AN EXPERIMENTAL STUDY OF DISPERSIVE SPREADING

Zimmermann and Miksad

Report No. 46 January 1978

ATMOSPHERIC
SCIENCE GROUP

The University of Texas
College of Engineering
Austin, Texas 78712

ATMOSPHERIC SCIENCE GROUP
College of Engineering
The University of Texas
Austin, Texas

Report No. 46 January 1978

AN EXPERIMENTAL STUDY OF DISPERSIVE SPREADING

by

K. A. Zimmermann

and

R. W. Miksad

Prepared for the

Techniques Development Laboratory
National Weather Service
National Oceanic and Atmospheric Administration
U. S. Department of Commerce

ACKNOWLEDGMENTS

This research and publication has been financed in part with Federal funds from the National Oceanic and Atmospheric Administration under contract No. T-35429. The contents do not necessarily reflect the views and policies of the National Oceanic and Atmospheric Administration, nor does mention of trade names or commercial products constitute endorsement or recommendation for use.

The help of the Bureau of Engineering Research of the University of Texas at Austin is also acknowledged.

ABSTRACT

A laboratory apparatus is used to study the problem of the relative dispersion of a cluster of oil patches on a turbulent ocean surface. The apparatus, an oscillating grid water tank, is tested to determine whether a horizontally homogeneous surface turbulent field is generated. Dispersion experiments are conducted in which the movements of the oil patches are simulated with discrete passive tracers.

The data from the laboratory dispersion experiments fit closely with actual oceanic dispersion measurements. It is concluded that the apparatus generates a simplified small scale version of the ocean's turbulent surface layer. It is shown that the relative diffusion theory can be used to predict the dispersion of oil patches relative to their collective center of mass. The variance of these patches about their mutual center of gravity (and hence, the area of the field of patches) varies with time, roughly as t^3.

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

The problem of water pollution from oil spills has grown steadily with the increase in the importation of overseas oil and with the further development of off-shore oil production in the United States. Lax safety standards and poorly trained oil tanker crews resulted in a record number of mishaps in 1976. In a two week period at the end of 1976, five spills occurred in the New England area. The worst accident took place off Nantucket Island, Massachusetts, when the Argo Merchant went aground, breaking up and spilling its 28,000 metric ton cargo of number 6 fuel oil. After several weeks, the area of the slick was approximately equal to the land area of the state of Massachusetts (21,000 km^2). Although stricter safety standards have been adopted by many countries to prevent accidents, it is impossible to reduce the probability of an oil spill mishap to zero. For this reason, we must develop methods for predicting the behavior of spilled oil on the ocean. This paper deals with an experimental investigation of dispersive spreading of oil spills.

There is little data available from actual oil spills since most accidents occur under the worst weather conditions and at the most inaccessible locations. Furthermore, environmentalists would be unfavorably impressed by an in situ experiment with controlled releases of oil even though useful information would be gained. It is necessary then, to rely on relevant theoretical models and laboratory experiments to understand oceanic oil spreading.

Theoretical and experimental work by Fay (1969) and Hoult (1972) on oil spreading in calm seas indicates that immediately after release, a continuous oil slick progresses through three sequential stages of "dynamic spreading", i.e., spreading due to the properties of the oil and the volume of the spill. The various stages of spreading in the Fay-Hoult model may be classified according to the dominant force balance involved:

Stage 1. (Gravity-Inertia) An outward directed horizontal pressure gradient force due to gravity is opposed by the inertia of the oil, causing the size of the slick to vary with time as:

$$\ell \cong \left(\frac{\Delta\rho}{\rho} g \forall\right)^{1/4} t^{1/2} \qquad A \sim t$$

Stage 2. (Gravity-Viscous) The horizontal force due to gravity is retarded by the viscous drag from the no-slip boundary layer at the oil-water interface, giving:

$$\ell \cong \left(\frac{\Delta\rho}{\rho} g \frac{\forall^2}{\nu^{1/2}}\right)^{1/6} t^{1/4} \qquad A \sim t^{1/2}$$

Stage 3. (Surface tension-viscous) Finally, the net outward surface tension force is opposed by the viscous drag, giving:

$$\ell \cong \left(\frac{\sigma^2}{\rho^2 \nu}\right)^{1/4} t^{3/4} \qquad A \sim t^{3/2}$$

where ℓ = radius of oil pool
$\Delta\rho = \rho_{H_2O} - \rho_{oil}$
ρ = density of water
t = time
g = acceleration due to gravity
$\forall$ = volume of slick
ν = kinematic viscosity of water
A = slick area
σ = net surface tension

Immediately after a spill, oil spreads across the water due to a horizontal pressure gradient force (i.e., Stage 1). The oil floats in the water similar to an iceberg; roughly 90 percent below and 10 percent above the water surface. The difference in pressure in the slick versus the pressure in the water around it results in a slight outward directed force. During this stage, the slick area increases proportional to t.

The time of transition from the gravity-inertia stage to the gravity-viscous stage depends on the volume of the spill. For the case of a large spill similar to the volume of the Torrey Canyon accident, Fay has calculated the transition to occur approximately one hour after the spill. (See Figure 1-1.) As the slick continues to spread and the thickness of the oil layer becomes small, the spreading becomes dominated by the effects of the surface tension forces. This may occur over a week after a large volume spill takes place.

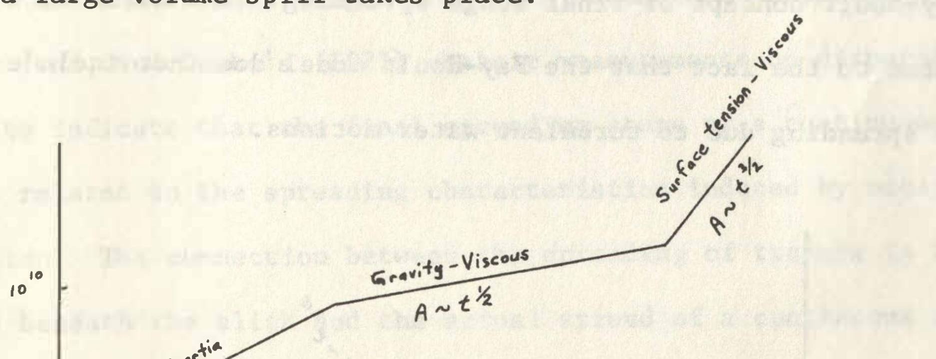


Figure 1-1. Slick Area Versus Time (Fay 1969).

In this last stage of the Fay-Hoult model, the slick area grows proportional to $t^{3/2}$. Because most oil spills are composed of a mixture of oil, the properties of the oil will change as some components evaporate into the air and others dissolve into the water. Fay assumes the final slick size will be reached when the net surface tension force acting on the slick becomes zero.

A comparison of the Fay-Hoult model with the limited amount of data on actual spills shows that their model not only underpredicts the final size of the slick, but also the long time spreading rates. Drapeau's (1974) data from a controlled oil release experiment in the St. Lawrence Estuary, for example, indicates that during the final stages of spreading, the area of their continuous slick increased with time as t^n where $2.65 < n < 3.15$. (See Figure 1-2.) This does not agree with the $t^{3/2}$ Fay-Hoult concept of final stage spreading. The difference may be attributed to the fact that the Fay-Hoult model does not include dispersive spreading due to turbulent water motions.

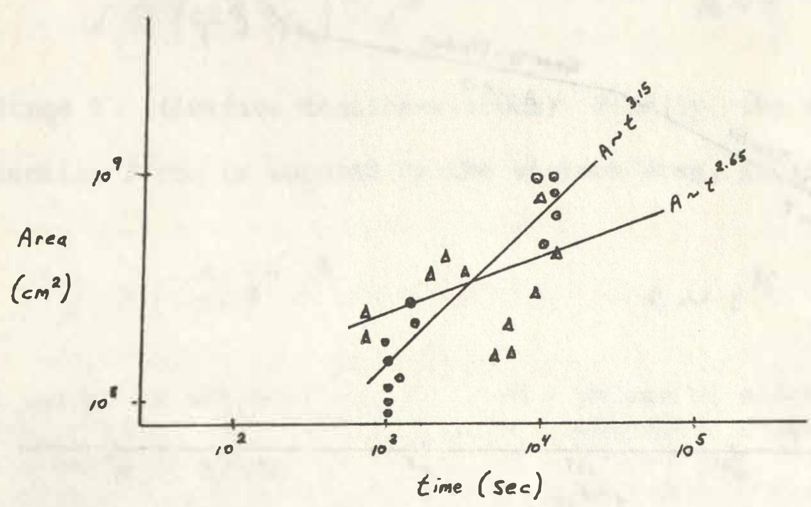


Figure 1-2. Results of Drapeau's (1974) Experiments.

"Dispersive spreading" is due to the action of random turbulent stresses acting on the surface of a slick. It is a weak phenomena which seems to be important only after dominant dynamic spreading forces have ceased to act on the slick. As we will discuss further, this final stage of slick dispersion seems to have some relation to the relative diffusion of discrete tracers in the turbulent ocean surface layer. For example, during the diffusion of a cluster of particles relative to their center of gravity, Batchelor (1952) has shown that the variance, s^2, of the distribution of particles in the cluster (which is a measure of area covered) increases with t^3 in a homogeneous turbulent field when the size of the cluster falls in the so-called turbulent inertial sub-range. Oceanic turbulence, Csanady (1973) states, may be considered horizontally homogeneous to a close approximation. The spreading rates measured by Drapeau are in reasonable agreement with Batchelor's relative diffusion predictions, and Okubo's (1971) oceanic measurements on dispersion. This seems to indicate that the final spreading stage of a continuous oil slick may be related to the spreading characteristics induced by oceanic relative diffusion. The connection between the spreading of tracers in the water column beneath the slick and the actual spread of a continuous slick floating on that surface has not been directly established yet. The physical link between slick spreading and water column dispersion must be due to the randomly directed shear stresses that turbulence within the water column exerts on the under-surface of the slick. This dispersive spreading is not accounted for in the Fay-Hoult model, and is in general an exceedingly difficult problem to study, either analytically, or experimentally.

We are concerned here with the problem of relative dispersion of oil patches (i.e., dispersion of the centroid of each patch relative to their mutual center of mass). Observations of oil slicks some time after a spill has occurred show that a slick will often break into various size patches or oil pancakes. Smith (1970) reported that this occurred after the Torrey Canyon accident. The individual area of these patches may remain relatively constant with time, but the distance between patches increases with time. The dominant dispersive mechanism for these stable size oil patches relative to one another must be oceanic and atmospheric turbulence. It is possible to simulate the movement of the centroids of these patches by using discrete passive tracers. By carefully choosing discrete tracers with a density similar to some crude oils, one may study some aspects of the turbulent dispersion of oil patches without using oil.

This paper will consider the application of a relative dispersion argument for the description of the spreading of a collection of oil patches and will present the results of experiments performed in a laboratory apparatus. Based on the techniques developed in this research project, the problem of dispersion within each continuous patch, relative to its own center of mass, is currently being investigated at The University of Texas at Austin.

CHAPTER 2

ABSOLUTE AND RELATIVE DIFFUSION

Turbulence is a complicated but fundamental mechanism in nature for diffusing or dispersing some transportable property of a fluid. The medium of turbulent diffusion is continuous in the sense that moving fluid parcels displace other parcels which in turn must displace others. This is quite different from molecular diffusion where discrete particles make random independent movements. This chapter briefly reviews absolute and relative diffusion theory and discusses the applicability of Richardson's relative diffusion theory to the description of the dispersion of oil patches on a water surface.

2.1 Absolute Diffusion

The turbulent diffusion of a cloud of marked particles may be studied in terms of the time-average concentration of particles at a fixed point in a stationary coordinate system, or in terms of the concentration at a specific point in a cloud measured relative to a frame of reference moving with the centroid of the cloud. The first method is referred to as "absolute" diffusion while the second is referred to as "relative" diffusion. Absolute diffusion can be described in terms of the motion of a single diffusing particle in a stationary, homogeneous turbulent field. By stationary, we mean that the 'characteristic velocity', $(\overline{v'^2})^{1/2}$, of an "eddy" or turbulent movement is reasonably invariant with time. In a homogeneous turbulent field, the characteristic velocity is reasonably uniform at all points in the field.

This one-particle Lagrangian description of diffusion was first formulated by Taylor (1922) in his famous paper "Diffusion by Continuous Movements". To illustrate Taylor's idea, consider an experiment in which a single marked particle is released from a point and followed over time t. (See Figure 2-1.)

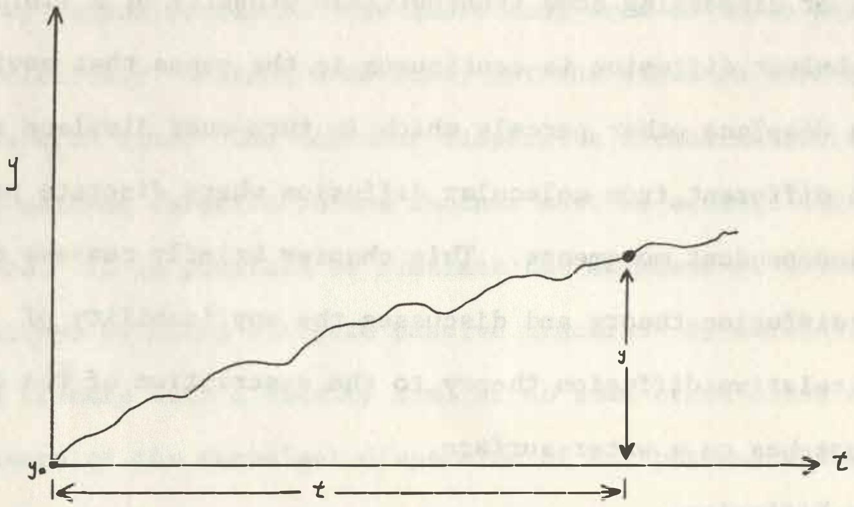


Figure 2-1. Motion of a Single Particle over Time t (from Gifford, 1970).

If the experiment is repeated many times, the mean square distance, $\overline{y^2}(t)$, may be found. This may be considered the statistical variance of the diffusion of a group of particles released at y_o at t_o. According to Gifford's (1970) derivation, Taylor's equation is:

$$\overline{y^2}(t) = 2\,\overline{v'^2}\int_0^t\int_0^{t_1} R(\tau)\,d\tau\,dt_1 \qquad (2.1)$$

The term, $R(\tau)$, is a one-point Lagrangian correlation coefficient. In its normalized form, it may be written as:

$$R(\tau) = \frac{\overline{v'(t)\,v'(t+\tau)}}{\overline{v'^2}} \qquad (2.2)$$

This is a measure of the correlation between the initial velocity of the particle and the velocity of the particle at time $t + \tau$. In a way, it is a measure of the ability of the particle to "remember" or retain its original movement characteristics. This is the fundamental difference between turbulent diffusion and molecular (or Fickian) diffusion. In Fickian diffusion, each discrete particle movement has a zero correlation (memory).

The autocorrelation coefficient becomes zero when τ is large enough for the particle to "forget" its initial movement and position. The limit of $R(\tau)$ rapidly approaches zero as τ becomes very large. So, the limit, according to Gifford (1970), is:

$$\overline{v'^2} \lim_{t \to \infty} \int_0^t R(t_1)\, dt_1 = K_1 \qquad (2.3)$$

and equation 2.1 becomes:

$$\overline{y^2(t)} \approx 2 K_1 t \qquad (2.4)$$

The constant, K, has the same dimensions as a diffusivity in Fick's law:

$$\frac{d\chi}{dt} = K_1 \frac{\partial^2 \chi}{\partial x^2} \qquad (2.5)$$

For large time intervals τ, Taylor's absolute diffusion theory approaches Fickian diffusion.

When $\tau = 0$, the autocorrelation coefficient must be equal to one. If τ is sufficiently small, then $R(\tau) \approx 1$. In this case, equation 2.1 gives:

$$\overline{y^2(t)} \approx \overline{v'^2}\, t^2 \qquad (2.6)$$

Here, the variance increases with t^2. This is a faster spreading rate than that predicted by Fickian diffusion where the variance increases with t. So, as we observe many times in nature, turbulent motions are more efficient than molecular motions for mixing a transportable fluid property.

2.2 Relative Diffusion

Taylor's absolute diffusion expression (Equation 2.1) is based on the averaging of an ensemble of experiments where the motion of a single particle is followed. The motions of other particles in the field are, by definition, independent. But if a group of particles, say a small cloud puff, start out very close to each other, then there will be a correlation between their movements, (i.e., a velocity fluctuation will move some or all of the particles in the cloud together). It was necessary then, for Richardson (1926) to present a different method to predict the dispersion of a cluster of particles relative to their mutual center of gravity.

The simplest model of a dispersing cloud of particles is the two-particle relative diffusion experiment shown in Figure 2-2.

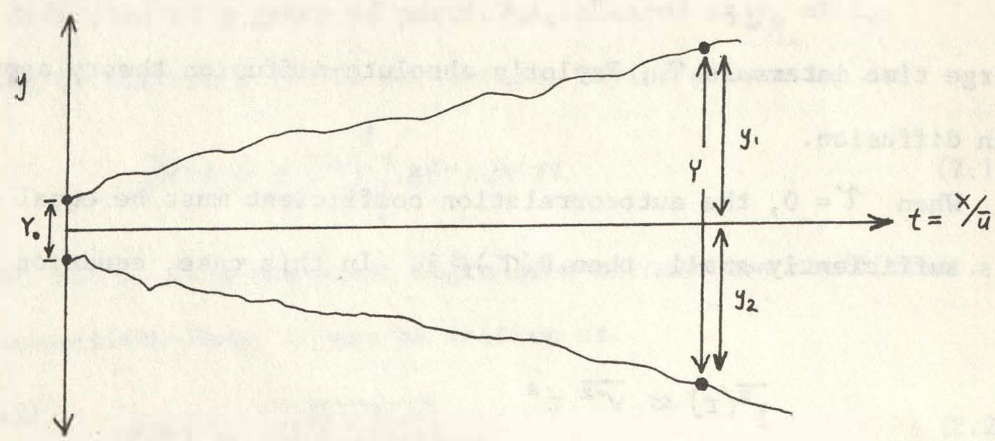


Figure 2-2. Motion of Two Particles over Time t (from Gifford, 1970).

Two particles are released at t_o with an initial separation of y_o. After time t, the separation of the two particles is $Y = y_1 - y_2$. Writing this in terms of the velocity fluctuations affecting each particle, we get (from Gifford, 1970):

$$Y = Y_o + \int_0^t v_1'(t)\,dt - \int_0^t v_2'(t)\,dt \tag{2.7}$$

The expression for the mean square separation of two particles may be derived in the same way as Taylor's one-particle equation. To make clear the distinction between relative and absolute diffusion, the variance for relative diffusion will be called s^2.

$$s^2 = Y_o^2 + 2\overline{v'^2}\int_0^t\int_0^t R(t_2 - t_1)\,dt_1\,dt_2 - 2\int_0^t\int_0^t \overline{v_1'(t_1)\,v_2'(t_2)}\,dt_1\,dt_2 \tag{2.8}$$

The dispersion of two particles involves the initial separation, y_o, the single particle Lagrangian time correlation, $R(\tau)$, and the two-particle Lagrangian correlation term, $\overline{v_1'(t_1)\,v_2'(t_2)}$.

According to this expression, two particles that initially have no separation will never move apart due to turbulence. Instead, the particles spread only by molecular movements. This important result led Richardson to make other qualitative deductions from the relative diffusion argument. Eddies, or turbulent movements, which are large compared to the particle separation distance tend to move the two particles along with little or no change in s^2. Very small eddies, much smaller than the distance between dispersing particles, will only oscillate each particle independently and not effectively increase s^2. The mean square separation distance is most effectively increased by eddies comparable in size to the distance between particles.

The phases of growth of a particle cluster undergoing relative diffusion have been summarized by Csanady (1972). As stated earlier, when the particle separation is less than the size of the smallest eddies, only molecular diffusion occurs and the variance increases in proportion to t.

$$\underline{s^2 \sim t}$$

When the distance between particles is comparable to the size of the eddies which fall in the so-called inertial subrange of turbulence, Batchelor (1950, 1952), showed the rate of relative diffusion, $\frac{ds^2}{dt}$, depends on the initial separation, the diffusing time, and the turbulent energy dissipation rate ε. As a result:

$$\overline{v'^2} \sim s^{2/3}\,\varepsilon^{2/3} \quad ; \quad t \sim s^{2/3}\,\varepsilon^{-1/3} \tag{2.9}$$

and so, $\underline{s^2 \sim t^3}$

The time rate of change of the variance, or the eddy diffusivity, in the inertial subrange is found by Batchelor (1952) to be:

$$\frac{1}{2}\,\frac{ds^2}{dt} \cong s^{4/3}\,\varepsilon^{1/3} \tag{2.10}$$

The eddy diffusivity increases with the 4/3 power of cloud size. This is exactly the same constant of proportionality that was found empirically by Richardson in 1926.

As the cloud continues to grow, all eddy sizes affect the spreading particles. Logically, the difference between 'relative' and 'absolute' diffusion disappears and we find that the variance only increases proportional to t.

$$\underline{(y^2 = s^2) \sim t}$$

2.3 Meandering

The center of gravity of a smoke plume diffusing in the air, or an oil slick dispersing in the ocean, will often visibly meander across an axis aligned with the mean flow. (See Figure 2-3.)

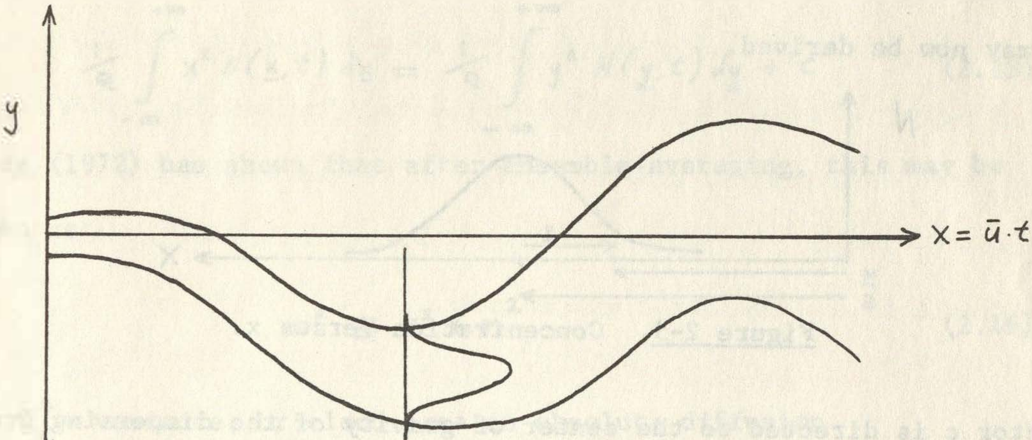


Figure 2-3. Plan View of Meandering Oil Slick.

If a time exposure photograph of the slick were made, it would, on average, show a smooth Gaussian distribution about the x-axis. (See Figure 2-4.) The concentration profile for a point x is also shown in Figure 2-4. This averaged dispersion is approximated by Taylor's absolute diffusion expression. The link between absolute and relative diffusion is the meandering term.

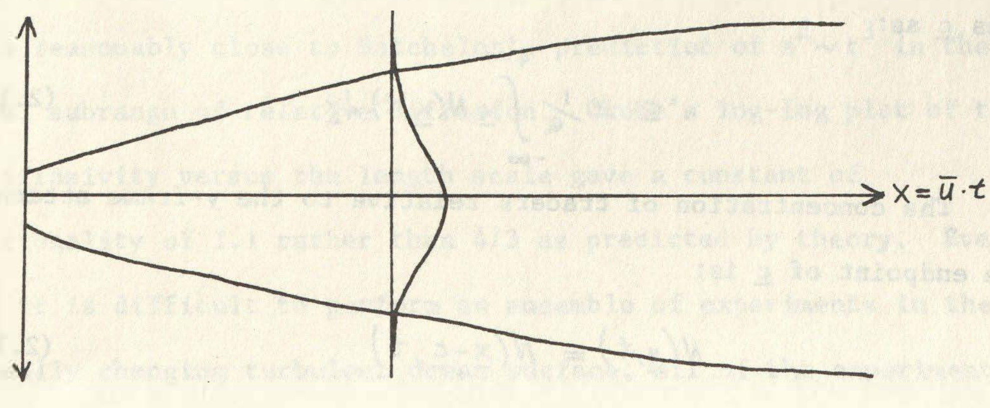


Figure 2-4. Time Averaged Oil Slick Dispersion.

