Short-Run Johansen Frontier-Based Industry Models: Methodological Refinements and Empirical Illustration on Fisheries

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Abstract

This contribution focuses on extending the current state of the art in a central resource allocation planning model known under the name of the short-run Johansen industry model in three ways. First, we correct a long-standing issue of the correct choice of weight variables on the capacity distribution by guaranteeing that these weights determine production combinations that belong to the production technology on which the plant capacity estimates are based in the first place. Second, we exploit the gap between average practice and best practice models by introducing an efficiency improvement imperative that allows for partial technical inefficiency when planning. Third, instead of only considering output-oriented plant capacity, we allow for alternative plant capacity concepts. In particular, we introduce an input-oriented plant capacity concept, and an alternative attainable output-oriented plant capacity concept that corrects a major empirical issue in the traditional output-oriented plant capacity notion. These methodological refinements are illustrated with a data set on U.S. fishing vessels by developing a planning model to curb overfishing.

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1 Introduction

The short-run Johansen (1972) industry or sectoral model is a planning tool which allows analysis of industry structure on a disaggregated basis from underlying ex post firm-level inputs and a single output. This model starts from a putty-clay model of production and investment decisions: ex-ante firms are free to choose among several production activities exhibiting smooth substitution possibilities, but ex post these firms face fixed coefficient technologies with capacities that are entirely conditioned by past investment decisions. The short-run Johansen industry model (SRJIM) nevertheless exhibits substitution possibilities when inputs and outputs can be reallocated across the units composing the industry. Over time, substitution and technical change can be traced via shifts in successive SRJIM. Surveys of this SRJIM are found in Førsund and Vislie (2016). Critical remarks on the whole SRJIM framework are available in Shephard (1974).

The short-run industry or ex post macro (Johansen's terminology) model is derived from the short-run ex post firm functions. It is a simple linear programming model with an objective function maximising the sum of firm outputs subject to capacity constraints related to the aggregate levels of inputs. The weight vectors are subject to an upper bound. Empirical applications of this SRJIM include the following examples in chronological order: Førsund, Gaunitz, Hjalmarsson, and Wibe (1980) analyse the Swedish pulp industry, Hildenbrand (1981) studies the Norwegian tanker fleet and the US electric power-generating industry; Førsund and Hjalmarsson (1983) analyse the Swedish cement industry; Førsund and Jansen (1983) reflect upon the Norwegian aluminum industry; Førsund, Hjalmarsson, and Eitrheim (1985) provide an international comparison of the cement industry in the Nordic countries comparing Denmark, Finland, Norway, and Sweden; the last four empirical chapters in Førsund and Hjalmarsson (1987) focus on a variety of sectors; Wibe (1995) studies the Swedish paper industry; Førsund, Hjalmarsson, and Summa (1996) scrutinise the Finnish brewery industry; and Førsund, Hjalmarsson, and Zheng (2011) develop an analysis for Chinese coal-fired electricity generation plants, among others.

Sengupta (1989) and Färe, Grosskopf, and Li (1992) are the first to establish a link between the SRJIM and frontier-based production theory that focuses on best practice instead of average practice (see also Dosi, Grazzi, Marengo, and Settepanella (2016) for some further links). Average practice analysis focuses on average behaviour, while best practice analysis concentrates on the best performing units on the boundary of the production possibility set. Dervaux, Kerstens, and Leleu (2000) innovate by developing an entirely non-parametric frontier-based approach to the SRJIM. This work improves two features. First, it transforms the single output case into a multiple outputs frontier framework.[1] Second, it substitutes the somewhat ad hoc specification of a capacity distribution in the traditional SRJIM by a non-parametric output-oriented (O-oriented) plant capacity

[1]However, in the traditional non-frontier literature Dosi, Grazzi, Marengo, and Settepanella (2016, Appendix B) also develop a multiple output-case. To the best of our knowledge, this multi-outputs approach has never been empirically implemented. Also Sengupta (1989, p. 49-50) outlines some possibilities to develop a multiple outputs approach: also these options have never been implemented empirically.

concept introduced in the literature by Färe, Grosskopf, and Kokkelenberg (1989) in the single output case and by Färe, Grosskopf, and Valdmanis (1989) in the multiple output case using a pair of O-oriented efficiency measures inspired by Johansen (1968).[2] Relaxing the single-output restriction substantially enlarges the scope of empirical applications beyond the historically almost exclusive focus on industry studies. Furthermore, the frontier nature allows for a benchmarking perspective when adopting it for social planning purposes.

Empirical applications of this generalised frontier-based SRJIM include the following examples: Dervaux, Kerstens, and Leleu (2000) analyse French surgery units in 1605 hospitals, Kerstens, Moulaye Hachem, Van de Woestyne, and Vestergaard (2010) provide an analysis of a German bank branch network and how it can be restructured, Färe, Grosskopf, Kerstens, Kirkley, and Squires (2001) provide a first study on how to reduce overfishing in the northwest USA Atlantic sea scallop fishery, Kerstens, Squires, and Vestergaard (2005) and Kerstens, Vestergaard, and Squires (2006) develop a plan to curb overfishing in the Danish fishery fleet under a variety of scenarios with quota and fishing days, while Lindebo (2005), Tingley and Pascoe (2005) and Yagi and Managi (2011) develop a similar plan for the North Sea, Scottish and Japanese fishing fleets, among others.

Note that the frontier-based SRJIM is but one example of a stream of literature on central resource allocation models in the frontier framework. Central resource reallocation models cover a heterogeneous variety of models reallocating some inputs and/or outputs across space and/or time while eventually accounting for multiple objectives (e.g., efficiency, effectiveness, equality). To the best of our knowledge Färe, Grosskopf, and Li (1992) and Golany, Phillips, and Rousseau (1993) are among the first frontier-based central resource reallocation models. Other examples of these models can be found in the work by Athanassopoulos (1998), Golany and Tamir (1995), Korhonen and Syrjänen (2004), Lozano and Villa (2004), and Ylvinger (2000), among others.[3]

The purpose of this contribution is threefold. First, we want to remedy one remaining problem in the SRJIM: while the O-oriented plant capacity concepts is estimated at the extremes of the empirical data range in the technology, there is currently no guarantee that the scaling of these plant capacity inputs and outputs remains technically feasible by remaining within the frontier technology. By contrast, all frontier-based central resource allocation models in the literature meet this requirement. This problem is illustrated using a numerical example and a general remedy is proposed. Second, we bridge the gap between traditional average practice and more recent best practice (frontier) models by introducing an efficiency improvement imperative that allows for some form of technical inefficiency in the planning process. Third, following Dervaux, Kerstens, and Leleu (2000) we make sure that the capacity distributions are based on nonparametric specifications that

[2]Johansen (1972) introduces the capacity distribution as a mechanism to derive optimal factor proportions in a dynamic setting. He and followers like Muysken (1985) and Seierstad (1985) explicitly introduce the capacity distribution notion as a continuous or discrete or mixed statistical distribution of the input coefficients when plants are used at full capacity.

[3]A selective survey of these frontier-based central resource allocation models is found in Mar-Molinero, Prior, Segovia, and Portillo (2014), while more complete and up to date reviews are published in White and Bordoloi (2015) and Afsharian, Ahn, and Harms (2021).

are compatible with the nonparametric nature of the SRJIM. Furthermore, we seek to widen the methodological choices open to the users of the SRJIM by introducing two new plant capacity concepts that are less problematic than the O-oriented plant capacity concept proposed in Dervaux, Kerstens, and Leleu (2000).

On the one hand, we follow Cesaroni, Kerstens, and Van de Woestyne (2017) who define a new input-oriented (I-Oriented) plant capacity measure using a pair of I-oriented efficiency measures. On the other hand, we follow up on Kerstens, Sadeghi, and Van de Woestyne (2019b) who argue that the traditional O-oriented PCU may be unrealistic in that the amounts of variable inputs needed to reach the maximum capacity outputs may simply be unavailable at either the firm or the industry level. This problem is linked to what Johansen (1968) called the attainability issue and therefore Kerstens, Sadeghi, and Van de Woestyne (2019b) define a new attainable O-oriented (AO-oriented) PCU. Throughout this contribution, we contrast the traditional average practice-based SRJIM with the more recent frontier-based SRJIM to highlight both similarities and differences.

This contribution is structured as follows. Section 2 defines the basic technology and efficiency measures needed to define frontier-based plant capacity concepts. Furthermore, it defines the traditional O-oriented PCU as well as the alternative I-oriented PCU and the AO-oriented plant capacity measure. The next section 3 defines the deterministic nonparametric technologies that are used to compute these plant capacity concepts and that implicitly define the SRJIM. The basic frontier-based SRJIM is discussed in Section 4. This same section illustrates the problem that the scaling of the plant capacity inputs and outputs need not remain technically feasible by remaining within the technology. Thereafter, Section 5 develops three new SRJIM. First, we develop a revised version of the SRJIM based on the O-oriented plant capacity that does respect the technology. Second, we introduce two new plant capacity concepts in the SRJIM: either the AO-oriented PCU, or the I-oriented plant capacity measure. The differences between old and new SRJIM are empirically illustrated in Section 6 using convex and nonconvex technologies. A final Section 7 concludes.

2 Technology and Plant Capacity Notions: Basic Definitions

2.1 Technology and Efficiency Measures

This section introduces basic notation and defines the firm technology. Given an N-dimensional input vector $x \in \mathbb{R}^N_+$ and an M-dimensional output vector $y \in \mathbb{R}^M_+$, the production possibility set or technology T is defined as: $T = \{(x, y) | x \text{ can produce } y\}$. Associated with T, the input set denotes all input vectors x capable of producing a given output vector y: $L(y) = \{x | (x, y) \in T\}$. Analogously, the output set associated with T denotes all output vectors y that can be produced from a given input vector x: $P(x) = \{y | (x, y) \in T\}$.

Throughout this contribution, technology T satisfies a combination of the following assumptions:

(T.1) Possibility of inaction and no free lunch, i.e., $(0,0) \in T$ and if $(0,y) \in T$, then $y = 0$.

(T.2) T is a closed subset of $\mathbb{R}^N_+ \times \mathbb{R}^M_+$, i.e., $\partial T \subset T$ where the symbol ∂T denotes the boundary of T.

(T.3) Strong input and output disposal, i.e., if $(x,y) \in T$ and $(x',y') \in \mathbb{R}^N_+ \times \mathbb{R}^M_+$, then $(x',-y') \geq (x,-y) \Rightarrow (x',y') \in T$.

(T.4) T is convex.

Briefly discussing these technology axioms, it is useful to recall the following (see, e.g., Hackman (2008) for details). Inaction is feasible, and there is no free lunch. Technology is closed. This closedness of T guarantees the existence of efficient output and input vectors: see Theorem 2.1 in Kerstens and Sadeghi (2023) for more details. We assume free disposal of inputs and outputs in that inputs can be wasted and outputs discarded. Finally, technology is convex. In our empirical analysis not all axioms are simultaneously maintained.[4] In particular, an assumption distinguishing some of the technologies in the empirical analysis is convexity versus nonconvexity.

The radial input efficiency measure characterizes the input set $L(y)$ completely and is defined as:

$$DF_i(x,y) = \min\{\lambda \mid \lambda \geq 0, \lambda x \in L(y)\}. \tag{1}$$

This radial efficiency measure is smaller or equal to unity ($DF_i(x,y) \leq 1$), with efficient production on the boundary (isoquant) of $L(y)$ represented by unity, and has a cost interpretation (see, e.g., Hackman (2008)).[5]

The radial output efficiency measure offers a complete characterization of the output set $P(x)$ and is defined as:

$$DF_o(x,y) = \max\{\theta \mid \theta \geq 0, \theta y \in P(x)\}. \tag{2}$$

This efficiency measure is larger than or equal to unity ($DF_o(x,y) \geq 1$), with efficient production on the boundary (isoquant) of the output set $P(x)$ represented by unity, and has a revenue interpretation (e.g., Hackman (2008)).

In the short run, we can partition the input vector into a fixed and variable part. In particular, we denote ($x = (x^f, x^v)$) with $x^f \in \mathbb{R}^{N_f}_+$ and $x^v \in \mathbb{R}^{N_v}_+$ such that $N = N_f + N_v$. Similarly, a short-run technology $T^f = \{(x^f, y) \in \mathbb{R}^{N_f}_+ \times \mathbb{R}^M_+ \mid \text{there exists } x^v \text{ such that } (x^f, x^v) \text{ can produce at least } y\}$ and the corresponding input set $L^f(y) = \{x^f \in \mathbb{R}^{N_f}_+ \mid (x^f, y) \in T^f\}$ and output set $P^f(x^f) = \{y \mid$

[4] For instance, note that the convex flexible or variable returns to scale technology does not satisfy inaction.

[5] The input-oriented distance function, denoted as $D_i(\mathbf{x}, \mathbf{y}) : \mathbb{R}^N_+ \times \mathbb{R}^M_+ \to \mathbb{R}_+ \cup \{\infty\}$, is defined as follows:

$$D_i(\mathbf{x}, \mathbf{y}) = \sup_{\varphi} \Big\{ \varphi > 0 \mid \frac{\mathbf{x}}{\varphi} \in L(\mathbf{y}) \Big\}.$$

We can express $DF_i(x,y) = \frac{1}{D_i(\mathbf{x}_k, \mathbf{y}_k)}$ (see Färe and Lovell (1978) for a first statement). Since there is a one-to-one relationship between distance functions and efficiency measures, our focus in this contribution is on efficiency measures.

$(x^f, y) \in T^f\}$ can be defined. Note that technology T^f is obtained by a projection of technology $T \subset \mathbb{R}_+^N \times \mathbb{R}_+^M$ into the subspace $\mathbb{R}_+^{N_f} \times \mathbb{R}_+^M$ (i.e., by setting all variable inputs equal to zero).[6] By analogy, the same applies to the input set $L^f(y)$ and the output set $P^f(x^f)$.

Denoting the radial output efficiency measure of the output set $P^f(x^f)$ by $DF_o^f(x^f, y)$, this efficiency measure can be defined as follows:

$$DF_o^f(x^f, y) = \max\{\theta \mid \theta \geq 0, \theta y \in P^f(x^f)\}. \tag{3}$$

The sub-vector input efficiency measure reducing only the variable inputs is defined as follows:

$$DF_{vi}^{SR}(x^f, x^v, y) = \min\{\lambda \mid \lambda \geq 0, (x^f, \lambda x^v) \in L(y)\}. \tag{4}$$

The sub-vector input efficiency measure reducing only the fixed inputs is defined as follows:

$$DF_{fi}^{SR}(x^f, x^v, y) = \min\{\lambda \mid \lambda \geq 0, (\lambda x^f, x^v) \in L(y)\}. \tag{5}$$

Next, we need the following particular definition of a technology: $L(0) = \{x \mid (x, 0) \in T\}$ is the input set with zero output level.[7] The sub-vector input efficiency measure reducing variable inputs evaluated relative to this input set with a zero output level is as follows:

$$DF_{vi}^{SR}(x^f, x^v, 0) = \min\{\lambda \mid \lambda \geq 0, (x^f, \lambda x^v) \in L(0)\}. \tag{6}$$

2.2 Plant Capacity Notions

It is common to distinguish between technical or engineering capacity, and economic capacity. Johansen (1968) develops a technical approach through an informally defined plant capacity notion. This informal definition of plant capacity by Johansen (1968, p. 362) reads:"the maximum amount that can be produced per unit of time with existing plant and equipment, provided that the availability of variable factors of production is not restricted." This plant capacity notion is made operational by Färe, Grosskopf, and Kokkelenberg (1989) and Färe, Grosskopf, and Valdmanis (1989) using a pair of O-oriented efficiency measures. Now recall the definition of O-oriented PCU.

