

1 Characterization of the diurnal cycle of maximum rainfall in  
2 tropical cyclones

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6 **Abstract**

7 We analyze the diurnal cycle of maximum rainfall from ~300 TCs from March 2014  
8 through February 2017, by cross-referencing the path of tropical cyclones (TCs) and high-  
9 resolution rainfall estimates from IMERG (Integrated Multi-satellitE Rainfall from GPM -  
10 Global Precipitation Measurement mission). IMERG is a gridded satellite product that  
11 offers high-resolution rainfall estimates at a spatiotemporal resolution of  $0.1^{\circ} \times 0.1^{\circ}$  every  
12 30 minutes, which are particularly suitable for these analyses.

13 Because of the nature of the data, we use circular statistics. Circular statistics allows us to  
14 account for the natural periodicity of a random variable such as the time of the day at  
15 which maximum rainfall from TCs occurs. We follow the non-parametric approach of  
16 Mixtures of Von Mises-Fisher distribution (MvMF), which enables an easy-to-interpret  
17 parameter identification of multimodal and anisotropic distributions of the TC-rainfall. We  
18 stratify our analysis by storm duration, maturity, and intensity, basin of origin, radial

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19 proximity to the center of the storm, and whether the storm is over the ocean or land.

20 In general, and across all scales, we find that there are mainly two cycles of maximum TC-  
21 rainfall: one diurnal cycle with peaks at ~10 and ~22h (local time), and one semi-diurnal  
22 cycle with peaks at ~2 and ~5h (local time). Although in a smaller proportion, the latter  
23 exhibits a weak afternoon alternative, i.e., ~14 and ~18h (local time).

24 *Keywords:* Tropical Cyclones, Diurnal Cycle, Rainfall, Circular Statistics, IMERG

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## 25 **1. Introduction**

26 Tropical cyclones (TCs) are phenomena of paramount importance not only for the rain  
27 they produce but also for the havoc they unleash, both in coastal and inland areas (e.g.  
28 Czajkowski et al., 2017; Khouakhi et al., 2017). They are also considered the deadliest  
29 type of weather-related disasters, as the death toll from ~2,000 storms (from 1995 through  
30 2015) amounts to ~242,000 fatalities (UNISDR and CRED, 2017) or 251,384 (roughly  
31 equivalent to 40% of the total casualties from weather-related disasters) from 1980 to  
32 2000 according to (UNDP, 2004, p.37). For instance, in 2017 Hurricane Harvey brought  
33 almost 125,000m<sup>3</sup> of rain, spread over four U.S. states (Fritz and Samenow, 2017).  
34 Averaged over the Houston area, the lowest total precipitation in seven days brought by  
35 Hurricane Harvey was 700.2mm, which is more than double of any previous record  
36 (315.8mm for seven days of rainfall) between 1950-2016 (Risser and Wehner, 2017). Put  
37 into perspective, this amount of rainfall is the equivalent to the yearly average precipitation  
38 in Houston (Burian and Shepherd, 2005; Fritz and Samenow, 2017). Overall, Hurricane  
39 Harvey produced the largest rainfall ever recorded of any hurricane affecting the United  
40 States (e.g. Emanuel, 2017; NOAA-WPC, 2017; Samenow, 2017). The number of  
41 fatalities caused by this storm is reported to be ~80 people (e.g. Moravec, 2017; van

42 Oldenborgh et al., 2017).

43 The impact exerted by TCs comes from the high-wave storm surges, extreme winds, and  
44 floods and landslides associated with the torrential rains they produce (e.g. Mendelsohn et  
45 al., 2012; Peduzzi et al., 2012). Out of these three factors, we devote our attention to the  
46 characterization of heavy rainfall from TCs given its direct relation to flooding, which in the  
47 last two decades has affected ~2.3 billion people (UNISDR and CRED, 2017). This is  
48 equivalent to 56% of the people affected by weather-related disasters. Hence, the  
49 characterization of heavy rainfall from TCs provides essential information to assess and  
50 evaluate the impact from landfalling TCs, helping thus potential affected communities to  
51 be more resilient against such natural hazards. Several studies have focused on TC-  
52 rainfall characterization. For instance, (Prat and Nelson, 2016) studied the contribution of  
53 TCs to extreme daily rainfall, whereas (Prat and Nelson, 2013) established the  
54 contribution of TC-rainfall to the seasonal precipitation totals for the southeastern United  
55 States. (Jiang et al., 2008) analyzed the rainfall distribution from landfalling TCs in the  
56 north Atlantic basin. All of the above studies were based on about one decade of satellite  
57 data. (Lonfat et al., 2004; Rios Gaona et al., 2018) are global studies in which TC-rainfall  
58 is characterized and stratified by basin and intensity (among other features) also from  
59 global satellite data.

60 The focus of this work is to delve into the diurnal cycle of TC-rainfall maxima. The number  
61 of studies about the diurnal cycle of TC-rainfall have grown in recent years due to the  
62 widespread development and availability of satellite rainfall estimates. (Bowman and  
63 Fowler, 2015) carried out statistical analyses over 15 years of TMPA 3B42 (Tropical  
64 Rainfall Measurement Mission - TRMM Multisatellite Precipitation Analysis) and IBTrACS  
65 (International Best Track Archive for Climate Stewardship) data to investigate the diurnal

66 cycle of TC-rainfall, which they see as one potential component of precipitation variability  
67 in these storms. (Wu et al., 2015) studied the diurnal variations of oceanic TC-rainfall in  
68 their inner core and outer rainbands. Their study was also based on 15 years (1998-2012)  
69 of TMPA 3B42 data (1401 TCs), and focused only on oceanic storms (i.e., beyond 300km  
70 from coastlines). (Leppert II and Cecil, 2016) used TRMM's Microwave Imager (TMI) and  
71 Precipitation Radar (PR) to study the diurnal cycle of 208 storms in the Atlantic basin  
72 during the period 1998-2011. They stratified their analyses by radii (from 100 to 1000km,  
73 every 100km), by intensity (wind speed larger than 34 kt, and 64 kt), and by height (2, 8,  
74 and 10km). More recently, (O'Neill et al., 2017) examined cloud-resolving TC simulations  
75 to understand the wavelike diurnal cycle responses on quasi-steady TCs. They found  
76 evidence of diurnal wave propagation in the upper troposphere in eddy-temperature fields.  
77 (Tang et al., 2017) studied the sensitivity of hurricane Secondary Eyewall Formation (SEF)  
78 to solar insolation. Through a numerical simulation, (Navarro et al., 2017) determined the  
79 impact of periodic diurnal heating on a balanced vortex, highlighting the importance of  
80 clouds. The introductions of (Bowman and Fowler, 2015; Leppert II and Cecil, 2016;  
81 O'Neill et al., 2017) provide extensive literature (and recounted details) on the diurnal  
82 cycle of oceanic precipitation (e.g. Frank, 1977; Hai-Long et al., 2013), of TC-rainfall (e.g.  
83 Jiang et al., 2011; Wu et al., 2015), and of cloud-tops changes (e.g. Browner et al., 1977;  
84 Dunion et al., 2014; Kossin, 2002). Studies on the diurnal cycle of TC-rainfall contribute to  
85 the characterization and understanding of TC-rainfall variability from the diurnal insolation  
86 cycle on TCs. Such a variation is key to improve storm intensity prediction, and TC  
87 modelling on global climate systems, for instance.

88 Our work advances the knowledge of the diurnal cycle of maximum TC-rainfall because  
89 we use high-resolution satellite data and circular statistics. IMERG (Integrated Multi-

90 satellite Retrievals for GPM - Global Precipitation Measurement mission) is a follow-up on  
91 almost two decades on continuous rainfall monitoring at global scales from TRMM and its  
92 equivalent TMPA products (Huffman et al., 2007). IMERG is a gridded rainfall product with  
93 a spatiotemporal resolution of  $0.1^\circ \times 0.1^\circ$  every 30 minutes (Hou et al., 2014). Rainfall  
94 monitoring at high resolution from space nowadays serves as a key tool to develop and  
95 enhance societal applications such as fresh water availability, flood forecasting, landslide  
96 warning, water-borne disease propagation, and storm-tracking (Kirschbaum and Patel,  
97 2016; Stanley et al., 2017). The main advantage with regard to storm-tracking is that from  
98 global rainfall estimates such IMERG one can track the precipitation path of such large  
99 scale storms that often are difficult to even quantify from ground-based sensors like  
100 gauges and weather radars. The IBTrACS data set offers a detailed record of TC-tracks  
101 and maximum sustained windspeed (MSW) of all the TCs worldwide since 1842 (and up  
102 to March 2017). By combining these two data sets, we can obtain a detailed and accurate  
103 description of the spatiotemporal variability of rainfall from TCs. This allows us to study the  
104 diurnal cycle of maximum rainfall for all the TCs (259) worldwide in a span of 3 years  
105 (GPM launched its core satellite on February 2014).

106 In addition to high-resolution satellite data, we use circular statistics, which represents the  
107 appropriate statistical framework for analyses of this kind. In circular statistics the data  
108 under analysis is represented as points over a unit circle, which is the support for “circular”  
109 variables (Pewsey et al., 2013). In a circular space, all data is equally likely to be  
110 distributed over a segment equivalent to  $2\pi$ . This abstraction has the unique advantage to  
111 account for the intrinsic periodicity of a circular and/or directional variables, such as time  
112 of the day at which rainfall occurs or the azimuthal direction of the maximum sustained  
113 windspeed of a hurricane, for instance. A basic example is that of the average of a

114 random variable that took place at 01:00 and 23:00, for instance. A linear analysis will tell  
115 us that the average time of such a random variable is 12:00. Due to the proximity of 01:00  
116 and 23:00 in a 24-h circular space, the circular analysis will yield an average time of  
117 00:00, which is a more correct approximation of the true nature of the random variable  
118 under analysis.

119 Work on TC-rainfall via circular statistics has not been carried out so far. The common  
120 approach is to apply linear statistics to draw the cyclic patterns (e.g., Hu et al., 2017).  
121 Recent and related work on the implementation of circular analysis in hydrometeorological  
122 topics include those of (Dhakal et al., 2015; Masseran, 2015; Villarini, 2016). (Dhakal et  
123 al., 2015) developed a non-parametric (circular statistics) approach that optimizes the  
124 bandwidth(s) of a Von Mises distribution (Sec. 2). Their approach assessed the non-  
125 stationarity of 60 years of maximum daily precipitation at ten locations in the northeastern  
126 United States. (Masseran, 2015) used non-parametric circular statistics to better  
127 characterize the wind regime in the northern region of Borneo (Malaysia). From almost  
128 one year of hourly wind direction data (one station only), they found that the finite mixture  
129 of Von Mises–Fisher approach (Sec. 2) systematically outperforms the one based on non-  
130 negative trigonometric sums. From annual maximum instantaneous peak discharges  
131 (~7,500 gage stations with at least 30 years of data), (Villarini, 2016) applied circular  
132 statistics to study the seasonality of flooding across the continental United States. Other  
133 examples of developments and implementations of circular statistics in earth sciences  
134 (including mixtures of Von Mises–Fisher probability density functions - MVMF-PDFs)  
135 include those by (Lark et al., 2014; Oliveira et al., 2012). To the best of our knowledge, our  
136 work is the first of its kind that offers a comprehensive and quantitative characterization of  
137 the diurnal cycle of TC-rainfall maxima, analyzed via the circular statistics framework.

138 A detailed presentation of the theoretical framework of circular statistics is beyond the  
139 scope of this paper. For that matter, we point the interested reader to previous works  
140 carried out by (Fisher, 1993b; Mardia, 1972b; Mardia and Jupp, 2000; Pewsey et al.,  
141 2013), where deep and comprehensive formulations, details, and references on the theory  
142 of circular statistics can be found. Our approach relies on the R-packages `movMF` (Hornik  
143 and Grün, 2014), `circular` (Agostinelli and Lund, 2017), and `Directional` (Tsagris et al.,  
144 2017). R is computing language and environment for statistical analysis (R Core Team,  
145 2017).

146 We stratify our analysis by TC duration, maturity, and intensity, basin of origin, distance  
147 from the center, and whether the storm is over the ocean or land. A thorough analysis of  
148 yet another characteristic of TC-rainfall such as the diurnal cycle of maximum TC-rainfall  
149 gets us closer to more realistic representations and models of the rainfall associated with  
150 TCs. We consider our approach a better assessment of the diurnal cycle because not only  
151 the available high-resolution data we use but also the circular framework offers a more  
152 accurate and appropriate approach for the statistical description of TC-rainfall maxima.

