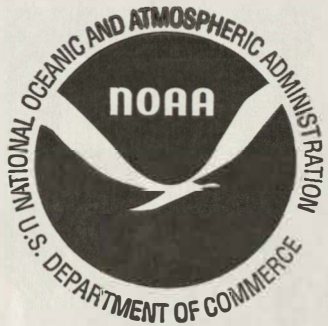


U. S. DEPARTMENT OF COMMERCE  
WEATHER BUREAU

Training Paper No. 13

# The Application of Routine Transosonde Data to Analysis and Forecasting



## Weather Bureau Training Papers

No. 1. Notes, July 1948.

No. 2. Lectures to Professional Interns, 1946-1947, March 1949.

No. 3. Introducing the New Observer to the Weather Bureau, April 1949.

No. 4. Training the New Observer, April 1949.

No. 5. Primary Training Manual for Supplementary Aeronautical Weather Reports, April 1949.

No. 6. Training Guide in Pilot Balloon Observations, March 1, 1951; revised May 1, 1951; 2nd revision February 1952.

No. 7. Training Guide in Rawins and Rabals, March 1, 1951; revised February 1952 (Second Edition).

No. 8. Training Guide in Radiosonde Observations, March 1, 1951; revised January 1952; Third Edition, August 1953.

No. 9. Training Guide in Surface Aviation Observations, July 1951.

No. 10. Training Guide in Radar Meteorological Observations, September 1953.

No. 11. Basic Supervision, 1954.

No. 12. Training Aid for Synoptic Observations, revised August 1955.

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U. S. DEPARTMENT OF COMMERCE

Sinclair Weeks, Secretary

WEATHER BUREAU

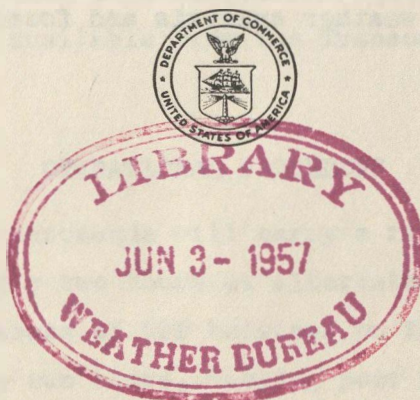
F. W. Reichelderfer, Chief

Training Paper No. 13

# The Application of Routine Transosonde Data to Analysis and Forecasting

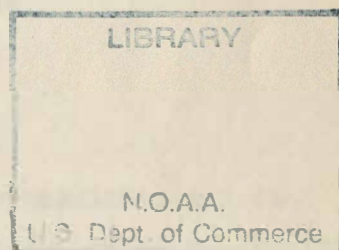
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## PREFACE

This Training Paper is offered for the guidance of meteorological personnel in preparation for the release by the Navy of Transosondes (Trans-oceanic-sonde constant level balloons) on a routine basis, commencing in the spring of 1957. Included in the text is a description of the basic principles of the Transosonde, the types of data derived therefrom, and a discussion of the practical utilization of these data for the purpose of improved weather analysis and forecasting.



# THE APPLICATION OF ROUTINE TRANSOSONDE DATA TO ANALYSIS AND FORECASTING

## INTRODUCTION

The long-range Navy Constant Level Balloons (CLB) or Transosondes are designed to float approximately along a constant height or constant pressure surface in the atmosphere. This is accomplished by means of a ballast system activated by a barometer attached to the balloon train, ballast being released when the environmental pressure exceeds a predetermined value. The Transosondes are also equipped with radio transmitters which broadcast meteorological intelligence and serve as a signal source for their positioning by radio direction finding (RDF) techniques.

Commencing in the late spring of 1957, Transosondes set to float at 300 mb. (30,000 feet) will be released from Japan on a routine basis. The rate of release of the balloons will be dependent upon the difficulties involved in their launching and tracking, but it is probable that (on the average) there will be, at any one time, about four Transosondes at 300 mb. in the sector of the hemisphere between Japan and Europe.

The following information will be available at 2-hour intervals from the preliminary Transosonde flights:

- (1) The most probable position of the Transosonde.
- (2) The radius of the circle (centered on the most probable position) within which the Transosonde is actually located.
- (3) The pressure at which the Transosonde is flying.

At a later date it is expected that additional information, such as temperature and humidity, will become available from the Transosondes.

## OPERATIONAL PROCEDURE

Positioning - Each Transosonde will carry a radio transmitter set to operate for 15 minutes every two hours at alternate frequencies of 6, 13, and 19 megacycles. The intercepts of RDF bearings on these signals will serve to position the balloon every two hours, barring poor radio propagation conditions,

aircraft emergencies, etc. Previous Transosonde flights were tracked in a highly satisfactory manner by the Federal Communications Commission (FCC) network of RDF stations. Subsequent flights will be tracked chiefly by the Navy RDF network.

The accuracy of the position fixes is a function of the number and proximity of RDF stations and the angle of intersection of the bearings. In order to provide an estimate of the accuracy of these fixes, the radius of the circle (about the most probable position) within which the Transosonde is actually located will be determined from the area of intercept of the bearings. As will be shown later in the text, these radii provide the means for estimating possible errors in the CLB velocity and velocity derivatives.

Flight Pressure - The pressure data telemetered from the Transosonde should not be neglected since, in addition to the relatively short-period vertical oscillations of the balloon due to intermittent release of ballast, there is a tendency on long flights for the balloon to ascend to higher elevations with time. Thus a Transosonde set to fly at approximately 300 mb. when released from Japan may have a mean floating pressure of 250 mb. upon its approach to Europe. This problem of ballast release requiring less bouyancy for a full volume balloon is currently under investigation by the Naval Research Laboratory, but until a solution is obtained care should be taken that the Transosonde data utilized may, justifiably, be applied to the 300-mb. surface.

Coding and Transmission of Data - The basic information to be obtained from the preliminary Transosonde flights includes the most probable position of the CLB (to the nearest 1/10 of a degree of latitude and longitude), the radius of the circle (about the most probable position) which includes the actual position of the CLB (in units of 20 nautical miles), and the pressure at which the CLB is flying (to the nearest 4 mb.). These data will be forwarded to the collection center at Honolulu for coding and entry on the conventional meteorological circuits and should be available to interested parties in the United States within 2 hours after Transosonde transmission time. For those requiring additional information about the code form and the method of transmission of data, reference may be made to Appendix 1.



## TRAJECTORY CONSIDERATIONS

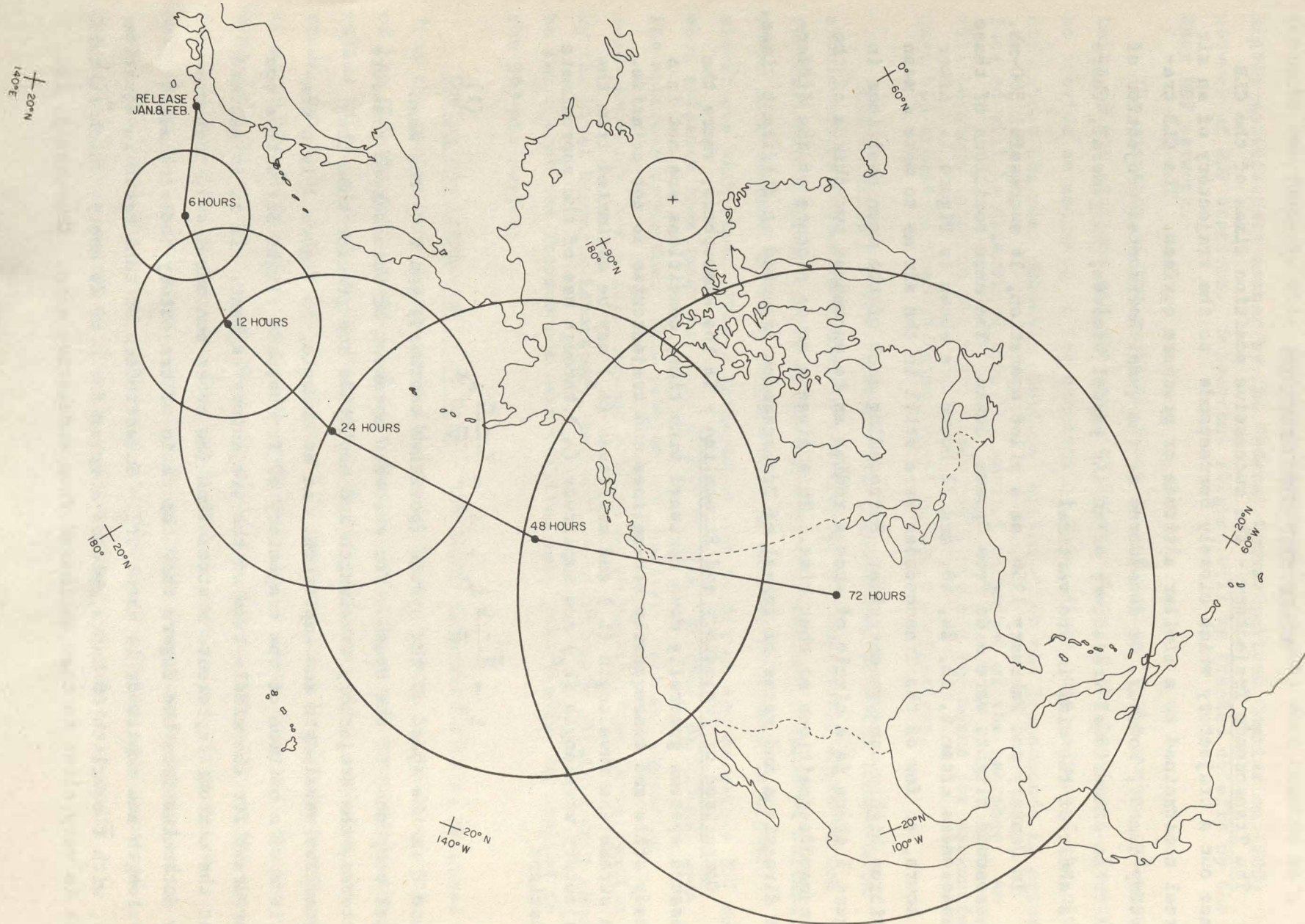
The Transosonde Trajectory - The successive position fixes of the CLB trace out a trajectory which closely corresponds to the trajectory of an air parcel constrained to a similar altitude or pressure surface. The CLB trajectory cannot, however, be considered as the quasi-horizontal projection of the three-dimensional trajectory of an air parcel because, in general, there is a shear of the wind in the vertical.

