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New theoretical formulation for the determination of radiance transmittance at the water-air interface: comment

Howard R. Gordon[1,2] **and Kenneth J. Voss**[1,*]

[1] *Department of Physics, University of Miami, Coral Gables, FL 33124, USA*
[2] *hgordon@miami.edu*
[*] *kvoss@miami.edu*

Abstract: We challenge a recent paper in this journal suggesting that the well-established formula governing the transmittance of radiance across a refracting interface needs revision [Optics Express, **25**(22) 27086 (2017)]. We provide a simple example of radiative transfer across an interface showing that the accepted formula is correct.

OCIS codes: (010.4450) Oceanic optics; (010.4455) Oceanic propagation; (010.5620) Radiative transfer; (010.5630) Radiometry; (010.7340) Water.

References and links

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1. Introduction

The upward radiance, just beneath the water surface, due to backscattered solar radiation in natural waters, $L_u^-(\theta_w, \phi_w)$ propagating in a direction specified by the angles (θ_w, ϕ_w), contains information regarding the constituents of the water [1]. As such it is of considerable interest to the remote sensing community. This subsurface radiance leads to an above-surface radiance exiting the water, the water-leaving radiance $L_u^+(\theta_a, \phi_a)$, propagating in the direction (θ_a, ϕ_a). It is well established in the literature [2–4] that these above and below upwelling radiances are related by

$$L_u^+(\theta_a, \phi_a) = \frac{L_u^-(\theta_w, \phi_w)}{n_w^2} t_f^-(\theta_w, \phi_w) \tag{1}$$

where n_w is the refractive index of water, $t_f^-(\theta_w, \phi_w)$ is the Fresnel transmittance of the air-water interface (from the water side), and Snell's law, $n_w \sin\theta_w = \sin\theta_a$ and $\phi_w = \phi_a$, relates the angles in air and water. Henceforth, the superscripts + or − on a radiance (or irradiance) indicates that it is above or below the interface, respectively, while on a transmittance they indicate the direction of propagation of the incident radiation: from the air side (+) or from the water side (−). In addition, the subscripts u and d, when employed, stand for upward and downward, respectively.

In a recent publication in this journal Dev and Shanmugan [5] called Eq. (1) into doubt and made the claim that the transmittance factor (referred to there as $\tau_{w,a}$) should be replaced by a factor that depended on the optical properties the water, rather than on just n_w and (θ_w, ϕ_w). In support of that claim they examined a hypothetical situation in which radiance is incident on a water body having no absorption, so that all photons entering the water body eventually escape.

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Their conclusion was, that because of multiple surface reflections of photons within the water, in this particular case the transmittance factor must be unity rather than given by Eq. (1). We challenge this conclusion.

To demonstrate that Eq. (1) is correct, we will apply it to a situation in which radiative transfer in a water body is modeled in a simple manner. We assume a totally transparent atmosphere. The water body is modeled as a thin, totally transparent (non-absorbing and non-scattering) layer of water with refractive index n_w bounded below by a lambertian-reflecting surface of albedo (irradiance reflectivity) R. The lambertian-reflecting surface is employed to simulate the bulk of the water body, which reflects downwelling radiance and produces upwelling radiance. The purpose of the thin transparent layer is to provide the refractive index discontinuity between the water and the air and to allocate a region in which multiple reflections between the bulk of the water body and the surface can be enumerated. Multiple reflections play a central role in our arguments and in those of [5].

In natural waters, the irradiance ratio $R = E_u^-/E_d^-$ can be expressed in terms of the inherent optical properties of the medium according to $R \approx f b_b/a$, where a is the absorption coefficient and b_b the backscattering coefficient of the water plus constituents [1]. The quantity f depends somewhat on the scattering phase function of the particles and on the solar zenith angle. For small solar zenith angles and most oceanic waters, $f \approx 0.33$ [1]. The assumption of the lambertian-nature of the hypothetical surface means that the radiance distribution reflected from the bulk of the water is uniform. Generally it is not, but this assumption is made to simplify the computations here.

Now, consider illuminating the water from above by irradiance E_d^+ incident on the surface. For $R = 1$, the situation is identical to that considered in [5], Section 2.1.1, i.e., all photons will eventually exit the water. We will compute the radiance exiting the surface by explicitly keeping track of the contribution of multiple reflections from the interface to $L_u^-(\theta_w, \phi_w)$ and then use Eq. (1) to propagate the resulting radiance through the air-water interface. From $L_u^+(\theta_a, \phi_a)$ we then determine the associated upwelling irradiance E_u^+, and show that all photons *do* in fact exit the water when $R = 1$, i.e., energy is conserved if Eq. (1) is employed as written.

