Choosing between adaptation and prevention with an increasing probability of a pandemic

Kevin C. Berry[a*]

David Finnoff[b]

a Yale University, School of Forestry and Environmental Studies,

195 Prospect Street, New Haven, CT 06511, United States, kevin.berry@yale.edu

b University of Wyoming, Department of Economics and Finance,

1000 E, University Avenue, Laramie, WY 82071, United States, finnoff@uwyo.edu

*Corresponding author

Highlights

- The risk of pandemic disease is increasing, in particular the risk of pandemic influenza
- Investment in preemptive risk mitigation provides economic savings to domestic policymakers
- Investment in both domestic adaptation and capital used abroad for prevention is optimal
- Excluding humanitarian benefits, a rational self-interested policymaker should provide foreign aid as insurance for their own country against pandemics
- The mixture of investment should respond to the technical relationships between capital stocks and risk

Abstract

The risk of pandemics is increasing, driven by changes in human behavior and climate, both of which are difficult for policymakers to control. There are two main strategies available for reacting to these changes. This paper considers the decision to invest in either adaptation (domestic) capital or prevention (foreign) capital before a pandemic in an interval of time when pandemic risk is increasing. This paper demonstrates how relatively small investments in the two strategies can provide large savings through smaller expected future damages. The technical relationships between adaptation, prevention and risk also determine the optimal mixture of investment over time. As risk increases, the technical relationships between these three stocks causes the optimal mixture of strategies to change over time.

Keywords: endogenous risk; prevention schedule; optimal control; infectious disease

JEL Codes: Q28, Q29, Q54

1. Introduction

When faced with external threats, nations have a tendency to circle the wagons and protect themselves first. In the past, this may have been the safest course of action. As the world becomes more interconnected, events in one nation have greater implications for others. Pandemic prevention is the quintessential global public good - prevention in remote regions of the world can help protect major cities on every continent (World Bank, 2013). Yet there are strong incentives to locate investment domestically in attempt to protect one's own country. There are additional complications as the risk of pandemics is increasing, driven by anthropogenic[1] and climatic forces, and poses a grave threat to human welfare (Cohen, 2000). These threats can consist of the spread of vectors into new geographies as well as the encroachment of humans into fragile ecosystems and disease hotspots. Influenza, with 10 pandemics over the last 300 years poses the greatest mortality threat and poses a tremendous risk to economic activity, with the 1918 pandemic arguably being the greatest natural disaster of all time (Morse et al., 2012; Osterholm, 2005). Although the background factors driving the risk of pandemics are largely exogenous to any specific policy maker, their influence can be tempered through investment in infrastructure and human capital, with the net effect (at least partially) endogenous.

Investments in risk reduction are typically some combination of adaptation (investments that reduce losses) and prevention (investments that makes losses less likely) (Ehrlich and Becker, 1972; Kane and Shogren, 2000; Shogren and Crocker, 1999). Determining the best mix of adaptation and prevention requires a knowledge of how the investments affect the risk and the technical relationships between the strategies (Zemel, 2015). Risk reduction in the most vulnerable regions includes targeted infrastructure and rapid response teams to contain and prevent outbreaks from becoming pandemics (Hufnagel et al., 2004). Developing countries are often targeted for these public health interventions due to the disproportionate impacts of infectious disease that they face, as well as the protection these interventions provide to

[1] Globalization and increased contact with new and novel diseases through land use changes and urbanization has been shown to lead to increasing pandemic risk (Cohen, 2000; Morse et al., 2012).

wealthier nations (Dawood et al., 2012)[2]. The technical relationships between the alternative investments result in their marginal returns being interconnected – for example, investments in prevention capital where the risk is building will make it less likely that adaptation capital in the United States will ever be used, and adaptation capital reduces the consequences of not investing in prevention.

The 2009 H1N1 pandemic started circulating in Mexico and the United States in April 2009, causing 60.8 million cases in the United States before April 2010 (Girard et al., 2010; Shrestha et al., 2011). Globally it is estimated to have caused 280,000 deaths, disproportionately in developing countries, likely due to other underlying diseases and weaker healthcare systems (Dawood et al., 2012). The 2009 H1N1 influenza pandemic was the first activation of the provisions of the International Health Regulations (IHR) (Fineberg, 2014). Vaccines were initially unavailable, and the pandemic disproportionately impacted younger people , possibly due to a lack of previous exposure to that particular strain of influenza (Shrestha et al., 2011). The World Health Organization (WHO) and Centers for Disease Control and Prevention (CDC) worked to speed vaccine development and distribute stockpiles of antivirals and other medicines (CDC, 2010; WHO, 2011). The WHO helped to distribute 3 million courses of antiviral drugs within 72 countries to slow the spread of the disease and limit its impact (Fineberg, 2014).Domestically the CDC deployed the Strategic National Stockpile of medical supplies, including personal protective equipment and antiviral drugs, which includes some 50 million treatment course of antiviral drugs (CDC, 2010).

Investments in risk reduction capacity significantly tempered the consequences of the H1N1 pandemic. As a result of the capacity built in response to the 2003 SARS outbreak, the H1N1 pandemic only had an economic impact of less than .5% of global GDP (World Bank, 2013).[3] Yet

---

[2] In the United States the national plan for pandemics focuses on the detection and containment of outbreaks internationally, as well as provisions domestically to reduce their spread and impact (Morse et al., 2012). The most recent pandemic, H1N1 in 2009, demonstrates the potential effectiveness of these investments.

[3] Estimates of a severe pandemic reach $3 trillion in immediate economic damages (roughly 4.8% of global GDP) (Jonas, 2013). Estimates of damages in the United States range from .6% to 5.5% of GDP (McKibbin & Sidorenko, 2007). These damages are disproportionately driven by demand and supply shocks due to avoidance behavior and high worker absenteeism (Jonas, 2013; World Bank, 2013). The direct costs of an influenza pandemic that caused 89,000 to 207,000 deaths in the United States had been estimated at only $71 billion to $166 billion excluding lost

the possible panic and lost labor of future pandemics have the potential to cause greater economic damages than recent experience, with a real potential for catastrophe (Jonas, 2013; World Bank, 2013). Influenza is also not the only threat – the list of emerging and reemerging diseases includes Chikungunya fever, Dengue fever, Ebola, tuberculosis, cholera, malaria as well as HIV, and SARS (Lindgren et al., 2012; McMichael, 2004; Morse et al., 2012). While our analysis is focused on influenza, there has been a recent influx of vector-borne diseases in the Western Hemisphere, including the current Zika virus outbreak (Fauci and Morens, 2016).

In spite of these risks, many countries lack the capacity to meet a surge in demand for healthcare services and supplies, and require greater investment into vaccination technology to speed their development during a crisis (Osterholm, 2005). The United States alone is 100,000 nurses short of what would be needed during a large pandemic and lacks critical supplies and infrastructure, such as an adequate number of beds, with many emergency facilities already operating at capacity (Bartlett and Borio, 2008). Vaccine development is hampered due to the difficulty in predicting the timing and magnitude of pandemics. As information on specific strains of viruses typically only becomes available after an outbreak occurs, vaccines are not available in the initial stages (Morse, 2007; Osterholm, 2005). The lack of preparedness has been made clear by many countries not meeting the core capacities outlined in the International Health Regulations (Morse et al., 2012; Ross et al., 2015) due to lack of resources and competing priorities. One estimate of the cost to bring capacity in public veterinarian and human health systems up to international standards is $3.4 billion a year, even though these investments have been estimated to yield annual benefits of $37 billion (World Bank, 2013).

We consider the capacity of pandemic risk reduction as stocks of capital that provide society the ability to prevent the likelihood of a pandemic and the ability to adapt to the consequences of a pandemic if it should occur. Given the location-specific emergence of pandemic risk, prevention capacity can entail actions in regions where the pandemic is likely to emerge, and affects the probability of a pandemic occurring. In contrast, adaptation capacity measures the ability (for example) of domestic actions in the United States to reduce domestic damages due

---

productivity and other economic costs (Meltzer, Cox, & Fukuda, 1999). Avoidance behavior and adverse economic shocks due to the 2003 SARS outbreak (with only 8,000 cases) cost $30 to $100 billion (Bartlett & Borio, 2008).

to economic interruptions, morbidity and mortality. Preemptive investments made over time build these capital stocks.

The best mix of pandemic prevention and adaptation capital and their investment pattern over time are derived using a Poisson jump process to model a pandemic as a sudden damaging event (Reed and Heras, 1992; Zemel, 2015). Similar work demonstrated the importance of endogenous versus exogenous hazard rates (De Zeeuw and Zemel, 2012), while Zemel(2015) examined the optimal mixture of prevention and adaptation capital when faced with a stock of pollution that can lead to sudden environmental damages, in the face of a constant background hazard. Investment in prevention(Berry et al., 2015) and adaptation capital (Tsur and Withagen, 2013) when facing an exogenously increasing hazard rate have been examined separately. In both cases, there are welfare gains to be had if investment responds to exogenous changes in the probability of an event occurring.

This paper extends this literature by including both an increasing background hazard and the tradeoff between adaptation and prevention. Importantly, we include the interaction between returns to alternative strategies and the technological response of these strategies to an increasing risk. When the risk exogenously increases, stocks of either capital should be built up first with a large one time investment, and then the stock should added to over time by investing above and beyond what is needed to replace depreciation. Investments and the mixture of strategies in the portfolio of risk reduction capital necessarily depend on the background hazard rate as well as how the marginal returns to each activity change with the background hazard rate.

The paper proceeds as follows: the model and solution procedure are introduced in section 2, and several analytical conclusions are drawn. A numerical exercise follows in section 3, with a discussion of results in section 4.

