wave height, were offered as predictors of wave height and period. The first four variables selected as predictors of wave height, in order of their selection, were wind speeds with 6-, 18-, 0-, and 12-h lags. The regression equation which contains these four predictors and yields a correlation of 0.74 with wave height is:

$$HT_T = 0.350 + 0.058\ WS_{T-6} + 0.070\ WS_{T-18} + 0.088\ WS_T + 0.050\ WS_{T-12}, \quad (1)$$

where HT_T is significant wave height in half-meters at time T. WS is wind speed in kt with lag times of 0, 6, 12, and 18 hours. The wind speed with a 0-h lag is weighed heaviest followed by the wind speed with an 18-h lag.

When HT_T, from equation 1, is squared and offered as a predictor, the correlation coefficient increased to 0.76. The derived wave equation which contained this predictor is:

$$WHT_T = 2.047 + 0.105\ HT_T^2, \quad (2)$$

where WHT_T is significant wave height in half-meters at time T and HT_T is computed with equation (1).

Wave period has a much lower correlation with wind speed. As with wave height, the set of potential predictors of wave period contained wind speeds with lag times of 0 through 30 hours at 6-h intervals. Wind speeds with 6- and 12-h lags did not reduce significantly the variance of the wave period. The derived wave period equation with predictors listed in order of selection is:

$$PER_T = 7.601 + 0.042\ WS_{T-18} - 0.025\ WS_T, \quad (3)$$

where PER_T is wave period in seconds at time T. WS is wind speed in kt with lag times of 18 and 0 hours. The correlation coefficient associated with this equation is only 0.18.

Since I had wind measurements at only one buoy, it was difficult to determine the relationship between fetch and wave height and period. I did, however, tabulate categories of wave height by categories of wind direction. I classified wind directions, which are recorded in tens of degrees, as either blowing from land (overland winds) or blowing from water (overwater winds). Winds with directions between 145° and 325° are defined as overwater winds (Fig. 2). Table 3 shows the number of wave height measurements tabulated by categories of wave height and wind direction. Overwater winds outnumber overland winds by about 200. When I tabulated wave period according to wind direction (Table 4), I found that the longer period waves (12-13 s) were associated with overland winds. This unexpected finding may be due to swell which is included in the wave measurement.

b. Temperatures

Air and sea surface temperatures recorded to a tenth of a degree (C), were the next predictors investigated. Air temperatures varied from 3.8° to 18.6°C, while sea surface temperatures ranged in value from 8.4° to 18.7°C. The correlation coefficient associated with air temperature and wave height is -0.36. Air temperature and wave period have a correlation of -0.26. The correlation coefficient associated with sea surface temperature is a little lower, -0.33 for wave height and -0.23 for wave period. A derived predictor, air-sea temperature difference, gave a much lower correlation, -0.17 for height and -0.15 for period. The majority (95 percent) of the temperature differences (air minus sea) were greater than -3.0°C, indicating near neutral or stable conditions. It appears that much of the time overwater winds are moderated by the sea surface temperature.

c. Persistence

The best predictor of wave height and period was persistence. Correlation coefficients associated with wave heights varied from 0.90 for persistence at 6 hours to 0.48 for persistence at 48 hours. For wave period, persistence at 6 and 48 hours had correlations of 0.74 and 0.30, respectively. For a different look at 12-h persistence for wave height and period, see Tables 5 and 6. The number of waves tabulated by categories of height and time, where times are T and T-12 hours, are shown in Table 5. Shown in Table 6 is a similar tabulation for wave period. Notice that wave heights and periods persist in more than 50 percent of the cases.

C. Wave Forecast Equations

The wave height and period forecast equations (equations (2) and (3), page 3) were rederived with additional predictors. Ten new forecast equations, five height equations and five period equations, were derived. Four equations of each set contain wave and wind measurements as predictors. For example, one equation contains wave measurements with 12-h lags as the most recent measurements, a second equation contains wave measurements with 24-h lags as the most recent measurements, and so on to lag times of 48 hours. When wave measurements are not available, the fifth equation of each set can be used to predict wave height and period. The predictors in these equations are winds and air temperature. Wave height and period forecast equations are shown in Appendixes A and B, respectively. The predictor, HT, is wave height which is computed by equation (1) page 3.

