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ABSTRACT

A simple method for estimating the rain volume over an area (for convective rains) was investigated. The method does not require precipitation rates or amounts obtained from gages or from radar data but considers only the rain events *per se*. The method requires only area coverage data, which could be obtained from gage or (preferably) radar data. The key element in the method is the existence of a good correlation between rainfall areas and rain volumes. Such correlations have been found in several sets of data, and for different climate conditions.

The present analysis is based on rain gage and radar data from western North Dakota from the summer of 1972. The area covered hourly with rain was estimated using recording rain gage data for a 6,800 km^2 area and radar echoes obtained from a 10-cm radar for a 38,700 km^2 area. The maximum echo area during any one scan in one hour seems to be the hourly radar parameter best correlated with the daily rain volume data. A quantity called the Integrated Rainfall Coverage was calculated from either gage or radar data and was found to be well correlated with the rain volume ($r = 0.955$).

The rain volume estimate obtained from radar data should be adjusted according to the type of rain. The radar estimates are smaller than corresponding gage estimates for instability rains and larger for frontal rains. This study suggests that the accuracy of the simplified method approaches that of methods using radar reflectivity data and may have operational value in some special situations.

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LIST OF DEFINITIONS

1. Average echo coverage: The average of the scan echo area coverages over the one-hour period.

2. Average rainfall coverage: The integrated rainfall coverage divided by the product of the total area and time interval considered.

3. Gage Estimated Rainfall (GER): The rainfall amount measured by a rain gage at its location.

4. Integrated rainfall coverage: The sum $\sum_{i=1}^{n} a_i \Delta t$ over n successive time periods Δt of the amount of area a_i where precipitation was detected during the ith time period.

5. Maximum echo coverage: The highest echo coverage on any one scan during the hour.

6. Radar Estimated Rainfall (RER): The rainfall amount calculated from radar echoes at each gage location using an optimized Z-R relationship.

7. Rain event: The detection of precipitation at a given location during a specified time period by either a radar set or a rain gage.

1. INTRODUCTION

The purpose of this report is to describe a simple method for estimating the amount of convective rain falling over areas of the order of 10,000 km^2 using weather radar data. The need for such operational data in hydrology, weather modification experiments, and agriculture is evident.

Although radar has been used to detect precipitation for more than 30 years (Maynard, 1945), it has been slow to supplant conventional techniques for measuring rainfall. Difficulties in obtaining accurate radar calibrations and processing the large amounts of data involved in radar measurement of rainfall are in part responsible for this. However, uncertainties regarding the relationships to be assumed between rainfall rate R and radar reflectivity factor Z in specific situations are a more serious deterrent to the adoption of radar as a means of measuring rainfall. Atlas (1964), Cataneo and Stout (1968), Stout and Mueller (1968), and Battan (1973) present different comprehensive tabulations of Z-R relationships obtained from various sources. The plethora of values in these tables illustrates the diversity of the relationships available, but provides little help in selecting the relationship most suitable to a particular situation.

Many authors have shown that variations in latitude, geographical, orographical, and synoptic conditions result in differences in radar estimated rainfall (e.g., Byers, 1948; Stout and Mueller, 1968; Estoque and Fernandez-Partagas, 1974). In addition, there are complicating effects due to microwave attenuation, evaporation of the raindrops, vertical air motions, particle shape and fall speed variations, and advection of the rain while falling from the radar's sampling volume to the ground. The combined effects may limit the accuracy of radar estimates of areal rainfall to not better than 50% (Atlas and Chmela, 1957; Hildebrand *et al.*, 1979).

The problem of variability in the Z-R relationship becomes particularly acute when radar measurements are used to evaluate cloud seeding experiments designed to stimulate rainfall. Not only are the investigators interested in small variations in the rainfall from each storm, but they must also consider the possibility that the Z-R relationship itself may be affected by the cloud seeding (Woodley, 1970; Cataneo, 1971).

Nevertheless, radar can map precipitation much more completely in space and time than can any feasible network of rain gages on the surface. This capability is so valuable that experimenters in atmospheric physics, weather modification, hydrology, and agriculture are coming to rely heavily upon radar for evaluation purposes (Woodley *et al.*, 1975; Dennis *et al.*, 1975a).

Current techniques of measuring rainfall include the use of rain gages, the use of radar, and even the use of satellite data (Scofield, 1978). Various techniques for combining these data have been derived to take advantage of the different capabilities of the instruments (e.g., Brandes, 1975).

Huff (1970, 1971) showed that gage measurements of area-mean rainfall should be accurate to within about 5% for gage densities >1 gage per 50 km^2 and for rainfall rates >10 mm hr^{-1}. On the other hand, Hildebrand et al. (1979) concluded that for low gage densities, <1 gage per 250-300 km^2, and for some climates gage-radar measurements of area-mean convective rainfall may be more accurate than gage-only measurements. The work of Woodley et al. (1974, 1975) with Florida data, using an approach involving small areas with dense rain gage groupings (called clusters), suggested that the gaging requirement for adjustment of radar estimates of rainfall over an area of 13,000 km^2 would be about 40 gages arranged into several clusters of 7 to 10 gages each.

Rain gages are frequently regarded as a costly, troublesome, time consuming method of measuring rainfall. Thus, in spite of the measurement techniques now available, the problem of measuring areal convective rainfall still remains open. The work reported here deals with a simple method of obtaining rainfall estimates that may have value in many situations.

2. REVIEW OF COMMON METHODS FOR OBTAINING Z-R RELATIONSHIPS

The method ordinarily used to derive Z-R relationships from observations of the two variables is to perform a regression analysis on logarithms of the two quantities. The result is a power-law relationship

$$Z = AR^b \qquad \text{or} \qquad R = A^{-1/b} Z^{1/b} \tag{1}$$

where Z can be given in $mm^6\ m^{-3}$ and R in $mm\ hr^{-1}$. A and b are sometimes called the Marshall-Palmer parameters. In logarithmic form, the relation (1) becomes:

$$\log R = -\frac{1}{b} \log A + \frac{1}{b} \log Z \quad . \tag{2}$$

Actually the relation (2) is used for plotting the scatter diagrams showing values of R and Z and fitting the resulting data points with straight lines.

Smith *et al.* (1975a) tried to use a relation of the form:

$$\log R = -\frac{1}{b} \log A + \frac{1}{b} \log Z + \gamma(\log Z)^2 \quad . \tag{3}$$

The quadratic term in (3) provides for possible curvature of the Z-R relationship when the variables are plotted on log-log paper. This contingency was intended to account for the effects of significant amounts of hail in some of the radar observations. However, the values found for the coefficient γ were essentially equal to zero.

The Z-R relationships can be investigated in two essentially different ways. One is the so-called "direct" method, in which the reflectivity factor Z is measured by a weather radar and the rate or amount of rainfall by a rain gage. The values are then plotted against one another on a log-log plot to determine the parameters A and b of (1). The "indirect" method involves measuring the raindrop size distributions, then separately computing values for the reflectivity and rainfall rate from these. The total reflectivity factor is obtained using a relation of the form

$$Z = \Sigma N(D)D^6 \tag{4}$$

where D is the raindrop diameter and N(D) the number concentration of drops of diameter D.

The main difficulty with Z-R relationships determined by the indirect method, as pointed out by Stout and Mueller (1968), is that the raindrop size distributions are measurable in a volume of a few cubic meters, whereas radar observes the reflectivity in a volume of the order of 10^8 m^3. A further drawback with the indirect method is that it requires the raindrop fall speeds to be known also. Other approximations are the drop sphericity (Seliga and Bringi, 1976), uniformity of the drop size distribution (which has to be maintained constant over the calculation time), etc.

The direct method itself has two disadvantages, among others (see Sec. 1):

(a) the radar reflectivity is measured at heights often exceeding 1000 m, whereas rain falls at the surface; and

(b) the radar measurements represent horizontal areas of at least 10^4-10^5 m^2 (for typical horizontal beam width and pulse duration), whereas rain gages monitor rain amounts over areas of the order of 10^{-1}-10^{-2} m^2.

Such problems cause the computed A and b parameters in the relationship (1) to vary greatly. According to Stout and Mueller (1968), when Z values as calculated by Dumoulin and Doherty for the 1 mm hr^{-1} rate are compared, they differ up to a factor of 10. If we assume a measured Z value of 10^5 mm^6 m^{-3} (50 dBz), the rain rates calculated from the two relationships differ by a factor 5. Thus, for heavier rainfall rates, differences of 500% appear between Dumoulin's and Doherty's relationship. This situation is not very satisfactory. For improving the radar rainfall estimates, many local and regional A and b parameters have been calculated. Various optimized or combined gage-radar rainfall measurement techniques have also been used in efforts to obtain better results.