2. Model

Consider a domestic policymaker charged with mitigating the risk of a pandemic that occurs at time given by the random variable T. Time is partitioned in the model into two periods, the ex-ante period before a pandemic has occurred, $t < T$, and the ex-post period when the pandemic occurs, $t \geq T$, and causes economic damage. The variable T has the distribution $F = 1 - e^{-\int_0^t \psi(b(t),N(t))ds}$ and density$f = \psi(b(t), N(t))e^{-\int_0^t \psi(b(t),N(t))ds}$. The density and distribution are related to the hazard function $\psi(b(t), N(t))$ which is the probability of a pandemic in the next instant, given one has not already occurred. The hazard rate is a measure of risk faced by the policymaker. The hazard rate depends on an exogenous background risk, b(t), $\psi_b > 0$, which is increasing, $\delta b/\delta t(t) \geq 0$, to a new constant level $\bar{b}$ which it reaches at time $t = \theta$, $\delta b(\theta)/\delta t = 0$, due to a wide variety of anthropogenic and climatic factors(Cohen, 2000; Morse et al., 2012). The hazard rate can be reduced by investment in prevention capital, $N(t)$, to delay a pandemic $\psi_N < 0$. This capital is located in "hotspots" in foreign countries where disease emergence is most likely and includes investment in rapid response teams and capacity. Examples include the new pandemic emergency facility proposed by the President of the World Bank to channel funds to organizations attempting to contain disease outbreaks before they become pandemics, as well as in capital to assist lower income countries in meeting the International Health Regulations standards (Ross et al., 2015).

In the ex-ante interval, domestic managers can also invest in a stock of adaptation capital $A(t)$ located domestically which reduces the local economic cost of a pandemic in the ex-post period. This consists of domestic investments (i.e. in the United States) such as containment units, investing in local hospitals, or otherwise increasing capacity in the U.S. to meet the needs of a future pandemic (Bartlett and Borio, 2008). By focusing on the decisions of domestic U.S. policymakers' we can distinguish between prevention capital and adaptation capital by location as well as purpose. For instance, investment in hospitals in Southeast Asia is prevention capital, while hospitals in Omaha, Nebraska are adaptation capital. Spending on expanding the capacity of GOARN (Global Outbreak Alert & Response Network) is prevention (intended to counter pandemic early on) and investments in first responders in the United States, are adaptation. For analytical clarity it is assumed that there is a clear distinction between the two investments.

In the ex-post interval, a pandemic has occurred at time T causing lump-sum damages which are reduced by the existing stock of adaptation capital at that instant, $A(T)$. Damages include economic damages from avoidance behavior and lost productivity, possible long term effects from infection, costs from treatment and hospitalization and surge spending on new resources to respond to a pandemic in the US. Preemptive ex-ante investment in adaptation capital influences realized economic damages. Given the preemptive focus of the problem, we abstract from ex-post optimization, and take the expected present value of the ex-post system at time T to be

$$-D\big(A(T)\big)e^{-rT} \qquad (1)$$

where $D'\big(A(T)\big) < 0$ and $D"(A(T)) > 0$. Ex-post damages are assumed to be independent of the probability of a pandemic. This model does not include the possibility that adaptation capital is incorrectly targeted, which would scale the marginal benefit of adaptation capital up or down.

Before a pandemic, $t < T$, the policymaker decides on the cost minimizing combination of investment in prevention $n(t)$ or adaptation $a(t)$ over the interval before a pandemic occurs,

$$\int_0^T (-n(t) - a(t))e^{-rt}dt \qquad (2)$$

where the timing of T is uncertain. The stocks of both prevention and adaptation capital are governed by the capital accumulation equations

$$\dot{A} = a - \zeta A \qquad (3)$$

and

$$\dot{N} = n - \delta N. \qquad (4)$$

Both stocks are increased by the related flow of investment and depreciate at constant rates; ζ for adaptation, and δ for prevention. Depreciation includes wear and tear on equipment, as well as changes in technology and antimicrobial resistance. For example, hospitals and facilities in developing countries may depreciate due to climate and difficulties in performing maintenance. Investment in vaccination might decay faster in one country or another due to population turnover. The effectiveness of treatments depreciates over time as microbes evolve to become resistant to treatment, as well as to find new hosts and adapt to changes in climate.

The policymaker's stochastic optimization problem is to maximize the expected sum of ex-ante expenditures and the ex-post value function

$$E_T\left\{\int_0^T(-n(t)-a(t))e^{-rt}dt - D\big(A(T)\big)e^{-rT}\right\} \qquad (5)$$

where E_T denotes expectations on the random variable T, the arrival time of a pandemic, and the problem is maximized subject to (3) and (4). Following the transformations in Reed and Heras(1992) and Zemel (2015), it is possible to write the stochastic optimal control problem as

$$\int_0^\infty\{(-n(t)-a(t))I(t<T) - D\big(A(T)\big)I(t=T)\}e^{-rt}\,dt \qquad (6)$$

where the indicator function $I(t<T)$ holds the value 1 when $t<T$ and 0 otherwise, and similarly $I(t=T)$ has value 1 when $t=T$ and 0 otherwise. T is a terminal time, so that at that instant agents must pay the expected value of damages from that Tinto the infinite future. We can then use the definitions of the distribution and density to rewrite the problem

$$\max_{\text{n,a}}\ \int_0^\infty\{(-n-a) - D\big(A(t)\big)\psi(b(t),N(t))\}e^{-\int_0^t(r+\psi(b(s),N(s))ds}\ \ dt \qquad (7)$$

Which is maximized subject to equations (3) and (4), the exogenous background hazard, the dynamics of the cumulative hazard function (8),

$$\dot{y}(t)=\int_0^t\psi\big(b(t),N(t)\big)dt\,,\dot{b}(t)\geq 0,\dot{b}(\theta)=0, \qquad (8)$$

and constraints on the maximum level of investment, $a\leq\bar{a}$ and $n\leq\bar{n}$. The upper bounds $\bar{a}$ and $\bar{n}$ reflect budget constraints and frictions when investing in capital stocks. We assume that at timeθ the background hazard rate will reach a new 'normal', $\bar{b}$, and cease to increase.

2.1 Hamiltonian and Maximum Principle

The objective function is maximized using the conditional current value Hamiltonian (henceforth simply the 'Hamiltonian') defined in Reed and Heras (1992). The Hamiltonian, suppressing time notation, is

$$H=-n-a-D(A)\psi\big(b,N(t)\big)\ +\rho_1[n-\delta N]+\rho_2[a-\zeta A]+\rho_3\psi(b,N). \qquad (9)$$

The benefit maximizing path of investment in prevention and adaptation capital is given by the Pontryagin and Boltyanskii (1962) maximum principle, which require investment in the respective capital stocks, *n* and *a,* balance the marginal cost of investment (both \$1) with the expected marginal benefit of investment, or

$$-1 + \rho_1 \begin{cases} > 0 \\ = 0, \\ < 0 \end{cases} \quad (10)$$

$$-1 + \rho_2 \begin{cases} > 0 \\ = 0. \\ < 0 \end{cases} \quad (11)$$

In equations (10) and (11) the marginal benefit of investment in either capital stock is the shadow value of the respective capital stock, given by the conditional costate variable. The costate variable is the value of an additional unit of capital stock in either a lower hazard rate or lower damages. If the marginal cost is always greater than the marginal benefit for one or both types of capital, it is optimal to not invest in that stock. If the marginal benefit is always greater than the marginal cost for one or both stocks investment should be made at the maximum possible rate, $\bar{a}$ and $\bar{n}$. When investment is made at the maximum rate in one or both stocks they will grow to their maximum feasible levels $\bar{A} = \frac{\bar{a}}{\zeta}$ and/or $\bar{N} = \frac{\bar{n}}{\delta}$. If the marginal benefit is exactly equal to the marginal cost at some non-boundary level the optimal decision is on a singular arc. It is possible to be on a singular arc for only prevention capital or only adaptation capital, or for both simultaneously. We refer to these different cases as either the 'singular solution' when both capital stocks are on a singular arc, or as a 'partial singular solution' when only one capital stock is on a singular arc, and the other is at an extreme value. A welfare analysis comparing maximum paths with singular and partial singular solutions is included below. For the remainder of this section the discussion will focus on singular solutions and partial singular solutions.

The maximum principle requires optimal intertemporal management of the system, given by a required evolution of the costate variables (where subscripts denote partial derivatives),

$$\dot{\rho}_1 = (r + \psi(b,N) + \delta)\rho_1 + \psi_N(b,N)\big(D\big(A(t)\big) - \rho_3\big) \qquad (12)$$

$$\dot{\rho}_2 = (r + \psi(b,N) + \zeta)\rho_2 + \psi(b,N)D_A\big(A(t)\big) \qquad (13)$$

$$\dot{\rho}_3 = \big(r + \psi(b,N)\big)\rho_3 - \mathrm{n} - \mathrm{a} - \psi(b,N)D\big(A(t)\big) \qquad (14)$$

Using the solutions to (12), (13), and (14) it is possible to sign the costate variables[4], so that on the optimal solution $\rho_1, \rho_2 > 0$ and $\rho_3 < 0$ (Zemel, 2015). Integrating equation (14) and applying the infinite horizon transversality condition has the solution

$$-\rho_3 = e^{rt+y(t)} \max_{n,a} \int_t^{\infty} \big[-n(\tau) - a(\tau) - \psi\big(b(\tau), N(\tau)\big)D\big(A(\tau)\big)\big] e^{-r\tau - y(\tau)} d\tau. \qquad (15)$$

The costate equation ρ_3 is the expected present value of costs and damages from an optimally managed system (Reed and Heras, 1992). The value of remaining in the ex-ante period without a pandemic in any instant is given by $\big(D\big(A(t)\big) - \rho_3\big)$, which is the instantaneous value of damages from a pandemic occurring conditional on the level of adaptation capital, less the expected value of damages if a pandemic is delayed.