Definition 2.1. The O-oriented PCU_o is defined as follows:

$$PCU_o(x, x^f, y) = \frac{DF_o(x, y)}{DF_o^f(x^f, y)},$$

[6]See Cesaroni, Kerstens, and Van de Woestyne (2019, p. 388 and following) for more details about this projection.

[7]As already pointed out in Cesaroni, Kerstens, and Van de Woestyne (2019, p. 388), $L(0)$ can also be defined as $L(y_{min}) = \{x \mid (x, y_{min}) \in T\}$, whereby $y_{min} = \min_{k=1,\dots,K} y_k$ takes the minimum in a component-wise manner for every output y over all observations K.

where $DF_o(x, y)$ and $DF_o^f(x^f, y)$ are output efficiency measures including (excluding) the variable inputs as defined before in (2) and (3).

O-oriented PCU has an upper limit of unity, since $1 \leq DF_o(x, y) \leq DF_o^f(x^f, y)$, $0 < PCU_o(x, x^f, y) \leq 1$. Färe, Grosskopf, and Kokkelenberg (1989) distinguishes between a biased ($DF_o^f(x^f, y)$) and unbiased ($PCU_o(x, x^f, y)$) plant capacity measure depending on whether the measure ignores (adjusts for) inefficiency. By taking the ratio of efficiency measures, existing inefficiency is eliminated yielding a cleaned concept of O-oriented PCU.[8]

Recently, Kerstens, Sadeghi, and Van de Woestyne (2019b) argue that the O-oriented $PCU_o(x, x^f, y)$ is unrealistic because the variable inputs needed to reach capacity output may be unavailable. This is linked to what Johansen (1968) called the attainability issue. Hence, Kerstens, Sadeghi, and Van de Woestyne (2019b) define a new AO-oriented PCU level.

Definition 2.2. An AO-oriented PCU $APCU_o$ at a certain level $\bar{\lambda} \in \mathbb{R}_+$ is defined by

$$APCU_o(x, x^f, y, \bar{\lambda}) = \frac{DF_o(x, y)}{ADF_o^f(x^f, y, \bar{\lambda})},$$

where the AO-oriented efficiency measure ADF_o^f at level $\bar{\lambda} \in \mathbb{R}_+$ is defined by

$$ADF_o^f(x^f, y, \bar{\lambda}) = \max\{\varphi \mid \varphi \geq 0, 0 \leq \lambda \leq \bar{\lambda}, \varphi y \in P(x^f, \lambda x^v)\} \qquad (7)$$

Again, for $\bar{\lambda} \geq 1$, since $1 \leq DF_o(x, y) \leq ADF_o^f(x^f, y, \bar{\lambda})$, notice that $0 < APCU_o(x, x^f, y, \bar{\lambda}) \leq 1$. Also, for $\bar{\lambda} < 1$, since $1 \leq ADF_o^f(x^f, y, \bar{\lambda}) \leq DF_o(x, y)$, notice that $1 \leq APCU_o(x, x^f, y, \bar{\lambda})$.

One can again distinguish between a so-called biased plant capacity measure ($ADF_o^f(x^f, y, \bar{\lambda})$), and an unbiased attainable PCU measure ($APCU_o(x, x^f, y, \bar{\lambda})$), whereby the latter is cleaned from any inefficiency. Kerstens, Sadeghi, and Van de Woestyne (2019b) pragmatically experiment with values of $\bar{\lambda} \in \{0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5\}$[9], and note that if expert opinion cannot determine a plausible value, then it may be better to opt for an I-oriented plant capacity measure that does not suffer from the attainability issue.

Cesaroni, Kerstens, and Van de Woestyne (2017) define an I-oriented plant capacity measure using a pair of I-oriented efficiency measures.

Definition 2.3. The I-oriented PCU_i is defined as follows:

$$PCU_i(x, x^f, y) = \frac{DF_{vi}^{SR}(x^f, x^v, y)}{DF_{vi}^{SR}(x^f, x^v, 0)},$$

where $DF_{vi}^{SR}(x^f, x^v, y)$ and $DF_{vi}^{SR}(x^f, x^v, 0)$ are the sub-vector input efficiency measures defined in

[8] Computational issues are discussed in Section 4.

[9] Notice that $\bar{\lambda} < 1$ is added for completeness sake. Normally there is no need to reduce variable inputs below their currently available levels.

(4) and (6), respectively.

Since $0 < DF_{vi}^{SR}(x^f, x^v, 0) \leq DF_{vi}^{SR}(x^f, x^v, y)$, notice that $PCU_i(x, x^f, y) \geq 1$.[10] Thus, I-oriented PCU has a lower limit of unity. Similar to the previous cases, one can distinguish between a so-called biased plant capacity measure ($DF_{vi}^{SR}(x^f, x^v, 0)$) and an unbiased $PCU_i(x, x^f, y)$, the latter being cleaned of any inefficiency. Graphical illustrations of plant capacity Definitions 2.1, 2.2 and 2.3 are in Appendix A. Cesaroni, Kerstens, and Van de Woestyne (2019) also define an input-based and output-based long-run plant capacity concept whereby both fixed and variable inputs can adjust. Furthermore, Kerstens, Sadeghi, and Van de Woestyne (2019a) empirically illustrate that both engineering and economic capacity concepts differ systematically when estimated using convex and nonconvex technologies.

As stated earlier, the average practice single output SRJIM suffer in practice from a rather ad hoc specification of capacity distributions (as recently admitted in Dosi, Grazzi, Marengo, and Settepanella (2016, fn 13)). It should be stressed that some substantial efforts are available in the literature to derive a more satisfactory solution for this state of affairs: Muysken (1985) develops continuous capacity distribution, while Seierstad (1985) develops any form of the capacity distribution (discrete, continuous, or a mixture). However, it is clear that the above frontier-based technical or engineering plant capacity concepts are quite appealing since these can easily be computed relative to deterministic nonparametric technologies (see below). For detailed formulations of the mathematical programs to compute these three PCU concepts, see Kerstens, Sadeghi, and Van de Woestyne (2020, Appendix B.1).

Kerstens and Sadeghi (2023) have theoretically investigated the existence question regarding the above plant capacity notions at the firm level and at the industry level. For the O-oriented, the AO-oriented, and the I-oriented plant capacity measures the question as to the existence at the firm level poses no problem: all these concepts are well defined for variable returns to scale technologies. However, at the industry level the picture changes: the O-oriented and the AO-oriented plant capacities may not exist, while the I-oriented plant capacity notion is the only one that always exists.

These theoretical results have drastic consequences for the use of the SRJIM as a planning model. The frontier-based SRJIM based on O-oriented plant capacities, as defined in Dervaux, Kerstens, and Leleu (2000), loses much of its appeal. The alternative SRJIM developed here based on the AO-oriented plant capacity can mitigate this problem under certain conditions. Clearly, the alternative SRJIM developed here based on the I-oriented plant capacity notion is the only solution free of any reservations.

[10] Kerstens, Sadeghi, and Van de Woestyne (2019a, Proposition B.1) prove that $DF_{vi}^{SR}(x^f, x^v, 0) = DF_{vi}^{SR}(x^f, x^v, y_{min})$, where y_{min} is as defined supra.

3 Deterministic Nonparametric Technologies: Definitions

Having introduced all efficiency measures needed to define various plant capacity concepts, we now turn to the algebraic definition of the technologies relative to which plant capacities are estimated. In the literature cited above, the fact that the SRJIM is not explicitly considered as a technology has led to the problem that the scaling of capacities need not respect the technology. Therefore, in this contribution we explicitly develop the deterministic nonparametric technologies relative to which plant capacities are computed and that implicitly define the SRJIM.

Given data on K observations ($k = 1, \cdots, K$) consisting of a vector of inputs and outputs $(x_k, y_k) \in \mathbb{R}_+^N \times \mathbb{R}_+^M$, a unified algebraic representation of convex and nonconvex nonparametric frontier technologies under the flexible or variable returns to scale assumption is as follows:

$$T^{\Lambda} = \left\{ (x, y) \mid x \geq \sum_{k=1}^{K} z_k x_k, y \leq \sum_{k=1}^{K} z_k y_k, (z_1, \ldots, z_K) \in \Lambda \right\}, \tag{8}$$

where

(i) $\Lambda \equiv \Lambda^{\mathrm{C}} = \left\{ (z_1, \ldots, z_K) \mid \sum_{k=1}^{K} z_k = 1 \text{ and } z_k \geq 0 \right\}$;

(ii) $\Lambda \equiv \Lambda^{\mathrm{NC}} = \left\{ (z_1, \ldots, z_K) \mid \sum_{k=1}^{K} z_k = 1 \text{ and } z_k \in \{0, 1\} \right\}$.

The activity vector $(z_1, \ldots, z_K)$ of real numbers summing to unity represents the convexity axiom. This same constraint with each vector element being a binary integer represents nonconvexity. The convex technology satisfies axioms (T.1) (except inaction) to (T.4), while the nonconvex technology adheres to axioms (T.1) to (T.3). It is now useful to condition the above efficiency measures relative to these nonparametric frontier technologies by distinguishing between convexity (convention C) and nonconvexity (convention NC). This firm technology allows us to compute a series of frontier-based concepts of plant capacity to which we now turn.

4 Short-run Johansen Industry Model: Basic Version

Following Dervaux, Kerstens, and Leleu (2000), this model permits reallocation of production among units by explicitly allowing technical efficiency and capacity utilisation improvements using two phases. Phase one computes capacity outputs and corresponding inputs. In phase two, the SRJIM is constructed with parameters from phase one. As explained below, this SRJIM does not inherit the technology properties used to compute plant capacity.

In phase one, the short-run O-oriented radial technical efficiency measure $DF_o^f(x_p^f, y_p)$ (i.e., the denominator in Definition 2.1) of firm p, ($p = 1, \ldots, K$), with fixed inputs $x_p^f \in \mathbb{R}_+^{N_f}$ and outputs

$y_p \in \mathbb{R}^M_+$ requires the following program:

$$\begin{aligned} DF^f_o(x^f_p, y_p) = \quad & \max_{\varphi, z, x^v} \quad \varphi \\ & s.t \quad \sum_{k=1}^{K} z_k y_k \geq \varphi y_p, \\ & \qquad \sum_{k=1}^{K} z_k x^f_k \leq x^f_p, \\ & \qquad \sum_{k=1}^{K} z_k x^v_k \leq x^v, \\ & \qquad z = (z_1, \dots, z_K) \in \Lambda, \\ & \qquad \varphi \geq 0, x^v \geq 0, \end{aligned} \tag{9}$$

where Λ determines the convex or nonconvex assumption of the technology defined in (8). Assume that φ^* is the optimal value of short-run O-oriented model (9). To find a solution that maximizes slacks and surpluses, the following model is solved for all p firms:

$$\begin{aligned} & \max_{S^+, S^-, z, x^v} \quad 1_M.S^+ + 1_{N_f}.S^- \\ & s.t \quad \sum_{k=1}^{K} z_k y_k - S^+ = \varphi^* y_p, \\ & \qquad \sum_{k=1}^{K} z_k x^f_k + S^- = x^f_p, \\ & \qquad \sum_{k=1}^{K} z_k x^v_k \leq x^v, \\ & \qquad z = (z_1, \dots, z_K) \in \Lambda, \\ & \qquad x^v \geq 0, S^+ \geq 0, S^- \geq 0, \end{aligned} \tag{10}$$

with $1_M = (1, \dots, 1) \in \mathbb{R}^M$ and $1_{N_f} = (1, \dots, 1) \in \mathbb{R}^{N_f}$. From model (10), an optimal activity vector $z^{p*} = (z^{p*}_1, \dots, z^{p*}_K)$ is provided for firm p under evaluation. Capacity outputs and the optimal fixed and variable input levels can be computed:

$$\hat{y}^*_p = \sum_{k=1}^{K} z^{p*}_k y_k; \quad x^{f*}_p = \sum_{k=1}^{K} z^{p*}_k x^f_k; \quad x^{v*}_p = \sum_{k=1}^{K} z^{p*}_k x^v_k. \tag{11}$$

Depending on the sector, it might be wise to adjust capacity outputs to account for technical inefficiencies. Realistic planning procedures in a second-best setting may allow for some form of inefficiency in production along part of the planning horizon (see Peters (1985)). This basic intuition may be modeled by modifying the capacity output in the second stage of the SRJIM based on observed technical inefficiency, which may eventually be remedied by an O-oriented efficiency improvement imperative (α^{out}_p). Technically efficient firms ($DF_o(x_p, y_p) = 1$) require no such adjustment. When technical inefficiency is (partially) tolerated, and assuming the O-oriented efficiency improvement imperative or correction factor is less than or equal to unity ($\frac{1}{DF_o(x_p,y_p)} \leq \alpha^{out}_p \leq 1$), the modification of capacity output in (11) can be considered as follows:

$$y_p^* = \alpha_p^{out} \sum_{k=1}^{K} z_k^{p*} y_k. \tag{12}$$

When inefficiencies are partially or completely accepted, capacity outputs decrease and the industry needs additional firms. When no adjustment for inefficiency is made in the planning process, then the O-oriented efficiency improvement imperative or correction factor is simply fixed at unity ($\alpha_p^{out} = 1$). Firms are required to shift away from their maximum capacity when the efficiency improvement imperative (α_p^{out}) moves away from unity.

In a second phase, these 'optimal' frontier results at the firm level are parameters in the SRJIM. The SRJIM minimises the use of fixed inputs in a radial way (using $DF_{fi}^{SR}(x^f, x^v, y)$ from (5)) such that the total production of outputs is at least at the current total level by reallocating production between firms. Reallocation is allowed based on the frontier production outputs and input usage of each firm. In the short run, current plant capacities cannot be exceeded. The formulation of the multi-output and frontier-based SRJIM (hereafter referred to as the basic version (bv)) is specified as:

$$\begin{array}{lll} \min\limits_{\theta^{bv}, w_k^{bv}, X^v} & \theta^{bv}, & \\ s.t. & \sum\limits_{k=1}^{K} w_k^{bv} y_k^* \geq Y, & \\ & \sum\limits_{k=1}^{K} w_k^{bv} x_k^{f*} \leq \theta^{bv} X^f, & \\ & \sum\limits_{k=1}^{K} w_k^{bv} x_k^{v*} \leq X^v, & \\ & 0 \leq w_k^{bv} \leq 1, & k = 1, ..., K, \\ & \theta^{bv} \geq 0, X^v \geq 0, & \end{array} \tag{13}$$

where

$$Y = \left(\sum_{k=1}^{K} y_{k1}, \ldots, \sum_{k=1}^{K} y_{kM} \right) \text{ and } X^f = \left(\sum_{k=1}^{K} x_{k1}^f, \ldots, \sum_{k=1}^{K} x_{kN_f}^f \right). \tag{14}$$

After solving model (13), the vector $(w_p^{bv^*} x_p^{f*}, w_p^{bv^*} x_p^{v*}, w_p^{bv^*} y_p^*)$ can be a target for firm p where $w_p^{bv^*}$ is an optimal solution of model (13) and x_p^{f*}, x_p^{v*} and y_p^* are obtained from the relations (11). Note that the variable inputs X^v in model (13) are a vector of decision variables.