In the derivation of the wave height equations, screening of winds, temperatures, wave heights, and wave periods was stopped when a predictor reduced the variance of the predictand less than one percent. When I used this "cut-off" criterion in the derivation of the wave period equations, wind direction, which varied from 10° to 360°, was selected as a predictor in three of the equations. Because of the selection of this "seemingly meaningless" predictor, I stopped screening for predictors at the predictor before the wind direction was selected.

The three equations where screening was stopped before wind direction was selected as a predictor are denoted with asterisks. Wave periods calculated by the equation which contains no wave measurements are approximately the same as averaged wave period of the dependent sample (8 s).

4. EVALUATION

I tested all equations on measured independent data. These data consisted of 1097 measurements of wave heights and 1063 measurements of wave period which were recorded at the buoy from November 3, 1977 through October 31, 1978. Wave height and period statistics, correlation coefficients and root-mean-square errors (RMSE's), were computed from dependent and independent data. An additional statistic, mean algebraic error (MALE), was computed from independent data.

Table 7 contains the statistics computed for wave height. For dependent and independent data, equation (a) which contained measured wave data with a 12-h lag had the highest correlation with the measured wave height and the lowest RMSE. The correlation coefficients decreased when wave measurements with longer lag times were used. Equation (e) which did not use measured wave data as predictors had the lowest correlation. The small positive MALE's indicate that the equations slightly underestimated wave heights. There is little difference between the RMSE's computed from dependent data and those computed from independent data.

Wave period statistics, shown in Table 8, were not nearly as good as the statistics computed for wave heights. The correlation coefficients associated with wave period equations are 0.20 to 0.50 lower than those associated with the wave height equations. Biases, as indicated by small negative MALE's, are very small. Notice that the statistics indicate that the equations, except for equation (f) which contains wave data with a 12-h lag, performed better with independent data.

5. SUMMARY AND FUTURE PLANS

This derivation and evaluation of wave forecast equations revealed the following:

1. wave heights and periods persist over time periods of up to 48 hours,
2. wave height is highly correlated with wind speeds,
3. average wave period is poorly correlated with wind speeds,
4. air-sea temperature difference is not a good predictor of wave height or wave period, and
5. useful wave height forecast equations can be derived from buoy measurements.

In coordination with Ocean Service Units and WSFO's with wave forecast responsibilities, I plan to derive wave forecast equations at buoy locations where MOS wind equations have been derived. I also plan to investigate the relationships between wave heights measured by buoys moored in deep water and wave heights measured by buoys anchored in shallower water.

6. ACKNOWLEDGEMENTS

A special thanks to Cindy Boggio for making many of the computer runs needed to process the buoy data. I also thank Larry Burroughs for his helpful discussion on air and sea surface temperatures. Measured data were provided by NDBO through the National Climatic Center.

7. REFERENCES

Britton, G. P., 1981: An Introduction to Sea State Forecasting. (Edited by Kenneth E. Lilly, Jr.), National Weather Service, NOAA, U.S. Department of Commerce, 208 pp.