A capital theoretic approach provides additional insight. Substitute the prevention capital first-order condition (10) into the prevention capital costate equation (12) on the prevention singular arc where $\dot{\rho}_1 = 0$ and ρ_1=1. The costate equation becomes an arbitrage condition

$$r + \psi(b,N) + \delta = -\psi_N(b,N)(D(A) - \rho_3). \qquad (16)$$

The left hand side of (16) is the required return on a unit of prevention capital, the right hand side the real return. The required return is what a policymaker could earn by divesting a unit of capital and investing the proceeds in an alternative investment, possibly the control of another

[4]Equations (12) and (13), integrating and using the infinite horizon transversality condition, have the solutions $\rho_1 = e^{(r+\delta)t+y(t)} \int_t^{\infty} \psi_N(b,N)[\rho_3 - D(A)]e^{-(r+\delta)\tau - y(\tau)} d\tau$ and $\rho_2 = e^{(r+\zeta)t+y(t)} \int_t^{\infty} -\psi(b,N)D_A(A)e^{-(r+\zeta)\tau - y(\tau)} d\tau$. Intuitively the costate variables reflect the discounted marginal benefit of either type of capital into the infinite future. For example, ρ_1 is the reduction in risk, weighted by the benefit of remaining in an uninvaded state into the future for a unit of prevention capital. The costate ρ_2 is the marginal reduction in expected damages from one unit of adaptation capital into the infinite future. Both are discounted not only by the market return on an alternative investment and the depreciation rate but also the probability of a pandemic. These can be confirmed with time differentiation using the Leibniz rule. The costate values of both types of capital are positive as long as it is preferable to avoid a pandemic ($\rho_3 < D\big(A(t)\big)$ using (16) below), as investment in either mitigates the expected cost of a pandemic.

risk, or a financial asset. The required return is the marginal cost of a unit of prevention capital and includes the market rate of return on alternative investments, r, plus premiums for the risk of a pandemic occurring, $\psi(b, N)$, and depreciation of the capital stock δ. The real return of a unit of prevention capital, given by the right hand side of (16) is the reduction in expected future damages and is the marginal benefit of an additional unit of prevention capital. It consists of the reduction in the probability of a pandemic occurring in the next instant, given it has not yet occurred, $\psi_N(b, N)$, weighted by marginal cost of transitioning to the ex-post period and incurring the costs of a pandemic , $D(A) - \rho_3$. If the real return is greater than the required return, prevention capital is better performing asset and a rational policymaker should sell alternative investments and purchase more prevention capital. If the required return is greater than the real return, a manager should optimally sell prevention capital and purchase the alternative investment.

Intertemporal arbitrage is complicated by marginal impact of prevention capital on the hazard rate, $\psi_N(b, N)$, changing as the background probability of a pandemic, $b(t)$, changes (unless $\psi_{Nb}(b, N) = 0$). The direction of change may be positive or negative. As the risk of a pandemic increases, the next dollar of investment may become more effective because it becomes easier to identify high risk “hotspots”. It is also possible as risks increase investment in prevention becomes less effective. Risk driven by globalization may become more difficult to mitigate as countries become more open to the outside world.

Real returns from prevention capital are also dependent on the stock of adaptation capital, as demonstrated in equation (16). The marginal benefit of investing in prevention capital is reduced by adaptation capital because the difference in value between the optimally controlled ex-ante situation and the potential damages from a pandemic is smaller. Similar to self-insurance, a larger stock of adaptation capital provides benefits in the ex-post period by increasing the value of a post-outbreak world. This comes at a cost of forgone benefits in the ex-ante period. Policymakers trade benefits in the present for reduced uncertainty in the future, reducing the value of self-protection.

A similar intertemporal arbitrage condition for holding adaptation capital can be derived by substituting the first-order condition (11) into costate equation (13) and assuming we are on the singular arc, $\rho_2 = 1, \dot{\rho}_2 = 0$,

$$r + \psi(b, N) + \zeta = -\psi(b, N)D_A(A) \qquad (17)$$

Equation (17) requires a balance of the required return on adaptation capital with its real return. The required return again consists of the market return on alternative investments, r, a premium for the hazard rate, $\psi(b, N)$, and a premium for the depreciation rate of adaptation capital, ζ. The real return is the decrease in expected damages in the ex-post world $D_A(A)$, weighted by the probability that the world transitions into the ex-post state, $\psi(b, N)$. Comparisons of the required and real returns again guide the policy maker's asset portfolio. The required and real returns of adaptation capital depend on investment in prevention capital in equation (17). Stocks of prevention capital reduce the probability of a pandemic occurring, which in turn reduces the real return to adaptation capital. Prevention capital also reduces the required hazard rate premium, and thus the required return on capital because adaptation capital can be expected to reduce the expected damages of a pandemic for a longer ex-ante period.

The final optimality conditions consist of the state equations, (3), (4) and (8) and the maximum constraints on investment. Maximum investment rates are due to friction when attempting to rapidly build capital stocks, and represent either budget constraints or physical limitations. Solution methods for the boundary solutions are straightforward. Solutions for singular arcs are outlined below.

2.2 Solutions

There are seven possible strategies that can be pursued by policymakers. The extreme solutions consist of investing in neither capital stock ($n = a = 0$), either at the maximum rate ($n = \bar{n}$ and $a = 0$ or $n = 0$ and $a = \bar{a}$), or both at maximum rates ($a = \bar{a}$ and $n = \bar{n}$). These solutions are included in the numerical exercise, and consist of most rapid approach paths to the extreme levels of their respective capital stocks.

The singular solutions and partial singular solutions follow the same pattern. Partial singular solutions assume one capital stock is on a singular arc and the other is set at an extreme. The singular solutions assume both capital stocks are on their respective singular arcs. For welfare calculations for partial singular solutions, investment in one capital stock will be constrained to zero, although policymakers may inherit a stock of capital[5]. The optimal paths for singular investment in both capital stocks are derived below. The partial singular solutions are nested within the singular solution.

2.2.1 Singular arcs

The (partial) singular solutions require the respective first-order conditions hold with equality. Imposing this condition on (10) and (11) provides $\rho_1 = \rho_2 = 1$ and $\dot{\rho}_1 = \dot{\rho}_2 = 0$, the conditions to be on their respective capital stock's singular arcs. To derive the singular solution we impose the conditions to be on the prevention capital singular arc on (12). Solving for ρ_3,

$$\rho_3 = \frac{(r+\psi(b,N)+\delta)}{\psi_N} + D(A). \qquad (18)$$

Time differentiating (18) finds

$$\dot{\rho}_3 = \frac{(\psi_b \dot{b} + \psi_N \dot{N})\psi_N - (\psi_{NN}\dot{N} + \psi_{Nb}\dot{b})\,(r+\psi(b,N)+\delta)}{\psi_N^2} + D_A(A)\dot{A}. \qquad (19)$$

Setting this equal to the previous definition of $\dot{\rho}_3$ in (14) allows us to solve for $\dot{N}$,

$$\dot{N} = \frac{\psi_b\psi_N - \psi_{Nb}(r+\psi+\delta)}{\psi_{NN}(r+\psi+\delta) - 2\psi_N^2}\dot{b} + \frac{\left(1+D_A(A)\right)\psi_N^2}{\psi_{NN}(r+\psi+\delta) - 2\psi_N^2}\dot{A} - \frac{\psi_N^2[(r+\psi)\rho_3 - \delta N - \zeta A - \psi D(A)]}{\psi_{NN}(r+\psi+\delta) - 2\psi_N^2}. \qquad (20)$$

Substituting for $\dot{N}$ in (20) from Equation (4), we can find a feedback rule for the optimal investment in prevention capital (21), which depends on the current stocks of prevention and adaptation capital, and how the adaptation capital and background risk are changing over time

$$n = \delta N + \frac{\psi_b\psi_N - \psi_{Nb}(r+\psi+\delta)}{\psi_{NN}(r+\psi+\delta) - 2\psi_N^2}\dot{b} + \frac{\left(1+D_A(A)\right)\psi_N^2}{\psi_{NN}(r+\psi+\delta) - 2\psi_N^2}\dot{A} - \frac{\psi_N^2[(r+\psi)\rho_3 - \delta N - \zeta A - \psi D(A)]}{\psi_{NN}(r+\psi+\delta) - 2\psi_N^2}.$$

(21)

[5] Comparative dynamic analysis shows the general result holds, even with preexisting stocks of the alternative capital stock.

We derive a feedback rule for adaptation capital by using the conditions to be on the adaptation singular arc from (11) upon (13). We then time differentiate and substitute for $\dot{A}$ using equation (3) to find

$$a = \zeta A - \frac{(r+\zeta)\psi_N}{\psi^2 D_{AA}}\dot{N} - \frac{(r+\zeta)\psi_b}{\psi^2 D_{AA}}\dot{b}. \qquad (22)$$

Equation (22) is the feedback rule for the optimal singular investment in adaptation capital, as a function of the current capital stocks and how the prevention capital stock and background risk are changing over time. Together equations (21) and (22) define how the singular levels of investment in adaptation and prevention capital change over time in the (partial) singular solutions.

If policymakers seek to follow one of the (partial) singular solutions, their choice of investment in one stock will be impacted by the level of the other. If policymakers begin off the optimal path with too little of one or both stocks of capital, they should make large lump sum investments to reach capital stocks on the path. If policymakers begin with too much of one type of capital and too little of the other, they should transfer assets between the two to reach an optimal combination. For example, a policymaker may seek to apply the singular solution, but have an adaptation capital stock that is greater than what is called for and an insufficient prevention capital stock. This policymaker might continue to invest in adaptation capital above what the optimal level would otherwise be while building up their prevention capital stock. They would eventually invest in less than the amount of depreciation in the excess capital stock, and allow it to fall to an optimal level.

To solve this system of feedback rules it is necessary to derive endpoint conditions. This is done with two steady-state curves where the conditions for either adaptation capital or prevention capital to be on their respective singular arcs hold (Zemel, 2015). On these curves the system has reached time θ where for $t \geq \theta$ the background hazard rate remains constant so that the problem becomes time-autonomous.