153 This paper is organized as follows: Section 2 briefly describes the data we use and  
154 introduces the conceptual framework of circular statistics, and its implementation. Section  
155 3 presents the results and discussion alongside. Summary and conclusions are provided  
156 in Section 4.

## 157 **2. Data and Methodology**

158 Our data set is similar to that of (Rios Gaona et al., 2018), in which they analyzed 166 TCs  
159 for the period of March 2014 through March 2016. Hence, the analysis comes from the  
160 merging of two data sets: IBTrACS, and IMERG V04 Final.

161 The IBTrACS (v03r10) is a worldwide collection of TC best-track data (Knapp et al., 2010).  
162 Developed by the National Climatic Data Center (NCDC) jointly with the World Data  
163 Center for Meteorology, it is a comprehensive project that gathers information from all the  
164 Regional Specialized Meteorological Centers (RSMCs) and Tropical Cyclone Warning  
165 Centres (TCWCs) members of the World Meteorological Organization (WMO), and other  
166 national agencies (IBTrACS data is freely available from the server  
167 <ftp://eclipse.ncdc.noaa.gov/pub/ibtracs/>). The IBTrACS data set contains several attributes  
168 or variables. One of them is the seven basins in which the Earth's surface is divided into  
169 from a TC perspective: North Atlantic (NA), Eastern Pacific (EP), Western Pacific (WP),  
170 Northern Indian Ocean (NI), Southern Indian Ocean (SI), South Pacific (SP), and South  
171 Atlantic (SA). Attributes such as MSW, the time at landfall (if available), and the longitude  
172 and latitude of the storm centers (from which later we interpolate the TC track at 30-  
173 minute resolution) are also included in this data set. The temporal resolution of IBTrACS is  
174 6 hourly (00:00, 06:00, 12:00, and 18:00 UTC).

175 IMERG is a gridded rainfall product (Level 3) from the GPM mission. This high-resolution  
176 product provides rainfall intensities with a spatiotemporal resolution of  $0.1^{\circ} \times 0.1^{\circ}$  every 30  
177 minutes between  $60^{\circ}\text{N}$ – $60^{\circ}\text{S}$ . It is obtained by processing (i.e., intercalibration, merging,  
178 and spatiotemporal interpolation) all the microwave precipitation estimates available from  
179 the GPM constellation (Huffman et al., 2017b). IMERG also incorporates infrared data  
180 from geostationary satellites, and it is calibrated with global gauge analyses of  
181 precipitation (Schneider et al., 2015a, 2015b). With three “flavors”, IMERG products are  
182 developed to address different user requirements of latency and accuracy, i.e., Early Run  
183 (near-real-time), Late Run (reprocessed near-real-time), and Final Run (post-real-time).  
184 Technical insights on IMERG and its recent update IMERG V04 (Final) can be found in



185 (Huffman et al., 2017a, 2017b, 2017c). The availability of IMERG–Final goes from 12  
186 March 2014 to the present with a latency of four months. This availability limits the number  
187 of TCs that one can potentially analyze. IMERG products contain several attributes  
188 (subsets). We only focus on the *precipitationCal* subset which offers the most accurate  
189 rainfall estimates. From here onwards, we refer to IMERG V04 Final (*precipitationCal*)  
190 only as IMERG. (GPM rainfall datasets are freely available at the NASA (National  
191 Aeronautics and Space Administration) portal [http://pmm.nasa.gov/data-](http://pmm.nasa.gov/data-access/downloads/gpm)  
192 [access/downloads/gpm](http://pmm.nasa.gov/data-access/downloads/gpm)).

193 Any circular variable (or observation) represented as a unit vector  $\mathbf{x}$  is equivalent to  
194 complex number  $z = e^{i\theta} = \cos \theta + i \sin \theta$ , where  $i = \sqrt{-1}$ . Such a unit vector can be  
195 placed in the complex plane with its real component ( $\cos \theta$ ) on the horizontal axis, and its  
196 imaginary component ( $i \sin \theta$ ) on the vertical axis. For a graphic interpretation consult  
197 (Mardia and Jupp, 2000, Fig. 2.1; Pewsey et al., 2013, Fig. 3.1).

198 Summary statistics such as the sample mean direction ( $\bar{\theta}$ ) and resultant length ( $\bar{R}$ ), and  
199 circular variance ( $V$ ) can be computed from circular data on the complex plane. The  
200 sample mean direction indicates the direction of the mean resultant (unit) vector of the  
201 sample (Pewsey et al., 2013, ch.3). It is given by  $\bar{\theta} = \tan^{-1}(b/a) \in [0, 2\pi)$ , where  $b =$   
202  $n^{-1} \sum_{j=1}^n \sin \theta_j$ , and  $a = [\sum_{j=1}^n \cos \theta_j]/n$  (only valid for  $a \wedge b \neq 0$ ),  $\theta_j$  represents the angle of  
203 a unit vector  $j$  with regard to the chosen zero/north, and  $n$  the sample size. As noted by  
204 (Pewsey et al., 2013, ch. 3), the sample mean direction is a good measure of central  
205 location for unimodal samples that are close to symmetric. The sample mean resultant  
206 length is defined by  $\bar{R} = \sqrt{b^2 + a^2} \in [0, 1]$ , and it is used as a measure of “concentration”  
207 for unimodal circular data (Pewsey et al., 2013, ch.3). If all the unit vectors  $j$  are identical  
208 then  $\bar{R} = 1$ . Conversely, the more  $\bar{R}$  approaches 0, the more evenly spread around the

209 unit circle the data is. A particular case where  $\bar{R} = 0$  may imply that all unit vectors cancel  
210 each other out, meaning that they all are evenly directed in the complex plane. The  
211 sample circular variance is defined as  $V = 1 - \bar{R} \in [0,1]$ .  $\kappa$  is the concentration parameter,  
212 equivalent to the “reliability” ( $\sigma^{-2}$ ) of a normal distribution (Murray and Morgenstern,  
213 2010). It can be obtained by linear interpolation from tabulated values of  $\bar{R}$  (e.g., Mardia,  
214 1972b, p.298, Table Appendix 2.3) or  $n$  and  $\bar{R}$  (e.g., Mardia and Jupp, 2000, p.364, Table  
215 Appendix 2.5).

216 Several tests have been developed to evaluate or infer the uniformity and symmetry  
217 conditions of the sample. (Pewsey et al., 2013, ch. 5) strongly recommend the Rayleigh  
218 test for departure from uniformity in unimodal circular distributions. For multimodal  
219 departures from uniformity, they advise omnibus tests such as Kuiper's  $V_n$ , Watson's  $U^2$ ,  
220 and Rao's spacing test, for instance. All the previous tests are for continuous circular data  
221 (i.e., data not grouped into bins). (Dhakal et al., 2015) noticed that while the Rayleigh is  
222 powerful against unimodal alternatives of uniformity (but not against multimodal  
223 alternatives, as suggested by (Pewsey et al., 2013)), the Rao's spacing and Kuiper's  $V_n$   
224 tests are consistent against unimodal and multimodal alternatives of uniformity. A  
225 parametric bootstrap adaptation of the Watson's  $U^2$  test is one alternative to test the  
226 goodness-of-fit of a specified distribution (Agostinelli and Lund, 2017; Tsagris et al., 2017).  
227 A sample can also be tested for two types of symmetry on the unit circle, reflective  
228 symmetry and  $\ell$ -fold symmetry. A distribution is reflectively symmetric about an angle  $\theta$  if  
229 the reflection of the distribution over such an angle is identical to the original distribution  
230 (Pewsey et al., 2013, ch.4). If a distribution is identical to the original distribution after  
231 being rotated through an angle  $2\pi/\ell$ , such a distribution is said to be  $\ell$ -fold symmetric. For  
232 simplicity, we only test for reflective symmetry. A mathematical description of all these

233 tests is beyond the scope of the present work. Still, we perform all of the above-mentioned  
234 tests (Sec. 3) to improve on the summary statistics, and gain a better perspective on the  
235 underlying distribution from which the sample is potentially drawn.

236 Our data exhibits multimodality (Fig. 1), therefore we follow the approach of a finite  
237 mixture of unimodal Von Mises–Fisher (MvMF) distributions. Non-parametric approaches  
238 (e.g., MvMF) offer more complex alternatives to account for the multimodality and  
239 asymmetry of irregular samples. The Von Mises distribution is a classic model in circular  
240 statistics, and it is considered the “equivalent” to the normal distribution model for linear  
241 data (Fisher, 1993b; Pewsey et al., 2013, ch. 4). It is also the most common, and more  
242 investigated approach given its easy-to-interpret parameters (Pewsey et al., 2013, ch. 4).  
243 The Cardiod, Wrapped Cauchy, Von Mises, Jones-Pewsey family, and Inverse Batschelet  
244 family models are alternative unimodal distributions developed to fit continuous circular  
245 data. For more details see (Pewsey et al., 2013, ch. 4).

246 As clearly presented and explained by (Qin et al., 2016) (see also Dhillon and Sra, 2003;  
247 Hornik and Grün, 2014), the  $D$ -variate Von Mises–Fisher distribution of a  $D$ -dimensional  
248 unit random vector  $x$  {for  $x \in \mathbb{R}^D$ , in the unit hypersphere  $\mathbb{S}^{D-1}$ , and  $\|x\| = 1$ } follows the  
249 probability density function:

$$f(x|\mu, \kappa) = C_D(\kappa) \cdot \exp(\kappa \cdot \mu^T x). \quad (1)$$

250 In Eq. (1),  $\kappa$  {for  $\kappa \geq 0$ } is the concentration parameter that quantifies how tightly the  
251 distribution is around the mean direction  $\mu$  {for  $\|\mu\| = 1$ },  $\mu^T x$  is the cosine similarity  
252 between  $x$  and  $\mu$ , i.e.,  $\cos(x - \mu)$  {for  $x$  and  $\mu$  expressed in radians}; and  $C_D(\kappa)$  is a  
253 normalizing constant defined as:

$$C_D(\kappa) = \frac{\kappa^{D/2-1}}{(2\pi)^{D/2} \cdot I_{D/2-1}(\kappa)}, \text{ where} \quad (2)$$

$$I_{D/2-1}(\kappa) := \sum_{s=0}^{\infty} \frac{1}{s! \cdot \Gamma(s + D/2 - 1 + 1)} \cdot \left(\frac{\kappa}{2}\right)^{2s+D/2-1}.$$

254 In Eq. (2),  $I_{D/2-1}(\kappa)$  is the infinite series form (Arfken et al., 2013, Eq. (14.99)) of the  
 255 modified Bessel function of the first kind with order  $D/2 - 1$  and argument  $\kappa$ , and  $\Gamma(s +$   
 256  $D/2 - 1 + 1) := (s + D/2 - 1)!$  the gamma function. In our case,  $D = 2$ , and the  $D$ -variate  
 257 Von Mises–Fisher distribution (Eq. (1)) reduces to the Von Mises distribution for the unit  
 258 circle (Fisher, 1993a; Mardia, 1972a; Pewsey et al., 2013).

259 The simplest and most common approach in multimodal probability density functions for  
 260 circular statistics is that of a finite mixture of Von Mises–Fisher (MvMF), which is given by,  
 261 e.g., (Qin et al., 2016):

$$f(x|\{\alpha, \mu, \kappa\}_{h=1}^H) = \sum_{h=1}^H \alpha_h \cdot f(x|\mu_h, \kappa_h), \text{ for} \quad (3)$$

$$0 \leq \alpha_h \leq 1, \text{ and } \sum \alpha_h = 1.$$

262 In Eq. (3),  $\alpha_h$  is the mixing proportion of the  $H$ -unimodal Von Mises–Fisher distributions  
 263 (PDFs). This equation has no analytical solution, hence its parameters are computed via  
 264 Maximum Likelihood Estimates under an Expectation Maximization framework (Banerjee  
 265 et al., 2005; Dhillon and Sra, 2003). The interested reader is pointed to the numerical  
 266 solution implemented by (Hornik and Grün, 2014), given that is from the R-package  
 267 `movMF` that we compute the parameters  $(\alpha_h, \mu_h, \kappa_h)$  of the assumed MvMF–PDFs.