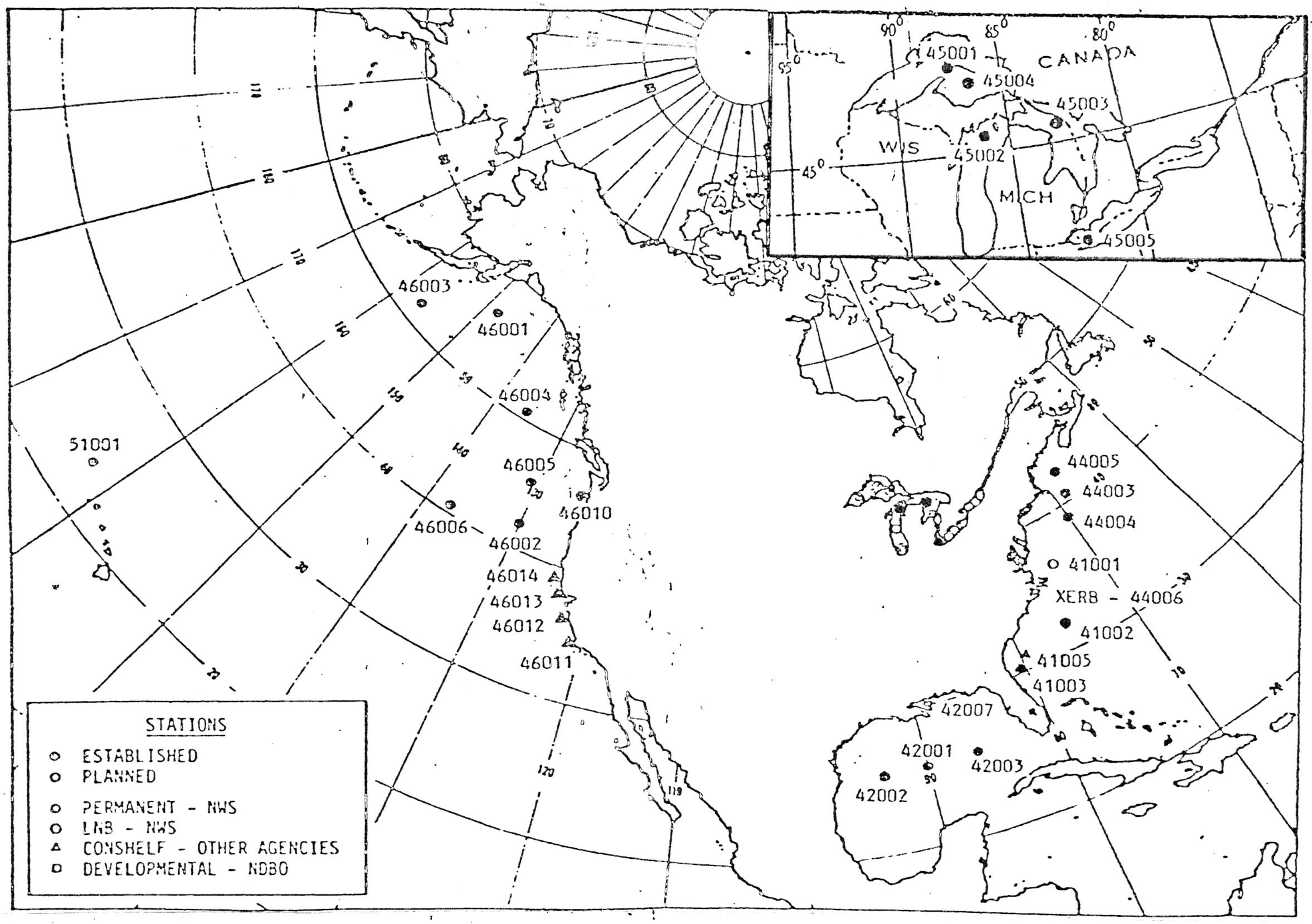

Figure 1. Locations of buoys that were measuring hourly wind and wave data in June 1981 and planned sites for future deployments (NOAA Data Buoy Office).