3. PURPOSE OF THE PRESENT STUDY

Some other approaches to the determination of Z-R relationships are discussed in the literature, but most of them will be found to be variants of one or the other of the techniques described above. The problems inherent in those techniques have led us to search for an alternative procedure for obtaining radar rainfall estimates.

We have investigated a technique for estimating convective rain volumes over an area from radar data without invoking a Z-R relationship. Byers' (1948) statement that the rain volume contributed by a rainstorm in a given locale bears a linear relationship to the rain area was taken as a clue here. If the slope is near unity, as seems to be the case, the implication is that for a given area coverage, including the entire storm period, the areal mean depth of rain is relatively constant. This areal mean depth can be multiplied by the area of the storm, as indicated by the echo coverage, to obtain the total radar estimated rain volume.

Dennis *et al.* (1975a) found also a good linear relationship (under appropriate transformations) between each pair of the rain volume, area coverage, and cloud depth. Thus methods which consider only the sizes of the storms may be able to give useful estimates of the rain volumes. The implication that we draw from this relationship between the volume of rain falling over an area and the areal extent of the showers producing it is that the reflectivity factors (Z-values) and the associated rainfall rates must follow some statistical distribution which is fairly well defined for a given geographic location, climate, and degree of organization of the convective activity. This statistical distribution should bear a close relation to the cloud volume.

The method discussed here does not require precipitation rates or point accumulations, which could be obtained from gages or from radar data, but considers only the rain events *per se*. A "rain event" as used here means the detection of precipitation at a given location during a specified time period (we used one hour) by either a radar set or a rain gage. Thus the method incorporates information about both the areal extent and the duration of the precipitation. The method requires only area coverage data, which are readily obtained from radar data. Adjustments for different synoptic conditions can be made but are not vital. Correction for advection of rainfall as it descends from the radar beam height to the ground is not needed, and the use of a suitable threshold for echo area calculation would reduce any need for an evaporation correction. The method has been tested for convective rain only.

4. SOURCE OF DATA

The data used in this study came from the North Dakota Pilot Project (NDPP), a randomized cloud seeding experiment to study weather modification techniques in the northern Great Plains (Dennis et al., 1975b). The radar data were collected with the NCPR-1 digital weather radar data system, which is described more fully by Smith et al. (1975a). It incorporated an S-band radar set with antenna beamwidth of about 2.5°. Data were logged whenever echoes were present in the McKenzie County target area between 1000 and 2200 CDT (Fig. 1). The data acquisition was controlled largely on an on-line minicomputer. The radar data used consisted of survey scans covering 360° in azimuth. The scans were made at 2° elevation at nominal 5-min intervals (there were actually 8 - 14 scans hourly). The reflectivity threshold for the radar data was about 15 dBz. As shown by Dixon and Smith (1978), the echo area coverage statistics are not very sensitive to the threshold used until it reaches perhaps 25 - 30 dBz. (That may not be true if the beam gets high enough to intercept a significant amount of storm anvil area.)

Networks of 22 recording rain gages (weighing type) and 80 conventional gages were spread over the project area of about 6,800 km^2 (McKenzie County). The recording gages were equipped with 12-hr charts that were read to yield rainfall accumulations for 1-hr intervals beginning and ending on each clock hour. The choice of the 1-hr interval was a compromise among many considerations, including possible clock errors, the effects of wind and wind shear, the difficulty of reading small rainfall amounts from the charts, and the complications introduced where a mixture of storm types contributed to the precipitation during the sampling interval. The conventional gages were read twice daily, at 0800 and 2000 CDT.

The raw radar data, essentially integrated logarithmic video signal levels, were recorded on tape in a range-azimuth data grid. They were later converted to equivalent radar reflectivity factors (Z values) using calibration information also stored on the tape. The reflectivity factor was then abstracted for each gage location at each survey scan time. Hourly radar estimated rainfall (RER) values were calculated at each recording gage location using the Z-R relationship $Z = 155R^{1.88}$ developed by an optimization technique (Smith et al., 1975b).

In this study we worked with two different sets of data. One set involved radar data covering the entire radar surveillance area, 38,700 km^2, while the other involved both gage and radar data for a sub-area, the McKenzie County area (Fig. 1). Using the McKenzie County data we analyzed 280 gage-hour events (occurring on 18 days with heavy rains from May-August) recorded by both radar and the 22 recording rain gages to determine the area coverage of the rainfall. For these 18 days, the daily areal rainfall was estimated both from the conventional gage data, taking advantage of a program developed by the NOAA Hydrologic Research Laboratory, and from the radar data (Smith et al., 1975b). The rain volume estimates for the 0800-2000 period computed using the 80 conventional gages available were considered the basic "ground truth" data.

Comprehensive rain gage data were not available for the entire radar surveillance area. Analyzing only the radar data, we were able to add 8 more days to the data sample for the surveillance area, giving 26 days in all.

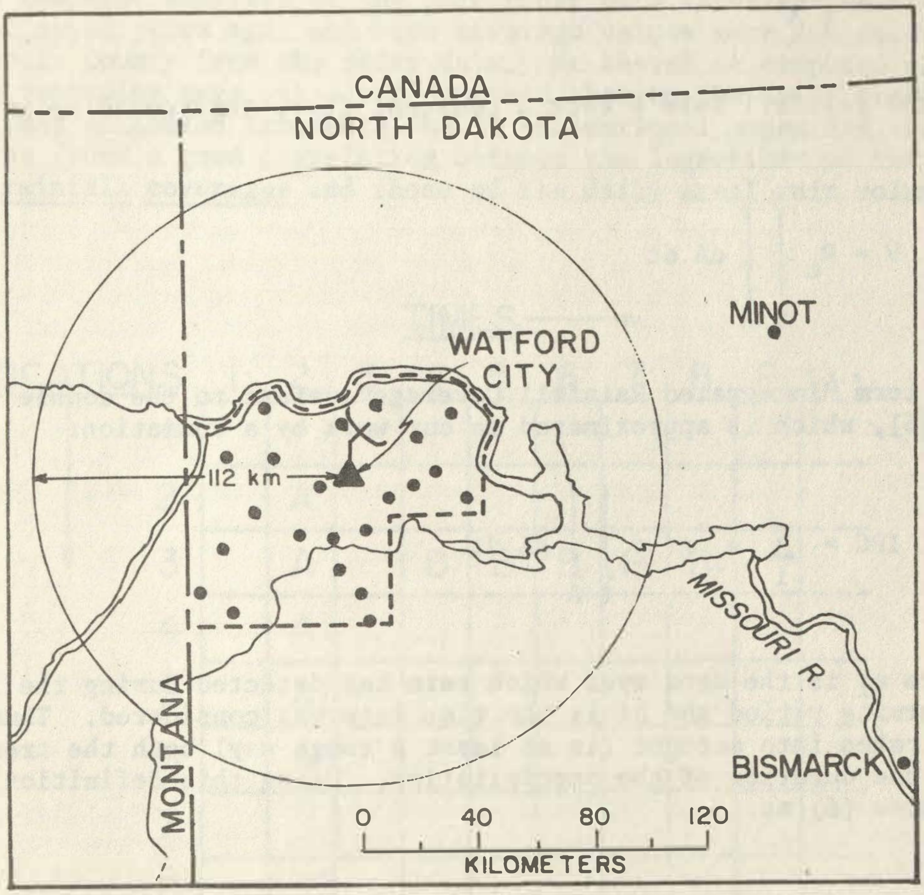

Fig. 1: Map showing the location of the North Dakota Pilot Project area centered at the Watford City radar site. The region within the 112 km radar surveillance radius was the source of the radar echo data used for this study. The area within the Montana border, the Missouri River, and the dashed lines is the McKenzie County area. Dots show the locations of recording rain gages.

5. RAINFALL COVERAGE - RAIN VOLUME CORRELATIONS

Our investigations confirmed earlier findings by Byers (1948) and several other investigators (see Sec. 5.3) that the volume of rain falling from a convective system is strongly correlated with the area covered by the system and its duration. This idea was employed by introducing a quantity called the Integrated Rainfall Coverage (IRC).

The rain volume V over an area A during time t is given by

$$V = \int_t \int_A R \, dA \, dt \tag{5}$$

If the rainfall rate R were a constant (R_c), this could be written as

$$V = R_c \int_t \int_A dA \, dt \tag{6}$$

The term "Integrated Rainfall Coverage" refers to the double integral in (6), which is approximated in our work by a summation:

$$IRC = \sum_i a_i \, \Delta t \simeq \int_t \int_A dA \, dt \tag{7}$$

where a_i is the area over which rain was detected during the ith observing period and Δt is the time interval considered. Thus the IRC takes into account (in at least a rough way) both the areal extent and the duration of the precipitation. Using this definition, we may rewrite (6) as:

$$V \simeq R_c \times IRC \tag{8}$$

The time interval used for this work was one hour, so the IRC has units of km^2 hr. While R_c was treated as a constant in the foregoing development, the data analysis that follows does not require that condition.