In January and February 1956, as a pilot operation, 16 successful 300-mb. Transosonde flights were made from Oppama, Japan. The mean positions of these Transosondes after 6, 12, 24, 48, and 72 hours are shown in Figure 1. After 72 hours, so few of the Transosondes were still in the air as to make a mean position highly unrepresentative. Surrounding each of the mean positions in Figure 1 there is a circle of such a radius as to encompass two-thirds of the Transosonde positions at that time. This gives a crude picture of the different directions and rates of travel of Transosondes launched at different times.

Comparison of Trajectory and Streamline - In the atmosphere, where the pressure systems generally move eastward with time, conditions are not in a steady state and consequently streamlines and trajectories do not coincide. The streamline wavelength ( $L_s$ ) and amplitude ( $A_s$ ) may be estimated from the trajectory wavelength ( $L_t$ ) and amplitude ( $A_t$ ) through use of the approximate equations

$$L_s = \frac{\bar{V} - C}{\bar{V}} L_t, \quad A_s = \frac{\bar{V} - C}{\bar{V}} A_t \quad (1)$$

where  $C$  is the speed of the system (positive eastward) and  $\bar{V}$  is the mean zonal component of the speed. For eastward movement of wave-shaped pressure patterns, the trajectory wavelength and amplitude are greater than the streamline wavelength and amplitude. As an example, in Figure 2 is a comparison of a portion of the trajectory of Transosonde flight 36, with a contour drawn for the middle time of the trajectory segment. If it is assumed that the contour represents a streamline (no cross contour flow), then one may determine from the figure that the ratio of trajectory and streamline wavelength and amplitude is about 3:2. Substitution of this ratio in equation (1), with  $\bar{V}$  equal to 60 knots, yields a value for  $C$  of 20 knots, which in this case is very close to that estimated from successive maps. Conversely, if



MEAN POSITION AND STANDARD DEVIATION OF 1956 300 MB. TRANSOSONDES  
FIG. 1



one has some knowledge of the speed with which the system is moving eastward, then one may estimate the contour wavelength and amplitude from the trajectory wavelength and amplitude.

Trajectory Extrapolation to 500 mb. - Since 500-mb. maps are more widely used than 300-mb. maps, it would be desirable to extrapolate the Transosonde trajectories to 500 mb. In general, there is little turning of the wind with height in the layer 500-300 mb. and therefore the wind direction obtained from the trajectory at 300 mb. is a good approximation to the wind direction at 500 mb. Consequently, the troughs and ridges at 500 mb. should be well delineated by the Transosonde trajectory at 300 mb. However, the variation of wind speed with height is often very large in the 500-300-mb. layer. It is illogical to use the thermal wind equation to extrapolate the speed downward, since, at least over the oceans where the extrapolation would be most useful, the temperature field is even less well known than the pressure field. The only suggestion offered here is to assume a linear change of speed between surface and 300 mb. and from this to pick off the 500-mb. speed. Obviously, the errors in such a procedure may be great. There is little doubt that the usefulness of Transosonde data is considerably impaired because of the restrictions preventing flights at levels below 30,000 feet.

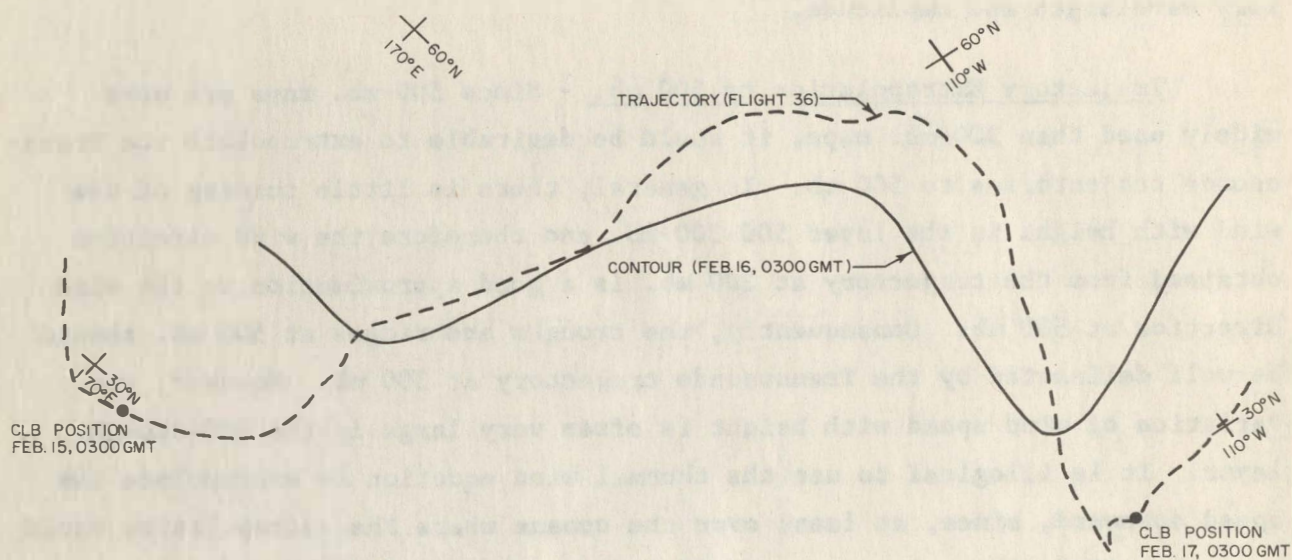
#### ESTIMATION OF THE VELOCITY AND VELOCITY ERROR FROM CONSTANT LEVEL BALLOONS

The Velocity - The most probable value for the speed of the CLB, and hence for the wind speed, is obtained by dividing the (straight line) distance (S) between position fixes by the time (t) it takes the CLB to move this distance. Thus the wind speed (V) is given by

$$V = S/t, \quad (2)$$

and its value may easily be computed "in the head" by a method presented in a subsequent subsection entitled "Suggested Computation and Plotting Procedures".

The most probable value for the direction of movement of the CLB, and hence for the wind direction, is obtained from the direction of the straight line connecting the position fixes.



COMPARISON OF CONTOUR (STREAMLINE) AND TRAJECTORY (FLIGHT 36)

FIG. 2

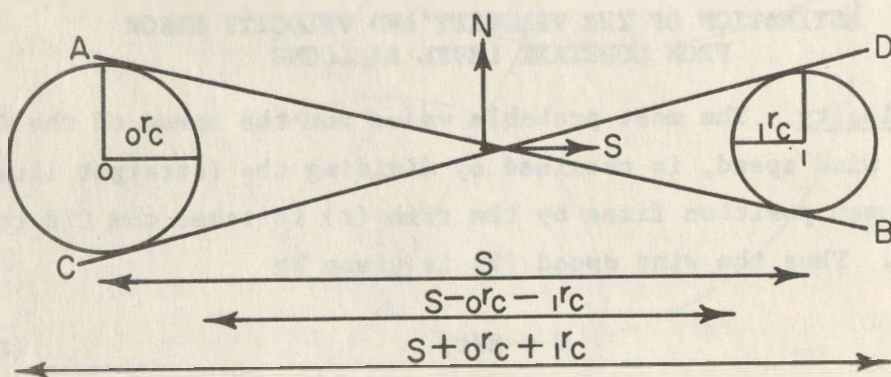
SPEED AND DIRECTION ERRORS AS FUNCTION OF RADII OF CIRCLES ( $r_c$ ) WITHIN WHICH CLB ACTUALLY LOCATED.

