# Measuring Capital Value in a Commercial Fishery: A distance function approach.

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## 1.0 Introduction

Government intervention and management of activity associated with the harvest of marine fisheries is typically justified based on a well-known triad of interconnected influences -- the common pool nature of the resource, "overcapitalization" of fishing fleets, and the depletion of fish stocks. Among these three, the term "overcapitalization" is likely the least understood and is usually equated to the number of vessels in a fishery. While simple vessel counts provide some information about capital in a fishery, a better measure is the total value of all the individual parts of a fishing vessel. A vessel is made up of a hull, engine, electronics and other pieces of equipment, which taken together comprise the capital stock. On a vessel basis, the capital stock value can increase when new pieces of equipment are added to a vessel, or older equipment is replaced by newer equipment. It can also decline as vessels age, and parts deteriorate. In a fishery, overcapitalization occurs when the aggregate level of capital stock is too high relative to what is needed to harvest the resource [1].

A complete accounting of capital stock in a fishery gives managers information about the level of investment in a fishery which is more revealing than simple vessel counts [2]. For example, electronics upgrades increase the level of capital without increasing vessel counts. It also allows capital user cost to be estimated, which is an important component of total cost in an economic analysis. Capital user cost in a time period is simply the capital value times an appropriate interest rate, plus the change in capital value. The first term is also referred to as the opportunity cost of capital, while the second term includes re-investment plus depreciation cost [3]. Without an initial estimate of capital value, user cost cannot be estimated. Economic profit includes capital user cost, and profitability change compared to a benchmark value is an indicator of relative economic well-being for vessels in a fishery. For a complete picture of investment, and to properly account for all costs in economic models, it is therefore imperative to calculate a measure of vessel capital value and track changes in this level through time.

Fishing vessels are made up of various pieces of equipment configured to operate together just as land based factories are usually comprised of land, buildings and equipment used on a production line. In order to measure the value of capital in a fishery, dedicated surveys are typically required to inventory the equipment used, value each piece and aggregate the values into a single measure of monetary worth. Usually this effort falls into the realm of fishery agencies because fishing vessels as operating units are generally too small a part of national economies to be broken out separately in terms of accounts used in the calculation of GDP by national statistical agencies. Since the measurement of fish resources usually consumes a large portion of fishery agency budgets, the measurement of vessel capital may not be a priority. Surveys are by nature expensive and time consuming.

Conceptually the measurement of fishing vessel capital is no different than the practices used for land based industries. Specifically, the Perpetual Inventory Method (PIM) has become the most widely accepted standard for valuing capital stocks[4, 5]. Although this method has not been used to value fishing capital in the United States, it has been used in Europe [4]. Simply put, the PIM method values each part of the capital stock (vessel, engine, and electronics), and aggregates the individual components into one value, which is considered a benchmark value. Once the benchmark value is established, subsequent year's value is calculated as the benchmark value, plus additional capital investment and any revaluation of capital, minus depreciation [4].[1] Depreciation is usually calculated using established formulas. It should be noted that the work in Europe is based on a much broader concept of capital than simply the value of the fishing vessel,

[1] For further discussion about PIM, see [6] L.R. Christensen, D.W. Jorgenson, The measurement of US real capital input, 1929–1967, Review of Income and Wealth 15(4) (1969) 293-320., and [3] B.M. Balk, Measuring and decomposing capital input cost, ibid.57(3) (2011) 490-512..

as shore side infrastructure, permit and quota value are also included in the calculation of capital value [4]. This has some advantages in that capital is valued at the firm level rather than the vessel level, recognizing that fishing firms may own more than one vessel, and their assets also include shore side support. It also presents additional challenges when the capital costs are allocated to individual fishing vessels. There are several methods for distributing shore side costs to vessels, and one method needs to be chosen with the recognition that other approaches may be just as valid.

In the northeast United States, data have not been collected to use the PIM method to set an initial capital value, nor to track changes in capital stock. Consequently, this study departs from the PIM approach, and instead adopts a method based on estimates of an input distance function which utilizes vessel sale prices to determine vessel capital value similar to the study by Kirkley and Squires [2], and Daures [7]. It differs in that it uses a distance function to model the transformation of vessel characteristics into vessel value rather than a hedonic approach. The distance function is a multiple input, multiple output representation of technology estimated with parametric or nonparametric methods. This study uses linear programming (LP) methods to estimate the distance function, as opposed to econometric methods used in previous studies [2, 7, 8].

The distance function model yields shadow prices of vessel attributes which are used to derive an estimate of capital value. Additionally, the value of permits which are often included when vessels are sold are estimated separately from the physical vessel capital value are derived, and a measure of vessel performance in terms of value is provided.  Results from the model are then used in two different estimations. First, the total capital and permit value for commercially permitted vessels in the northeastern United States is estimated based on the shadow prices obtained from the model. These values can be considered "benchmark" values for future calculation of capital and permit value. Secondly, shadow prices for the vessel attributes are used to construct a capital quantity index for vessels permitted in the northeast squid, mackerel and butterfish (SMB) fishery which had landings in any year between 1996 and 2015. This allows an examination of trends in the total quantity of capital used in a fishery over a fairly lengthy time period. The Lowe quantity index is chosen for this part of the analysis, and is particularly well-suited because it is a fixed weight index employing constant prices to construct aggregates representing the capital stock value. This avoids having to calculate the shadow prices on a yearly basis, particularly if sale data by year are limited, which is the situation with the data used in this study. Trends in the quantity index are then compared to a simple index of vessel numbers to see if the two indices exhibit similar trends.

