Estimation of size-transition matrices with and without moult probability for Alaska golden king crab using tag–recapture data

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Abstract

Size-structured population dynamics models are used for stock assessments of hard to age invertebrate species, such as crabs, and size-transition matrices play an important role in modeling growth in those models. Crabs grow by moulting and then incrementing in size. Therefore, the size-transition matrix should ideally contain sub-models for the probability of moult and the growth increment. Size-transition matrices were estimated in an integrated model setting, which included tagging data. Various models, including those that explicitly model moult probability and that include the moult implicitly in the size-transition model, were applied to data for golden king crab (*Lithodes aequispinus*) in the eastern Aleutian Islands region. Several diagnostic statistics (e.g., covariance matrix, likelihood, AIC, mean growth increment metric, sensitivity of estimates of mature male biomass, and the fits to the tag recapture, catch-per-unit effort, and length frequency data) were used to investigate the implications of the way growth was modelled. Overall, the fit of the integrated model that included the moult probability sub-model was better than that of the model that did not include this sub-model. However, the trends in key stock assessment outputs did not differ markedly between approaches for modelling growth even though the estimates of the size-transition matrices differed.

Keywords: Alaska, golden king crab, growth, size-transition matrix

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Highlights

- **Crab growth was modelled with and without the moult probability sub-model in the size-transition model.**
- **The estimates of the size-transition matrices differed among approaches.**
- **Overall, the fit of the integrated model that included the moult probability sub-model was better than which did not include this sub-model.**
- **Key stock assessment outputs did not differ markedly between approaches for modelling growth.**

1. Introduction

Crustaceans are hard to age, and size-structured population dynamics models have consequently often been used for stock assessment purposes for these species (e.g., Zheng et al., 1995; Chen et al., 2005; Haist et al., 2009; Punt et al., 2013, this volume). In age- or stage-structured models, the population dynamics process is described by cohorts moving through different ages or stages over time. On the other hand, the process of ageing in size-structured models is described by growth in size through a suitable growth function, and the population dynamics process is described by cohorts moving through different size-classes over time (Punt et al., this volume).

The size-transition matrix, which governs the probability of animals moving from one size class to the others, plays an important role in size-structured models. The moult increment and the time between moult events have been modelled separately and combined to compute size-transition matrices for crustacean growth (e.g., Zheng et al., 1995; McGarvey and Feenstra, 2001). The size-transition matrix has also been estimated disregarding moulting (i.e., assuming that the animals moult with certainty at each time step or by lumping the moult probability and moult increment processes together) (e.g., Punt et al., 1997; Haist et al., 2009). The moult increment has often been modeled assuming a linear increment from pre- to post-moult size (e.g., Punt et al., 1997; Chen et al., 2003; Montgomery et al., 2009; Hillary, 2011; Zheng and Siddeek, 2014). The variability in moult increment has been accounted for by assuming a probability distribution, such as the normal or a gamma (e.g., Zheng et al., 1995; Punt et al., 1997). The time period between the two consecutive moults has been modeled using a variety of functions (Chang et al., 2012); for example, a logistic function for Bristol Bay red king crab (*Paralithodes camtschaticus*) (Zheng et al., 1995).

Tag release-recapture and size frequency data have been used to estimate the parameters of size-transition matrices, either within or outside stock assessment models (Fournier et al., 1990; Punt et al., 1997; Zheng et al., 1998; Lloyd-Jones et al., 2014).

Golden king crab, *Lithodes aequispinus*, support a valuable commercial fishery in the Aleutian Islands, Alaska. This stock has been managed as two sub-stocks; the eastern and the western Aleutian

Islands sub-stocks. The fisheries on the two sub-stocks are managed with a constant annual total allowable catch. There is no annual fishery-independent stock abundance survey, and the status of the fishery is assessed largely based on catch, catch-per-unit-effort, and catch size-composition data (Pengilly, 2014). A male-only size-structured assessment model, with size measured by carapace length, is under development for the two golden king crab sub-stocks in the Aleutian Islands (Siddeek et al., 2014). Several tagging experiments have been conducted in the eastern Aleutian Islands region (e.g., Watson, et. al., 2002).