FIG. 3



The Velocity Error - In addition to knowledge of the most probable wind speed and direction, it is desirable to obtain an estimate of the possible errors in these quantities. Such an estimate may be obtained from the radii of the circles (centered on the most probable position fixes) within which the CLB is actually located. For example, let the radius of this circle at fix 0 be given by  ${}_0r_c$  while that at fix 1 is given by  ${}_1r_c$  (Fig. 3). If the actual position of the CLB can be anywhere within these circles then, while the most probable distance traveled by the CLB in the time interval (t) is given by S, the distance could vary from  $S - {}_0r_c - {}_1r_c$  to  $S + {}_0r_c + {}_1r_c$ . Therefore, from equation (2), the maximum possible error in wind speed  $(\Delta V)_M$  would be given by

$$(\Delta V)_M = \pm \left( \frac{{}_0r_c + {}_1r_c}{t} \right) . \quad (3)$$

In Figure 3 the most probable wind direction is given by the straight line connecting the position fixes 0 and 1, but if the actual location of the CLB may be anywhere within the circles of radius  ${}_0r_c$  and  ${}_1r_c$ , then the wind may have any direction intermediate to that represented in the figure by lines A-B and C-D. Expressed mathematically, the maximum possible error in direction  $(\Delta \theta)_M$  would be given by

$$(\Delta \theta)_M = \pm \tan^{-1} \left( \frac{{}_0r_c + {}_1r_c}{S} \right) . \quad (4)$$

There is a very small probability that the CLB would be located in such extreme positions within the circles of radius  ${}_0r_c$  and  ${}_1r_c$ . Probably of much greater importance for the purpose at hand is the estimation of the errors in speed and direction not exceeded 90 percent of the time, rather than the maximum possible errors. In order to obtain an estimate of the errors not exceeded 90 percent of the time, it is necessary to carry through some statistics (Appendix 2). Here we merely state the result that, assuming the distance between position fixes is large compared to the circle radius ( $S > r_c$ ), and that the actual CLB positions are normally distributed within these circles, 90 percent of the time the error in speed  $(\Delta V)_{90}$  (knots) is satisfied by the inequality

$$(\Delta V)_{90} < \frac{.6 \sqrt{({}_0r_c)^2 + ({}_1r_c)^2}}{t} , \quad (5)$$

and 90 percent of the time the error in direction  $(\Delta\theta)_{90}$  (degrees) is satisfied by the inequality

$$(\Delta\theta)_{90} < \frac{36 \sqrt{(r_c^0)^2 + (r_c^1)^2}}{\bar{V}t} \quad (6)$$

where  $r_c^0$  and  $r_c^1$  are the radii of the circles (in nautical miles) about the position fixes involved in the determination of the velocity,  $t$  is the time interval (in hours) for which the velocity is evaluated, and  $\bar{V}$  is the mean speed in knots along the trajectory segment. A table which yields the errors in wind speed and direction not exceeded 90 percent of the time for code values of  $r_c^0$  and  $r_c^1$  is presented in a subsequent subsection entitled "Suggested Computation and Plotting Procedures".

Time Interval for Velocity Calculations - It may be noted from equations (5) and (6) that the error in wind speed and direction is inversely proportional to the time interval ( $t$ ). Therefore, other things being equal, the longer the time interval for which the wind speed and direction are evaluated, the more accurate their values. However, the longer the time interval the more smoothing is performed, and the less representative of the true wind is the calculated wind at any point. Moreover, the assumption that the straight line distance between two position fixes actually yields the distance traveled by the CLB becomes crude in regions of curved flow as the time interval becomes longer.

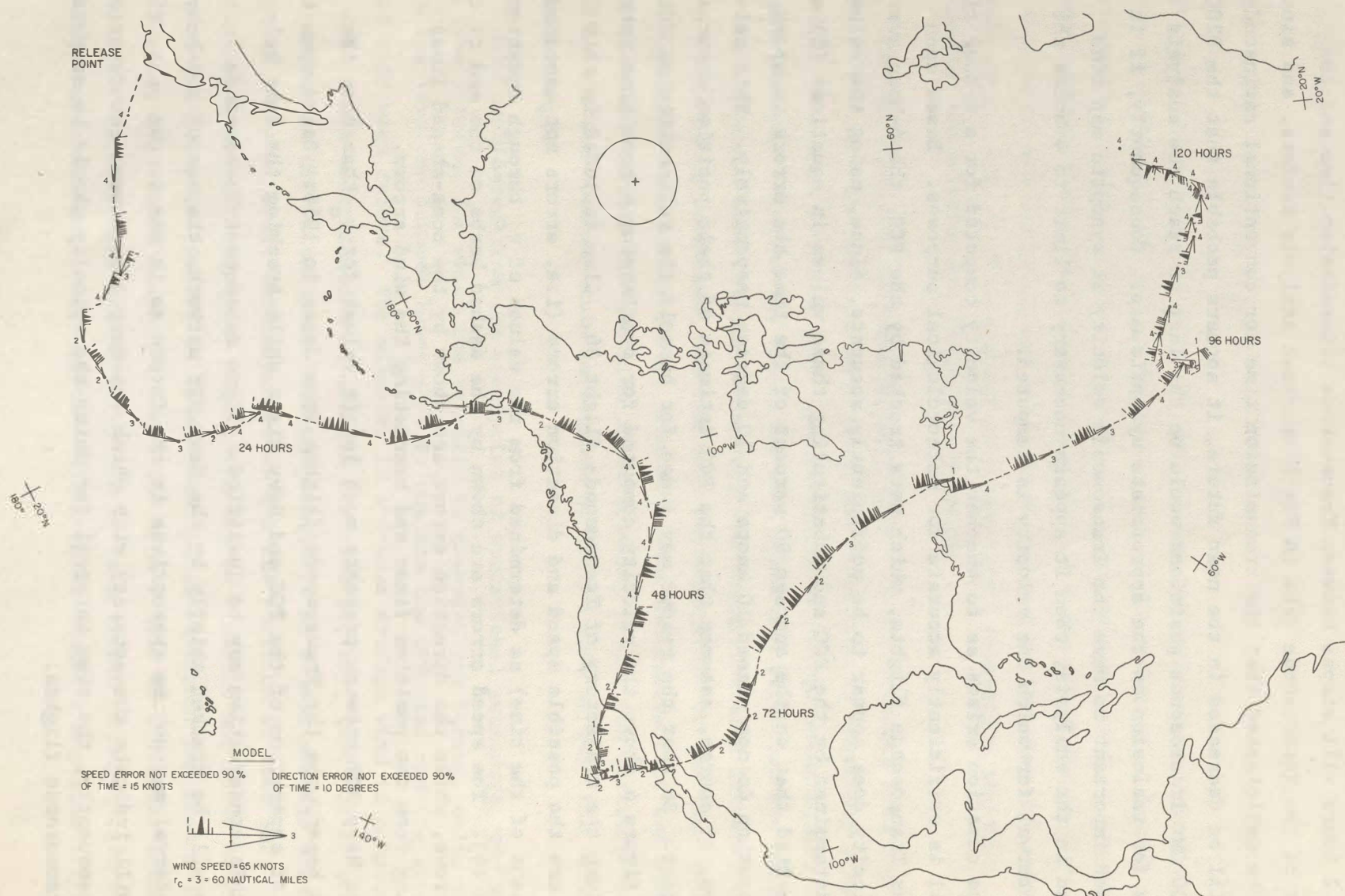
When considered from the standpoint of operational procedure, the shorter the time interval for which the velocity is computed the better, owing to the time lag involved between being able to compute the CLB wind and the time to which this wind refers. For example, if a 4-hour average wind is computed, one must await determination of the CLB position at, say, 1400 GMT in order to compute the CLB velocity at 1200 GMT, while if an 8-hour average wind is computed one must await determination of the CLB position at 1600 GMT. It is desirable that the Transosonde data referring to synoptic map times be available to the analysis centers no later than the conventional rawinsonde data. At present, that would mean that these CLB data should be available within 3 hours after synoptic map time if they are to be included on maps based on first transmission data, and within 5 hours after synoptic map time if they



are to be included on maps based on second transmission data. Since approximately 2 hours will elapse between Transosonde transmission time and the receipt of the Transosonde data in North American analysis centers, and since it may be anticipated that the transmission time for conventional rawinsonde data will be decreased in the near future, it appears probable that the 0200 and 1400 GMT Transosonde positions would be the last to reach the analysis centers for inclusion on the appropriate synoptic map. Consequently, if it is deemed important to have the Transosonde velocity at synoptic map time included in the analysis, then it appears necessary to limit to 4 hours the time interval for which the velocity is computed.

The question arises as to whether the velocity computed for a 4-hour time interval is sufficiently accurate for meteorological purposes. Based upon previous Transosonde flights, which were tracked by the FCC, the 4-hour average velocity does appear to be sufficiently accurate, since, using the values of  $r_c$  determined by the FCC and substituting these values in equations (5) and (6), we find that on the average 90 percent of the time the errors in speed and direction do not exceed 10 knots and 10 degrees respectively. This calculation, of course, assumes that the FCC estimate of their position accuracy is correct. So that the reader may judge for himself the reliability of FCC fixes, Figure 4 shows the velocity computed for overlapping 4-hour time intervals along the trajectory of Transosonde flight 36. Also included in this figure are the possible speed and direction errors (i.e. errors not exceeded 90 percent of the time) as determined from the values of  $r_c$  through equations (5) and (6). The speed errors are shown by the dashed barbs on the end of the wind arrows, while the direction errors are shown by the cone-shaped lines emanating from the position fixes and bracketing the wind arrows.