Consider an experiment in which a number of discrete tracers are released at the origin of a fixed axis system. Let the concentration of tracers at some point x at time t be $N(\underline{x},t)$, where $\underline{x}$ is a position vector. (See Figure 2-5.) Following Csanady (1973), the meandering term may now be derived.

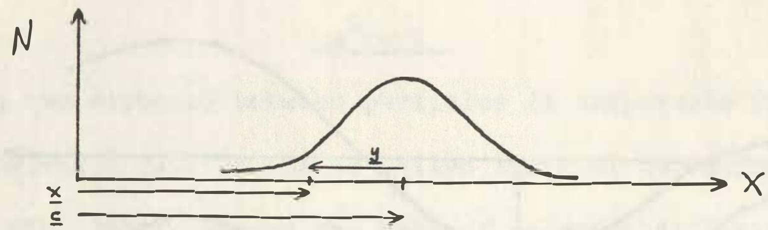


Figure 2-5. Concentration Versus x.

A vector $\underline{c}$ is directed to the center of gravity of the dispersing group of particles, while $\underline{y}$ is a relative position vector defined as:

$$\underline{y} = \underline{x} - \underline{c} \tag{2.11}$$

The total mass of tracers is found by integrating the concentration distribution over all x vectors:

$$Q = \int_{-\infty}^{+\infty} N(\underline{x},t)\, d\underline{x} \tag{2.12}$$

The centroid, given by the first moment of the distribution, defines $\underline{c}$ as:

$$\underline{c} = \frac{1}{Q} \int_{-\infty}^{+\infty} \underline{x}\, N(\underline{x},t)\, d\underline{x} \tag{2.13}$$

The concentration of tracers relative to the y-frame attached at the endpoint of $\underline{c}$ is:

$$N(\underline{y},t) = N(\underline{x}-\underline{c},t) \tag{2.14}$$

Now, by using the parallel axis theorem, the second moment, or the variance, of the patch concentration relative to the y-frame may be linked to the variance of the patch concentration in the fixed axis.

$$\frac{1}{Q}\int_{-\infty}^{+\infty} x^2 N(\underline{x},t)\,d\underline{x} = \frac{1}{Q}\int_{-\infty}^{+\infty} y^2 N(\underline{y},t)\,d\underline{y} + c^2 \qquad (2.15)$$

Csanady (1972) has shown that after ensemble-averaging, this may be written as:

$$\sigma^2 = s^2 + m^2 \qquad (2.16)$$

where σ^2 = variance, or cloud area in absolute diffusion
s^2 = cloud area relative to a moving frame of reference
m^2 = meandering term

2.4 Previous Experimental Data on Relative Diffusion

Experiments on the diffusion of dyes and the dispersion of tracers on the ocean indicate that the theory of relative diffusion is an accurate predictor of oceanic turbulent spreading. Okubo (1971) summarized the results of many different experiments over a wide range of diffusing times and patch sizes. After plotting s^2 versus t on a log-log scale, he found the best fit straight line had a slope of 2.3. This is reasonably close to Batchelor's prediction of $s^2 \sim t^3$ in the inertial subrange of relative diffusion. Okubo's log-log plot of the eddy diffusivity versus the length scale gave a constant of proportionality of 1.1 rather than 4/3 as predicted by theory. Even though it is difficult to perform an ensemble of experiments in the continually changing turbulent ocean surface, all of the experimental results are in reasonable agreement with relative diffusion theory.

The experiments of Stommel (1949) also supported Richardson's 4/3 power diffusivity law. Stommel distributed two rows of markers on the ocean. The distance between the rows was varied over three decades of length (from 10^1 to 10^4 cm). A photograph was made just after the markers had been distributed. A second photograph was taken after a specified time interval. From the two photographs, identical marker pairs were tracked and the change in separation was measured. The scale of length for each experiment was determined by averaging the distance between measured marker pairs.

$$\ell = \frac{\overline{(\ell_1 + \ell_0)}}{2} \qquad (2.17)$$

A measure of the eddy diffusivity was found by using Richardson's original experimental method.

$$F = \frac{\overline{(\ell_1 - \ell_0)^2}}{2T} \qquad (2.18)$$

Stommel's log-log plot of F versus ℓ showed a close fit to a line with slope equal to 4/3.

Batchelor (1952) has pointed out that statistically it is more meaningful to relate the eddy diffusivity to the mean square distance between dispersing pairs rather than the distance ℓ between pairs. In other words,

$$F \sim (\overline{s^2})^{2/3} \qquad (2.19)$$

Even with this adjustment, the available experimental data for oceanic diffusion closely agrees with relative diffusion theory.

The "real-life" experiments of Stommel and Okubo are limited by the small number of test repetitions that can be made before a change occurs in the character of the ocean's turbulent surface layer. For this reason, a laboratory apparatus was designed and built by us to produce a stationary homogeneous surface layer of turbulence. With this apparatus, many repetitions of experiments were made to insure that we obtained an honest ensemble average. The following chapters discuss the experimental apparatus and the results we obtained with it.

CHAPTER 3

EXPERIMENTAL APPARATUS

The oscillating grid water tank used in the turbulent diffusion experiments is based on a design developed by Turner (1968) and used by Thompson (1969) for studying turbulent mixing across a density interface. (See Figure 3-1.) Thompson statistically analyzed velocity measurements he made above the oscillating grid and showed that a reasonably homogeneous turbulent field was generated. Based on Turner's success, a nearly identical but larger grid was constructed for our experiments. Turner's tank was relatively small with a water surface area of 645 square centimeters. The tank used in our experiments has roughly 7900 square centimeters of surface area.

3.1 Initial Design

Turbulent surface water movements are generated by oscillating a horizontal grid at a specified distance below the water surface. The base of the tank is square, 89 x 89 cm, and the height of the tank is 46 cm. The tank walls are made of clear Plexiglas. A hinged Plexiglas cover on top of the tank prevents air currents from influencing the flow at the water surface.

Initially, the grid's shape was square, 87.6 x 87.6 cm, with 5 cm square openings. The mass of this brass and Plexiglas grid was approximately 6.5 kilograms. During testing, a vibration problem was discovered to be caused by the oscillating motion of the heavy grid. This led to a modification of the grid design.

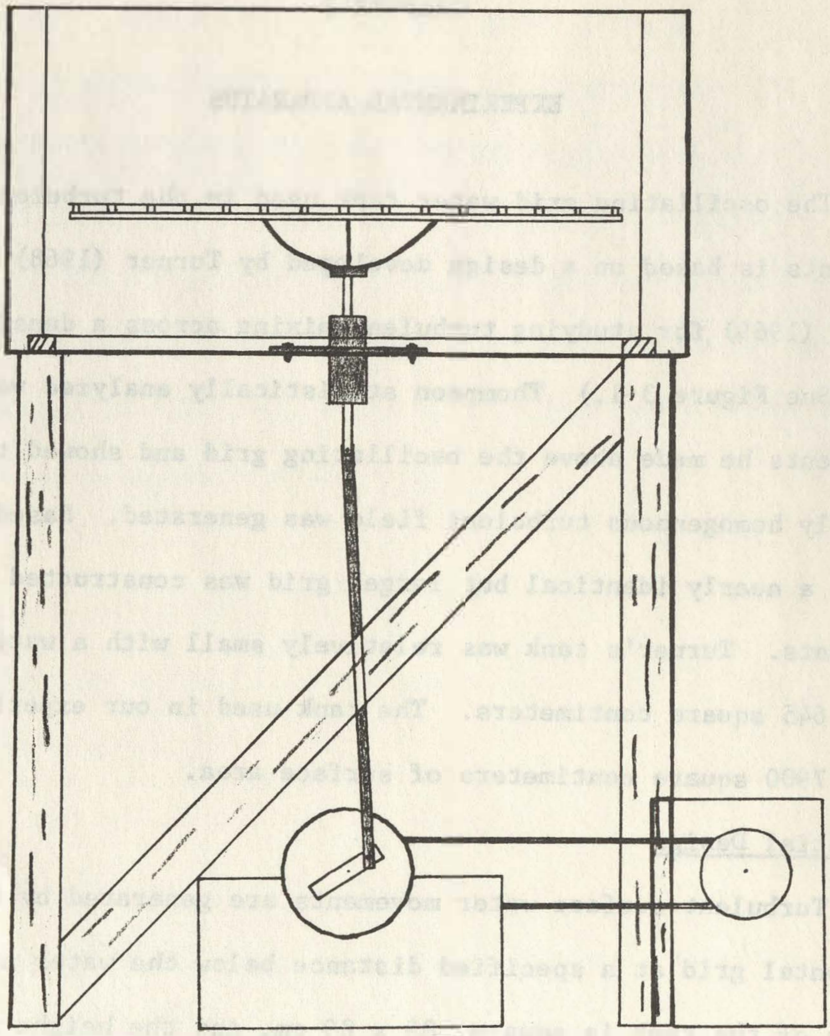


Figure 3-1. Diagram of Laboratory Apparatus.

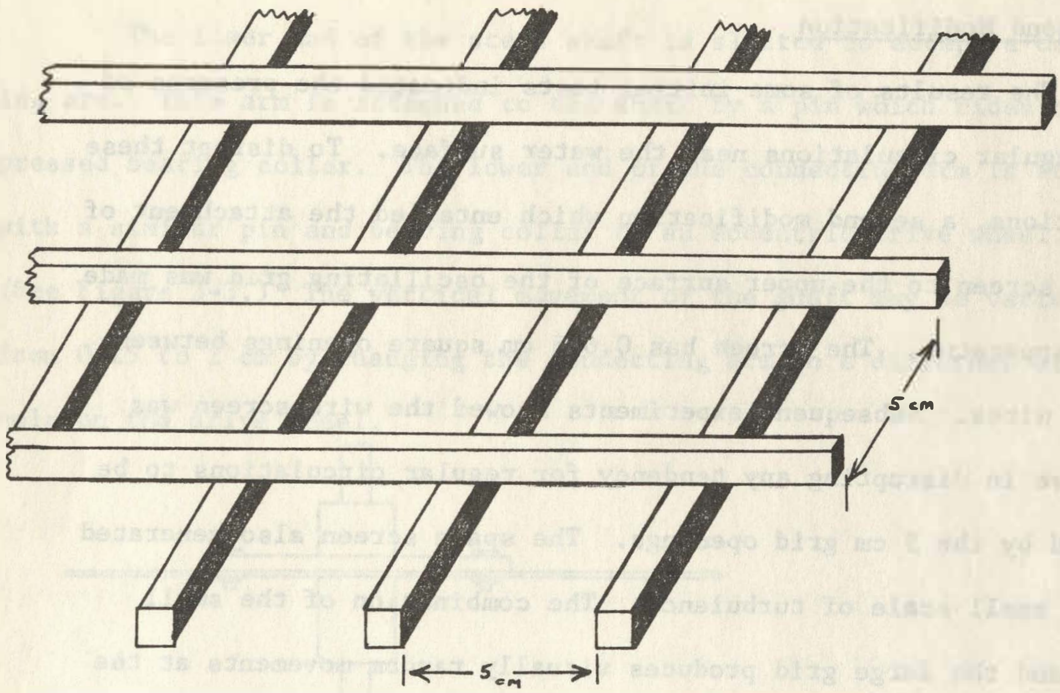


Figure 3-2. Diagram of the Overlapping Plexiglas Bars.

3.2 First Modification

A new lighter grid was made with a circular shape, 71.8 cm diameter, and with 5 cm square openings. Two square brass bars, 0.95 x 0.95 x 71.8 cm, and 24 square Plexiglas bars are overlapped to form the grid. (See Figure 3-2.) The mass of the new grid was 50 percent less than the mass of the original grid. Consequently, tests with the new circular oscillating grid have shown no vibration problems.

A Lexan plastic cylinder, 78.6 cm diameter, and 0.16 cm in wall thickness, provides a circular boundary around the new grid. The clear plastic cylinder is supported 1.9 cm above the floor of the tank. This permits a weak flow to exist between the inside and outside of the cylinder in order to dampen surface upwellings and provide an almost smooth water surface.

3.3 Second Modification

The results of some initial tests indicated the presence of small regular circulations near the water surface. To disrupt these circulations, a second modification which entailed the attachment of a space screen to the upper surface of the oscillating grid was made to the apparatus. The screen has 0.635 cm square openings between 1.19 mm wires. Subsequent experiments showed the wire screen was effective in disrupting any tendency for regular circulations to be produced by the 5 cm grid openings. The space screen also generated its own small scale of turbulence. The combination of the small screen and the large grid produces visually random movements at the water surface.

3.4 Final Design of Apparatus

Satisfactory results were obtained using the combination of the large Plexiglas grid and the small wire screen. The new grid is supported at a mean distance of 18 cm above the floor of the tank. Four brass arms rigidly connect the screen-grid to the top of a brass piston. The cylindrical piston is machine-pressed onto a 2.5 cm diameter steel shaft which exits the bottom of the tank through a brass collar and base-plate. The steel shaft rides on two sets of low friction Thompson linear bearings. The shaft and bearings are protected from water leakage by a rubber bellows-seal made from a motorcycle inner tube.

The lower end of the steel shaft is slotted to accept a connecting arm. This arm is attached to the shaft by a pin which rides on a pressed bearing collar. The lower end of the connecting arm is attached with a similar pin and bearing collar to an eccentric drive wheel. (See Figure 3-3.) The vertical movement of the shaft may be varied from 0.25 to 2 cm by changing the connecting arm to a different off-set hole on the drive wheel.

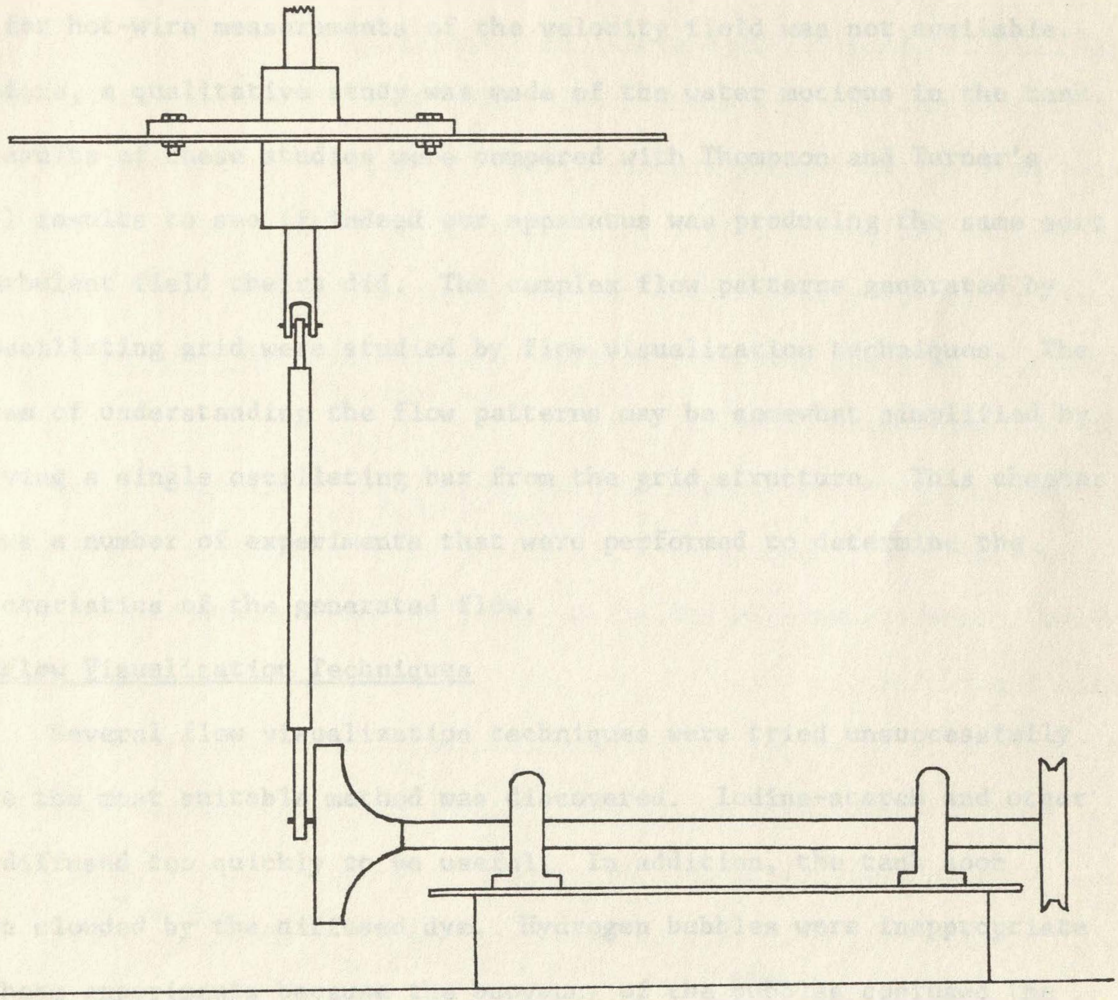


Figure 3-3. Diagram of Shaft, Connecting Arm, Eccentric Drive Wheel, and Pulley.

The drive wheel shaft is supported horizontally by two Dodge ball bearing pillow blocks. A pulley is attached to the opposite end of the shaft. A belt connects the drive wheel shaft to a Graham variable speed transmission which is powered by a 1/2 hp General Electric motor. The speed of the motor was monitored during every experiment by a General Radio Company Strobatac strobe light. After a warm-up period, the speed of the motor remained virtually constant.

CHAPTER 4

THE FLOW PATTERN AROUND AN OSCILLATING GRID

Before proceeding with surface turbulent diffusion experiments in our tank, it was necessary to confirm in some way the presence of a horizontally homogeneous layer of turbulence at the water surface. Equipment for hot-wire measurements of the velocity field was not available. Therefore, a qualitative study was made of the water motions in the tank. The results of these studies were compared with Thompson and Turner's (1975) results to see if indeed our apparatus was producing the same sort of turbulent field theirs did. The complex flow patterns generated by the oscillating grid were studied by flow visualization techniques. The problem of understanding the flow patterns may be somewhat simplified by observing a single oscillating bar from the grid structure. This chapter reviews a number of experiments that were performed to determine the characteristics of the generated flow.

4.1 Flow Visualization Techniques

Several flow visualization techniques were tried unsuccessfully before the most suitable method was discovered. Iodine-starch and other dyes diffused too quickly to be useful. In addition, the tank soon became clouded by the diffused dye. Hydrogen bubbles were inappropriate for these experiments because the buoyancy of the bubbles confused the vertical velocity observations. Extra fine aluminum powder suspended in water proved to be the best method for observing and photographing the flow patterns.

The technique of utilizing aluminum powder in water was developed by Clutter, et al (1959). Approximately three grams of powder is washed in a solution of Photoflo (Eastman Kodak). This removes some of the aluminum stearate and allows the powder to mix with water. The solution is introduced into the tank below the water surface. Illumination of the aluminum particles is accomplished by projecting a narrow slit focused beam of light through the tank.

4.2 Flow Pattern around a Single Oscillating Bar

A complicated pattern of flow is present around an oscillating grid made of overlapping angular bars. To simplify the problem, an experiment was performed using a single bar from the grid. The square brass bar, 0.95 x 0.95 x 87.6 cm, was supported horizontally in the tank 12.7 cm below the water surface. The position of the center of the bar may be described as:

$$y_c = -\tfrac{1}{2}s \cos (2 \pi \nu t)$$

where s is the length of the stroke and ν is the frequency in Hertz. (See Figure 4-1.)

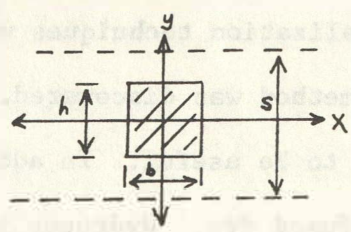


Figure 4-1. Cross Section of Bar.

In this experiment, the bar oscillated vertically with 0.5 cm and 1.0 cm strokes and at frequencies from 1 to 6 Hertz. Two distinct flow patterns resulted from the two different strokes.

The angular shape of the bar caused it to leave a wake as it oscillates, even at a relatively short stroke length of 0.5 cm. Schlichting's (1932) early experimental and theoretical work on oscillating circular cylinders applies only in smooth no-wake cases. In such a case, the complicated boundary layer velocity field may be represented as shown in Schlichting's diagram. (See Figure 4-2.)

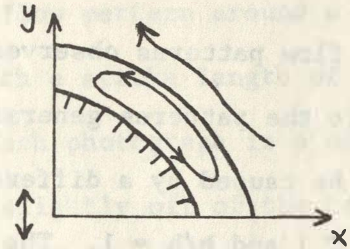


Figure 4-2. Flow Pattern around Schlichting's (1932) Oscillating Cylinder.

In Schlichting's experiment, an 8 cm diameter circular cylinder was oscillated in water at 0.5 Hz with a 1.8 cm stroke. The Reynolds number is of order 1000 and the ratio s/b = .22 (where b is the diameter of the cylinder).

Similar patterns of flow were observed in the acoustic streaming experiments of Andres and Ingard (1952). The flow pattern around a cylinder vibrating in air was found to be similar to Schlichting's pattern when the Reynolds number was relatively high (R = 200) and reversed from Schlichting at low Reynolds numbers ($R < 10$). Carriere (1929) observed the same flow reversal in his early experiments with oscillating cylinders in air. From Andres and Ingard's paper, the low Reynolds number flow is shown in Figure 4-3.

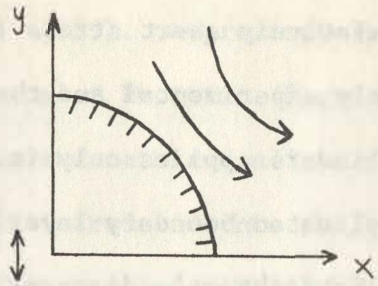


Figure 4-3. Flow Pattern around a Cylinder Oscillating at a Low Reynolds Number (Andres 1952b).

The two opposite flow patterns observed around an oscillating angular bar are similar to the patterns generated by the no-wake cases but will now be shown to be caused by a different process. Consider first the case where $s/h \geq 1$ and $b/h = 1$. The movement of the square bar with $s = 1.0$ cm and $s/h = 1.05$ is shown in Figure 4-4.

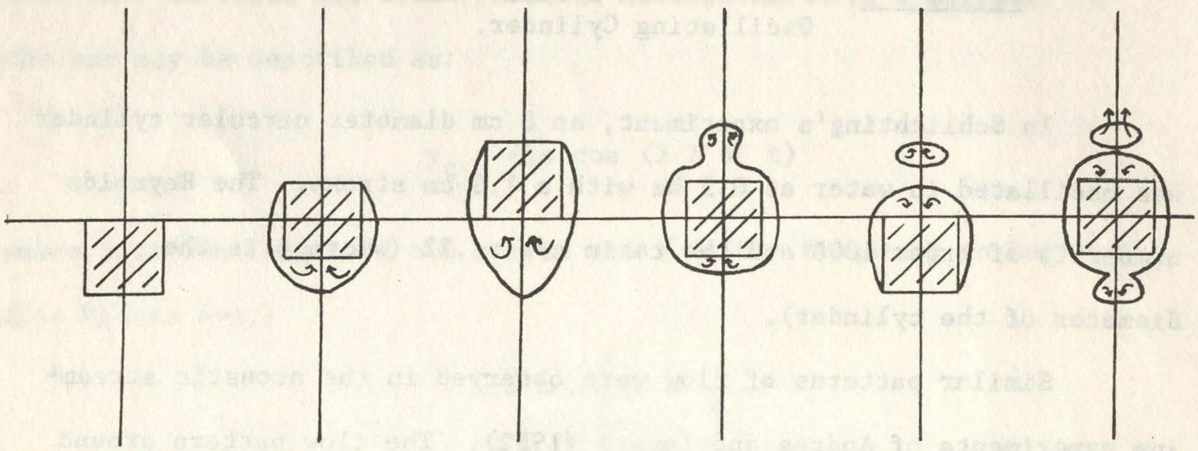


Figure 4-4. Flow around an Oscillating Square Bar. Stroke = 1 cm.

A wake is formed behind the bar as it pushes upward. When the bar reverses direction, it begins to move through the wake of the upstroke. Since the boundary layer is thin, only a small part of the previous upstroke wake is cancelled by vorticity of the opposite sign as it passes through the downstroke boundary layer. As the bar reaches the bottom of the downstroke, the small volume of fluid separates from the wake. On the return upstroke, the bar passes through the downstroke wake, transferring it to the bottom of the bar. So a net transfer of fluid from near the origin of the x-y axis toward the top and bottom of the bar occurs with $s/h \geq 1$.