This paper examined approaches for estimating size-transition matrices for the eastern Aleutian Islands sub-stock. Specifically, we investigated the effect of (1) including or (2) ignoring the moult probability sub-model when developing the size-transition matrix on stock assessment-related outputs (henceforth referred to as approaches 1 and 2). We considered the case in which the estimation of the size-transition matrix was integrated into the stock assessment, as is becoming common practice (Punt et al., this volume). A number of diagnostic statistics (e.g., covariance matrix, likelihood, AIC, mean growth increment metric, sensitivity of estimates of mature male biomass, and the fits to the tag recapture, catch-per-unit effort, and length frequency data) were used to investigate the implications of the choice between the two size-transition matrix estimators.

2. Materials and methods

2.1 Data sources

The data sets included in analyses for the eastern Aleutian Islands golden king crab fishery are: 1) commercial pot fishery retained catch (1985/86 to 2012/13 fishing seasons), 2) total catch estimated from observer samples and fish ticket landings data (1990/91 to 2012/13), 3) groundfish fishery bycatch (1995/96 to 2012/13), 4) retained catch length compositions (1985/86 to 2012/13), 5) total catch length compositions (1990/91 to 2012/13), 6) groundfish bycatch length compositions (1990/91 to 2012/13), 7) standardized catch-per-unit effort (CPUE) index from observer samples (1995/96 to 2012/13), and 8) tag release-recapture lengths from releases during 1991, 1997, 2000, 2003, and 2006.

The Alaska Department of Fish and Game (ADF&G) uses fish tickets to record details of a fishing trip such as fishing area, dates of fishing, number of trap hauls made, and total crab catch by species. Since the entire landed catch is recorded on the fish tickets, the annual retained catch by species was estimated by adding the fish ticket landings records for the entire fishing season. Onboard observers count and measure all crabs caught in the sampled pots and categorize the catch as females, sublegal males, retained legal males, and non-retained legal males. Annual mean nominal CPUE of retained and total crabs were estimated considering all sampled pots within each season. The annual total (landed and discarded) catch in the fishery was estimated as the product of the observer nominal total CPUE and the total effort (number of pot lifts).

In relation to the tagging data, rectangular, king crab pots were used to capture crabs for tagging in all experiments, with the exception of the 1991 experiment where smaller, conical pots were used. The tags were released during summer (July –September) before the fishery started. Location, date, and fishing depth were recorded for each pot retrieved. Upon pot retrieval, the carapace lengths (CL) of crabs were measured to the nearest millimeter and shell condition recorded. Isthmus-loop (“spaghetti”) tags were used to tag crabs (Gray, 1965) larger than 90 mm CL, and tagged crabs were released on or adjacent to the capture location. The majority of tag recaptures were obtained from the fishers during the commercial pot fishery. The tagging data used in the analyses comprised CL-at-release, CL-at-recapture, and time-at-liberty. Table 1 provides a brief summary of the tag release data and Table 2 provides the number of recaptures grouped by time (in years)-at-liberty. The tagging records were restricted to male golden king crab releases in the size-range 101 – 185 mm CL given the size-range included in the population dynamics model. There were 27,131 tagged crab releases in this size-range with a 6.33% return rate.

Paul and Paul (2000) observed captive crab in the lab moulting every month of the year with the highest frequencies moulting during May–October. We assumed that tag returns that were at-liberty for at least 9 months would likely have gone through the first moult cycle after release and disregarded recaptures with growth increment < -5 mm CL (an arbitrary cut off point) because they were assumed to be errors. We assumed that growth increments between 0 to -5mm to be measurement errors and set those increments to 0 (Table 2). Less than 1% of the data were discarded.

2.2 Size transition matrix

Appendix A outlines key aspects of the stock assessment (see Siddeek et al. [2014] for full details). The assessment is based on the 'integrated' approach to stock assessment; with parameter estimation implemented using AD Model Builder (Fournier et al., 2012). The structure of the size-transition matrix for moult increment is described in Punt et al. (this volume). It is a square nXn matrix subject to the condition $\sum_{j=1}^{n} X_{i,j} = 1$ where n is the number of size-classes (each of 5 mm CL) and $X_{i,j}$ is the probability of growing from size-class i to each size-class j, subject to the constraint that $X_{i,j} = 0$ for $j < i$. We assumed size transition to occur from July 1 in year t to July 1 in year t+1. Following Zheng et al. (1995), we used a linear model to predict growth increment from pre-moult length. A variety of probability functions, such as normal, Student's t, and gamma can be used to describe the distribution of observed growth increments (Sullivan et al., 1990; Hillary, 2011). We chose the normal distribution with a constant standard deviation (σ) (Hamazaki and Zheng, 2014). Independent analysis of golden king crab tag release-recapture data indicated no systematic increase in the spread of moult increments by size (Siddeek et al., 2005; Watson et al., 2002). Hence the assumption of constant standard deviation is plausible. The two model variants considered in this paper differ in terms of how the matrix **X** is developed. Both variants can be derived from the general formulation:

$$X_{i,j} = \begin{cases} 0 & if\ j < i \\ m_i P_{i,j} + (1 - m_i) & if\ j = i \\ m_i P_{i,j} & if\ j > i \end{cases} \tag{1}$$

where

$$P_{i,j} = \begin{cases} \int_{-\infty}^{j_2 - \tau_i} N\,(x\,|\mu_i, \sigma^2)\,dx & if\ \ j = i \\ \int_{j_1 - \tau_i}^{j_2 - \tau_i} N\,(x\,|\mu_i, \sigma^2)\,dx & if\ \ i < j < n\ , \\ \int_{j_1 - \tau_i}^{\infty} N\,(x\,|\mu_i, \sigma^2)\,dx & if\ \ \ i = n \end{cases}$$

$N(x|\mu_i, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(\frac{x-\mu_i}{\sqrt{2}\sigma})^2}$, and

μ_i is the mean growth increment for crabs in size-class *i*:

$$\mu_i = a + b * \tau_i. \qquad (2)$$

a, *b*, and σ are estimable parameters, j_1 and j_2 are the lower and upper limits of the receiving length-class *j* (in mm CL), and τ_i is the mid-point of the contributing length-class *i*. The quantity m_i is the moult probability for size-class *i*, modeled either as 1 for all size-classes or as a logistic function of size (Zheng et al., 1995):

$$m_i = \frac{1}{1 + e^{c(\tau_i - d)}} \qquad (3)$$

where *c* and *d* are estimable parameters.

Thus, approach 1 estimates three parameters (*a*, *b*, and σ) while approach 2 estimates five parameters (*a*, *b*, σ, *c*, and *d*). There was a high correlation between the estimates of parameters *a* and *b* (correlation coefficient 0.99). Therefore, we re-parameterized each length-class as $(\frac{L_i - \bar{L}}{\bar{L}})$, where L_i = mid CL of the i-th length-class and $\bar{L}$ = the mean length of the size range considered in the model, and re-fitted the model, which reduced the correlation coefficient between *a* and *b* to 0.0942 for approach 1 and to 0.488 for approach 2. Re-parameterization also reduced the correlation between moult probability parameters *c* and *d*, and σ to less than 0.5. Parameters *a*, *c*, and *d* were log-transformed to keep them positive.

2.3 Diagnostic measures

2.3.1 Mean growth increment metric

The size-transition matrix is an output from the assessment and its distributional properties are consequently unknown. Therefore, traditional chi-square statistics cannot be used to compare the size-transition matrices from the two approaches. Jafry and Schuermann (2003) discussed the use of L^2 (Euclidean) cell-by-cell distance metric for matrix comparison. Although this approach is likely to detect the differences among matrix elements, it does not identify the differences in the spread of the non-zero matrix elements, which is crucial to understand whether the estimated size-transition matrices are biologically realistic. Therefore, we developed a mean growth increment metric, $\overline{G_k}$, as follows:

$$\overline{G_k} = \frac{\sum_{i=1}^{n} \frac{\sum_{j=i}^{n} g_i X_{k,ij}}{n-i}}{n} \qquad (4)$$

where g_i = 0, 5, 10, …. mm CL (cumulative bin size increment), $X_{k,i,j}$ is the i ,j -th element of the *k*-th matrix (*k* = 1 and 2 for the two approaches). We then computed the difference between $\overline{G_k}$ for the two approaches, $D = \overline{G_1} - \overline{G_2}$

Although *D* is a simple metric difference, there is no statistical test to determine its significance. We consequently developed a test based on a randomization test. This involved generating a limited number (100) of bootstrap *D* values by resampling individual tag-recaptures with replacement while keeping the rest of the data sets the same and fitting the assessment model for each bootstrap data set. We computed the 2.5 and 97.5 percentile limits of the bootstrapped *D* values using Efron's method (Efron and Tibshirani, 1986) to test whether *D* was significantly different from 0.