The Navy RDF network presents much larger values for  $r_c$  than does the FCC, as based upon the Transosonde flights from Japan in 1956. Based upon the degree of separation of the FCC and Navy fixes while tracking the same balloon, this conservatism may be justified. Since subsequent Transosonde flights will be tracked chiefly by the Navy RDF network, the use of a 4-hour time interval may not be appropriate in the future as it was in the past. Time alone will indicate the accuracy with which the Navy RDF network can function, and consequently, the time interval for which the velocity should be computed from Transosonde flights.



TRANSOSONDE TRAJECTORY — FLIGHT 36 (300 MB)

FIG. 4



Suggested Computational and Plotting Procedures - For the purpose of this subsection, it is assumed that the evaluation of the velocity for a 4-hour time interval will prove feasible. The changes in the computational procedure, if it becomes necessary to use a 6- or 8-hour time interval, are minor, and will be indicated in this text.

Utilizing a 4-hour time interval for the computation of the speed from Transosonde flights, we find that the speed is given by

$$V = 15S \quad (7)$$

where  $S$  is the distance (in degrees of latitude) traveled by the Transosonde in 4 hours, and  $V$  is in knots. If the speed is computed for a 6- or 8-hour time interval, the coefficient 15 in equation (7) is replaced by 10 and 7.5, respectively. The multiplication by 15 "in the head" is most easily accomplished through multiplication by 10 and then by  $10/2$ , and the summing of the results. Thus, if the CLB moves the equivalent of 12.4 degrees of latitude in 4 hours, the speed along this segment of the trajectory is found to be

$$V = 15 \times 12.4 = 10 \times 12.4 + (10/2) \times 12.4 = 124 + 62 = 186 \text{ knots}$$

This value of the speed should be plotted as a conventional wind barb at the intermediate fix, with a direction parallel to the straight line between the two position fixes bracketing this intermediate fix. This procedure is illustrated in Figure 5. In most cases the distance traveled by the CLB in 4 hours can be obtained with sufficient accuracy from the fixes plotted on the synoptic map, since with map parallels at 1-degree intervals and with the aid of dividers, the travel distance may be estimated to within  $2/10$  of a degree of latitude, which, for a 4-hour time interval, corresponds to a speed error of only 3 knots.

The possible errors in wind speed and direction (i.e. the errors not exceeded 90 percent of the time) as a function of the radii of the circles ( $r_c$ ) within which the CLB is actually located, are not easily computable, and therefore Table 1 gives these errors as a function of the code figure for  $r_c$  and the mean speed along the trajectory segment ( $\bar{V}$ ). Table 1 is valid only for velocities computed for a 4-hour time interval; if the velocity is computed for a 6- or 8-hour time interval, the numbers in the body of the table should be multiplied by  $2/3$  and  $1/2$  respectively. The table is set up so as

TABLE 1.- SPEED AND DIRECTION ERRORS FOR FOUR HOUR TRAVEL TIME OF CLB AS FUNCTIONS OF RADII ( $r_c$ ) OF CIRCLES (CENTERED ON PROBABLE CLB POSITION) WITHIN WHICH CLB IS ACTUALLY LOCATED.

$r_c$ (Code figure)									
1	2	3	4	5	6	7	8	9	
5	5	10	10	15	20	20	25	$\geq 25$	1
	10	10	15	15	20	20	25	$\geq 30$	2
		15	15	15	20	25	25	$\geq 30$	3
			15	20	20	25	25	$\geq 30$	4
				20	25	25	30	$\geq 30$	5
					25	30	30	$\geq 30$	6
						30	30	$\geq 35$	7
							35	$\geq 35$	8
								$\geq 40$	9

SPEED ERROR (NEAREST 5 KNOTS)  
NOT EXCEEDED 90% OF TIME  
FOR GIVEN  $r_c$ .

r (Code figure)									
1	2	3	4	5	6	7	8	9	
5 5 0 0	10 5 5 0	10 5 5 5	15 10 5 5	20 10 5 5	20 10 5 5	25 15 10 5	30 15 10 5	30 15 10 10	1
	10 5 5 5	15 5 5 5	15 10 5 5	20 10 5 5	25 10 10 5	25 15 10 5	30 15 10 5	35 15 10 10	2
		15 10 5 5	20 10 5 5	20 10 5 5	25 10 10 5	30 15 10 5	30 15 10 5	35 15 10 10	3
			20 10 5 5	25 10 10 5	25 15 10 5	30 15 10 5	35 15 10 10	35 20 10 10	4
				25 15 10 5	30 15 10 5	30 15 10 10	35 20 10 10	35 20 10 10	5
					30 15 10 5	35 15 10 10	35 20 10 10	40 20 15 10	6
						35 20 10 10	40 20 15 10	40 20 15 10	7
							40 20 15 10	45 20 15 10	8
								45 25 15 10	9

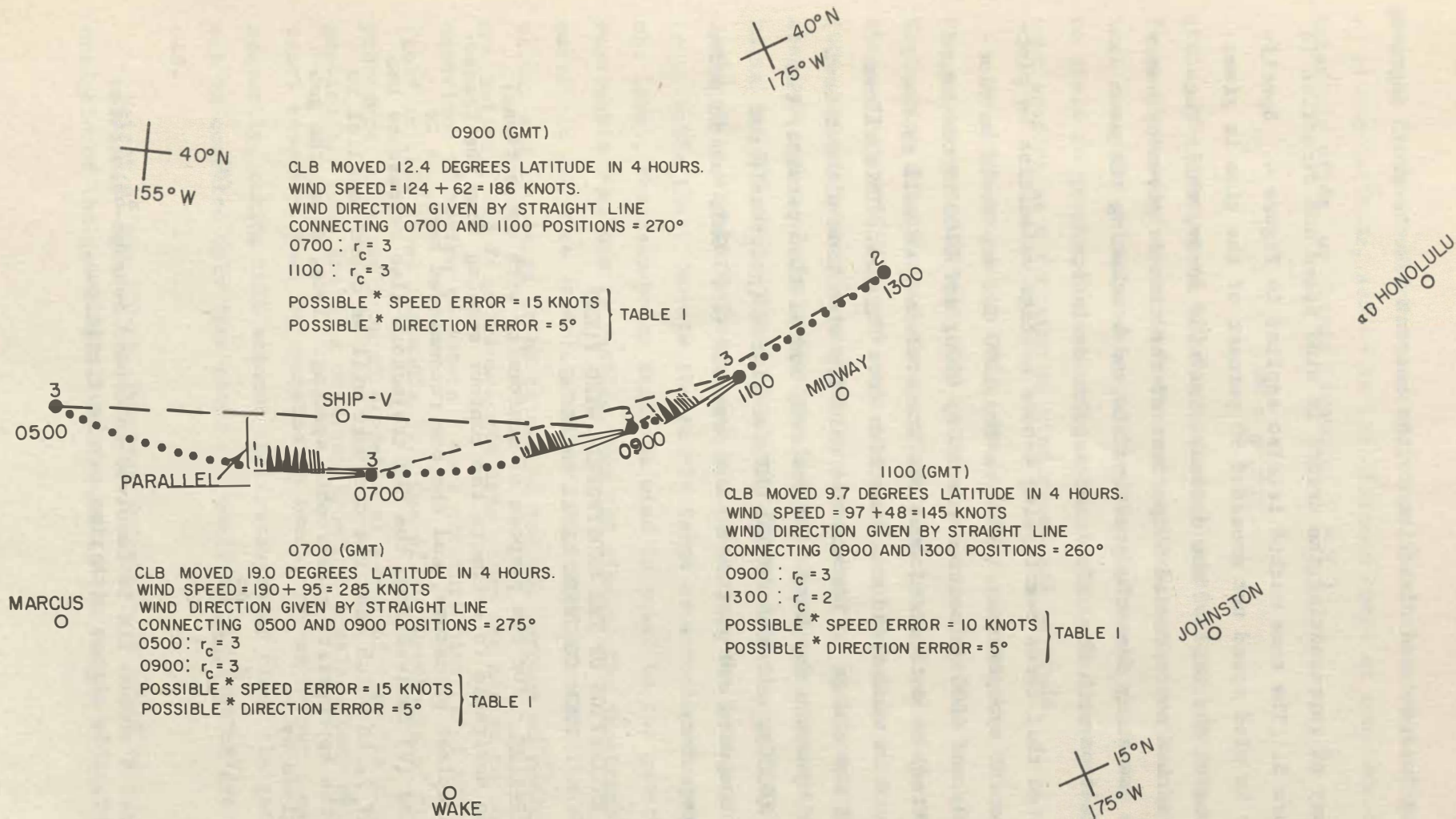
$\bar{V} =$ 

50	100
150	200

 KNOTS

DIRECTION ERROR (NEAREST 5°) NOT EXCEEDED 90% OF TIME FOR GIVEN  $r_c$  AND GIVEN MEAN WIND SPEED ( $\bar{V}$ ).





\* ERROR NOT EXCEEDED 90% OF THE TIME

FLIGHT 23 — JAN. 17, 1956  
 ILLUSTRATION OF COMPUTATIONAL AND PLOTTING PROCEDURE  
 FIG. 5

to give the errors in speed and direction to the nearest 5 knots and 5 degrees, respectively.