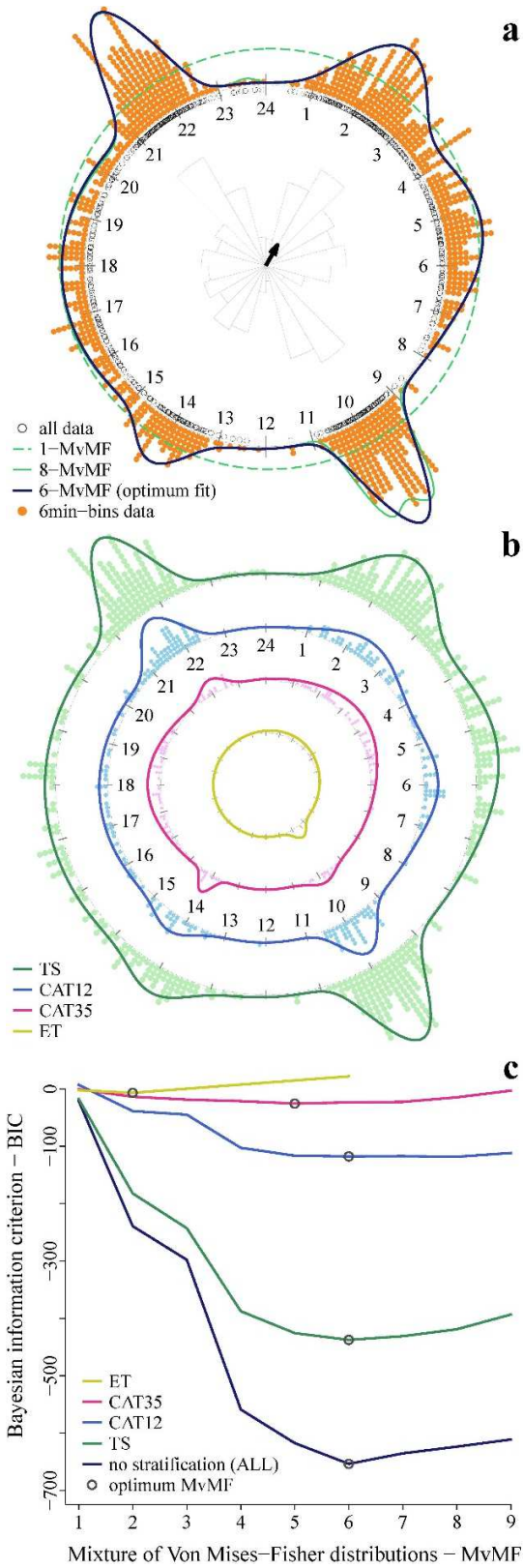
268 An objective assessment of the optimal/best mixture (number) of  $H$ -unimodal Von Mises–  
 269 Fisher distributions is that of Akaike's or Bayesian information criterion (AIC or BIC,  
 270 respectively, (Pewsey et al., 2013, p.114, ch.6)). The idea behind these criteria is to select

271 the least complex of all models providing equally good fits (i.e., parsimony). We use the  
272 BIC criterion to select the number of distributions for a given MvMF–PDFs.

273 As with (Rios Gaona et al., 2018), we downscaled IBTrACS attributes to 30-minute  
274 IMERG native resolution. We interpolated the 6-hour TC-centers to 30-minute resolution  
275 via cubic spline interpolation of latitudes and longitudes. Hence, the interpolated TC-  
276 centers are not absolutely accurate. Nevertheless, the variability generated by such a  
277 method has no practical effect on the radii-averaged precipitation (Bowman and Fowler,  
278 2015). For every 30-minute TC-center, we extracted IMERG rainfall up to a radius of  
279 1,000km, every 7km from the TC center outwards (i.e., 0km, 7km, 14km ...). For each  
280 radius, we averaged all the rainfall depth from the TC center up to the radius under  
281 consideration. Following (Bowman and Fowler, 2015; Wu et al., 2015), we compute the  
282 local time (LST,  $T_{LST}$ ) of all radii as the difference from their UTC ( $T_{UTC}$ ) with regard to their  
283 longitude  $\lambda \in [-180^\circ, +180^\circ]$ , i.e.,  $T_{LST} = [T_{UTC} + \lambda(12/180)]$ . For each storm we select  
284 the LSTs at which all maximum rainfall averages occur.

285 We stratify our analysis into six categories: storm duration, storm development, storm  
286 intensity, basin of origin, radial proximity to the TC center, and surface (land or ocean).  
287 The basin-of-origin and surface categories are entirely based on the coordinates of the TC  
288 center. A TC is considered to be over land if its center is geographically located over land,  
289 regardless of its proximity to the shore. We define three radial intervals to further stratify  
290 our analysis of maximum rainfall with regard to its proximity to the TC center. The storm-  
291 duration category is based on the day, relative to the storm beginning, from which a LST  
292 (or maximum TC-rainfall) was sampled, whereas the storm-development category is  
293 based on the quartile from which a given LST was sampled. The intensity-category is  
294 based on the MSW of the storm (for a given center). The MSW for a given 30-minute TC

295 center corresponds to the previous 6-hourly step stored in the IBTrACS. We re-  
296 categorized the TC intensity into four categories based on the Saffir-Simpson Hurricane  
297 Scale (SSHS - Simpson, 1974): for  $MSW < 64$  kt ( $33.1 \text{ m}\cdot\text{s}^{-1}$ , TS), for  $64 \leq MSW < 96$  kt  
298 ( $33.1 \leq MSW < 49.4 \text{ m}\cdot\text{s}^{-1}$ , CAT12), for  $MSW \leq 96$  kt (CAT35), and extra-tropical cyclones  
299 (ET).



300

301 Figure 1: a. Circular distribution for 6-min-bins data (orange dots), and continuous data (black  
302 circles), for a 1024-value sample of maximum rainfall per storm. The dark blue continuous curve  
303 indicates the optimal MvMF-PDFs (6 mixtures), whereas the continuous green curve represents a  
304 fit of 8 mixtures, and the dashed green curve a fit of 1 vMF-PDF. The direction and magnitude of  
305 the black arrow are the sample mean direction ( $\bar{\theta}$ ), and the sample mean resultant length ( $\bar{R}$ ),  
306 respectively. A 1-h-bin circular histogram is also plotted. b. Circular distributions and best-fit MvMF-  
307 PDFs for TCs stratified by intensity, i.e., TS (green), CAT12 (blue), CAT35 (pink), and ET (gold). c.  
308 Bayesian information criterion (BIC) against a given number of MvMF, for the intensities in panel b,  
309 and the optimal mixture in panel a. (dark blue curve). The circles indicates the lowest point of the  
310 related BIC curve, which represents the optimal number of vMF mixtures that best describes the  
311 sample multimodality, i.e., 2 for ET, 5 for CAT35, and 6 for CAT12, TS, and all data (no stratification  
312 applied).

### 313 3. Results and Discussion

314 The summary statistics for the sample of 1024 unit vectors that represent the LSTs at  
315 which maximum precipitation (per storm for all the 259 TCs under analysis) occurs are:  
316  $\bar{\theta}=1.952$  hours or 0.5111 radians (sample mean direction),  $\bar{R}=0.131$  (sample mean  
317 resultant length), and  $V=0.8693$  (sample circular variance). The concentration parameter  
318 ( $\kappa$ ) is 0.26375. Bear in mind that as the sample of average rainfall per TC is really large  
319 (multiple radii per several TC-centers), each storm can potentially have several rainfall  
320 estimates of equal maximum value (especially if one uses up to two significant  
321 figures/digits in the rounding up). This is why in this case we have a 1024-maximum  
322 sample for 259 TCs.

323 The  $p$ -value for the Rayleigh was 0, which indicates the rejection of the null hypothesis of  
324 uniformity. The  $p$ -values for the Kuiper's  $V_n$ , Watson's  $U^2$ , and Rao's spacing tests were  
325 smaller than 0.01, 0.01, and 0.001, respectively, which led us to the rejection of the null  
326 hypothesis of uniformity. The "goodness-of-fit" test for the grouped data as presented in  
327 Fig. 1-a, i.e., 6-min bins yielded  $p$ -values of 0.026, and 0.743 for the null hypotheses of  
328 uniform, and Von Mises distribution, respectively. Therefore, the null hypothesis of a  
329 uniform distribution that fits the sample is rejected (with a 2.6% significance level). The  $p$ -  
330 value for the reflective symmetry test was 0.053, which implies the rejection of the null



331 hypothesis of an assumed reflectively symmetric distribution at the 5.3% significance  
332 level. The result of all these tests, jointly with a visual evaluation of Fig. 1-a, suggests that  
333 a-priori assumptions of isotropy, unimodality and reflective symmetry do not hold for our  
334 sample. Hence, we must turn to non-parametric circular statistics to evaluate and quantify  
335 the multimodality present in the diurnal cycle of maximum TC-rainfall (Fig. 1, for instance).

336 Fig. 1-a shows the distribution of the 1024 samples for TC-rainfall maxima, grouped into 6-  
337 min bins. In both distributions, either continuous or stacked, one can see how the data is  
338 not equally distributed over the circular space (anisotropy and multimodality). Maximum  
339 TC-rainfall tends to concentrate roughly around five times, i.e., ~2, 5, 10, 14, and ~22h;  
340 and somewhat spread between 15 and 21h (~18h average). This figure also highlights  
341 how summary statistics are misleading if some a-priori knowledge on the type-of  
342 distribution is not known. In Sec. 2, we established the MvMF distributions as the  
343 appropriate approach for a non-parametric multimodal fitting given its easy-to-interpret  
344 parameters. A visual inspection of Fig. 1-a reveals that, most likely, a mixture of 5 vMF-  
345 PDFs should be sufficient to describe well the sample distribution.

346 Fig. 1-c shows the BIC values for different number of unimodal vMF-PDFs (mixtures). Six  
347 is the optimal number of unimodal vMF-PDF for the overall distribution, i.e., no  
348 stratification implemented. As seen from Fig. 1-a, a single unimodal vMF-PDF is not  
349 suitable to identify the diurnal cycle. On the other hand, an 8-MvMF-PDF offers a quite  
350 detailed, and parametrized distribution at the expenses of parsimony. Nevertheless, Fig.  
351 1-c tells us that only 6 vMF-PDFs are necessary to accurately account for the  
352 multimodality of the sample's distribution, and thus to identify the diurnal cycle of TC-  
353 rainfall. This can be seen from Fig. 1-a on how the continuous dark blue curve (6 vMF-  
354 PDFs) simply and accurately comprises the information gathered by the continuous green

355 curve (8 vMF-PDFs). Fig. 1-c also shows the BIC values for the stratification of TC-rainfall  
356 maxima given the intensity category (i.e., TS (green curve), CAT12 (blue curve), CAT35  
357 (pink curve), and ET (gold curve)). The optimal mixture for each of these distributions is 6,  
358 6, 5, and 2, respectively. This optimal fit can also be seen in Fig. 1-b, in which the MvMF-  
359 PDFs are plotted over the binned sample distribution of each intensity category.