2.3.2 Observed vs. predicted plots of derived variables

For each approach, we compared the observed vs predicted number of tag recaptures by size-class, the observed vs predicted annual CPUE index, and the observed vs predicted retained catch size-compositions (see Equations A10, A5, and A6). The trend in mature male biomass (MMB; Equation A11) and its terminal year value are important stock assessment outputs for setting annual total allowable catch. We therefore compared the trends in MMB for the two approaches.

2.3.3 Comparison of the overall fit using the likelihood and AIC values

We compared the fits for the two approaches using AIC:

$$AIC = -\ln(MLH) + 2 * p \quad (5)$$

where *MLH* is the likelihood corresponding to the maximum likelihood estimates, and *p* is the number of estimated parameters (Burnham and Anderson, 2002).

3. Results

3.1 Diagnostic measures

The highest correlation among the parameters (0.49; ln(*a*) and *b* for approach 2) was fairly low (Table 3). Approach 2 had a much lower negative log likelihood for the tagging data and AIC than approach 1 (Table 4), owing primarily to an improved ability to fit the data. The two approaches replicate the tag-recapture and catch-rate indices about equally well (Figure 1). The predicted trends in mature male biomass [i.e., biomass of male crabs > 50% maturity length of 120.8 mm CL (Otto and Cummiskey, 1985)] were very similar for the two approaches. However, the confidence limits of the time series of mature male biomass differed slightly (Figure 2). There were also slight differences in some years in terms of predicted retained catch length compositions between the two approaches (Figure 3). Nevertheless, overall, the time series of length composition fits were satisfactory for the two approaches.

Table 5 shows clear differences between the estimated size-transition matrices from the two approaches. The diagonal elements were higher for approach 2 than for approach 1 as a result of the additional non-moulting probability in the former. The spread of the growth increment distribution was wider for approach 1 than that for approach 2. The mean growth increment metric difference (*D*) indicated a significant positive difference (best fit value 0.0849, 95% bootstrap CI 0.0165-0.1719) between the two size-transition matrices.

4. Discussion

We developed two ways to represent size-transition matrices [without (approach 1) and with (approach 2) the moult probability sub-model], and applied these to data for eastern Aleutian Islands golden king crab to investigate their effects on fit diagnostics and management-related model outputs. AIC favoured the estimator with the moult probability sub-model. The size-transition matrix elements were biologically meaningful when the estimator included the moult probability sub-model. However, the trends in management-related model outputs did not differ noticeably among approaches. This may have been due to the confounding with other parameters (e.g., selectivity, catchability).

We followed an integrated modeling approach to estimate the size-transition matrices. Whether the size-transition matrix should be estimated within or outside the stock assessment model is debatable. Chen et al. (2005) estimated growth matrices for the American lobster (*Homarus americanus)* outside the assessment model while Zheng et al. (1998) did so for Norton Sound red king crab. There are advantages and disadvantages to estimating the size-transition matrix within an assessment model. The advantages include: (1) when the tag recapture data are incomplete (i.e., not covering all length-classes) other information such as size composition data could inform matrix elements, and (2) the probability of recapturing a tagged animal depends on fishery selectivity and it is not straightforward to account for this without including the tagging data in the assessment model. This is because fishery selectivity is estimated simultaneously with the size-transition matrix if the entire analysis is 'integrated'. One notable potential disadvantage associated with estimating the size-transition matrix in an assessment is that other parameters in the model (e.g., natural mortality, catchability, selectivity) could be confounded with the parameters of the size-transition matrix.

Unless tag-recapture data were separated by moult and non-moult stages, it would be difficult to estimate the moult probability unequivocally and the moult probability parameters would likely to be confounded with the growth increment parameters. Unlike red king crab, golden king crab may have an asynchronous moult cycle (Otto and Cummiskey, 1985; Blau and Pengilly, 1994; Watson et al., 2002). Consequently, it would be difficult to separate the tag recaptures into moult and non-moult stages by year. Because of this difficulty, we attempted to reduce the influence of moult probability parameters on growth increment parameters by reparametrizing the growth increment model by standardizing the length-classes. This reduced the correlation among the growth increment and moult probability model parameters.

We considered a fixed length-class interval (5 mm CL) as is common practice in other Alaska crab size-structured stock assessments (e.g., Zheng et al., 1995) and did not explore any other size-class widths. This could be a future research topic. We also followed Punt et al.'s (1997) approach of computing the distribution of growth increment starting from one point (i.e., middle length of a length-class) to other size-classes rather than one size-class to other size-classes (Hillary, 2011). We considered this difference to be minor and did not explore it further in this paper.