A possible way of representing the errors in wind speed and direction is presented in Figure 5. The same method is also applied in Figure 4. Specifically, the error in wind speed not exceeded 90 percent of the time is given by the dashed barbs on the tail of the conventional wind arrow, while the error in wind direction not exceeded 90 percent of the time is given by means of V-shaped lines emanating from the position fix and bracketing the most probable wind direction with the calculated angular deviation.

It is suggested that three overlapping 4-hour average velocities be plotted on the most recent synoptic map. Thus on the 1200 GMT map would be plotted the 1200, 1000, and 0800 velocities (or 1100, 0900, and 0700 if odd hour fixes are transmitted) as determined from the Transosondes, as well as the corresponding errors in wind speed and direction (see Fig. 5). This allows for an estimate of the change in Transosonde velocity with time without deviating too far from synoptic map time.\* If desired, and if time permits, the 0200 (0100), 0400 (0300), and 0600 (0500) GMT velocities (previously not available) may be computed and plotted on the previous (2400 GMT) map in order to complete the trajectory.

#### ESTIMATION OF THE GEOSTROPHIC WIND FIELD FROM CONSTANT LEVEL BALLOONS

The Contour Spacing - For the purpose of conformity with conventional analysis, it may be desirable to estimate the contour spacing from the Transosonde data. The simplest procedure, and the one recommended here, is to assume that the wind ( $V$ ) derived from the CLB trajectory also represents the geostrophic wind ( $V_g$ ), in which case the contours will be drawn parallel to the CLB wind with the appropriate geostrophic spacing. Thus, since the geostrophic wind is given by

$$V_g = \frac{g}{2\Omega \sin\phi} \frac{dz}{dn} \quad (8)$$

---

\* However, since a common fix is involved in 4-hour average velocities 4 hours apart, this change with time may be fictitious.



where  $g$  is the acceleration of gravity,  $\Omega$  is the earth's angular velocity,  $\phi$  is the latitude, and  $dn$  is the distance apart of contours drawn for height intervals of  $dz$ , we obtain (assuming  $V = V_g$ )

$$dn = \frac{150}{V \sin \phi} \quad (9)$$

giving the spacing of 400-foot-interval contours ( $dn$ ) in degrees of latitude for a given wind speed in knots and a given latitude. Nearly all meteorological analysis charts have a geostrophic wind scale which may easily be utilized to yield  $dn$  graphically. Consequently no table is presented here of the contour spacing as a function of wind speed and latitude.

An estimate of the error committed by assuming the CLB velocity to equal the geostrophic velocity may be obtained from previous Transosonde flights. Neglecting friction and the vertical advection of velocity, it was found from these flights (650 cases) that the mean absolute value of the angle of indraft (the angle between the contours and the wind) is 11 degrees while the mean absolute value of the difference between speed and geostrophic speed is 18 knots (at 300 mb.). Since in almost all conceivable cases it is the wind field which should be known with precision, not the geostrophic wind field, it is probable that, despite the fairly large geostrophic deviations at the 300-mb. level, the assumption that the wind is equal to the geostrophic wind is reasonable when one is passing from the wind to the geostrophic wind (but certainly not vice versa). However, if for some reason the above ageostrophic values are too large to be tolerated for the purpose of constructing the 300-mb. height field from Transosonde data, then one may go to the (approximate) equations of motion to obtain a more accurate picture of the geostrophic wind field at this level (Appendix 3). One of the drawbacks of this more precise procedure, at least from an operational point of view, is the fact that the geostrophic wind field is obtained from CLB accelerations which, for sufficient accuracy, must be computed at least over an 8-hour time interval. Consequently, taking into account the transmission time, the geostrophic field can be computed only from those CLB position fixes which are nearly 8 hours old.

Height of the 300-mb. Pressure Surface - It would be of great help in the analysis of the geostrophic wind field if the height of the 300-mb. pressure

surface at the position of the Transosonde were known in addition to the contour spacing. By equating the change of kinetic energy of the CLB and the change of potential energy due to cross-contour flow, it can be shown that (neglecting friction and the vertical advection of velocity) part of the change of the height of the 300-mb. surface following the CLB is given by

$$\Delta z_1 = \frac{-(v_1^2 - v_0^2)}{2g}, \quad (10)$$

where  $\Delta z_1$  is the height change of the 300-mb. surface at the position of the CLB due to the change in speed of the CLB from  $v_0$  to  $v_1$ , and where  $g$  is the acceleration of gravity. Putting this equation into more usable form we obtain

$$\Delta z_1 = -.04(v_1^2 - v_0^2), \quad (11)$$

where  $\Delta z_1$  is the height change in feet for given speed changes (in knots). Unfortunately there is another term which contributes to the change of the height of the 300-mb. pressure surface at the position of the CLB. This term involves the local height change along the trajectory and is given by

$$\Delta z_2 = \int_{t_1}^{t_2} \frac{\partial z}{\partial t} dt, \quad (12)$$

which can only be approximated from trajectory data. Since the value of the integrated local height change along the path of the CLB is usually positive to the east of ridges in the pressure field (assuming eastward movement of the pressure systems) and negative to the east of troughs, the CLB is generally located at a larger value for the height of the 300-mb. surface after completing its southward movement than after completing its northward movement, regardless of the change of speed along the trajectory. No attempt to take into account the local height change term ( $\Delta z_2$ ) can be particularly successful; however, an approximation of its value can be obtained by assuming that the pressure systems move zonally without amplification and that therefore (substituting from the meridional component of the geostrophic wind equation)



$$\Delta z_2 = \int_{t_1}^{t_2} \frac{\partial z}{\partial t} dt = -C \int_{t_1}^{t_2} \frac{\partial z}{\partial x} dt \approx -C \int_{t_1}^{t_2} \frac{f v_g}{g} dt \approx -C \int_{t_1}^{t_2} \frac{f v}{g} dt, \quad (13)$$

where  $C$  is the speed of the pressure systems (positive eastwards) and  $v$  is the meridional component of the velocity (assumed equal to  $v_g$ ). Putting this equation in more usable form we obtain

$$\frac{\Delta z_2}{12 \text{ hours}} \approx - \frac{.50C \bar{v} \sin \phi}{(\text{knots})}, \quad (14)$$

giving part of the height change following the CLB in terms of feet per 12 hours for mean values of the speed of the pressure system eastward ( $C$ ), the mean meridional component of the velocity ( $v$ ), and the mean latitude ( $\phi$ ) during the 12-hour period. In the practical utilization of equation (14),  $v$  may be obtained from the trajectory data, whereas  $C$  must be estimated from successive Transosonde flights or given a reasonable value by some other method. For example, if during the past 12 hours  $\bar{v} = -40$  knots and  $C = 20$  knots at a mean latitude of 43 degrees, then  $\Delta z_2 \approx 300$  feet for the 12-hour period. For comparison, if during the same 12-hour period the speed of the CLB decreases from 90 to 40 knots, then  $\Delta z_1 \approx 300$  feet. Therefore, in this case, if the CLB started out at a position where the height of the 300-mb. pressure surface was 30,000 feet, then after 12 hours the contour through the position of the CLB would be estimated to have a value of approximately 30,600 feet.

Conversely, if one knows the height of the 300-mb. pressure surface at the beginning and end of a trajectory segment, then by subtracting the height change ( $\Delta z_1$ ) due to the change in CLB speed between the trajectory extremities, one may estimate the mean value of the local height change along the trajectory.

# ESTIMATION OF THE VERTICAL VELOCITY FROM CONSTANT LEVEL BALLOONS

Procedure - The mean vertical motion along the Transosonde trajectory ( $\bar{W}$ ) may be determined from the formula (assuming adiabatic flow)

$$\bar{W} = - \frac{dT/dt}{\gamma_p - \bar{\gamma}} \quad (15)$$

where  $dT/dt$  is the change of environmental temperature with time following the CLB,  $\gamma_p$  is the process lapse rate (usually assumed to be dry adiabatic at 300 mb.) and  $\bar{\gamma}$  is the mean lapse rate along the trajectory segment. As yet, temperature data are not available from the Transosondes, and will probably not be available for some time, so that the practical utilization of equation (15) is dependent upon extrapolation of radiosonde temperatures to the position of the Transosonde. In addition, radiosonde data must be used to obtain an idea of the mean lapse rate ( $\bar{\gamma}$ ) along the trajectory segment. Therefore, it is only practical, at present, to evaluate the mean vertical motion over a trajectory segment of 12 hours duration, and then only in regions of good radiosonde data. Putting equation (15) into usable form we obtain

$$\bar{W} = \frac{- .25(T_1 - T_0)}{1 - a} \quad (16)$$

where  $\bar{W}$  is the mean vertical motion in centimeters per second,  $T_1 - T_0$  is the difference in temperature (in degrees Celsius) at positions of the CLB 12 hours apart, and  $a$  is the ratio of the lapse rate to the dry adiabatic lapse rate ( $1^\circ\text{C}/100$  meters). If the temperature at the position of the CLB becomes warmer, there is descending motion over that portion of the trajectory, and vice versa. The errors in extrapolating temperatures to the position of the CLB are more serious than errors in the positioning of the CLB by RDF techniques. As can be seen from equation (16), for a mean lapse rate along the trajectory equal to  $3/4$  the dry adiabatic lapse rate (a good mean value at 300 mb.), an error in extrapolated temperature of  $1^\circ$  Celsius will introduce an error of 1 centimeter per second in the mean vertical velocity over a 12-hour period.