We begin by solving for the prevention capital steady-state curve, shown in Figure 1a. Set (14) equal to zero and substitute the steady state levels of investment, $n = \delta N$ and $a = \zeta A$

$$0 = \big(r + \psi(b, N)\big)\rho_3 - \delta\text{N} - \zeta\text{A} - \psi(b, N)D(A). \tag{23}$$

Solve (23) for ρ_3 and substitute this into (12) evaluated at the steady state to define the prevention steady-steady state curve, or N-steady state curve in Figure 1. Combinations of adaptation capital and prevention capital on this curve satisfy the conditions for a steady state and a partial singular solution in prevention capital.

The second steady state curve is defined by evaluating (13) on the singular arc for adaptation capital and steady state background hazard rate. Combinations of adaptation capital and prevention capital on this curve in Figure 1a satisfy the conditions for a steady state and a partial singular solution in adaptation. Points where these two curves cross in Figure 1a are combinations of adaptation and prevention capital where conditions for a double singular steady state hold. Where the curves cross, all conditions for a steady state hold ($\dot{A} = \dot{N} = \dot{b} = \dot{\rho}_1 = \dot{\rho}_2 = \dot{\rho}_3 = 0$). These intersections are shown in Figure 1a, and there are two candidate singular solutions for our parameterization. There are also two partial singular solutions shown in Figure 1b (one with only adaptation capital, one with only prevention capital). Both partial singular solutions end on their respective capital stock's steady state curve where they intersect their respective axis.

To examine how a change in the exogenous probability of a pandemic impacts the optimal mix of prevention and adaptation capital stocks, examine equation (12) divided by (13) assuming a singular solution,

$$\frac{r+\psi(b,N)+\delta}{r+\psi(b,N)+\zeta} = \frac{\psi_N(b,N)(D(A)-\rho_3)}{\psi(b,N)D_A(A)}. \tag{24}$$

The optimal mix of prevention capital and adaptation capital depends on the technological characteristics of the risk mitigating technology and the technological substitute and compliment relationships between adaptation and prevention, as well as how investment in the other capital stock responds to the change in background probability of a pandemic. For instance, as the depreciation rate of either capital stock increases, the relative required return of that capital stock rises due to the increased depreciation premium. This requires a larger real return to that strategy, and a reduction in the size of that stock. This reduction in the size of the stock will also change the relative balance of the two capital stocks and may lead to a change in the other stock at the same time. The relative real returns are even more interconnected than the required returns. For instance, a change in the marginal return of prevention, ψ_N, due to a new technology, will lead to both a change in the real return of prevention capital and the relative return to adaptation capital. It will also impact the required return of both strategies by changing the required premium for risk. Similarly, a change in the marginal return of adaptation capital, D_A, will be magnified by changes to the marginal benefit of prevention.

Another example would be a change in the exogenous background hazard. Differentiating (24) with respect to the background risk,

$$\frac{\psi_b(b,N)[\zeta-\delta]}{(r+\psi(b,N)+\zeta)^2} = \frac{[\psi\psi_{Nb}-\psi_N\psi_b]D_A(A)(D(A)-\rho_3)}{(D_A(A)\psi)^2} \qquad (25)$$

The left hand side includes how the ratio of required returns changes with a change in the hazard rate, where the relative change in the two ratios depends on the different depreciation rates, $[\zeta - \delta]$. If the depreciation rate for prevention capital is larger, an increase in background risk causes the left hand side, the relative required return of prevention capital to adaptation capital, to decrease. The right hand side must also decrease for the first-order conditions to continue to hold with equality. This requires a relatively larger stock of prevention capital. If it is assumed both capital stocks depreciate at the same rate, there is no change in the relative required returns. With the assumption that remaining in a pre-pandemic system is preferable $(D(A) - \rho_3 > 0)$ the change in the right hand side depends on the sign of the terms in the square brackets. The far right term is unambiguously negative, $D_A(A)(D(A) - \rho_3) < 0$. The sign of the square bracketed terms depends on whether prevention efforts become more effective as background risk increases $\psi_{Nb}(b, N)$. If background risk makes prevention more effective, the relative marginal benefit of prevention increases, leading to more prevention investment. We examine this in our numerical example below, where our parameterization assumes higher risk makes prevention more effective. If an increase in risk makes prevention less effective, the effect is ambiguous and depends on the relative magnitudes of $\psi\psi_{Nb}$ and $\psi_N\psi_b$.

2.2.2 Prevention only partial singular solution

First examine the partial singular solution where managers invest only in prevention so that $\dot{\rho}_1 = 0,\ \rho_1 = 1,\ a(t) = 0$. The feedback rule for prevention capital is modified so that $a(t)$=0 in (21) however it is not true that $A(t)$=0, as it is possible that some stock of adaptation capital is inherited by our policymaker. On this path the adaptation capital stock monotonically depreciates over time to nothing. In this case only the N-steady state curve is relevant, and a solution should end on this curve.

As the background probability of a pandemic increases, investment in prevention increases, leading to a larger prevention capital stock. In order to reach the optimal prevention capital schedule it is necessary to follow a most rapid approach path to the optimal path, and this depends on the initial background probability of a pandemic and existing adaptation capital stock. After this initial investment in prevention capital, it is necessary to invest more than the amount required to replace depreciation to build up the capital stock to compensate for an increase in the background probability of a pandemic. Because there is no investment in adaptation, the adaptation stock depreciates away over time.

Examine the costate equation for adaptation evaluated when there is no adaptation capital stock, so that the marginal benefit of the next unit is at its highest possible level, $A = 0$. If it is initially optimal to invest only in prevention, it may be optimal to invest in adaptation in the future. To begin investing in adaptation, it is necessary for $\dot{\rho}_2 > 0$. This requires that the return on the shadow value of adaptation capital, $(r + \psi(b, N) + \zeta)$, weighted by the value of adaptation capital, ρ_2, be greater than the expected dividend (or instantaneous benefit) of adaptation capital $\psi(b, N)D_A(A)$, which consists of the increase in expected value of the ex-post state.

$$\dot{\rho}_2 = (r + \psi(b, N) + \zeta)\rho_2 + \psi(b, N)D_A(A) \quad (26)$$

Using the solution for ρ_2, the value of adaptation capital will rise over time. This is because ρ_2 is equal to the marginal benefit of adaptation capital from the current time into the future, discounted by the market rate as well as premiums for the hazard rate and depreciation. This necessarily includes the marginal benefit in the current period, and given r and ζ are strictly positive, the return on prevention capital $(r + \psi(b, N) + \zeta)\rho_2$ is always greater than the per period dividend $D_A(A)\psi(b, N)$. This implies that ceteris paribus it is preferable to invest in a mixture of adaptation capital and prevention capital, rather than only prevention capital.

The impact of a change in the prevention stock on the growth of the shadow value of adaptation capital is found using the derivative of (13) with respect to N,

$$\frac{\partial \dot{\rho}_2}{\partial N} = \psi_N \rho_2 + \psi_N D_A(A) \ = \psi_N(\rho_2 + D_A(A)). \tag{27}$$

An increase in prevention capital has a mixed impact on the incentive to invest in adaptation. The marginal cost of adaptation capital falls because a smaller rate of return is required on investment to compensate for the risk of a pandemic occurring. The marginal benefit of adaptation capital is reduced by the change in the probability of adaptation capital being used in the future. The overall sign of the derivative is ambiguous, and depends on if the shadow value of adaptation capital and is greater than the marginal change in damages from adaptation capital.

The impact of a rise in the background probability of a pandemic is found with the derivative of (21) with respect to b,

$$\frac{\partial \dot{\rho}_2}{\partial b} = \left(\rho_2 + D_A(A)\right)\left(\psi_b + \psi_N \frac{\partial N}{\partial b}\right) \tag{28}$$

Again, the impact depends on the relative magnitude of the shadow value of adaptation capital in comparison to the marginal reduction in damages from adaptation capital (first term on the right). There are two effects in (23) that follow a change in background risk. The first reflects how an increase in background risk directly increases the hazard, increasing the marginal value of adaptation capital. The second is how a change in background risk changes the prevention stock and indirectly changes the hazard. If an increase in background risk leads to a larger prevention stock, the indirect effect accentuates the direct effect, putting an upward pressure on the change in marginal value of adaptation capital. The indirect effect will attenuate the direct effect if an increase in background risk leads to a smaller prevention stock.

2.2.3 Adaptation partial singular solution without prevention

Now examine the situation where only adaptation is on its singular arc ($\rho_1 < 1, \rho_2 = 1\,, \dot{\rho}_2 = 0$). In this case investment in prevention capital is set equal to zero, while adaptation is assumed to evolve at the singular rate. Again the prevention capital stock will only decrease over time as it depreciates, but is not necessarily always zero because policymakers may inherit a stock of capital. In order for it to become optimal to also invest in prevention capital, the shadow value of prevention capital must increase over time. The dynamics of the costate value of prevention capital are given by equation (12), which in the adaptation capital partial singular solution is

$$\dot{\rho}_1 = (r + \psi(b, N) + \delta)\rho_1 + \psi_N(b, N)(D(A) - \rho_3). \qquad (29)$$

The first term represents the required return of prevention capital. This is the return on a comparable asset, where ρ_1 is the value of the prevention capital stock. The return includes the market rate of return, r, plus premiums for the risk of a collapse ψ and depreciation δ. For the value of the asset to grow, this return must be greater than the expected dividends received in that period from the prevention capital stock, given by the reduced chance of outbreak and transition to lower welfare, $\psi_N(b, N)(D(A) - \rho_3)$. Because ρ_1 is the integral of this marginal benefit now and into the future, ceteris paribus the value of prevention capital will increase.