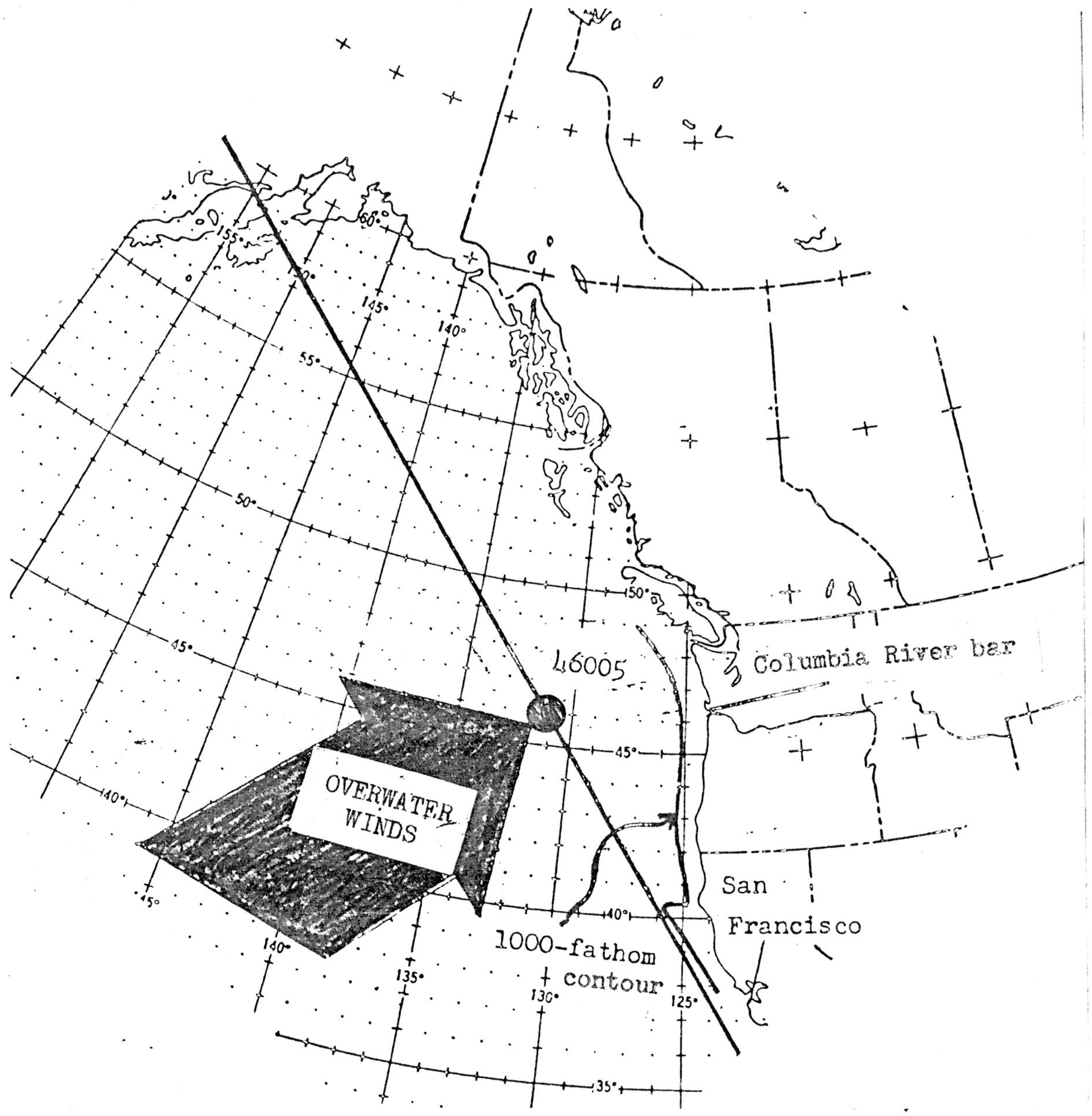

Figure 2. Location of buoy 46005 and the 1000-fathom contour. Winds with directions between 145° and 325° are defined as overwater winds.