Plate 1 shows the flow pattern around a square bar oscillating at 1, 2, 4, and 6 Hertz with a stroke length of 1 cm. In this case, $s/h = 1.05$ and $b/h = 1$. Each photograph is a one second exposure. The camera was positioned slightly off of the center of the bar so that a 2.0 cm light streak from the bar's edge would be visible for a convenient length scale.

The strength of the outflow from the top and bottom of the bar clearly increases with the frequency of oscillation. Eddies or swirls occasionally form some distance from the corners of the bar. In addition, interesting wiggle-streaks are visible on the flow of fluid toward the bar. These wiggle-streaks occur in the region of potential flow which oscillates in phase with the bar.

Thompson (1969) observed a 1.27 x 1.27 cm square bar in a smaller but otherwise identical experimental tank. When the bar oscillated with s = 1.70 cm, $s/h = 1.34$, he produced a similar pattern of outflow from the top and bottom of the bar as our apparatus produced with s = 1.0 cm and $s/h = 1.05$. However, when Thompson used s = 1.28 cm and $s/h = 1.0$, he reported a confused and unorganized flow pattern.

The critical value of s/h for flow reversal should be of order unity. Our single bar experiment was repeated with s = 0.5 cm and s/h = 0.53. When s/h is less than one, the trailing edge of the bar does not cross the origin. However, even though the stroke is short, the bar still leaves a wake as it moves. Figure 4-5 illustrates a cycle of movement.

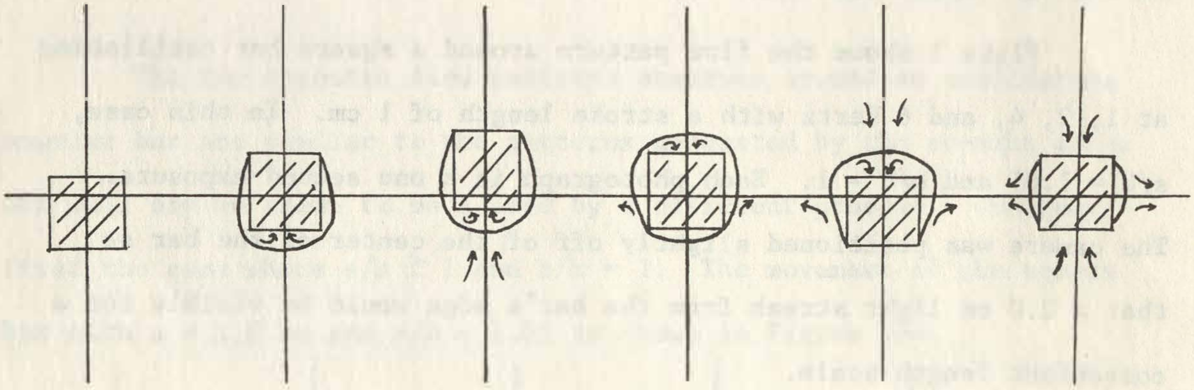


Figure 4-5. Flow around an Oscillating Square Bar. Stroke = 0.5 cm.

The fluid in the wake is not transferred to the opposite end of the bar after the bar reverses direction. Instead, it reaches only to the x-axis before the bar again reverses its direction. The result is a transfer of fluid from the top and bottom of the bar to the sides of the bar. Thompson observed his square bar oscillating with s/h = 0.52, where s = 0.68 cm and b = h = 1.27 cm, and reported a pattern of four outflows from the corners of the bar as shown in Figure 4-6.

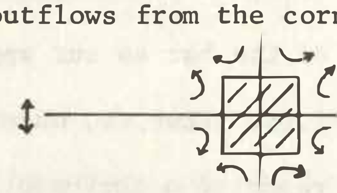


Figure 4-6. Thompson's (1969) Flow Pattern for s/h = 0.52.

Plate 2 shows the flow pattern around the bar when s = 0.5 cm and f = 1, 2, 4, and 6 Hertz. These photographs are one second exposures. The light streak represents 1.5 cm and the shadow is 1.0 cm. Much of the kinetic energy imparted to the water is dissipated viscously very close to the sides of the bar in the area of intense convergence. As a result, the efficiency of this stirring mechanism is low. However, if a grid is made of overlapping square bars and oscillated with s/h less than one, then the outflow from the sides of the bars would presumably mix in a random fashion. This may be one reason why Thompson and Turner obtained a nearly horizontally homogeneous turbulent layer above the grid.

4.3 Flow Pattern around an Oscillating Grid

Several experiments were conducted to determine the character of the flow around the oscillating grid. Because the grid is made of overlapping square bars, it is impossible to predict the kind of velocity field that will be present at a given distance above the grid at a given time. Even at low oscillation frequencies, the flow is complicated by reflections, and divergences of upward jets at the water surface. However, a stationary circulation pattern was observed with the original grid oscillating with s = 1 cm and f = 1 Hertz.

Photograph A of Plate 3 shows the stationary circulations that were observed. In this experiment, the water depth was 6.4 cm above the mean position of the grid. A narrow vertical light beam was projected through the tank to illuminate the aluminum particles. The beam passes halfway between two bars that run from left to right in the photograph. This photograph clearly shows a steady downward flow toward the midpoints of the open spaces in the grid. Outward streaming occurs above and below each bar. Even though the mean distance from the grid to the water surface is not the same as the distance from the grid to the floor of the tank, the stationary pattern was almost symmetric about the plane of the grid. This circulation pattern disappears as soon as the frequency is increased to 2 Hertz as seen in Photograph B of Plate 3.

Although the stationary patterns disappear at higher frequencies, there was still a tendency for the formation of steady downward motions toward the grid-space midpoints. For this reason, the 1 cm stroke that was used in some of Thompson and Turner's (1975) experiments was rejected in favor of a 0.5 cm stroke.

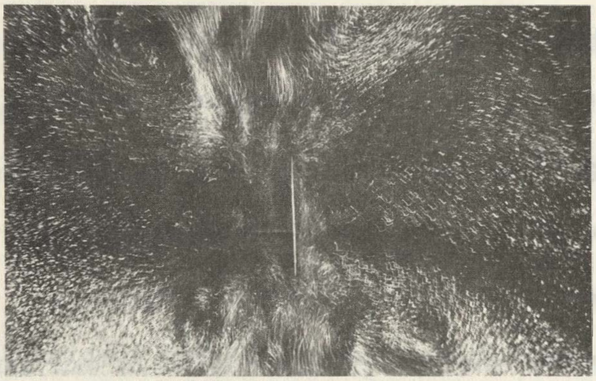
Since the flow pattern around a single bar reverses when s = 0.5 cm, it was assumed that the pattern around an oscillating grid would be considerably more random with s = 0.5 cm compared to s = 1.0 cm. Presumably the horizontal outflow from the middle of the square bars would mix randomly in the grid openings. Essentially this appears to be true. Photographs A and B of Plate 4 illustrate the vertical and horizontal flow around the grid when s = 0.5 cm and f = 6 Hertz. In this experiment, the water depth to the mean position of the grid was 10.5 cm. All of the remaining experiments were run at this water level. Photograph A of Plate 4 shows there is still a pattern of weak downflow into the grid spaces. The top view seen in Photograph B was made with a flat horizontal light beam projected through the tank just below the water surface. A number of circular patterns are visible with a diameter approximately equal to the grid openings (5 cm).

The circular pattern is the result of slow upward jets of water intersecting with the water surface. The circular boundary is visible where the water motion becomes horizontal before it curves downward. The circular patterns also indicate that a slow circulation is present. The discovery of these quasi-stationary horizontal circulations led us to add a wire screen to the top of the grid.

The dimensions of the wire screen are discussed in section 3.3. Aluminum powder was again added to the water in a flow visualization experiment to observe the water motion above the screen-grid combination. Photographs C and D in Plate 4 show the vertical and horizontal aspect of water motion above the screen-grid. The screen provides a smaller scale of motion that interacts with the flow generated by the large square bars.

Photograph C of Plate 4 shows the additional water motions that are generated by the screen. The screen is effective in blocking the slow downward streams that were present with only the large grid. Visually, the side view indicates random motions are present between the screen-grid and the water surface.

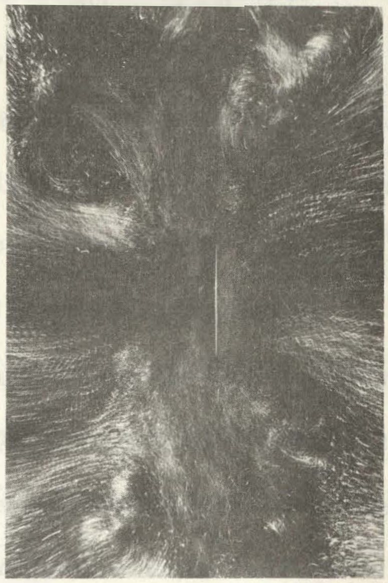
Photograph D shows the horizontal water motions just below the surface of the water. The small circular patterns that were visible in Photograph B are not present above the oscillating screen-grid. Analysis of sequential photographs of the horizontal and vertical motions qualitatively indicates a random flow pattern in general agreement with the studies and results of Turner and Thompson. As we will see, actual diffusion experiments seem to bear out the true horizontal homogeneous turbulent nature of our surface layer flow field.



A. f = 1 Hz

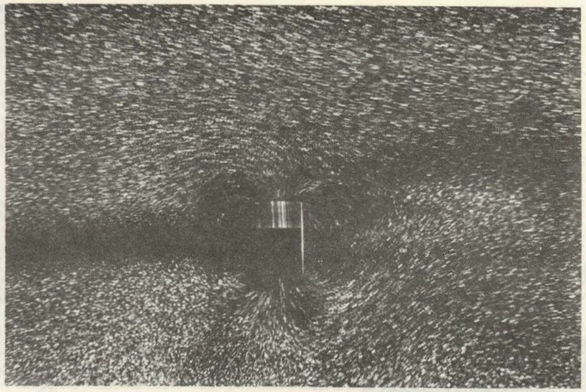
B. f = 2 Hz

C. f = 4 Hz

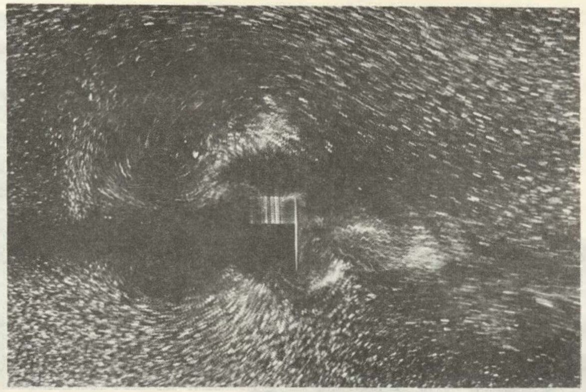


D. f = 6 Hz

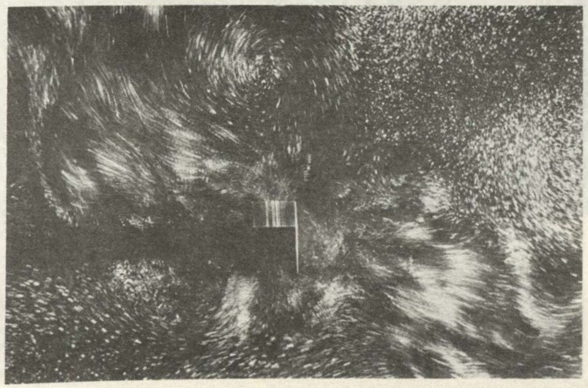
Plate 1. Flow pattern around a vertically oscillating square bar. Stroke = 1 cm.



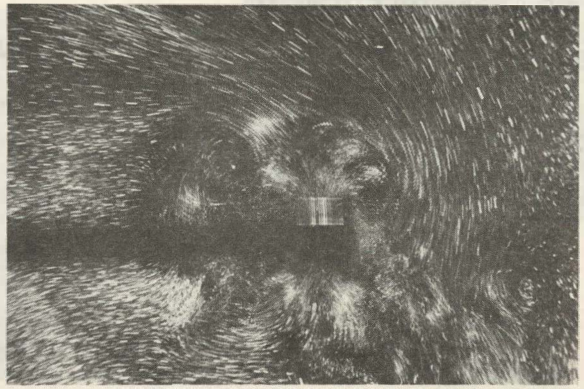
A. f = 1 Hz



B. f = 2 Hz



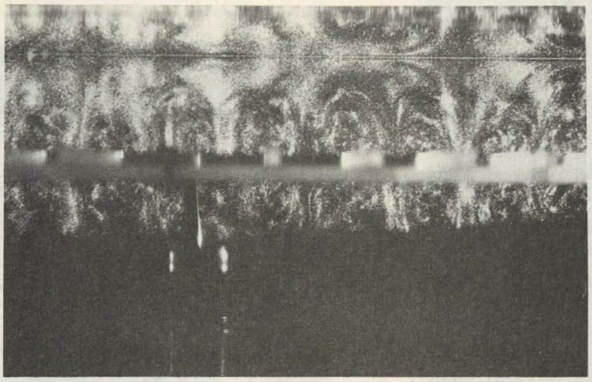
C. f = 4 Hz



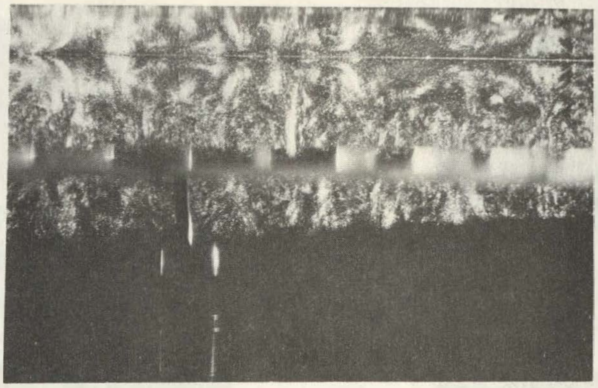
D. f = 6 Hz

Plate 2. Flow Pattern around a vertically oscillating square bar.

Stroke = 0.5 cm.



A. f = 1 Hz



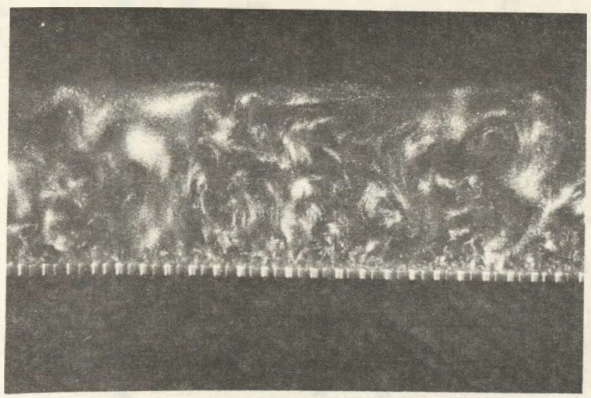
B. f = 2 Hz

Plate 3. Flow pattern around an oscillating grid.

Stroke = 1 cm.

A. Side view of oscillating grid.

B. Top view of oscillating grid.



C. Side view of screen-grid.

D. Top view of screen-grid.

Plate 4. Flow pattern around an oscillating grid.

Stroke = 0.5 cm. Frequency = 6 Hz.

CHAPTER 5

DISPERSION EXPERIMENTS

The flow visualization experiments discussed in the preceding chapter qualitatively showed that away from the tank walls, a reasonably horizontal homogeneous turbulent field is present at the water surface in our tank. If this is correct, then a simple dispersion experiment should quantitatively show that the rate of diffusion (or the eddy diffusivity) increases with an increase in the particle-pair separation distance. As mentioned in Chapter 2, Richardson empirically found that his "neighbor" diffusivity, F, is proportional to the 4/3 power of the length scale in the inertial subrange. In our laboratory tank, the scale of the energy containing eddies above the grid is estimated to be of order L, the 5 cm grid spacing. Since the water depth above the grid was approximately 2L, the characteristic eddy size may be greater at the surface due to divergence in the water column. Therefore, the inertial subrange in our tank is estimated to extend from 10 cm down to the Kolmogorov scale eddies.

Batchelor (1952) and Lin (1960) have confirmed the 4/3 power law by theoretical arguments. However, in his 1952 paper, Batchelor states his belief that, statistically, it is not good to relate F with ℓ, the random distance between dispersing particle pairs. Instead, Batchelor suggests using the mean square separation distance. So Richardson's 4/3 power law becomes:

$$F \sim (\overline{\ell^2})^{2/3}$$

As a result, a log-log plot of F versus $\overline{\ell^2}$ should give a straight line with a slope of 2/3.

The dispersion experiments that we conducted in the laboratory apparatus are similar to field experiments performed by Stommel at Woods Hole in 1949. In his experiments, two parallel lines of passive markers were dropped onto the ocean. Marker pairs with similar separation were tracked over a given time interval in order to find the change in distance between particles. By tracking a number of pairs with approximately the same separation, Stommel was able to determine the mean square change in separation during the time interval. Using Richardson's method, he computed a neighbor diffusivity by:

$$F = \frac{\overline{(\ell_1 - \ell_0)^2}}{2T} \qquad (5.1)$$

where, ℓ_1 = distance between the marker pair at time t_1.
ℓ_0 = distance between the marker pair at time t_0.
$T = t_1 - t_0$

Richardson originally proposed making the neighbor diffusivity a function of the initial separation of two dispersing particles. However, it was necessary for Stommel to make the diffusivity a function of the mean distance between each particle-pair since the initial separation was not fixed. So the mean length scale of each group of similar pairs was computed by:

$$\bar{\ell} = \frac{\overline{(\ell_1 + \ell_0)}}{2} \qquad (5.2)$$

The same method of computing F was used in our laboratory dispersion experiments. However, following Batchelor's suggestion, F was made a function of the mean square distance between particle-pairs.

$$\overline{l^2} = \left[\tfrac{1}{2}\left(l_1 + l_0\right)\right]^2 \qquad (5.3)$$

In these laboratory experiments, a cluster of six or ten lens-shaped polyethylene pellets was dropped onto the water surface. High density polyethylene plastic supplied by Phillips Petroleum Company was used because its specific gravity, 0.93, is similar to that of typical crude oil. Pellets of similar size, each with an approximate mass of 0.01 grams, were used in the experiments. The geometry of the deployed cluster is shown in Figure 5-1.

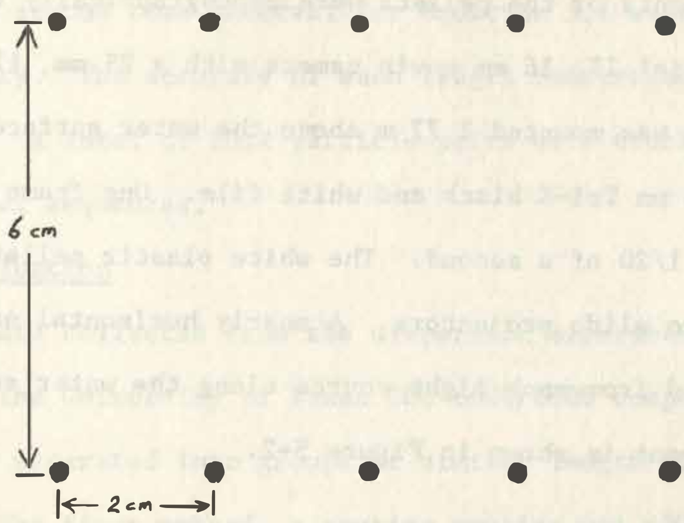


Figure 5-1. Cluster Geometry.

All of the pellets were dropped simultaneously in this pattern by a sliding plate multiple shutter device. Alternate shutter holes were used when only six pellets were dropped. The multiple shutter device was held just above the water surface before releasing the pellets. Therefore, the pellets had a minimum downward velocity upon entering the water.

The laboratory tank was operated in the same manner as the successful final flow visualization experiment discussed in section 3.3. The screen-grid oscillated at 6 Hz with a 0.5 cm stroke. The depth of water above the mean position of the grid was 10.5 cm (roughly two grid scale lengths). Before each experiment, the grid was oscillated for one hour to eliminate any "start-up" effects. Dimensionally, the start-up time may be characterized as L^2/ν , where ν is the kinematic viscosity of water, and L is a length. If the water depth and stroke length are used as the length scales, then:

$$t \doteq \frac{(10.5\ \text{cm})\,(0.5\ \text{cm})}{(0.01\ \text{cm}^2/\text{sec})} = 525\ \text{sec}$$

The movements of the pellets were photographically recorded with a Cine-Kodak Special II, 16 mm movie camera with a 25 mm, f1.9 Ektar lens. The camera was mounted 1.77 m above the water surface. The film used was Kodak 16 mm Tri-X black and white film. One frame was exposed every second for 1/20 of a second. The white plastic pellets were illuminated by two slide projectors. A nearly horizontal narrow-slit beam was projected from each light source along the water surface. The lighting arrangement is shown in Figure 5-2.

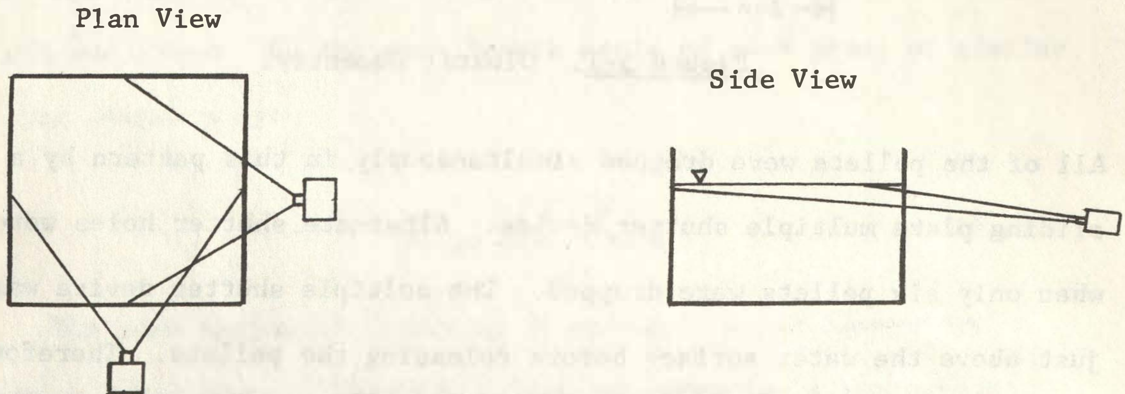


Figure 5-2. Lighting Arrangement.

A typical cluster dispersion sequence is shown in Plate 5 on page 50. These six photographs were taken from alternate frames of the first 12 seconds of one experimental run, hence they illustrate the cluster spreading at two second intervals. Generally, the experiments were most successful when only six pellets were used in a dispersion sequence. With ten pellets, there was a greater tendency for two of the pellets to be attracted to each other because of surface tension effects.

The distance between dispersing particle-pairs was measured from a 'real-size' projection of the movie frames. Measurements were made from alternate frames of each sequence of the spreading clusters to give a two second time interval for Equation 5.1 when calculating the diffusivity. The accuracy of each length measurement is estimated to be $\pm$ 1 mm. A total of 2032 particle-pairs were tracked and measured from 21 cluster sequences.

5.2 Data Evaluation

The data collected from the dispersion experiments were evaluated on The University of Texas CDC 6600/6800 computer system. The data were separated into groups of similar length scales by two methods. In the first method, a sorting routine was used to separate the data pairs into one centimeter intervals according to the average value of each ℓ_0 and ℓ_1 pair. In the second method, the data pairs were grouped according to the initial separation, ℓ_0.

Method I. The first sorting method grouped the data pairs in a manner similar to Stommel's experiment. The mean value of ℓ_0 and ℓ_1 for each pair of numbers is used to separate the pairs into one centimeter intervals. For example, the interval from 4 cm to 5 cm contains all of the pairs with a mean separation such that:

$$4 \leq \frac{(\ell_1 + \ell_0)}{2} \leq 5 \text{ cm}$$

Pairs with an integer value were included in the two adjacent intervals. The data pairs which were sorted by this method are given in Appendix A.

These sorted pairs of data were used to compute the mean separation distance, the mean square separation distance, and the eddy diffusivity for each interval using equations 5.2, 5.3, and 5.1 respectively. The values of these computed quantities appear in Table 5-1. The measurement of a particularly elusive quantity such as an eddy diffusivity is difficult even under laboratory conditions. Therefore, it is not surprising that the distribution of the individual eddy diffusivity values which are averaged for each length interval results in a fairly large standard deviation. For example, the separation interval from 6 cm to 7 cm contains 235 measured ℓ_0 and ℓ_1 pairs. The computed mean square separation is 42.8 cm^2. The diffusivity was found to be 0.053 cm^2/sec with a standard deviation of 0.096 cm^2/sec. Data from Stommel's experiments show a similar magnitude of error in the eddy diffusivity calculation.