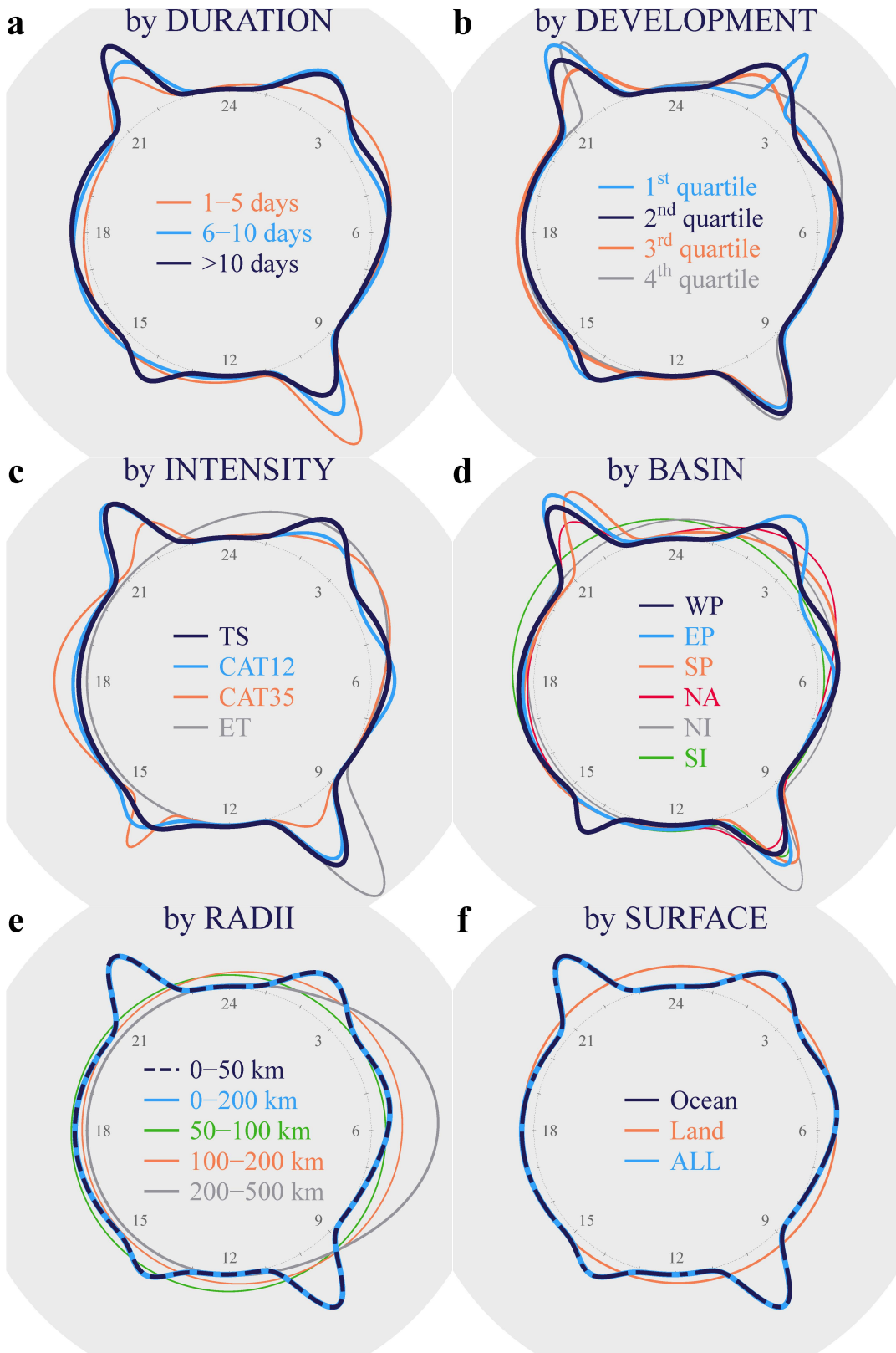
360 Numerical solutions for small-size samples often yield large values of  $\kappa$ , which  
361 consequently yield infinite ( $\infty$ ) values of probability  $f(x|\mu, \kappa)$  (Eq. (1)). Hence, the MvMF-  
362 PDFs here presented (e.g., Table 1) correspond to the lowest possible BIC obtained (from  
363 1 to 9 components) for which all of its unimodal vMF-PDFs are finite and/or defined.

364 The four predominant mean times ( $\mu$ 's) of the diurnal cycle of TC-rainfall maxima are 2.22,  
365 5.20, 9.88, and 21.75h (Fig. 1-a, and Table 1 - ALL rows). These times account for the  
366 largest  $\kappa$ 's and  $\alpha$ 's, i.e., the concentration parameter and mixing proportion of each  
367 unimodal vMF-distribution, which describe ~72% of the MvMF-PDF of TC-rainfall maxima.  
368 About 22% of the distribution is described by the mean direction  $\mu=17.80$ h, which is  
369 spread between ~15 and ~21h. This distribution has the lowest concentration parameter  
370 ( $\kappa=2.36$ ), which is an indication of how sparse the sample is around its  $\mu$ . The remaining  
371 ~6% belongs to  $\mu=14.02$ h, with a high  $\kappa$  despite its lowest contribution to the MvMF-PDF.  
372 Hence, it seems that there are two main cycles of rainfall maxima in TCs: one with mean  
373 directions ( $\mu$ ) of 9.88 and 21.75h (note the ~12h of difference), referred to as the ~10–22h  
374 diurnal cycle; the other with mean directions of 2.22 and 5.20h, referred to as the ~2–5h  
375 (or ~22–2–5h) semi-diurnal cycle. This latter is also perceived, although very slightly, in  
376 the afternoon hours, i.e., ~14–18h (or ~10–14–18h) semi-diurnal cycle. (Navarro et al.,  
377 2017) showed a cycle in storm intensity that reaches its peak in the “early hours” of the  
378 morning, and lags a periodic response of ~6h from latent heat. Their results suggests that

379 the axisymmetric TC diurnal cycle is primarily a balanced response driven by periodic  
380 heating. Such a signal is a function of the local solar time, which can help to explain our  
381 similar results regardless of stratification by basin or type of surface. (Navarro et al., 2017)  
382 also hint at the extension of the cycles, arguing that long diurnal periods exhibit a more  
383 balanced solution with greater impact on the storm intensity, whereas short diurnal periods  
384 project onto inertia–buoyancy waves, radiating energy away from the region of heating.

385 The general (ALL) MvMF-PDF is quite representative of the samples over the ocean, as  
386 97.4% of all TC-rainfall maxima correspond to TC-centers located over ocean. This can be  
387 seen from Fig. 2-f (ALL and Ocean curves) and from the very similar MvMF-PDF  
388 parameters in Table 1 - ALL and Ocean rows. For maximum TC-rainfall from TC-centers  
389 located inland, its mean direction is 2.06h with  $\kappa=0.45$  (Table 1 - Land rows). Even though  
390 nothing conclusive can be inferred or concluded from such a small sample (~2.6% of  
391 data), it is widely known that TCs weaken as they move inland, which complicates the  
392 identification of a diurnal cycle for inland maximum TC-rainfall. (O'Neill et al., 2017) found  
393 that over land and on average, tropical rainfall rates reach their maximum in the afternoon.  
394 According to (Dai, 2001; Wu et al., 2015), convective precipitation over land tends to peak  
395 in the late afternoon to early evening hours (most likely to a direct response to daytime  
396 heating of the surface and the planetary boundary layer), whereas over oceans the peak  
397 is reached in the early morning hours. They did not explore ocean-land stratification  
398 though. On the other hand, (Bowman and Fowler, 2015) carried out an ocean-land, basin  
399 and intensity stratification. They suggested that either the TC-land interaction (landfall)  
400 has little incidence on the diurnal behavior of the storms, or that as storms move inland  
401 they retain their oceanic pattern/cycle until the land fraction is too large for this pattern to  
402 be present. More generally, (Bowman and Fowler, 2015) found a diurnal variation of TC-

403 rainfall with peak rainfall at ~06:00 LST, and a minimum at ~18:00 LST. Although our  
404 results do show a lag of 12h for both diurnal and semi-diurnal cycles, our results do not  
405 show maxima at ~06:00 LST but rather at ~9.88h (LST) (or even at ~5.20h). Nevertheless,  
406 and as seen in Table 1 - ALL rows, several maxima are scattered around  $\mu=17.80$ h. A  
407 different and larger sample (i.e., 15 years of TMPA-3B42 data), and an alternative  
408 approach (i.e., characterization of the diurnal cycle of rainfall in terms of Fourier harmonics  
409 by sines and cosines fitting via least squares regression), may be the reasons behind the  
410 discrepancy between our maxima (~9.88h) and the one (~06:00) of (Bowman and Fowler,  
411 2015). Such diurnal cycles of oceanic precipitation with maxima in early morning hours  
412 (Bowman and Fowler, 2015) are common in studies such as (Gray and Jacobson Jr.,  
413 1977; Kraus, 1963; Serra and McPhaden, 2004), just to cite a few. (Jiang et al., 2011) also  
414 performed an ocean-land stratification in which they found two peaks for the diurnal  
415 variation of TC-rainfall over land: one at ~01:30–07:30 LST, and the other one at 16:30–  
416 19:30 LST (minimum at 10:30–13:30 LST). They also found maximum TC-rainfall at  
417 04:30–07:30 LST (and minimum ~19:30–22:30 LST) for non-stratified analyses. As seen  
418 from Fig. 2-f and Table 1 - ALL rows, the maximum by (Jiang et al., 2011) is consistent  
419 with the second peak of the ~2–5h semi-diurnal cycle, i.e.,  $\mu=5.20\pm 1$ h. With regard to our  
420 land stratification, our results showed an absence of any diurnal cycle (Table 1 - by  
421 SURFACE rows, and Fig. 2-f). Radial stratification, as suggested by (Bowman and Fowler,  
422 2015), is an alternative to reduce the impact on the diurnal cycle amplitude of averaged  
423 TC-rainfall involving potentially non-raining areas (at large radii). Nevertheless, a  
424 sensitivity analysis carried out by (Wu et al., 2015) indicates that the decrease in average  
425 rainfall and diurnal variation in the outer rainbands is not attributable to such non-raining  
426 averages.



428 Figure 2: Mixtures of Von Mises-Fisher probability density functions (MvMF-PDFs) for stratifications  
429 by storm duration (a), storm development (b), storm intensity (c), basin of origin (d), radii proximity  
430 (e), and surface (f). The light-grey region indicates the maximum  $\kappa$  among all stratification, so  
431 individual concentration parameters among all MvMF-PDFs can be visually compared.

432 Short-living storms only develop a semi-diurnal cycle, whereas intermediate and long-  
433 lasting TCs develop stable diurnal and semi-diurnal cycles of maximum rainfall. A  
434 temporal stratification of maximum TC-rainfall with regard to the number of days a given  
435 TC lasts shows that for short-living TCs, namely TCs that last up to five days, there is  
436 mainly a ~10–22h diurnal cycle (Fig. 2-a). Its  $\mu$ 's are clustered around 9.90 and 21.52h,  
437 with  $\kappa$ 's of 98.15 and 65.51, respectively (Table 1 - by DURATION rows). Fig. 2-a also  
438 shows a ~10–14h semi-diurnal cycle ( $\mu=14.01$  with  $\kappa=73.61$ ) for long-lasting storms, and a  
439 much less marked one ( $\kappa=3.47$ ) for short-living storms. As we show later, this ~10–14h  
440 semi-diurnal cycle is mainly characteristic of TCs developed in the WP basin (Fig. 2-d).  
441 Intermediate and long-lasting storms (i.e., TCs lasting more than 10 days) follow a similar  
442 dynamic in terms of both diurnal and semi-diurnal cycles. Their ~10–22h diurnal cycles  
443 show  $\mu$ 's of 9.89 and 9.87h, and 21.77 and 21.76h, respectively for intermediate and long-  
444 lasting TCs. As the storm lasts longer (e.g., more than 10 days), maximum TC-rainfall  
445 seems to be more concentrated at the end of the diurnal cycle than at its beginning. This  
446 can be seen from the larger values of  $\kappa$ , i.e., 60.14 and 116.60 at ~22h than those of  
447 108.12 and 67.88 at ~10h (Table 1 - by DURATION rows, and Fig. 2-a). Intermediate and  
448 long-lasting TCs have similar ~22–2–5h semi-diurnal cycles (i.e.,  $\mu$ 's of 2.20 and 5.74h for  
449 intermediate TCs, and  $\mu$ 's of 2.23 and 4.97h for long-lasting TCs).

450 A second temporal stratification showed that as the storms reach their end, maximum TC-  
451 rainfall develops earlier in the day (rather than later). When the duration of all TCs were  
452 stratified into quartiles, the first quartile (i.e., the first 25% of any storm, Fig. 2-b – light  
453 blue curve, and Supplemental Fig. 2 - q1) distinctively showed one diurnal ~10–22h cycle

454 ( $\mu$ 's of 9.86 and 21.72h), and two semi-diurnal cycles, which are distinctively marked  
455 around the  $\mu$ 's of 2.42 and 14.15h. The ~5h (or ~17h) component of the semi-diurnal cycle  
456 does not fully develop for this quartile (Supplemental Fig. 2 - q1). Bear in mind that ~22-  
457 and ~2-h  $\mu$ 's are only characteristic of intermediate and long-lasting storms (Fig. 2-a). The  
458  $\kappa$ 's for this first quartile are larger for ~22–2h (i.e., 92.50, and 511.50) than for ~10–14h  
459 (i.e., 76.98, and 40.06). This implies a larger concentration of maximum TC-rainfall during  
460 “night” (~22–2–5h semi-diurnal cycle) than “day” hours (~10–14–18h semi-diurnal cycle).  
461 As the storms develop, i.e., 2<sup>nd</sup> and 3<sup>rd</sup> quartiles, the larger concentration of maximum TC-  
462 rainfall shifts from ~22 to ~10h. Fig. 2-b (or Supplemental Fig. 2 - q2 or - q3) shows how  
463 the  $\mu$ 's for the 2<sup>nd</sup> and 3<sup>rd</sup> quartiles (i.e., 9.90 and 9.87h, respectively) present larger  $\kappa$ 's  
464 (85.41 and 96.25) than those for the  $\mu$ 's of 21.66 and 21.83h ( $\kappa$ 's of 91.63 and 51.08,  
465 respectively for the 2<sup>nd</sup> and 3<sup>rd</sup> quartiles). It appears that the ~2–5h semi-diurnal cycle is  
466 mostly characteristic of the 2<sup>nd</sup> and 3<sup>rd</sup> quartiles; stronger for the 2<sup>nd</sup> quartile, and  
467 weakening for the 3<sup>rd</sup> one, with more samples concentrated at ~2 than at ~5h (Table 1 - by  
468 DEVELOPMENT rows; and Supplemental Fig. 2 - q2 and - q3). The ~10–14h semi-diurnal  
469 cycle is not present anymore in the 3<sup>rd</sup> and 4<sup>th</sup> quartiles. The absence of multimodality for  
470 the last (4<sup>th</sup>) quartile, given also the extension of the sample, indicates that as the storm  
471 vanishes, no diurnal or semi-diurnal cycle of maximum TC rainfall can be identified  
472 (Supplemental Fig. 2 - q4). Our results for this alternative temporal stratification agree with  
473 the suggestion by (Navarro et al., 2017) that the magnitude of the diurnal signal may vary  
474 throughout the lifetime of the storm. According to them, results in the literature are mixed  
475 about the impact of TC diurnal cycle with regard to storm maturity, i.e., some advocate for  
476 impacts in the developing (early) stages (e.g., Hobgood, 1986; Melhauser and Zhang,  
477 2014; Sundqvist, 1970), while others for impacts in the mature (late) stages (e.g., Craig,

478 1996; Tang and Zhang, 2016; Tuleya and Kurihara, 1981). Our results lay in both groups  
479 as the 2<sup>nd</sup> and 3<sup>rd</sup> (and even the 1<sup>st</sup>) quartiles show diurnal and semi-diurnal cycles.