One criticism of the current analysis may be that the size-transition matrix was treated as time invariant. Growth patterns can change over time. However, limited tagging data and the complex moulting cycle of golden king crab compelled us to follow a simple approach. We also did not investigate the tagging effect on crab growth. Tagging may temporarily retard growth as found in shrimp (Penn, 1975) and lobster (Punt, personal observation).

5. Acknowledgments

We are grateful to CAPAM for inviting us to present this paper at their Growth Workshop. We thank Doug Pengilly, Heather Fitch, Leland Hulbert, William Gaeuman, and Robert Foy for providing various data for this study and helpful review. We also thank Steve Martel, Mark Maunder, Johnoel Ancheta, and Dave Fournier for improving earlier versions of the model and ADMB code. Comments made by two anonymous reviewers and the editors also helped to improve the manuscript. This work is partially funded by NOAA Award NA14NMF4370119. The findings and conclusions in this paper are those of the author(s) and do not necessarily represent the views of the National Marine Fisheries Service (NMFS), NOAA. This manuscript is contribution PP-278 of the ADF&G, Commercial Fisheries Division, Juneau, Alaska.

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Appendix: Key equations include the stock assessment model for golden king crab

Basic population dynamics

The annual male abundances by size are modeled using the equation:

$$N_{t+1,j} = \sum_{i=1}^{j}[N_{t,i}e^{-M} - (\hat{C}_{t,i} + \widehat{D}_{t,i} + \widehat{Tr}_{t,i})e^{(y_t-1)M}]X_{i,j} + R_{t+1,j} \quad \text{(A1)}$$

where $N_{t,i}$ is the number of male crab in size-class *i* on 1 July (the start of biological year) of year *t*; $\hat{C}_{t,i}$, $\hat{D}_{t,i}$, and $\widehat{Tr}_{t,i}$ are respectively the model-predicted fishery retained, pot fishery discard dead, and groundfish fishery discard dead catches in size-class *i* during year *t*; $\widehat{D}_{t,i}$ is estimated from the total ($\hat{T}_{t,i}$) and the retained ($\hat{C}_{t,i}$) catch (Eqn A2c). $X_{i,j}$ is the size-transition matrix; y_t is elapsed time period from 1 July to the mid–point of the fishery during year *t*; $R_{t,j}$ is the number of recruits; and *M* is instantaneous rate of natural mortality.

The catches are predicted using the equations

$$\hat{T}_{t,j} = \frac{F_t s_{t,j}^T}{Z_{t,j}} N_{t,j}e^{-y_t M}(1 - e^{-Z_{t,j}}) \quad \text{(A2a)}$$

$$\hat{C}_{t,j} = \frac{F_t s_{t,j}^T s_{t,j}^r}{Z_{t,j}} N_{t,j}e^{-y_t M}\left(1 - e^{-Z_{t,j}}\right) \quad \text{(A2b)}$$

$$\widehat{D}_{t,j} = 0.2(\hat{T}_{t,j} - \hat{C}_{t,j}) \quad \text{(A2c)}$$

$$\widehat{Tr}_{t,j} = 0.8\frac{F_t^{Tr} s_j^{Tr}}{Z_{t,j}} N_{t,j}e^{-y_t M}(1 - e^{-Z_{t,j}}) \quad \text{(A2d)}$$

where $Z_{t,j}$ is total fishery-related mortality on animals in size-class *j* during year *t*:

$$Z_{t,j} = F_t s_{t,j}^T + F_t^{Tr} s_j^{Tr} \quad \text{(A3)}$$

F_t is the full selection fishing mortality by the pot fishery during year *t*, F_t^{Tr} is the full selection fishing mortality in the trawl fishery during year *t*, $s_{t,j}^T$ is the total selectivity for animals in length-class *j* by the pot fishery during year t, s_j^{Tr} is the selectivity for animals in length-class j by the trawl fishery, and $s_{t,j}^{r}$ is the probability of retention for animals in length-class j by the pot fishery during year t.

Selectivity and retention

Selectivity and retention are both assumed to be logistic functions of length. Selectivity depends on the two fishing periods (1985/86–2004/05 and 2005/06–2012/13) for the pot fishery:

$$S_{k,i} = \frac{1}{1+e^{\left[-\ln(19)\frac{\tau_i-\theta_{k,50}}{\theta_{k,95}-\theta_{k,50}}\right]}} \qquad \text{(A4)}$$

where $\theta_{k,95}$ and $\theta_{k,50}$ are the parameters of the selectivity / retention pattern for period k.