A logarithmic plot of F versus $\overline{\ell^2}$ is shown in Figure 5-3 on page 46. Diffusivity values computed from fewer than 16 measurements are excluded from the graph. The rate of turbulent dispersion in our tank clearly increases with increasing mean square particle separation up to approximately $\overline{\ell^2}$ = 120 cm^2. The value of F decreases with increasing values of $\overline{\ell^2} > 120$ cm^2.

MEAN LENGTH (CM.)	MEAN SQUARED LENGTH (CM. SQRD.)	DIFFUSIVITY (CM SQRD/SEC)	NUMBER OF PAIRS
.72	.5	.0030	5
1.68	2.9	.0198	43
2.54	6.5	.0306	73
3.54	12.6	.0379	138
4.51	20.4	.0522	144
5.54	30.7	.0601	161
6.54	42.8	.0528	235
7.46	55.8	.0681	232
8.51	72.4	.1126	178
9.48	90.0	.0838	157
10.48	109.8	.1209	102
11.54	133.3	.1158	89
12.47	155.7	.0984	85
13.57	184.1	.0725	59
14.47	209.5	.0598	61
15.46	239.1	.0632	54
16.52	272.8	.0580	40
17.41	303.3	.0426	29
18.50	342.5	.0293	28
19.45	378.4	.0282	28
20.55	422.2	.0427	11
21.42	458.9	.0983	9
22.45	504.1	.0763	8
23.58	556.2	.0299	25
24.45	597.9	.0980	10
25.42	646.2	.0872	8
26.56	705.4	.0757	7
27.57	760.2	.0345	10
28.08	788.7	.0092	3
0	0	0	0

Table 5-1. Method I Results.

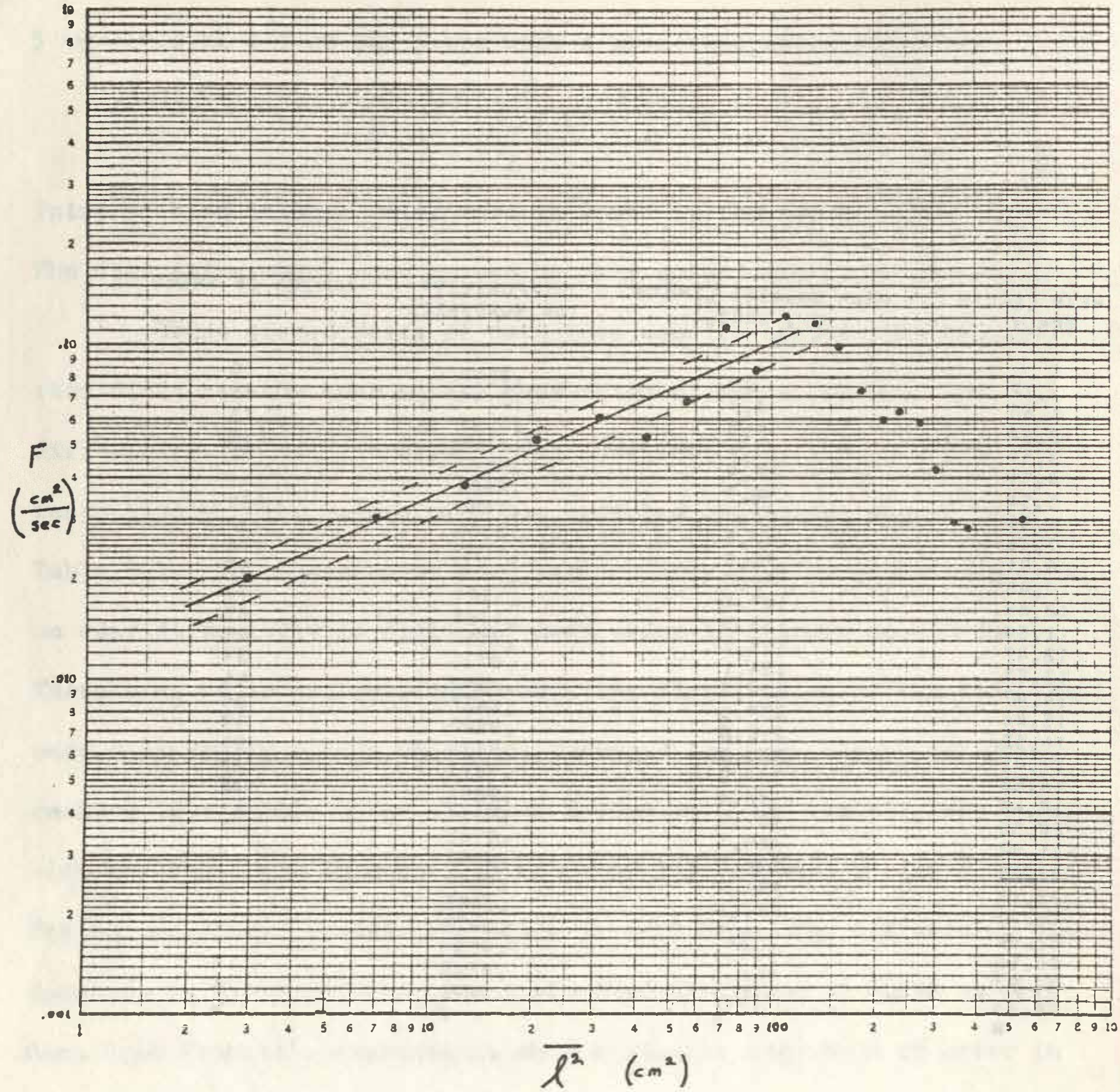


Figure 5-3 Diagram of eddy diffusivity vs. mean square separation.

An examination of the first ten plotted points strongly suggests a linear relationship between the logarithmic values of F and $\overline{l^2}$. The correlation coefficient of these ten points is 0.97. The best fitting straight line through these points was found by linear regression. The equation of this regression line is:

$$\log(F) = 0.458 \log(\overline{l^2}) - 1.91 \quad ; \quad \overline{l^2} < 120 \text{ cm}^2$$

The dashed lines which bound the best fitting line through the plotted points in Figure 5-3 represent one standard deviation, or the 68 percent confidence limit.

The slope of the line through the first ten points is in reasonable agreement with the theoretical value of 2/3. The decrease in the eddy diffusivity values for particle separations greater than approximately 11 cm may be due to the limited extent of the inertial subrange of turbulence generated in our tank. Based on our observations, the energy containing eddies at the surface are estimated to have a 'characteristic size' of two times the large grid spacing, or approximately 10 cm. This factor of two seems to be due to surface reflections and divergences. Therefore, Richardson's relative diffusion theory should be applicable, in our tank, from $\overline{l^2} \approx 100$ cm^2 down to the mean square length of the Kolmogorov scale eddies. It is curious that the plotted points of decreasing diffusivities for $\overline{l^2} > 120$ cm^2 are nearly linear. The correlation coefficient of the last ten points is 0.94. The equation of the best fit straight line is:

$$\log(F) = -1.1 \log(\overline{l^2}) + 1.35 \quad ; \quad \overline{l^2} > 120 \text{ cm}^2$$

Method II. A second method was used to group the data in order to test the results found by the first sorting method. In this case, the data pairs were grouped according to the initial separation distance in a manner similar to Richardson's proposed experiment in his 1926 paper. Since the estimated error in the measurement of the initial separation is ± 0.1 cm, each group contained pairs with a measurement of $\ell_0 \pm 0.1$ cm. For example, the group of data for $\ell_0 = 5.6$ cm included all pairs with an initial separation of 5.6 cm ± 0.1 cm. The interval between the sorted ℓ_0 groups is 0.2 cm. Therefore, a diffusivity value was found for $\ell_0 = 1.0, 1.2, \ldots$ 15.0, using equation 5.1. Data pairs sorted by this method are given in Appendix B. The calculated diffusivity values may be found in Table 5-2.

A logarithmic plot of this data is given in Figure 5-4 on page 51. The squared initial separation is plotted on the abscissa while the computed diffusivity values are on the ordinate. Values for $\ell_0^2 < 110$ cm^2 are plotted for each ℓ_0 group with 16 or more data pairs. The correlation coefficient of these points is 0.89. The equation of the least-squares line through the points is:

$$\log(F) = 0.475 \log(\ell_0^2) - 1.92 \qquad ; \ \ell_0^2 < 110 \text{ cm}^2$$

The 68 percent confidence limits of the points giving this line are represented by the dashed lines.

The results of the data pairs grouped by this second method are in close agreement with the first sorting technique. Since the interval between groups is smaller in the second method, there are fewer pairs that are used in each diffusivity calculation. Therefore, a greater scatter of the points from a linear relationship would be expected.

5.3 Summary of Results

The calculated eddy diffusivity values from measurements of passive tracers in our tank increase approximately in proportion to the 2/3 power of the mean square distance between particles in a spreading cluster. These results are in reasonable agreement with the relative diffusion theory, and indicate a reasonably horizontal homogeneous turbulent surface layer is generated by the oscillating grid.

INITIAL LENGTH (cm)	LENGTH SQUARED (cm^2)	DIFFUSIVITY (cm^2/sec)	NUMBER OF PAIRS
1.0	1.0	.0708	3
1.2	1.4	.0465	5
1.4	2.0	.0283	9
1.6	2.6	.0319	12
1.8	3.2	.0103	22
2.0	4.0	.0227	15
2.2	4.8	.0539	22
2.4	5.82	.0258	22
2.6	6.8	.0254	24
2.8	7.8	.0311	22
3.0	9.0	.0352	24
3.2	10.2	.0428	38
3.4	11.6	.0370	42
3.6	13.0	.0393	53
3.8	14.4	.0500	51
4.0	16.0	.0524	43
4.2	17.6	.0553	52
4.4	19.4	.0500	50
4.6	21.2	.0478	47
4.8	23.0	.0530	45
5.02	25.0	.0634	35
5.22	27.0	.0637	43
5.42	29.2	.0540	42
5.62	31.4	.0624	49
5.8	33.6	.0592	49
6.0	36.0	.0600	47
6.2	38.4	.0554	64
6.4	41.0	.0635	83
6.6	43.6	.0618	79
6.82	46.2	.0485	89
7.0	49.0	.0649	82
7.2	51.8	.0676	79
7.4	54.8	.0645	73
7.62	57.8	.08112	73
7.8	60.8	.12272	60
8.0	64.0	.10482	45
8.2	67.2	.10522	46
8.4	70.6	.11362	51
8.6	74.0	.1232	50
8.8	77.4	.10442	46
9.0	81.0	.06712	39
9.2	84.6	.09122	38
9.4	88.4	.1001	56
9.6	92.2	.09732	59
9.8	96.0	.1009	38
10.0	100.0	.11532	30
10.2	104.0	.0782	32
10.4	108.2	.1136	30
10.6	112.4	.1012	26
10.8	116.6	.1171	21
11.0	121.0	.1218	19
11.2	125.4	.1646	29
11.4	130.0	.1076	24
11.6	134.6	.0532	23
11.8	139.2	.0587	26
12.0	144.0	.1186	26
12.2	148.8	.0699	25
12.4	153.8	.0679	20
12.6	158.8	.0283	27
12.8	163.8	.07322	17
13.0	169.0	.11272	13
13.2	174.2	.1125	17
13.4	179.6	.0770	102
13.6	185.0	.07842	22
13.8	190.4	.04622	21
14.0	196.0	.04212	13
14.2	201.6	.06372	22
14.4	207.4	.05712	24
14.6	213.2	.0688	21
14.8	219.0	.1071	14
15.0	225.0	.0568	17

Table 5-2. Method II Results.

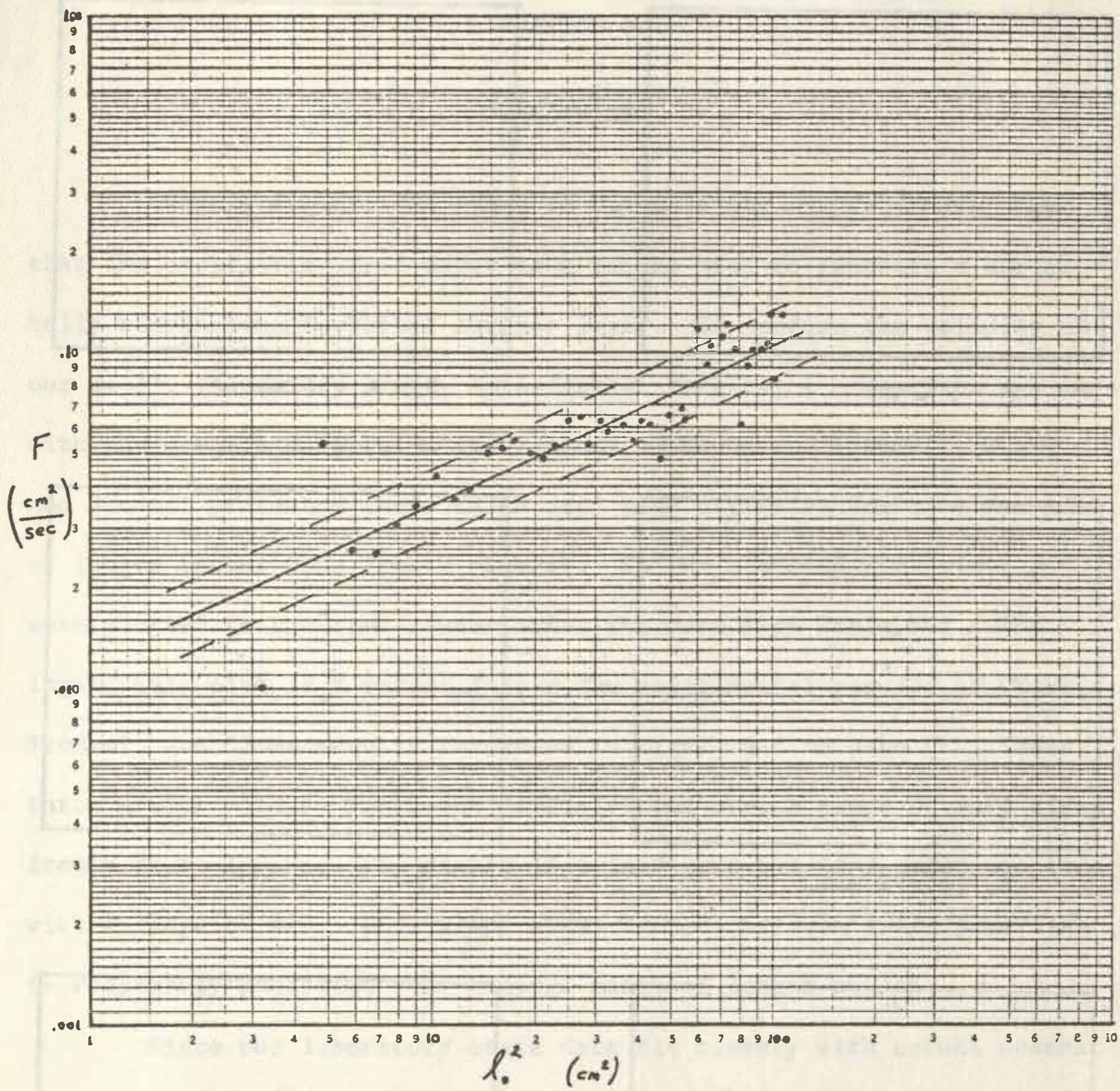


Figure 5-4 Diagram of eddy diffusivity vs. initial separation squared.

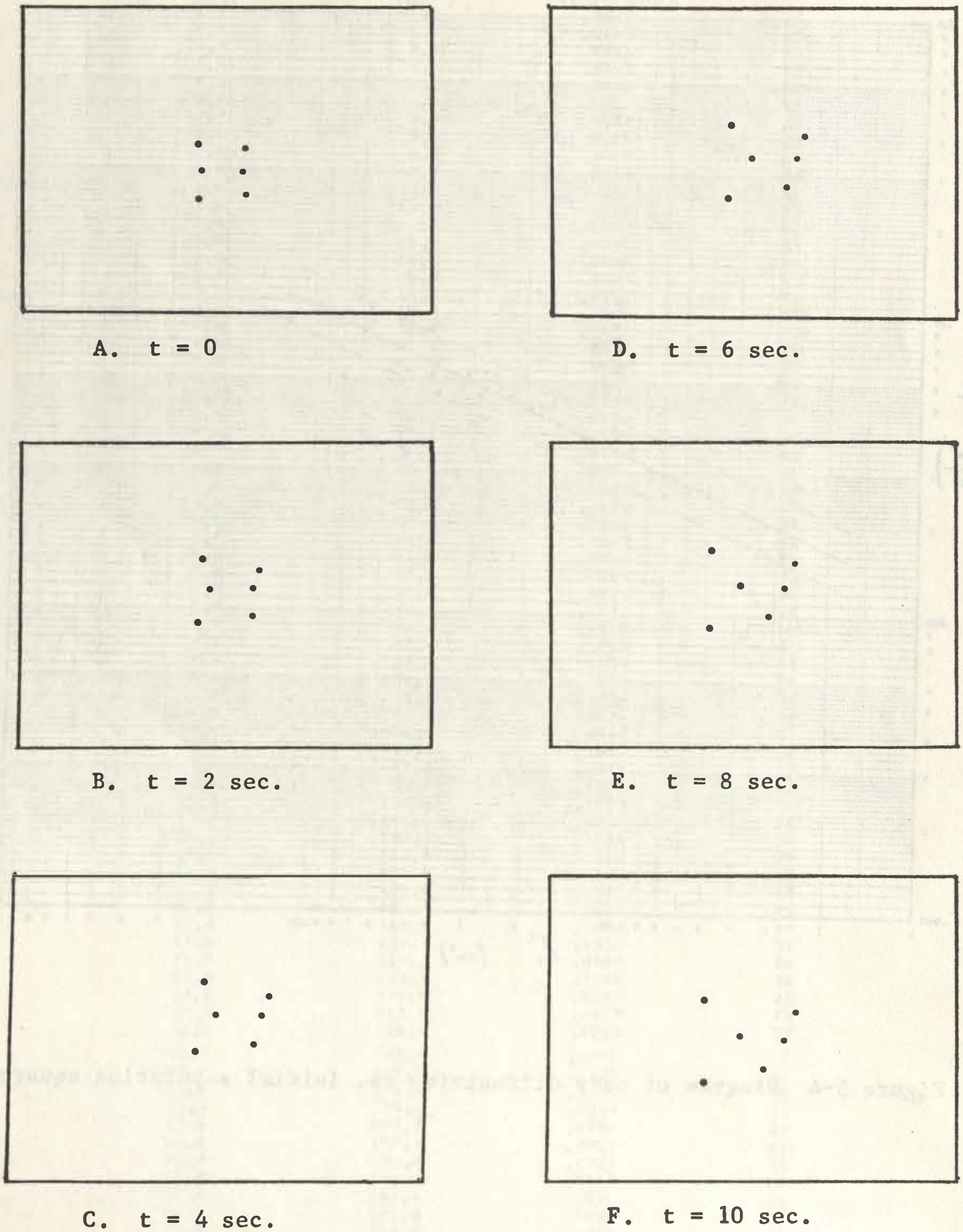


Plate 5. Cluster spreading sequence.

CHAPTER 6

CONCLUSIONS

The experiments described in the previous chapters have shown that the oscillating grid water tank can be used to generate a horizontally homogeneous turbulent surface layer. To confirm the validity of our small, laboratory scale, experimental results, a comparison was made with the oceanic dispersion experiments of Okubo and Stommel. Okubo and Stommel presented their calculated eddy diffusivities as a function of length rather than length squared. So our computed diffusivities were plotted in the same manner using the data from Table 5-1. The logarithmic plot of F versus ℓ from the experimental results of Okubo, Stommel, and Zimmermann is presented in Figure 6-1 on page 55. These three groups of data represent diffusivities over a range of eddy sizes from 2 cm to 100 km. The dashed line in Figure 6-1 is a reference line with a slope of 4/3. This graph shows that Richardson's 4/3 power law is reasonably satisfied over a large range of length scales.

Since our laboratory scale data fit closely with actual oceanic dispersion measurements, it is reasonable to conclude that we are generating a simplified small scale version of the ocean's turbulent surface layer. It is a simplified version in the sense that our horizontally homogeneous turbulent field is stationary and has no discernable mean flow superimposed on it.

In our experiments, we used passive tracers which simulate small stable oil patches. It was shown that the relative diffusion theory can be used to predict the dispersion of these patches relative to their collective center of mass. It was found that the spreading rate of these patches is approximately proportional to the 2/3 power of the mean square distance between patches. In other words,

$$F = \frac{1}{2}\frac{ds^2}{dt} \sim (s^2)^{2/3}$$

and then by integration, we find:

$$s^2 \sim t^3$$

The variance of these patches about their mutual center of gravity (and hence, the area of the field of patches) varies roughly with t^3.

The results of these discrete tracer experiments are now being used as a basis to test the effects of a surface turbulent field on the spreading of a continuous slick of oil. Experiments on this problem are now being conducted in the oscillating grid tank at The University of Texas at Austin.

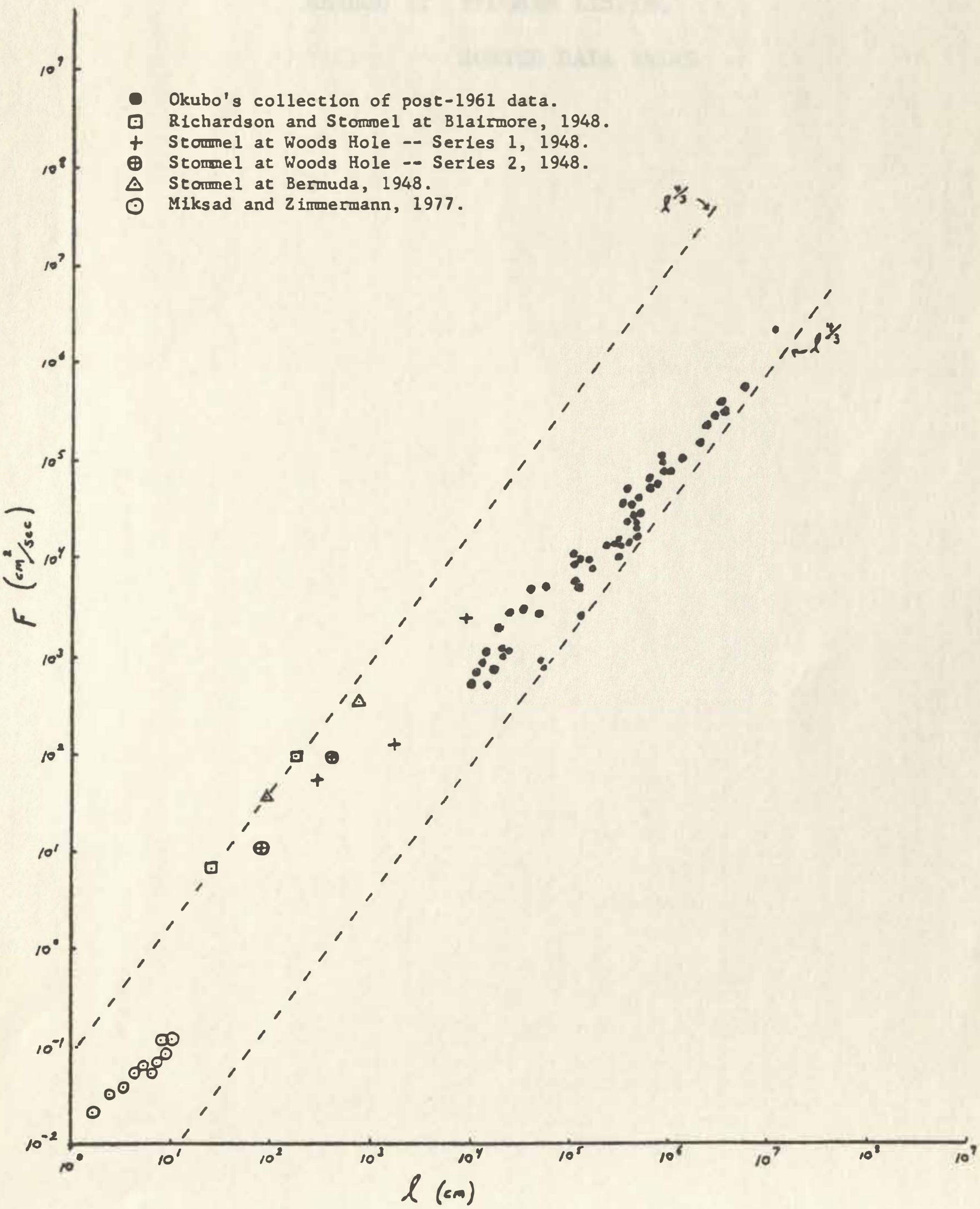


Figure 6-1. Diagram of eddy diffusivity vs. length scale.