480 The stratification by intensity shows that the overall (unstratified) MvMF-PDF is roughly  
481 based on the TS category. This can be seen from Fig. 1-b and Fig. 2-c, and the similar  
482 parameters in Table 1 - TS and ALL rows. For the TS category, the diurnal (~10–22h) and  
483 semi-diurnal (~2–5h) cycles of maximum TC-rainfall are equally distinctive. This can be  
484 seen from its  $\kappa$ 's of 36.88, 9.31, 85.00, and 87.92, respectively for  $\mu$ 's of 2.23, 5.30, 9.89,  
485 and 21.72h. All of the above four vMF unimodal distributions account for ~75% of the  
486 mixture (~20% is described by  $\mu=17.54$ h, with  $\kappa=1.65$  the lowest for any category of  
487 intensity). TS is the only category with a weak ~10–14h semi-diurnal cycle around  
488  $\mu=13.90$ h. As with the stratification by surface, this behavior is expected for the TS  
489 category, which accounts for 61.1% of the maximum rainfall analyzed. CAT12 and CAT35  
490 account for 24.7% and 13.1% of the sample, respectively. The MvMF-PDF for CAT12 is  
491 mainly described by a strong ~10–22h diurnal cycle with  $\mu$ 's of 9.89 and 21.65h (Table 1 -  
492 CAT12 rows). This strong diurnal cycle is responsible for ~39% of the distribution. About  
493 33% of this MvMF-PDF comes from the less marked ~2–5h semi-diurnal cycle with  $\mu$ 's of  
494 2.46 and 5.98h. Given that the largest proportion of the MvMF-PDF for CAT35 (~74%)  
495 comes from very diffused ( $\kappa$ 's of 5.87 and 6.31) mean directions ( $\mu$ 's of 3.57 and 18.04h,  
496 respectively), we can ascertain the absence of any (semi-) diurnal cycle for extremely  
497 intense TCs. One of the causes for this absence might be related to the random nature of  
498 such an extreme maximum rainfall. This is yet to be proven, as there are few CAT35  
499 samples (13.1%). Still, our results for CAT35 are in line with (Leppert II and Cecil, 2016)  
500 who associated intense TCs with a weaker diurnal signal. They based this assertion on  
501 studies such as (Browner et al., 1977; Hobgood, 1986). With only 1.1% of ET storms, no



502 conclusive non-parametric analysis was possible for the ET category. Similar to (Bowman  
503 and Fowler, 2015), (Wu et al., 2015) stratified oceanic TC-rainfall (inner core, i.e., 0-  
504 100km, and outer rainbands, i.e., 100-500km) by weak (CAT1 and TS), and strong (CAT2-  
505 5) storms. (Wu et al., 2015) found that the daily maximum is reached at 02:30–05:00 UTC  
506 (inner core), and at 05:00–08:00 UTC (rainbands) for weak storms, whereas for strong  
507 storms these periods are 01:30–04:00 UTC (inner core) and 04:00–12:00 UTC  
508 (rainbands). In our case, all intensity categories encompass the early periods for either the  
509 inner core or rainbands, i.e.,  $\sim 2.7 \pm 0.9$ h and  $\sim 5.6 \pm 0.4$ h, even though the early-hours  
510 shifting is not as large as the one in (Wu et al., 2015). Nevertheless, our analysis does not  
511 show correspondence with their late periods. With regard to inner core or rainbands, we  
512 later show that these periods remain “the same” for 0-200km (or 0-50km).

513 With regard to the stratification by basin, the two main diurnal cycles of maximum TC-  
514 rainfall are provided by the Pacific basin with 75.3% of the sample. The sample  
515 percentages for the SP, EP, WP, SI, NI, and NA basins are 15.4, 21.2, 38.7, 8.5, 8.1, and  
516 8.1%, respectively. With  $\mu$ 's of 2.33, 5.36, 9.91, and 21.60h, the WP basin follows the  
517  $\sim 10$ – $22$ h diurnal and  $\sim 2$ – $5$ h semi-diurnal cycles (Fig. 2-d – dark blue curve). It is also the  
518 only basin with a distinctive  $\mu = 14.12$ h, which suggests a late and light  $\sim 10$ – $14$ – $18$ h semi-  
519 diurnal cycle. For detailed  $\kappa$  values see Table 1 - by BASIN rows. WP is the basin that  
520 resembles the overall (ALL) MvMF-PDF the most (Fig. 2-f - light blue curve, and Fig. 1-a -  
521 continuous dark blue curve). This is probably due to its highest percentage of sampled  
522 TCs (38.7%). (Jiang et al., 2011) also found that the WP basin has the largest number of  
523 deepest and most intense tropical cyclone precipitation, cloud, and convective cell  
524 features. The EP and SP basins follow a pattern similar to the WP basin. Both present a  
525 distinctive  $\sim 10$ – $22$ h cycle, and a less marked  $\sim 2$ – $5$ h semi-diurnal cycle, practically absent

526 in the SP basin. As seen from Supplemental Fig. 1, it appears that the ~2–5h semi-diurnal  
527 cycle is something really characteristic of the north Pacific basin (i.e., WP and EP basins;  
528 see also Table 1 - by BASIN rows). The non-parametric analysis is less conclusive for the  
529 SI, NI, and NA basins, which add the remaining quarter (24.7%) of the sample altogether.  
530 The Indian basin (SI and NI) presents a distinctive mean time ~10h, with  $\kappa$ 's of 123.52 and  
531 113.71, respectively for SI and NI. The difference between the SI and NI basins is that the  
532 latter shows a scattered sample ( $\kappa=0.83$ ) at  $\mu=2.77$ h, whereas the scattered sample for  
533 the former is at  $\mu=20.66$ h ( $\kappa=0.80$ ). Note the discordance of these  $\mu$ 's from the diurnal or  
534 semi-diurnal cycles. Such a disagreement may be influenced by non-parametric analyses  
535 based on few samples (8.3% on average for each Indian basin). The NA basin (Fig. 2-d -  
536 red curve, and Supplemental Fig. 1 - NA) has also one of the lowest samples (8.1%),  
537 which does not allow a clear identification of any diurnal cycle.

538 When the analysis was stratified by radii, the general pattern (ALL) mimicked that of the  
539 maximum TC-rainfall within 200km radii (Fig. 2-e – light blue curve). Thus, the ~10–22h  
540 diurnal cycle, and the ~2–5h and ~14–18h semi-diurnal cycles are mainly present within  
541 200km from the TC-center ( $\mu$ 's of 2.22, 5.19, 9.88, 14.02, and 21.75h). For radii between  
542 200 and 500km, there is only a weak ( $\kappa=3.33$ )  $\mu=5.73$ h. This, and the fact that no  
543 maximum TC-rainfall was beyond 500km radii suggest that TC-maximum rainfall only  
544 develops within 200km radii. We stratify radii further down to three more intervals within  
545 200km, i.e., 0-50, 50-100 (not presented here), and 100-200km. As shown in Table 1 (by  
546 RADII rows) and Fig. 2-e, 50km is descriptive of the overall behavior for maximum TC-  
547 rainfall within 200km, as these two radii (0-50 and 0-200km) follow almost identical diurnal  
548 and semi-diurnal cycles. Almost the entire sample (98.2%) is in the 0-50km range,  
549 whereas 99.7% is within 200km. (Wu et al., 2015) suggested that the outward propagation

550 of the diurnal signals is associated with the internal structure of TC convective systems,  
551 regardless of the basins where they develop. Recent work presented by (Leppert II and  
552 Cecil, 2016) is somewhat in agreement with our results for radial stratification. They found  
553 that for 100-500-km radii TC-rainfall (clouds) peaks in the morning (01:30–07:30 LST),  
554 and that the minimum is reached between 10:30–19:30 LST. From Fig. 2-e, one can see  
555 the similarities of these peaks with the ~2–5h semi-diurnal cycle for radii shorter than  
556 500km (or even 200km). Nevertheless, in what they call “minimum”, we have the ~10–22h  
557 diurnal cycle. Our ~22–2–5h semi-diurnal cycle also appears in their inner core (0-100km,  
558 “with a maximum at 22:30–04:30 LST”) associated with only upper levels (8-10km).  
559 According to (Leppert II and Cecil, 2016), the peak between 01:30–07:30 LST is also  
560 found in several previous studies (e.g., Bowman and Fowler, 2015; Lajoie and  
561 Butterworth, 1984; Muramatsu, 1983).

562 The MvMF-PDFs in Fig. 2-e represent PDFs of maximum TC-rainfall for several radii. As  
563 presented in (Rios Gaona et al., 2018), the average maximum TC-rainfall for the intervals  
564 0-200km, 200-500km, and 500-1000km are 48.07, 11.61, and 3.41mm, respectively.  
565 Likewise, the average maximum TC-rainfall for the intervals 0-50km, 50-100km, and 100-  
566 200km are 48.04, 27.87, and 19.26mm, respectively. About 48mm of rainfall either for 0-  
567 50 or 0-200km confirms the vast representativeness of TC-rainfall maxima just within the  
568 first 50km from the TC-center. Such detailed statistics close to the TC-centers are  
569 possible thanks to the high resolution rainfall retrievals offered by IMERG.

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572

Table 1: Mean times  $\mu$ , concentration parameters  $\kappa$  and mixing proportions  $\alpha$  for up-to 6 MvMF-PDFs for un- and stratified TC-rainfall maxima. All the curves shown in Fig. 2 can be reconstructed if the parameters presented in this table are plug into Eqs. (3), (2), and (1). The 'size' is the percentage of a given category relative to its stratification.