The integrated rainfall coverage concept is illustrated by Fig. 2, which depicts the array of possible rain events. The size of the array is determined by the number of possible rain locations and the number of time intervals (here, 12 one-hour periods). The integrated rainfall coverage is determined by the total number of the elements in this array (assuming equal-sized "locations") for which rain was observed. Thus the rain occurrence represented by the A's in Fig. 2 makes the same contribution to the integrated coverage as that represented by the B's.

We worked with the NDPP sets of data as follows:

5.1 McKenzie County Data

The computer analysis of the NDPP radar data tapes was carried out about seven years ago, and area coverage values were not computed for McKenzie County from the radar data. We therefore computed them from the recording gage data, and compared them to the daily areal rain volumes estimated from both the 80 conventional gages and the radar. We found a good correlation between the logarithms of the average rainfall coverages and those of the daily areal rain volumes.

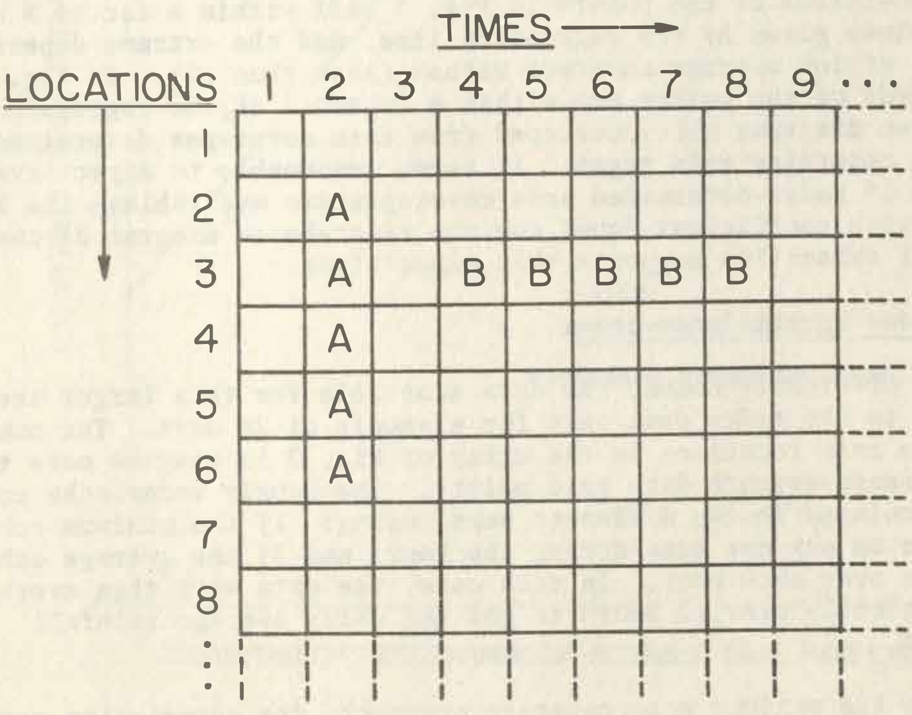

Fig. 2: Schematic illustration of the array of possible rain locations and time intervals involved in determining the integrated rainfall coverage (IRC). The A's and B's represent two distinct rain occurrences that contribute equally to the IRC.

The term "average rainfall coverage" refers to the integrated rainfall coverage divided by the product of the total area considered (here, 6,800 km^2) and the total time interval considered (here, 12 hours).

Figure 3 shows the logarithm of the rain gage estimated daily rain volume in McKenzie County plotted against the logarithm of the average rainfall coverage. Figure 4 shows a similar plot except that the radar estimated rain volume was plotted on the ordinate instead of the gage estimated rain volume. The plots were made using data for 18 days. For both diagrams, the coefficients of determination are $r^2 = 0.79$ ($r = 0.89$), which means that 79% of the variability of the rain volume can be explained by a linear relationship between the variables. These diagrams show the distribution of values corresponding to rains from subpolar and temperate air masses (marked by dots) in the lower section, and values corresponding to rains from subtropical air masses (marked by crosses) in the upper part (see Sec. 6 for details concerning the synoptic adjustment of the radar estimates).

The diagrams of Figs. 3 and 4 can be used to estimate the daily rain volume if hourly reports of rain events in the area are available. The procedure is to enter the abscissa with the observed average rainfall coverage and determine the estimated volume from the point of intersection of a vertical line at that point with the regression line. More than two-thirds of the points in Fig. 3 fall within a factor 3 of the rain volume given by the regression line, and the extreme departures are all at low average coverage values (less than 2%). In Fig. 4, two-thirds of the points are within a factor 2 of the regression line. These two diagrams were developed from area coverages determined with only 22 recording rain gages. It seems reasonable to expect even better results if radar-determined area coverages are available. The higher correlation coefficient found for the radar-based diagram discussed in the next subsection supports this expectation.

5.2 Radar Surveillance Area

As previously noted, the data available for this larger area were limited to the radar data only for a sample of 26 days. The number of possible rain locations in the array of Fig. 2 is now the more than 28,000 range-azimuth data grid points. The hourly radar echo coverage was calculated in two different ways, using: 1) the maximum echo coverage on any one scan during the hour; and 2) the average echo coverage over each hour. In each case, the data were then averaged arithmetically over 12 hours to get the daily average rainfall coverage.

For the maximum echo coverage approach, the correlation coefficient between the daily average echo coverage and the radar estimated rain volume was 0.91. For the average echo coverage approach, the coefficient was somewhat less, $r = 0.86$. Thus the maximum echo coverage

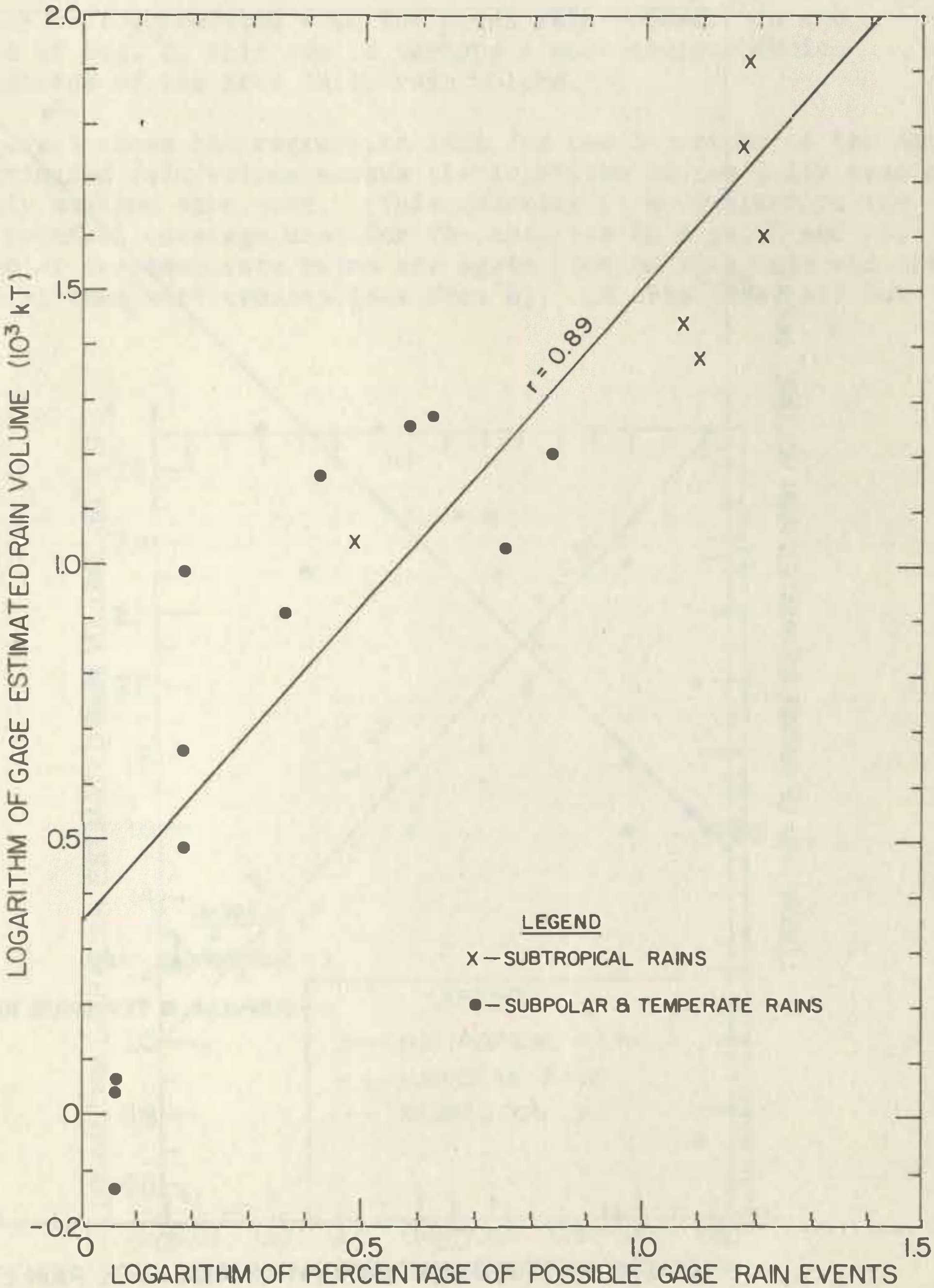

Fig. 3: Log-log plot of gage-estimated daily rain volume for McKenzie County, North Dakota vs. the average rainfall coverage.