The frontier-based SRJIM (13) focuses on reducing fixed inputs by a scalar θ^{bv}. This is shown in the empirical application in Dervaux, Kerstens, and Leleu (2000) which sought to minimize surgery units. The same motivation applies to empirical applications curbing overfishing in fisheries where output quotas are imposed to guarantee biological sustainability. While fixed inputs can normally not be reduced by definition, one can mothball either temporarily or definitively particular vessels. It is trivial to define an alternative SRJIM that maximises all industry outputs similar to (2): see, e.g., Färe and Grosskopf (2003, p. 109-115) for an output-oriented approach based on directional distance functions.[11]

Geometrically, this SRJIM (13) is a set consisting of a finite sum of line segments, or *zonotopes* (see Hildenbrand (1981, p. 1096)).[12] More precisely, assuming divisibility and additivity of production processes, the industry technology is geometrically represented by the space formed by the finite sum of all the line segments linking the origin and the points representing each production unit (see Dosi, Grazzi, Marengo, and Settepanella (2016, p. 877)). Furthermore, Dosi, Grazzi, Marengo, and Settepanella (2016, footnote 3) remark that convexity comes as a result of the chosen analytical framework: it is not an assumption of some underlying theory of production.

The activity vector $w = (w_1, \dots, w_K)$ indicates which portions of the line segments representing the firm capacities are effectively used to produce outputs from given inputs. The bounds on the activity vector w ($0 \leq w_k \leq 1$) reflect the assumption of constant returns to scale up to full capacity for individual production units (see Hildenbrand (1981, p. 1096)). The optimal solution to this simple LP gives the combination of firms that can produce the same or more outputs with less or the same use of fixed inputs at the aggregate level. In the following Proposition, we prove that model (13) has finite optimum value.

Proposition 4.1. *Model* (13) *is always feasible and has finite optimal value.*

The proofs of Proposition 4.1 and the other propositions are given in Appendix C.

In brief, average and best practice SRJIM share a similar formal structure of the SRJIM. The main difference is that only the best practice version is consistent with the idea of an industry frontier, while the average practice version does not ensure estimation of an industry frontier given uncertainties surrounding the underlying ad hoc capacity estimates.

In the putty-clay framework with limited substitution ex post, Johansen (1972) assumes embodied technical change in the successive vintages of capital. This typically leads to co-existing units of different vintages with different unit costs. One may wonder whether co-existing vintages prevents one from speaking about technical inefficiencies, implying that the frontier version of the SRJIM is questionable. For instance, Belu (2015) illustrates in a putty-clay vintage model where recent vintages are modeled as more efficient than older ones that basic production frontier models may not detect the imputed distribution of inefficiencies. However, we conjecture that the metafrontier framework initiated by O'Donnell, Rao, and Battese (2008) and corrected by Kerstens, O'Donnell, and Van de Woestyne (2019) can provide a way out: for a discrete number of vintages each group technology represents a single vintage and the metaproduction technology is the union of all group technologies. This framework affects both the plant capacity estimates and the SRJIM solution. Since vintages play no role in our empirical application, we leave out the details of such a metafrontier vintage framework for future work.

[11] Färe and Grosskopf (2003) define a model similar to (13), except that they ignore the first phase and base capacities on observed inputs and outputs.

[12] One may also benefit from consulting the work of Koopmans (1977), Hildenbrand (1983) or Settepanella, Dosi, Grazzi, Marengo, and Ponchio (2015).

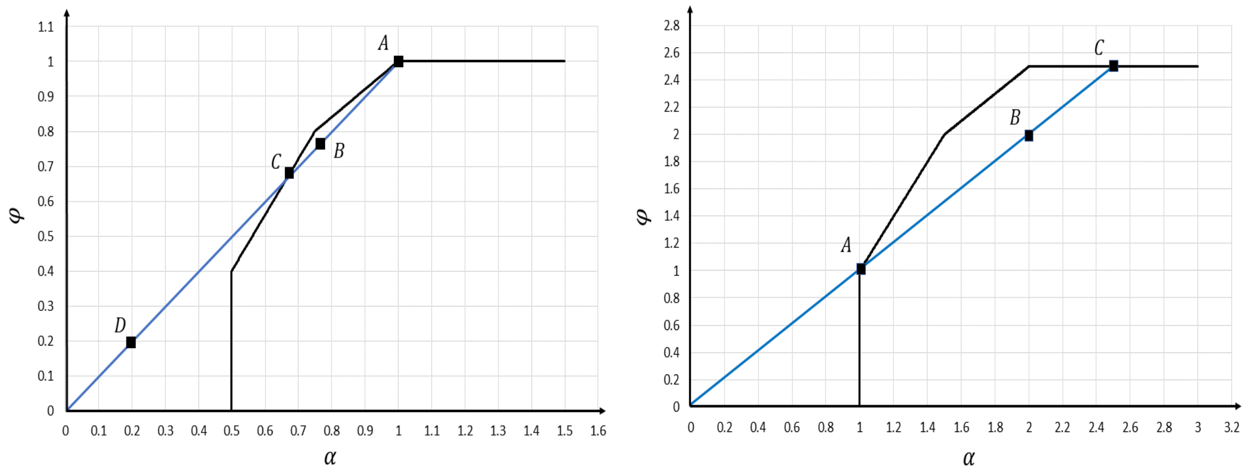
Furthermore, to impose minimal assumptions on the frontier technology when estimating plant capacity utilisation measures, as well as on the short-run industry model, we dispense with the traditionally maintained convexity axiom. Following Afriat (1972) and Deprins, Simar, and Tulkens (1984) we employ a strongly disposable variable returns to scale nonconvex production technology in addition to the more traditional convex production technology. Nonconvex models are known to provide a tighter fit with the data.

Some may object that social planning based on an SRJIM is too demanding: perhaps, one should allow for some amount of technical inefficiency persisting among firms. But, as shown in Kerstens, Vestergaard, and Squires (2006) and as developed infra, it is straightforward to adjust the frontier-based short-run Johansen (1972) industry model to allow for some technical inefficiency.

Additionally, there are some subtle differences between average practice and best practice models. Average practice models ignore fixed inputs, while best practice models do not. As a matter of fact, in average practice models the fixed inputs indirectly determine the capacities. Furthermore, some of the average practice authors assume cost minimization (e.g., Hildenbrand (1983, p. 175)). Indeed, average practice models need input prices to determine the cost per output, while many best practice models depend solely on physical inputs and outputs. It is relatively easy to demonstrate that the feasible set of the multi-output SRJIM (13) under certain conditions is comparable to the multi-output average practice zonotope set in Dosi, Grazzi, Marengo, and Settepanella (2016, Appendix B).

Finally, we mention a series of methodological refinements of the SRJIM. First, it has been rather common to trace how the short-run average practice Johansen (1972) industry production function has evolved over time (Førsund and Hjalmarsson (1983), Førsund and Jansen (1983), Førsund and Hjalmarsson (1987), Wibe (1995)). Second, Dosi, Grazzi, Marengo, and Settepanella (2016) define a normalized volume of the zonotope as a measure of industry heterogeneity. These authors also propose a measure of productivity change based on the zonotope's main diagonal, and assess the role of firm entry and exit on industry level productivity growth (see Settepanella, Dosi, Grazzi, Marengo, and Ponchio (2015) for technical details). Both these developments so far do not seem to have been implemented in a frontier context.

To provide some intuition, we graphically show using 13 fictitious observations (Appendix B) with two inputs (one variable, one fixed) and a single output, that by solving model (13) the optimal weight vector w^{bv^*} does not guarantee the projected point is part of the technology. Figure 1a presents a two dimensional representation of this three dimensional technology. The horizontal axis shows the amount of simultaneous change in fixed and variable inputs (α) for the target point 13 in a radial way while the vertical axis shows the amount of changes in outputs (φ). For observation 13, $(\alpha, \varphi) = (1, 1)$ since $(x_{13}^{v*}, x_{13}^{f*}, y_{13}^{*}) = (6, 4, 5)$. Consequently, the target point of observation 13 is depicted as the black solid box (label A). Based on these results, we must scale down point A by a factor 0.2 resulting in the target point $(1.2, 0.8, 1)$ for which $(\alpha, \varphi) = (0.2, 0.2)$. The corresponding point is labeled D in Figure 1a: obviously, point D does not belong to the technology and is thus



(a) Output-oriented case (b) Input-oriented case

Figure 1: Intersection of the technology with the plane going through the origin and the output- and input-oriented target point of observation 13

infeasible.

Anticipating further developments in Section 5, the revised version of the SRJIM will only consider the line segment between points A and C in Figure 1a. The new SRJIM based on the I-oriented plant capacity will in Figure 1b start from point A and only considers solutions on the line segment between points A and C.

5 Output-, Attainable Output-, and Input-oriented Short-run Johansen Industry Models: New Proposals

This section develops methodological refinements to the basic SRJIM. We first correct the SRJIM such that the scaling of the plant capacity inputs and outputs remains technically feasible.Thereafter, we develop a new SRJIM based on the AO-oriented plant capacity concept. Finally, we develop a new SRJIM based on the I-oriented plant capacity notion.

5.1 Short-run Johansen Industry Model with Output-oriented Capacity Measures: A Revised Version

This model requires two steps. Starting from models (9) and (10), an optimal firm p activity vector z^{p*} is provided. Capacity output and its optimal use of fixed and variable inputs x_p^{f*} and x_p^{v*} can be computed by means of equation (11) and optimal outputs y_p^* can be obtained by equation (12).

In step two, these 'optimal' frontier results (capacity output, variable and fixed inputs) at the firm level are used as parameters in the SRJIM (hereafter also referred to as the revised version (rv)):

$$
\begin{aligned}
\min_{\theta^{rv},w^{rv},X^v} \quad & \theta^{rv} \\
s.t. \quad & \sum_{k=1}^{K} w_k^{rv} y_k^* \geq Y, \\
& \sum_{k=1}^{K} w_k^{rv} x_k^{f*} \leq \theta^{rv} X^f, \\
& \sum_{k=1}^{K} w_k^{rv} x_k^{v*} \leq X^v, \\
& w^{rv} = (w_1^{rv}, \ldots, w_K^{rv}) \in \Gamma^{rv}, \\
& \theta^{rv} \geq 0, X^v \geq 0.
\end{aligned}
\tag{15}
$$

where

$$
Y = \left(\sum_{k=1}^{K} y_{k1}, \ldots, \sum_{k=1}^{K} y_{kM} \right) \text{ and } X^f = \left(\sum_{k=1}^{K} x_{k1}^f, \ldots, \sum_{k=1}^{K} x_{kN_f}^f \right),
$$

and

$$
\Gamma^{rv} = \{(w_1, \ldots, w_K) \mid w_k \leq 1, (w_k x_k^{f*}, w_k x_k^{v*}, w_k y_k^*) \in T^{\Lambda},\ k = 1, \ldots, K\}. \tag{16}
$$

This set Γ^{rv} determines the feasible weights $(w_1, \ldots, w_K)$ such that the target points $(w_p x_p^{f*}, w_p x_p^{v*}, w_p y_p^*)$, $(p = 1, \ldots, K)$, belong to the technology. Note that for feasible weights $(w_1, \ldots, w_K) \in \Gamma^{rv}$, we have $w_p \leq 1$ for all $p = 1, \ldots, K$. Therefore in model (15), the decision variable w_p^{rv} scales down the target point $(x_p^{f*}, x_p^{v*}, y_p^*)$ of firm p and respects the technology. Note that in model (15), the vector X^v of variable inputs are decision variables. To obtain a lower bound L_p^{rv} for $w_p^{rv}, (p = 1, ..., K)$, we need to solve model (17):

$$
\begin{aligned}
L_p^{rv} = \quad \min_{\delta,z} \quad & \delta \\
s.t. \quad & \sum_{k=1}^{K} z_k y_k \geq \delta y_p^*, \\
& \sum_{k=1}^{K} z_k x_k^f \leq \delta x_p^{f*}, \\
& \sum_{k=1}^{K} z_k x_k^v \leq \delta x_p^{v*}, \\
& z = (z_1, \ldots, z_K) \in \Lambda, \\
& \delta \geq 0,
\end{aligned}
\tag{17}
$$

where y_p^*, x_p^{f*} and x_p^{v*} are defined in (11). By solving model (17), output and input capacity targets are scaled down such that they become feasible within the technology. Therefore, model (17) can be interpreted as reducing the capacity targets to obtain the lower bound of weights, while respecting the technology. This relaxes the assumption of constant returns to scale up to full capacity in the basic version of the model.

Note that the main difference between the basic version (13) and the revised version (15) of the SRJIM is in the range of the weights $(w_1, \ldots, w_K)$: in model (13) we have $0 \leq w_k^{bv} \leq 1$, while in model (15) we have $L_k^{rv} \leq w_k^{rv} \leq 1$. Therefore, after solving model (15), the vector $(w_p^{rv^*} x_p^{f*}, w_p^{rv^*} x_p^{v*}, w_p^{rv^*} y_p^*)$, where $w_p^{rv^*}$ is an optimal solution of model (15), can be a target for firm p which belongs to the technology T^Λ.

Contrasting the basic (bv) and revised version (rv) of the SRJIM yields the following result:

Proposition 5.1. *In technology* (8)*, we have:*

(i) Model (15) *is always feasible and it has finite optimal value.*

(ii) Assume that $(\theta^{bv^*}, w^{bv^*})$ *and* $(\theta^{rv^*}, w^{rv^*})$ *are an optimal solution of models* (13) *and* (15)*,respectively, then we have:* $\theta^{bv^*} \leq \theta^{rv^*}$ *and* $w_p^{bv^*} \overset{\geq}{\underset{<}{=}} w_p^{rv^*}$.

(iii) If $\theta^{bv^*} < \theta^{rv^*}$*, then for all multiple optimal solutions of model* (13)*, there exists* $k \in \{1, \ldots, K\}$ *such that the corresponding target point* $(w_k^{bv^*} x_k^{f*}, w_k^{bv^*} x_k^{v*}, w_k^{bv^*} y_k^*)$ *does not belong to the technology.*

(iv) If $\theta^{bv^*} = \theta^{rv^*}$*, then there is at least one optimal solution of model* (13) *for which the corresponding target points of all observed units belong to the technology.*

Interpreting Proposition 5.1, the fact that $\theta^{bv^*} \leq \theta^{rv^*}$ shows the empirical relevance of relaxing the hypothesis of constant returns to scale up to full capacity. Furthermore, it also shows that if we have $\theta^{bv^*} < \theta^{rv^*}$, then for every multiple optimal solution of the basic version of the SRJIM (13), there is at least one observation for which its target point does not respect the technology. Also, the relation $\theta^{bv^*} = \theta^{rv^*}$ guarantees one optimal solution of the basic version of the SRJIM (13) such that all corresponding target points of observations belong to the technology.

It is important to note that the relation $\theta^{bv^*} = \theta^{rv^*}$ does not guarantee that all multiple optimal solutions of model (13) lead to target points belonging to the technology. Even if $\theta^{bv^*} = \theta^{rv^*}$, the possibility exists of having a target point of some observations not respecting the technology.

By solving model (13) on the data of the numerical example in Table B.1, we obtain $\theta^{rv^*} = 0.660$. Hence, we have $0.638 = \theta^{bv^*} < \theta^{rv^*} = 0.660$. Therefore, based on Proposition 5.1, for every multiple optimal solution of the basic version of the SRJIM (13), there is at least one observation for which its target point does not respect the technology.