MvMF-PDFs	#1	#2	#3	#4	#5	#6	size [%]
ALL (no stratification)							
$\mu$	2.2152418	5.2016072	9.8812249	14.0237249	17.8024248	21.7446129	
$\kappa$	31.98304	8.89786	82.76682	68.67366	2.35992	82.38932	100
$\alpha$	0.1818	0.1798	0.1795	0.0632	0.2218	0.1739	
by DURATION							
1-5 days	$\mu$ 3.8888570	-	9.9019170	14.4064328	-	21.5235256	
	$\kappa$ 3.91834	-	98.14502	3.46633	-	65.50892	15.8
	$\alpha$ 0.3931	-	0.2657	0.1873	-	0.1538	
6-10 days	$\mu$ 2.1977949	5.7379428	9.8905352	-	16.4961242	21.7734378	
	$\kappa$ 31.34776	8.99902	108.11746	-	2.73278	60.13451	44.7
	$\alpha$ 0.2141	0.1548	0.1700	-	0.2669	0.1942	
>10 days	$\mu$ 2.2267869	4.9729403	9.8672777	14.0073784	18.1866170	21.7614043	
	$\kappa$ 33.72356	9.92566	67.87566	73.60951	3.11870	116.60151	39.5
	$\alpha$ 0.1852	0.1771	0.1475	0.0714	0.2376	0.1813	
by DEVELOPMENT							
1 <sup>st</sup> quartile	$\mu$ 2.4184217	4.0470154	9.8587574	14.1444514	17.3995804	21.7164537	
	$\kappa$ 511.49821	3.30833	76.98216	40.05593	4.72186	92.49673	32.7
	$\alpha$ 0.0734	0.2841	0.1804	0.1028	0.1403	0.2191	
2 <sup>nd</sup> quartile	$\mu$ 2.1408816	5.4999594	9.8999525	14.0022682	17.8312735	21.6622133	
	$\kappa$ 33.17070	15.49447	85.40689	75.88738	2.82371	91.63058	31.8
	$\alpha$ 0.2314	0.1624	0.1894	0.0669	0.1775	0.1724	
3 <sup>rd</sup> quartile	$\mu$ 1.9331084	4.6909351	9.8675160	-	16.9885625	21.8251026	
	$\kappa$ 44.60595	6.55601	96.25009	-	2.26269	51.07975	29.2
	$\alpha$ 0.1217	0.2206	0.1602	-	0.3325	0.1650	
4 <sup>th</sup> quartile	$\mu$ -	3.6825855	10.0059731	-	16.6991725	21.9284509	
	$\kappa$ -	4.54927	99.61211	-	2.56488	283.27545	6.3
	$\alpha$ -	0.4831	0.1827	-	0.2154	0.1188	
by RADII							
0-50 km	$\mu$ 2.1983336	5.1768204	9.8823541	14.0114995	17.7677399	21.7395741	
	$\kappa$ 32.71734	9.12278	83.12776	67.38022	2.45511	82.79723	98.2
	$\alpha$ 0.1815	0.1780	0.1802	0.0623	0.2215	0.1765	
0-200 km	$\mu$ 2.2158813	5.1849830	9.8802406	14.0236901	17.8014777	21.7446034	
	$\kappa$ 31.67820	9.23182	82.30998	68.71572	2.35887	82.38592	99.7
	$\alpha$ 0.1821	0.1774	0.1802	0.0634	0.2225	0.1744	
100-200 km	$\mu$ -	-	-	12.8738133	-	-	
	$\kappa$ -	-	-	0.18349	-	-	0.5
	$\alpha$ -	-	-	1.0	-	-	
200-500 km	$\mu$ -	5.7290486	-	-	-	-	
	$\kappa$ -	3.33307	-	-	-	-	0.3
	$\alpha$ -	1.0	-	-	-	-	
by INTENSITY							
TS	$\mu$ 2.2271917	5.2946123	9.8851433	13.9043633	17.5407806	21.7197800	
	$\kappa$ 36.88254	9.31251	85.00220	59.11905	1.65370	87.92400	61.1
	$\alpha$ 0.1984	0.1647	0.1985	0.0606	0.1923	0.1854	
CAT12	$\mu$ 2.4581381	5.9759664	9.8883872	14.2248417	17.7298958	21.6490429	
	$\kappa$ 19.95348	19.52417	87.33768	66.70176	3.47275	74.19870	24.7
	$\alpha$ 0.1891	0.1381	0.1772	0.0794	0.2063	0.2099	
CAT35	$\mu$ 3.5711101	-	9.9043271	14.0923809	18.0361222	22.1192259	
	$\kappa$ 5.87049	-	71.11366	266.51113	6.30509	79.32346	13.1
	$\alpha$ 0.3973	-	0.0742	0.0777	0.3469	0.1039	
ET	$\mu$ 2.5263721	-	9.6360580	-	-	-	
	$\kappa$ 3.26545	-	71.66865	-	-	-	1.1
	$\alpha$ 0.6397	-	0.3603	-	-	-	
by BASIN							
SP	$\mu$ 3.7529389	-	9.7617842	15.9540656	-	22.0136527	
	$\kappa$ 3.90692	-	110.68994	3.46186	-	96.25777	15.4
	$\alpha$ 0.4128	-	0.1787	0.2142	-	0.1943	
SI	$\mu$ -	-	9.8441757	-	-	20.6582418	
	$\kappa$ -	-	123.52417	-	-	0.79691	8.5
	$\alpha$ -	-	0.1347	-	-	0.8653	
WP	$\mu$ 2.3264704	5.3600035	9.9105106	14.1227946	17.2293971	21.6014794	
	$\kappa$ 34.66562	9.89724	78.75968	137.32522	3.78443	94.58224	38.7
	$\alpha$ 0.1839	0.1775	0.1683	0.0622	0.2231	0.1850	
NI	$\mu$ 2.7692542	-	9.9647873	-	-	-	
	$\kappa$ 0.83253	-	113.70709	-	-	-	8.1
	$\alpha$ 0.7786	-	0.2214	-	-	-	
EP	$\mu$ 2.4953669	5.8594334	9.9006361	-	16.5093351	21.6812694	
	$\kappa$ 40.66527	32.67055	117.70027	-	1.42725	89.48307	21.2
	$\alpha$ 0.2445	0.0713	0.1646	-	0.3035	0.2161	
NA	$\mu$ 3.2040271	-	9.9497323	15.1860953	-	21.6652349	
	$\kappa$ 5.83075	-	52.51851	1.80111	-	63.59554	8.1
	$\alpha$ 0.4628	-	0.1672	0.2139	-	0.1561	
by SURFACE							
Ocean	$\mu$ 2.2491078	5.2375259	9.8708787	14.0332012	17.7416074	21.7369835	
	$\kappa$ 31.40649	9.78592	82.29653	66.17639	2.37856	81.33687	97.4
	$\alpha$ 0.1845	0.1739	0.1808	0.0647	0.2176	0.1785	
Land	$\mu$ 2.0573905	-	-	-	-	-	
	$\kappa$ 0.45273	-	-	-	-	-	2.6
	$\alpha$ 1.0	-	-	-	-	-	

#### 573 4. Summary and Conclusions

574 The goal of this work was to quantitatively assess the diurnal cycle of maximum TC-  
575 rainfall by means of non-parametric circular statistics. To do so, we cross-referenced the  
576 IBTrACS (v03r10) and IMERG (V04) data sets to accurately account for high-resolution  
577 rainfall within a 2,000km-wide swath along the path of a given TC. We analyzed 259 TCs  
578 that occurred from March 2014 through February 2017 (~3 years of data). The IMERG  
579 data set is a gridded satellite product of high spatiotemporal rainfall estimates ( $0.1^\circ \times 0.1^\circ$   
580 every 30 minutes), which makes it very suitable for analyses related to the diurnal cycle of  
581 TC rainfall. Circular statistics is a mathematical framework that allows statistical analyses  
582 accounting for the intrinsic periodicity of circular/directional variables. In our case, such a  
583 circular (random) variable is the time of the day for which a TC-rainfall maximum occurred.  
584 We modelled the multimodality and anisotropy of TC-rainfall maxima using the finite  
585 mixtures (aggregations) of unimodal Von Mises-Fisher distributions (MvMF-PDFs), which  
586 is the most common approach, given its easy-to-interpret parameters (e.g., mean  
587 direction/time  $\mu$ , concentration parameter  $\kappa$ , and mixing proportion  $\alpha$ ). We stratified our  
588 analysis by storm duration, maturity, and intensity, basin of origin, proximity of the TC-  
589 rainfall maxima to the storm center (i.e., by radii), and whether the TC center was over the  
590 ocean or land.

591 On average, when no stratification is implemented over the 259 TCs here analyzed, there  
592 are two main cycles of maximum rainfall: one with mean directions ( $\mu$ ) of 9.88 and 21.75h  
593 ( $\kappa$ 's of 82.77 and 82.39, respectively), referred as the ~10–22h diurnal cycle; the other one  
594 with  $\mu$ 's of 2.22 and 5.20h ( $\kappa$ 's of 31.98 and 8.90, respectively), referred as the ~2–5h  
595 semi-diurnal cycle. This semi-diurnal cycle appears to be also present at afternoon hours,

596 i.e., ~14–18h ( $\mu$ 's of 14.02 and 17.80h with respective  $\kappa$ 's of 68.67 and 2.36).

597 Ocean, TS (tropical storms, i.e., MSW < 64kt), WP (West Pacific), 0-200km (or even 0-  
598 50km) radii, long-lasting (i.e., storms with duration longer than 10 days), and 2<sup>nd</sup> quartile  
599 are the stratifications that resemble the general MvMF-PDF of the ~10–22, and ~2–5h  
600 cycles the most. All of these particular stratifications average  $\mu$ 's of 2.23, 5.26, 9.89, and  
601 21.70h, with standard deviations of 0.061, 0.178, 0.017, and 0.060, respectively. This  
602 correspondence is mainly attributed to the large influence each of these stratifications  
603 exert on the sample. That is, out of the sample of 2014 values (of maximum TC-rainfall  
604 per storm), 97.4% comes from Ocean, 61.1% from TS, 38.7% from WP, 98.2% from radii  
605 smaller than 50km, 39.5% from storms lasting more than 10 days, and 31.8% from all their  
606 2<sup>nd</sup> quartiles.

607 Short-lived TCs (i.e., 1-5 days) mainly develop a diurnal cycle (of maximum rainfall)  
608 around  $\mu$ 's of 9.90 and 21.52h (i.e., ~10–22h diurnal cycle). Intermediate, and long-lasting  
609 (i.e., 5-10 days, and longer than 10 days, respectively) TCs develop both diurnal (~10–  
610 22h) and semi-diurnal (~2–5h) cycles. Long-lasting storms show a weak ~14–18h semi-  
611 diurnal at  $\mu=14.01$ h, which is mainly characteristic of TCs from the WP basin. As storms  
612 last longer, the maximum TC-rainfall distribution concentrates more at the end of the  
613 diurnal cycle (~22h) than at its beginning (~10h).

614 At an earlier stage of a storm (1<sup>st</sup> quartile) there is also a larger concentration of maximum  
615 TC-rainfall at the end of the diurnal cycle ( $\mu=21.72$ h) than at its beginning ( $\mu=9.86$ h). This  
616 concentration shifts towards the beginning of the ~10–22h diurnal cycle as the storm  
617 progresses to its 2<sup>nd</sup> and 3<sup>rd</sup> quartiles. The ~22–2–5h semi-diurnal cycle is mainly  
618 characteristic of the 2<sup>nd</sup> ( $\mu$ 's of 2.14 and 5.50h) and 3<sup>rd</sup> ( $\mu$ 's of 1.93 and 4.69h) quartiles. As  
619 the storm matures this semi-diurnal cycle weakens. No particular diurnal or semi-diurnal

620 cycle is developed at the end of the storm (4<sup>th</sup> quartile).

621 While CAT12 storms show a distinctive ~10–22h diurnal cycle ( $\mu$ 's of 9.89 and 21.65h),  
622 CAT35 storms show an absence of any (semi-) diurnal cycles, which might be attributed to  
623 the very random nature of such an extreme maximum rainfall. It is reminded that CAT12  
624 and CAT35 respectively account for 24.7 and 13.1% of the sample, and that nothing  
625 conclusive can be said for ET storms as only few samples of TC-rainfall maxima were  
626 within this category (1.1%).

627 All the Pacific basins (i.e., WP, EP - East Pacific, and SP - South Pacific) show a  
628 distinctive ~10–22h diurnal cycle. On average, their  $\mu$ 's are at 9.86 and 21.77h, with WP  
629 the basin with the larger number of samples (38.7% of TC-rainfall maxima). WP is the only  
630 basin with two distinctive semi-diurnal cycles, i.e., ~2–5h (with  $\mu$ 's of 2.33 and 5.36h), and  
631 ~14–18h (with  $\mu$ 's of 14.12 and 17.23h).

632 Neither the North Atlantic (NA) nor the Indian basins (SI - South Indian, and NI - North  
633 Indian) showed any distinctive diurnal and/or semi-diurnal cycles. Nevertheless, it appears  
634 that both Indian basins have predominant  $\mu$ 's ~9.91h, with a tendency for the SI basin to  
635 distribute maximum rainfall at  $\mu=20.66$ h, and at  $\mu=2.77$ h for the NI basin. These different  
636 values of  $\mu$ 's can be attributed to the lower number of samples the non-parametric  
637 analysis was based on (e.g., NA, NI and SI represent just 24.7% of the sample).

638 When the analysis was stratified by radii, the ~10–22h diurnal and ~2–5h semi-diurnal  
639 cycles are rather similar among the 0-50 and 0-200km radii, given that 99.7% of the  
640 sample is within 200km, and 98.2% within 50km radii. This is a clear indication that the  
641 diurnal cycle of maximum TC-rainfall takes place within the first 50km from the TC-center  
642 (outwards). No maximum TC-rainfall was found for radii beyond 300km.