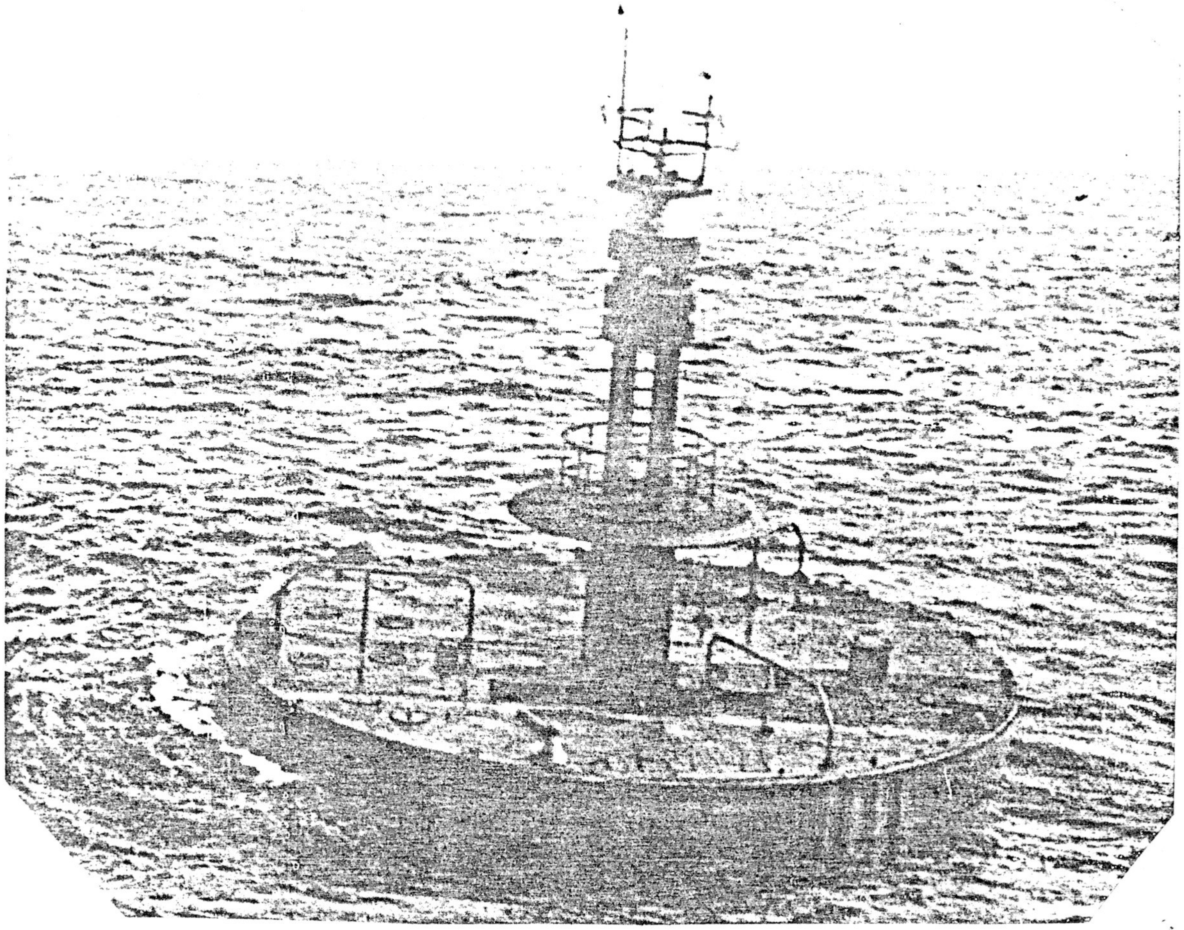

Figure 3. An environmental monitoring buoy (photograph courtesy of NOAA Data Buoy Office).