Results show that the estimated value of all commercially permitted fishing vessels in the Northeast region in 2016 was between \$606.6 and \$769.7 million, with the capital stock estimated to be worth between \$555.8 and \$700.2 million and the permit value between \$50.8 and \$69.5 million. Fiberglass vessels were the most valuable group in aggregate because they were the largest, while steel hulled vessels were the most valuable vessels on a per vessel basis. Wood hulled vessels were the least valuable vessels. Trends in the capital quantity index for the SMB fishery were similar to those found using a simple vessel count, but the magnitude of change was different between the two indices in individual years.

## 2.0 Methods

In order to derive shadow prices for the vessel attributes, parameter values from an input distance function are estimated using linear programming (LP) methods. A full explanation of the distance function derivation can be found in Appendix 1. For each observation in the data, the input distance function efficient value is one, and the LP model seeks to minimize deviations from one subject to non-negativity constraints for the distance function and its partial derivatives with respect to the inputs (attributes) for each observation. The distance function chosen has a translog specification. As an example of what this would look like, the translog distance function equation for a vessel with three attributes (z=1,2,3) and one value term (v), is:

$$\ln D_i(v,z) = \alpha_o + \alpha_1 \ln v + 1/2\alpha_{11}(\ln v)^2 + \sum_{n=1}^{3} \beta_n \ln z_n$$

$$+1/2\sum_{n=1}^{3}\sum_{n'=1}^{3} \beta_{nn'}(\ln z_n)(\ln z_{n'}) + \sum_{n=1}^{3} \gamma_n (\ln z_n)(\ln v)$$

This specific form also requires additional constraints be placed on the parameters:

$$\beta_1 + \beta_2 + \beta_3 = 1$$

$$\sum_{n'=1}^{3} \beta_{nn'} = \sum_{n=1}^{3} \gamma_n = 0$$

$$\beta_{nn'} = \beta_{n'n}, n = 1,2,3; n' = 1,2,3.$$

Empirically, the  general empirical approach taken in Färe, Grosskopf, Lovell and Yaisawarng [9] is adopted, which employed a non-parametric programming model developed by Aigner and Chu [10], and allows estimation of  the distance function as a (deterministic) non-parametric frontier function. Letting $k = 1, \ldots, K$ index observations, the following LP model is solved:

$$\min \sum_{k=1}^{K} [\ln D_i(v^k, z^k) - \ln 1] \qquad (1)$$

Subject to:

$$\sum_{n=1}^{N} \beta_n = 1 \qquad (2)$$

$$\sum_{n'=1}^{N}\beta_{nn'} = 0, n = 1.....N \qquad (3)$$

$$\sum_{n=1}^{N}\gamma_n = 0 \qquad (4)$$

$$\beta_{nn'} = \beta_{n'n} \qquad (5)$$

$$\beta_n + \sum_{n'=1}^{N}\beta_{nn'} * \ln(z_{kn'}) + \gamma_n * \ln(v) \geq 0, n = 1,2,3; k = 1...K \qquad (6)$$

Based on the beta and gamma parameters returned by the model, the value (or price) for each input (n) is calculated as shown in equation 7:

$$p_n = \frac{v}{z_n}(\beta_n + \beta_{n1}\ln z_1 + \beta_{n2}\ln z_2 + \beta_{n3}\ln z_3 + \gamma_n \ln v), \qquad (7)$$

Here, v is the vessel value and z an input value, such as length. Note that both of these are observed values. If there are three separate observed inputs, such as length, engine horsepower and hold capacity, a shadow price is calculated for each. The shadow price times the quantity of the input equals input value, and summing the value for all inputs gives the value of the vessel. The sum of all vessel values yields the value of the fleet.

In addition to being able to calculate the value of each vessel, and then sum values to a bigger group of vessels, the shadow prices can also be used to construct a quantity index of capital value. With enough yearly data, a price index can also be created. Because the data used in this project does not have enough yearly data, only the quantity index is constructed. There are several different types of indices which can be constructed, but for this analysis the Lowe quantity index is chosen. The Lowe quantity index is a fixed weight index which uses prices as weights, and the prices don't need to be from the specific year in question. The Lowe quantity index is specified as follows:

$$Q^{Lo}(x^1, x^0; p^b) \equiv p^b \cdot x^1 / p^b \cdot x^0$$

Here, $p^b$ is a vector of prices from any period b, while $x^1$ and $x^0$ are vectors of quantities from time period 0 and 1 [11]. For this study, prices ($2009) for each part of the vessel capital stock are based on an average shadow price returned by the model during the time period 2000-2013. Being able to use a single mean price for each vessel attribute is advantageous for this study because as noted below, there are not enough yearly sale price data to estimate shadow prices on a yearly basis.

## 2.1 Data.

Vessel price data were obtained from sale listings found in the Athearn Marine web page, National Fishermen and Commercial Fisheries News during the summer of 2013. Characteristics about each vessel taken from the listings included the advertised sale price, vessel length, gross tonnage, horsepower and hull material (i.e. fiberglass, steel, or wood). Additionally, the number of limited access permits held by each vessel along with the vessel age was obtained from federal permit files. Years included in the analysis were between 2000 and 2013, and all sale prices were converted to 2009 values using the GDP implicit price deflator. For-hire vessels which carry recreational anglers were not included in the data collection effort, so the results only apply to commercial vessels.