As illustrated in Figure 1a, the traditional O-oriented SRJIM (13) scales down point A to obtain the target point D, located outside of the technology. But, by implementing the revised SRJIM (15), the target point A translates to the solid black box B: this remains technically feasible by remaining within the technology (see Appendix D, section D.1).

5.2 Short-run Johansen Industry Model with Attainable Output-oriented Efficiency Measure: New Proposal

As mentioned in Section 2.2, the original O-oriented $PCU_o(x, x^f, y)$ has no variable input limitations. However, in most empirical settings this is unrealistic and we limit the variable inputs available at either the firm or the industry level (see Kerstens, Sadeghi, and Van de Woestyne (2019b) for details). Thus, $APCU_o(x, x^f, y, \bar{\lambda})$ is a more realistic alternative PCU measure provided a reasonable level $\bar{\lambda}$ is chosen.

The AO-oriented efficiency measure $ADF_o^f(x_p^f, y_p, \bar{\lambda})$ at level $\bar{\lambda} \in \mathbb{R}_+$ is computed by:

$$
\begin{aligned}
ADF_o^f(x_p^f, y_p, \bar{\lambda}) = \quad & \max_{x^v, \varphi, z} \quad \varphi \\
& s.t \quad \sum_{k=1}^{K} z_k y_k \geq \varphi y_p, \\
& \qquad \sum_{k=1}^{K} z_k x_k^f \leq x_p^f, \\
& \qquad \sum_{k=1}^{K} z_k x_k^v = x^v, \\
& \qquad x^v \leq \bar{\lambda} x_p^v, \\
& \qquad z = (z_1, \ldots, z_K) \in \Lambda, \\
& \qquad x^v \geq 0.
\end{aligned}
\tag{18}
$$

In model (18), the scalar $\bar{\lambda}$ is varied over some part of the interval $(0, \infty)$. But, when $\bar{\lambda} < 1$, then model (18) may be infeasible. However, Kerstens, Sadeghi, and Van de Woestyne (2019b) determine the complete feasible interval for $\bar{\lambda}$ by defining three critical points. For our purpose, we only need two critical points:

Definition 5.1. For a given observation (x_p, y_p), the following two critical points C_P^1 and C_P^2 can be defined.

$$C_P^1 = DF_{vi}^{SR}(x_p^f, x_p^v, 0), \tag{19}$$

and

$$C_P^2 = DF_{vi}^{SR}(x_p^f, x_p^v, y_p). \tag{20}$$

Note that C_P^1 and C_P^2 make up the components of the I-oriented $PCU_i(x, x^f, y)$ in Definition 2.3. Furthermore, Kerstens, Sadeghi, and Van de Woestyne (2019b) have proven that for every observation (x_p, y_p): if $\bar{\lambda} < C_P^1$, then model (18) is infeasible.

Assume that φ^* is the optimal value of model (18), then the following model can be solved to find a solution maximizing slacks and surpluses:

$$
\begin{aligned}
\max_{x^v, S^+, S^-, z} \quad & 1_M.S^+ + 1_{N_f}.S^- \\
s.t \quad & \sum_{k=1}^{K} z_k y_k - S^+ = \varphi^* y_p, \\
& \sum_{k=1}^{K} z_k x_k^f + S^- = x_p^f, \\
& \sum_{k=1}^{K} z_k x_k^v = x^v, \\
& x^v \leq \bar{\lambda} x_p^v, \\
& z = (z_1, \dots, z_K) \in \Lambda, \\
& x^v \geq 0, S^+ \geq 0, S^- \geq 0.
\end{aligned}
\tag{21}
$$

The method is developed in two steps. First, from model (21) an optimal activity vector $z^{p^*} = (z_1^{p*}, \dots, z_K^{p*})$ is provided for firm p under evaluation yielding capacity output and optimal fixed and variable inputs:

$$
y_p^* = \alpha_p^{out} \sum_{k=1}^{K} z_k^{p*} y_k; \quad x_p^{f*} = \sum_{k=1}^{K} z_k^{p*} x_k^f; \quad x_p^{v*} = \sum_{k=1}^{K} z_k^{p*} x_k^v. \tag{22}
$$

Moreover, the O-oriented efficiency improvement imperative or correction factor α_p^{out}, which indicates the portion of adjustment for the technical inefficiency of firm p, is less than or equal to unity ($\frac{1}{DF_o(x_p, y_p)} \leq \alpha_p^{out} \leq 1$). This is repeated for all firms $p = 1, \dots, K$.

In a second step, these 'optimal' frontier results (capacity output, variable and fixed inputs) at the firm level are used as parameters in the below SRJIM (hereafter referred to as the attainable version (att)):

$$
\begin{aligned}
\min_{\theta^{att}, w^{att}, X^v} \quad & \theta^{att} \\
s.t. \quad & \sum_{k=1}^{K} w_k^{att} y_k^* \geq Y, \\
& \sum_{k=1}^{K} w_k^{att} x_k^{f*} \leq \theta^{att} X^f, \\
& \sum_{k=1}^{K} w_k^{att} x_k^{v*} \leq X^v, \\
& w^{att} = (w_1^{att}, \dots, w_K^{att}) \in \Gamma^{att}, \\
& \theta^{att} \geq 0, X^v \geq 0,
\end{aligned}
\tag{23}
$$

where

$$
Y = \left(\sum_{k=1}^{K} y_{k1}, \dots, \sum_{k=1}^{K} y_{kM} \right) \text{ and } X^f = \left(\sum_{k=1}^{K} x_{k1}^f, \dots, \sum_{k=1}^{K} x_{kN_f}^f \right),
$$

and

$$
\Gamma^{att} = \{(w_1, \dots, w_K) \mid w_k \leq 1, \ (w_k x_k^{f*}, w_k x_k^{v*}, w_k y_k^*) \in T^{\Lambda}, \ k = 1, \dots, K\}, \tag{24}
$$

where y_p^*, x_p^{f*} and x_p^{v*} are now defined in (22) instead of (11). Note that the variable inputs X^v in model (23) is a vector of decision variables. Set Γ^{att} determines the feasible area of weights $(w_1, \ldots, w_K)$ such that the target point $(w_p x_p^{f*}, w_p x_p^{v*}, w_p y_p^*)$, where $p = 1, \ldots, K$, belongs to the technology.

The constraints $w_k \leq 1,\ (k = 1, ..., K)$, in set Γ^{att} guarantee that the obtained target points $(w_p x_p^{f*}, w_p x_p^{v*}, w_p y_p^*)$ can be magnified at most by $\bar{\lambda}$ which is an attainable level of variable inputs defined in model (18). Therefore, in model (23) decision variable w_k scales down the target point $(x_k^{f*}, x_k^{v*}, y_k^*)$ of firm p such that the technology is respected. Note that we have no relation between θ^{att^*} and θ^{rv^*} in optimality.

To obtain a lower bound $L_p^{att}, (p = 1, ..., K)$, for w_p^{att} in model (23) we need to solve model (17) where y_p^*, x_p^{f*} and x_p^{v*} are now defined in (22) instead of (11).

Note that the attainable SRJIM (23) can lead to infeasibilities in practical applications. Proposition 5.2 proves some necessary and sufficient conditions for which model (23) is feasible.

Proposition 5.2. *In technology* (8)*, we have:*

(i) Model (23) *is feasible if and only if* $\sum_{k=1}^K y_k^* \geq Y$*.*

(ii) If $C_k^2 \leq \bar{\lambda}$ *for all* $k = 1, \ldots, K$*, then model* (23) *is feasible.*

(iii) If we remove constraint $(w_1^{att}, \ldots, w_K^{att}) \in \Gamma^{att}$ *in model* (23)*, then model* (23) *is always feasible.*

(iv) If model (23) *is infeasible under the convex case, then it is infeasible under the nonconvex case.*

Based on Proposition 5.2, if there is an $m \in \{1, ..., M\}$ such that $\sum_{k=1}^K y_{km}^* < \sum_{k=1}^K y_{km}$, then model (23) is infeasible. Also, if model (23) is infeasible, then there is some $k \in \{1, ..., K\}$ such that we have $C_k^2 > \bar{\lambda}$. However, since $C_k^2 \leq 1$, if we assume that $\bar{\lambda} \geq 1$, then the attainable SRJIM (23) is feasible. Finally, when the attainable SRJIM need not comply with the technology, this model is always feasible. Again, the problem of infeasibility is potentially worse under nonconvexity.

After solving model (23), the vector $(w_p^{att^*} x_p^{f*}, w_p^{att^*} x_p^{v*}, w_p^{att^*} y_p^*)$ can be a target for firm p which belongs to the technology (8), and in which $w_p^{att^*}$ is an optimal solution of model (23) and x_p^{f*}, x_p^{v*} and y_p^* are obtained from the relations (22). Note that if in the SRJIM (23) instead of minimising the fixed inputs, we maximise the outputs in a radial way by reallocating production between firms, then Proposition 5.2 becomes redundant.

5.3 Short-run Johansen Industry Model with Input-oriented Capacity Measures: New Proposal

The I-oriented short-run efficiency measure $DF_{vi}^{SR}(x_p^f, x_p^v, 0)$ is computed by optimizing the following program [13]:

$$
\begin{aligned}
DF_{vi}^{SR}(x_p^f, x_p^v, 0) = \quad & \min_{\theta, z} \quad \theta \\
& s.t \quad \sum_{k=1}^{K} z_k y_k \geq y_{min}, \\
& \qquad \sum_{k=1}^{K} z_k x_k^f \leq x_p^f, \\
& \qquad \sum_{k=1}^{K} z_k x_k^v \leq \theta x_p^v, \\
& \qquad z = (z_1, \ldots, z_K) \in \Lambda, \\
& \qquad \theta \geq 0.
\end{aligned} \tag{25}
$$

Note that the observed output levels on the right-hand side of the output constraints are set equal to y_{min}. These output levels are compatible with any output levels where production is initiated and differs from zero. The reader is referred to Kerstens, Sadeghi, and Van de Woestyne (2019a, Proposition B.1) for additional interpretations (see also supra). Therefore, in model (25), one can put y at the right-hand side of the first constraint and make it a decision variable (instead of y_{min}). In so doing, we are symmetric with the O-oriented model (9) where the variable inputs are decision variables. Assume that θ^* is the optimal value of model (25), the following model can be solved which maximizes slacks and surpluses:

$$
\begin{aligned}
\max_{z, S^+, S^{v-}, S^{f-}} \quad & 1_M.S^+ + 1_{N_f}.S^{f-} + 1_{N_v}.S^{v-} \\
s.t \quad & \sum_{k=1}^{K} z_k y_k - S^+ = y_{min}, \\
& \sum_{k=1}^{K} z_k x_k^f + S^{f-} = x_p^f, \\
& \sum_{k=1}^{K} z_k x_k^v + S^{v-} = \theta^* x_p^v, \\
& z = (z_1, \ldots, z_K) \in \Lambda, \\
& S^+ \geq 0, S^{v-} \geq 0, S^{f-} \geq 0,
\end{aligned} \tag{26}
$$

with $1_{N_v} = (1, \ldots, 1) \in \mathbb{R}_+^{N_v}$.

Similar to the O-oriented SRJIM above, we proceed in two steps. First, from model (26) an optimal activity vector $z^{p*} = (z_1^{p*}, \ldots, z_K^{p*})$ is provided for firm p under evaluation allowing computation

[13] One can put y at the right-hand side of the first constraint and make it a decision variable (instead of 0). In so doing, we are symmetric with the O-oriented model (9) where the variable inputs are decision variables. ???

of capacity output and its optimal levels of fixed and variable inputs:

$$y_p^* = \sum_{k=1}^{K} z_k^{p*} y_k; \quad x_p^{f*} = \sum_{k=1}^{K} z_k^{p*} x_k^f; \quad x_p^{v*} = \alpha_p^{inp} \sum_{k=1}^{K} z_k^{p*} x_k^v. \tag{27}$$

This has to be repeated for all firms $p = 1, \ldots, K$. The I-oriented efficiency improvement imperative or correction factor α_p^{inp}, which indicates the portion of adjustment for variable I-oriented technical inefficiency of firm p is greater than or equal to unity ($1 \leq \alpha_p^{inp} \leq \frac{1}{DF_{vi}^{SR}(x^f,x^v,y)}$).

In a second step, these 'optimal' frontier results (capacity output and capacity variable and fixed inputs) at the firm level are used as parameters in the below SRJIM (hereafter referred to as the I-oriented version (inp)):

$$\begin{aligned}
\min_{\theta^{inp}, w^{inp}, X^v} \quad & \theta^{inp} \\
s.t. \quad & \sum_{k=1}^{K} w_k^{inp} y_k^* \geq Y, \\
& \sum_{k=1}^{K} w_k^{inp} x_k^{f*} \leq \theta^{inp} X^f, \\
& \sum_{k=1}^{K} w_k^{inp} x_k^{v*} \leq X^v, \\
& w^{inp} = (w_1^{inp}, \ldots, w_K^{inp}) \in \mathit{\Gamma}^{inp}, \\
& \theta^{inp} \geq 0, X^v \geq 0.
\end{aligned} \tag{28}$$

where

$$Y = \left(\sum_{k=1}^{K} y_{k1}, \ldots, \sum_{k=1}^{K} y_{km} \right) \text{ and } X^f = \left(\sum_{k=1}^{K} x_{k1}^f, \ldots, \sum_{k=1}^{K} x_{kN_f}^f \right), \tag{29}$$

and

$$\mathit{\Gamma}^{inp} = \{(w_1, \ldots, w_K) \mid w_k \geq 1, (w_k x_k^{f*}, w_k x_k^{v*}, w_k y_k^*) \in T^\Lambda,\ k = 1, \ldots, K\}. \tag{30}$$

This set $\mathit{\Gamma}^{inp}$ determines the feasible weights $(w_1, \ldots, w_K)$ such that the target points $(w_p x_p^{f*}, w_p x_p^{v*}, w_p y_p^*)$ belong to the technology. Note that for the weights $(w_1, \ldots, w_K) \in \mathit{\Gamma}^{inp}$, we have $w_p \geq 1$ for all $p = 1, \ldots, K$. Therefore, in model (28) decision variable w_k scales up the target point $(x_k^{f*}, x_k^{v*}, y_k^*)$ of firm p such that the technology is respected. Note that θ^{inp^*} cannot be compared to θ^{bv^*}, θ^{rv^*} and θ^{att^*} in optimality.

To obtain an upper bound U_p^{inp}, where $p = 1, ..., K$, for w_p^{inp} we need to solve the next model

(31):

$$
\begin{aligned}
U_p^{inp} = \quad & \max_{\delta,z} \quad \delta \\
& s.t. \quad \sum_{k=1}^{K} z_k y_k \geq \delta y_p^*, \\
& \qquad\;\; \sum_{k=1}^{K} z_k x_k^f \leq \delta x_p^{f*}, \\
& \qquad\;\; \sum_{k=1}^{K} z_k x_k^v \leq \delta x_p^{v*}, \\
& \qquad\;\; z = (z_1, \ldots, z_k) \in \Lambda, \\
& \qquad\;\; \delta \geq 0,
\end{aligned}
\tag{31}
$$

where y_p^*, x_p^{f*} and x_p^{v*} are defined in (27). By solving this model we scale up the output and input capacity targets such that they become feasible within the technology. Notice that in all previous models based on O-oriented plant capacity we start from output and input capacity targets that are situated in point A at the horizontal section in Figure 1a, while here we start from I-oriented plant capacity targets that are situated at the vertical section in Figure 1a: in Figure 1b one can note another point A at the vertical section.