Catch-per-unit-effort (CPUE)

The retained catch CPUE is predicted using the equation

$$\widehat{CPUE}_t^r = q_t \sum_j s_j^T s_j^r \left(N_{t,j} - 0.5\left[\widehat{C_{t,j}} + \widehat{D_{t,j}} + \widehat{Tr_{t,j}}\right]\right)e^{-y_t M} \qquad \text{(A5)}$$

where q is the fishery catchability.

Size composition

The size composition of the retained catch is predicted by the equation

$$\hat{P}_{t,j} = \frac{\hat{C}_{t,j}}{\sum_j^n \hat{C}_{t,j}} \qquad \text{(A6)}$$

The size composition likelihood is formulated using the robust normal for proportions and the negative log likelihood is,

$$LL = 0.5 \sum_t \sum_j \ln\left(2\pi {\sigma_{t,j}}^2\right) - \sum_t \sum_j ln\left[exp\left(-\frac{(P_{t,j}-\hat{P}_{t,j})^2}{2{\sigma_{t,j}}^2}\right) + 0.01\right] \quad \text{(A7)}$$

where $P_{t,j}$ is the observed proportion of crabs in size-class j in the catch during year *t* and $\sigma^2_{t,j}$ is the variance of $P_{t,j}$,

$\sigma^2_{t,j} = \left[\left(1 - P_{t,j}\right)P_{t,j} + \frac{0.1}{n}\right]/S_t$ and S_t is the effective sample size for year *t*.

The effective sample size was calculated iteratively following McAllister and Ianelli's (1997) formula (stage-2 weighting, Francis, 2011).

Tagging data

Let $V_{j,t,y}$ be the number of males that were released in year t that were in length-class j when they were released and were recaptured after y years, and $\underline{\tilde{V}}_{j,t,y}$ be the vector of recaptures by length-class from the males that were released in year t that were in length-class j when they were released and were recaptured after y years. The multinomial likelihood of the tagging data is then:

$$\ell nL = \sum_t \sum_j \sum_y \sum_i \tilde{V}_{j,t,y,i} \ell n \hat{\rho}_{j,t,y,i} \quad \text{(A8)}$$

where $\hat{\rho}_{j,t,y,i}$ is the proportion in size-class i of the recaptures of males which were released during year t that were in size-class j when they were released and were recaptured after y years:

$$\underline{\hat{\rho}}_{j,t,y} \propto \underline{s}^T [\mathbf{X}]^y \underline{\Omega}^{(j)} \quad \text{(A9)}$$

where $\underline{\Omega}^{(j)}$ is a vector with $V_{j,t,y}$ at element j and 0 otherwise, and s^T is the total selectivity vector (Punt et al., 1997).

This likelihood function is predicted on the assumption that all recaptures are in the pot fishery and the reporting rate is independent of the size of crab. The expected number of recaptures in size-class l is given by:

$$r_l = \sum_t \sum_j \frac{s_l [\mathbf{X}^t]_{j,l}}{\sum_{l'} s_{l'} [\mathbf{X}^t]_{j,l'}} \sum_k V_{j,k,t} \qquad \text{(A10)}$$

The last term, $\sum_k V_{j,k,t}$, is the numbers recaptured of male crabs that were released in size-class j after t time-steps . The term $\sum_j \frac{s_l [\mathbf{X}^t]_{j,l}}{\sum_{l'} s_{l'} [\mathbf{X}^t]_{j,l'}} \sum_k V_{j,k,t}$ is the predicted number of animals recaptured in length-class l that were at liberty for t time-steps.

Mature male biomass (MMB)

Mature male biomass on the (assumed) 15 February spawning time (NPFMC 2007) in the following year is computed using equation

$$MMB_t = \sum_{j=mature\ size}^{n} \{N_{j,t} e^{y'M} - (\hat{C}_{j,t} + \hat{D}_{j,t} + \widehat{Tr}_{j,t}) e^{(y_t - y')M}\} w_j \qquad \text{(A11)}$$

where y' is the elapsed time period from 1 July to 15 February and w_j is the weight of crab of size j.

Table 1. Summary of the eastern Aleutian Islands male golden king crab tag release data with the release size range.