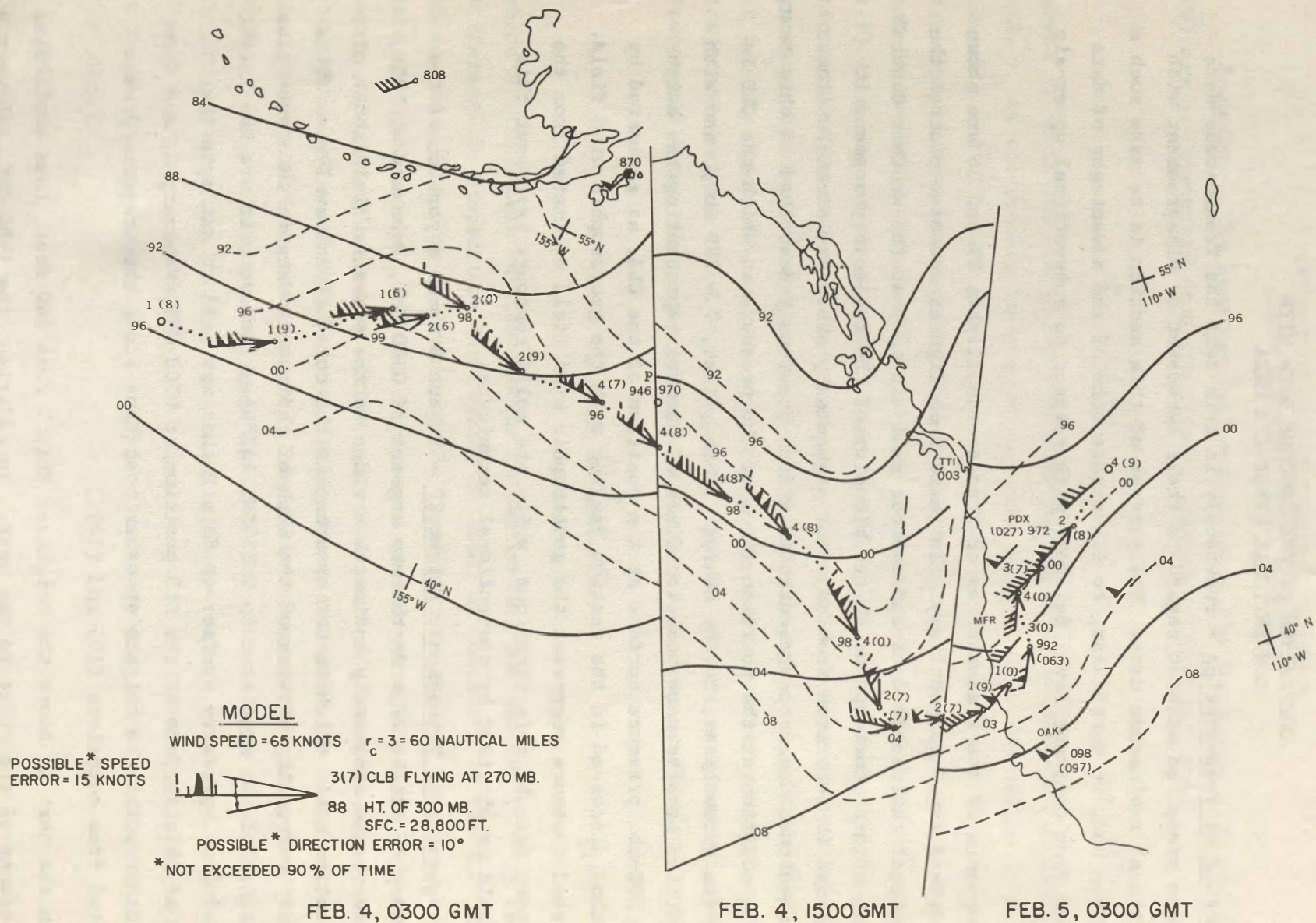
## COMPARISON OF TRANSOSONDE DATA WITH CONVENTIONAL UPPER-AIR DATA

Method of Presentation - Previously in this text the Transosonde data have been presented without regard to their agreement or disagreement with conventional rawinsonde data. The purpose of this section is to make such a comparison and, in particular, to illustrate some of the advantages of data obtained from Constant Level Balloons with respect to conventional upper-air data.

Segments of the trajectories of Transosonde flights 27 and 36 are shown in Figures 6 and 7 respectively. In these same figures are also plotted the conventional rawinsonde data and contour analysis made at the Weather Central. Each map segment corresponds to the midpoint of the trajectory segment it covers, and is separated from adjacent segments by arbitrary straight lines. The Transosonde data are presented in a form previously described in this text, with the addition at the position fixes of the pressure at which the CLB is flying (in parentheses, to the nearest 10 mb.; i.e., 7 = 270 mb.), and with the addition at alternate position fixes of numbers representing the height of the 300-mb. pressure surface at the position of the CLB, as estimated by the method discussed in the section dealing with the geostrophic wind field. The dashed contours represent the geostrophic wind field estimated from the trajectory data by this technique, while the solid contours represent this wind field as obtained by conventional techniques.

Discussion of Transosonde Flight 27 - Figure 6 shows a portion of the trajectory of flight 27. On the map segment of 0300 GMT, February 4, 1956, the Transosonde trajectory indicates a ridge in the pressure field south of the Aleutian chain which is not apparent in the conventional analysis. This is hardly surprising in view of the lack of radiosonde data in this area plus the absence of the wind at Ship P (50°N, 145°W). (However, it was the height of the 300-mb. pressure surface at Ship P that was used to estimate the height of this surface at the CLB position at 0700 GMT, February 4, and from this anchor point the heights at other positions along the trajectory were estimated from equations (10) and (14).)

In the next 12 hours the height at Ship P rose 240 feet, thus confirming the presence of the ridge to the west. In addition, the 1500 GMT, February 4



FLIGHT 27  
FIG. 6



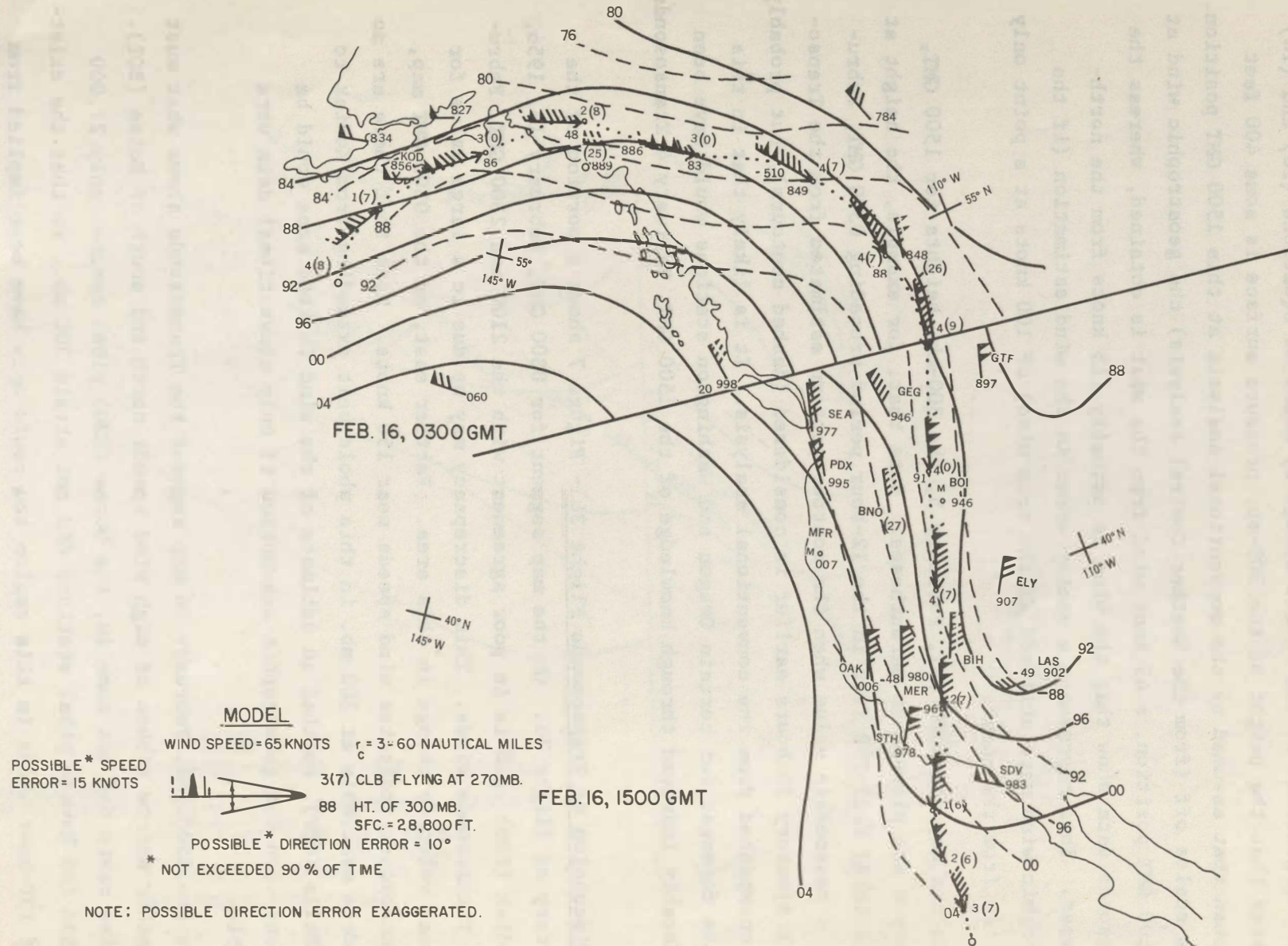
Transosonde wind suggests the existence of a rather deep trough off the Oregon coast. Computation of the height change by means of equations (10) and (14) indicates that the height of the 300-mb. pressure surface is some 400 feet less than that assumed by the conventional analysis at this 1500 GMT position. If one scales off (from the Weather Central analysis) the geostrophic wind at the 1500 GMT position, a 45-knot wind from the west is obtained, whereas the Transosonde data show that the wind is actually 115 knots from the north-northwest. This represents a vector error in the wind estimation (if the geostrophic wind were utilized as the true wind) of 100 knots at a point only 300 miles from the coast.