Taking a derivative of (29) with respect to $A(t)$ shows how the growth in the value of prevention capital $\dot{\rho}_1$ changes as the level of adaptation capital changes.

$$\frac{\partial \dot{\rho}_1}{\partial A} = \psi_N\left(D_A(A) - {\rho_3}_A\right) \qquad (30)$$

Additional investment in adaptation has a mixed effect on the incentive to invest in prevention capital. More adaptation capital makes a pandemic less damaging, $D_A(A) < 0$. Adaptation capital also impacts the value of the optimally controlled ex-ante system. Assuming investment in adaptation follows an optimal path, it must necessarily not lower the value of that system relative to no investment at all, so that ${\rho_3}_A \leq 0$ (because $-\rho_3$ is equal to the expected net present value of benefits). Whether $\frac{\partial \dot{\rho}_1}{\partial A}$ is positive or negative will depend on the relative magnitude of two effects. The increase in the value of an optimally controlled ex-ante system (from the derivative of (15) with respect to A) consists of both an increase in the expected ex-post value ($\psi(b,N)D_A(A)$) and the higher necessary flow of investment to maintain the higher stock, a. This suggests that $D_A(A) > -\rho_{3_A}$ is the most likely case, and $\frac{\partial \dot{\rho}_1}{\partial A} > 0$. When only on the adaptation singular arc the growth rate of the costate value for prevention capital increases with more adaptation capital, so that it is again optimal to invest in a combination of strategies.

Taking the derivative

$$\frac{\partial \dot{\rho}_1}{\partial b} = \psi_b(b,N)\rho_1 + (\psi_{Nb}(b,N)\left(\frac{D_A}{r} - \rho_3\right) + \psi_N(b,N)\left(\frac{D_A}{r} - \rho_{3_A}\right)\frac{\partial A}{\partial b} \qquad (31)$$

there are several effects in (31) that determine if an increase in the background risk of a pandemic raises or lowers the incentive to invest in prevention capital. First, the higher background probability raises the hazard rate and the required return on prevention capital, $\psi_b(b,N)\rho_1$. Second, the rise in the background hazard rate can either increase or decrease the effectiveness of investments in prevention, $\psi_{Nb}(b,N)$, which will depend on whether risk and prevention are technological compliments or substitutes and is ambiguous. This is weighted by the expected value of avoiding a pandemic. Finally, a rise in the background risk can either increase or decrease the investment in adaptation, $\frac{\partial A}{\partial b}$, which will change the benefit of remaining in the ex-ante state of the world, $\left(\frac{D_A}{r} - \rho_{3_A}\right)$, which weights the marginal impact of investing in prevention $\psi_N(b,N)$. If a higher probability of a pandemic increases adaptation capital, this effect will increase the growth rate in the value of prevention capital. Overall whether or not an increase in the background probability of a pandemic makes it optimal to begin investing in prevention depends on the technological characteristics of the prevention and adaptation technologies. Due to the difficulty in finding analytical results, a numerical exercise is included in section 3.

2.2.4 Multiple Steady States and Skiba Thresholds

Given that there are multiple candidate singular solutions, partial singular solutions and multiple possible extreme solutions, it is necessary to evaluate which solution is locally or globally dominant. A Skiba plane that divides the phase space between the candidate steady states can be derived using condition (32) (Mäler et al., 2004; Skiba, 1978). Equation (38) compares the value of immediately transitioning from any initial point to one of the two optimal paths (A^0, N^0, b^0).

$$-\rho_3^i\left(b^0, N^i, A^i\right) - \left(N^0 - N^i\right) - \left(A^0 - A^i\right) = -\rho_3^j(b^0, N^j, A^j) - \left(N^0 - N^j\right) - \left(A^0 - A^j\right) \quad (32)$$

This value consists of the expected net present value of benefits on a candidate path, net the required changes in capital stocks to move onto that path given the current level of risk. The Skiba threshold consists of all points where the value of the path to either steady state is equal. This curve determines which path a policymaker would prefer to be on for a given starting point, and is evaluated numerically in the next section.

3. Numerical Exercise

Due to the difficulty in deriving analytical results, a numerical exercise is included. The parameters for this exercise are given in Table 1, and represent the global threat of a pandemic outbreak of influenza, where a severe pandemic causes $3 trillion in economic damages (4.8% of global GDP) if there is no stock of adaptation capital (Jonas, 2013; World Bank, 2013). We use the global GDP estimate even though we are focused on a self-interested domestic policymaker because domestic damages are likely to be 60% from lost economic activity and aversion behavior, and for a large economy these global economic effects are likely to be proportional to domestic damages (Jonas, 2013). This is large compared to the 2009 H1N1 pandemic, which caused damages of less than .5% of global GDP. The 2009 pandemic was also impacted by mitigation efforts that were started in response to the 2003 SARS epidemic. Estimates of potential damages range from a pandemic .5% to 5.5% of GDP (Jonas, 2013; McKibbin and Sidorenko, 2007). These losses are due to changes in population, morbidity and mortality and lost economic activity.

The background probability of a pandemic is assumed to be equal to .1, greater than the historic rate of 3 outbreaks over the last 100 years to reflect the increasing nature of the risk (Meltzer et al., 1999). Numerical results, including final capital stocks and savings are included in Table 2.The effectiveness of prevention and adaptation capital are calibrated so that the singular solution capital stocks are of a magnitude consistent with previous large scale public health interventions (roughly \$2-\$4 billion), such as smallpox eradication (\$2.1 billion in 2014 USD) and (unsuccessful) Polio eradication (\$7 billion) (Keegan et al., 2011). This is similar to estimates of what it would cost to bring the international health system up to a set of minimum standards, costing \$3.4 billion per year (World Bank, 2013). The depreciation rates are chosen to be consistent with typical rates used in economic analysis, and the discount rate is similar to that used by the Stern report, as both problems focus on large public goods with long lasting impacts (Stern and Treasury, 2006).

3.1 Baseline

We begin by defining a baseline case where policymakers do not invest in any mitigation strategy. The expected present value of net benefits is given by the costate variable for the cumulative hazard, as defined in (15),

$$\rho_3 = -e^{rt+y(t)} \max_{n,a} \int_t^{\infty} \left[-n(\tau) - a(\tau) - \psi\big(b(\tau), N(\tau)\big) D\big(A(\tau)\big)\right] e^{-r\tau - y(\tau)} d\tau. \tag{33}$$

We define our baseline by setting $n = a = N = A = 0$, so that the effective hazard rate is the background probability of a pandemic, and damages are equal to their unmitigated level. The expected present value of damages reflects the per-period flow of costs before a pandemic and damages due to the uncontrolled probability of a shift into the ex-post time period, where there is no adaptation capital to mitigate damages.

For this case, the expected present value of damages using the parameters chosen above is equal to \$2.5 trillion, consisting of the expected value of one pandemic causing \$3 trillion in damages at some point in the future, with an uncertain time of occurrence. This value is used to calculate savings from investment in control strategies in the next several sections.

3.2 Solutions on both singular arcs

Solutions where investment is made on both the adaptation and prevention singular arcs are considered singular solutions (partial singular solutions are when investment is only on one singular arc). For our current parameterization, there are two feasible singular solutions that satisfy the optimality conditions. In the first solution the final steady state stock of adaptation capital is equal to \$8.093 billion and the final steady state prevention capital stock is equal to \$505 million. The expected value of damages from a pandemic at the first instant in time is \$52.39 billion and grows over time. Implementing this singular solution leads to a savings of \$2.448 trillion at the first instant. At the steady state, a constant investment is made to maintain stocks of both prevention and adaptation capital. The second candidate singular solution leads to a steady state adaptation capital stock of \$4.297billion and prevention capital stock of \$2.174 billion. The initial expected value of damages due to a pandemic is \$49.15 billion and there are savings from mitigation at this steady state equal to \$2.451 trillion. The greater potential savings from implementing this singular solution mean this solution is the cost minimizing choice. The dynamics of the two capital stocks and investments in those stocks are shown in Figure 2 In both singular solutions large investments are made initially so that capital stocks reach the path. Once on the path the level of investment increases until an inflection point is reached. At this point investment in capital begins to fall until investment just replaces depreciation at the steady state. Additionally, while initially investment is focused on adaptation after a certain point, when the amount of investment begins to fall it becomes more focused in prevention. This is shown in Figure 3, and is a result of our choice of hazard function, where prevention becomes relatively more effective as the probability of an outbreak increases. This is an example of our theoretical results, where an increase in the relative marginal benefit of prevention capital leads to a substitution towards investment in prevention.

The expected value of benefits on both paths were evaluated by deriving a Skiba threshold given by (32) (Mäler et al., 2004; Skiba, 1978). This involved evaluating the expected damages of both paths at every point. The second singular solution (\$4.297 billion, \$2.174 billion) always dominates the first, so that even if a policymaker begins on the optimal path for the first singular solution they would prefer to make investments in the capital required to switch paths.

Figure 4 also shows a comparative dynamic result for an increase in the depreciation rate for both adaptation and prevention capital stocks, which are increased from $\delta = \zeta = .05$ to $\delta = \zeta = .075$. We compare investment in the capital stocks with our baseline case. The first panel in Figure 4 shows that the higher depreciation rate leads to overall lower stocks of both kinds of capital, so that less permanent investments lead to more money being spent just on replacing depreciation. In the second panel there is more investment in adaptation capital even to maintain the lower capital stock. There is simultaneously less investment in prevention capital, leading to the much lower capital stock. This change in the mixture of investment can be seen in the third panel where the mixture of investment shifts towards adaptation along the entire path. The shorter lived we consider capital stocks to be, the more investment should shift towards only adaptation capital. If the increase in depreciation rates is large enough, investment should only be made in adaptation capital.

3.3 Partial singular solution, only prevention capital

Partial singular solutions are defined as when investment is on only one singular arc, and the other capital stock is set equal to zero. When it is only optimal to invest in prevention capital the stock of prevention capital evolves as shown in Figure 2. A large initial investment is required first to build up the capital stock and get on the optimal path of prevention capital. As the risk increases, the flow of investment also increases above the depreciation rate. This causes the stock to grow over time as risk increases, until it reaches its steady state level of $17.535 billion. The expected damages of a pandemic from the initial point where risk is negligible over the entire path is equal to $176.63 billion. This increased expected value of the system leads to initial savings of $2.323 trillion.