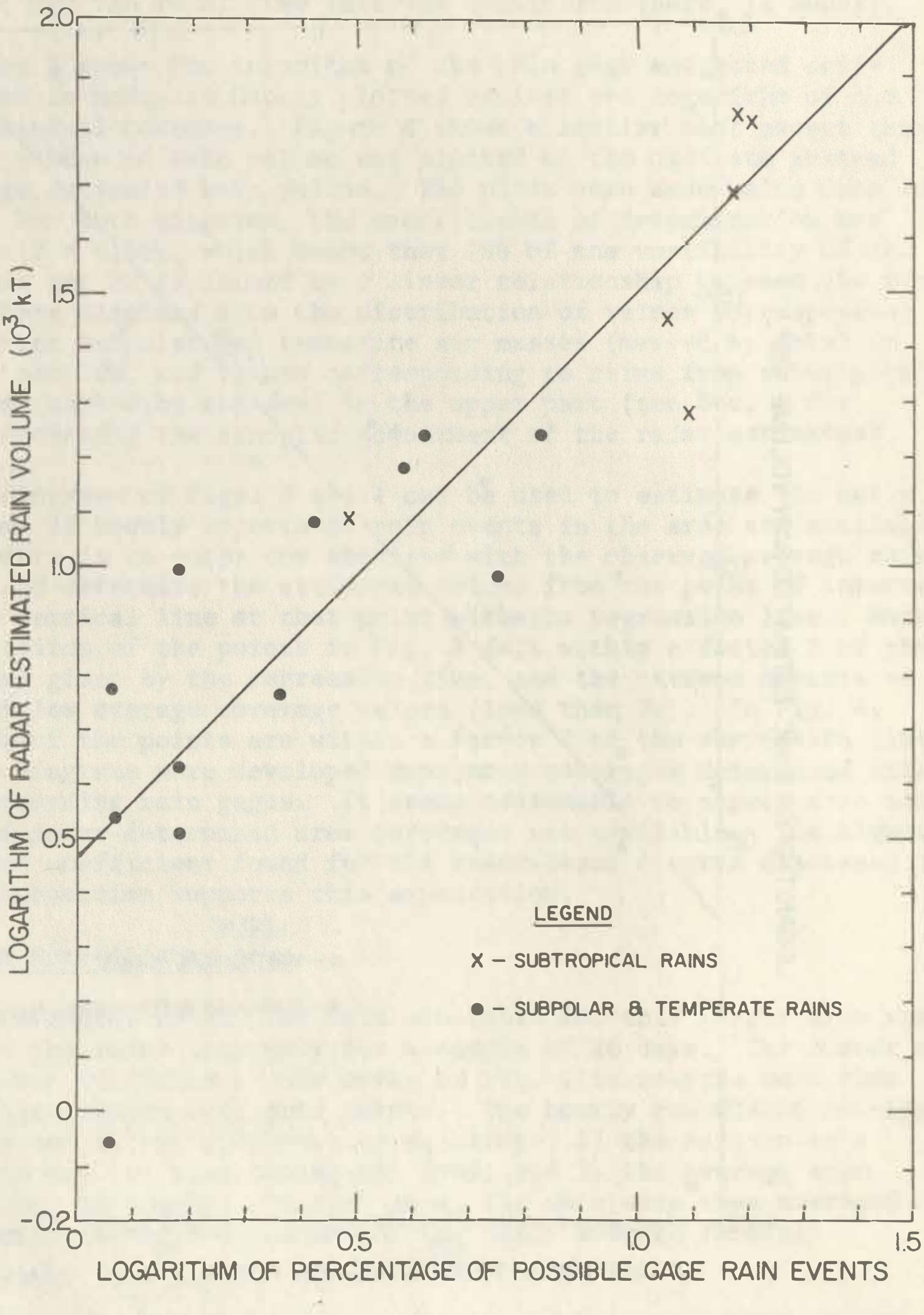

Fig. 4: Log-log plot of radar-estimated daily rain volume for McKenzie County, North Dakota vs. the average rainfall coverage.

during each hour seems to be a better predictor of rain volume. As the correlation coefficient between two variables is invariant under a scale transformation, the sum of the maximum hourly echo coverages is equally well correlated with the areal rain volume. In the framework of Fig. 2, this sum is perhaps a more obvious choice as a predictor of the area daily rain volume.

Figure 5 shows the regression line for the logarithm of the daily radar estimated rain volume versus the logarithm of the daily mean of the hourly maximum echo area. (This quantity is equivalent to the average rainfall coverage used for the abscissa in Figs. 3 and 4). The subpolar and temperate rains are again plotted with dots and the subtropical ones with crosses (see Sec. 6). In this case, all but

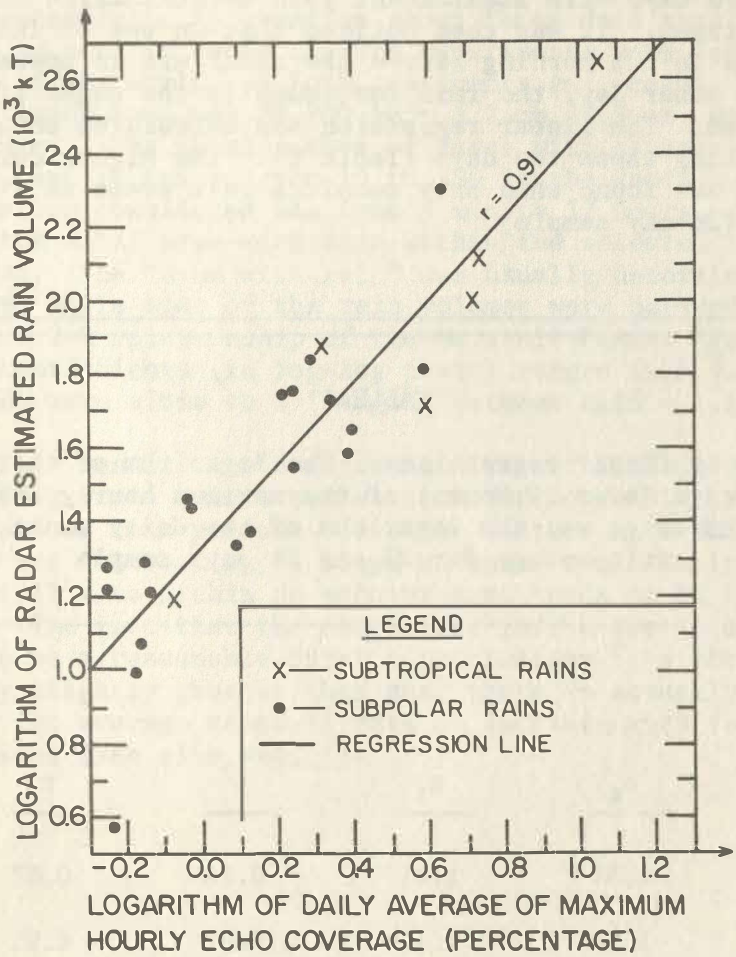

Fig. 5: Log-log plot of radar estimated rain volume vs. average echo coverage for the entire radar surveillance area and for 26 days.

one of the data points falls within a factor of two of the rain volume estimate given by the regression line. We want to underline that these results were obtained without considering synoptic stratification of the data.

We also calculated the daily average of the echo coverage excluding the hourly echo coverages of less than 1% and 0.1% of the total surveillance area. These exclusions were made with the thought that evaporation would be important for such small showers, so that the echoes might represent virga rather than rain at the ground. The results showed no improvement by excluding hours with very small echo coverage, but rather the reverse. Thus, in using this approach, any area with rain has to be considered, no matter how small.