On the map for 0300 GMT, February 5, the 300-mb. heights for 1500 GMT, February 4 are plotted in parentheses. Note that, for example, the height at Medford (MFR) fell 700 feet in the 12-hour period preceding 0300 GMT, February 5, a reasonable value when the contour pattern estimated from the Transosonde trajectory 12 hours earlier is considered (dashed contours), but probably quite unexpected from the conventional analysis. It is likely that in this case the forecast for certain Oregon and Washington stations would have been considerably improved through knowledge of the 1500 GMT, February 4 Transosonde wind.

Discussion of Transosonde Flight 36 - Figure 7 shows a portion of the trajectory of flight 36. On the map segment for 0300 GMT, February 16, 1956, the Kodiak (KOD) wind is in poor agreement with the 2100 and 2300 GMT, February 15 Transosonde winds. This discrepancy may be due to a large value for the local velocity change in this area. Farther east, on the 0300 GMT map, the Transosonde indicates wind speeds near 150 knots. Note that there are no rawin data available at 300 mb. in this whole belt extending from Kodiak to Great Falls (GTF), so that an estimate of the wind in this area would be dependent upon the geostrophic assumption if only conventional data were available.

On the 1500 GMT, February 16 map segment the Transosonde shows what must be a rather narrow ribbon of high wind speeds north and south of Boise (BOI). The Boise rawin did not come in, the Burns (BNO) pibal reached only 27,000 feet, and the Nevada pibal stations did not attain 300 mb., so that the existence of 150-knot winds in this region too could only have been implied from





the geostrophic assumption using conventional data. Farther south, the 140-knot wind indicated by the Bishop (BIH) double theodolite appears representative, and note how well the 125-knot wind obtained from the Transosonde at 1500 GMT fits between the 140-knot Bishop wind and the 110-knot Santa Maria (STH) wind. This example shows the usefulness of CLB winds even over land areas where the problem is usually not lack of observation stations but the difficulty in obtaining data (particularly wind data) at the 300-mb. level under jet stream conditions.

Route Errors in the Wind at 300 mb. - For flight purposes, it is the accuracy of the wind along the whole flight path that is important, not the accuracy of the wind at one particular point. In order to obtain an estimate of the route errors introduced by conventional methods, the mean speed along the entire trajectory of Transosonde flight 36 was evaluated and compared with the mean geostrophic wind speed along this same trajectory as obtained from the Weather Central analysis. The mean Transosonde wind along the 13,000-nautical mile path was 101 knots while the mean geostrophic wind from the Weather Central analysis was 79 knots. Presumably, the reason the speed differences do not cancel out is due to flight 36 being embedded near the core of the "jet" where it might be expected that the conventional analysis, through smoothing, would consistently underestimate the wind strength.

#### SUMMARY

The primary purpose of this paper has been to give field meteorologists some idea of how to use the routine Transosonde flights soon to become available; in other words, to give them a "feel" for this new and somewhat different type of information. Therefore, this paper has dealt solely with the highly practical problems of the Transosonde data - how to use them? What do they mean?, etc. There has been little mention herein of the important meteorological research results available from this new atmospheric probe. For those interested in a broader outlook concerning the Constant Level Balloon, and a discussion of some of the scientific findings therefrom, a list of references is appended.



It is likely that the future of the Transosonde will be largely dependent upon the success of the routine series of flights commencing in the spring of 1957. Therefore, it is only reasonable to ask that the field meteorologists give this technique a fair trial, carefully weighing the advantages and disadvantages of the system, so that finally a consensus may be obtained concerning the applicability of Transosondes to the problem of weather analysis and forecasting.

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## APPENDIX 1

Transmission of Data - According to present plans, the most probable position of the Transosonde and the radius of the circle (about the most probable position) within which the balloon is actually located will be determined within 15 minutes after Transosonde transmission time. This information, plus the flight pressure of the CLB, will then be fed into radio teletypewriter circuit 0 at Honolulu when gaps occur in the regular transmission schedule. The data designator UT will be used to identify messages containing Transosonde data.\* Upon arrival in San Francisco the information will await a gap in land-line circuit 0 for teletypewriter transmission to analysis centers. It is estimated that the information will be available to all interested parties in North America within 2 hours after Transosonde transmission time.

Transosonde Code Form - The full code form for transmission of Transosonde data is as follows:

8/9IIIr<sub>c</sub> YQL L<sub>a</sub> L<sub>a</sub> L<sub>a</sub> L<sub>o</sub> L<sub>o</sub> L<sub>o</sub> GG hhPPP OTTUU (1ddff) (2L<sub>a</sub>"L<sub>a</sub>"L<sub>o</sub>"L<sub>o</sub>") (4/3H<sub>d</sub>H<sub>d</sub>H<sub>d</sub>H<sub>d</sub>)

where:

Group 8/9IIIr<sub>c</sub>

8 indicates Pacific area Transosonde release, 9 Atlantic area release.  
III is the balloon release number.

r<sub>c</sub> is the radius of the circle within which the CLB is actually located (r<sub>c</sub> is reported in units of 20 nautical miles and describes the position circle about the point reported for L<sub>a</sub>L<sub>a</sub>L<sub>a</sub> and L<sub>o</sub>L<sub>o</sub>L<sub>o</sub>)

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\* Subject to WMO approval

Code FigureNautical Miles

0	0
1	20
2	40
3	60
4	80
5	100
6	120
7	140
8	160
9	180 and above

Group YQL L L  
a a a

Y is day of week (GMT), (1 = Sunday, 2 = Monday, etc.).

Q is octant of globe (0 = 0°W - 90°W, 1 = 90°W - 180°W, etc., in Northern Hemisphere).

L L L is CLB latitude position in tenths of degrees.  
a a a

Group L L L L GG  
o o o o

L L L L is CLB longitude position in tenths of degrees.  
o o o o

GG is GMT time of observation to nearest quarter hour (for observation time within 7-1/2 minutes of quarter hour add 25 to hour time, within 7-1/2 minutes of half hour add 50 to hour time, within 7-1/2 minutes of three quarters hour add 75 to hour time.)

Group hhPPP

hh is true altitude of pressure surface in thousands and hundreds of geometric feet.

PPP is pressure altitude of CLB in whole millibars.

Group OTTUU

O is group indicator figure.

TT is temperature in whole degrees Celsius (add 50 for negative temperatures).

UU is relative humidity (tens and units).

Group lddff

l is group indicator figure.

dd is wind direction in tens of degrees.

ff is wind speed in knots (add 50 to direction if speed exceeds 100 knots).

Group 2L''L''L''L''  
a a o o

2 is group indicator figure.

L''L'' is latitude in degrees to which the mean CLB wind applies.  
a a

L''L'' is longitude in degrees to which the mean CLB wind applies.  
o o

Group 4/3H<sub>d</sub>H<sub>d</sub>H<sub>d</sub>H<sub>d</sub>

4 indicates positive D value, 3 negative D value ( $D = Z - Z_p$  = actual CLB height-pressure height).

H<sub>d</sub>H<sub>d</sub>H<sub>d</sub>H<sub>d</sub> is D value in feet.

The preliminary Transosondes to be released from Japan in the late spring of 1957 will carry neither radio altimeters nor temperature or humidity measuring devices. Therefore, hh in the fourth group will be replaced by XX, the 4/3 group will be omitted, and the group with the 0 designator will be omitted. Furthermore, at the present time, it seems unlikely that groups lddff and 2L''L''L''L'' will be included in the message. Therefore, the preliminary message forms will probably be of the type

8IIIr<sub>c</sub> YQL L L L L L L GG XXPPP.  
c a a a o o o o

For example, the message

80014 22354 66753 XX287

means that Pacific release Transosonde flight 1 on Monday at 0330 GMT is floating at 287 mb. with a most probable position at latitude 35.4°N, longitude 166.7°E and with an actual position within a circle of 80 nautical miles radius centered on this most probable position.