3.4 Partial singular solution, only adaptation capital

The time paths of the background probability of outbreak, adaptation capital and investment in adaptation capital are shown in Figure 4. When policymakers decide to only invest in adaptation capital, the expected value of the system is $83.32 billion. This implies savings of $2.416 trillion[6] at the initial point from higher expected GDP into eternity. In this case, the stock of adaptation capital is initially built up with an impulse of investment, and then increases through a flow of investment that is above what is required to replace depreciation,

[6] These savings are significantly higher than investing only in prevention. This is due to the delayed nature of benefits from prevention capital. Adaptation capital impacts the damages from a pandemic, which have a constant unmitigated level. Prevention capital impacts the hazard rate, which is increasing over time but initially low. This means that prevention is initially riskier, because its some of its benefit does not occur until after the hazard rate has risen. (Finnoff et al., 2007).

until it reaches the steady state level of $15.998 billion when the background probability of a pandemic reaches the steady state level of .1.

3.5 Singular vs bang-bang solutions

A brief analysis of the choice between singular and bang-bang solutions is included to consider the optimality of alternative solutions, where investments are made at some maximum rate. For this analysis, investment in either adaptation or prevention is made at the boundary levels $\bar{a}$ and/or $\bar{n}$, which assumed to be $200 billion[7]. When investing in both prevention and adaptation at the same time, it is possible to spend $400 billion total. The expected net benefits for three cases were derived using (18) above. In order to be comparable, they were derived at the same initial risk as the singular paths. The three cases considered include when investments are made in both capital stocks, when investment is only made in prevention, and finally when investment is only made in adaptation. All three cases involve the capital stocks approaching a steady state level and then remaining there indefinitely. These steady state levels are defined as $\bar{A} = \frac{\bar{a}}{\zeta}$ and $\bar{N} = \frac{\bar{n}}{\delta}$, respectively.

The expected value of these paths as well as the singular solutions is included in Table 2. The singular solutions are always preferable to the bang-bang solutions, as they have lower present values of expected damages. Further, it is optimal to invest in both risk mitigation strategies as this solution has the lowest expected present value of damages.. It is possible that policymakers find themselves constrained to a smaller control set, and are able to invest either only domestically (adaptation capital) or only overseas (prevention capital). This can be due to misconceptions of the risk, such as voters believing pandemics only occur in poorer countries and thus investment should be overseas, or political constraints such as voters not wanting their money to be spent on noncitizens. Our results would suggest that being constrained to domestic investment is less damaging. Taking into account political considerations, we do have evidence that even in the double singular solution where investment is heavily skewed to adaptation the small increase in prevention leads to large savings. This is shown by the difference between the inferior singular solution and the adaptation only partial singular

[7] Sensitivity analysis on the boundary was performed, and analytically the results remain the same, while the exact difference between the singular and maximum investment paths may differ slightly.

solution, where moving to the inferior singular solution leads to an additional $3 billion in savings from lower expected damages. If investment is non-recoverable, or the required increase in prevention capital is too costly, it may still be optimal to remain in the partial singular solution and avoid switching costs. Regardless of the choice to switch, investing at a singular solution level is always preferable to investing at some upper bound. A sensitivity analysis of the upper boundary chosen was performed, and this result is not sensitive to the boundary chosen. Intuitively, the cure can be worse than the disease if nations overspend on risk mitigation, as those resources could be put to better use elsewhere.

4. Discussion

There are two main strategies for controlling the risk of pandemics to the United States. It is possible to either invest in prevention to make pandemics less likely to occur, or instead in adaptation to reduce the damages of a pandemic. The relative returns to these two strategies depend on the technologies available to policymakers as well as the level of damages and risk faced by managers. This paper has explored the trade-offs between two methods of risk reduction and their respective cost savings. While a combination of risk reduction strategies can lead to the greatest savings, there is a possibility that governments face political constraints when deciding how to invest in risk mitigation. Given the geographic distinction between adaptation and prevention capital, our work implies a self-interested US policymaker should invest sums in healthcare abroad that are comparable to the investments made domestically. If that policymaker is constrained to investment only in the United States, there are significant savings from extending programs overseas, or shifting existing investment abroad.

The United States and global community already make investments in public and veterinary health. The current world health capital stock can be thought of to consist of public health systems in all developing countries, and has a value of $450 million (World Bank, 2013). Our estimates for optimal investment in this global stock are in line with previous work, which has predicted a cost of $3.4 billion yearly to bring all systems up to minimum international standards (World Bank, 2013). Estimates by the Global Health Risk Framework Commission call for an incremental $4.5 billion per year in spending for what we refer to health capital (Gostin et al., 2016). We also find much larger savings due to investing in these mitigation strategies, because our work takes into account the increasing nature of risk. Repeating our analysis using the WHO estimates of risk (1% annually (Jonas, 2013) of a $3 trillion outbreak) we find expected savings of $1.3 trillion when taking into account the recurring nature of the risk. We also find that when taking into account only the incentives of the United States to invest as a form of prevention, it should be spending less than the $3.4 billion necessary to entirely mitigate the risk, due in part to its ability to mitigate the domestic consequences of a pandemic. This reflects the global public good nature of international health systems, and suggests noncooperative investment as a natural and important extension of this work.

For domestic adaptation capital, the National Institute for Allergies and Infectious Disease has a budget of $4.7 billion in 2016[8], targeted toward research and development for new therapies, vaccines, diagnostic techniques and technologies. This can be thought of as attempts to reduce the background hazard rate. The clearest analogy to prevention capital is the Centers for Disease Control and Prevention (CDC) which had a budget of $699 million in 2016 for combating emerging and zoonotic infectious disease, up from $405 million in 2015, in part in response to the threat of Ebola. The response to Ebola has been criticized for being slow and inefficient, allowing the outbreak to persist and expand (Heymann et al., 2015), although it is likely that the existing institutions reduced the impact of the disease relative to the case where each nation was left to fend for itself. The CDC also has $448 for Global Health, and $1.382 billion for Public Health Preparedness and Response in 2016[9]. This capital is split between domestic and international activities, but is of the same magnitude as our estimates. Our results suggest that these investments should be increased, and that there is room for larger investments both domestically and internationally, however relatively more of increased investment should be made domestically, when ignoring the benefits accruing overseas.

We also find that when investments are made at their optimal level, the expected damages from a pandemic rival those from the 'mild' 2009 influenza pandemic. In our optimally controlled singular solution, expected damages from a pandemic are $49.15 billion, similar in

[8] https://officeofbudget.od.nih.gov/pdfs/FY17/31-Overview.pdf
[9] http://www.hhs.gov/about/budget/budget-in-brief/cdc/index.html#a4

magnitude to the actual damages from the 2009 pandemic of ~$72 billion. This suggests that large prevention and adaptation capital stocks, built in response to the 2003 SARS outbreak, helped mitigate the 2009 pandemic. Optimal investment in both prevention and adaptation capital stocks could lead to even more savings in the future. These investments should only increase as the risk of pandemics increases due to globalization, urbanization, and human behavior as well as climate change. Additionally, these investments should be maintained between outbreaks to avoid allowing outbreaks to become out of control.

While our work is focused on influenza, there has been a recent influx of vector-borne viral diseases to the Western Hemisphere, with the most recent being the Zika virus (Fauci and Morens, 2016). Zika has now circled the globe and been tied to terrible health consequences, causing it to be declared a Public Health Emergency in February 2016 (Fauci and Morens, 2016; WHO, 2016). The WHO response has included work on coordination, surveillance, caring for infected individuals, vector control and community engagement, as well as research on the outbreak – the functions of our prevention capital stock (or adaptation, if Zika reaches the United States). Existing capital and coordination efforts allowed for this rapid response, even though it required the WHO and World Bank Group to provide a temporary flow of funding (WHO, 2016). These efforts are vital, as there are no Zika vaccines in advanced development, and epidemics appear randomly so that preemptive vaccination makes little sense, and reactive vaccination is too slow (Fauci and Morens, 2016). It is likely that Zika virus will not be as damaging as it otherwise would have been, given the international response and existing organizations such as GOARN and the WHO that are available for international coordination and response. The seemingly increasing pace of emergence for these diseases implies that the background risk to any one nation is also rising. Our analysis suggests that nations should respond by increasing investment in both their domestic capital and their efforts to fight disease elsewhere. It also suggests that if prevention is becoming relatively more efficient, the relative balance of prevention and adaptation should shift towards international efforts.

Investments in risk mitigation, whether domestic or international, are insurance against the threat of pandemics to the United States, omitting any other benefits to other nations. These investments in the two strategies must change as the risk increases, with the overall capital stock rising in response to the risk, and the mixture of strategies also responding. This analysis provides insight into the role of an increasing risk of outbreak, and the anthropogenic forces that drive the risk of disease outbreak which must be considered when determining the optimal mix of mitigation strategies. Even ignoring humanitarian benefits, international public health aid provides insurance to the United States against the threat of pandemics.

Acknowledgements: This publication was made possible by grant number 1R01GM100471-01 from the National Institute of General Medical Sciences (NIGMS) at the National Institutes of Health and NOAA CSCOR Grant no. NA09NOS4780192. Its contents are solely the responsibility of the authors and do not necessarily represent the official views of NIGMS or NOAA CSCOR.

5. References

Bartlett, J.G., Borio, L., 2008. The current status of planning for pandemic influenza and implications for health care planning in the United States. Clin. Infect. Dis. 46, 919–925.