Therefore, model (31) can be interpreted as expanding the capacity targets to obtain the upper bound of weights while respecting the technology. Note that all weights $w_k^{inp} \geq 1$ since the optimal solution starts out from the vertical section in Figure 1b and moves up to the right in input-output space, while all previous models based on O-oriented plant capacity start from output and input capacity targets that are situated at the horizontal section in Figure 1a and move down to the left in input-output space. Hence, in model (31) we need to scale up capacity outputs and capacity variable and fixed inputs to meet all requirements.

Note that the I-oriented SRJIM (28) can lead to infeasibilities in practical applications. But, if there are no upper bounds in the I-oriented short-run Johansen industry model (28) (i.e., we do not need to respect the technology by ignoring constraint $(w_1^{inp}, \ldots, w_K^{inp}) \in \Gamma^{inp}$ in model (28)), then model (28) is always feasible. Proposition 5.3 proves some necessary and sufficient conditions for which model (28) is feasible.

Proposition 5.3. *In technology* (8)*, we have:*

(i) Model (28) *is feasible if and only if* $\sum_{k=1}^{K} U_k^{inp} y_k^* \geq Y$.

(ii) If we remove constraint $(w_1^{inp}, \ldots, w_K^{inp}) \in \Gamma^{inp}$ *in model* (28)*, then model* (28) *is always feasible.*

(iii) If model (28) *is infeasible under the convex case, then it is infeasible under the nonconvex case.*

After solving model (28), the vector $(w_p^{inp^*} x_p^{f*}, w_p^{inp^*} x_p^{v*}, w_p^{inp^*} y_p^*)$ can be a target for DMU_p which belongs to the technology (8) where $w_p^{inp^*}$ is an optimal solution of model (28) and x_p^{f*}, x_p^{v*}

and y_p^* are obtained from the relations (27).

To foster understanding, the reader may consult the numerical example in Appendix D.3. It is now shown graphically that by solving the I-oriented SRJIM (28) one obtains a solution that again respects the technology.

Figure 1b shows the intersection of the technology with the plane passing through the origin and the I-oriented target point of observation 13, i.e., point $(x_{13}^{v*}, x_{13}^{f*}, y_{13}^*) = (2, 2, 2)$ which is obtained from equation (27). The horizontal axis shows the amount of simultaneous changes in fixed and variable inputs (α) for the I-oriented target point 13 in a radial way and the vertical axis shows the amount of changes in outputs (φ). Therefore, for $(\alpha, \varphi) = (1, 1)$ we have $(x_{13}^{v*}, x_{13}^{f*}, y_{13}^*) = (2, 2, 2)$ (black solid box A).

Note that by implementing the I-oriented SRJIM (28) by using the numerical data in Table B.1, we have $\theta^{inp^*} = 0.81$. In this case, the target point A (i.e., the target point of unit 13) remains unchanged at point A in Figure 1b (see Appendix D, section D.3).

6 Empirical Illustration

6.1 Data

Our sample is from 170 fishing vessels operating in the northwest Atlantic Ocean during a single year (exact year not disclosed for confidentiality purposes). All vessels use similar technology and catch their fish by dragging a net behind their vessels just off the ocean floor. Catches were grouped into three distinct categories based on species: flatfish, roundfish, and "other". There are three fixed inputs: vessel length, engine horsepower, and vessel gross tonnage. The only variable input is days spent at sea.

Table 1 presents basic descriptive statistics. Vessels are between 36 and 88 feet in length (average 63). Their horsepower ranges from 180 to 1,380 (494 average) and their tonnage is between 5 and 199 (average 90). On average, these vessels fish 67 days per year with a range between 2 and 242 days. Their average roundfish catch is 99,113 pounds with a range between zero and 750,976. Flatfish catch is between 9 and 265,617 pounds (average 50,602). The "other" category average catch is 154,253 pounds with a range between 299 and 1,462,807 pounds.

An important remark needs to be made with respect to the sole variable input time spent at sea in days. Based on equation (11) we have $x_p^{v*} = \sum_{k=1}^{K} z_k^{p*} x_k^v$ and since $\sum_{k=1}^{K} z_k^{p*} = 1$, then $\min_{k=1,\dots,K} x_{kn}^v \leq x_{pn}^{v*} = \sum_{k=1}^{K} z_k^{p*} x_{kn}^v \leq \max_{k=1,\dots,K} x_{kn}^v$ for all $n = 1, \dots, N_v$. Hence, we have $2.222 \leq x_{p1}^{v*} \leq 242.195$ for all $p = 1, \dots, K$. Thus, the optimal amount of variable inputs is always bounded by the minimum and maximum levels of observed variable inputs in the data, and it can certainly not reach the absolute upper bound of 365 days in the year analysed.

Table 1: Descriptive Statistics for 170 Observed Data

	Fixed input 1 Horsepower	Fixed input 2 Length	Fixed input 3 Tonnage	Variable input Days	Output 1 Roundfish	Output 2 Flatfish	Output 3 Other
Average	494.4824	62.67194	90.14706	67.79868	99113.2254	50601.95	154252.701
St. Dev.	210.1697	14.60609	54.59042	66.21814	154640.012	54758.96	233021.661
Min	180	35.8	5	2.222	0	9	299
Max	1380	88.4	199	242.195	750976	265616.9	1462806.89

Table 2 reports the descriptive statistics of I-oriented, O-oriented and AO-oriented PCU for our vessels using convex and non-convex technologies. These results reflect output- and I-oriented efficiency improvement imperatives of unity (i.e., $\alpha_p^{out} = \alpha_p^{int} = 1$). The main motivation to differentiate between convex and non-convex technologies is that recently Kerstens, Sadeghi, and Van de Woestyne (2019a) revealed significant differences between convex and non-convex PCU. Note that for both the AO-oriented efficiency measure $ADF_o^f(x^f, y, \bar{\lambda})$ and the AO-oriented PCU $APCU_o(x, x^f, y, \bar{\lambda})$, we have chosen $\bar{\lambda} = 2$.

Table 2: Descriptive Statistics of Input and Output Plant Capacity Utilisation for 170 DMUs in both Convex and Non-convex Cases

Convex	$DF_{vi}(x^f, x^v, y)$	$DF_{vi}(x^f, x^v, 0)$	$PCU_i(.)$	$DF_o(.)$	$DF_o^f(.)$	$PCU_o(.)$	$ADF_o^f(.)$	$APCU_o(.)$
Average	0.576	0.201	16.557	2.283	8.056	0.631	3.892	0.712
St. Dev.	0.242	0.279	21.297	1.735	14.286	0.342	3.777	0.246
Min	0.109	0.009	1.000	1.000	1.000	0.022	1.000	0.134
Max	1.000	1.000	108.999	11.546	129.824	1.000	28.865	1.000
Nonconvex								
Average	0.984	0.222	28.120	1.056	3.866	0.679	1.454	0.862
St. Dev.	0.064	0.300	30.095	0.230	10.792	0.344	1.189	0.220
Min	0.543	0.009	1.000	1.000	1.000	0.014	1.000	0.094
Max	1.000	1.000	108.999	2.675	129.558	1.000	11.282	1.000

Analyzing Table 2, first we conclude that on average the $PCU_i(x, x^f, y)$ indicates that one needs 16.55 times more variable inputs (days) with current outputs than with zero outputs under C, while under NC one employs 28.12 times more variable inputs (days). Second, on average the biased PCU measure $DF_o^f(x^f, y)$ indicates that outputs can be increased 8.05 times under C and 3.86 times under NC. There is substantial variation in $DF_o^f(x^f, y)$ as indicated by the standard deviation and range: the maximum increase in outputs amounts to 129.824 times under C and 129.558 under NC. Third, on average the unbiased PCU measure $PCU_o(x, x^f, y)$ indicates that current outputs are 63% of maximal plant capacity outputs under C and 67% under NC. Heterogeneity in $PCU_o(x, x^f, y)$ is large as indicated by the standard deviation and the range: the minimum of 2.2% under C and 1.4% under NC are quite low. Fourth, for the biased attainable PCU measure $ADF_o^f(x^f, y, \bar{\lambda} = 2)$ the average of the output magnification under C is higher than under NC. For a twofold increase in variable inputs (i.e., $\bar{\lambda} = 2$), we obtain on average a 3.892 output magnification under C and 1.454 under NC. Fifth, the average of $APCU_o(x, x^f, y, \bar{\lambda} = 2)$ is smaller under C than under NC.

In conclusion, the different PCU measures behave substantially different under C and NC technologies. This is in line with earlier results reported by Kerstens, Sadeghi, and Van de Woestyne (2019a).

6.2 Key Results

Turning to the results of the four SRJIM, Table 3 shows basic descriptive statistics of their efficiency scores (θ), weights (w_p), lower and upper bounds (L_p and U_p), the number of units for which their weights coincide to their lower bound ($\#w_p = L_p$), the number of units for which their weights coincide to their upper bound ($\#w_p = U_p$), and the number of units which are located outside of the technology ($\#\ DMU_p \notin T$). The rows of Table 3 include results under the convex and nonconvex cases.

Table 3: The results of weights, lower and upper bounds for all methods

		Weights				Lower or upper bound						
Convex	θ	Average	St. Dev.	Min	Max	Average	St. Dev.	Min	Max	$\#\ w_p = L_p$	$\#\ w_p = U_p$	$\#\ DMU_p \notin T$
bv	0.3	0.330	0.466	0	1					111	54	117
rv	0.84	0.937	0.108	0.5802	1	0.9366	0.1076	0.580	1	170	170	0
att	0.82	0.946	0.104	0.5802	1	0.9464	0.1040	0.580	1	170	170	0
inp	Inf	Inf	Inf	Inf	Inf	61.0550	25.4301	1	116.19	Inf	Inf	Inf
Nonconvex												
bv	0.35	0.350	0.474	0	1					109	56	114
rv	0.92	0.996	0.025	0.817	1	0.996	0.025	0.817	1	170	170	0
att	0.91	0.995	0.033	0.6858	1	0.995	0.033	0.686	1	170	170	0
inp	Inf	Inf	Inf	Inf	Inf	14.567	35.205	1	116.19	Inf	Inf	Inf

bv: basic version of O-oriented SRJIM
rv: revised version of O-oriented SRJIM
att: AO-oriented SRJIM
inp: I-oriented SRJIM

We draw the following conclusions from Table 3. First, fixed inputs can be reduced by 70% in the basic version (bv), but only 16% in the revised version (rv). This dramatic difference is because 117 of the 170 vessels are not part of the frontier technology, an issue largely ignored in the SRJIM literature. This is due to low average weights in the basic version compared to the revised version. In the revised version all 170 observations have weights equal to their lower bound. Second, applying a nonconvex technology slightly attenuates these results: fixed inputs can be reduced by 65% in the basic version and by just 8% in the revised version. Average weights are higher under nonconvexity in both versions.

Third, opting for an AO-oriented PCU slightly improves the results compared to the revised version of the O-oriented PCU because capacity inputs and outputs are somewhat reduced. Under convexity fixed inputs can be reduced by 16% in the revised version and by 18% in the attainable case, while in the nonconvex case fixed inputs can be reduced by 8% in the revised version and by 9% in the attainable case. While the average weight slightly increases under convexity, it marginally decreases under nonconvexity. Also in the attainable version all 170 observations have weights equal to their lower bound. Fourth, the I-oriented SRJIM (28) is infeasible for this empirical application under both convex and nonconvex cases. Thus, it is impossible to scale up the I-oriented capacity targets of units such that these are capable to generate the current aggregate output levels while respecting the technology. The reader should realise that the I-oriented SRJIM (28) does yield a solution for the numerical example (see Appendix D), but that the configuration of the empirical data leads to an infeasibility. More detailed results are found in Appendix E.

We think it is safe to conclude the following from our empirical illustration. First, the basic version of the SRJIM is both conceptually wrong and leads to overly optimistic reductions in fixed inputs. Secondly, the degree of reallocation is somehow conditioned on the type of PCU to which one adheres. Our results indicate that the traditional O-oriented PCU may still be a bit too optimistic compared to the AO-oriented PCU that leads to fewer reductions in fixed inputs. Regrettable, the conceptually appealing I-oriented SRJIM results in an infeasible solution for our data.

7 Conclusions

This contribution has provided a cursory review of the historic development of the SRJIM, and distinguishes between the traditional average practice version and the more recent best practice or frontier-based version. The goals of this contribution are twofold. First, we remedy a remaining problem in the Johansen (1972) SRJIM by relaxing the assumption of constant returns to scale up to full capacity for individual production units. Hence, capacity inputs and outputs remain technically feasible and remain within the technology. Second, we have opened up the methodological choices of the SRJIM by introducing a new I-oriented PCU, and an AO-oriented PCU.

In order to demonstrate our findings, we provided a basic numerical example to illustrate the differences and similarities between these modeling options, as well as an empirical illustration using US based fishery data. Both these illustrations have shown the viability of our new modeling options.

To conclude, we mention some avenues for future research. One possibility is to further extend the choice of PCU by including a graph-oriented plant capacity concept (see Kerstens, Sadeghi, and Van de Woestyne (2020)) or some of the new plant capacity concepts introduced in Kerstens and Sadeghi (2023). Furthermore, this presentation may perhaps benefit from introducing a directional distance function to unify all types of specialised efficiency measures that are currently employed.

Another avenue is to trace the evolution of the frontier-based SRJIM over time. Finally, the link between a vintage-based model and the metafrontier framework as the union of several vintage group technologies needs to be worked out in more detail.

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Appendices: Supplementary Material

A Graphical Illustrations

Now we try to clarify Definitions 2.1, 2.2 and 2.3 with the help of a two-dimensional Figure A.1 which depicts a single variable input and an output space. In particular, Figure A.1 shows a total product curve for given variable inputs as the polyline *abcd* and its horizontal extension at d. We focus on observation e. Note that observations are represented by squares and projection points by circles.

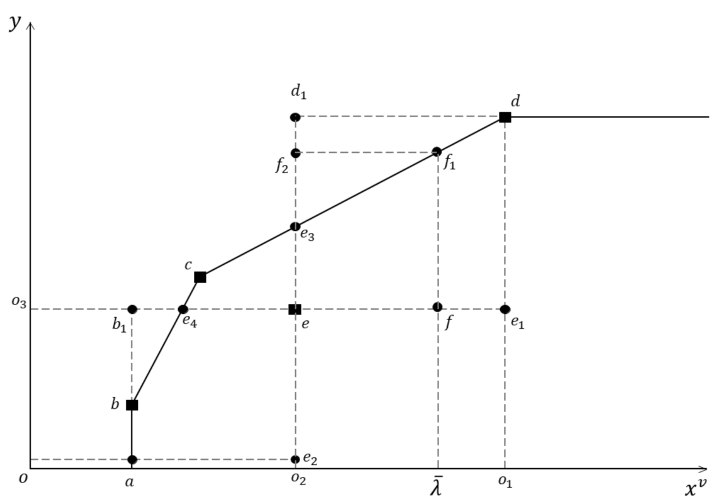


Figure A.1: Total product curve: Output-oriented, attainable output-oriented and I-oriented plant capacities

The O-oriented $PCU_o(x, x^f, y)$ compares point e to its vertical projection point e_3 on the frontier on the one hand, and the translated point e_1 that consumes more variable inputs to its vertical projection point on the horizontal frontier segment emanating from point d with maximal outputs on the other hand. Clearly, the maximal output d can be labeled the plant capacity output. Thus, the unbiased $PCU_o(x, x^f, y)$ is somehow linked to the distance e_3d_1, whereby point d_1 is simply the translation of the maximal output at point d to the output level comparable with point e.