Overall, there were 380 vessel sale prices with data which could be linked to permit information. The final sale price is not known, which means there is some uncertainty about the value term used in the model if vessels sell for less than the asking price. Vessels were initially stratified by hull material (steel, fiberglass or wood), and the deflated sale prices ($2009) for each group were plotted (Figure 1). This initial stratification showed that steel hulled vessels had a higher median sale price than either wood or fiberglass vessels, which also confirmed the findings of Kirkley and Squires [2]. The median value for the fiberglass and wood hulls are both below the 25% quartile for the steel hulled vessels, but within the lower interquartile range. Because of the large difference in sale price between steel hulled vessels and the other two vessel types, steel vessels were modeled separately from the other two vessel types. Similarly, because wood hulled vessels had a lower median value than fiberglass vessels, wood and fiberglass vessels were modeled separately. Finally, three vessels which had missing vessel characteristics were removed from the analysis, reducing the total number of vessels to 377.

[Figure 1. here]

Vessels with fiberglass hulls averaged 38 feet in length, 18.6 tons, had engine horsepower of 391, were 15 years old and averaged 1.4 limited access permits. This was the largest group in the data with 262 vessels (Table 1). Vessels with steel hulls were the next largest vessel group (68 vessels). On average, steel hulled vessels were 74 feet in length, 105 tons, and with engine horsepower of 583. They were 28.6 years old, and held 4.1 limited access permits on average. The smallest category was wood vessels (47 vessels). These averaged 46.7 feet in length with engine horsepower of 338, and were 27.5 tons. They were on average 32.5 years old, and owned 1.7 permits (Table 1).

[Table 1. here]

The input distance function model had a single output variable, which was deflated sale price. The input variables used were vessel length, engine horsepower, a transformed age variable and number of permits. Because tonnage and vessel length are highly correlated with one another, tonnage was not used. Price was an increasing function of vessel length, which simply means bigger vessels had a higher price (Figure 2). An initial review of the data showed that price and age of the vessel were inversely related (Figure 2). However, in order to make age an increasing value and be consistent with the properties of the input distance function, age value was transformed by subtracting its value from 100. This meant that a 10 year old vessel would have a higher age input value (90) then one that is 40 years old (60). Price also was correlated with the number of permits, but the relationship did not seem to be as strong as price and length (Figure 2). The permits variable used in the model was simply the total number of limited access permit categories held by the vessel plus one. One was added to the number of permits because some vessels did not have any associated permits, and since we are using a log model, the variable for number of permits could not be zero.

[Figure 2. here]

## 3.0 Results

The average distance function scores indicate how far observations are from the efficient frontier, with a distance function score of one indicating an efficient (i.e. resides on the frontier) observation. As a group, steel vessels had the lowest average distance function score (1.12) with 12 observations considered to be efficient (Table 2). These were followed by fiberglass vessels with an average score of 1.12 and 10 efficient observations, and wood vessels with an average score of 1.29 and seven efficient observations.

Results were then examined to see if the signs on the estimated coefficients were consistent with prior expectations. In the context of a distance function, a positive beta coefficient means that as the input increases, the vessel in question is further from the sale price (v) frontier. Alternatively, holding sale price (v) constant, an increase in inputs means the observation becomes inefficient; it is no longer on the efficient frontier. A priori, the beta coefficients are expected to be positive because graphical analysis showed a generally increasing price given greater input use (figure 2). For the steel hulled vessels, all beta coefficient were positive, confirming expectations. However, the beta coefficient was negative for horsepower and length on fiberglass vessels, indicating that as horsepower and length increase the observation is closer to the efficient frontier. This was unexpected, but given the large beta value for the transformed age variable, with a constraint on the sum of the betas being equal to one (equation 2), means that the values for beta for some of the other inputs need to be less than zero for the constraint to hold. These were the only two unexpected parameter estimates from all three model runs. Consistent with expectations, both the transformed age and number of limited access permits were positive for all vessel types. Recall that the transformed age meant that higher values were associated with newer vessels. More limited access permits are also consistent with a higher sales price, since limited access permits have value since they are not issued to a vessel without the vessel having catch history in the permit category.

The mean shadow price of the inputs revealed that steel hulled vessels had higher values for their inputs than either fiberglass or wooden vessels, with the exception of length (Table 2). Both fiberglass and wood hulled vessels had a mean shadow price for length that was twice that of steel hulled vessels. Steel hulled vessels shadow prices for both age and then number of limited access permits were both substantially higher than the other two hull types. This is consistent with prior data analysis which showed that steel vessels sold at a higher price than wood or fiberglass vessels.

[Table 2. here]

In order to estimate the value of the commercial fishing fleet in the northeast U.S., mean shadow prices from the model were combined with permit data. Vessel characteristics on length, horsepower, hull type and age were obtained from 2016 permit data, along with the number of limited access commercial fishing permits held.  Mean shadow prices were multiplied by the vessel characteristic for each vessel, and then summed to obtain individual vessel values. Total value is the sum of the physical vessel capital value and the permit value. It is likely that the permit value is an underestimate because vessels with limited access permits have increased in value during recent years, particularly scallop limited access permits. There were some vessels which did not have a wood, steel or fiberglass hull (identified as ‘other’ in the permit data). These vessels were assigned wood hulled vessel shadow prices as that would be the minimal level of value expected.