643 The level of detail reached in this work was possible due to high- resolution and quality  
644 data sets such as IBTrACS and IMERG. Despite their intrinsic and potential deficiencies,  
645 the combination of such data sets offers a comprehensive record and a rather accurate  
646 evaluation of TC-rainfall. An accurate description (or modelling) of the diurnal cycle of  
647 maximum rainfall from TCs further propels better and more accurate TC-rainfall models,  
648 which in the end serves to increase our resilience against this type of natural hazards and  
649 their catastrophic consequences.

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654 **References**

- 655 Agostinelli, C. and Lund, U.: {R} package `\texttt{circular}`: Circular Statistics (version 0.4-  
656 93), [online] Available from: <https://r-forge.r-project.org/projects/circular/>, 2017.
- 657 Arfken, G. B., Weber, H. J. and Harris, F. E.: Chapter 14 - Bessel Functions, in  
658 Mathematical Methods for Physicists (Seventh Edition), edited by G. B. Arfken, H.  
659 J. Weber, and F. E. Harris, pp. 643–713, Academic Press, Boston., 2013.
- 660 Banerjee, A., Dhillon, I. S., Ghosh, J. and Sra, S.: Clustering on the Unit Hypersphere  
661 Using Von Mises-Fisher Distributions, *J. Mach. Learn. Res.*, 6, 1345–1382 [online]  
662 Available from: <http://www.jmlr.org/papers/volume6/banerjee05a/banerjee05a.pdf>,  
663 2005.
- 664 Bowman, K. P. and Fowler, M. D.: The Diurnal Cycle of Precipitation in Tropical Cyclones,  
665 *J. Clim.*, 28(13), 5325–5334, doi:10.1175/JCLI-D-14-00804.1, 2015.
- 666 Browner, S. P., Woodley, W. L. and Griffith, C. G.: Diurnal Oscillation of the Area of  
667 Cloudiness Associated with Tropical Storms, *Mon. Weather Rev.*, 105(7), 856–864,  
668 doi:10.1175/1520-0493(1977)105<0856:DOOTAO>2.0.CO;2, 1977.
- 669 Burian, S. J. and Shepherd, J. M.: Effect of urbanization on the diurnal rainfall pattern in  
670 {H}ouston, *Hydrol. Process.*, 19(5), 1089–1103, doi:10.1002/hyp.5647, 2005.
- 671 Craig, G. C.: Numerical experiments on radiation and tropical cyclones, *Quart. J. Roy.*  
672 *Meteor. Soc.*, 122(530), 415–422, doi:10.1002/qj.49712253006, 1996.
- 673 Czajkowski, J., Villarini, G., Montgomery, M., Michel-Kerjan, E. and Goska, R.: Assessing  
674 Current and Future Freshwater Flood Risk from North Atlantic Tropical Cyclones via  
675 Insurance Claims, , 7(art. no. 41609), doi:10.1038/srep41609, 2017.
- 676 Dai, A.: Global Precipitation and Thunderstorm Frequencies. Part II: Diurnal Variations, *J.*  
677 *Clim.*, 14(6), 1112–1128, doi:10.1175/1520-  
678 0442(2001)014<1112:GPATFP>2.0.CO;2, 2001.
- 679 Dhakal, N., Jain, S., Gray, A., Dandy, M. and Stancioff, E.: Nonstationarity in seasonality  
680 of extreme precipitation: A nonparametric circular statistical approach and its  
681 application, *Water Resour. Res.*, 51(6), 4499–4515, doi:10.1002/2014WR016399,  
682 2015.

683 Dhillon, I. S. and Sra, S.: Modeling data using Directional Distributions. [online] Available  
684 from: [http://www.cs.utexas.edu/users/inderjit/public\\_papers/tr03-06.pdf](http://www.cs.utexas.edu/users/inderjit/public_papers/tr03-06.pdf), 2003.

685 Dunion, J. P., Thorncroft, C. D. and Velden, C. S.: The Tropical Cyclone Diurnal Cycle of  
686 Mature Hurricanes, *Mon. Weather Rev.*, 142(10), 3900–3919, doi:10.1175/MWR-D-  
687 13-00191.1, 2014.

688 Emanuel, K.: Assessing the present and future probability of Hurricane Harvey’s rainfall,  
689 *Proc. Natl. Acad. Sci.*, 114(48), 12681–12684, doi:10.1073/pnas.1716222114,  
690 2017.

691 Fisher, N. I.: Models, in *Statistical Analysis of Circular Data*, pp. 39–58, Cambridge  
692 University Press., 1993a.

693 Fisher, N. I.: *Statistical Analysis of Circular Data*, Cambridge University Press, Cambridge,  
694 Great Britain., 1993b.

695 Frank, W. M.: The Structure and Energetics of the Tropical Cyclone I. Storm Structure,  
696 *Mon. Weather Rev.*, 105(9), 1119–1135, doi:10.1175/1520-  
697 0493(1977)105<1119:TSAEOT>2.0.CO;2, 1977.

698 Fritz, A. and Samenow, J.: Harvey unloaded 33 trillion gallons of water in the {U.S.},  
699 [online] Available from: [https://www.washingtonpost.com/news/capital-weather-gang/wp/2017/08/30/harvey-has-unloaded-24-5-trillion-gallons-of-water-on-texas-and-louisiana/?utm\\_term=.e32e1837ee3d](https://www.washingtonpost.com/news/capital-weather-gang/wp/2017/08/30/harvey-has-unloaded-24-5-trillion-gallons-of-water-on-texas-and-louisiana/?utm_term=.e32e1837ee3d), 2017.

700  
701

702 Gray, W. M. and Jacobson Jr., R. W.: Diurnal Variation of Deep Cumulus Convection,  
703 *Mon. Weather Rev.*, 105(9), 1171–1188, doi:10.1175/1520-  
704 0493(1977)105<1171:DVODCC>2.0.CO;2, 1977.

705 Hai-Long, S., Qing-Hong, Z. and Bin, X.: Diurnal Variation of Tropical Cyclone Rainfall in  
706 the Western North Pacific in 2008--2010, *Atmos. Ocean. Sci. Lett.*, 6(2), 103–108,  
707 doi:10.1080/16742834.2013.11447064, 2013.

708 Hobgood, J. S.: A Possible Mechanism for the Diurnal Oscillations of Tropical Cyclones, *J.*  
709 *Atmos. Sci.*, 43(23), 2901–2922, doi:10.1175/1520-  
710 0469(1986)043<2901:APMFTD>2.0.CO;2, 1986.

711 Hornik, K. and Grün, B.: {movMF}: An {R} Package for Fitting Mixtures of von Mises-

712 Fisher Distributions, *J. Stat. Softw.*, 58(10), 1–31, doi:10.18637/jss.v058.i10, 2014.

713 Hou, A. Y., Kakar, R. K., Neeck, S., Azarbarzin, A. A., Kummerow, C. D., Kojima, M., Oki,  
714 R., Nakamura, K. and Iguchi, T.: The Global Precipitation Measurement  
715 Mission, *Bull Amer Meteor Soc*, 95(5), 701–722, doi:10.1175/BAMS-D-13-00164.1,  
716 2014.

717 Hu, H., Duan, Y., Wang, Y. and Zhang, X.: Diurnal Cycle of Rainfall Associated with  
718 Landfalling Tropical Cyclones in China from Rain Gauge Observations, *J. Appl.*  
719 *Meteorol. Climatol.*, 56(9), 2595–2605, doi:10.1175/JAMC-D-16-0335.1, 2017.

720 Huffman, G. J., Bolvin, D. T., Nelkin, E. J., Wolff, D. B., Adler, R. F., Gu, G., Hong, Y.,  
721 Bowman, K. P. and Stocker, E. F.: The TRMM Multisatellite Precipitation  
722 Analysis (TMPA): Quasi-Global, Multiyear, Combined-Sensor Precipitation  
723 Estimates at Fine Scales, *J Hydrometeor*, 8(1), 38–55, doi:10.1175/JHM560.1,  
724 2007.

725 Huffman, G. J., Bolvin, D. T. and Nelkin, E. J.: Integrated Multi-satellite Retrievals  
726 for GPM (IMERG) Technical Documentation., 2017a.

727 Huffman, G. J., Bolvin, D. T., Braithwaite, D., Hsu, K., Joyce, R., Kidd, C., Nelkin, E. J.,  
728 Sorooshian, S., Tan, J. and Xie, P.: NASA Global Precipitation Measurement  
729 Integrated Multi-satellite Retrievals for GPM (IMERG). Algorithm  
730 Theoretical Basis Document, version 4.6, 2017b.

731 Huffman, G. J., Bolvin, D. T., Nelkin, E. J. and Stocker, E. F.: V04 IMERG Final Run  
732 Release Notes. NASA Tech. Doc., 2017c.

733 Jiang, H., Halverson, J. B. and Zipser, E. J.: Influence of environmental moisture on  
734 TRMM-derived tropical cyclone precipitation over land and ocean, *Geophys. Res.*  
735 *Lett.*, 35(17), n/a--n/a, doi:10.1029/2008GL034658, 2008.

736 Jiang, H., Liu, C. and Zipser, E. J.: A TRMM-Based Tropical Cyclone Cloud and  
737 Precipitation Feature Database, *J. Appl. Meteorol. Climatol.*, 50(6), 1255–1274,  
738 doi:10.1175/2011JAMC2662.1, 2011.

739 Khouakhi, A., Villarini, G. and Vecchi, G. A.: Contribution of Tropical Cyclones to Rainfall  
740 at the Global Scale, *J. Clim.*, 30(1), 359–372, doi:10.1175/JCLI-D-16-0298.1, 2017.

741 Kirschbaum, D. and Patel, K. G.: Precipitation data key to food security and public health,  
742 Eos (Washington, DC), 97(7), 10 [online] Available from: [https://eos.org/meeting-](https://eos.org/meeting-reports/precipitation-data-key-to-food-security-and-public-health)  
743 [reports/precipitation-data-key-to-food-security-and-public-health](https://eos.org/meeting-reports/precipitation-data-key-to-food-security-and-public-health), 2016.

744 Knapp, K. R., Kruk, M. C., Levinson, D. H., Diamond, H. J. and Neumann, C. J.: The  
745 International Best Track Archive for Climate Stewardship (IBTrACS),  
746 Bull. Amer. Meteor. Soc., 91(3), 363–376, doi:10.1175/2009BAMS2755.1, 2010.

747 Kossin, J. P.: Daily Hurricane Variability Inferred from GOES Infrared Imagery, Mon.  
748 Weather Rev., 130(9), 2260–2270, doi:10.1175/1520-  
749 0493(2002)130<2260:DHVIFG>2.0.CO;2, 2002.

750 Kraus, E. B.: The Diurnal Precipitation Change over the Sea, J. Atmos. Sci., 20(6), 551–  
751 556, doi:10.1175/1520-0469(1963)020<0551:TDPLOT>2.0.CO;2, 1963.

752 Lajoie, F. A. and Butterworth, I. J.: Oscillation of High-Level Cirrus and Heavy Precipitation  
753 around Australian Region Tropical Cyclones, Mon. Weather Rev., 112(3), 535–544,  
754 doi:10.1175/1520-0493(1984)112<0535:OOHLCA>2.0.CO;2, 1984.

755 Lark, R. M., Clifford, D. and Waters, C. N.: Modelling complex geological circular data with  
756 the projected normal distribution and mixtures of von Mises distributions, Solid  
757 Earth, 5(2), 631–639, doi:10.5194/se-5-631-2014, 2014.

758 Leppert II, K. D. and Cecil, D. J.: Tropical Cyclone Diurnal Cycle as Observed by TRMM,  
759 Mon. Weather Rev., 144(8), 2793–2808, doi:10.1175/MWR-D-15-0358.1, 2016.

760 Lonfat, M., Jr., F. D. M. and Chen, S. S.: Precipitation Distribution in Tropical Cyclones  
761 Using the Tropical Rainfall Measuring Mission (TRMM) Microwave Imager: A Global  
762 Perspective, Mon. Weather Rev., 132(7), 1645–1660, doi:10.1175/1520-  
763 0493(2004)132<1645:PDITCU>2.0.CO;2, 2004.

764 Mardia, K. V.: 3 - Basic Concepts and Theoretical Models, in Statistics of Directional  
765 Data, edited by Z. W. Birnbaum and E. Lukas, pp. 39–79, Academic Press., 1972a.