The AO-oriented plant capacity measure $APCU_o(x, x^f, y, \bar{\lambda})$ compares point e to its vertical projection point e_3 on the frontier on the one hand, and the translated point f that consumes at most a fraction $\bar{\lambda}$ more variable inputs to its vertical projection point at point f_1 with maximal outputs at level $\bar{\lambda}$ on the other hand. Clearly, the maximal output f_1 at level $\bar{\lambda}$ can be labeled the attainable plant capacity output. Thus, the unbiased attainable plant capacity measure $APCU_o(x, x^f, y, \bar{\lambda})$ is somehow linked to the distance e_3f_2, whereby point f_2 is simply the translation of the maximal output at point f_1 to the output level comparable with point e.

The I-oriented $PCU_i(x, x^f, y)$ focuses on a sub-vector of variable inputs and compares point e

to its horizontal projection point e_4 on the frontier on the one hand, and the translated point e_2 (consuming equal amounts of variable inputs but at a zero outputs level) to its horizontal projection point on the vertical frontier segment ab with zero outputs on the other hand. Clearly, the minimal variable input a yielding zero output can be labeled the plant capacity input. Thus, the unbiased $PCU_i(x, x^f, y)$ is somehow linked to the distance $b_1 e_4$, whereby point b_1 is the translation of the variable input at point b to the variable input level comparable with point e.

B Numerical Example on the Infeasibility of Model (13) with the Technology

To provide some intuition, we graphically show that by solving model (13) the optimal weight vector w^{bv^*} does not guarantee that the projection point is part of the technology. Consider a numerical example containing 13 fictitious observations with two inputs generating a single output: one input is variable, the other one is fixed. The first four columns of Table B.1 contain these data. By solving model (13), we obtain $\theta^{bv^*} = 0.638$, where θ^{bv^*} is the optimal value of θ^{bv}. Columns 5 to 7 of Table B.1 show the inputs and outputs targets defined in equation (11) which are obtained by solving model (10). The vector $w^{bv^*} = (w_1^{bv^*}, \ldots, w_K^{bv^*})$ is an optimal solution of model (13) and is reported in column 8. The final target points of inputs and outputs obtained by solving model (13) (i.e., points $(w_p^{bv^*} x_p^{f*}, w_p^{bv^*} x_p^{v*}, w_p^{bv^*} y_p^*)$ corresponding to firm p) are presented in the last three columns.

As can be seen in Table B.1, the value of $w_k^{bv^*}$ for all units is unity except for units $4, 5, 6$ and 13. For these four units, we have $w_4^{bv^*} = w_5^{bv^*} = w_6^{bv^*} = 0$ and $w_{13}^{bv^*} = 0.2$. Therefore, for units $4, 5$ and 6, the target points are located at the origin. However, the target point of unit 13 is $(1.2, 0.8, 1)$: this point does not belong to the production possibility set. We show this by reporting the result of the refined O-oriented SRJIM in Section D.1.

Table B.1: Inputs and outputs targets obtained by solving model (13)

DMU_p	x_p^v	x_p^f	y_p	x_p^{v*}	x_p^{f*}	y_p^*	$w_p^{bv^*}$	$w_p^{bv^*} x_p^{v*}$	$w_p^{bv^*} x_p^{f*}$	$w_p^{bv^*} y_p^*$
1	3	3	2	5	3	4	1	5	3	4
2	2	5	2	6	4	5	1	6	4	5
3	2	7	2	6	4	5	1	6	4	5
4	5	2	2	2	2	2	0	0	0	0
5	10	2	2	2	2	2	0	0	0	0
6	2	2	2	2	2	2	0	0	0	0
7	3	7	4	6	4	5	1	6	4	5
8	3	4	4	6	4	5	1	6	4	5
9	5	3	4	5	3	4	1	5	3	4
10	9	3	4	5	3	4	1	5	3	4
11	5	5	5	6	4	5	1	6	4	5
12	6	4	5	6	4	5	1	6	4	5
13	4	6	5	6	4	5	0.2	1.2	0.8	1

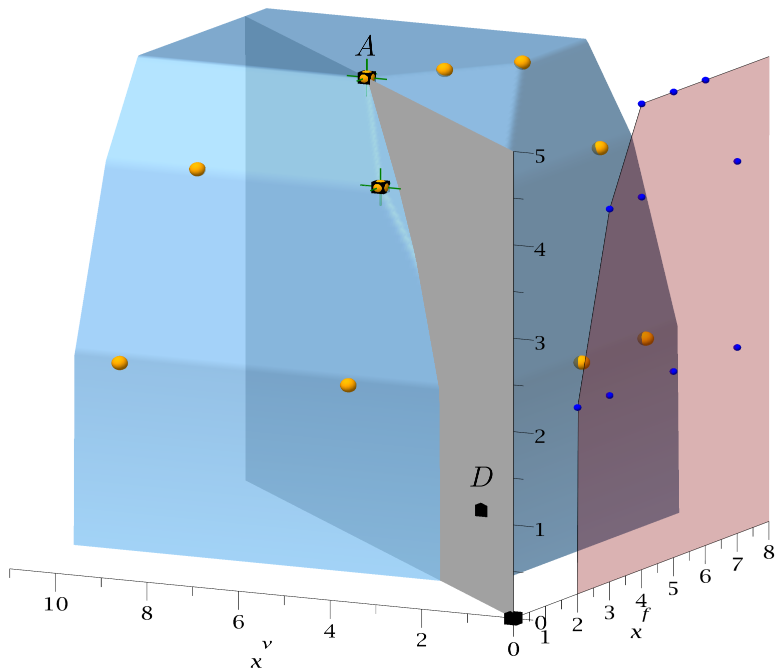
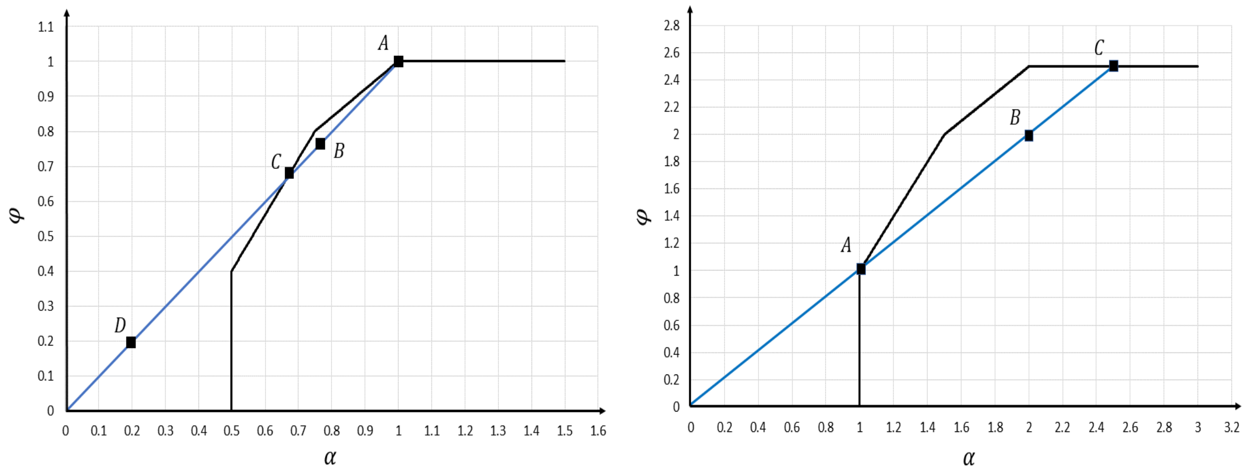


Figure B.1: 3-dimensional view of the convex frontier for numerical example

Also, we can visualize this infeasibility problem in Figures B.1 and B.2a. A three-dimensional representation of the technology resulting from these 13 fictitious observations is provided by Figure B.1. This technology consists of two inputs (variable input x^v and fixed input x^f) and one output (y) and is visible by means of its convex boundary. The original observations are visible by means of orange spheres. The projection of the frontier in the vertical plane $x^v = 0$ is visualised by the transparent red region positioned on the x^f axis. The projection of the original observations in the vertical plane $x^v = 0$ is indicated by blue boxes. The optimal 3D points obtained from equation (11) (i.e., $(x_p^{v*}, x_p^{f*}, y_p^*)$) are denoted with green crosses. Finally, the targets points obtained after applying model (13) (i.e., $(w_p^{bv^*} x_p^{f*}, w_p^{bv^*} x_p^{v*}, w_p^{bv^*} y_p^*)$) are illustrated with black boxes.

The gray intersecting plane passes through the origin and the O-oriented target point $(x_{13}^{v*}, x_{13}^{f*}, y_{13}^*) = (6, 4, 5)$ of observation 13 (label A). Based on the results of Table B.1, since $w_{13}^{bv^*} = 0.2$, this target point scales down by 0.2 times to $(1.2, 0.8, 1)$ depicted by the black square (label D) in the gray intersecting plane. Obviously, this point does not belong to the technology.

To even better illustrate this technological infeasibility, we present in Figure B.2a the intersection of the gray plane and the boundary of technology of Figure B.1. The horizontal axis shows the amount of simultaneous change in fixed and variable inputs (α) for the target point 13 in a radial way while the vertical axis shows the amount of changes in outputs (φ). For observation 13, $(\alpha, \varphi) =$



(a) Output-oriented case (b) Input-oriented case

Figure B.2: Intersection of the technology with the plane going through the origin and the output- and input-oriented target point of observation 13

$(1, 1)$ since $(x_{13}^{v*}, x_{13}^{f*}, y_{13}^{*}) = (6, 4, 5)$. Consequently, the target point of observation 13 is depicted as the black solid box (label A). Again based on the results of Table B.1, we must scale down point A by a factor 0.2 resulting in the target point $(1.2, 0.8, 1)$ for which $(\alpha, \varphi) = (0.2, 0.2)$. The corresponding point is labeled D in Figure B.2a. Geometrically, this scaling factor corresponds with the ratio of Euclidean distances $\|0D\|/\|0A\| = 0.2 = w_{13}^{bv^*}$. Obviously, this point D does not belong to the technology and is thus not feasible.

Based on the results of Table B.1, the O-oriented target points of units $2, 3, 7, 8, 11$ and 13 are identical. Therefore, the intersection of the technology with the plane passing through the origin and the O-oriented target point $(x_p^{v*}, x_p^{f*}, y_p^*)$ of these observations are the same as illustrated in Figure B.2a. The value of $w_k^{bv^*}$ for these units is unity. Therefore, the target point of these units, except unit 13, remains unchanged at point A in Figure B.2a.

C Proofs of Propositions

Proof of Proposition 4.1: Assume that $(S^{+^*}, S^{-^*}, z_k^*)$ is an optimal solution of model (10). Since $S^{+^*} \geq 0$ and $\varphi^* \geq 1$, we have:

$$y_p^* = \alpha_p^{out} \hat{y}_p^* = \alpha_p^{out} \sum_{k=1}^{K} z_k^* y_k \geq \alpha_p^{out} \sum_{k=1}^{K} z_k^* y_k - S^{+^*} \geq \frac{1}{\varphi^*} \sum_{k=1}^{K} z_k^* y_k - S^{+^*} = y_p.$$

By summation on p, we have $\sum_{p=1}^{K} y_p^* \geq \sum_{p=1}^{K} y_p = Y$. Moreover, let $\theta^{bv^*} = \frac{\sum_{k=1}^{K} x_k^{f*}}{X^f}$, $X^{v^*} = \sum_{k=1}^{K} x_k^{v*}$ and $w_k^{bv^*} = 1 (k = 1, \ldots, K)$, then $(\theta^{bv^*}, X^{v^*}, w_k^{bv^*})$ is a feasible solution of model (13) with the finite objective function. Therefore, model (13) has a finite optimum value.

Proof of Proposition 5.1:

(i) The proof is similar with the proof of Proposition 4.1.

(ii) Suppose that the vector $(\theta^{rv^*}, w^{rv^*}, X^{v^*})$ is an optimal solution of model (15). Since the target points $(y_p^*, x_p^{f*}, x_p^{v*})$ for models (13) and (15) are the same, hence $(\theta^{rv^*}, w^{rv^*}, X^{v^*})$ is a feasible solution for model (13). Therefore, $\theta^{bv^*} \leq \theta^{rv^*}$ because this kind of model (13) is a minimising problem. To complete the proof, note that we have $w_p^{bv^*} \gtreqless w_p^{rv^*}$ because the results of the numerical as well as empirical examples show that $w_p^{bv^*}$ can be equal, bigger or smaller than $w_p^{rv^*}$.

(iii) Assume that $\theta^{bv^*} < \theta^{rv^*}$ and $(w_1^{bv^*}, \ldots, w_K^{bv^*})$ is an optimal solution of model (13). This optimal solution is not a feasible solution of model (15), because if we assume that $(w_1^{bv^*}, \ldots, w_K^{bv^*})$ is a feasible solution of model (15), then we have $\theta^{rv^*} \leq \theta^{bv^*}$ and based on the part (i), we have $\theta^{bv^*} \leq \theta^{rv^*}$. Hence, we have $\theta^{rv^*} = \theta^{bv^*}$ which it is a contradiction because we assume that $\theta^{bv^*} < \theta^{rv^*}$. Therefore, we have $(w_1^{bv^*}, \ldots, w_K^{bv^*}) \notin \Gamma^{rv}$. Based on equation (16), there is $k \in \{1, \ldots, K\}$ such that $(w_k^{bv^*} x_k^{f*}, w_k^{bv^*} x_k^{v*}, w_k^{bv^*} y_k^*) \notin T^\Lambda$.

(iv) Assume that $(w_1^{rv^*}, \ldots, w_K^{rv^*})$ is an optimal solution of model (15) with the optimal value θ^{rv^*}. Therefore, it is a feasible solution of model (13) with the objective value θ^{rv^*}. Assume that θ^{bv^*} is an optimal value of model (13). Since we assume that $\theta^{bv^*} = \theta^{rv^*}$, hence $(w_1^{rv^*}, \ldots, w_K^{rv^*})$ is an optimal solution of model (13) and for this optimal solution we have $(w_1^{rv^*}, \ldots, w_K^{rv^*}) \in \Gamma^{rv}$. Thus based on equation (16), we have $(w_k^{rv^*} x_k^{f*}, w_k^{rv^*} x_k^{v*}, w_k^{rv^*} y_k^*) \in T^\Lambda$ for all $k \in \{k = 1, \ldots, K\}$.

Proof of Proposition 5.2:

(i) Suppose that model (23) is feasible and $(w_1^{att^*}, ..., w_K^{att^*})$ is an optimal solution for decision variables $(w_1^{att}, ..., w_K^{att})$. Hence, we have $\sum_{k=1}^K w_k^{att^*} y_k^* \geq Y$. Since $w_k^{att^*} \leq 1$, we have $\sum_{k=1}^K y_k^* \geq Y$.
Now, assume that $\sum_{k=1}^K y_k^* \geq Y$. Letting,

$$w_k^{att^*} = 1, \theta^{att^*} = \max_{n=1,\ldots,N_f} \frac{\sum_{k=1}^K x_{kn}^{f*}}{\sum_{k=1}^K x_{kn}^{f}}, X^{v*} = \sum_{k=1}^K x_k^{v*}.$$

Hence, $(w_k^{att^*}, \theta^{att^*}, X^{v*})$ is a feasible solution of model (23).