There were 3,557 permitted commercial vessels in the Northeast region in 2016, comprised of 2,470 fiberglass vessels, 716 steel vessels, 262 wood vessels and 109 vessels classified as “other” (Table 3). Based on the mean shadow price of the inputs, the total value of the fleet was estimated to be $688.3 million in 2016 dollars, 95% CI [606.6, 769.7] (Table 3). The capital stock was estimated to be worth $628.1 million, 95% CI [$555.8, $700.2] and the permit value $60.1 million, 95% CI [50.8, 69.5] (Table 3). Examination of values by hull type using mean shadow prices revealed that the 716 steel vessels had a capital stock value of $286.3 million ($399,890 per vessel), a permit value of $34.3 million ($47,954 per vessel), and a total value of $320.7 million, 95% CI [272.0, 369.3] (Table 3). These were the most valuable vessels both overall and on a per vessel basis. Fiberglass vessels had a capital stock value of $309.1 million ($125,130 per vessel) and a total permit value of $24.5 million ($9,916 per vessel) for a total value of $333.6 million 95% CI [309.8, 357.1]. Although the lower CI for the fiberglass vessels is greater than the lower CI for steel vessels, the fiberglass fleet was three times as large as the steel hulled fleet. Wood vessels had a capital stock value of $26.0 million ($99,079 per vessel) and a total permit value of $0.9 million ($3,492 per vessel) for a total value of $26.8 million, 95% CI [19.5, 34.2]. The vessels which were classified as “other” were the smallest group, and had a total capital stock value of $6.8 million ($62,099 per vessel) and a permit value of $0.44 million ($4,040 per vessel) for a total value of $7.2 million, 95% CI [5.3, 9.1].

[Table 3 here]

### 3.1 The Lowe Capital Quantity Index

Capital and permit value estimates shown above give a snapshot estimate of the asset value of the fleet in 2016. These can form a baseline value estimate which can be further refined as more data becomes available. A further advantage of the model is that the same shadow values used to calculate total capital value can also be used to examine how capital levels have fluctuated in a fishery through time.  A stated in the methods section, the Lowe quantity index is particularly attractive because it is a fixed weight index where the prices used as weights can be from any time period. This means that the shadow prices which reflect the years 2000-2013 can be used in prior years.

In order to demonstrate how the quantity index will work, mean shadow prices along with the shadow prices at the 5% and 95% CI for the vessel inputs were used and applied to vessels which landed squid, mackerel and butterfish in the northeast region between 1996 and 2016 using  otter trawl gear. The focus here is on the active vessels, rather than permitted vessels as permit categories and requirements were not constant over the full 20 year time period. The quantity index uses the mean capital value in 2009 as the base time period, meaning all changes are in relation to the value in 2009, which was $78.8 million ($2009). A value greater than one indicates that the capital value is greater than 2009, and a value less than one shows the capital value has declined compared to 2009. An upper and lower bound index was also constructed using the 95% CI values for the shadow prices relative to the 2009 mean value. In other words, the index number constructed for the lower (upper) bound used the lower (upper) shadow price value divided by the 2009 mean capital value.  Finally, a vessel count index was constructed based on active vessel counts between 1996 and 2015 with 2009 as the base year. This was to compare trends between the two types of indices.

Overall, both the capital value and vessel indices trended downward for the 20 year time period (Figure 3). Because the mean capital value index and the vessel index trend were nearly identical, only the lower and upper confidence intervals for the capital value index are shown in figure 3. In 2015, the vessel index was between 0.76 and 1.02, which was a decline of nearly half from the 1996 index which was between 1.47 and 2.02. . The vessel index had declined from 1.71 to 0.91, which was also nearly a 50% drop. Regardless of the magnitude of the index in 2015, trends for all indices during the study period were down, indicating disinvestment from the SMB fishery.

[Figure 3. here]

## 4.0 Summary and Conclusions

Measuring the capital value of fishing vessels and fleets is important to track investment and disinvestment in fisheries, and to provide information on capital consumption in economic models. Unfortunately, there has been little attention paid to collecting information on capital value of fishing fleets in the northeastern United States. This is regrettable because regular measurement of capital values can provide better information on investment in a fishery than simple vessel counts. It also provides a basis for calculating capital user cost in a given year, which is needed to determine economic profitability.

This work presented a method for estimating capital value using an input distance function. Results from the model were used to construct a benchmark capital value for commercial fishing vessels in the northeastern United States. The methodology depends on collecting information about vessel sale price and attributes from publicly available data sources, and combining those data with federal permit information on the number of permits and vessel age. It is recognized that advertised sale price may differ from actual sale price. The listed sale price reflects the owner's perceived value of their vessel, which may not reflect what a buyer is willing to pay for the vessel and permits. With no publicly available data source showing actual sale price, asking price was assumed to be a reasonable measure of vessel value.

Shadow prices for vessel characteristics were then used to construct a Lowe capital quantity index for vessels operating in the squid, mackerel and butterfish fishery over a 20 year time period. Since the number of observations per year for each hull type was small, the Lowe quantity index was a good choice to use because it is a fixed weight index. This means that an average shadow price for each vessel characteristic from any time period can be used to weight the vessel characteristics. Whether the shadow prices are based on actual versus listed sale price does not matter, since the trend should be the same no matter which set of prices were used. This is a real advantage of the Lowe index, particularly when the number of observations per year don't allow yearly estimation of the input distance function model.