Berry, K., Finnoff, D., Horan, R.D., Shogren, J.F., 2015. Managing the endogenous risk of disease outbreaks with non-constant background risk. J. Econ. Dyn. Control 51, 166–179. doi:10.1016/j.jedc.2014.09.014

CDC, 2010. The 2009 H1N1 Pandemic: Summary Highlights, April 2009-April 2010 [WWW Document]. URL http://www.cdc.gov/h1n1flu/cdcresponse.htm

Cohen, M.L., 2000. Changing patterns of infectious disease. Nature 406, 762–767. doi:10.1038/35021206

Dawood, F.S., Iuliano, A.D., Reed, C., Meltzer, M.I., Shay, D.K., Cheng, P.-Y., Bandaranayake, D., Breiman, R.F., Brooks, W.A., Buchy, P., 2012. Estimated global mortality associated with the first 12 months of 2009 pandemic influenza A H1N1 virus circulation: a modelling study. Lancet Infect. Dis. 12, 687–695.

De Zeeuw, A., Zemel, A., 2012. Regime shifts and uncertainty in pollution control. J. Econ. Dyn. Control 36, 939–950.

Ehrlich, I., Becker, G.S., 1972. Market insurance, self-insurance, and self-protection. J. Polit. Econ. 80, 623–648.

Fauci, A.S., Morens, D.M., 2016. Zika Virus in the Americas—Yet Another Arbovirus Threat. N. Engl. J. Med.

Fineberg, H. V, 2014. Pandemic Preparedness and Response — Lessons from the H1N1 Influenza of 2009. N. Engl. J. Med. 370, 1335–1342. doi:10.1056/NEJMra1208802

Finnoff, D., Shogren, J.F., Leung, B., Lodge, D., 2007. Take a risk: preferring prevention over control of biological invaders. Ecol. Econ. 62, 216–222. doi:10.1016/j.ecolecon.2006.03.025

Girard, M.P., Tam, J.S., Assossou, O.M., Kieny, M.P., 2010. The 2009 A (H1N1) influenza virus pandemic: A review. Vaccine 28, 4895–4902.

Gostin, L.O., Mundaca-Shah, C.C., Kelley, P.W., 2016. Neglected Dimensions of Global Security: The Global Health Risk Framework Commission. JAMA 315, 1451–1452.

Heymann, D.L., Chen, L., Takemi, K., Fidler, D.P., Tappero, J.W., Thomas, M.J., Kenyon, T.A., Frieden, T.R., Yach, D., Nishtar, S., 2015. Global health security: the wider lessons from the west African Ebola virus disease epidemic. Lancet 385, 1884–1901.

Hufnagel, L., Brockmann, D., Geisel, T., 2004. Forecast and control of epidemics in a globalized world. Proc. Natl. Acad. Sci. U. S. A. 101, 15124–15129.

Jonas, O.B., 2013. Pandemic risk.

Kane, S., Shogren, J.F., 2000. Linking adaptation and mitigation in climate change policy. Clim. Change 45, 75–102.

Keegan, R., Dabbagh, A., Strebel, P.M., Cochi, S.L., 2011. Comparing measles with previous eradication programs: enabling and constraining factors. J. Infect. Dis. 204, S54–S61.

Lindgren, E., Andersson, Y., Suk, J.E., Sudre, B., Semenza, J.C., 2012. Monitoring EU emerging infectious disease risk due to climate change. Science (80-.). 336, 418–

419.

Mäler, K.-G., Xepapadeas, A., De Zeeuw, A., 2004. The economics of shallow lakes. Springer.

McKibbin, W.J., Sidorenko, A.A., 2007. The Global Costs of an Influenza Pandemic. Milken Inst. Rev. 18–27.

McMichael, A.J., 2004. Environmental and social influences on emerging infectious diseases: past, present and future. Philos. Trans. R. Soc. B Biol. Sci. 359, 1049–1058.

Meltzer, M.I., Cox, N.J., Fukuda, K., 1999. The economic impact of pandemic influenza in the United States: priorities for intervention. Emerg. Infect. Dis. 5, 659–671.

Morse, S.S., 2007. The US pandemic influenza implementation plan at six months. Nat. Med. 13, 681–684.

Morse, S.S., Mazet, J.A.K., Woolhouse, M., Parrish, C.R., Carroll, D., Karesh, W.B., Zambrana-Torrelio, C., Lipkin, W.I., Daszak, P., 2012. Prediction and prevention of the next pandemic zoonosis. Lancet 380, 1956–1965. doi:10.1016/S0140-6736(12)61684-5

Osterholm, M.T., 2005. Preparing for the next pandemic. N. Engl. J. Med. 352, 1839–1842.

Pontryagin, L.S., Boltyanskii, V.G., 1962. Mathematical Theory of Optimal Processes. John Wiley & Sons Inc., New York.

Reed, W.J., Heras, H.E., 1992. The conservation and exploitation of vulnerable resources. Bull. Math. Biol. 54, 185–207. doi:10.1016/50092-8240(05)80022-9

Ross, A.G.P., Crowe, S.M., Tyndall, M.W., 2015. Planning for the Next Global Pandemic. Int. J. Infect. Dis. 38, 89–94.

Shogren, J.F., Crocker, T.D., 1999. Risk and its consequences. J. Environ. Econ. Manage. 37, 44–51.

Shrestha, S.S., Swerdlow, D.L., Borse, R.H., Prabhu, V.S., Finelli, L., Atkins, C.Y., Owusu-Edusei, K., Bell, B., Mead, P.S., Biggerstaff, M., 2011. Estimating the burden of 2009 pandemic influenza A (H1N1) in the United States (April 2009–April

2010). Clin. Infect. Dis. 52, S75–S82.

Skiba, A.K., 1978. Optimal growth with a convex-concave production function. Econom. J. Econom. Soc. 527–539.

Stern, N.H., Treasury, H.M., 2006. Stern Review: The economics of climate change. HM treasury London.

Tsur, Y., Withagen, C., 2013. Preparing for catastrophic climate change. J. Econ. 110, 225–239.

WHO, 2016. Zika virus microcephaly and Guillain-Barré Syndrome Situation Report 26 February 2016.

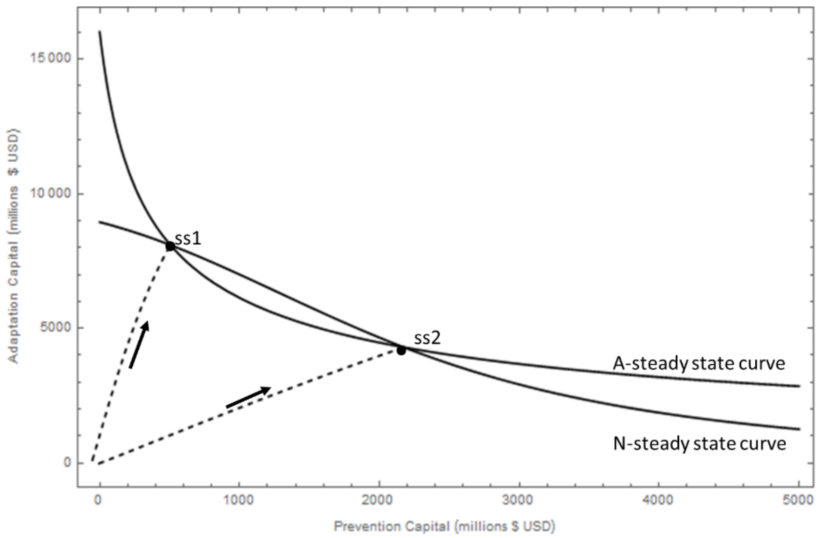
WHO, 2011. Pandemic influenza A(H1N1) - draft donor report.

World Bank, 2013. World development report 2014. Risk and Opportunity. Managing Risk for Development. Washington, DC.

Zemel, A., 2015. Adaptation, mitigation and risk: An analytic approach. J. Econ. Dyn. Control 51, 133–147. doi:10.1016/j.jedc.2014.10.001

Figure Captions

Figure 1. Panel (a) shows the steady state curves for combinations of adaptation and prevention capital (solid curves). The two possible optimal paths are also shown to the candidate singular solution steady states, ss1 and ss2 (dashed lines). It is important to note that on the optimal paths the probability of a pandemic outbreak is increasing, while on the steady state curves the probability is held constant at the steady state level. The steady states when investing only in one capital stock (setting the alternative to zero) are shown in panel (b).



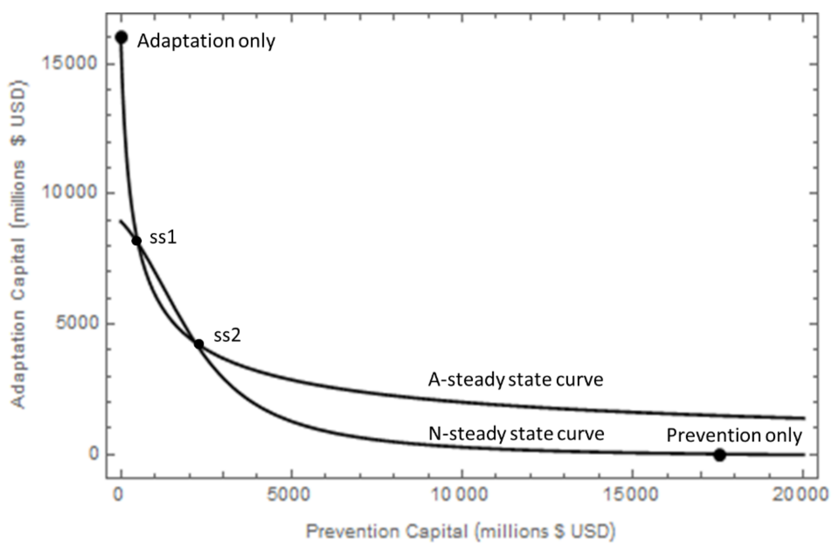


Figure 2. Prevention capital is the dashed line, adaptation capital is the solid line. Thick lines are the dominant singular solution (labelled ss2), thin lines are the inferior singular solution (labelled ss1). First figure is stock, both increasing to the steady state. The second figure is investment over risk, increasing to inflection point then following. Third figure is relative mix of adaptation and prevention. Starting at lower left point, increasing and then falling back with slightly more prevention.