(ii) Suppose that $C_k^2 \leq \bar{\lambda}$, then $ADF_o^f(x_p^f, y_p, \bar{\lambda}) = \varphi^* \geq 1$. For this reason, assume that $(z_k^*, \theta^* = C_k^2)$ is an optimal solution of model (20). Since $C_k^2 x_p^v \leq \bar{\lambda} x_p^v$, hence $(\hat{z}_k = z_k^*, \hat{x}^v = C_k^2 x_p^v, \hat{\theta} = 1)$ is a feasible solution of model (18) with objective value $\hat{\theta} = 1$. Therefore, $ADF_o^f(x_p^f, y_p, \bar{\lambda}) \geq 1$ because this kind of model is a maximising problem. Thus, based on model (21), we have $\sum_{k=1}^K y_k^* \geq Y$. Hence, based on part (i), model (23) is feasible.

(iii) If we define $(w_k^{att^*}, \theta^{att^*}, X^{v^*})$ as follows:
$w_1^{att^*} = \max_{m=1,\ldots,M} \frac{\sum_{k=1}^K y_{km}}{y_{1m}^*}$ and $w_k^{att^*} = 0$ for all $k = 2, \ldots, K$,
$\theta^{att^*} = \max_{n=1,\ldots,N_f} \frac{w_1^{att^*} x_{1n}^{f*}}{\sum_{k=1}^K x_{kn}^f}$,
$X^{v^*} = w_1^{att^*} x_1^{v*}$.
Then, $(w_k^{att^*}, \theta^{att^*}, X^{v^*})$ is a feasible solution of model (23).

(iv) Based on the technology (8), since we have $T^{NC} \subseteq T^C$, therefore, if $T^C = \emptyset$, then $T^{NC} = \emptyset$.

Proof of Proposition 5.3

(i) Suppose that model (28) is feasible and $(w_1^{inp^*}, ..., w_K^{inp^*})$ is an optimal solution for decision variables $(w_1^{inp}, ..., w_K^{inp})$. Hence, we have $\sum_{k=1}^K w_k^{inp^*} y_k^* \geq Y$. Since $w_k^{inp^*} \leq U_k^{inp}$, we have $\sum_{k=1}^K U_k^{inp} y_k^* \geq Y$.
Now, assume that $\sum_{k=1}^K U_k^{inp} y_k^* \geq Y$. Letting,

$$w_k^{inp^*} = U_k^{inp}, \theta^{inp^*} = \max_{n=1,\ldots,N_f} \frac{\sum_{k=1}^K U_k^{inp} x_{kn}^{f*}}{\sum_{k=1}^K x_{kn}^f}, X^{v*} = \sum_{k=1}^K U_k^{inp} x_k^{v*}.$$

Hence, $(w_k^{inp^*}, \theta^{inp^*}, X^{v*})$ is a feasible solution of model (28).

(ii) If we define $(w_k^{inp^*}, \theta^{inp^*}, X^{v^*})$ as follows:
$w_1^{inp^*} = \max_{m=1,\ldots,M} \frac{\sum_{k=1}^K y_{km}}{y_{1m}^*}$ and $w_k^{inp^*} = 0$ for all $k = 2, \ldots, K$,
$\theta^{inp^*} = \max_{n=1,\ldots,N_f} \frac{w_1^{inp^*} x_{1n}^{f*}}{\sum_{k=1}^K x_{kn}^f}$,
$X^{v^*} = w_1^{inp^*} x_1^{v*}$.
Then, $(w_k^{inp^*}, \theta^{inp^*}, X^{v^*})$ is a feasible solution of model (28).

(iii) Based on the technology (8), since we have $T^{NC} \subseteq T^C$, therefore, if $T^C = \emptyset$, then $T^{NC} = \emptyset$.

D Numerical Example

D.1 Section 5.1: Short-run Johansen Industry Model with Output-oriented Capacity Measures: A Revised Version

We illustrate the ease of implementing this revised SRJIM with O-oriented capacity measures by using the numerical data in Table B.1. By solving model (13) on the data of the numerical example in Table B.1, we obtain $\theta^{rv^*} = 0.660$. Hence, we have $0.638 = \theta^{bv^*} < \theta^{rv^*} = 0.660$. Therefore, based on Proposition 5.1, for every multiple optimal solution of the basic version of the SRJIM (13), there is at least one observation for which its target point does not respect the technology.

As illustrated in Figure 1a, the traditional O-oriented SRJIM (13) scales down point A to obtain the target point D which is located outside of the technology. But, by implementing the revised SRJIM (15), the target point A translates to the solid black box B: this remains technically feasible by remaining within the technology.

Table D.1 reports input and output targets obtained by solving model (15). The first four columns show the target points of units obtained by relation (11). The lower bound L_p^{rv} and the amounts $w_p^{rv^*}$ are reported in the fifth and sixth columns, respectively. The final targets of inputs and outputs obtained by solving model (15) (i.e., points $(w_p^{rv^*} x_p^{f*}, w_p^{rv^*} x_p^{v*}, w_p^{rv^*} y_p^*)$ corresponding to firm p) are presented in the 7-th, 8-th and 9-th columns. To see the magnification of the variable inputs we report the ratio of variable inputs of the target point over the current variable inputs (i.e., $\frac{w_p^{rv^*} x_p^{v*}}{x_p^v}$) in the very last column.

Table D.1: Inputs and outputs targets obtained by solving model (15)

DMU_p	x_p^{v*}	x_p^{f*}	y_p^*	L_p^{rv}	$w_p^{rv^*}$	$w_p^{rv^*} x_p^{v*}$	$w_p^{rv^*} x_p^{f*}$	$w_p^{rv^*} y_p^*$	$\frac{w_p^{rv^*} x_p^{v*}}{x_p^v}$
1	5	3	4	1	1	5	3	4	1.667
2	6	4	5	0.667	0.667	4	3	3.333	2
3	6	4	5	0.667	0.667	4	3	3	2
4	2	2	2	1	1	2	2	2	0
5	2	2	2	1	1	2	2	2	0.2
6	2	2	2	1	1	2	2	2	1
7	6	4	5	0.667	0.667	4	3	3.333	1.333
8	6	4	5	0.667	0.667	4	3	3.333	1.333
9	5	3	4	1	1	5	3	4	1
10	5	3	4	1	1	5	3	4	0.556
11	6	4	5	0.667	0.786	5	3	4	0.943
12	6	4	5	0.667	0.786	5	3	3.929	0.786
13	6	4	5	0.667	0.762	5	3.048	3.810	1.143

Analyzing the results in Table D.1, we can draw the following conclusions. First, the minimum amount of lower bound $w_p^{rv^*}$ is 0.667 and its maximum amount remains 1. Comparing with the results of Table B.1, this new method puts all target points in the production possibility set by excluding weights below the lower bound $w_p^{rv^*}$ of 0.667. Second, the optimal amount $w_p^{rv^*}$ of units $2, 3, 7$ and 8 coincides with their lower bounds, and the amount of $w_p^{rv^*}$ of units $11, 12$ and 13 is situated between their lower and upper bounds. Furthermore, the amount of $w_p^{rv^*}$ for the remaining units is unity such that these units reach their upper bounds. For unit 13, we have $L_{13}^{rv} = 0.667$: this means that if we put $w_{13}^{rv^*} < 0.667$, then the obtained target point $(w_{13}^{rv^*} x_{13}^{v*}, w_{13}^{rv^*} x_{13}^{f*}, w_{13}^{rv^*} y_{13}^*)$ does no longer belong to the production possibility set. Note that in the previous Table B.1, since we have $w_{13}^{bv^*} = 0.2 < 0.667 = L_{13}^{rv}$, the obtained target of unit 13 in the basic version is situated outside the production possibility set. As illustrated in Figure 1a, the traditional O-oriented SRJIM (13) scales down the target of unit 13 (i.e., point A) to obtain the target point D which is located outside of the technology.

To solve this problem of the infeasibility of point D in Figure 1a, we have now modified the

SRJIM (13) such that the scaling of this point A remains technically feasible by remaining within the frontier technology by only moving along the segment AC. We can show this feasibility again by reference to Figure 1a. Note that since $w_{13}^{rv^*} = 0.762$, we now scale down the point $(x_{13}^{v*}, x_{13}^{f*}, y_{13}^{*}) = (6, 4, 5)$ (solid black box A) by 0.762 times to obtain the target point $(w_{13}^{rv^*} x_{13}^{v*}, w_{13}^{rv^*} x_{13}^{f*}, w_{13}^{rv^*} y_{13}^{*}) = (5, 3.048, 3.810)$ in Figure B.1. As can be seen in Figure 1a, the latter target point translates to point $(0.762, 0.762)$ that is represented by the solid black box B: this remains technically feasible by remaining within the technology.

Note that based on the results of Table B.1, the O-oriented target point of units $2, 3, 7, 8, 11$ and 12 are identical with unit 13. Therefore, the intersection of the technology with the plane that passes through the origin and the O-oriented target point $(x_p^{v*}, x_p^{f*}, y_p^{*})$ of these observations are the same as illustrated in Figure 1a. The amount of $w_p^{rv^*}$ for the units $2, 3, 7$ and 8 coincides with their lower bounds. Hence, their target points are located on point C in Figure 1a. The amount of $w_p^{rv^*}$ of units 11 and 12 is situated between its lower and upper bounds and these units have the same behavior as unit 13.

Finally, the last column of Table D.1 indicates the amounts by which the variable inputs can be magnified. There is rather a large amount of variation in these variable inputs. Indeed, the range is broad: the minimum change in variable inputs amounts to 0.2 times and the maximum increase in variable inputs amounts to 2 times.

D.2 Section 5.2: Short-run Johansen Industry Model with Attainable Output-oriented Efficiency Measure: New Proposal

Note furthermore that by implementing the AO-oriented SRJIM (23) by using the numerical example in Table B.1, we have $\theta^{att^*} = 0.70$ which is higher than θ^{bv^*} and θ^{rv^*}. In this case, the target point A translates to the solid black box C in Figure 1a: this remains technically feasible by remaining within the boundary of the frontier technology.

Table D.2 reports the results of the SRJIM with AO-oriented efficiency measure on the numerical example. It is structured in a way similar to the previous Table D.1. For this numerical example, we have chosen $\bar{\lambda} = 2$. Thus, we believe that an increase of the variable inputs with a factor more than 2 is implausible. We make three observations. First, as can be seen in the last column of Table D.2, the variable input can be magnified by maximum 1.667 times. Only for the first unit this magnification is 1.667 and for the other units, it is smaller than 1.667. Second, the optimal amount $w_p^{att^*}$ of units $2, 3, 7$ and 13 coincides with their lower bounds. The amount of $w_p^{att^*}$ of units $8, 11$ and 12 is situated between their lower and upper bounds. Furthermore, the amount of $w_p^{att^*}$ for the remaining units is unity such that these units reach their upper bounds. Third, for none of the observations in this numerical example we reach the upper bound $\bar{\lambda} = 2$.

Note that based on the results of Table D.2, the AO-oriented target point of units $7, 8, 11, 12$ and 13 are $(x_p^{v*}, x_p^{f*}, y_p^{*}) = (6, 4, 5)$. Therefore, the intersection of the technology with the plane

Table D.2: Inputs and outputs targets obtained by solving model (23)

DMU_p	x_p^{v*}	x_p^{f*}	y_p^*	L_p^{att}	$w_p^{att^*}$	$w_p^{att^*}x_p^{v*}$	$w_p^{att^*}x_p^{f*}$	$w_p^{att^*}y_p^*$	$\frac{w_p^{att^*}x_p^{v*}}{x_p^v}$
1	5	3	4	1	1	5	3	4	1.667
2	4	5	4.667	0.6	0.6	2	3	2.8	1.2
3	4	6	5	0.667	0.667	3	4	3.333	1
4	5	2	2	1	1	5	2	2	1
5	2	2	2	1	1	2	2	2	0.200
6	2	2	2	1	1	2	2	2	1
7	6	4	5	0.667	0.667	4	3	3	1
8	6	4	5	0.667	0.829	5	3	4	1.658
9	5	3	4	1	1	5	3	4	1
10	5	3	4	1	1	5	3	4	0.556
11	6	4	5	0.667	0.778	5	3	4	0.933
12	6	4	5	0.667	0.833	5	3	4.167	0.833
13	6	4	5	0.667	0.667	4	2.667	3.333	1

that passes through the origin and the AO-oriented target point $(x_p^{v*}, x_p^{f*}, y_p^*)$ of these observations are the same as illustrated in Figure 1a. Note that since $w_{13}^{att^*} = 0.667$, we need to scale down the point $(x_{13}^{v*}, x_{13}^{f*}, y_{13}^*) = (6, 4, 5)$ (solid black box A) by 0.667 times to obtain the target point $(w_{13}^{att^*}x_{13}^{v*}, w_{13}^{att^*}x_{13}^{f*}, w_{13}^{att^*}y_{13}^*) = (4, 2.667, 3.333)$ in Figure 1 and its projection $(0.677, 0.677)$ in Figure 1a (solid black box C). Also, unit 7 has the same behavior as unit 13. The amount of $w_p^{att^*}$ of units $8, 11$ and 12 is situated between their lower and upper bounds.

D.3 Section 5.3: Short-run Johansen Industry Model with Input-oriented Capacity Measures: New Proposal

Table D.3 reports the results for the SRJIM with I-oriented plant capacity. It is structured in a similar way as Tables D.1 and D.2. The only difference between these tables is in the 5-th column: while in Tables D.1 and D.2 this column reports the lower bound of w_p, in Table D.3 it shows the upper bound for w_p^{inp} (i.e., U_p^{inp}).

Analyzing the results in Table D.3, we draw the following conclusions. First, the upper bound w_p^{inp} of all units is 2.5. This new method keeps all target points within the production possibility set by excluding weights above the upper bound $w_p^{inp^*}$ of 2.5. Second, the optimal amount $w_p^{inp^*}$ of units $3, 4$ and 5 are bigger that unity and coincides with their upper bounds. The amount of $w_p^{inp^*}$ of unit $6, 7, 8$ and 10 is situated between its lower and upper bounds. Furthermore, the amount of $w_p^{inp^*}$ for the remaining units $1, 2, 9, 11, 12$ and 13 is unity such that these units reach their lower bound that is smaller than their upper bound.

Based on the results of Table D.3, since $U_{13}^{inp} = 2.5$, hence it can be scaled up 2.5 times such that its target point remains technologically feasible. But, since we have $w_{13}^{inp^*} = 1$, therefore unit 13 remains unchanged at point A in Figure 1b. Note that based on the results of Table D.3, the I-oriented target points of all units are identical with the I-oriented target point of unit 13. Therefore,

Table D.3: Inputs and outputs target obtained by solving of model (28)

DMU_p	x_p^{v*}	x_p^{f*}	y_p^*	U_p^{inp}	$w_p^{inp^*}$	$w_p^{inp^*} x_p^{v*}$	$w_p^{inp^*} x_p^{f*}$	$w_p^{inp^*} y_p^*$
1	2	2	2	2.5	1	2	2	2
2	2	2	2	2.5	1	2	2	2
3	2	2	2	2.5	2.5	5	5	5
4	2	2	2	2.5	2.5	5	5	5
5	2	2	2	2.5	2.5	5	5	5
6	2	2	2	2.5	2.417	4.833	4.833	4.833
7	2	2	2	2.5	1.833	3.667	3.667	3.667
8	2	2	2	2.5	2	4	4	4
9	2	2	2	2.5	1	2	2	2
10	2	2	2	2.5	1.75	3.5	3.5	3.5
11	2	2	2	2.5	1	2	2	2
12	2	2	2	2.5	1	2	2	2
13	2	2	2	2.5	1	2	2	2

the intersection of the technology with the plane that passes through the origin and the I-oriented target points $(x_p^{v*}, x_p^{f*}, y_p^*)$ of all observations are the same as illustrated in Figure 1b. The amount of $w_p^{inp^*}$ for the units $1, 2, 9, 11$ and 12 is unity, hence these units have the same behavior as unit 13 and their targets remain unchanged at point A in Figure 1a.