A comparison of a Lowe quantity index of vessel capital for vessels fishing with trawl gear in the northeast squid, mackerel and butterfish fishery during the period 1996-2015, with a simple index of vessel numbers showed a large decline in capital since 2009, signaling disinvestment from the fishery. The vessel index and mean vessel capital index were nearly identical and is suggestive of no new vessels entering the fishery. If newer, larger vessels were replacing older vessels the capital stock index would likely be higher. For this specific fishery, a simple vessel count showed the same trend as the capital value trend and could be used if vessel value data were not available.

More attention needs to be paid to estimation of capital values in commercial fisheries both as an indicator of investment, and as data that can be used in economic analysis. A better understanding of capital value will give a clearer picture on the economic health of commercial fishing fleets operating in U.S. fisheries. Capital value information is needed to completely understand the costs of various management alternatives, particularly capital user cost. The distance function approach shown here can give a "snapshot" estimate of capital value, and a partial view of how capital value has changed over time, but by itself does not provide a picture of how investment or disinvestment will evolve in the future. Additionally, as fishing vessels become part of multi-vessel fleets, or become more integrated with the processing sector, a more complete picture of capital would include shore side infrastructure. Regular surveys which periodically gather information on capital investment would be a positive development for constructing complete capital accounts for firms operating in marine fisheries.

Appendix 1.

In this section[2] the model used for pricing characteristics of a fishing vessel is presented. From mathematics, it is known the gradient vector of a function belongs to the dual space of its variables. In economics Shephard's Lemma is a classic example, which states that the gradient vector of a cost function with respect to input prices are the associated input quantities. In this work Shephard's dual Lemma is used, which states that the gradient vector of the input distance function (i.e., the dual of the cost function) with respect to input quantities yields the associated input prices.

More formally, let the value of a vessel be given by $v \geq 0$ and let $z \in \Re_{+}^{N}$ be a vector of characteristics of the vessel and its value. Stated in a production framework:

$$L(v) = \{z : z \text{ generates value } v\}.$$

Let $p \in \Re_{+}^{N}$ denote the unknown prices of the vessel characteristics, which are solved for and whose characteristics determine the value of the vessel:

$$v = pz.$$

$L(v)$ is the set of all inputs required to produce value $v$, and it can be given a specific function representation as a Shephard input distance function, i.e.,

$$D_i(v, z) = \sup\{\theta : z / \theta \in L(v)\}.$$

Under some mild assumptions the distance function[3] is a complete representation of the set $L(v)$, i.e.,

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[2] This section draws on earlier work by R Färe, S. Grosskopf, C.K. Lovell, S. Yaisawarng. "Derivation of Shadow Prices for Undesirable Outputs: a Distance Function Approach, *The Review of Economics and Statistics* !1993) 374-380, and more closely on R. Färe, S. Grosskopf, C. Shang, and R.Sickles. "Pricing Characteristics: an Application of Shephard's Dual Lemma." . Accessed August 1, 2016.

[3] See [12] R. Färe, D. Primont, Multi-Output Production and Duality: Theory and Applications, Boston: Kluwer Academic Publishers1995.. We note that this distance function is homogenous of degree +1 in z.

$$D_i(v,z) \geq 1 \text{ if and only if } z \in L(v).$$

The distance function takes a value of one when the observation is on the best practice frontier, i.e., technically efficient, and values greater than one if the observation is inefficient.

The unknown shadow prices can be calculated from the observed data on $z$ and $v$ using Shephard's dual lemma

$$p = \frac{v \nabla_z D_i\,(v,z)}{D_i(v,z)}$$

where $\nabla_z D_i(v,z)$ is the gradient vector of the distance function with respect to the characteristics.

To verify the relationship between the shadow prices $p$ and the expression above, the cost function, which is dual to the input distance function, is introduced:

$$C(v,p) = \min_z \{pz : D_i(v,z) \geq 1\}.$$

Rewriting this in Lagrangian form yields:

$$C(v,p) = \min_z \; pz - \lambda(D_i(v,z) - 1)$$

where $\lambda$ is the Lagrangian multiplier. The first order conditions associated with this problem are:

$$p - \lambda \nabla_z D_i(v,z) = 0.$$

To interpret $\lambda$, consider the following problem:

(i) $\bar{C}(v,p,\alpha) = \min_z \; pz - \lambda(D_i(v,z) - \alpha)$

(ii) $= \min_z \; pz - \lambda(\alpha D_i(v, z/\alpha) - 1)$ (by homogeneity)

(iii) $= \alpha(\min_{\hat{z}} \; p\hat{z} - \lambda(D_i(v,\hat{z}) - 1))$

where $\hat{z} = z / \alpha$ and the last expression follows from the homogeneity of $D_i(v, z)$ in z. From this last expression it follows from (i) that

$$\partial \bar{C} / \partial \alpha = \lambda,$$

and from (iii) it follows that:

$$\partial \bar{C} / \partial \alpha = C(v, p),$$

thus

$$\lambda = C(v, p),$$

i.e., the optimal Lagrangian $\lambda$ equals the value function $C(v, p)$ [13, 14]. Using this to substitute for $\lambda$ in the first order conditions results in the following shadow pricing rule:

$$\text{(iv)} \qquad p = C(v, p)\nabla_z D_i(\text{v,z})$$

In order to make this operational, p needs to be derived from an observable expression for $C(v, p)$ which itself contains the desired input prices. If both sides of the above expression are multiplied by $z$, and Euler's theorem is applied, the result is:

$$v = pz = C(v, p) D_i(v, z)$$

which can be rearranged as $v / D_i(v, z) = C(v, p)$. Substituting for $C(v, p)$ into (iv) yields:

$$p = \frac{v \nabla_z D_i\,(v, z)}{D_i(v, z)}$$

This can be used to estimate the shadow prices using only data on $v$ and $z$ which are both observed.