APPENDIX A

METHOD I: PROGRAM LISTING

SORTED DATA PAIRS

```
      PROGRAM OIL(INPUT,OUTPUT)
      DIMENSION A(6),B(300,2,30)
      DO 1 I=1,300
      DO 1 J=1,2
      DO 1 K=1,30
    1 B(I,J,K)=0.0
      PRINT 2
    2 FORMAT(*1*,51X,*SORTED DATA PAIRS  (CENTIMETERS)*)
      READ 5,N
    5 FORMAT(I5)
      DO 200 I=1,N
      READ 10,(A(J),J=1,6)
   10 FORMAT(6F5.1)
      DO 150 J=1,5
      AVG=(A(J)+A(J+1))/2.0
      DO 40 K=1,30
      C=K
      IF (C.EQ.AVG) GO TO 90
      IF (C.GE.AVG) GO TO 110
   40 CONTINUE
   90 K=K+1
      DO 95 L=1,300
      IF (B(L,1,K).LT.0.001) GO TO 100
   95 CONTINUE
  100 B(L,1,K)=A(J)
      B(L,2,K)=A(J+1)
      K=K-1
  110 CONTINUE
      DO 120 L=1,300
      IF (B(L,1,K).LT.0.001) GO TO 130
  120 CONTINUE
  130 B(L,1,K)=A(J)
      B(L,2,K)=A(J+1)
  150 CONTINUE
  200 CONTINUE
      DO 305 K=1,30
      M=K-1
      PRINT 210,M,K
  210 FORMAT(*0*,I3,1X,*TO*,I3,* CM.*)
      DO 290 L=1,300
      IF (B(L,1,K).LT.0.001) GO TO 300
  290 CONTINUE
  300 M1=L-1
      PRINT 301,((B(L,J,K),J=1,2),L=1,M1)
  301 FORMAT(* *,20X,5(F4.1,2X,F4.1,10X))
  305 CONTINUE
      DO 400 K=1,30
      SUM=SUMQ=DIFQ=0.0
      DO 350 L=1,300
      IF (B(L,1,K).LT.0.001) GO TO 360
      X=(B(L,1,K)+B(L,2,K))/2.0
      SUM=SUM+X
      SUMQ=SUMQ+X*X
      DIF=(B(L,1,K)-B(L,2,K))
      DIFQ=DIF*DIF+DIFQ
  350 CONTINUE
  360 L=L-1
      IF (L.EQ.0) GO TO 372
      AVGE=SUM/FLOAT(L)
      AVGQ=SUMQ/FLOAT(L)
      T=2.0
      DIFU=DIFQ/FLOAT(L)/2.0/T
      GO TO 378
  372 AVGE=AVGQ=DIFU=0.0
  378 PRINT 380,AVGE,AVGQ,DIFU,L
  380 FORMAT(///,*  MEAN LENGTH SCALE*,3X,F5.2,2X,*CM.*,/,*  MEAN SQUARE
     $D LENGTH*,1X,F5.1,2X,*SQR CM.*,/,*  DIFFUSIVITY*,7X,F7.4,2X,*SQR C
     $M. PER SEC*,/,*  NUMBER OF PAIRS*,7X,I3)
  400 CONTINUE
      STOP
      END
```

SORTED DATA PAIRS (CENTIMETERS)

.9	1.0	.8	.8	.8	.7	.7	.5	.5	.5
1.9	1.6	1.6	1.5	1.9	1.0	1.0	1.5	1.5	1.6
1.6	2.2	1.5	2.1	1.9	1.9	1.9	1.8	2.0	1.9
1.9	1.8	1.0	1.6	1.6	1.0	1.3	.9	1.0	1.2
1.2	1.5	1.5	1.7	1.1	1.9	1.9	2.0	2.1	1.0
1.8	1.6	1.7	1.4	1.4	1.6	1.6	1.8	1.8	1.0
1.8	1.7	1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.7
1.7	1.9	1.6	1.7	1.7	1.0	1.0	1.8	1.8	1.8
1.8	1.7	1.6	.8	1.8	2.1	2.0	2.0	2.1	1.8
1.8	1.0	1.8	1.8	1.8	1.7				
2.8	3.1	3.0	2.8	3.2	2.6	2.6	2.2	2.2	2.0
2.0	2.2	2.2	2.6	3.3	2.7	2.7	2.2	2.2	1.9
3.1	2.9	2.9	2.7	2.7	3.4	3.0	2.4	2.4	2.2
2.2	1.9	3.1	2.9	2.9	2.8	2.1	2.4	2.4	2.5
2.5	2.8	2.8	3.2	2.9	2.5	2.2	3.2	2.1	2.7
2.7	2.9	2.9	2.7	2.7	2.2	2.3	2.5	2.5	2.6
2.6	2.5	2.5	2.7	2.7	2.9	2.2	2.4	2.4	2.2
2.2	1.9	2.2	2.3	2.3	2.0	1.0	2.4	2.8	2.4
2.4	2.5	2.5	2.7	2.7	2.3	2.0	3.0	2.3	3.1
2.0	2.2	2.2	2.3	2.3	2.2	3.0	2.5	2.5	2.1
3.0	2.7	2.7	2.6	2.6	2.5	2.5	2.6	2.6	2.6
3.0	3.0	2.3	2.0	2.8	3.1	2.3	3.1	2.1	2.5
2.5	2.5	2.5	2.3	2.3	2.0	2.8	2.4	2.4	2.2
2.2	2.0	2.0	2.0	2.5	2.1	3.1	2.7	2.7	2.4
2.9	2.7	2.7	2.5	2.5	2.4				
3.5	2.9	3.1	3.5	3.5	4.4	3.5	4.2	3.9	3.7
3.7	3.3	3.3	3.1	3.1	3.0	3.3	4.0	3.3	2.7
3.9	3.6	3.6	3.4	3.4	3.3	3.3	3.1	3.1	2.9
4.0	3.9	3.9	3.9	3.9	4.0	4.2	3.7	3.7	3.2
3.0	2.9	3.0	3.5	3.5	3.3	3.4	3.0	3.9	3.3
3.3	3.2	3.2	3.1	3.1	2.9	3.8	4.0	4.0	3.9
3.9	3.8	3.8	3.9	3.9	4.0	3.9	3.7	3.7	3.6
3.6	3.6	3.1	3.6	3.6	3.7	3.7	3.0	3.8	4.1
4.0	3.6	3.6	3.4	3.4	3.5	3.5	3.5	3.5	3.6
3.6	3.6	3.6	3.5	3.5	3.2	3.8	3.9	3.8	3.5
3.5	3.5	3.5	3.6	3.6	3.9	2.0	3.2	3.2	3.8
2.7	3.5	3.5	3.5	3.5	3.3	3.3	2.9	3.2	4.5
3.4	3.8	2.8	3.4	3.4	3.6	3.8	4.1	4.1	3.7
3.1	3.2	3.2	3.3	3.3	3.3	3.3	3.3	4.0	3.0
3.3	4.5	3.7	3.2	3.2	3.2	3.2	3.3	3.3	3.5
3.0	3.0	3.0	3.1	3.1	3.2	3.2	3.5	3.5	3.6
3.1	3.3	3.3	3.5	3.5	3.0	3.1	3.8	3.4	3.2
3.2	3.1	3.1	3.2	3.2	3.4	3.4	3.6	3.8	3.7
3.7	3.7	3.7	3.0	3.0	4.0	3.8	2.8	3.7	3.5
3.5	3.4	3.4	3.2	3.2	3.2	3.2	3.0	3.6	3.5
3.5	3.6	3.6	4.1	4.1	3.7	3.7	3.0	3.5	3.5
3.5	3.8	3.0	4.2	3.5	3.7	3.7	4.1	3.7	4.3
3.5	3.2	3.2	3.2	3.2	3.5	3.5	3.9	4.0	3.8
3.3	3.7	3.7	3.6	3.5	3.7	3.7	4.2	4.0	3.8
3.8	3.8	3.8	4.0	3.6	4.3	4.0	3.6	3.6	3.1
3.9	3.2	3.2	2.9	4.1	3.7	3.7	3.7	3.7	4.2
3.5	3.9	4.0	3.8	3.8	3.6	3.6	3.3	3.3	3.1
3.1	3.5	3.5	4.1	3.9	2.9				
4.0	4.6	4.2	4.4	4.4	4.5	4.5	4.8	4.8	5.2
4.1	4.7	4.7	5.2	4.3	5.3	3.7	4.7	4.3	4.7
4.7	5.2	4.0	4.5	4.5	4.7	4.7	4.6	4.5	4.3
4.3	4.2	4.2	4.3	4.3	4.5	4.5	5.0	4.7	4.0
5.4	4.6	4.6	4.5	4.5	4.5	4.5	4.5	4.5	4.6
4.1	5.1	4.3	5.0	4.1	4.2	4.2	4.0	4.8	4.2
4.2	5.3	4.4	4.4	4.4	4.7	4.7	5.3	5.0	4.8
4.8	4.6	5.2	4.7	4.7	4.5	4.5	4.2	4.2	3.9
4.1	4.3	4.8	4.0	4.1	4.6	4.6	5.2	3.9	4.3
4.3	4.0	4.8	3.8	5.1	4.9	4.2	4.5	4.5	4.9
3.8	4.3	4.3	5.0	4.5	4.8	4.8	4.7	4.7	4.4

4.4	4.1	4.2	4.3	4.3	4.6	4.6	5.1	4.5	4.9
4.5	3.7	3.8	4.4	4.4	4.8	4.0	4.2	3.8	4.3
4.3	4.9	4.1	4.5	4.5	5.2	4.3	4.7	4.7	4.6
4.8	4.8	4.8	4.7	4.7	4.7	3.7	4.4	4.4	5.0
4.9	4.5	4.5	4.3	4.3	4.1	4.8	4.5	4.5	4.3
3.8	4.2	4.2	4.9	4.4	4.9	3.7	4.4	4.4	5.6
4.9	4.7	4.7	5.0	4.4	5.2	4.1	4.4	4.4	5.0
3.7	4.3	4.3	4.9	3.9	4.2	4.2	4.5	4.5	5.0
4.2	4.6	4.6	4.7	4.7	4.7	4.7	4.8	4.8	4.8
4.1	4.3	4.3	4.5	4.5	4.7	4.7	4.9	4.9	5.0
4.0	4.3	4.3	4.3	4.3	4.4	4.4	4.6	4.6	4.8
4.2	4.3	4.3	4.6	4.6	5.1	4.0	4.3	4.3	4.7
4.7	5.2	4.2	4.2	4.2	4.5	4.5	5.0	4.3	4.7
4.7	5.1	3.9	4.6	4.3	4.1	4.1	4.0	4.2	4.7
4.7	5.1	5.1	4.8	4.8	4.8	4.8	4.9	4.0	4.3
4.3	4.8	4.3	4.0	4.1	4.0	5.2	4.7	4.7	4.6
4.6	4.1	3.9	4.3	4.3	4.6	4.6	4.7	4.7	4.3
4.1	4.4	4.1	4.6	4.6	5.1	5.2	3.9		

4.8	5.2	5.2	5.8	5.3	6.1	6.0	5.9	5.9	5.7
5.7	5.6	4.7	5.0	5.2	5.7	5.7	6.1	5.0	5.7
4.6	5.5	5.5	6.3	5.5	5.5	5.5	4.7	5.8	5.9
5.9	6.0	6.0	5.9	5.9	5.7	5.7	5.5	5.4	4.6
5.1	5.7	5.7	6.3	5.0	5.7	6.2	5.7	5.7	5.2
6.0	5.5	5.5	4.0	5.5	5.6	5.5	6.0	5.3	6.2
4.7	5.3	5.3	5.9	6.4	5.5	5.5	5.3	5.3	5.0
6.1	5.7	5.7	5.2	5.3	6.0	5.5	6.5	5.6	5.9
5.6	4.8	6.0	5.9	5.9	6.0	5.0	5.8	5.2	5.6
5.6	6.0	4.8	5.6	5.6	6.3	6.0	5.3	5.3	5.0
5.0	5.1	5.1	5.3	5.3	5.7	5.8	5.0	6.2	5.7
5.7	5.4	5.4	5.1	5.1	4.9	5.5	5.6	5.6	6.1
6.1	5.6	5.6	5.5	5.5	5.5	5.5	5.5	5.5	5.6
4.9	5.2	5.2	5.5	5.5	5.7	5.0	5.9	5.1	5.6
5.2	5.8	5.8	5.2	4.9	5.2	5.2	5.4	5.4	5.2
5.7	5.6	5.6	5.7	5.7	5.7	5.7	5.8	5.8	5.3
4.8	5.3	5.4	5.1	5.1	5.3	5.3	5.6	5.6	5.7
5.7	5.0	5.3	5.4	5.4	5.5	5.0	6.1	4.9	5.3
5.3	5.8	5.0	6.2	6.2	5.3	5.3	5.0	5.0	5.1
5.1	5.4	5.4	5.9	5.2	5.6	5.6	6.0	5.8	5.6
5.6	6.2	5.9	5.3	5.3	4.8	4.9	5.0	4.9	5.2
5.2	5.3	5.3	5.4	5.4	5.2	5.1	6.1	6.2	5.0
5.8	5.4	4.4	5.6	5.6	4.9	5.0	5.4	5.7	6.0
5.2	6.0	5.0	5.8	4.9	5.5	5.5	6.3	5.0	5.4
5.4	5.6	5.8	6.1	5.1	5.7	5.7	6.0	6.0	6.0
6.0	5.8	5.8	5.7	5.7	5.5	5.5	5.3	5.2	5.8
5.8	6.2	5.0	5.4	5.4	5.9	5.1	5.5	5.5	5.8
5.8	6.1	4.9	5.4	5.4	5.7	5.7	6.0	5.1	5.4
6.2	5.6	5.6	5.1	6.0	5.7	5.7	5.5	5.5	5.3
5.3	5.0	5.7	6.3	6.1	5.9	5.9	5.5	4.9	5.2
5.2	5.3	5.3	5.3	5.3	5.0	4.8	5.3	5.3	5.7
5.7	6.1	5.8	5.2	5.9	5.9	5.1	5.3	6.2	5.8
6.5	5.2								

6.2	5.5	6.5	6.8	6.8	6.9	6.9	6.8	6.8	6.7
5.8	6.7	6.1	6.9	6.8	6.3	6.3	6.1	6.1	6.2
6.2	6.0	6.5	6.3	6.8	6.7	6.7	6.2	6.2	6.0
6.0	6.1	6.7	6.9	6.9	6.8	6.8	6.5	6.5	6.3
6.3	5.9	5.8	7.0	6.1	6.2	7.1	6.7	6.7	6.5
6.5	6.4	6.4	6.4	6.9	6.8	6.8	6.7	6.7	6.6
6.6	6.9	6.8	7.9	6.2	6.4	6.4	6.7	6.4	6.3
6.3	6.6	6.6	6.9	6.9	7.0	7.0	7.0	6.3	7.3
7.2	6.5	6.5	5.5	6.8	6.8	6.8	6.9	6.9	7.0
7.0	7.0	6.3	6.7	5.7	6.3	6.3	7.0	5.7	6.4
6.4	7.0	6.7	6.5	6.5	6.4	6.4	6.3	6.3	6.5
6.5	6.8	7.0	6.5	6.5	6.4	6.4	6.2	5.8	7.2
6.0	6.2	6.2	6.3	6.3	6.3	6.2	7.0	5.9	6.4
6.6	6.1	6.0	6.1	6.1	6.2	6.2	6.4	6.4	6.6
5.5	6.5	6.5	7.2	5.9	6.5	6.5	7.2	6.6	7.2
6.4	7.0	6.1	6.2	6.2	6.1	6.1	6.0	5.8	6.3
6.3	6.9	6.0	6.3	7.0	6.9	6.9	6.7	6.7	6.7
6.7	6.5	6.5	6.4	5.9	6.7	6.7	7.3	6.5	7.2
5.8	6.3	6.3	6.7	6.7	6.8	6.8	6.8	6.3	6.2
6.1	6.4	6.4	6.4	6.4	6.1	6.9	7.1	5.8	6.4
6.4	6.9	6.9	7.0	7.0	6.8	6.8	6.6	6.6	6.5
6.1	6.8	6.6	7.3	6.5	6.9	6.5	7.3	6.9	6.6
5.6	6.5	5.8	6.2	6.2	6.5	6.5	6.7	6.7	6.9

6.4	6.1	6.1.	6.3.	6.3	6.7	6.7	7.1	6.4	7.4
5.9	6.5	6.5	7.2	6.7	7.3	6.7	7.0	7.0	6.9
6.9	6.0	6.0	6.7	6.7	6.9	6.9	7.0	6.4	7.5
6.3.	6.9	6.5	7.4	6.2	6.0	6.0	6.5	6.5	6.8
6.8	7.2	6.6	7.1	6.4	6.5	6.5	6.3	6.3	6.3
6.3	6.1	6.1	6.0	7.0	6.9	6.9	6.0	6.8	6.9
6.1	6.5	6.5	6.7	6.3	7.0	5.8	6.2	6.8	6.8
6.8	7.1.	6.5	7.2	6.3	6.7	6.7	7.1	6.8	7.2
6.0	6.3.	6.3	6.5	6.5	7.0	6.2	7.1	6.4	5.9
6.7	7.0	6.1	6.3	6.3	6.2	6.2	5.8	5.6	6.9
6.9	5.6	6.0	6.2	6.2	6.3	6.3	6.4	6.4	6.5
6.4	6.6	6.0	6.7	6.7	6.0	6.0	6.6	6.6	6.3
6.0	6.4	6.4	6.7	6.7	6.9	6.3	7.3	6.6	6.8
6.8	7.0	7.0	6.8	6.8	6.6	6.6	6.6	6.1	6.6
6.6	6.9	6.7	6.7	6.7	6.7	6.7	6.8	6.0	6.9
6.0	7.0	6.7	6.8	6.8	6.9	6.9	7.1	6.0	6.0
5.0	6.2	6.3	6.6	6.6	6.8	6.8	7.1	6.0	6.3
6.3	6.0	6.7	6.9	7.0	6.2	6.8	6.0	5.7	6.3
6.3.	7.0	6.2	6.3	6.3	6.3	6.3	6.1	6.1	5.9
6.5	7.1.	7.1	6.7	6.7	6.2	6.3	6.3	6.3	6.4
6.4	6.6.	6.6	6.7	6.7	6.6	7.0	6.4	7.2	6.5
6.5	5.0.	5.9	6.3	6.3	7.0	6.5	6.5	6.5	6.5
6.5	6.5	6.5	6.5	6.5	6.4	6.9	6.7.	6.7	6.7
6.7	6.5	6.5	6.4	6.4	6.0	6.9	6.2	6.2	5.8

7.2	8.7	7.2	8.2	6.7	7.4	5.9	7.8	7.4	7.9
7.0.	7.5	7.5	7.3	7.3	6.0	7.0	8.3	7.1	7.6
7.6	7.7	7.7	7.7	7.7	7.9	7.3	7.1	8.0	7.4
6.9	7.3	7.9.	7.8	7.8	8.0	6.7	7.4	7.4	8.2
7.0.	7.0	7.3	8.2	6.9	7.2	7.2	7.5	7.5	7.9
7.5	6.0	7.0.	7.0	6.7	7.5	7.5	8.0	7.0	7.3
7.0	7.4	7.2.	7.7	7.5	8.4	7.2	8.3	7.0	7.8
7.2	7.1	7.1	7.1	7.1	7.2	7.2	7.1	7.1	7.0
7.2	7.7	7.2	7.7	7.7	8.2	7.2	7.4	7.4	7.6
7.6	8.0	7.0	7.4	7.4	7.6	7.6	7.7	7.7	8.1
6.7	7.5	7.5	8.2	7.0	7.5	7.5	8.0.	7.1	8.3.
6.9	7.3	7.3	7.6	6.7	7.3	7.3	7.8	7.0	8.1
7.2	7.5	7.5	7.6	7.6	7.5	7.5	7.1	7.2	8.4
6.9	7.1	7.1	7.7	6.0	7.6	7.6	8.4	7.3	7.9
6.7	7.6	7.6	8.4	6.9	7.2	7.2	7.2.	7.2	7.6
7.6	7.9	7.3	8.3	6.8	7.5	7.5	7.6	7.6	7.7
7.7	8.2	8.0	7.3	7.3	6.9	6.6	7.9	7.1	7.3
7.4	7.7	7.7	7.8	7.8	7.9	7.2	7.7.	7.7	8.2
6.7	7.3	7.3	8.2	6.0	7.3	7.3	7.2.	7.2	7.1
7.1	7.3	7.3	7.6	7.0	7.5	7.5	8.0	7.5	8.4
6.9	7.4	7.4	7.7	7.7	8.0	7.4	8.1.	6.8	7.0
6.0	7.3	7.3	8.0	6.8	7.2	7.2	7.5	7.5	7.8
7.4	8.3	6.9	7.7	7.7	8.3	7.4	8.6	7.1	7.3
7.3	7.6	7.6	7.8	7.0	8.1	7.0	7.9	7.1	7.5
7.5	7.5	7.5	7.6	7.6	7.5	7.5	7.5	6.9	7.2
7.2	7.2	7.2	7.5	7.5	7.7	7.7	8.0	7.0	7.4
7.4	7.6	7.6	7.8	7.0	7.9	7.1	7.5	7.5	7.6
7.6	7.5	7.2	8.0	7.2	7.3	7.3	7.6	7.6	7.6
7.6	7.5	7.5	7.4	7.5	7.0	7.0	7.1	7.1	7.4
7.4	7.8	7.8	8.2	7.3	7.0	7.0	7.1	7.1	7.3
7.3	7.7	7.7	8.2	7.1	7.6	7.6	8.4	6.0	7.2
7.2	7.5	7.5	7.8	7.1	8.1	7.3	8.1	8.0	7.6
7.6	7.4	7.3	8.2	7.0	7.2	7.2	7.3	7.3	7.6
7.6	7.0	6.9	8.0	7.4.	7.5	7.5	7.5	7.5	7.4
7.4	7.4	7.4	7.6	7.0.	8.0	7.0	7.5	7.5	8.2.
7.6	6.4	6.9	7.3	7.3	7.5	6.9	7.2	7.0	7.8
7.1	7.3	7.3.	7.6	7.6	8.1	7.5	7.3	7.3	7.2
7.2	7.2	7.2	7.1	7.1.	7.0	6.9	7.1	7.1	7.4.
7.4	7.5	7.3	7.3.	7.3	7.2	7.2	7.4	7.4	7.5
7.5.	7.8	8.1	7.8	7.0	7.4	7.4	7.2	7.2	7.0.
7.0.	7.2	7.7	7.6	7.6	7.7	7.7	7.8	7.8	8.1
6.4	7.7	7.1	7.4	7.4	7.7	7.0	7.3	7.3	7.5
7.5	8.0	6.9	7.4	7.4	7.0	7.0	8.1	7.0	7.8
7.1.	7.7	7.8	7.7	7.7	7.5	7.5	7.1	7.0	7.1
7.1	7.2	7.2	7.1	7.1	7.0	7.6	8.2	8.0	7.2
7.0	7.8	7.8	7.9	6.7	7.8	7.9	7.8	7.8	8.1
7.6.	6.9	7.9	6.5						