## APPENDIX 2

Estimation of Errors in Wind Speed and Direction - For the purpose of simplifying the estimation of the possible errors in wind speed and direction obtained from Constant Level Balloons, it will be assumed that the errors in speed are solely a function of errors in position fixes along the direction of flow (direction S) while the errors in direction are solely a function of errors in position fixes normal to the direction of flow (direction N). This is a reasonable approximation as long as the distance between fixes (S) is large compared to the radii ( $r_c$ ) of the circles (centered on the position fix) within which the CLB is actually located. It is then assumed that the actual positions of the CLB follow a normal distribution along these axes such that 68 percent of the actual CLB positions lie along the axes within  $\sigma_{0r_c}$  of the position 0 and within  $\sigma_{1r_c}$  of the position 1 (see Fig. 3). On the basis of the normal distribution, and the assumption that 99 percent of the CLB positions fall within the circles of radius  $r_c$ , it can be shown that  $\sigma_{r_c} = r_c/2.6^*$ . Since the standard error of the difference (S) between two quantities is the square root of the sum of their standard errors, we have

$$\sigma_S = \sqrt{(\sigma_{0r_c})^2 + (\sigma_{1r_c})^2} \quad (17)$$

Now

$$\sigma_V = \frac{\sigma_S}{t}, \quad (18)$$

since the time interval (t) is known relatively accurately, and therefore substituting for the standard error of position as estimated from  $r_c$ ,

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\* Brooks, C. E. P., and N. Carruthers, Handbook of Statistical Methods in Meteorology, London, H. M. Stationery Office, 1953, p. 196.

$$\sigma_V = \frac{.4 \sqrt{(\sigma_0 r_c)^2 + (\sigma_1 r_c)^2}}{t} \quad (19)$$

where  $\sigma_V$  is the standard error of the speed. Of more significance for the purpose at hand is the speed error exceeded only 10 percent of the time ( $1.6 \sigma_V$ ), and we obtain

$$1.6 \sigma_V = \frac{.6 \sqrt{(\sigma_0 r_c)^2 + (\sigma_1 r_c)^2}}{t} \quad (20)$$

In an analogous fashion, since

$$\sigma_\theta \approx \frac{\sigma_N}{S}, \quad (21)$$

where  $N$  is distance normal to the direction ( $S$ ), we may write

$$\sigma_\theta \approx \frac{\sqrt{(\sigma_0 r_c)^2 + (\sigma_1 r_c)^2}}{S} \quad (22)$$

where  $\sigma_\theta$  is the standard error in direction. Consequently

$$1.6 \sigma_\theta \approx \frac{36 \sqrt{(\sigma_0 r_c)^2 + (\sigma_1 r_c)^2}}{V t} \quad (23)$$

Equations (20) and (23) give, respectively, the speed error in knots and the direction error in degrees not exceeded 90 percent of the time for values of  $r_c$  expressed in nautical miles, time intervals expressed in hours, and mean speed expressed in knots.

## APPENDIX 3

Estimation of the Deviation of Wind Speed and Geostrophic Wind Speed -

Neglecting friction and the vertical advection of velocity, the normal equation of motion may be written

$$K_H V^2 = V \frac{d\theta}{d\tau} = - 2 \Omega \sin\varphi (V - V_g \cos i), \quad (24)$$

where  $K_H$  is the trajectory curvature,  $d\theta/d\tau$  is the angular velocity,  $\varphi$  is latitude,  $\Omega$  is the earth's angular velocity,  $V$  is wind speed,  $V_g$  is geostrophic wind speed, and  $i$  is the angle of indraft (the angle between wind and geostrophic wind, assumed positive if the flow is toward low pressure). For the values of angle of indraft generally found in the free atmosphere,  $\cos i$  remains very close to 1, and we may, with little approximation, write the normal equation of motion in the form

$$V - V_g = - \frac{V}{2 \Omega \sin\varphi} \frac{d\theta}{d\tau}. \quad (25)$$

Putting the equation into usable form ( $d\theta$  in degrees,  $d\tau$  in hours) we obtain

$$V - V_g = \frac{.03 \bar{V}}{\sin\varphi} \frac{d\theta}{d\tau}. \quad (26)$$

Thus, for a mean wind speed of 60 knots at a latitude of  $43^\circ$ , a direction change of  $-40^\circ$  (anticyclonic turning) in 8 hours will correspond to a wind speed 14 knots greater than the geostrophic wind speed.

Estimation of Error in Speed Deviation - In order to determine the possible error in the value obtained for the deviation of wind speed and geostrophic wind speed, we proceed as in Appendix 2 with the recognition that the largest source of error in its determination lies in the estimation of the angular velocity  $d\theta/d\tau$ . Thus, since



$$\sigma_{d\theta/d\tau} = \frac{\sqrt{(\sigma_{\theta_1})^2 + (\sigma_{\theta_5})^2}}{\tau}, \quad (27)$$

where  $\tau$  is the time interval between directions of the CLB used to estimate the angular velocity and  $\sigma_{\theta_1}$  and  $\sigma_{\theta_5}$  represent the standard errors in these directions, we obtain by substituting from equation (22), (assuming  $V$  is constant along the trajectory segment)

$$1.6 \sigma_{d\theta/d\tau} = \frac{.6}{t\tau V} \sqrt{({}_0r_c)^2 + ({}_2r_c)^2 + ({}_4r_c)^2 + ({}_6r_c)^2}, \quad (28)$$

where  ${}_0r_c$  and  ${}_2r_c$  are the radii of the circles around those positions from which  $\theta_1$  is determined and  ${}_4r_c$  and  ${}_6r_c$  are the radii of the circles around those positions from which  $\theta_5$  is determined. Finally, substituting from equation (25), we obtain

$$1.6 \sigma_{(V-V_g)} = \frac{1.1}{t\tau \sin \phi} \sqrt{({}_0r_c)^2 + ({}_2r_c)^2 + ({}_4r_c)^2 + ({}_6r_c)^2}, \quad (29)$$

giving the error not exceeded 90 percent of the time for the difference between speed and geostrophic speed (in knots) as a function of the time interval  $t$  (in hours) between fixes used to estimate the CLB direction, the time interval  $\tau$  (in hours) between directions used to estimate angular velocity, the radii ( $r_c$ ) of the circles (in nautical miles) around those position fixes utilized for the determination of the angular velocity, and the latitude  $\phi$ . In this text it is recommended that  $t = 4$  hours,  $\tau = 8$  hours, and therefore, if  $r_c$  has a value of 40 miles around all those fixes utilized to estimate the angular velocity, at latitude  $43^\circ$ , 90 percent of the time the error in  $V - V_g$  will not exceed 4 knots.

Estimation of the Angle of Indraft - Neglecting friction and the vertical advection of velocity, the tangential equation of motion may be written

$$\frac{dv}{dt} = 2 \Omega \sin \phi V_g \sin i, \quad (30)$$

where  $\frac{dv}{d\tau}$  is the rate of change of speed and the other parameters have the meaning indicated at the beginning of this Appendix. Solving for the angle of indraft ( $i$ ) and putting the equation in usable form ( $d\tau$  in hours), we obtain

$$i = \sin^{-1} \left( \frac{1.9}{V_g \sin \phi} \frac{dv}{d\tau} \right) . \quad (31)$$

For example, if the wind speed at latitude  $43^\circ$  increases by 40 knots over an 8-hour time interval ( $d\tau = 8$ ) and the mean geostrophic wind along this trajectory segment is 60 knots, then the mean value of the angle of indraft along the trajectory segment is  $13^\circ$ , with the wind cutting toward low pressure.

Estimation of Error in Angle of Indraft - In order to determine the possible error in the value obtained for the angle of indraft we proceed as previously with the recognition that the largest source of error in its determination lies in the estimation of  $\frac{dv}{d\tau}$ . Thus, since

$$\sigma_{dv/d\tau} = \frac{\sqrt{(\sigma_{V_1})^2 + (\sigma_{V_5})^2}}{\tau} \quad (32)$$

where  $\tau$  is the time interval between speeds of the CLB used to estimate the rate of change of speed, we obtain by substituting from equation (19)

$$1.6 \sigma_{dv/d\tau} = \frac{1.6}{t\tau} \sqrt{(0r_c)^2 + (2r_c)^2 + (4r_c)^2 + (6r_c)^2} \quad (33)$$

where  $0r_c$  and  $2r_c$  are the radii of the circles around those positions from which  $V_1$  is determined and  $4r_c$  and  $6r_c$  are the radii of the circles around those positions from which  $V_5$  is determined. Finally, substituting from equation (31), we obtain

$$1.6 \sigma_i = \sin^{-1} \left\{ \frac{1.1}{V_g t\tau \sin \phi} \sqrt{(0r_c)^2 + (2r_c)^2 + (4r_c)^2 + (6r_c)^2} \right\} \quad (34)$$

giving the error of the angle of indraft (in degrees) not exceeded 90 percent of the time as a function of the time interval  $t$  (in hours) between fixes used to estimate the CLB speed, the time interval  $\tau$  (in hours) between

speeds used to estimate the rate of change of speed, the radii ( $r_c$ ) of the circles (in nautical miles) around those position fixes utilized for the determination of the rate of change of speed, and the latitude  $\varphi$ . Again, using  $t = 4$  hours,  $\tau = 8$  hours, and assuming all values of  $r_c$  are equal to 40 miles, then at latitude  $43^\circ$  with a mean geostrophic wind speed along the trajectory of 60 knots, 90 percent of the time the error in the angle of indraft will not exceed  $4^\circ$ .