The next step is to parameterize the input distance function, and this is done following the approach used by Färe and Sung [15].They showed that a homogeneous generalized quadratic function can take either a translog or mean of order $\rho$ specific functional form. Let

$$\rho^{-1}(F(x_1, x_2)) = \alpha_o + \alpha_1 f(x_1) + \alpha_2 f(x_2) + b_1 f(x_1) f(x_1) + b_2 f(x_2) f(x_2) + b_3 f(x_1) f(x_2)$$

be a generalized quadratic function. If it is homogeneous of degree +1 in its characteristics like the input distance function, then $F(x_1, x_2)$ above must be either of translog or mean of order $\rho$ functional form. Since the latter only has second order terms, the translog is chosen because it contains both first and second order terms and is therefore more flexible. For a vessel where there are three attributes (z=1,2, 3) and one value term (v), the specific form which is estimated is:

$$\ln D_i(v,z) = \alpha_o + \alpha_1 \ln v + 1/2\alpha_{11}(\ln v)^2 + \sum_{n=1}^{3} \beta_n \ln z_n$$

$$+1/2\sum_{n=1}^{3}\sum_{n'=1}^{3} \beta_{nn'}(\ln z_n)(\ln z_{n'}) + \sum_{n=1}^{3} \gamma_n (\ln z_n)(\ln v)$$

with the parameter constraints:

$$\beta_1 + \beta_2 + \beta_3 = 1$$

$$\sum_{n'=1}^{3} \beta_{nn'} = \sum_{n=1}^{3} \gamma_n = 0$$

$$\beta_{nn'} = \beta_{n'n}, n = 1,2,3; n' = 1,2,3$$

.

Here, v is the value of the asset, $z_n$ is a characteristic of the asset, such as vessel length, and there are *n* number of separate characteristics. The alpha, beta and gamma terms are parameters which are estimated. The first two constraints impose homogeneity of degree +1 in inputs, the third set imposes symmetry. The shadow price for say $z_1$ is then estimated as:

$$p_1 = \frac{v}{z_1}(\beta_1 + \beta_{11} \ln z_1 + \beta_{12} \ln z_2 + \beta_{13} \ln z_3 + \gamma_1 \ln v),$$

and similarly for characteristics $z_2$ and $z_3$.

Empirically, the general empirical approach taken in Färe, Grosskopf, Lovell and Yaisawarng [9] is adopted, which employed a non-parametric programming model developed by Aigner and Chu [10], and allows estimation of the input distance function as a (deterministic) non-parametric frontier function. Letting $k = 1, \ldots, K$ index observations, the following problem is solved:

$$min \sum_{k=1}^{K} [\ln D_i(v^k, z^k) - \ln 1]$$

subject to:

$$(i) \ln D_i(v^k, z^k) \geq 0, k = 1, \ldots, K$$

$$\frac{\partial \ln D_i(v^k, z^k)}{\partial \ln z_n^k} \geq 0, n = 1, \ldots, N, k = 1, \ldots, K$$

as well as the homogeneity and symmetry constraints summarized above.