Two more days with significant rain were at first considered in the calculations. It was then noticed that on one of the days, the rain started in the morning before the radar was in operation, and that on the other day, the rain continued in the night after the radar was shut down. The linear regression was calculated both considering and eliminating these two days (Table 1). The higher correlation coefficient was found when only complete rain event data were considered (26-day sample).

TABLE 1

Results of linear regressions. The logarithm of the daily average (over 12 hours) of the maximum hourly echo coverage vs. the logarithm of the daily radar rain volume for 28 and 26 days sample

No. of Days	a_0	a_1	r^2	r	Std. Err. of Estimate (S_{yx})
28	1.33	1.01	0.76	0.87	0.21
26	1.33	1.11	0.82	0.91	0.19

Because the radar echo intensity is related to the rain rate by a power-law relationship, we also calculated the averages of the echo coverages using the geometric mean. Different sets of data were considered by excluding again the very small showers in the calculations. The best fits were obtained, in this case also, using the maximum hourly echo coverage and the daily mean as the basic data. However, the echo area-rain volume correlation showed no improvement by using the geometric mean, but rather the reverse. We therefore concluded that the arithmetic mean coverage is the appropriate value.

5.3 Results From Other Studies

To substantiate our finding concerning the rain area-rain volume relationship, we looked for other, similar analyses, and found a total of seven (Byers, 1948; Dennis et al., 1975a, 1980; Doneaud et al., 1979, 1980; Hudlow et al., 1979; Voit et al., 1974), including ours. The most characteristic information about these data sets is summarized and compared to ours in Table 2. The experiments were located in different parts of the world, on both continents and oceans. The maximum radar surveillance area was 38,700 km^2; 3.2-cm, 5.3-cm, and 10-cm radars were used. The total number of cases with convective rains was close to 860, and it ranged from 18 to 350 in the various experiments. The time duration considered was from 5 min to 12 hours. In each experiment, the total area with rain within the selected time interval was considered. The "area with rain" was usually determined from radar echo coverage, while some of the rain volumes were derived from radar data through a Z-R relationship of the Marshall-Palmer type. The correlation coefficients (in log-log plots) ranged from 0.86 to 0.99, and the slopes were close to 1 (ranging between 0.95 - 1.13).

The earlier results are consistent with our finding concerning the area coverage-rain volume correlation for convective rains, even though the various rain periods and other factors were different. The slope values are interesting; a slope of unity would imply that the average rainfall rate within an echoing area tends to be independent of its size. The fact that the slopes are very close to unity suggests this to be a reasonable first approximation. As the slopes are generally slightly greater than one, there is actually a slight tendency for the average rainfall rate to increase with the size of the echoing area (see also Sec. 7).

TABLE 2

Rain area-rain volume correlations

Author	Location	Interval	Surface Area (10^3 km^2)	Time Duration	No. of Cases	Radar Type	Gage Density	Type of Correlation	Synoptic Stratif.	Coefficient of Correlation	Remarks
H.R. Byers et al.	Florida USA	Summer 1946	38	1-6 hours	28	MEW/CPS-1 10.0 cm	1 gage per 4.54 km^2	Linear on semi-log paper	No	0.94	Echo areas were used; average echo time (individual clouds)
F. Ya Voit et al.	Ukraine USSR	Summer 1961-72	10	15 min-5 hours	99	MRL-1 3.2 cm	1 gage per 15 km^2	Multilinear with rain area and volume in a log-log form	No	0.86	Echo and rain gage areas were used for one storm
A.S. Dennis et al.	South Dakota USA	June-Aug 1969-70	9	1 hour	33 no-seed 29 salt 18 silver iodide	Modified Nike/Ajax 3.2 cm	1 gage per 90.8 km^2	Linear on log-log paper after a cube-root transformation	No	0.97 0.99 0.97	Average echo areas with Z_e>30 dBz were used (individual clouds)
A.A. Doneaud et al.	North Dakota USA	May-Aug 1972	7	12 hours	18	NCPR-1 10.0 cm	1 gage per 318 km^2	Linear on a log-log plot	Four types of rain	0.91	Rain gage areas, in % of the target area, were used (for one storm or more)
A.A. Doneaud et al.	North Dakota USA	May-Aug 1972	38	12 hours	26	NCPR-1 10.0 cm	-	Linear on a log-log plot	Four types of rain	0.93	Echo areas were used in % of the target area (for one storm or more)
M. Hudlow et al.	Atlantic Ocean Trop & Equat Zone	June-Sep 1974 (GATE)	38	5 min	∿350 (both radars)	Oceanographer and Researcher 5.3 cm	-	Linear on log-log paper	No	∿0.95	Echo areas with Z_e>24 dBz were used (individual clouds)
A.S. Dennis et al.	Montana USA	Summer 1977	38	1-3 hours	255	SWR-75 5.3 cm	-	Linear on log-log paper	No	0.95	Echo areas were used (individual cloud complexes)

6. SYNOPTIC ADJUSTMENT OF THE RADAR RAINFALL ESTIMATES

Synoptic stratification of the estimated rainfall amounts was based on a morning sounding from Watford City (Fig. 1) and the mesoscale surface, synoptic surface, and upper level maps. Considering the prevailing flow (average between 500 mb and 700 mb wind vectors), the vertical wind shear, the humidity, the stability, and the frontal movement, we established four types of rain: 1) frontal rain from subpolar or temperate air masses; 2) instability rain from subpolar or temperate air masses; 3) frontal rain from subtropical air masses; and 4) instability rain from subtropical air masses. The frontal rains were all cold front rains except for one warm front case with a very small area coverage. Table 3 summarizes the results of the synoptic stratification for the hourly rain events at the recording gage locations. Subtropical rains gave on the average about 4.5 times more rain events than those from subpolar or temperate air masses.

TABLE 3

Synoptic stratification of McKenzie County data; 280 gage-hour events on 18 rain days, summer 1972

Types of Rain	No. of Rain Days	No. of Events	Mean of Logarithm GER/RER	Mean GER (mm)
Frontal, subpolar	7	50	-0.121	1.52
Instability, subpolar	5	38	0.068	2.08
Frontal, subtropical	3	111	-0.068	2.39
Instability, subtropical	3	81	0.112	2.61
Frontal	10	161	-0.105	2.12
Instability	8	119	0.083	2.44

An air mass was considered subtropical for an average humidity (mixing ratio) $\overline{H} \geq 7.5$ g kg^{-1}, and temperate or subpolar for an average humidity $\overline{H} < 7.5$ g kg^{-1}, in the 900-600 mb layer. The air mass stability was specified in terms of four indices, obtained from the Watford City morning soundings, as follows: 1) the Best Lindex Index; 2) the Showalter Index; 3) the K-index; and 4) the Total-total Index (National Weather Service, 1977).

A polar diagram was drawn to indicate the effect of wind direction (Fig. 6). The maximum percent area covered with rain for each day was plotted versus the prevailing wind. The percentage of area covered with rain was plotted at the end of the vector representing the prevailing flow direction for the day. Two dominant wind sectors were found: the SSE (non-zonal flow) with the largest rain coverage, mostly occurred in August with subtropical rains; and the WNW (zonal flow) with somewhat reduced rain coverage, mostly occurred in June with subpolar rains.

Table 3 suggests that the rain volume estimates derived from radar data should be adjusted according to the synoptic conditions. It shows that frontal rains give negative values of the mean log ratio (the logarithm of the ratio GER/RER for a given gage-hour event), while instability rains give positive mean values. That is, the radar estimates tend to be larger than the corresponding gage estimates for the frontal rains and smaller for the instability rains. The average log ratio on the days with frontal rains is -0.105 (Table 3), while the average log ratio for instability rains is +0.083. A two-sample rank test performed on the data rejects the null hypothesis that the mean log ratio under instability rains is less than, or equal to, the mean under frontal rains, for a significance level of $\alpha = 0.001$. It was concluded that the radar estimated rain volumes should be multiplied by about 0.8 (the antilog of -0.105) in the case of frontal rains and by about 1.2 in the case of instability rains to bring them into better agreement with the gage estimates.

These results are consistent with earlier findings by Atlas and Chmela (1957) that heavy showers and large coefficients in the Z-R equation occurred more frequently with cold fronts. Cataneo and Stout (1968) found smaller coefficients with cold frontal rains than with warm frontal rains. However, their observations were made in humid continental climates.

To test these adjustments of rain volume estimates obtained from radar data in the cases of instability and frontal rains, we calculated adjusted values of the daily areal volumes estimated for McKenzie County by multiplying by 1.2 and 0.8, respectively. Figure 7 shows the raw and adjusted radar estimated rain volumes compared to the estimates based on the 80 conventional rain gages; arrows show the direction and magnitude of the adjustment in each case. The agreement between the radar and gage estimates is improved on 12 out of 18 days, including the six cases with the heaviest rains.