766 Mardia, K. V.: Statistics of Directional Data, 1st ed., edited by Z. W. Birnbaum and E.  
767 Lukas, Academic Press, London, Great Britain. [online] Available from:  
768 [https://www.elsevier.com/books/statistics-of-directional-data/mardia/978-0-12-](https://www.elsevier.com/books/statistics-of-directional-data/mardia/978-0-12-471150-1)  
769 [471150-1](https://www.elsevier.com/books/statistics-of-directional-data/mardia/978-0-12-471150-1), 1972b.

- 770 Mardia, K. V and Jupp, P. E.: Directional Statistics, John Wiley & Sons, Ltd., West  
771 Sussex, PO19 1UD England., 2000.
- 772 Masseran, N.: Modeling a Multi-modal Distribution of Wind Direction Data in Kudat,  
773 Malaysia, Pakistan J. Stat. Oper. Res., 11(2), 195–212,  
774 doi:10.18187/pjsor.v11i2.974, 2015.
- 775 Melhauser, C. and Zhang, F.: Diurnal Radiation Cycle Impact on the Pregenesis  
776 Environment of Hurricane {K}arl (2010), J. Atmos. Sci., 71(4), 1241–1259,  
777 doi:10.1175/JAS-D-13-0116.1, 2014.
- 778 Mendelsohn, R., Emanuel, K., Chonabayashi, S. and Bakkensen, L.: The impact of  
779 climate change on global tropical cyclone damage, Nat. Clim. Chang., 2(3), 205–  
780 209, doi:10.1038/nclimate1357, 2012.
- 781 Moravec, E. R.: Texas officials: {H}urricane {H}arvey death toll at 82, “mass casualties  
782 have absolutely not happened,” [online] Available from:  
783 [https://www.washingtonpost.com/national/texas-officials-hurricane-harvey-death-](https://www.washingtonpost.com/national/texas-officials-hurricane-harvey-death-toll-at-82-mass-casualties-have-absolutely-not-happened/2017/09/14/bff3ffea-9975-11e7-87fc-c3f7ee4035c9_story.html?utm_term=.d9e372423251)  
784 [toll-at-82-mass-casualties-have-absolutely-not-happened/2017/09/14/bff3ffea-9975-](https://www.washingtonpost.com/national/texas-officials-hurricane-harvey-death-toll-at-82-mass-casualties-have-absolutely-not-happened/2017/09/14/bff3ffea-9975-11e7-87fc-c3f7ee4035c9_story.html?utm_term=.d9e372423251)  
785 [11e7-87fc-c3f7ee4035c9\\_story.html?utm\\_term=.d9e372423251](https://www.washingtonpost.com/national/texas-officials-hurricane-harvey-death-toll-at-82-mass-casualties-have-absolutely-not-happened/2017/09/14/bff3ffea-9975-11e7-87fc-c3f7ee4035c9_story.html?utm_term=.d9e372423251), 2017.
- 786 Muramatsu, T.: Diurnal Variations of Satellite-Measured {T\$\_{\text{BB}}\$} Areal  
787 Distribution and Eye Diameter of Mature Typhoons, J. Meteorol. Soc. Japan. Ser. II,  
788 61(1), 77–90, doi:10.2151/jmsj1965.61.1\_77, 1983.
- 789 Murray, R. F. and Morgenstern, Y.: Cue combination on the circle and the sphere, J. Vis.,  
790 10(11), 1–11, doi:10.1167/10.11.15, 2010.
- 791 Navarro, E. L., Hakim, G. J. and Willoughby, H. E.: Balanced Response of an  
792 Axisymmetric Tropical Cyclone to Periodic Diurnal Heating, J. Atmos. Sci., 74(10),  
793 3325–3337, doi:10.1175/JAS-D-16-0279.1, 2017.
- 794 NOAA-WPC: STORM SUMMARY NUMBER 20 FOR TROPICAL DEPRESSION HARVEY  
795 PRELIMINARY RAINFALL TOTALS AND WIND REPORTS, 2017.
- 796 O’Neill, M. E., Perez-Betancourt, D. and Wing, A. A.: Accessible Environments for Diurnal-  
797 Period Waves in Simulated Tropical Cyclones, J. Atmos. Sci., 74(8), 2489–2502,  
798 doi:10.1175/JAS-D-16-0294.1, 2017.

799 van Oldenborgh, G. J., van der Wiel, K., Sebastian, A., Singh, R., Arrighi, J., Otto, F.,  
800 Haustein, K., Li, S., Vecchi, G. and Cullen, H.: Attribution of extreme rainfall from  
801 Hurricane Harvey, August 2017, *Environ. Res. Lett.*, 12(12), 124009,  
802 doi:10.1088/1748-9326/aa9ef2, 2017.

803 Oliveira, M., Crujeiras, R. M. and Rodríguez-Casal, A.: A plug-in rule for bandwidth  
804 selection in circular density estimation, *Comput. Stat. & Data Anal.*, 56(12), 3898–  
805 3908, doi:10.1016/j.csda.2012.05.021, 2012.

806 Peduzzi, P., Chatenoux, B., Dao, H., De Bono, A., Herold, C., Kossin, J., Mouton, F. and  
807 Nordbeck, O.: Global trends in tropical cyclone risk, *Nat. Clim. Chang.*, 2(4), 289–  
808 294, doi:10.1038/nclimate1410, 2012.

809 Pewsey, A., Neuhäuser, M. and D., R. G.: *Circular Statistics in {R}*, 1st ed., Oxford  
810 University Press, Oxford, United Kingdom. [online] Available from:  
811 <http://circstatinr.st-andrews.ac.uk/index.html>, 2013.

812 Prat, O. P. and Nelson, B. R.: Mapping the world's tropical cyclone rainfall contribution  
813 over land using the {TRMM} {M}ulti-satellite {P}recipitation {A}nalysis, *Water*  
814 *Resour. Res.*, 49(11), 7236–7254, doi:10.1002/wrcr.20527, 2013.

815 Prat, O. P. and Nelson, B. R.: On the Link between Tropical Cyclones and Daily Rainfall  
816 Extremes Derived from Global Satellite Observations, *J. Clim.*, 29(17), 6127–6135,  
817 doi:10.1175/JCLI-D-16-0289.1, 2016.

818 Qin, X., Cunningham, P. and Salter-Townshend, M.: Online Trans-dimensional Von  
819 Mises-Fisher Mixture Models for User Profiles, *J. Mach. Learn. Res.*, 17(1), 7021–  
820 7071 [online] Available from: <http://dl.acm.org/citation.cfm?id=2946645.3053482>,  
821 2016.

822 R Core Team: *R: A Language and Environment for Statistical Computing*, [online]  
823 Available from: <https://www.r-project.org/>, 2017.

824 Rios Gaona, M. F., Villarini, G., Zhang, W. and Vecchi, G. A.: The added value of  
825 {IMERG} in characterizing rainfall in tropical cyclones, *Atmos. Res.*, 209, 95–102,  
826 doi:10.1016/j.atmosres.2018.03.008, 2018.

827 Risser, M. D. and Wehner, M. F.: Attributable human-induced changes in the likelihood  
828 and magnitude of the observed extreme precipitation during Hurricane Harvey,

829 Geophys. Res. Lett., n/a--n/a, doi:10.1002/2017GL075888, 2017.

830 Samenow, J.: Nearly 65 inches of rain fell from {H}urricane {H}arvey in {T}exas, shattering  
831 {U.S.} storm record, [online] Available from:  
832 [https://www.washingtonpost.com/news/capital-weather-](https://www.washingtonpost.com/news/capital-weather-gang/wp/2017/08/29/harvey-marks-the-most-extreme-rain-event-in-u-s-history/?utm_term=.1c10e1f2ef49)  
833 [gang/wp/2017/08/29/harvey-marks-the-most-extreme-rain-event-in-u-s-](https://www.washingtonpost.com/news/capital-weather-gang/wp/2017/08/29/harvey-marks-the-most-extreme-rain-event-in-u-s-history/?utm_term=.1c10e1f2ef49)  
834 [history/?utm\\_term=.1c10e1f2ef49](https://www.washingtonpost.com/news/capital-weather-gang/wp/2017/08/29/harvey-marks-the-most-extreme-rain-event-in-u-s-history/?utm_term=.1c10e1f2ef49), 2017.

835 Schneider, U., Becker, A., Finger, P., Meyer-Christoffer, A., Rudolf, B. and Ziese, M.:  
836 {GPCC} {F}ull {D}ata {R}eanalysis {V}ersion 7.0 at 0.5\$^{\circ}\$: Monthly Land-  
837 Surface Precipitation from Rain-Gauges built on {GTS}-based and Historic Data, ,  
838 doi:10.5676/DWD\_GPCC/FD\_M\_V7\_050, 2015a.

839 Schneider, U., Becker, A., Finger, P., Meyer-Christoffer, A. and Ziese, M.: {GPCC}  
840 {M}onitoring {P}roduct: Near Real-Time Monthly Land-Surface Precipitation from  
841 Rain-Gauges based on {SYNOP} and {CLIMAT} data, ,  
842 doi:10.5676/DWD\_GPCC/MP\_M\_V5\_100, 2015b.

843 Serra, Y. L. and McPhaden, M. J.: In Situ Observations of Diurnal Variability in Rainfall  
844 over the Tropical Pacific and Atlantic Oceans, *J. Clim.*, 17(18), 3496–3509,  
845 doi:10.1175/1520-0442(2004)017<3496:ISOODV>2.0.CO;2, 2004.

846 Simpson, R. H.: The Hurricane Disaster-Potential Scale, *Weatherwise*, 27(4), 169–186,  
847 doi:10.1080/00431672.1974.9931702, 1974.

848 Stanley, T., Kirschbaum, D. B., Huffman, G. J. and Adler, R. F.: Approximating Long-Term  
849 Statistics Early in the {G}lobal {P}recipitation {M}easurement Era, *Earth Interact*,  
850 21(3), 1–10, doi:10.1175/EI-D-16-0025.1, 2017.

851 Sundqvist, H.: Numerical simulation of the development of tropical cyclones with a ten-  
852 level model. Part II, *Tellus*, 22(5), 504–510, doi:10.1111/j.2153-  
853 3490.1970.tb00516.x, 1970.

854 Tang, X. and Zhang, F.: Impacts of the Diurnal Radiation Cycle on the Formation,  
855 Intensity, and Structure of Hurricane Edouard (2014), *J. Atmos. Sci.*, 73(7), 2871–  
856 2892, doi:10.1175/JAS-D-15-0283.1, 2016.

857 Tang, X., Tan, Z.-M., Fang, J., Sun, Y. Q. and Zhang, F.: Impact of the Diurnal Radiation  
858 Cycle on Secondary Eyewall Formation, *J. Atmos. Sci.*, 74(9), 3079–3098,

859 doi:10.1175/JAS-D-17-0020.1, 2017.

860 Tsagris, M., Athineou, G. and Sajib, A.: Directional: Directional Statistics, [online] Available  
861 from: <https://cran.r-project.org/package=Directional>, 2017.

862 Tuleya, R. E. and Kurihara, Y.: A Numerical Study on the Effects of Environmental Flow  
863 on Tropical Storm Genesis, *Mon. Weather Rev.*, 109(12), 2487–2506,  
864 doi:10.1175/1520-0493(1981)109<2487:ANSOTE>2.0.CO;2, 1981.

865 UNDP: Reducing disaster risk: A challenge for development, [online] Available from:  
866 [http://www.undp.org/content/dam/undp/library/crisis%5C  
867 prevention/disaster/asia\\_pacific/Reducing%5C Disaster%5C risk%5C a%5C  
868 Challenge%5C for%5C development.pdf](http://www.undp.org/content/dam/undp/library/crisis%5Cprevention/disaster/asia_pacific/Reducing%5CDisaster%5Crisk%5Ca%5CChallenge%5Cfor%5Cdevelopment.pdf), 2004.

869 UNISDR and CRED: The human cost of weather-related disasters 1995-2015, [online]  
870 Available from:  
871 [https://www.unisdr.org/2015/docs/climatechange/COP21\\_WeatherDisastersReport  
872 \\_2015\\_FINAL.pdf](https://www.unisdr.org/2015/docs/climatechange/COP21_WeatherDisastersReport_2015_FINAL.pdf), 2017.

873 Villarini, G.: On the seasonality of flooding across the continental United States, *Adv.  
874 Water Resour.*, 87, 80–91, doi:10.1016/j.advwatres.2015.11.009, 2016.

875 Wu, Q., Ruan, Z., Chen, D. and Lian, T.: Diurnal variations of tropical cyclone precipitation  
876 in the inner and outer rainbands, *J. Geophys. Res. Atmos.*, 120(1), 1–11,  
877 doi:10.1002/2014JD022190, 2015.

878