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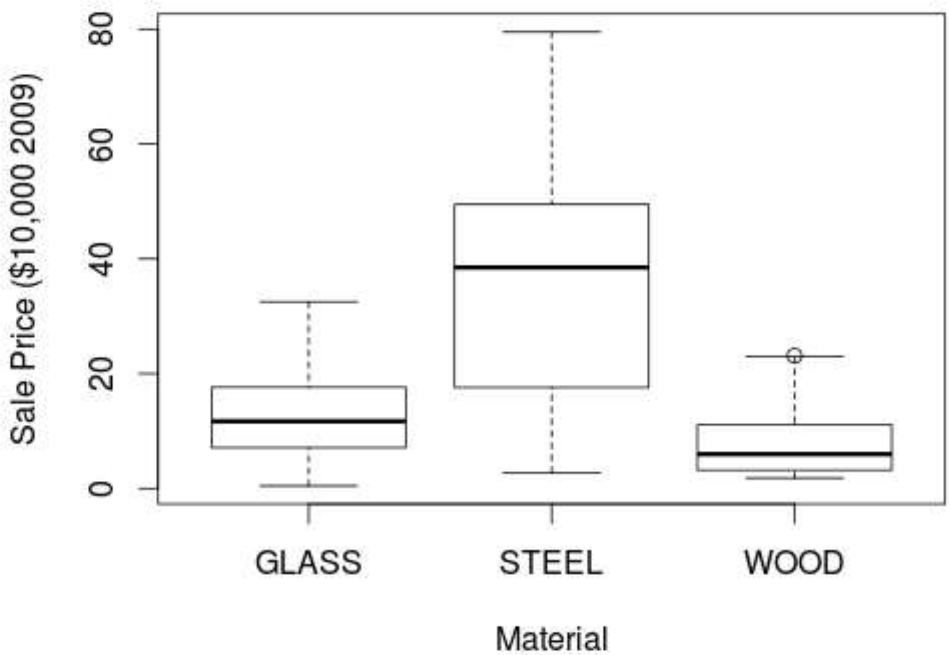
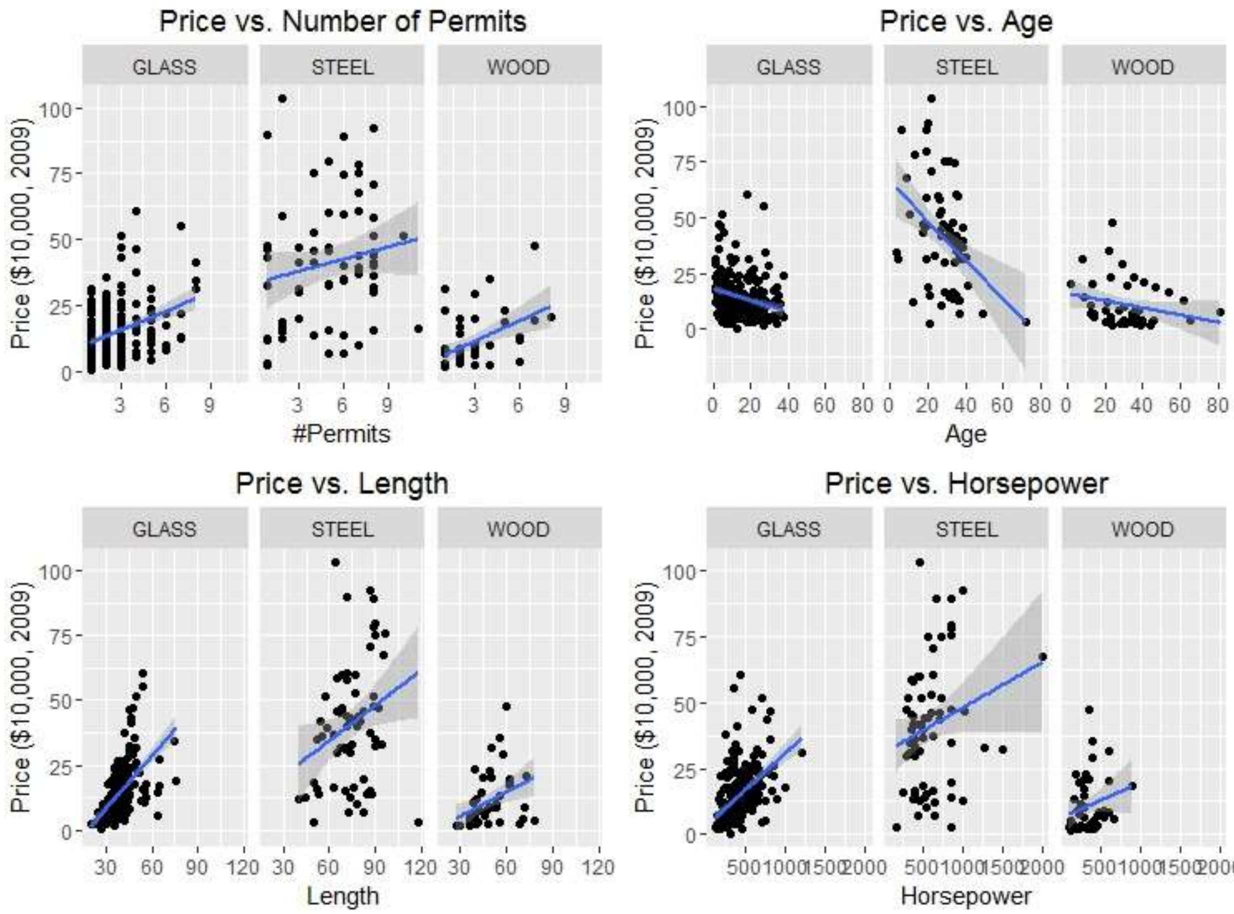
Figure 1. Plot of sale price by hull material for vessels in sample.

Figure 2. Vessel price trends plotted against number of permits, age, length and horsepower.

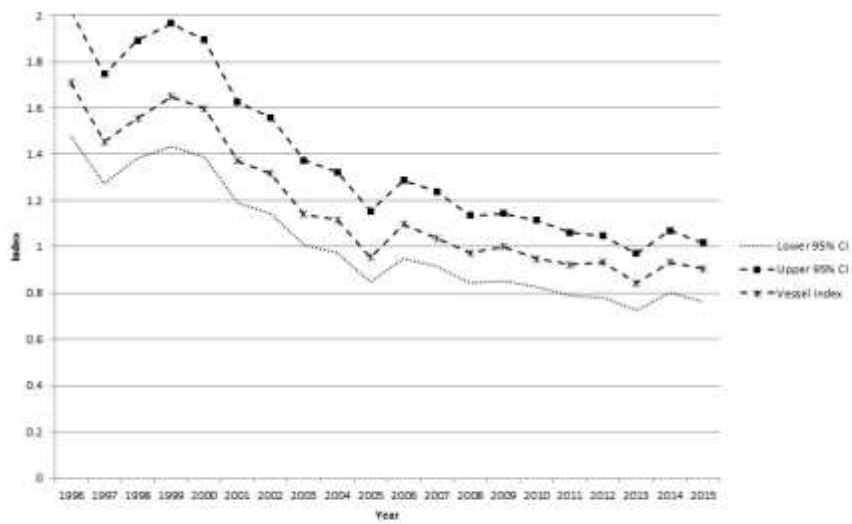
Figure 3. 95% CI for Vessel Capital index (2009=1) and vessel quantity index (2009=1) 1996-2015