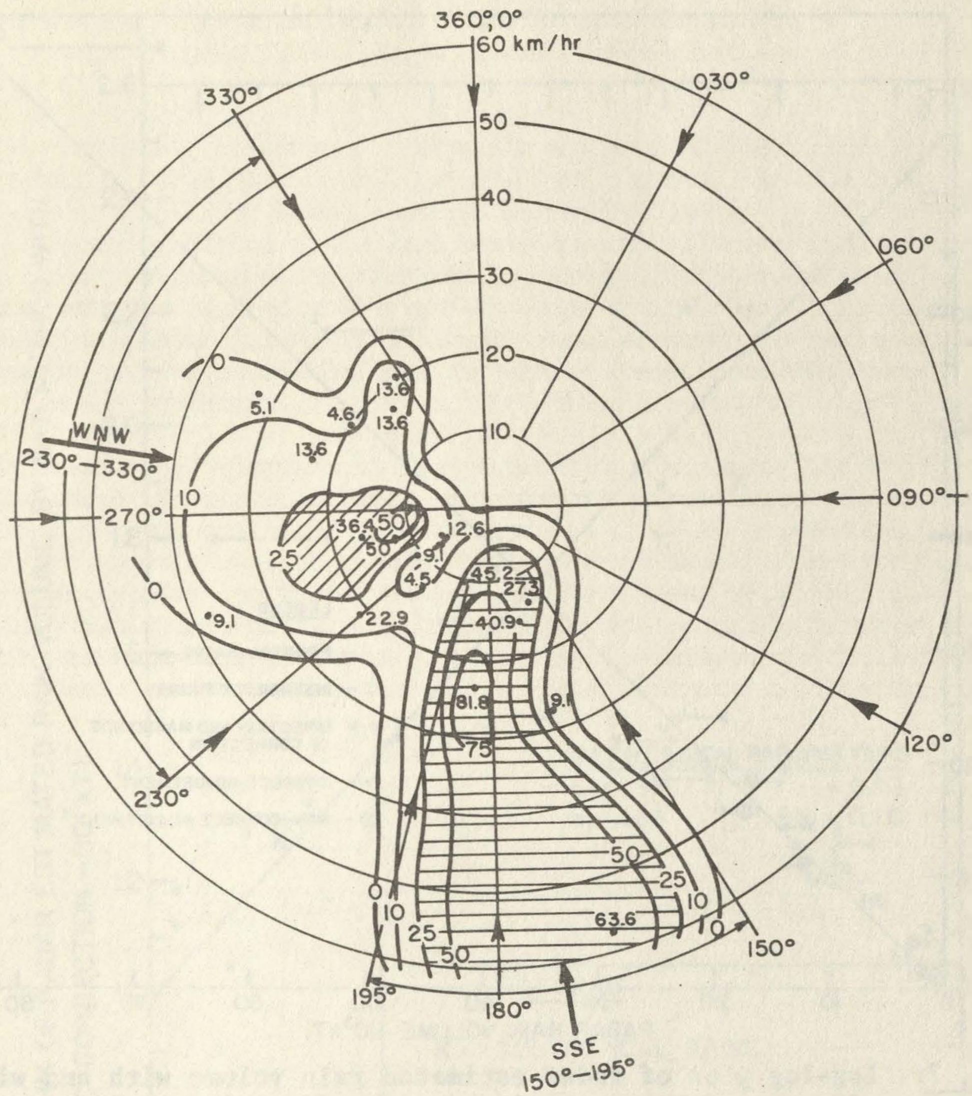

Fig. 6: Maximum rainfall as a function of the mean wind.

The synoptic adjustment technique was developed using rain gage and radar data for the McKenzie County area. We also checked the adjustments by applying them to radar rainfall estimates for the entire radar surveillance area, of which McKenzie County was only a part. In the absence of an extensive rain gage network covering the larger area, we compared the rain volumes estimated on the basis of echo area coverage using Fig. 5 to those derived from another similar diagram obtained from the radar data using the optimized Z-R relationship, with the latter estimates adjusted for synoptic type.

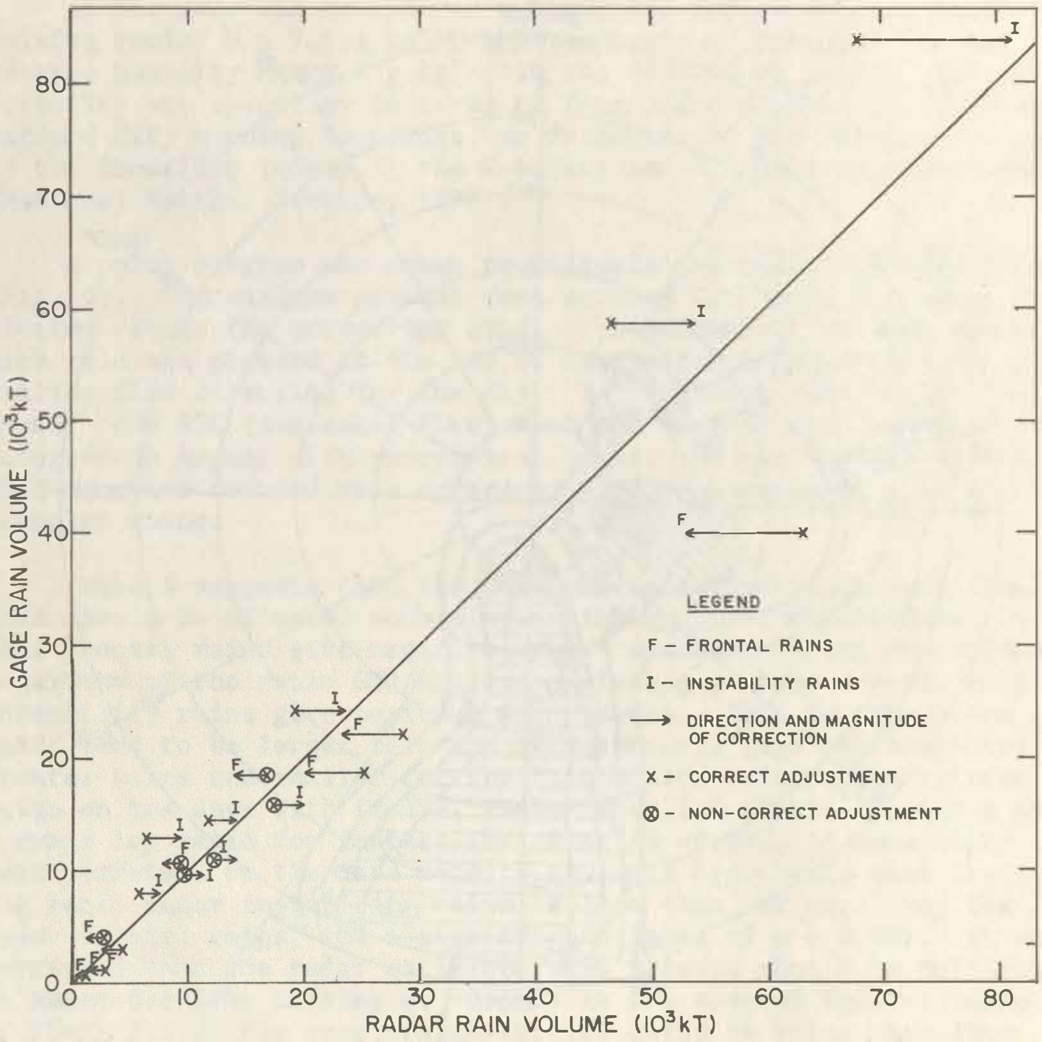

Fig. 7: Log-log plot of radar estimated rain volume with and without synoptic adjustment vs. gage estimated rain volume for McKenzie County.

We obtained a correlation coefficient of 0.93 using the adjusted Z-R estimates, compared to the 0.91 obtained by using non-adjusted values. The regression line relating the logarithm of the echo coverage to the logarithm of the adjusted radar rain volume is given in Fig. 8. All but three of the points in Fig. 8 have rain volumes within a factor 1.6 of those given by the regression line, and the logarithmic standard error of estimate was 0.16, compared to the 0.20 obtained without the synoptic adjustment. This again suggests that the synoptic adjustment improves the radar rainfall estimates based on a single Z-R relationship, although independent comparison with gage data would be needed to substantiate the point.