8.2	8.7	8.7	9.0	7.8	8.7	7.9	8.4	8.4	8.7
8.7	8.8	8.8	9.2	7.9	8.8	7.8	9.7	8.3	9.5
7.9	8.3	8.6	9.1	9.0	8.6	8.6	8.0	9.1	8.3
8.3	7.8	8.0	8.5	7.9	8.6	8.6	8.6	8.6	8.3
8.3	8.0	8.2	9.3	8.2	8.8	7.9	9.6	7.9	8.5
8.5	9.3	8.9	8.6	8.4	9.4	7.7	8.6	8.4	9.0
8.3	9.3	7.8	8.6	7.8	9.0	8.7	9.0	7.7	8.4
8.4	9.1	8.0	8.5	8.1	8.4	8.2	8.7	8.0	8.6
8.6	9.1	8.3	9.3	8.2	8.7	8.7	8.9	8.0	8.2
8.2	8.3	8.3	8.4	8.4	8.8	8.1	8.3	8.4	8.8
7.7	8.5	8.5	9.4	7.6	8.4	8.4	9.6	7.9	8.6
7.6	8.4	8.4	8.9	8.3	9.5	8.2	8.9	7.8	8.7
7.8	8.3	8.3	8.0	7.9	8.5	8.5	9.0	7.9	8.2
8.2	8.5	8.2	9.0	8.0	8.2	8.2	8.3	8.4	9.3
8.0	8.2	8.1	8.7	8.7	8.9	8.9	9.0	7.8	8.6
8.6	9.2	8.0	8.7	8.7	9.1	8.3	9.2	7.7	8.3
8.3	8.8	8.8	9.2	7.9	8.5	8.5	9.3	7.4	8.6
8.6	9.3	7.9	8.6	8.6	9.0	7.5	8.8	8.0	8.2
8.2	8.4	8.0	8.7	8.9	8.9	8.9	8.9	8.9	9.0
9.0	9.0	7.8	8.2	7.6	8.4	8.4	9.4	7.8	8.4
8.4	9.2	8.1	8.8	8.1	8.9	8.0	8.5	8.5	8.6
8.6	8.8	8.8	9.0	8.5	8.5	8.5	8.3	8.3	8.0
8.6	8.8	8.8	9.0	9.0	9.0	9.0	8.9	8.9	8.7
8.2	8.7	8.7	9.2	8.0	8.8	8.6	9.2	8.3	9.6
7.9	8.4	8.4	9.0	8.7	9.3	8.0	8.7	8.7	9.2
7.8	8.8	8.2	8.8	8.2	8.9	7.9	9.0	7.6	8.4
8.4.	9.3	7.9	8.8	8.1	8.6	8.6	9.3	7.9	8.3
8.3	8.4	8.4	8.0	8.4	8.3	8.3	8.2	7.7	8.7
8.3	8.8	8.8	9.1	8.2	8.4	8.4	8.6	8.6	9.0
7.6	9.3	7.9	8.3	8.3	9.1	8.2	9.3	8.1	8.5
7.7	8.9	8.0	8.7	8.1	8.5	7.8	8.8	8.4	9.4
8.3	8.6	8.6	9.0	7.8	8.5	8.5	9.1	7.7	8.5
8.5	9.2	8.6	8.6	8.6	9.1	8.2	8.6	8.6	9.3
8.1	8.9	7.8	8.7	8.5.	9.1	8.1	8.7	7.6	8.7
8.3	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	9.4
9.3	8.5	8.5	7.6	8.9	7.9				

8.7	10.0	9.0	9.1	9.1	9.1	8.8	9.2	8.8	10.0
9.1	9.8	9.3	10.1	10.2	9.8	9.8	9.3	9.3	8.9
9.4	10.2	8.6	9.5	9.5	10.3	9.0	9.5	9.5	9.8
9.8	10.0	9.3	10.0	9.0	9.8	9.4	9.7	9.0	9.5
9.5	10.1	8.7	9.4	9.4	10.0	9.1	9.5	9.5	9.8
9.8	9.9	9.7	9.7	9.7	9.6	9.6	9.6	9.6	9.6
9.6	9.9	9.3	10.1	8.9	9.2	9.2	9.4	9.4	9.8
8.8	9.3	8.8	9.3	9.3	9.7	9.7	10.2	9.4	10.2
8.4	9.6	8.6	9.7	8.9	9.6	9.6	10.1	8.7	9.5
9.5	10.4	9.0	9.5	9.5	10.0	9.8	9.7	9.7	10.1
9.3	9.9	9.2	9.5	9.5	9.5	9.2	9.6	9.6	9.9
9.9	9.7	8.8	9.2	9.2	9.3	9.3	10.2	9.3	9.7
9.7	9.6	9.6	9.5	9.0	9.3	9.3	9.2	8.8	9.5
9.5	10.1	8.7	9.8	9.3	9.4	9.4	10.0	9.3	8.9
9.0	9.0	8.8	9.5	9.5	9.7	9.7	9.7	8.9	9.8
9.0	9.3	8.7	9.7	9.4	10.2	9.0	9.0	9.2	9.5
9.5	10.0	9.2	9.4	9.4	9.4	9.4	9.4	9.4	9.3
9.0	9.5	9.5	10.3	8.7	9.3	9.3	9.7	9.7	10.0
9.2	9.6	9.6	9.9	8.8	9.6	9.6	10.4	8.8	9.4
9.4	9.9	8.9	9.3	9.3	9.6	9.6	9.6	9.6	9.5
9.3	10.2	9.0	10.1	9.3	10.2	8.8.	9.7	9.7	10.3
8.7	9.8	9.1	9.3	9.3	9.6	9.0	9.8	9.0	9.4
9.4	9.3	9.3	10.7	8.5	9.7	9.1	9.9	9.3	10.0
8.9	9.9	8.7	9.6	8.8	9.5	9.5	10.3	9.4	10.3
9.6	10.3	9.0	9.2	9.2	9.4	9.4	9.6	9.0	9.5
9.1	9.8	10.0	9.7	9.7	9.4	9.4	8.8	9.3	10.1
8.9	9.5	9.5	10.0	8.7	9.4	9.4	10.0	9.1	9.3
9.3	9.5	9.5	9.7	9.7	10.0	8.7	9.5	9.5	9.7
8.7	10.0	9.9	9.8	9.8	9.9	9.9	9.7	9.7	9.5
9.5	9.3	10.0	9.9	9.9	9.7	9.7	9.5	9.5	9.2
9.2	8.9	9.8	9.2	10.1	9.8	9.8	10.2	10.0	9.6
9.6	9.4	9.4	9.4						

10.0	11.3	10.0	11.2	9.7	11.1	10.5	11.2	9.6	11.2
10.1	11.3	10.1	10.2	10.2	9.8	10.3	10.9	9.8	10.5
10.5	11.1	10.3	10.6	10.6	11.1	10.8	10.9	9.8	10.5
10.5	11.1	9.8	10.8	10.1	10.5	9.9	10.2	10.2	10.5
10.1	10.7	10.7	11.2	10.3	10.9	10.5	11.1	9.6	10.5
9.7	11.0	9.5	11.2	10.4	11.5	9.9	10.3	10.2	13.0
10.8	11.0	10.1	10.2	10.2	10.0	9.8	10.7	10.0	10.7
9.8	10.9	9.7	10.6	10.6	11.4	10.2	10.8	10.8	11.0
9.6	10.5	10.5	11.2	11.0	10.8	10.8	10.5	10.3	11.0
10.0	10.2	10.2	10.2	9.6	10.4	10.4	11.1	10.2	10.9
10.1	10.9	10.2	11.0	9.7	10.3	9.8	10.8	9.9	11.1
10.3	10.6	10.6	10.7	10.7	10.0	10.8	10.7	10.7	10.5
9.3	10.7	9.7	10.5	10.5	11.3	9.9	10.7	10.7	11.3
10.0	10.6	10.6	10.0	10.0	11.0	10.5	11.1	9.9	10.6
10.6	10.9	10.3	10.0	10.3	11.2	10.3	11.3	10.5	10.4
10.4	10.4	10.4	10.3	10.3	10.3	10.3	10.2	9.8	10.6
10.6	11.1	10.1	10.2	10.2	10.0	10.1	10.8	10.0	10.3
10.3	10.7	10.0	10.4	10.0	11.4	10.7	10.0	11.0	10.7
10.3	10.5	10.5	10.6	10.5	10.3	10.3	9.8	11.3	10.6
11.2	10.0	10.8	10.4	10.4	10.1	9.8	10.2	10.2	10.7
11.0	10.5	10.5	10.0						

11.2	12.4	11.1	12.1	10.4	12.0	11.2	11.9	10.7	11.5
11.6	12.3	11.2	12.1	11.3	12.6	11.1	12.0	11.1	11.6
11.6	11.6	11.6	11.5	11.1	11.7	10.0	11.5	11.6	12.0
10.9	11.4	11.4	12.2	11.1	11.7	11.7	12.0	11.2	12.8
11.5	12.5	10.7	11.4	11.4	12.1	10.9	12.4	10.6	11.4
11.4	11.9	11.2	12.1	11.3	11.9	11.0	11.2	11.2	11.7
11.2	11.7	11.7	11.8	10.4	11.8	11.1	11.4	11.4	11.2
11.2	11.0	11.1	11.7	10.9	11.5	11.5	11.9	10.6	12.0
10.9	11.7	11.0	11.5	10.0	11.8	11.1	12.1	10.7	11.9
11.3	11.8	11.8	12.2	10.7	11.3	11.1	11.4	11.4	11.7
11.7	11.7	11.7	11.8	10.8	11.3	11.2	12.0	11.3	12.2
11.4	12.5	11.2	12.4	12.2	11.5	11.5	10.7	11.8	11.5
11.5	11.0	12.2	11.6	11.9	11.5	11.5	11.3	11.3	10.9
11.9	12.1	11.1	11.4	11.4	11.7	11.7	11.3	11.8	11.9
11.9	11.7	11.3	11.5	11.5	11.8	11.8	12.2	12.1	11.8
11.8	11.7	11.7	11.6	11.5	11.7	11.7	11.8	11.8	11.7
11.7	11.5	11.5	11.2	12.0	11.3	12.1	11.9	11.9	11.5
11.5	11.2	11.2	10.8	12.3	11.6	11.6	11.0		

11.3	12.8	12.4	13.4	12.1	12.8	12.0	13.6	11.9	12.5
12.5	13.0	12.3	13.0	12.1	12.6	12.6	12.8	12.6	13.2
12.0	12.8	12.0	12.5	12.5	13.1	12.0	12.6	12.6	13.2
12.0	12.3	12.3	12.5	11.2	12.8	11.5	12.5	11.9	12.3
12.1	13.1	11.9	12.2	12.2	12.2	12.2	12.1	12.1	12.3
11.8	13.1	11.9	12.2	12.0	13.2	11.7	12.6	11.8	12.7
12.1	13.2	11.9	12.8	11.8	12.2	12.0	13.0	12.2	13.1
12.5	13.0	12.4	13.4	12.2	12.5	12.5	12.2	13.1	12.7
12.7	12.3	12.3	11.8	12.2	12.5	12.5	12.8	12.8	13.2
12.2	13.3	13.1	12.8	12.8	12.7	12.7	12.6	12.6	12.5
12.5	12.2	12.2	11.8	12.5	12.5	12.5	12.6	12.6	12.7
12.7	12.7	12.7	12.7	13.0	12.7	12.7	12.3	12.3	12.3
12.7	12.3	12.3	11.9	11.9	12.1	12.1	12.2	12.2	12.5
12.5	12.6	12.6	12.9	13.1	12.6	12.6	12.2	12.9	12.2
13.5	12.2	12.6	12.9	13.1	12.8	12.8	12.4	12.8	13.2
11.8	12.2	12.2	12.5	12.5	12.3	12.7	12.5	12.5	12.1
13.2	12.5	12.5	12.0	12.1	12.1	12.1	11.9	13.0	12.3

12.8	13.8	12.8	13.4	13.0	13.6	13.0	13.6	13.6	14.2
13.2	13.5	13.5	13.4	12.8	13.7	13.1	13.9	13.1	14.3
13.2	14.3	13.2	14.4	12.8	13.7	13.1	14.0	13.4	14.2
13.8	14.2	13.6	13.8	13.8	13.8	13.8	13.5	13.5	13.1
12.8	13.2	13.2	13.5	13.5	13.8	13.8	13.8	13.8	13.7
13.7	13.7	13.7	13.7	13.7	14.0	14.1	13.5	13.5	12.9
13.3	13.8	13.8	14.1	14.0	13.6	14.1	13.5	13.5	13.1
13.6	13.2	13.2	13.0	13.8	13.1	13.7	12.9	14.0	13.7
13.7	13.6	13.6	13.7	13.7	13.6	13.6	13.7	13.2	13.9
14.3	13.7	12.9	13.3	13.3	13.1	13.6	13.8	13.8	14.1
13.7	14.3	12.8	13.2	13.2	13.6.	13.6	13.8	13.8	13.9
13.9	13.9	13.8	13.2	13.9	13.5	13.5	13.0		

13.6	14.9	13.9	14.6	14.6	15.3	14.3	15.4	14.3	15.4
14.4	15.3	14.2	14.9	13.8	14.2	14.2	14.5	14.5	14.5
14.5	14.5	14.5	14.7	14.7	15.0	15.0	14.8	14.8	14.5
14.5	14.1	14.1	14.1	14.1	14.2	14.2	14.2	14.2	14.1
14.0	14.0	14.4	14.9	14.2	14.5	14.5	14.9	14.1	14.3
14.3	14.3	14.3	14.5	14.5	14.4	14.4	14.3	14.3	14.8
14.6	14.4	14.4	14.1	15.1	14.8	14.5	14.3	14.3	14.2
14.2	14.3	14.3	14.6	14.6	15.0	15.8	14.5	14.5	13.8
14.6	15.2	15.1	14.5	14.5	13.7	14.5	14.7	14.7	14.9
14.9	14.9	14.9	14.8	14.8	14.8	14.7	13.5	13.9	14.6
15.2	14.7	14.7	14.3	14.3	13.7	14.1	14.4	14.4	14.5
14.5	14.1	13.7	14.3	14.3	14.8	15.0	14.6	14.6	14.2
14.2	13.9								

14.9	16.1	15.4	16.6	15.4	16.5	14.9	15.4	15.0	15.1
15.1	15.3	15.3	15.3	15.3	15.3	15.3	15.8	14.9	15.3
15.3	15.3	15.3	15.4	14.9	15.7	15.0	15.5	15.5	16.2
16.0	15.7	15.7	15.4	15.4	15.1	15.1	15.8	15.7	15.2
15.2	15.2	15.2	15.4	15.4	15.7	15.7	15.9	15.2	15.6
15.6	16.0	15.8	15.1	15.7	14.7	14.6	15.5	15.5	16.4
16.3	15.6	15.6	15.2	15.3	15.6	15.6	15.7	15.7	16.2
14.8	15.6	15.6	16.3	15.4	15.7	15.7	15.9	15.9	15.8
15.8	15.4	15.4	15.8	14.9	15.2	15.2	15.3	15.3	15.5
15.5	15.3	15.3	15.8	15.7	15.5	15.5	15.8	15.1	15.3
15.3	15.4	15.4	15.5	15.5	15.7	15.7	15.7		

16.1	17.3	15.4	15.6	17.2	16.8	16.8	16.5	16.5	16.5
16.5	16.8	16.8	17.0	17.8	17.3	15.7	16.5	16.5	17.1
16.0	16.3	16.3	16.5	16.5	16.3	16.3	16.0	15.8	16.4
16.4	17.0	16.8	16.5	16.5	16.9	16.8	16.3	16.3	16.7
15.7	17.3	17.8	16.6	16.6	16.4	16.4	16.5	16.5	17.0
16.6	16.5	16.5	15.7	16.4	17.1	16.2	16.5	16.5	16.7
16.1	16.1	16.1	15.1	16.1	16.4	16.4	16.7	16.7	17.2
16.3	16.7	15.7	16.4	16.4	16.7	16.7	16.9	17.8	17.8

17.3	18.2	17.8	17.2	17.2	16.8	17.8	17.8	17.0	17.1
17.1	17.4	17.4	17.7	17.7	17.7	17.7	17.6	17.6	17.3
17.8	17.6	17.6	18.0	18.8	17.9	17.5	18.2	16.7	17.3
17.3	17.8	17.8	18.2	17.3	17.0	16.8	17.4	17.4	17.9
16.9	17.2	17.2	17.6	17.9	17.5	17.5	17.1	17.1	17.8
17.0	17.0	17.0	17.2	17.2	17.5	17.5	18.8		

18.2	18.8	18.8	19.0	19.1	18.9	18.9	19.8	18.4	18.3
18.3	17.8	18.9	18.6	18.6	18.3	18.3	18.1	18.1	18.8
18.2	18.7	18.7	19.2	17.8	18.2	18.3	18.6	18.6	18.5
18.5	18.4	18.4	18.4	18.4	18.2	18.8	18.4	17.9	18.5
18.5	19.1	18.7	18.6	18.6	18.4	18.4	17.9	18.8	18.4
18.4	18.7	18.7	18.8	18.8	19.1				

19.0	19.1	19.1	19.3	19.3	19.6	19.6	19.8	19.8	19.9
19.9	19.9	19.9	19.7	19.7	19.5	19.5	19.3	19.3	19.1
19.1	18.9	19.8	19.2	19.2	19.5	19.5	19.6	19.6	20.8
19.2	19.5	19.5	19.6	20.4	19.6	19.6	18.8	19.1	19.7
18.9	19.6	19.6	20.0	19.1	19.1	19.1	19.2	19.2	19.3
19.3	19.5	19.5	19.6	19.6	19.7				

20.8	20.2	20.8	21.1	21.1	20.4	20.4	19.6	20.6	21.0
20.4	20.6	20.6	20.8	20.8	21.1	20.8	20.2	20.2	20.6
20.6	20.9								

20.8	22.0	21.1	21.6	21.6	21.9	21.7	21.1	21.0	21.4
21.4	21.8	21.1	21.3	21.3	21.5	21.8	22.8		

23.1	22.9	22.9	22.7	22.8	22.8	21.9	22.2	22.2	22.3
21.8	22.3	22.3	22.7	22.0	23.1				

23.9	23.5	23.5	23.1	23.1	22.9	24.1	23.9	23.9	23.7
23.7	23.5	23.5	23.4	23.1	23.3	23.3	23.4	23.4	23.3
23.3	23.3	23.3	23.0	23.0	23.8	23.8	23.8	23.8	23.8
23.8	23.7	23.7	23.4	24.0	23.8	23.8	23.7	23.7	23.8
23.0	23.7	23.7	23.6	22.0	23.7	23.7	24.3	23.1	24.1
24.4	23.9	24.2	24.1	24.1	23.9	24.5	24.8	24.8	25.0
23.7	24.3	24.3	25.3	23.9	24.6	24.6	25.4	24.1	25.1
25.0	25.3	25.3	25.5	25.5	25.7	24.6	25.4	25.4	26.1
25.1	25.7	24.8	25.5	25.5	26.3				
26.0	26.5	26.5	27.2	26.0	26.4	26.4	26.9	26.1	26.7
26.7	27.2	26.3	26.9						
27.2	27.7	27.7	28.0	26.9	27.3	27.3	27.6	27.6	27.7
27.3	27.7	27.7	28.0	28.0	27.9	26.9	27.5	27.5	27.9
28.0	28.3	28.0	28.1	28.1	29.0				
0	0								

APPENDIX B

METHOD II: PROGRAM LISTING

SORTED DATA PAIRS

```
      DIMENSION A(6),B(100,2,150)
      DO 1 I=1,100
      DO 1 J=1,2
      DO 1 K=1,150
    1 B(I,J,K)=0.0
      PRINT 2
    2 FORMAT(1H1,10X,*INITIAL LENGTH*,5X,*LENGTH SQUARED*,5X,*DIFFUSIVIT
     $Y*,5X,*NUMBER OF PAIRS*,//)
C
C     SORTING PAIRS BY L KNOT
C
      READ 5,N
    5 FORMAT(I5)
      DO 60 I=1,N
      READ 10,(A(J),J=1,6)
   10 FORMAT(6F5.1)
      DO 50 J=1,5
      IF (A(J).LT.0.9) GO TO 50
      IF (A(J).GT.15.1) GO TO 50
      DO 20 K=10,150,2
      S=FLOAT(K)/10.+0.1
      BUG=A(J)-S
      BIG=ABS(BUG)
      IF (BIG.LT.0.001) GO TO 30
      IF (A(J).LT.S) GO TO 40
   20 CONTINUE
      GO TO 50
   30 K=K+2
      DO 31 L=1,100
      IF (B(L,1,K).LT.0.001) GO TO 32
   31 CONTINUE
   32 B(L,1,K)=A(J)
      B(L,2,K)=A(J+1)
      K=K-2
   40 CONTINUE
      DO 41 L=1,100
      IF (B(L,1,K).LT.0.001) GO TO 42
   41 CONTINUE
   42 B(L,1,K)=A(J)
      B(L,2,K)=A(J+1)
   50 CONTINUE
   60 CONTINUE
C
C     FIND DIFFUSIVITIES
C
      DO 200 K=10,150,2
      AM=FLOAT(K)/10.
      DIFQ=0.0
      DO 110 L=1,100
      IF (B(L,1,K).LT.0.001) GO TO 120
      DIF=AM-B(L,2,K)
  110 DIFQ=DIF*DIF+DIFQ
  120 L=L-1
      IF (L.EQ.0) GO TO 130
      T=2.0
      DIFU=DIFQ/FLOAT(L)/2.0/T
      GO TO 140
  130 DIFU=0.0
  140 AMSQ=AM*AM
      PRINT 150,AM,AMSQ,DIFU,L
  150 FORMAT(1H ,15X,F4.1,15X,F5.1,13X,F5.4,13X,I3)
  200 CONTINUE
      STOP
      END
```

SORTED DATA PAIRS (CENTIMETERS)

.9	1.0	1.0	1.2	1.1	1.9				
1.3	.9	1.2	1.5	1.1	1.9	1.3	1.3	1.3	1.7
1.5	1.6	1.5	2.1	1.3	.9	1.5	1.7	1.4	1.6
1.4	1.4	1.4	1.3	1.3	1.3	1.3	1.7		
1.6	1.5	1.5	1.6	1.6	2.2	1.5	2.1	1.6	1.8
1.5	1.7	1.7	1.4	1.6	1.8	1.7	1.9	1.6	1.7
1.7	1.8	1.6	.8						
1.9	1.6	1.9	1.8	1.8	1.5	1.9	1.9	1.9	1.8
1.9	1.8	1.8	1.6	1.8	2.4	1.9	2.0	1.8	1.6
1.7	1.4	1.8	1.8	1.8	1.7	1.7	1.9	1.7	1.8
1.8	1.8	1.8	1.8	1.8	1.7	1.8	2.1	1.8	1.9
1.8	1.8	1.8	1.7						
2.0	2.2	1.9	1.6	1.9	1.8	2.1	2.4	2.1	2.7
1.9	1.9	1.9	1.8	2.0	1.9	1.9	1.8	1.9	2.0
2.0	2.2	2.1	1.8	2.1	2.5	2.0	2.0	2.1	1.8
2.2	2.0	2.2	2.6	2.2	1.9	2.2	1.9	2.1	2.4
2.2	3.2	2.1	2.7	2.3	2.5	2.2	2.4	2.2	1.9
2.2	2.3	2.3	2.8	2.3	3.1	2.2	2.3	2.3	2.2
2.1	1.8	2.3	2.8	2.3	3.1	2.1	2.5	2.3	2.0
2.2	2.0	2.1	1.8						
2.4	2.2	2.4	2.5	2.5	2.8	2.3	2.5	2.5	2.6
2.5	2.7	2.4	2.2	2.3	2.8	2.4	2.5	2.5	2.7
2.3	3.1	2.3	2.2	2.5	2.1	2.5	2.6	2.3	2.8
2.3	3.1	2.5	2.5	2.5	2.3	2.3	2.0	2.4	2.2
2.5	2.1	2.5	2.4						
2.6	2.2	2.7	2.2	2.7	3.0	2.5	2.8	2.7	3.5
2.7	2.9	2.7	2.2	2.5	2.6	2.6	2.5	2.5	2.7
2.7	2.9	2.5	2.7	2.7	2.8	2.5	2.1	2.7	2.0
2.6	2.5	2.5	2.6	2.6	2.6	2.5	2.5	2.5	2.3
2.5	2.1	2.7	2.4	2.7	2.5	2.5	2.4		
2.8	3.1	2.7	2.2	2.9	2.7	2.7	3.0	2.9	2.8
2.8	3.2	2.7	3.5	2.9	2.5	2.7	2.9	2.9	2.7
2.7	2.2	2.7	2.9	2.8	3.4	2.8	2.4	2.7	2.8
2.8	3.0	2.7	2.6	2.8	3.1	2.8	2.4	2.7	2.4
2.9	2.7	2.7	2.5						
3.1	3.5	3.1	3.0	3.0	2.8	3.1	2.9	2.9	2.7
3.0	3.5	3.0	2.4	3.1	2.9	2.9	2.8	3.1	3.6
2.9	2.5	2.9	2.7	3.1	3.2	3.0	2.5	3.0	2.7
3.0	3.0	3.0	3.1	3.1	3.2	3.1	3.3	3.1	3.8
3.1	3.2	3.1	2.7	2.9	2.7	3.1	3.5		
3.1	3.5	3.3	3.1	3.1	3.0	3.2	2.6	3.3	2.7
3.3	3.1	3.1	2.9	3.3	3.2	3.2	3.1	3.1	2.9
3.1	3.6	3.2	3.8	3.3	2.9	3.2	4.6	3.1	3.2
3.2	3.3	3.3	3.3	3.3	3.3	3.3	4.5	3.2	3.2
3.2	3.3	3.3	3.5	3.1	3.2	3.2	3.5	3.1	3.3
3.3	3.5	3.1	3.8	3.2	3.1	3.1	3.2	3.2	3.4
3.2	3.2	3.2	3.0	3.2	3.2	3.2	3.5	3.1	2.7
3.2	2.9	3.3	3.1	3.1	3.5				
3.5	2.8	3.5	4.0	3.5	4.2	3.3	3.1	3.3	2.7
3.4	3.3	3.3	3.1	3.5	3.8	3.4	3.0	3.3	3.2
3.4	3.5	3.5	3.5	3.5	3.6	3.5	3.2	3.5	3.5
3.5	3.6	3.5	3.5	3.5	3.3	3.3	2.9	3.4	3.8
3.4	3.8	3.3	3.3	3.3	3.3	3.3	4.5	3.3	3.5
3.5	3.6	3.3	3.5	3.5	3.8	3.4	3.2	3.4	3.6
3.5	3.4	3.4	3.2	3.5	3.6	3.5	3.5	3.5	3.8
3.5	3.7	3.5	3.2	3.5	3.9	3.5	3.7	3.5	3.9
3.3	3.1	3.5	4.1						