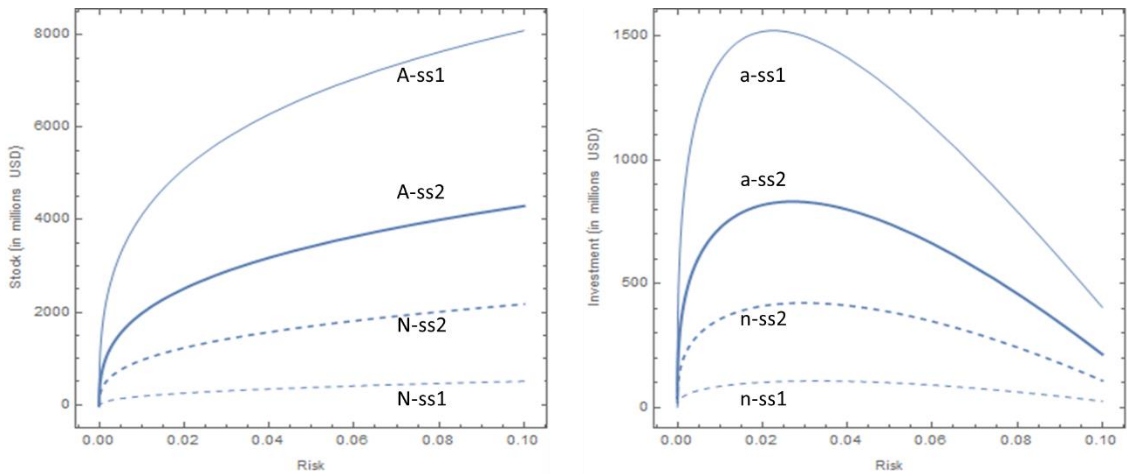


Figure 3. Figure 3 shows the relative mix of investment in adaptation and prevention capital over the time path to either steady state ss1 or ss2.

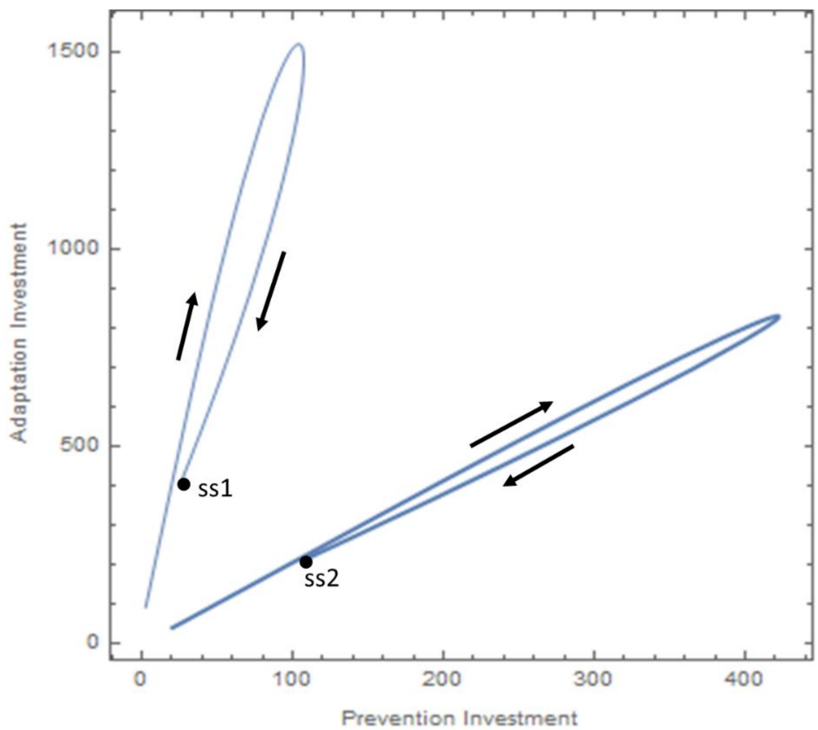


Figure 4. Figure 4 shows the optimal paths of investment and capital stocks, as well as the relative mix of investment in adaptation and prevention capital over the time path. Thin lines are paths with higher depreciation rates.

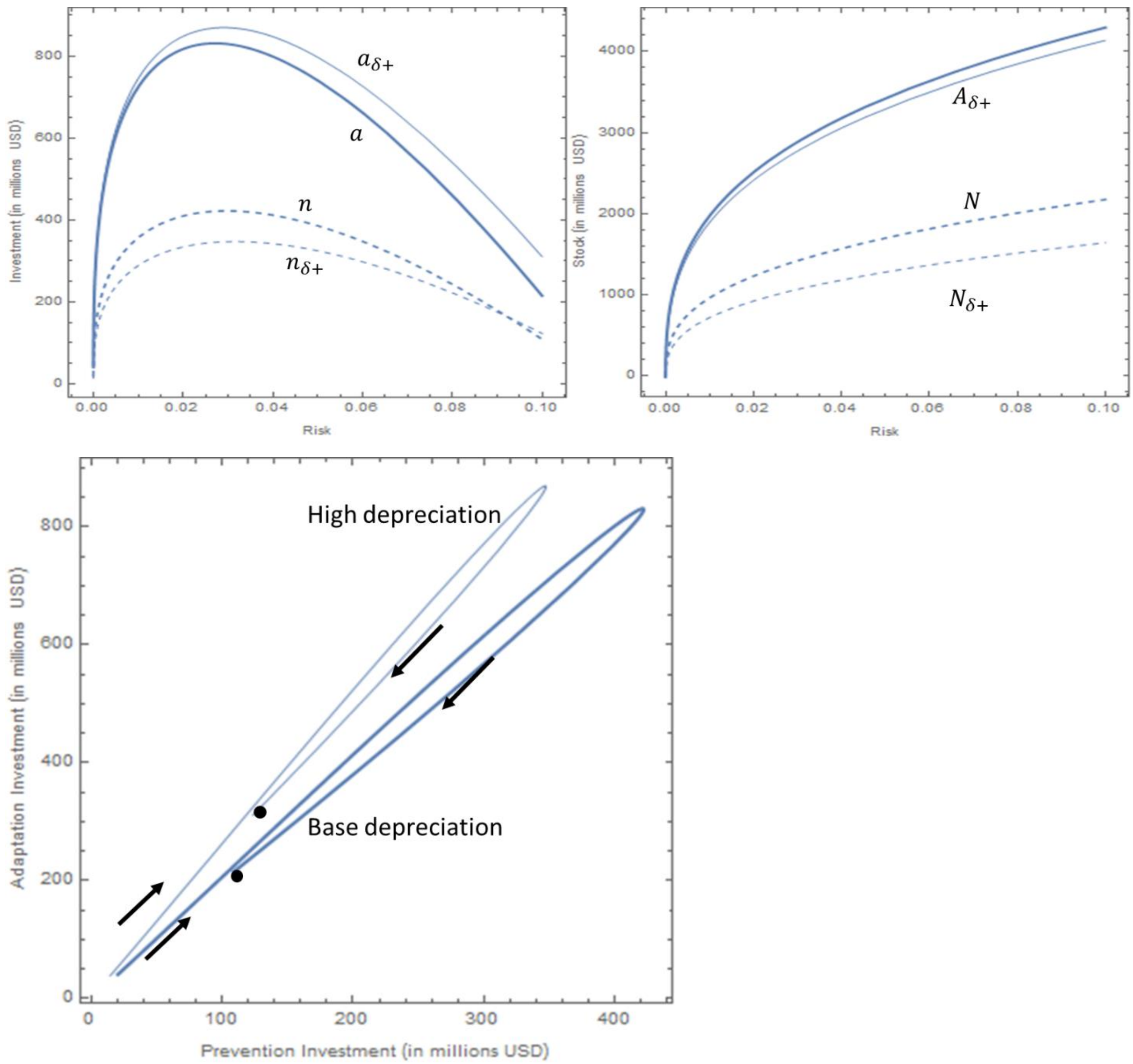


Table 1. Parameters used in numerical exercises.

Parameter	Definition	Value or functional form
c_n	Effectiveness of prevention capital	.015
c_a	Effectiveness of adaptation capital	.0072
$\frac{D(A)}{r}$	Value of optimally controlled ex-post system	$-\frac{\widehat{D}}{1+c_a N(t)}$
$\psi\big(N(t), b(t)\big)$	Hazard function	$\frac{b(t)}{1+c_n N(t)}$
δ	Depreciation rate of prevention capital	.05
ζ	Depreciation rate of adaptation capital	.05
$\widehat{D}$	Damages without adaptation capital	\$3,000,000 (millions)
r	Discount rate	.01
$\bar{b}$	Steady state probability of outbreak	.1
$\dot{b}(t)$	State equation for steady state probability of outbreak	$b(t) * \frac{1-b(t)}{\bar{b}}$

Table 2. Initial welfare savings from investing in mitigation strategies, steady state capital stocks and per period expenditure in the steady state. By investing in a combination of prevention and adaptation capital, it is possible to save a greater amount with a significantly smaller level of total investment. For welfare analysis it is assumed when one control is set at an extreme and the other at the singular arc, that extreme is no capital stock.

Strategy	**Expected Damages**	**Initial savings**	**Steady state capital stock**
No control	\$2.5 trillion	0	0
Prevention only	\$176.63 billion	\$2.323 trillion	\$17.535 billion
Adaptation only	\$83.32 billion	\$2.416 trillion	\$15.998 billion
Both Ss1	\$52.39 billion	\$2.448 trillion	\$8.093 billion Adaptation \$505 million Prevention
Both Ss2	\$49.15 billion	\$2.451 trillion	\$4.297 billion Adaptation \$2.174 billion Prevention
Boundary Prevention	\$20.1 trillion	N/A	\$400 billion
Boundary Adaptation	\$3.76 trillion	N/A	\$400 billion
Boundary Both	\$4.03 trillion	N/A	\$400 billion Adaptation \$400 billion Prevention