The optimal amount $w_p^{inp^*}$ of unit $3, 4$ and 5 is bigger than unity and coincides with their upper bounds. Therefore, the target point of these three units translates to point $(2.5, 2.5)$ that is represented by the solid black box B in Figure 1b. The amount of $w_p^{inp^*}$ of units $6, 7, 8$ and 10 is situated between their lower and upper bounds. For example, if we focus on unit 8, since $w_8^{inp^*} = 2$, hence we must scale up point A by 2 times to obtain the target point B of unit 8 (i.e., $(4, 4, 4)$ in Figure B.1 and its projection $(2, 2)$ in Figure 1b).

E Empirical Illustration: Supplementary Material

E.1 Output-oriented Short-run Johansen Industry Model: Basic Version

Table E.1 shows basic descriptive statistics for all normalised inputs and outputs defined in equation (11) which are obtained by solving model (10). The rows of this table include two parts: first part shows the results under convex case, and the second shows the results under non-convex case. In both parts, we report the arithmetic averages, the standard deviation, the minima and maxima depending on the context.

Turning to the analysis of Table E.1, we can draw several conclusions. First, the average magnification of three fixed inputs are smaller and close to unity under both C and NC. Second, the result indicates that the variable input can be magnified by at least 3.46 times under C and 3.05 times under NC, on average. Also, the range is broad: the maximum increase in variable input

Table E.1: Descriptive Statistics of Normalised Inputs and Outputs Defined in (11)

Convex	$\frac{x^{f*}_{p1}}{x^{f}_{p1}}$	$\frac{x^{f*}_{p2}}{x^{f}_{p2}}$	$\frac{x^{f*}_{p3}}{x^{f}_{p3}}$	$\frac{x^{v*}_{p1}}{x^{v}_{p1}}$	$\frac{y^{*}_{p1}}{y_{p1}}$	$\frac{y^{*}_{p2}}{y_{p2}}$	$\frac{y^{*}_{p3}}{y_{p3}}$
Average	0.922	0.926	0.907	3.465	273.639	10.528	16.029
St. Dev.	0.126	0.074	0.138	4.691	1136.602	33.802	55.466
Min	0.477	0.636	0.396	0.425	1.000	1.000	1.000
Max	1.000	1.000	1.000	46.697	11869.221	390.978	670.947
Nonconvex							
Average	0.885	0.928	0.815	3.056	211.902	21.029	8.685
St. Dev.	0.141	0.087	0.207	4.680	892.748	204.597	48.571
Min	0.450	0.660	0.323	0.844	1.000	1.000	1.000
Max	1.000	1.000	1.000	49.076	6015.026	2620.529	618.344

amounts to 46.70 times under C and 49.07 times under NC.[14] Third, the results show that three outputs can be magnified by at least 273.64, 10.53 and 16.03 times under C and 211.90, 21.03 and 8.68 times under NC, on average. There is also a great amount of variation, as indicated by the standard deviation, and the range is broad: for example the maximum increase in the first output amounts to 11869.22 times under C and 6015.03 times under NC.

Table E.2 shows the basic descriptive statistics for all normalised inputs and outputs obtained by solving model (13), i.e., points $(\frac{w^{bv^*}_p x^{f*}_p}{x^f_p}, \frac{w^{bv^*}_p x^{v*}_p}{x^v_p}, \frac{w^{bv^*}_p y^*_p}{y_p})$ corresponding to DMU_p where $w^{bv^*}_p$ is an optimal solution of model (13) and x^{f*}_{pn}, x^{v*}_{pn} and y^*_{pm} are obtained from the relations (11). The rows of this table include again two parts. The first part reports the results under the convex case, and the second part reports the results under the non-convex case. In both parts, we report the arithmetic averages, the standard deviation, the minima and maxima depending on the context.[15]

Table E.2: Descriptive Statistics of Normalised Inputs and Outputs Obtained by Solving Model (13)

Convex	$w^{bv^*}_p$	$\frac{w^{bv^*}_p x^{f*}_{p1}}{x^{f}_{p1}}$	$\frac{w^{bv^*}_p x^{f*}_{p2}}{x^{f}_{p2}}$	$\frac{w^{bv^*}_p x^{f*}_{p3}}{x^{f}_{p3}}$	$\frac{w^{bv^*}_p x^{v*}_{p1}}{x^{v}_{p1}}$	$\frac{w^{bv^*}_p y^{*}_{p1}}{y_{p1}}$	$\frac{w^{bv^*}_p y^{*}_{p2}}{y_{p2}}$	$\frac{w^{bv^*}_p y^{*}_{p3}}{y_{p3}}$
Average	0.335	0.299	0.303	0.301	1.319	151.982	2.838	4.771
St. Dev.	0.468	0.425	0.425	0.427	2.918	991.182	6.627	16.113
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Max	1.000	1.000	1.000	1.000	19.322	11869.221	45.386	146.072
Nonconvex								
Average	0.357	0.311	0.325	0.282	1.465	102.164	18.705	5.748
St. Dev.	0.476	0.422	0.436	0.394	4.574	586.743	204.704	48.382
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Max	1.000	1.000	1.000	1.000	49.076	6015.026	2620.529	618.344

Analyzing the results in Table E.2, we can draw the following conclusions. First, the average

[14] Based on equation (11) we have $x^{v*}_p = \sum_{k=1}^{K} z^{p*}_k x^v_k$ and since $\sum_{k=1}^{K} z^{p*}_k = 1$, then $\min_{k=1,\dots,K} x^v_{kn} \leq x^{v*}_{pn} = \sum_{k=1}^{K} z^{p*}_k x^v_{kn} \leq \max_{k=1,\dots,K} x^v_{kn}$ for all $n = 1, \dots, N_v$. Hence, based on the information in Table 1, we find that $2.222 \leq x^{v*}_{p1} \leq 242.195$ for all $p = 1, \dots, K$. Therefore, the optimal amount of variable inputs is always bounded by the minimum and maximum levels of observed variable inputs in the data.

[15] Note that the first output is zero for 6 DMUs: hence, we do not consider these DMUs in the descriptive statistics.

of $w_k^{bv^*}$ indicates that for scaling down capacity outputs and capacity variable and fixed inputs to meet all requirements, we need on average a 0.335 scaling under C and a 0.357 scaling under NC. Second, the minimum amount of $w_p^{bv^*}$ is zero, Therefore, for some units the target points are located on the origin.

E.2 Output-oriented Short-run Johansen Industry Model: Revised version

Table E.3 is structured in a similar way as Table E.2. In this table, the basic descriptive statistics for all normalised inputs and outputs obtained by solving model (15) are reported. The amounts of $w_p^{rv^*}$ and lower bound L_p^{rv} are reported in the second and third columns, respectively. To see the magnification of the fixed, variable inputs and outputs we report the ratio of their target point over their current amount, i.e., points $(\frac{w_p^{rv^*} x_p^{f*}}{x_p^f}, \frac{w_p^{rv^*} x_p^{v*}}{x_p^v}, \frac{w_p^{rv^*} y_p^*}{y_p})$ corresponding to DMU_p where $w_p^{rv^*}$ is an optimal solution of model (15) and x_p^{f*}, x_p^{v*} and y_p^* are obtained from the relations (11), in the fourth to tenth columns.

Table E.3: Descriptive Statistics of Normalised Inputs and Outputs Obtained by Solving Model (15)

Convex	L_p^{rv}	$w_p^{rv^*}$	$\frac{w_p^{rv^*} x_{p1}^{f*}}{x_{p1}^f}$	$\frac{w_p^{rv^*} x_{p1}^{f*}}{x_{p1}^f}$	$\frac{w_p^{rv^*} x_{p1}^{f*}}{x_{p1}^f}$	$\frac{w_p^{rv^*} x_{p1}^{v*}}{x_{p1}^v}$	$\frac{w_p^{rv^*} y_{p1}^*}{y_{p1}}$	$\frac{w_p^{rv^*} y_{p2}^*}{y_{p2}}$	$\frac{w_p^{rv^*} y_{p3}^*}{y_{p3}}$
Average	0.934	0.934	0.863	0.864	0.846	3.309	261.448	10.323	14.839
St. Dev.	0.109	0.109	0.164	0.119	0.157	4.658	1061.042	33.828	44.615
Min	0.580	0.580	0.383	0.576	0.396	0.425	0.580	0.580	0.580
Max	1.000	1.000	1.000	1.000	1.000	46.697	10919.902	390.978	517.788
Nonconvex									
Average	0.996	0.996	0.880	0.924	0.811	3.047	211.887	21.022	8.679
St. Dev.	0.026	0.026	0.141	0.088	0.206	4.679	892.751	204.598	48.571
Min	0.817	0.817	0.450	0.660	0.323	0.817	0.817	0.817	0.817
Max	1.000	1.000	1.000	1.000	1.000	49.076	6015.026	2620.529	618.344

Based on Table E.3, we draw the following conclusions. First, the minimum amount of lower bound L_p^{rv} as well as $w_p^{rv^*}$ is 0.580 times under C and 0.817 times under NC and their maximum amount remains 1 for both C and NC. Second, the amount of lower bound L_p^{rv} and $w_p^{rv^*}$ are identical for all units, hence the optimal amount $w_p^{rv^*}$ of all units coincides with their lower bounds. Third, since we have $w_p^{rv^*} \leq 1$, comparing with the results of Table E.1, all final target points obtained by solving model (15) are smaller than the target points obtained from equation (11). Fourth, comparing with the results of Table E.2, this new method puts all target points within the production possibility set by excluding weights below the lower bound $w_p^{rv^*}$ of 0.580 under C and 0.817 under NC. Fifth, comparing with the results of Table E.2, in the basic version of the O-oriented short run Johansen Industry model the average of $w_p^{bv^*}$ is 0.335 under both C and NC, while in the revised version of O-oriented short run Johansen Industry model the average of $w_p^{rv^*}$ is 0.934 under C and 0.996 under NC. It means that for most units, the target points obtained from model (13) do no longer belong to the production possibility set. Finally, the seventh column of Table E.3 indicates that the variable inputs can be magnified by at least 3.31 times under C and 3.05 times under NC, on average. There is a large amount of variation in the variable inputs. Indeed, the range is broad: the minimum changes in variable inputs amounts to 0.42 times under C and 0.82 times under NC and the maximum increase in variable inputs amounts to 46.70 times under C and 49.08 times under NC.

E.3 Attainable Output-oriented Short-run Johansen Industry Model

Table E.4 reports the results of the SRJIM with AO-oriented efficiency measure (23). It is structured in a similar way as the previous Table E.3. For this empirical example, we have chosen $\bar{\lambda} = 2$. Thus, we believe that an increase of the variable inputs with a factor more than 2 is implausible.

Table E.4: Descriptive Statistics of Normalised Inputs and Outputs Obtained by Solving Model (23)

Convex	L_p^{att}	$w_p^{att^*}$	$\frac{w_p^{att^*} x_{p1}^{f*}}{x_{p1}^f}$	$\frac{w_p^{att^*} x_{p2}^{f*}}{x_{p2}^f}$	$\frac{w_p^{att^*} x_{p3}^{f*}}{x_{p3}^f}$	$\frac{w_p^{att^*} x_{p1}^{v*}}{x_{p1}^v}$	$\frac{w_p^{att^*} y_{p1}^*}{y_{p1}}$	$\frac{w_p^{att^*} y_{p2}^*}{y_{p2}}$	$\frac{w_p^{att^*} y_{p3}^*}{y_{p3}}$
Average	0.944	0.944	0.857	0.847	0.762	1.476	34.246	5.304	7.029
St. Dev.	0.105	0.105	0.166	0.124	0.206	0.518	151.191	15.470	18.207
Min	0.580	0.580	0.383	0.576	0.226	0.425	0.580	0.580	0.580
Max	1.000	1.000	1.000	1.000	1.000	2.000	1781.082	182.033	221.958
Nonconvex									
Average	0.995	0.995	0.878	0.923	0.830	1.207	9.052	2.358	2.052
St. Dev.	0.033	0.033	0.168	0.110	0.238	0.324	49.919	6.221	2.494
Min	0.686	0.686	0.333	0.505	0.113	0.686	0.686	0.686	0.686
Max	1.000	1.000	1.000	1.000	1.000	1.994	610.176	62.861	19.585

We make three observations. First, the minimum amount of the lower bound L_p^{att} as well as $w_p^{att^*}$ is 0.580 times under C and 0.686 times under NC and their maximum amount remains 1 for both C and NC. Second, the amount of the lower bound L_p^{att} and $w_p^{att^*}$ are identical for all units, hence the optimal amount $w_p^{att^*}$ of all units coincides with their lower bounds. Third, as can be seen in the seventh column of Table E.4, the variable input can be magnified at least 1.476 times under C and 1.207 times under NC, on average, and it can be magnified by about maximum 2 times.

E.4 Input-oriented Short-run Johansen Industry Model

Table E.5 shows basic descriptive statistics for all normalised inputs and outputs defined in equation (27) which is obtained by solving model (26). Turning to the analysis of Table E.5, we can draw two conclusions. First, the average of magnification of all fixed, variable inputs and outputs except for the first output are smaller than unity under both C and NC. For the first output, there is a great amount of variation, as indicated by the standard deviation, and the range is broad: for example the maximum increase in the first outputs amounts to 1516.43 times under both C and NC. Second, the fixed input and three variable inputs can be magnified by maximum 1 times under both C and NC.

Table E.5: Descriptive Statistics of Normalised Inputs and Outputs Defined in Relation (27)

Convex	$\frac{x_{p1}^{f*}}{x_{p1}^{f}}$	$\frac{x_{p2}^{f*}}{x_{p2}^{f}}$	$\frac{x_{p3}^{f*}}{x_{p3}^{f}}$	$\frac{x_{p1}^{v*}}{x_{p1}^{v}}$	$\frac{y_{p1}^{*}}{y_{p1}}$	$\frac{y_{p2}^{*}}{y_{p2}}$	$\frac{y_{p3}^{*}}{y_{p3}}$
Average	0.8161	0.7346	0.4852	0.1907	13.6486	0.8339	0.3732
St. Dev.	0.2062	0.1614	0.2855	0.2730	120.1295	4.8842	1.3003
Min	0.2855	0.5079	0.1479	0.0092	0.0004	0.0112	0.0032
Max	1.0000	1.0000	1.0000	1.0000	1516.4267	61.1712	15.5652
Nonconvex							
Average	0.7953	0.7232	0.4738	0.2100	11.4405	0.9093	0.4244
St. Dev.	0.1967	0.1550	0.2678	0.2927	118.4416	5.9376	1.5092
Min	0.2855	0.5079	0.1479	0.0092	0.0000	0.0112	0.0031
Max	1.0000	1.0000	1.0000	1.0000	1516.4267	74.9765	15.5652

Note that the I-oriented SRJIM (28) is infeasible for this empirical application under both convex and nonconvex cases. Thus, it is simply impossible to scale up the I-oriented capacity targets of units such that these can generate the current aggregate output levels while respecting the technology.