3.5	2.8	3.5	4.0	3.5	4.2	3.7	4.7	3.7	3.3
3.6	3.4	3.7	3.2	3.5	3.8	3.7	3.6	3.6	3.6
3.6	3.7	3.7	3.8	3.6.	3.4	3.5	3.5	3.5	3.6
3.6	3.6.	3.6.	3.5	3.5	3.2.	3.5	3.5	3.5	3.6
3.6	3.9	3.5	3.5	3.5.	3.3.	3.7.	3.2.	3.5	3.6
3.5.	3.8.	3.7	3.7	3.7	3.8	3.7	3.5	3.5	3.4
3.6	3.5	3.5	3.6	3.6	4.1	3.7	4.4	3.7	3.0
3.5	3.5	3.5	3.8	3.7	4.4	3.5	3.7	3.7	4.1
3.7	4.3	3.5	3.2	3.5	3.9	3.7	3.6	3.5	3.7
3.7.	4.2	3.6	4.3	3.6	3.1.	3.7	3.7	3.7	4.2
3.5.	3.9	3.6	3.3	3.5	4.1				

3.7	4.7	3.9	3.7	3.7	3.3	3.8	4.0	3.9	3.6
3.9	3.9	3.9	4.0	3.7	3.2	3.8	2.9	3.9	3.3
3.8	4.0	3.9	3.8	3.8	3.9	3.9	4.0	3.9	3.7
3.7	3.6	3.7	3.8	3.8	4.1	3.8	3.9	3.9	4.3
3.8	3.5	3.8	4.3	3.8	4.1	3.7	3.2	3.8	4.4
3.8	3.7	3.7	3.7	3.7	3.8	3.8	4.0	3.8	2.8
3.8	4.3	3.7	3.5	3.7	4.4	3.7	3.0	3.8	4.2
3.7	4.4	3.7	4.1	3.7	4.3	3.9	4.2	3.9	4.6
3.8	3.7	3.7	3.6	3.7	4.2	3.8	3.8	3.8	4.0
3.9	3.2	3.7	3.7	3.7	4.2	3.9	4.3	3.8	3.6
3.9	2.9								

4.0	4.6	4.1	4.7	3.9	3.7	4.0	4.5	4.1	5.1
3.9	3.6	4.1	4.2	4.0	3.9	3.9	3.9	3.9	4.0
3.9	3.3	4.0	3.9	3.9	3.8	3.9	4.0	3.9	3.7
4.1	4.3	4.0	3.6	4.1	4.6	3.9	4.3	4.1	3.7.
4.0	3.0	4.0	4.2	4.1	4.5	4.1	3.7	4.1	4.4
3.9	4.2	4.1	4.3	4.0	4.3	4.0	4.3	3.9	4.6
4.1	4.0	4.0	3.8	4.0	3.8	4.0	4.3	4.1	4.0
4.0	3.6	3.9	3.2	4.1	3.7	3.9	4.3	4.1	4.0
4.0	3.8	4.1	4.6	3.9	2.9				

4.2	4.4	4.1	4.7	4.3	5.3	4.3	4.7	4.3	4.2
4.2	4.3	4.3	4.5	4.1	5.1	4.3	5.0	4.1	4.2
4.2	4.0	4.2	3.7	4.2	5.3	4.2	3.9	4.1	4.3
4.1	4.6	4.3	4.8	4.2	4.5	4.3	5.0	4.1	3.7
4.2	4.3	4.3	4.6	4.3	4.9	4.1	4.5	4.3	4.7
4.3	4.1	4.1	3.7	4.2	4.9	4.1	4.4	4.3	4.9
4.2	4.5.	4.2	4.6	4.1	4.3	4.3	4.5	4.3	4.3
4.3	4.4	4.2	4.3	4.3	4.6	4.3	4.7	4.2	4.2
4.2	4.5	4.3	4.7	4.3	4.1	4.1	4.0	4.2	4.7
4.3	4.8	4.3	4.8	4.1	4.0	4.1	3.7	4.3	4.6
4.1	4.0	4.1	4.6						

4.4	4.5	4.5	4.8	4.3	5.3	4.3	4.7	4.5	4.7
4.3	4.2	4.3	4.5	4.5	5.0	4.5	4.5	4.5	4.5
4.5	4.6	4.3	5.0	4.4	4.4	4.4	4.7	4.5	4.2
4.3	4.8	4.5	4.9	4.3	5.0	4.5	4.8	4.4	4.1
4.3.	4.6	4.5	4.9	4.5	3.7	4.4	4.8	4.3	4.9
4.5	5.2	4.3	4.7	4.4	5.0	4.5	4.3	4.3	4.1
4.5	4.3	4.4	4.9	4.4	5.6	4.4	5.2	4.4	5.0
4.3	4.9	4.5	5.0	4.3	4.5	4.5	4.7	4.3	4.3
4.3	4.4	4.4	4.6	4.3	4.6	4.3	4.7	4.5	5.0
4.3	4.7	4.3	4.1	4.3	4.8	4.3	4.8	4.3	4.6

4.5	4.8	4.7	5.2	4.7	5.0	4.7	5.2	4.5	4.7
4.7	4.6	4.6	4.3	4.5	5.0	4.6	5.5	4.7	4.0
4.6	4.5	4.5	4.5	4.5	4.5	4.5	4.6	4.7	5.3
4.7	4.5	4.5	4.2	4.6	5.2	4.5	4.9	4.5	4.8
4.7	4.4	4.6	5.1	4.5	4.9	4.5	3.7	4.5	5.2
4.7	4.8	4.7	4.7	4.5	4.3	4.5	4.3	4.7	5.0
4.5	5.0	4.6	4.7	4.7	4.7	4.7	4.8	4.5	4.7
4.7	4.9	4.6	4.8	4.6	5.1	4.7	5.2	4.5	5.0
4.7	5.1	4.7	5.1	4.7	4.6	4.6	4.1	4.6	4.7
4.7	4.3	4.6	5.1						

4.8	5.2	4.7	5.2	4.7	5.8	4.7	5.2	4.7	4.6
4.7	4.0	4.8	4.2	4.7	5.3	4.8	4.6	4.7	4.5
4.8	4.0	4.8	5.6	4.8	3.8	4.9	5.2	4.8	4.7
4.7	4.4	4.9	5.2	4.8	5.3	4.9	5.3	4.7	4.8
4.8	4.8	4.8	4.7	4.7	4.7	4.9	4.5	4.8	4.5
4.9	5.8	4.9	5.2	4.9	4.7	4.7	5.0	4.9	5.5
4.7	4.7	4.7	4.8	4.8	4.8	4.7	4.9	4.9	5.0
4.7	5.2	4.7	5.1	4.9	5.4	4.7	5.1	4.8	4.8
4.8	4.9	4.9	5.2	4.8	5.3	4.7	4.6	4.7	4.3

5.0	5.7	5.1	5.7	5.0	5.7	5.0	4.8	5.0	5.8
5.0	5.1	5.1	5.3	5.1	4.9	4.9	5.2	5.0	5.8
5.1	5.6	4.9	5.2	5.1	5.3	4.9	5.3	5.0	5.1
5.1	5.4	5.0	5.6	4.9	4.5	4.9	5.0	4.9	5.2
5.1	6.1	4.9	4.7	5.0	5.4	5.0	5.0	4.9	5.5
5.0	5.4	4.9	5.0	5.1	5.7	5.0	5.4	5.1	5.5
4.9	5.4	5.1	5.4	5.1	4.8	4.9	5.2	5.1	5.3
5.2	5.8	5.3	6.1	5.2	5.7	5.1	5.7	5.3	6.2
5.3	5.9	5.3	5.0	5.2	4.7	5.3	6.0	5.2	5.6
5.3	5.0	5.1	5.3	5.3	5.7	5.1	4.9	5.2	5.5
5.1	5.6	5.2	5.8	5.2	5.4	5.1	5.3	5.3	5.6
5.3	5.4	5.3	5.8	5.3	5.0	5.1	5.4	5.2	5.6
5.3	4.8	5.2	5.3	5.3	5.4	5.1	6.1	5.2	6.0
5.1	5.7	5.2	5.8	5.1	5.5	5.1	5.4	5.1	4.8
5.3	5.0	5.2	5.3	5.3	5.3	5.3	5.0	5.3	5.7
5.2	4.7	5.1	5.3	5.2	3.9				
5.3	6.1	5.5	6.3	5.5	4.7	5.4	4.6	5.5	4.8
5.5	5.6	5.3	6.2	5.3	5.9	5.5	5.3	5.3	5.0
5.3	6.0	5.5	6.5	5.3	5.0	5.3	5.7	5.4	5.1
5.5	5.6	5.5	5.5	5.5	5.5	5.5	5.6	5.5	5.7
5.4	5.2	5.4	5.1	5.3	5.6	5.3	5.4	5.4	5.8
5.3	5.8	5.3	5.0	5.4	5.9	5.3	4.8	5.3	5.4
5.4	5.2	5.5	6.3	5.4	5.6	5.5	5.3	5.4	5.9
5.5	5.8	5.4	5.7	5.5	5.3	5.3	5.0	5.3	5.3
5.3	5.0	5.3	5.7						
5.7	5.6	5.7	6.1	5.5	6.3	5.5	4.7	5.7	5.5
5.7	6.3	5.7	6.4	5.7	5.2	5.5	4.8	5.5	5.6
5.6	6.0	5.5	5.3	5.7	5.2	5.5	6.5	5.6	5.9
5.6	4.8	5.6	6.0	5.6	6.3	5.7	5.4	5.5	5.6
5.6	6.1	5.6	5.5	5.5	5.5	5.5	5.5	5.5	5.6
5.5	5.7	5.6	6.5	5.7	5.6	5.6	5.7	5.7	5.7
5.7	5.8	5.6	5.7	5.7	5.8	5.6	6.0	5.6	6.2
5.6	6.9	5.6	4.9	5.7	6.0	5.5	6.3	5.7	6.0
5.7	5.5	5.5	5.3	5.5	5.8	5.7	6.0	5.6	5.1
5.7	5.5	5.5	5.3	5.7	6.3	5.7	6.1		
5.8	6.7	5.9	5.7	5.7	5.6	5.8	7.0	5.7	6.1
5.8	5.9	5.9	6.0	5.9	5.7	5.7	5.5	5.7	6.3
5.7	6.4	5.7	5.2	5.8	7.2	5.9	6.4	5.7	5.2
5.9	6.5	5.9	6.0	5.8	6.3	5.9	6.7	5.8	5.8
5.8	6.3	5.7	5.4	5.8	6.4	5.8	6.2	5.9	6.5
5.7	5.6	5.7	5.7	5.7	5.8	5.8	5.8	5.7	5.8
5.8	6.1	5.8	6.2	5.9	5.3	5.8	5.4	5.7	6.0
5.8	6.1	5.7	6.0	5.8	5.7	5.7	5.5	5.8	6.2
5.8	6.1	5.7	6.0	5.7	5.5	5.7	6.3	5.9	5.5
5.7	6.1	5.8	5.2	5.9	5.9	5.9	6.3		
6.1	6.9	6.1	6.2	6.0	5.9	5.9	5.7	6.0	6.1
6.1	6.2	6.0	7.9	5.9	6.0	6.0	5.9	5.9	5.7
6.0	5.5	6.0	6.2	5.9	6.4	6.1	5.7	6.0	6.1
6.1	6.2	5.9	6.5	6.1	6.2	6.1	6.0	6.0	5.9
5.9	6.0	6.0	6.3	6.0	5.3	5.9	6.7	6.1	6.4
6.1	5.6	6.1	6.8	6.1	6.3	5.9	6.5	6.0	6.5
6.1	6.0	6.1	6.5	6.0	6.3	5.9	5.3	6.1	6.3
6.0	6.2	6.0	6.4	6.1	6.6	6.0	7.0	6.0	6.0
6.0	5.8	6.0	6.3	6.0	5.7	6.1	5.9	5.9	5.5
5.9	5.9	5.9	6.3						
6.2	6.5	6.1	6.9	6.3	6.1	6.1	6.2	6.2	6.0
6.2	6.0	6.3	5.9	6.1	6.2	6.2	6.4	6.3	6.6
6.3	7.3	6.3	6.7	6.3	7.0	6.3	6.5	6.2	5.7
6.2	6.3	6.3	6.3	6.2	7.0	6.1	5.7	6.1	6.2
6.2	6.4	6.1	6.2	6.2	6.1	6.1	6.0	6.3	6.9
6.3	6.7	6.3	6.2	6.2	5.7	6.1	6.4	6.1	5.6
6.1	6.8	6.2	6.5	6.1	6.3	6.3	6.7	6.3	6.9
6.2	6.8	6.3	6.3	6.3	6.1	6.1	6.0	6.1	6.5
6.3	7.0	6.2	5.3	6.3	6.7	6.3	6.5	6.2	7.1
6.1	6.3	6.3	6.2	6.2	5.8	6.2	6.3	6.3	6.4
6.3	7.3	6.1	6.6	6.3	6.6	6.3	6.8	6.2	5.6
6.3	7.0	6.2	6.3	6.3	6.3	6.3	6.1	6.1	5.9
6.3	6.3	6.3	6.4	6.3	7.0	6.2	5.8		

6.5	6.8	6.3	6.1	6.5	6.8	6.5	6.3	6.3	5.9
6.5	6.4	6.4	6.4	6.4	6.7	6.4	6.3	6.3	6.6
6.3	7.3	6.5	5.5	6.3	6.7	6.3	7.0	6.4	7.0
6.5	6.4	6.4	6.3	6.3	6.5	6.5	6.8	6.5	6.4
6.4	6.2	6.3	6.3	6.4	5.5	6.4	6.6	6.5	7.2
6.5	7.2	6.4	7.0	6.3	6.9	6.5	6.4	6.5	7.2
6.3	6.7	6.3	6.2	6.4	6.4	6.4	6.1	6.4	6.9
6.5	6.9	6.5	7.3	6.5	6.7	6.4	6.1	6.3	6.7
6.4	7.4	6.5	7.2	6.4	7.5	6.3	6.9	6.5	7.4
6.5	6.8	6.4	6.5	6.5	6.3	6.3	6.3	6.3	6.1
6.5	6.7	6.3	7.0	6.5	7.2	6.3	6.7	6.3	6.5
6.5	7.0	6.4	5.9	6.3	6.2	6.3	6.4	6.4	6.5
6.4	6.6	6.4	6.7	6.3	7.3	6.4	7.7	6.3	6.6
6.3	6.8	6.3	7.0	6.3	6.3	6.3	6.1	6.5	7.1
6.3	6.3	6.3	6.4	6.4	6.0	6.5	5.8	6.3	7.0
6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.4
6.5	6.4	6.4	6.0	6.5	5.2				

6.5	6.8	6.7	7.4	6.5	6.8	6.7	6.2	6.7	6.9
6.5	6.3	6.7	6.5	6.5	6.4	6.7	6.6	6.6	6.9
6.7	7.4	6.6	6.9	6.5	5.5	6.7	7.5	6.7	6.5
6.5	6.4	6.5	6.8	6.5	6.4	6.6	6.1	6.5	7.2
6.5	7.2	6.6	7.2	6.7	7.5	6.7	6.7	6.7	6.5
6.5	6.4	6.7	7.3	6.5	7.2	6.7	6.8	6.6	6.5
6.6	7.3	6.7	7.6	6.5	6.9	6.5	7.3	6.5	6.7
6.7	6.9	6.6	7.9	6.7	7.1	6.5	7.2	6.7	7.3
6.7	7.0	6.7	6.9	6.5	7.4	6.5	6.8	6.6	7.1
6.5	6.3	6.5	6.7	6.5	7.2	6.7	7.1	6.5	7.0
6.7	7.0	6.6	6.7	6.7	6.8	6.6	6.3	6.7	6.9
6.6	6.8	6.6	6.6	6.6	6.9	6.7	6.7	6.7	6.7
6.7	6.8	6.7	6.8	6.6	6.8	6.7	6.9	6.5	7.1
6.7	6.2	6.6	6.7	6.7	6.6	6.5	5.8	6.7	7.8
6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.4
6.7	6.7	6.7	6.5	6.5	6.4	6.5	5.2		

6.8	6.9	6.9	6.8	6.8	6.7	6.7	7.0	6.9	7.8
6.8	6.3	6.8	6.7	6.7	6.2	6.7	6.9	6.9	6.8
6.8	6.5	6.7	6.5	6.9	6.8	6.8	6.7	6.7	6.6
6.9	7.3	6.7	7.4	6.9	7.0	6.9	7.2	6.8	6.8
6.8	6.9	6.9	7.0	6.7	7.5	6.7	6.5	6.7	7.5
6.9	7.3	6.9	6.7	6.7	6.7	6.7	6.5	6.7	7.3
6.7	6.8	6.8	6.8	6.9	7.1	6.9	7.0	6.8	6.6
6.8	7.6	6.7	7.6	6.9	7.2	6.8	7.5	6.9	6.6
6.7	6.9	6.7	7.1	6.7	7.3	6.8	7.3	6.7	7.0
6.9	6.8	6.8	6.7	6.7	6.9	6.9	7.0	6.9	7.0
6.8	7.8	6.8	7.3	6.8	7.2	6.9	7.7	6.9	6.8
6.8	6.9	6.9	7.2	6.8	6.8	6.8	7.1	6.7	7.1
6.8	7.2	6.7	7.0	6.9	8.0	6.9	5.6	6.7	6.8
6.8	6.6	6.7	6.9	6.8	7.0	6.8	6.6	6.9	7.3
6.7	6.7	6.7	6.7	6.7	6.8	6.8	6.9	6.9	7.2
6.7	6.8	6.8	6.9	6.9	7.1	6.8	7.1	6.7	6.9
6.9	7.4	6.8	6.0	6.7	6.2	6.7	6.6	6.7	7.8
6.9	6.7	6.7	6.7	6.7	6.5	6.9	6.2		

6.9	6.8	6.9	7.8	6.9	6.8	7.0	8.3	7.1	7.6
7.1	6.7	6.9	6.8	6.9	7.3	6.9	7.0	7.0	7.0
6.9	7.2	6.9	7.0	7.0	7.0	7.0	7.3	7.0	7.4
7.0	6.5	7.0	7.8	7.1	7.1	7.1	7.2	7.1	7.0
7.0	7.4	7.0	7.5	7.1	8.3	6.9	7.3	7.0	6.9
6.9	6.7	6.9	7.1	7.1	7.7	6.9	7.0	7.0	6.8
6.9	7.2	6.9	6.6	7.1	7.3	7.1	7.3	7.0	7.5
7.0	6.9	6.9	6.8	6.9	7.0	6.9	7.4	6.9	7.7
7.1	7.3	7.0	7.9	7.1	7.5	7.0	6.9	6.9	6.8
6.9	7.2	7.0	7.4	7.1	7.5	7.0	7.1	7.1	7.4
7.0	7.1	7.1	7.3	7.1	7.6	7.1	8.1	7.0	7.2
6.9	8.0	6.9	5.6	7.0	8.0	7.0	6.8	7.0	7.5
6.9	7.3	6.9	7.2	7.0	7.8	7.1	7.3	7.1	7.0
6.9	7.1	7.1	7.4	7.0	7.2	7.1	7.4	7.0	7.3
6.9	7.4	7.0	6.2	7.0	7.8	7.1	7.7	7.1	6.7
7.0	7.1	7.1	7.2	7.1	7.0	7.0	6.4	7.0	7.0
6.9	6.7	6.9	6.2						

7.2	8.7	7.2	8.2	7.3	6.8	7.1	7.6	7.3	7.1
7.1	6.7	7.3	8.2	7.2	6.5	7.2	7.5	7.2	7.7
7.2	8.3	7.2	7.1	7.1	7.1	7.1	7.2	7.2	7.1
7.1	7.0	7.2	7.7	7.2	7.7	7.2	7.4	7.1	8.3
7.3	7.6	7.3	7.8	7.2	7.5	7.2	8.4	7.1	7.7
7.3	7.9	7.2	7.2	7.2	7.6	7.3	8.3	7.3	6.9
7.1	7.3	7.2	7.7	7.3	8.2	7.3	7.2	7.2	7.1
7.1	7.3	7.3	7.6	7.3	8.0	7.2	7.5	7.1	7.3
7.3	7.6	7.1	7.5	7.2	7.2	7.2	7.5	7.1	7.5
7.2	8.0	7.2	7.3	7.3	7.6	7.1	7.4	7.3	7.0
7.1	7.3	7.3	7.7	7.1	7.6	7.2	7.5	7.1	8.1
7.3	8.1	7.3	8.2	7.2	7.3	7.3	7.6	7.3	7.5
7.1	7.3	7.3	7.6	7.3	7.2	7.2	7.2	7.2	7.1
7.1	7.0	7.1	7.4	7.3	7.3	7.3	7.2	7.2	7.4
7.2	7.0	7.1	7.4	7.3	7.5	7.1	7.7	7.1	6.7
7.1	7.2	7.2	7.1	7.1	7.0	7.2	6.5		
7.4	7.9	7.5	7.3	7.3	6.8	7.3	7.1	7.4	8.2
7.3	8.2	7.5	7.9	7.5	6.8	7.5	8.4	7.5	8.4
7.4	7.6	7.4	7.6	7.5	8.2	7.5	8.0	7.3	7.6
7.3	7.8	7.5	7.6	7.5	7.1	7.3	7.9	7.3	8.3
7.5	7.6	7.3	6.9	7.4	7.7	7.3	8.2	7.3	7.2
7.3	7.6	7.5	8.0	7.5	8.4	7.4	7.7	7.4	8.1
7.3	8.0	7.5	7.8	7.4	8.3	7.4	8.6	7.3	7.6
7.5	7.5	7.5	7.6	7.5	7.5	7.5	8.8	7.5	7.7
7.4	7.6	7.5	7.6	7.3	7.6	7.5	7.4	7.5	7.0
7.4	7.8	7.3	7.0	7.3	7.7	7.5	7.8	7.3	8.1
7.3	8.2	7.3	7.6	7.4	7.5	7.5	7.5	7.5	7.4
7.4	7.4	7.4	7.6	7.5	8.2	7.3	7.5	7.3	7.6
7.5	7.3	7.3	7.2	7.4	7.5	7.3	7.3	7.3	7.2
7.4	7.5	7.5	7.8	7.4	7.2	7.4	7.7	7.3	7.5
7.5	8.0	7.4	7.8	7.5	7.1				
7.5	7.3	7.6	7.7	7.7	7.7	7.7	7.9	7.5	7.9
7.5	6.8	7.5	8.4	7.7	8.6	7.5	8.4	7.7	8.4
7.7	8.2	7.6	8.0	7.6	7.7	7.7	8.1	7.5	8.2
7.5	8.0	7.5	7.6	7.6	7.5	7.5	7.1	7.7	8.5
7.6	8.4	7.6	8.4	7.6	7.9	7.5	7.6	7.6	7.7
7.7	8.2	7.7	7.8	7.7	8.2	7.5	8.0	7.5	8.4
7.7	8.0	7.5	7.8	7.7	8.3	7.6	7.8	7.5	7.5
7.5	7.6	7.6	7.5	7.5	7.5	7.5	8.8	7.5	7.7
7.7	8.0	7.6	7.8	7.5	7.6	7.6	7.5	7.6	7.6
7.6	7.5	7.5	7.4	7.5	7.0	7.7	8.2	7.6	8.4
7.5	7.8	7.6	7.4	7.6	7.8	7.5	7.5	7.5	7.4
7.5	8.2	7.6	8.4	7.6	8.1	7.7	8.7	7.5	7.3
7.6	9.3	7.5	7.8	7.7	7.6	7.6	7.7	7.7	7.8
7.7	8.9	7.5	8.0	7.7	8.5	7.7	7.5	7.5	7.1
7.6	8.2	7.6	8.7	7.6	6.9				
7.8	8.7	7.9	8.4	7.9	8.8	7.8	9.7	7.8	7.5
7.7	7.7	7.7	7.9	7.9	8.3	7.8	7.8	7.8	8.0
7.9	8.6	7.9	9.6	7.9	8.5	7.7	8.6	7.8	8.6
7.8	9.0	7.7	8.4	7.7	8.2	7.7	8.1	7.8	8.1
7.7	8.5	7.9	8.6	7.7	8.2	7.8	8.7	7.8	8.3
7.9	8.5	7.7	7.8	7.8	7.9	7.9	8.2	7.7	8.2
7.7	8.0	7.8	8.6	7.7	8.3	7.9	8.5	7.8	8.1
7.9	8.6	7.7	8.0	7.8	7.9	7.8	8.2	7.7	8.2
7.8	8.4	7.9	8.4	7.8	8.8	7.9	9.0	7.8	8.8
7.9	8.3	7.7	8.7	7.9	8.3	7.8	7.4	7.7	7.6
7.7	7.8	7.8	8.1	7.7	8.9	7.8	8.1	7.8	8.8
7.8	8.5	7.7	8.5	7.8	7.7	7.7	7.5	7.8	7.9
7.8	8.7	7.9	7.8	7.8	8.1	7.9	6.5		
7.9	8.4	7.9	8.8	7.9	8.3	8.0	7.4	8.0	8.5
7.9	8.6	7.9	9.6	7.9	8.5	8.0	8.5	8.1	8.4
8.0	6.6	8.0	8.2	8.1	8.3	7.9	8.6	8.0	7.3
7.9	8.5	7.9	8.2	8.0	8.2	8.0	8.2	8.1	8.7
8.0	8.7	7.9	8.5	7.9	8.6	8.0	8.2	8.0	8.7
8.1	8.8	8.1	8.9	8.0	8.5	8.0	7.6	8.0	8.8
7.9	8.4	8.0	8.7	7.9	9.0	8.1	8.6	7.9	8.3
7.9	8.3	8.1	7.8	8.1	8.5	8.0	8.7	8.1	8.5
8.0	7.2	8.1	8.9	7.9	7.8	8.1	8.7	7.9	6.5