Sale Price ($10,000 2009)
0
20
40
60
80
GLASS
STEEL
WOOD
Material



Price vs. Number of Permits
Price vs. Age
Price vs. Length
Price vs. Horsepower
GLASS
STEEL
WOOD
Price ($10,000, 2009)
#Permits
Age
Length
Horsepower



Index
Year
Lower 95% CI
Upper 95% CI
Vessel Index

Table 1. Vessel values and characteristics by hull type

| | Min | Average | Max |
|---|---|---|---|
| ***Fiberglass*** | | | |
| N=262 | | | |
| Length | 20.0 | 38.8 | 76.0 |
| Horsepower | 110.0 | 397.2 | 1200.0 |
| Age | 1 | 14.8 | 38 |
| Value | 4,497 | 141,446 | 606,463 |
| Permits | 0 | 1.4 | 7.0 |
| ***Steel*** | | | |
| N=68 | | | |
| Length | 40 | 74.6 | 118 |
| Horsepower | 160 | 593 | 2000 |
| Age | 3 | 28.6 | 72 |
| Value | 27,180 | 410,458 | 1,032,822 |
| Permits | 0 | 4.1 | 10 |
| ***Wood*** | | | |
| N=47 | | | |
| Length | 27 | 47.6 | 78 |
| Horsepower | 115 | 347 | 900 |
| Age | 2 | 31.5 | 81 |
| Value | 18,000 | 107,022 | 474,623 |
| Permits | 0 | 1.7 | 7.0 |

Table 2. Input Beta Values and shadow prices returned from the Directional Distance Function Model

| Distance Function Value | Min | Mean | Max | Number of Efficient (d.f=1) |
|---|---|---|---|---|
| Glass | 1.0 | 1.19 | 1.86 | 10 |
| Steel | 1.0 | 1.12 | 1.41 | 12 |
| Wood | 1.0 | 1.29 | 2.09 | 7 |

| Beta Values | Age2 | Horsepower | Length | Permits |
|---|---|---|---|---|
| Glass | 1.57 | -0.15 | -0.6 | 0.18 |
| Steel | 0.29 | 0.57 | 0.09 | 0.06 |
| Wood | 0.4 | 0.01 | 0.27 | 0.32 |

| | Lower 95% CI | Winsored Mean | Upper 95% CI |
|---|---|---|---|
| ***Fiberglass*** | | | |
| Age | 519.8 | 543.9 | 568.0 |
| Horsepower | 26.3 | 29.0 | 31.6 |
| Length | 1513.6 | 1650.2 | 1786.7 |
| Permits | 3648.6 | 4006 | 4364.1 |
| ***Steel*** | | | |
| Age | 3302.0 | 3845.4 | 4388.7 |
| Horsepower | 33.5 | 40.9 | 48.2 |
| Length | 781.1 | 918.3 | 1055.5 |
| Permits | 6759.2 | 8471 | 10182 |
| ***Wood*** | | | |
| Age | 77.5 | 92.7 | 107.8 |
| Horsepower | 0.4 | 0.7 | 0.9 |
| Length | 1339.1 | 1870.9 | 2402.8 |
| Permits | 1038 | 1238 | 1438.2 |

Table 3. Total Value of Vessel Capital and Permits for 2016 Northeast Fishing Fleet

Capital Value

| | Vessels # | Lower 95% CI ( $2016) | Average ( $2016) | Upper 95% CI ( $2016) |
|---|---|---|---|---|
| Glass | 2470 | 287,530,253 | 309,070,027 | 330,459,389 |
| Steel | 716 | 244,558,044 | 286,321,542 | 328,031,229 |
| Wood | 262 | 18,773,676 | 25,958,595 | 33,130,514 |
| Other | 109 | 4,944,458 | 6,768,774 | 8,588,964 |
| Totals | 3557 | 555,806,432 | 628,118,939 | 700,210,097 |

Permit Value

| | | Lower 95% CI ( $2016) | Average ( $2016) | Upper 95% CI ( $2016) |
|---|---|---|---|---|
| Glass | 2470 | 22,304,975 | 24,491,701 | 26,679,039 |
| Steel | 716 | 27,398,022 | 34,335,082 | 41,272,142 |
| Wood | 262 | 766,975 | 914,829 | 1,062,682 |
| Other | 109 | 369,157 | 440,321 | 511,485 |
| Totals | 3557 | 50,839,129 | 60,181,933 | 69,525,348 |

Total Value

| | | Lower 95% CI ( $2016) | Average ( $2016) | Upper 95% CI ( $2016) |
|---|---|---|---|---|
| Glass | 2470 | 309,835,228 | 333,561,728 | 357,138,427 |
| Steel | 716 | 271,956,067 | 320,656,624 | 369,303,371 |
| Wood | 262 | 19,540,652 | 26,873,424 | 34,193,196 |
| Other | 109 | 5,313,615 | 7,209,096 | 9,100,450 |
| Totals | 3557 | 606,645,561 | 688,300,872 | 769,735,444 |