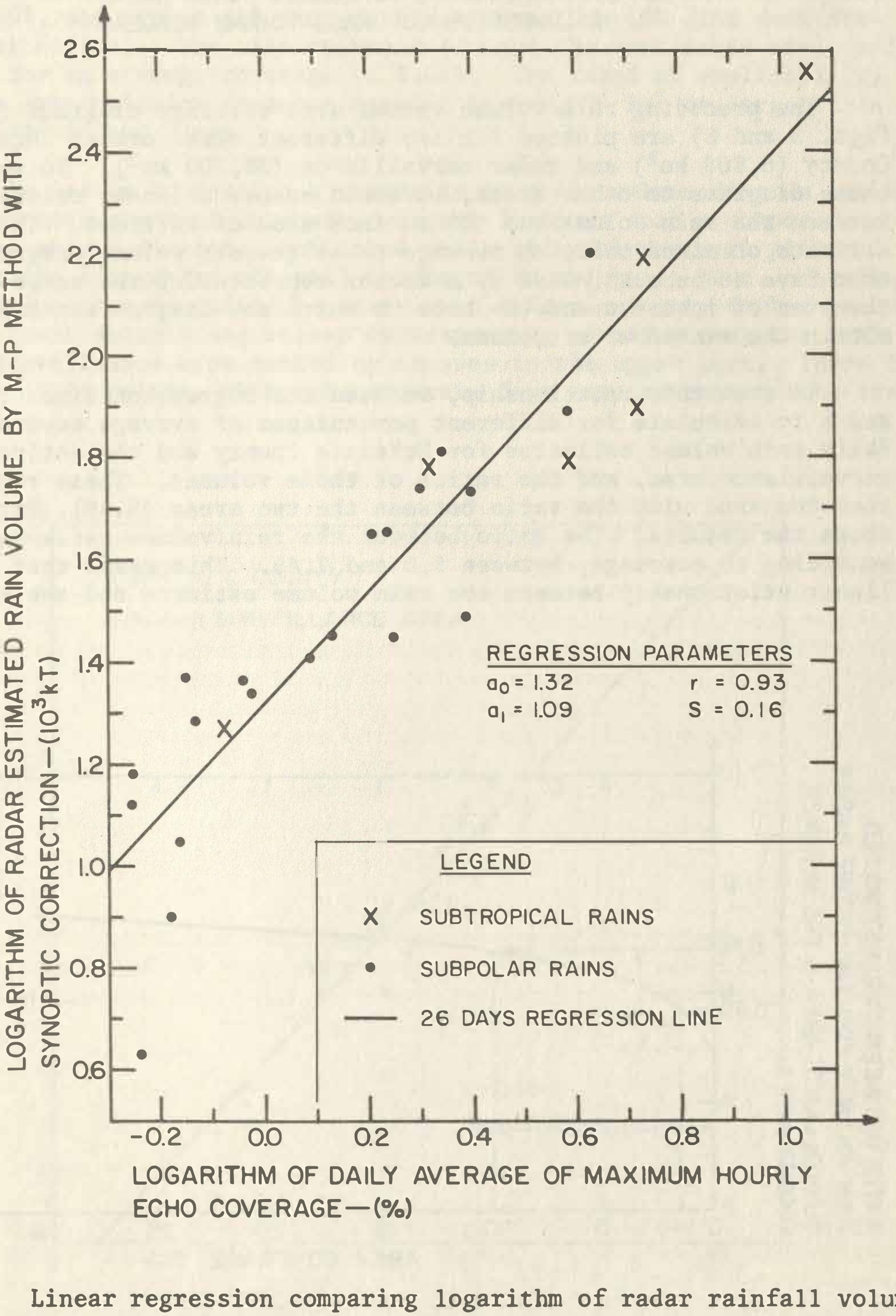

Fig. 8: Linear regression comparing logarithm of radar rainfall volume obtained by Marshall-Palmer type relationship with adjustment for synoptic type to logarithm of daily average of maximum hourly echo coverage. Study area was entire radar surveillance area.

7. APPLICATION TO DIFFERENT SIZED AREAS: A COMPOSITE COVERAGE-VOLUME DIAGRAM

The preceding rain volume versus area coverage diagrams (e.g., Figs. 3 and 8) are plotted for two different sized areas: McKenzie County (6,800 km^2) and radar surveillance (38,700 km^2). To apply these diagrams to other areas, we would assume a linear relationship between the rain volume and the surface area of interest. The volume estimate obtained using an average coverage-rain volume diagram would thus have to be multiplied by a factor representing the ratio between the area of interest and the area on which the diagram was based, to obtain the wanted water volume.

To test this relationship, we used the regression lines in Figs. 3 and 8 to calculate for different percentages of average coverage the daily rain volume estimates for McKenzie County and the entire radar surveillance area, and the ratios of those volumes. These ratios were then compared with the ratio between the two areas (5.69); Fig. 9 shows the results. The ratio between the rain volume estimates varies, according to coverage, between 5.0 and 7.45. This means that the linear relationship between the rain volume estimate and the average

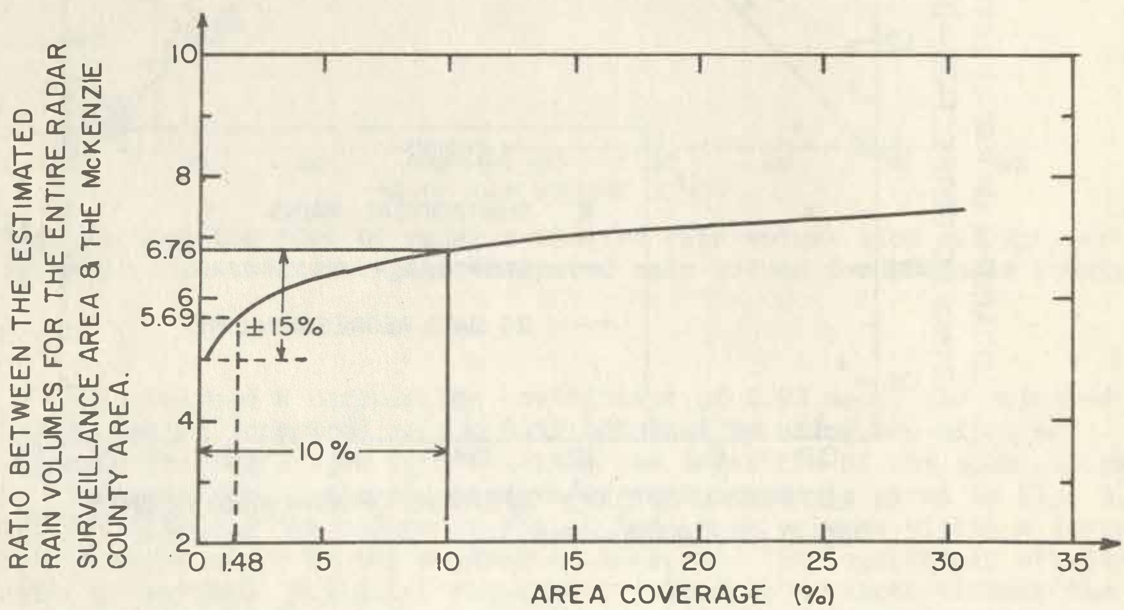

Fig. 9: Ratio between the estimated rain volumes for the entire radar surveillance area and the McKenzie County area vs. average coverage percentage.

coverage should hold (at least for areas up to 40,000 km^2) to within about 30%, and within 15% with coverage less than 10% (the most frequent situation). The rain volume ratio and the area ratio are equal (5.69) for an average coverage of 1.48%. The range of applicability of this extrapolation to areas smaller than 6,800 km^2 or larger than 40,000 km^2 is not known.

This led us to develop a composite coverage-volume diagram, using data for both areas of interest. Figure 10 shows such a composite coverage-volume diagram. Estimated daily rain volumes were plotted against the integrated rainfall coverages in a log-log plot. The diagram shows values corresponding to rains over McKenzie County marked by dots in the lower section and values corresponding to rains over the entire radar surveillance area marked by crosses in the upper part. There is a total of 44 points, although many rain days were common to both areas.