8.2	8.7	8.3	9.5	8.3	7.8	8.3	8.0	8.2	9.3
8.2	8.8	8.3	9.3	8.1	8.4	8.2	8.7	8.3	9.3
8.2	8.7	8.2	9.3	8.3	8.4	8.1	8.3	8.3	9.5
8.2	8.9	8.3	8.0	8.2	8.5	8.2	9.0	8.2	8.3
8.1	8.7	8.3	9.2	8.3	8.8	8.2	8.4	8.1	8.8
8.1	8.9	8.3	8.0	8.2	8.7	8.3	9.6	8.2	8.8
8.2	8.9	8.1	8.6	8.3	8.4	8.3	8.2	8.3	8.8
8.2	8.4	8.3	9.1	8.2	9.3	8.1	7.8	8.1	8.5
8.1	8.5	8.3	8.6	8.2	8.6	8.1	8.9	8.1	8.7
8.3	8.4								
8.4	8.7	8.3	9.5	8.3	7.8	8.3	8.0	8.5	9.3
8.4	9.4	8.4	9.0	8.3	9.3	8.4	9.1	8.3	9.3
8.3	8.4	8.4	8.8	8.4	8.8	8.5	9.4	8.4	9.6
8.4	8.9	8.3	9.5	8.3	8.0	8.5	9.0	8.4	9.3
8.3	9.2	8.3	8.8	8.5	9.3	8.4	9.4	8.4	9.2
8.5	8.6	8.5	8.5	8.5	8.3	8.3	8.0	8.3	9.6
8.4	9.0	8.4	9.3	8.3	8.4	8.4	8.4	8.4	8.3
8.3	8.2	8.3	8.8	8.4	8.6	8.5	9.7	8.3	9.1
8.4	9.4	8.3	8.6	8.5	9.1	8.5	9.0	8.5	9.1
8.3	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4
8.5	7.6								
8.7	10.0	8.7	9.0	8.7	8.8	8.6	9.1	8.6	8.0
8.6	8.6	8.6	8.3	8.5	9.3	8.6	9.5	8.7	9.0
8.7	9.4	8.6	9.1	8.7	8.9	8.5	9.4	8.6	9.7
8.7	9.5	8.5	9.0	8.7	8.9	8.6	9.2	8.7	9.1
8.5	9.3	8.6	9.3	8.6	9.0	8.7	9.0	8.5	8.6
8.6	8.8	8.7	9.7	8.5	8.5	8.5	8.3	8.6	8.8
8.7	9.2	8.6	9.2	8.7	9.3	8.7	9.2	8.6	9.3
8.7	9.8	8.6	9.0	8.5	9.7	8.7	9.6	8.6	9.0
8.5	9.1	8.5	9.0	8.6	8.6	8.6	9.1	8.6	9.3
8.7	9.4	8.5	9.1	8.7	9.5	8.7	10.0	8.5	7.6
8.7	10.0	8.7	9.0	8.7	8.8	8.8	9.2	8.8	10.0
8.9	8.6	8.7	9.0	8.7	9.4	8.7	8.9	8.9	9.2
8.8	9.3	8.8	9.3	8.9	9.6	8.7	9.5	8.7	8.9
8.9	9.0	8.7	9.1	8.8	9.2	8.8	9.5	8.7	9.8
8.9	8.9	8.9	8.9	8.9	9.0	8.8	9.5	8.9	9.0
8.8	9.0	8.7	9.7	8.8	9.0	8.9	8.7	8.7	9.2
8.7	9.3	8.7	9.2	8.8	9.6	8.8	9.4	8.9	9.3
8.8	9.7	8.7	9.8	8.8	9.1	8.9	9.9	8.7	9.6
8.8	9.5	8.9	9.5	8.7	9.4	8.7	9.5	8.7	10.0
8.9	7.9								
9.0	9.1	9.1	9.1	9.1	9.0	9.0	8.6	9.1	8.3
8.9	8.6	9.0	9.5	9.0	9.8	9.0	9.5	9.1	9.5
8.9	9.2	8.9	9.6	9.0	9.5	9.0	9.7	8.9	9.0
9.0	9.3	8.9	8.9	8.9	8.9	8.9	9.0	9.0	9.0
8.9	9.8	9.0	9.3	9.0	9.0	9.0	8.9	8.9	8.7
9.0	9.5	8.9	9.3	9.0	10.1	9.1	9.3	9.0	9.4
9.1	9.9	8.9	9.9	9.0	9.2	9.0	9.5	9.1	9.8
8.9	9.5	9.1	9.3	8.9	7.9				
9.1	9.1	9.1	9.0	9.1	8.3	9.3	10.1	9.3	8.9
9.3	10.0	9.1	9.5	9.3	10.1	9.2	9.4	9.3	9.7
9.3	9.9	9.2	9.5	9.2	9.6	9.2	9.3	9.3	10.2
9.3	9.7	9.3	9.2	9.3	9.4	9.3	8.9	9.2	9.5
9.2	9.4	9.3	9.7	9.2	9.6	9.3	9.6	9.3	10.2
9.3	10.2	9.1	9.3	9.3	9.6	9.3	10.7	9.1	9.9
9.3	10.0	9.2	9.4	9.1	9.8	9.3	10.1	9.1	9.3
9.3	9.5	9.2	8.9	9.3	8.5				
9.3	10.1	9.3	8.9	9.4	10.2	9.5	10.3	9.5	9.8
9.3	10.0	9.4	8.7	9.5	10.1	9.4	10.0	9.5	9.8
9.3	10.1	9.4	9.8	9.3	9.7	9.4	10.2	9.5	11.2
9.5	10.4	9.5	10.0	9.3	9.9	9.5	9.5	9.3	10.2
9.3	9.7	9.3	9.2	9.5	10.1	9.3	9.0	9.4	10.0
9.3	8.9	9.5	9.7	9.4	10.2	9.5	10.0	9.4	9.4
9.4	9.4	9.4	9.3	9.5	10.3	9.3	9.7	9.4	9.9
9.3	9.6	9.3	10.2	9.3	10.2	9.3	9.6	9.4	9.8
9.3	10.7	9.3	10.0	9.5	10.3	9.4	10.3	9.4	9.6
9.4	8.8	9.3	10.1	9.5	10.0	9.4	10.0	9.3	9.5
9.5	9.7	9.5	9.7	9.5	9.3	9.5	9.2	9.3	8.5
9.4	9.4								

9.7	11.1
9.5	9.8
9.6	9.9
9.5	11.2
9.6	9.9
9.7	9.7
9.7	10.0
9.7	10.3
9.7	9.4
9.7	9.5
9.7	11.1
9.8	10.8
9.7	10.2
9.7	9.6
9.7	10.0
9.9	10.7
9.9	9.8
9.7	9.5
10.0	11.3
10.1	10.5
10.1	10.2
9.9	10.7
10.1	10.8
9.9	9.7
10.1	11.3
10.1	10.5
10.1	10.2
10.2	10.9
10.3	11.2
10.2	10.0
10.1	9.8
10.4	12.0
10.5	11.1
10.4	11.0
10.5	11.1
10.4	10.4
10.3	10.5
10.5	11.2
10.7	11.2
10.6	12.0
10.5	11.3
10.5	10.4
10.5	10.0
10.7	11.6
10.7	11.4
10.9	11.7
10.7	11.9
10.8	10.4
11.1	12.1
11.1	11.7
11.1	11.7
11.1	11.4
11.3	12.8
11.3	12.6
11.2	12.8
11.1	11.4
1111	11.4
11.1	11.4
11.3	12.8
11.4	11.9
11.4	11.7
11.5	11.3
11.5	11.7
11.6	12.3
11.5	12.5
11.7	11.0
11.5	11.81
11.5	11.2

9.6	11.2
9.7	9.7
9.7	10.2
9.5	10.4
9.7	9.6
9.7	10.6
9.6	9.9
9.6	9.8
9.5	10.0
9.5	9.3
9.8	9.3
9.8	9.9
9.7	11.0
9.8	10.7
9.7	10.3
9.9	10.6
9.8	9.9
9.8	9.2
10.0	11.2
9.9	10.2
10.0	10.7
10.0	10.6
10.0	10.3
10.0	9.9
10.1	10.2
10.2	10.5
10.2	10.0
10.1	10.9
10.3	11.3
10.1	10.8
10.2	10.7
10.5	11.2
10.3	10.9
10.3	11.0
10.3	10.0
10.4	10.3
10.5	10.6 ✓
10.7	11.6
10.5	11.1
10.6	10.7
10.7	11.3
10.6	11.1
10.8	11.6
10.9	12.4
10.8	11.8
10.7	11.3
11.1	12.0
10.9	12.4
10.9	11.5
11.0	10.7
11.2	12.4
11.1	12.0
11.2	12.1
11.2	11.0
11.2	12.0
11.3	11.5
11.3	12.6
11.3	11.9
11.3	12.2
11.3	10.9
11.5	11.2
11.6	11.6
11.7	11.8
11.5	10.7
11.7	11.6
11.5	11.2

9.5	10.3
9.7	9.6
9.6	10.5
9.5	10.0
9.6	9.5
9.5	10.0
9.6	10.4
9.7	10.5
9.5	9.7
9.7	9.5
9.8	10.0
9.9	10.2
9.7	10.1
9.7	9.7
9.8	10.8
9.8	10.6
9.9	9.7
9.8	10.2
10.1	11.3
10.1	10.7
10.0	10.2
9.9	10.6
10.0	10.4
9.9	9.7
10.2	9.8
10.1	10.7
10.2	10.8
10.2	11.0
10.3	10.3
10.3	10.7
10.3	10.9
10.5	11.1
10.4	11.1
10.3	11.2
10.3	10.3
10.3	9.8
10.5	11.1
10.7	11.4
10.7	10.8
10.6	10.8
10.7	10.0
10.7	11.2
10.8	11.0
10.7	10.8
10.8	11.0
11.1	11.6
11.0	11.2
10.9	11.7
11.1	11.4
11.1	12.1
11.1	11.6
11.3	11.9
11.1	11.7
11.3	12.2
11.3	10.6
11.4	12.0
11.4	11.2
11.4	12.5
11.4	11.7
11.3	10.6
11.6	11.5
11.5	11.9
11.5	11.0
11.5	11.7
11.6	11.0

9.5	9.8
9.6	9.6
9.7	11.0
9.7	10.1
9.5	10.1
9.6	10.5
9.6	9.6
9.5	10.3
9.7	10.0
9.5	9.2
9.8	10.5
9.7	9.7
9.9	10.3
9.8	10.9
9.9	11.1
9.7	9.4
9.7	9.5
10.1	10.2
9.9	10.3
10.1	10.9
10.1	10.2
10.0	11.4
10.1	9.8
10.3	10.9
10.3	10.9
10.3	11.0
10.3	10.6
10.3	10.2
10.3	10.5
10.5	11.1
10.4	11.5
10.3	10.6
10.3	11.3
10.3	10.2
10.4	10.1
10.6	11.1
10.6	11.4
10.7	10.5
10.5	11.1
10.5	10.6
10.9	11.4
10.8	10.5
10.8	10.7
10.0	11.3
11.1	11.7
11.1	11.4
11.0	11.5
11.0	10.5
11.2	11.9
11.1	11.7
11.2	11.7
11.1	12.1
11.2	12.4
11.2	10.0
11.5	12.5
11.5	11.9
11.5	10.7
11.3	11.5
11.5	11.2
11.6	12.0
11.7	12.6
11.5	11.3
11.7	11.8

9.5	10.1
9.6	9.6
9.6	10.1
9.5	9.5
9.5	9.7
9.5	10.3
9.6	9.5
9.6	10.3
9.5	9.7
9.6	9.4
9.8	10.5
9.7	9.6
9.9	9.7
9.7	10.6
9.7	10.5
9.7	10.0
9.9	9.7
10.0	10.9
9.9	9.7
9.9	11.1
10.0	9.7
9.9	9.8
10.0	9.6
10.3	10.6
10.2	10.8
10.2	10.2
10.3	10.8
10.1	10.2
10.3	9.8
10.3	10.6
10.5	11.2
10.5	11.3
10.5	10.4
10.3	10.7
10.5	10.0
10.5	11.1
10.5	11.2
10.7	11.9
10.6	10.9
10.6	10.3
10.8	11.0
10.9	11.5
10.7	10.5
10.7	10.0
10.9	11.4
11.0	10.8
11.1	12.1
11.2	12.1
11.1	11.7
11.2	11.7
11.3	11.8
11.3	10.9
11.4	12.1
11.3	11.8
11.5	11.0
11.5	11.8
11.7	12.0
11.7	11.7
11.7	11.8
11.7	11.5

11.9	12.5
11.8	13.1
11.8	12.2
11.9	12.1
11.8	11.7
11.9	11.5
12.1	12.8
12.0	12.5
11.9	12.2
11.9	12.8
11.9	11.7
11.9	11.5
12.1	12.8
12.2	12.2
12.2	12.5
12.2	11.8
12.2	12.5
12.4	13.4
12.5	13.0
12.5	12.2
12.5	12.6
12.5	13.0
12.5	13.0
12.6	12.5
12.7	12.7
12.6	12.9
12.5	12.1
12.8	13.8
12.8	13.2
12.7	12.3
12.8	13.2
13.0	13.6
13.1	12.7
12.9	13.3
13.2	13.5
13.1	14.0
13.2	13.0
13.2	13.6
13.5	13.4
13.3	13.8
13.6	14.9
13.5	13.8
13.5	13.1
13.7	13.6
13.6	13.8
13.9	14.6
13.8	13.7
13.8	13.1
13.8	14.1
13.9	13.5
13.9	14.6
14.1	14.3
14.1	14.4
14.3	15.4
14.1	14.2
14.1	14.3
14.3	14.2
14.3	14.0
14.3	15.4
14.5	14.7
14.3	14.5
14.5	14.3
14.3	13.7

11.7	12.0
11.9	12.2
11.7	11.7
11.7	11.8
11.7	11.6
12.0	13.6
12.0	12.6
12.1	12.3
12.0	13.0
12.1	11.8
12.3	13.0
12.2	12.1
12.2	11.5
12.3	12.3
12.1	11.8
12.5	13.0
12.4	13.4
12.5	12.5
12.5	12.3
12.6	12.8
12.5	12.2
12.5	12.2
12.7	12.7
12.6	12.2
12.5	12.0
12.8	13.4
12.8	12.7
12.7	12.3
12.7	12.5
13.0	13.6
13.1	12.8
13.1	12.8
13.1	13.9
13.1	12.7
13.1	12.6
13.2	12.5
13.4	14.2
13.5	13.1
13.6	14.2
13.7	13.7
13.6	13.2
13.6	13.7
13.5	13.0
13.8	14.2
13.7	13.7
13.7	12.9
13.7	14.3
14.1	14.1
14.0	13.6
13.9	13.9
14.3	15.4
14.2	14.2
14.3	14.3
14.2	14.3
14.2	13.9
14.3	15.4
14.5	14.1
14.5	14.4
14.3	14.2
14.4	14.5

11.9	12.3
11.7	12.6
11.7	11.8
11.8	11.9
11.7	11.8
11.9	12.5
12.0	12.3
11.9	12.2
11.9	11.5
12.0	11.3
12.1	12.6
12.1	12.3
12.3	11.8
12.3	11.9
12.1	12.1
12.3	13.0
12.5	12.2
12.5	12.6
12.5	12.1
12.6	13.2
12.7	12.3
12.5	12.5
12.7	12.3
12.6	12.9
12.8	13.7
12.7	12.6
12.9	12.2
13.1	13.9
13.0	12.7
13.0	12.3
13.1	14.3
13.2	13.5
13.2	13.9
13.5	13.1
13.5	12.2
13.5	13.4
13.7	13.7
13.7	12.9
13.5	12.2
13.8	13.8
13.7	13.7
13.7	13.6
13.8	13.9
14.1	14.2
14.1	13.5
13.9	13.5
14.2	14.9
14.2	14.1
14.3	14.5
14.3	14.6
14.4	15.3
14.4	14.9
14.4	14.3
14.3	14.6
14.5	14.1

11.9	12.2
11.8	12.7
11.8	11.5
11.9	11.7
11.8	11.7
12.1	12.6
11.9	12.3
12.0	13.2
11.9	12.1
12.1	12.1
12.3	12.5
12.1	13.2
12.2	12.5
12.1	12.2
12.1	11.9
12.5	13.1
12.3	11.8
12.3	12.3
12.5	12.0
12.5	13.1
12.5	12.8
12.5	12.6
12.7	12.3
12.5	12.3
12.8	13.7
12.7	12.7
12.9	13.3
13.1	14.3
13.1	12.6
13.2	14.3
13.3	13.8
13.3	13.1
13.5	13.8
13.3	13.1
13.6	13.8
13.7	14.0
13.7	13.6
13.6	13.8
13.8	13.5
13.7	14.0
13.7	13.6
13.9	13.9
14.1	13.5
14.0	13.7
14.2	14.5
14.1	13.5
14.3	14.0
14.3	13.7
14.5	14.5
14.5	14.9
14.3	14.0
14.5	13.8
14.3	14.8

11.7	11.8
11.9	12.8
11.9	11.5
11.8	12.2
11.7	11.5
12.0	12.8
12.1	13.1
12.1	13.2
12.1	12.2
12.1	11.9
12.1	13.1
12.2	13.1
12.2	13.3
12.2	12.5
12.3	11.6
12.3	12.5
12.5	12.8
12.3	11.9
12.3	11.6
12.6	13.2
12.7	12.6
12.6	12.7
12.5	12.6
12.7	12.5
12.7	12.3
12.7	12.7
12.8	12.4
13.1	14.0
12.9	12.2
13.2	14.4
13.1	12.8
13.1	12.8
13.5	12.9
13.5	13.0
13.5	13.1
13.5	12.9
13.6	13.7
13.7	14.3
13.8	13.8
13.8	14.1
13.9	14.6
13.8	13.2
14.0	14.4
13.9	14.6
14.1	14.1
14.2	14.5
14.1	13.5
14.1	14.4
14.5	14.5
14.3	14.3
14.4	14.1
14.5	13.7

14.6	15.3	14.5	14.5	14.5	14.5	14.5	14.7	14.7	15.0
14.5	14.1	14.5	14.9	14.5	14.4	14.6	14.4	14.5	14.3
10.6	15.0	14.5	13.8	10.6	15.2	14.5	13.7	14.6	14.7
14.7	10.9	14.7	13.5	14.6	15.5	14.7	14.3	14.5	14.1
14.6	14.2								
14.9	16.1	14.9	15.4	14.7	15.0	14.8	14.5	14.9	15.3
14.9	15.7	14.7	14.9	14.9	14.9	14.9	14.8	14.8	14.8
14.7	13.5	14.7	14.3	14.8	15.6	14.9	15.2		
14.9	16.1	14.9	15.4	15.0	15.1	15.1	15.3	15.0	14.8
14.9	15.3	14.9	15.7	15.0	15.5	15.1	14.8	15.1	15.8
15.0	14.5	15.1	14.5	14.9	14.9	14.9	14.8	14.9	15.2
15.0	14.6	15.1	15.3						

BIBLIOGRAPHY

Andres, J. M., and U. Ingard, 1953a: Acoustic streaming at high Reynolds numbers. J. Acoustical Soc. of Amer., 25, 928-932.

Andres, J. M., and U. Ingard, 1953b: Acoustic streaming at low Reynolds numbers. J. Acoustical Soc. of Amer., 25, 932-938.

Batchelor, G. K., 1950: The application of the similarity theory of turbulence to atmospheric diffusion. Quart. J. Roy. Meteor. Soc., 76, p. 133.

Batchelor, G. K., 1952: Diffusion in a field of homogeneous turbulence, II. The relative motion of particles. Proc. Cambridge Phil. Soc., 48, 345-362.

Carriere, P.M.Z., 1929: Analyse ultramicroscopique des vibrations aerienne. J. de Physique et le Radium, Series 6, V. 10, 198-208.

Clutter, D. W., A.M.O. Smith, and J. M. Brazier, 1959: Technique of Flow Visualization using Water as the Working Medium. Report No. ES-29075, Douglas Aircraft Co., El Segundo, CA.

Csanady, G. T., 1973: Turbulent Diffusion in the Environment. Dordrecht-Holland, D. Reidel, 1-248.

Drapeau, G., W. Hamson, W. Bien, and P. Leinonon, 1974: Oil slick fate in a region of strong tidal currents. Proc. of 14th Coastal Eng. Conf., 3, 2245-2259.

Fay, J. A., 1969: The spread of oil slicks on a calm sea. Oil on the Sea, ed. D. P. Hoult, New York, Plenum Press, 53-63.

Gifford, F. A., Jr., 1968: An outline of theories of diffusion in the lower layers of the atmosphere. Meteorology and Atomic Energy, ed. D. H. Slade, U.S. Atomic Energy Comm., 83-94.

Hoult, D. P., 1972: Oil spreading on the sea. Annual Review of Fluid Mechanics, ed. M. Van Dyke, Palo Alto, Annual Reviews Inc., 341-368.

Lin, C. C., 1960: On a theory of dispersion by continuous movements. Proc. Natl. Acad. Sci., 46, 566-570.

Okubo, A., 1974: Some speculations on oceanic diffusion diagrams. Rapp. P. V. Re'un. Cons. int. Explor. Mer., 167, 77-85.

Richardson, L. F., 1926: Atmospheric diffusion shown on a distance neighbor graph. Proc. Roy. Soc. (A), 110, 709-737.

Schlichting, V. H., 1932: Berechnung ebener periodischer Grenzschichtstromungen. Physikalische Zeitschrift, 33, 327-335.

Smith, J. E., 1970: 'Torrey Canyon' Pollution and Marine Life. Cambridge, University Press, 150-162.

Stommel, H., 1949: Horizontal diffusion due to oceanic turbulence. J. Marine Res., V. 3, No. 3, 199-225.

Taylor, G. I., 1922: Diffusion by continuous movements. Proc. of London Math. Soc., Series 2, V. 20, Part 3, 196.

Thompson, S. M., 1969: Turbulent interfaces generated by an oscillating grid in a stably stratified fluid. PhD. thesis, Emmanual College, University of Cambridge.

Thompson, S. M., and J. S. Turner, 1975: Mixing across an interface due to turbulence generated by an oscillating grid. J. Fluid Mech., 67, Part 2, 349-368.

Turner, J. S., 1968: The influence of molecular diffusivity on turbulent entrainment across a density interface. J. Fluid Mech., 23, p. 639.