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North American Gravity Modelling

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When the National Geodetic Survey (NGS) made the decision that the North American Vertical Datum Network would be adjusted in terms of Potential Numbers ('potential heights'), a surface gravity value was to be provided at every benchmark of the network. The fact that this requirement could not be satisfied by observed gravity, the Surface Gravity Modelling project was initiated.

NGS has always had foresight regarding the importance of its sizeable gravity databank in its activities of geodesy. Considering that the data bank's gravity were collected from a variety of sources, some years before the North American Vertical Datum Network was perceived NGS effected a quality review campaign for its gravity holdings. Unfortunately, since there was no cutting-edge graphics apparatus available to NGS at that time, the finding of inconsistencies among the gravity values could be accomplished only by manually contouring among correctly positioned data marked on computer print-outs; nevertheless, this enterprise was important in itself and produced rather reliable datasets.

Regrettably, since observed gravity values were not available for all benchmarks of the data base of NGS, interpolated gravity became the substitutes for the lack of observed values. The Continental Gravity Model was constructed from the totality of 30' x 30' adjoining geographic square sub-models ('mosaics') which covered the conterminous United States, regions of the coastal and ocean areas (inclusive of ship gravity data) and the boundaries at Canada and Mexico. For gaining smooth transitions among model mosaics, the model margins were extended. The simplest widening of boundaries could have been done by moving the perimeters of all squares uniformly by 15 geographic minutes which would have doubled the number of data points but there was a high premium on gaining the least number of additional data for modeling. The number of needed gravity values for smooth transitions was variable; consequently, the margins were best extended by viewing gravity density plots which activity became a substantial task for the whole continental gravity model.

Essential preparatory activity for attaining the continental model was the selecting of a surface gravity modeling methodology which best represents the characteristics of the data, provides accurate interpolant for unstructured data, points would not need to lie on a structured grid, yet it has a simple kernel.

In the 1971 issue of the Journal of Geophysical Research a publication appeared from the pen of R. L. Hardy [ref.**1]** which had the title**:** 'Multiquadric equations of topographic and other irregular surfaces'**.** This technique soon became a popular interpolation / prediction tool for gravity since gravity can be just as irregular and scattered as topographic heights could possibly be**.** It does have an accurate interpolant as that of a radial basis function for unstructured data**,** and it has a simple kernel**.**

"In the multiquadric **(**MQ**)** method the data are modeled with polynomial equations**…** This involves representing the gravity data as a sum of individual cones, each contributing to form the model. The equation is of the form**:**

$$\mathbf{g_i = \Sigma_j (a[i,j] \cdot C_j) \quad j = 1, 2, \ldots n} \qquad (1)$$

where

$$\mathbf{a[i,j] = [(x(i) - x(j))^2 + (y(i) - y(j))^2 + D^2]^{1/2}} \qquad (2)$$

a [i,j] is an **n x n** ***symmetric matrix*** of distances from observed data points

($\mathbf{g_i}$ for **i = 1, 2, … n)** to all data points **(j = 1,2,...n)** having ***zero diagonal elements*****,** and **D** (delta) is a smoothing factor**.** $\mathbf{C_j}$ **'s** represent coefficients to be determined from the data**.** Delta has been omitted because a value of zero was found to produce the best values**.** Once the coefficients are known, the predicted value becomes

$$\mathbf{g = \Sigma i \{ \Sigma j \; (a[i,j] \cdot C_j) \} \quad j = 1, 2, \ldots n \quad i = 1, 2, \ldots n} \text{ "} \qquad (3)$$

One application of this method is the 'National Geodetic Survey (NGS) Gravity prediction Methodology' (*Tool Kit*)**.** As stated there**:** "*The process is designed to predict surface gravity at specified geographic position and topographic height***.** This application preselects the scattered data around the given geographic positions of gravity and topographic height for two concurrent predictions**.** "

There had been concerns that gravity could be obtained from a source of regular patterns**.** Therefore**,** the original concept of selection of observations over *high-resolution* grid patterns for gravity was examined and found demonstrably inadequate even for small data sets **[6];** thus**,** assembling the gravity data this way in forming the sub-models (mosaics) of Continental Gravity Model became untenable**.**

There were also concerns for the control of model surface at possibly large gaps in the gravity distributions. Hardy's MQ method proposed application *Modes* in which there are provisions for such cases**:** *Least squares mode***,** *Osculating mode* **[3].** These provisions add equations of condition to the coefficient matrix which place *large burdens* on the numerical process but do not take into consideration a very practical matter**:** a trend is frequently detectible in scattered, random data**.** When this trend is modeled by a best-fitting (least-squares) plane**,** this plane also stabilizes the surface at gaps of data and at the edges of model mosaics with much less numerical burden**.**