2. Radiative transfer computation

First, the incident irradiance E_d^+ is transmitted through the interface via the Fresnel transmittance t_f^+ evaluated at normal incidence. This transmitted irradiance then reflects from the Lambertian surface to yield an upward radiance $L_{u0}^- = E_d^+ t_f^+ R/\pi$, where the factor π results from the fact that the upward irradiance is totally diffuse. This is the component of $L_u^-(\theta_w, \phi_w)$ that has not yet been internally reflected from the interface (indicated by the subscript 0). It is then reflected downward leading to a downward radiance $L_{u0}^- r_f^-(\theta_d)$, where (θ_d, ϕ_d) is the downward direction of the reflected radiance incident from the direction (θ_w, ϕ_w) and r_f^- is the Fresnel reflectance. This leads to a downward irradiance $E_{d1}^- = \int_{\Omega_d} L_{u0}^- r_f^-(\theta_d) \cos\theta_d \, d\Omega_d = \pi \bar{r} L_{u0}^-$, where $\bar{r} \equiv 2\int_0^{\pi/2} r_f^-(\theta_d) \sin\theta_d \cos\theta_d \, d\theta_d$, is the internal reflectance for diffuse irradiance and has a value of approximately 0.475 for water. This downward irradiance is then reflected upward leading to a second upwelling radiance $L_{u1}^- = (\bar{r}R)L_{u0}^-$: the upward radiance due to photons that have been reflected once from the interface. Continuing this process, $L_{u2}^- = (\bar{r}R)L_{u1}^- = (\bar{r}R)^2 L_{u0}^-$, etc. The total radiance incident on the underside of the surface is then

$$L_u^- = L_{u0}^- + L_{u1}^- + L_{u2}^- \cdots = \frac{L_{u0}^-}{1-\bar{r}R} = \frac{E_d^+ t_f^+ R}{\pi(1-\bar{r}R)} \qquad (2)$$

(Note, if our hypothetical surface was not assumed to be lambertian, the formulation would be the same but the values of $\bar{r}R$ would be different for each internal reflection and the series in Eq.

(2) would not be summable in closed form.) The radiance L_u^- must then be transmitted though the interface to give the upward radiance above the surface. We use Eq. (1) to effect the transmission:

$$L_u^+(\theta_a, \phi_a) = \frac{t_f^-(\theta_w, \phi_w)}{n_w^2} L_u^-(\theta_w, \phi_w) = \frac{E_d^+ t_f^+ R}{\pi(1 - \bar{r}R)} \frac{t_f^-(\theta_w, \theta_a)}{n_w^2}. \tag{3}$$

To see if this is correct we compute the upward irradiance:

$$E_u^+ = \int_{\Omega_d} L_u^+(\theta_a, \phi_a) \cos\theta_a \, d\Omega_a = \frac{E_d^+ t_f^+ R}{\pi(1 - \bar{r}R)} \int_{\Omega_d} \frac{t_f^-(\theta_w, \theta_a)}{n_w^2} \cos\theta_a \, d\Omega_a. \tag{4}$$

Consider the last integral on the right hand side:

$$\int_{\Omega_d} \frac{t_f^-(\theta_w, \theta_a)}{n_w^2} \cos\theta_a \, d\Omega_a = \int_{\Omega_d} [1 - r_f^-(\theta_w, \theta_a)] \cos\theta_w \, d\Omega_w, \tag{5}$$

where we have used the fact that $\cos\theta_a \, d\Omega_a = n_w^2 \cos\theta_w \, d\Omega_w$ [5], and replaced the Fresnel transmittance by one minus the Fresnel reflectance. A simple computation shows that this integral is just $\pi(1 - \bar{r})$. Thus,

$$E_u^+ = \frac{E_d^+ t_f^+ (1 - \bar{r}) R}{(1 - \bar{r}R)} \tag{6}$$

and, adding this to the incident-beam irradiance reflected from the upper side of the water surface, i.e., $E_d^+(1 - t_f^+)$, the total upward irradiance just above the water surface is

$$E_u^+(\text{Total}) = \frac{E_d^+ t_f^+ (1 - \bar{r}) R}{(1 - \bar{r}R)} + E_d^+(1 - t_f^+). \tag{7}$$

We note that this is correct at both limits: $R = 0$ where $E_u^+(\text{Total}) = E_d^+(1 - t_f^+)$, i.e., the specularly reflected solar beam; and $R = 1$, where $E_u^+(\text{Total}) = E_d^+$. Clearly, we included all orders of reflection of photons from the underside of the interface, used Eq. (1), and arrived at the correct result in these two limits, in particular the $R = 1$ limit used in [5].

3. Discussion

After accounting for all orders of multiple reflections of upwelling radiance from the water side of the air-water interface in this simple example, we have shown that application of Eq. (1) yields an above-water radiance, that when integrated, provides the correct result for the reflected irradiance — specular and diffuse — in the two limiting cases $R = 0$ and $R = 1$.

Where was the error that the authors of [5] made in concluding that Eq. (1) was inadequate? We believe they incorrectly interpreted the radiance on the right-hand-side of Eq. (1) as being the radiance before multiple reflections, i.e., L_{u0}^- of Eq. (2). We note that L_u^- is the actual radiance that would be measured by a radiometer placed just beneath the water surface. The quantity L_{u0}^- cannot be directly measured, it must be calculated from Eq. (2).

4. Conclusion

Our conclusion is that Eq. (1) is correct, and there is no relation between the radiance transmittance of the interface, i.e., Eq. (1), and the optical properties of the water, other than n_w. If an investigator measures $L_u^-(\theta_w, \phi_w)$, then he/she can be confident that $L_u^+(\theta_a, \phi_a)$ is determined by Eq. (1).

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