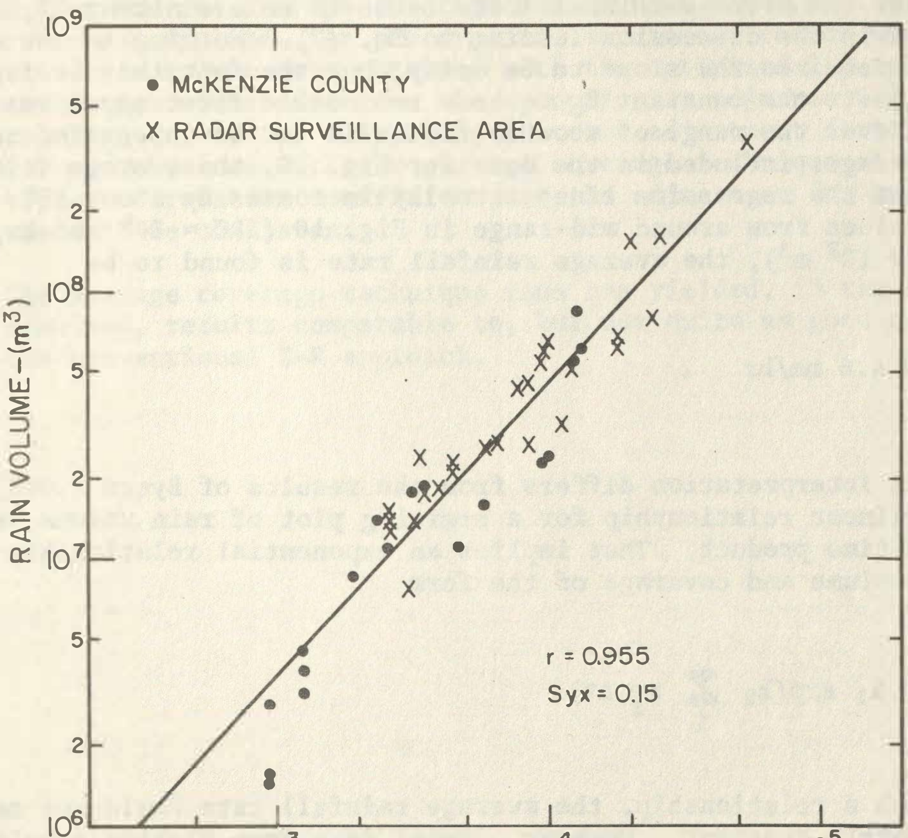

Fig. 10: Linear regressions comparing the integrated rainfall coverage (km^2 hr) to the rain volume. A log-log diagram is used.

Because the sources of both the coverages and the rain volumes are different, being gage data in one case and radar data in the other, the points are effectively independent.

The regression line has a slope of 1.1 and the correlation coefficient for the plot is r = 0.955. The (logarithmic) standard error of estimate for the regression line is 0.15; the antilog of 0.15 is a factor of 1.41, indicating that for two-thirds of the points, the rain volumes should fall within this factor of the regression line value. Seventy-five percent of the points in Fig. 10 fall within a factor of less than 1.5 of the rain volume given by the regression line, and the extreme departures are at low integrated coverage values (less than 1000 km^2 hr). While these errors are not small, they are comparable to those encountered using other methods of estimating rain volumes from radar data (e.g., Hildebrand et al., 1979) and with methods using satellite imagery (e.g., Griffith et al., 1981).

The slope value (1.1) is interesting. A slope of unity would imply that the average rainfall rate tends to be a constant R_c, as suggested in the discussion leading to Eq. (7). Nothing in the data analysis requires the slope to be unity, but the fact that it is close to 1 suggests the constant R_c to be a reasonable first approximation. However, over the range of about a factor 50 in the integrated rainfall coverages included in the data for Fig. 10, the average rainfall rate (from the regression line) actually increases by about 50%. Taking values from around mid-range in Fig. 10 (IRC = 10^4 km^2 hr; V = 4.6 x 10^7 m^3), the average rainfall rate is found to be

$$\overline{R} \approx 4.6 \text{ mm/hr} \quad .$$

This interpretation differs from the results of Byers (1948), who found a linear relationship for a semi-log plot of rain volume versus the area-time product. That implies an exponential relationship between volume and coverage of the form

$$V = k_1 \exp(k_2 \sum_i a_i \Delta t) \tag{9}$$

Under such a relationship, the average rainfall rate would not be even approximately constant. However, Byers' data from Florida involve coverages less than 200 km^2 hr, which is well below the range included in this work. For the high end of Byers' range, the average rainfall rate approaches 20 mm/hr, indicating the probable need to develop volume versus coverage diagrams appropriate for the region under consideration.

8. COMPARISON BETWEEN RAIN VOLUME ESTIMATES FROM AVERAGE-COVERAGE AND Z-R TECHNIQUES

To test our findings we compared both the rain volumes estimated from coverage data and those obtained from radar data using the optimized Z-R relationship with corresponding rain gage volumes. We considered the rain gage volumes to be the basic "ground truth" values. McKenzie County data were used for 18 days with heavy rains. The average coverage estimates were obtained from Fig. 4. The recording gage density was low, one gage per 318 km^2, but this was the only possible way to conduct the test. The test data are not strictly independent, and the test really answers only the question: "For the set of days plotted in Fig. 4, are the regression line values for the rain volume estimates closer than the original data points to the "ground truth" values?"

Table 4 shows the results of the volume comparisons expressed in logarithmic form. The correlation coefficient was higher for the Z-R method than for the average coverage method when the synoptic adjustment factor was not applied. As the synoptic adjustment factor was established using the rain volume data obtained with the optimized Z-R relationship, the correlation coefficient for the synoptically adjusted Z-R estimates was very high ($r = 0.98$). Applying the same adjustment factor to the rain volumes obtained with the average coverage technique also resulted in a higher correlation coefficient, so the adjustment for types of rain seems to be consistent.

The average coverage technique thus has yielded, in the sets of data examined, results comparable to, but not quite as good as, those from the conventional Z-R approach.

TABLE 4

Results of linear regressions. The logarithm of the daily rain gage volume was plotted against the logarithm of the daily radar estimated rain volume calculated in three different ways (McKenzie County data)

Correlations between the logarithm of the daily rain gage volume and:	No. of Days	a_0	a_1	r^2	r	Std. Err. of Estimate (S_{yx})	Accuracy* for a 0.95 Confidence Interval
Logarithm of the daily radar estimated rain volume (using the Z-R relationship)	18	-0.13	1.09	0.86	0.93	0.17	0.33/2.14
Logarithm of the daily radar estimated rain volume (using the rain volume vs. average coverage linear regressions)	18	-0.18	1.14	0.79	0.89	0.26	0.41/2.57
Idem with synoptic adjustment and 2 outliers omitted	16	0.11	0.92	0.86	0.93	0.16	0.31/2.09

*Acc = log x/x

9. THE RAINFALL ESTIMATING TECHNIQUE

The estimating technique is, in effect, a two-stage process. in the first stage, we obtain the integrated rainfall coverage. If radar data are available, the integrated rainfall coverage can be calculated by summing the maximum echo area coverage during any radar scan in each hour. This process can easily be computerized. The integrated rainfall coverage could also be determined, less satisfactorily, using hourly rain gage reports.

In the second stage, the rain volume is estimated from the integrated rainfall coverage using a graph like Fig. 10. The basic procedure is to enter the abscissa with the observed integrated rainfall coverage and determine the volume estimate from the point of intersection of a vertical line at that point with the regression line. The diagram presented here is applicable for the northern Great Plains continental temperate climate. Similar diagrams could be prepared using radar and rain gage data from areas with different climates.

10. FINAL REMARKS

The principal advantage of the integrated rainfall coverage technique is its simplicity. By using only the rain events (yes-no values) and not the rain rates obtained through a Z-R relationship of the Marshall-Palmer type, the problems associated with the variability of this relationship can be partially circumvented. The areas with rain are more accurately, inexpensively, and quickly determined by radar than by gages. Correction for advection of rainfall between the radar beam height and the ground is not needed and the use of a suitable reflectivity threshold for echo area calculation would reduce any need for an evaporation correction.

Intuition suggests that the area coverage ought to depend somewhat on the reflectivity factor threshold of the data (i.e., the minimum radar sensitivity), although the work of Dixon and Smith (1978) suggests the dependence to be weak. Certainly changes in the radar sensitivity will result in different determinations of the rain areas (and, in turn, the estimated rain volumes), but the associated errors may well be smaller than those involved in determining rain intensities for the entire area coverage using a Z-R relationship. This possibility should be more fully explored. The technique needs to be tested with independent data for verification. It is also not clear that the 1-hr time interval used here is the optimum choice, and other possibilities should be investigated. Further, there must be some minimum area for application of the method below which the integrated coverage usually is either zero or is determined mainly by the rain duration. The present analysis gives no idea of where this limit may lie.

The integrated coverage method is not proposed to supplant the conventional Z-R technique. If one has a well calibrated radar equipped with digital data processing equipment and a gage network adequate for adjustment purposes, there is little chance that this method can improve the rainfall estimates. However, we can visualize situations where recourse to this method might help one avoid drawing wrong conclusions on the basis of a poorly calibrated radar, a sudden change in radar performance, or a failure in the rain gage network. Where very accurate results are essential, the Z-R approach is preferable, but the simplicity of the echo coverage technique may be advantageous in a variety of situations. For example, some form of this technique might well be applicable with the Manually Digitized Radar (MDR) data currently available in the U.S. National Weather Service system (National Weather Service, 1977).

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