Since gravity is a gradient of a well-established potential function defined on the earth's surface**,** the proper interpolant for gravity is the one that represents this potential function**;** *yet***,** Hardy **&** Nelson assert of the suitability of their *multiquadric-biharmonic* (MQ-B) method **[2]** for using it as interpolant for observed gravity**.** This became debated for *the interpolant* <u>*not being harmonic*</u> **[4].** ***Jekeli*'**s closing view was that "the method is a viable numerical approach when <u>*harmonicity is not a premise*</u>, such as in the interpolation of a <u>function defined on the surface</u>"**.** Hardy's epitome of *"20 YEARS OF DISCOVERY 1968-1988"* **[5]** cited numerous studies of mathematicians and (geodetic) scientists on the efficiencies and desirability of using MQ[-B] *for [gravity] data modeling and interpolation***,** thus convincingly supported my recommendation to NGS management **[6]** for using MQ[-B] to the massive task ahead of us for producing the Continental Gravity Model**.** The proven assertion that the *matrix of coefficients is always solvable* **[5]** was significant assurance**.**

Gravity modeling for one sub-model commences when the observed gravity data are entered from its defined and limited surrounding area into computer memory**.**

1. Parameters of the plane representing the trend of full set of data are determined**.**
2. Subsequently**, 25** (6'x6') compartments define one-one point where the preliminary model points *are designated*
3. The observation equation**s (**eq.1) are formed with the coefficient matrix **(**eq. 2**)** where $\mathbf{g}_i$ designates the *reduced gravity to the plane;*
4. Given this gravity model and the solved $\mathbf{C}_j$ values they provide the means to calculate the corresponding variances between modeled and observed values.

The start of ***iterations*:**

The acceptable limit for root-mean-square of variances for gravity modelling was targeted to be 1-2 milligals but it could be also entered as input value. When this RMS was not met, the Gravity Modelling system performs these tasks**:**

a. *it sorts* the computed variances by magnitude **(** *in-memory* software)

b. selects the *positions at the 10 largest residuals***,** *adds the corresponding observed gravity* values **(**$\mathbf{g}_i$**)** **i = 1, n+1…10,** to the observation equations **(**eq. 1**).** Repeats (iterates) the process for solving the new set of **(**$\mathbf{C}_j$**)** **j = 1, n+1…10,** coefficients**,** and retests the RMS value of variances of model fit**.** When the newly calculated RMS value was still not within limits, *the iterated process will continue*. In case, the RMS value eventually fell within specified limits, this process plots the bar-charts of variances [on huge pages of printed paper!] for visual confirmation of its bell-shape which indicated the statistical stability of the model.

Hardy revealed that the MQ-B methodology "*was not fool-proof*" **[5]**, although ill-conditioned matrices were rare. The well-known 'track-data' problem occurred at the oceans' edges: long lines (i.e., non-scattered) and densely recorded gravity measurements on ships were disruptive. This was reasonably well solved by dropping some (~ every second) observation in a series of ship-data. – The dynamics of selected gravity depend on the variability of magnitudes and locations of observation points which influence the number of yet-needed iterations; never-the-less, *the numerical process was always stable*. For speed of computations, the observed gravity data were held in computer memory instead of on direct-access magnetic media. This made the *iterative mode* of the evolving-type modeling feasible. Additionally, it also assured that the number of coefficients of the MQ-B gravity models became minimum. There was a great demand for large memory capacity with this system. Finally, *model parameters* were transferred to the continental gravity data base. I modified and installed a very efficient data base structure for storing and accessing the gravity models' parameters; originally designed by Dr. Clyde Goad (also at NGS for some years), it was eminently applicable for geographically organized information and he used it very successfully for storing and accessing satellite orbits' parameters.

Reference was given to the final product of the Surface Gravity Modelling project in some sources, and the model is also offered for Surface Gravity Computation at any position via the Continental Modeled Gravity' *Tool Kit*. Its predicted values are also stored with the Bench Mark records of the NGS Data Base. The success of this massive Surface Gravity Modelling enterprise is confirmed by the existence of the National Vertical Network which was affirmed as Vertical Datum for Surveying and Mapping by the Federal Register Vol. 58 No. 120 dated Thursday, June 24, 1993 (page 34245).

A D D EN D U M

The Management of the National Geodetic Survey were granted the usage of NOAA's Honeywell ***'Super-Computer' at Suitland*** for the accomplishment of the Gravity Modelling project. *C*opies of full contents of NGS's Gravity Data Bank were sent and kept at the computer site on several large ***magnetic reels***. These were mounted on hardware drives when 5-6**…** sub-models were run on the hardware in sequence at a time**.** Per agreement**,** the gravity modeling project was accomplished mostly in the night-time shifts. This was a very demanding operation even to *super computers* at *that time* (i.e., from mid- 1980's)**.** It occurred occasionally that the computer site personnel held back the submitting for processing some 'computer runs' due to **"***not-available time/resource of such magnitude at this time***"** – **'***what in the world are you doing?'* was the frequent inquiry**.** Well**,** an enormous number of matrices needed be formed and 'inverted' in iterated mode **[**the availability of '*quantum computing'* probably would have been helpful**].**

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