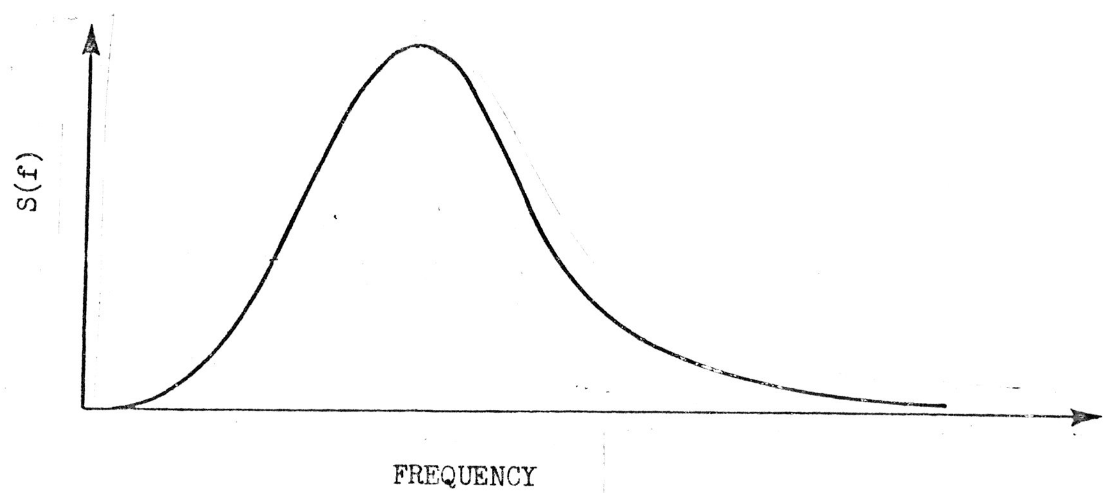

Figure 4. An example of a spectral energy curve. S(f) is the energy content of the wave system as a function of wave frequency.

Table 1. Features associated with buoy sensors (modified from Britton, 1981).

ENVIRONMENTAL BUOY SENSORS

Parameter[1]	Sensor Type	[4]Reporting Range	Sampling Interval	Averaging Period	Designed Accuracy
Wind speed	Vortex shedder	0 to 80 m/s	1 s	8.5 min	± 1 m/s
Wind direction	Vane and flux-gate compass	0 to 360°	1 s	8.5 min	± 10°
Air temperature	Thermistor	-15 to 49°C	8.5 min	8.5 min	± 0.1°C
Barometric pressure	Variable capacitance	900 to 1100 mb	4 s	8.5 min	± 0.1 mb
Significant wave height[2]	Accelerometer	0 to 281 m	0.67 s	20 min	± 0.5 m
Wave period[2,3]	Accelerometer	2 to 30 s	0.67 s	20 min	± 0.5 s
Wave spectra	Accelerometer	0.01 to 0.5 Hz (f = 0.005Hz)	0.67 s	20 min	---
Sea surface temperature	Thermistor	-15 to 49°C	8.5 min	8.5 min	± 0.1°C

[1]Each parameter reported every hour to the National Meteorological Center.
[2]Computed from wave spectra.
[3]Average or spectral peak.
[4]Reporting range of sensor is not necessarily the reporting range of the complete buoy package.

Table 2. Number of waves tabulated by categories of height and period for November 2, 1978 - October 29, 1979.

Wave Period (s)	Wave Height (half-meters)								TOTAL
	1-2	3-4	5-6	7-8	9-10	11-12	13-14	15-16	
≤5	94	31							125
6-7	172	305	166	16	1				660
8-9	40	106	125	110	29	11			411
10-11	2	25	35	25	15	11	7	3	123
12-13		6	1	5	2				14
TOTAL	308	473	327	146	47	22	7	3	1333

Table 3. Number of waves tabulated by categories of wave height and wind direction (overland and overwater).

Wind Category	Wave Height (half-meters)								TOTAL
	1-2	3-4	5-6	7-8	9-10	11-12	13-14	15-16	
Overland (330°-140°)	138	185	160	54	18	7	2	1	565
Overwater (150°-320°)	170	288	167	92	29	15	5	2	768
TOTAL	308	473	327	146	47	22	7	3	1333

Table 4. Number of waves tabulated by categories of wave period and wind direction (overland and overwater winds).

Wind Category	Wave Period (s)					TOTAL
	≤5	6-7	8-9	10-11	12-13	
Overland (330°-140°)	63	265	165	63	9	565
Overwater (150°-320°)	62	395	246	60	5	768
TOTAL	125	660	411	123	14	1333

Table 5. Number of waves tabulated by categories of height measured at times T and T-12 hours.

Wave Height (half-meters) at time T-12 h	Wave Height (half-meters) at time T								TOTAL
	1-2	3-4	5-6	7-8	9-10	11-12	13-14	15-16	
1-2	244	62	2						308
3-4	64	309	85	15	2	1			476
5-6		96	162	57	6	3	1	2	327
7-8		6	63	54	15	4	1		143
9-10			13	12	13	7	2		47
11-12			1	8	7	4	2		22
13-14			1		3	1	1	1	7
15-16					1	2			3
TOTAL	308	473	327	146	47	22	7	3	1333

Table 6. Number of waves tabulated by categories of period measured at times T and T-12 hours.