Release Year	Number Released	Release Size Range (mm CL)
1991	3,611	93 - 197
1997	7,659	87 - 187
2000	7,768	85 - 179
2003	6,170	88 - 186
2006	5,235	90 - 190
Total	30,443	

Table 2. Summary of the eastern Aleutian Islands male golden king crab tag recapture data. The summary pertains to the crab size range 101-185 mm CL that was considered in the size-structured model. The overall recovery rate was 6.33 %

Time-at-Liberty (years)	Number of Recoveries by Time-at-Liberty
1	936
2	491
3	214
4	51
5	13
6	12

Table 3. Correlation matrices of growth increment and molt probability parameters for approaches 1 and 2 for eastern Aleutian Islands golden king crab. Parameter symbols are defined in the text.

Approach 1:

	ln(a)	*b*	σ
ln(a)	1		
b	-0.0942	1	
σ	-0.4107	-0.2074	1

Approach 2:

	ln(a)	*b*	*ln(c)*	*ln(d)*	σ
ln(a)	1				
b	0.4880	1			
ln(c)	0.2840	0.2655	1		
ln(d)	-0.3431	-0.3020	-0.1555	1	
σ	-0.1823	0.0812	0.0586	0.0804	1

Table 4. Best fit negative log-likelihood and the contributions to the negative log-likelihood by each data source, number of parameters, and Akaike Information Criteria (AIC) for each approach.

	Approach 1	Approach 2	Difference
Negative log-likelihood			
Retained catch length compositions	-526.29	-525.60	-0.69
Retained catch CPUE index	-9.81	-10.17	0.36
Tag recaptures	554.37	338.08	216.29
Total	-992.20	-1213.80	221.60
Number of estimated parameters	108	110	2
AIC	-1768.39	-2207.60	439.21

Table 5. Estimate of the size-transition matrix for (a) approach (1) and (b) approach 2 for the golden king crab data from the eastern Aleutian Islands.

Table 5(a)

0.0216	0.0770	0.1899	0.2806	0.2484	0.1317	0.0418	0.0079	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000
	0.0344	0.1040	0.2226	0.2855	0.2195	0.1011	0.0279	0.0046	0.0005	0.0000	0.0000	0.0000	0.0000
		0.0529	0.1350	0.2509	0.2795	0.1866	0.0746	0.0178	0.0026	0.0002	0.0000	0.0000	0.0000
			0.0784	0.1685	0.2720	0.2631	0.1525	0.0529	0.0110	0.0014	0.0001	0.0000	0.0000
				0.1123	0.2024	0.2837	0.2383	0.1199	0.0361	0.0065	0.0007	0.0000	0.0000
					0.1557	0.2338	0.2845	0.2075	0.0907	0.0237	0.0037	0.0003	0.0000
						0.2087	0.2597	0.2745	0.1739	0.0660	0.0150	0.0020	0.0002
							0.2712	0.2776	0.2547	0.1401	0.0461	0.0091	0.0011
								0.3418	0.2853	0.2274	0.1086	0.0310	0.0059
									0.4185	0.2820	0.1952	0.0809	0.0233
										0.4983	0.2682	0.1612	0.0723
											0.5783	0.2453	0.1764
												0.6551	0.3449
													1.0000

Table 5(b)