Wave Period (s) at time T-12 h	Wave Period (s) at time T					TOTAL
	≤5	6-7	8-9	10-11	12-13	
≤5	74	45	5	1		125
6-7	43	478	126	20	2	669
8-9	7	127	224	45	2	405
10-11	1	10	53	50	6	120
12-13			3	7	4	14
TOTAL	125	660	411	123	14	1333

Table 7. Verification scores associated with wave height equations for the dependent sample (1333 sets) and an independent sample (1097 sets). MALE is the mean algebraic error. Wave height equations (a), (b), (c), (d), and (e) are listed in Appendix A.

Equation	Dependent		Independent		
	Correlation Coefficient	RMSE (half-meters)	Correlation Coefficient	RMSE (half-meters)	MALE (half-meters)
(a)	0.87	1.16	0.84	1.12	0.09
(b)	0.82	1.34	0.79	1.30	0.22
(c)	0.81	1.40	0.76	1.38	0.24
(d)	0.79	1.44	0.73	1.43	0.23
(e)	0.77	1.50	0.71	1.50	0.32

Table 8. Same as Table 7 except for wave period equations (listed in Appendix B) and an independent sample of 1063 sets of data.

Equation	Dependent		Independent		
	Correlation Coefficent	RMSE (s)	Correlation Coefficent	RMSE (s)	MALE (s)
(f)	0.68	1.21	0.65	1.15	-0.11
(g)	0.56	1.37	0.69	1.12	-0.05
(h)	0.39	1.52	0.40	1.39	-0.05
(i)	0.35	1.54	0.39	1.39	-0.04
(j)	0.26	1.59	0.37	1.41	-0.05

Appendix A

WAVE HEIGHT FORECAST EQUATIONS

$$WH_T = -0.188 + 0.628\ WH_{T-12} + 0.068\ WS_T + 0.052\ WS_{T-6} \qquad (a)$$

$$WH_T = -0.203 + 0.084\ HT_T^2 + 0.299\ WH_{T-24} + 0.180\ WP_{T-48} \qquad (b)$$

$$WH_T = -0.011 + 0.092\ HT_T^2 + 0.223\ WH_{T-36} + 0.176\ WP_{T-48} \qquad (c)$$

$$WH_T = 1.217 + 0.093\ HT_T^2 + 0.251\ WH_{T-48} \qquad (d)$$

$$WH_T = 3.384 + 0.099\ HT_T^2 - 0.104\ AT_T \qquad (e)$$

WH_T is wave height in half-meters at time T. WS is wind speed in knots and WP is wave period in seconds. Negative subscripts are lag time in hours. HT is wave height computed by equation (1) page 3. AT is air temperature in degrees C.

Appendix B

WAVE PERIOD FORECAST EQUATIONS

$$WP_T = 4.373 + 0.414\ WP_{T-12} + 0.295\ WH_{T-12} - 0.040\ WS_T - 0.030\ WS_{T-30} \qquad (f)$$

$$WP_T = 7.143 + 0.239\ WP_{T-24} + 0.108\ WH_{T-24} - 0.102\ WS_T + 0.088\ HT_T^2$$
$$- 0.086\ WS_{T-6} + 0.123\ WP_{T-48} - 0.071\ ST_T - 0.052_{T-18} \qquad (g)$$

$$*WP_T = 6.543 + 0.300\ WP_{T-36} - 0.090\ AT_T \qquad (h)$$

$$*WP_T = 6.998 + 0.250\ WP_{T-48} - 0.010\ AT_T \qquad (i)$$

$$*WP_T = 9.341 - 0.129\ AT_T \qquad (j)$$

WP_T is wave period in seconds at time T. Asterisks denote equations where screening was stopped before wind direction was selected as a predictor. WH is wave height in half-meters and WS in wind speed in knots. Negative subscripts are lag times in hours. HT is wave height computed by equation (1) page 3. ST and AT are sea surface temperature and air temperature respectively, in degrees C.