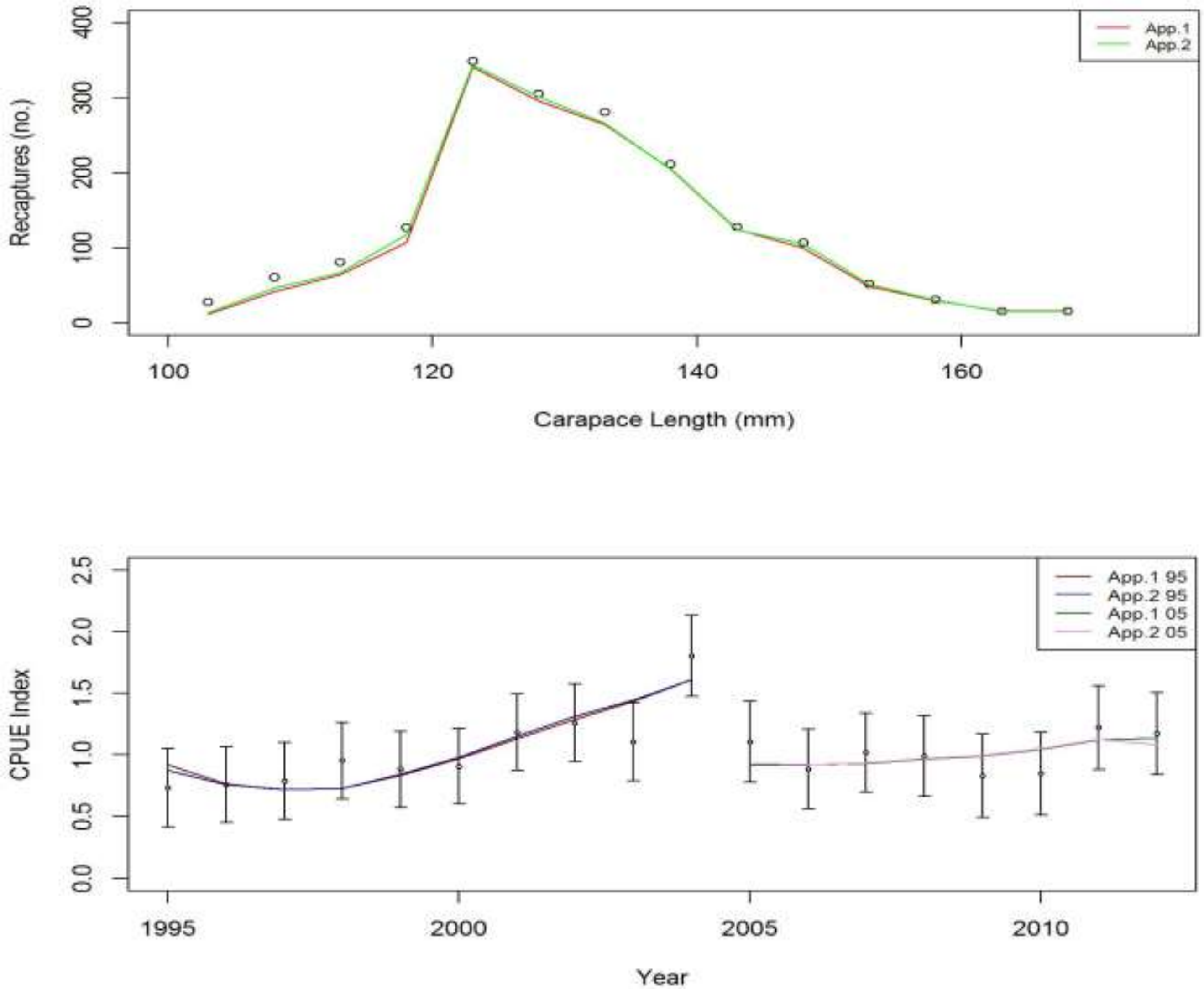
0.0359	0.0155	0.1958	0.4848	0.2435	0.0241	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0540	0.0188	0.2141	0.4775	0.2161	0.0192	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		0.0805	0.0226	0.2302	0.4628	0.1887	0.0150	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
			0.1184	0.0265	0.2426	0.4395	0.1614	0.0115	0.0002	0.0000	0.0000	0.0000	0.0000
				0.1707	0.0303	0.2491	0.4067	0.1345	0.0086	0.0001	0.0000	0.0000	0.0000
					0.2400	0.0335	0.2475	0.3643	0.1085	0.0062	0.0001	0.0000	0.0000
						0.3263	0.0355	0.2363	0.3135	0.0841	0.0043	0.0000	0.0000
							0.4263	0.0360	0.2152	0.2575	0.0621	0.0029	0.0000
								0.5327	0.0347	0.1863	0.2009	0.0436	0.0018
									0.6363	0.0317	0.1531	0.1488	0.0302
										0.7287	0.0275	0.1198	0.1241
											0.8048	0.0229	0.1723
												0.8636	0.1364
													1.0000

Figure Captions

Figure 1. Top panel: Predicted (Eqn A10, solid line) and observed tag recaptures (open circle) by length-class for approaches 1 (App.1) and 2 (App. 2). Lower panel: Observed (open circles with two standard errors) and predicted (colored solid lines) CPUE indices for approaches 1 (App.1 95, App.1 05) and 2 (App.2 95, App.2 05). 95 refers to the CPUE indices estimated for the 1995/96 – 2004/05 period and 05 refers to the CPUE indices estimated for the 2005/06 – 2012/13 period [see Siddeek et al. (2014) for justification to calculate separate sets of CPUE indices].

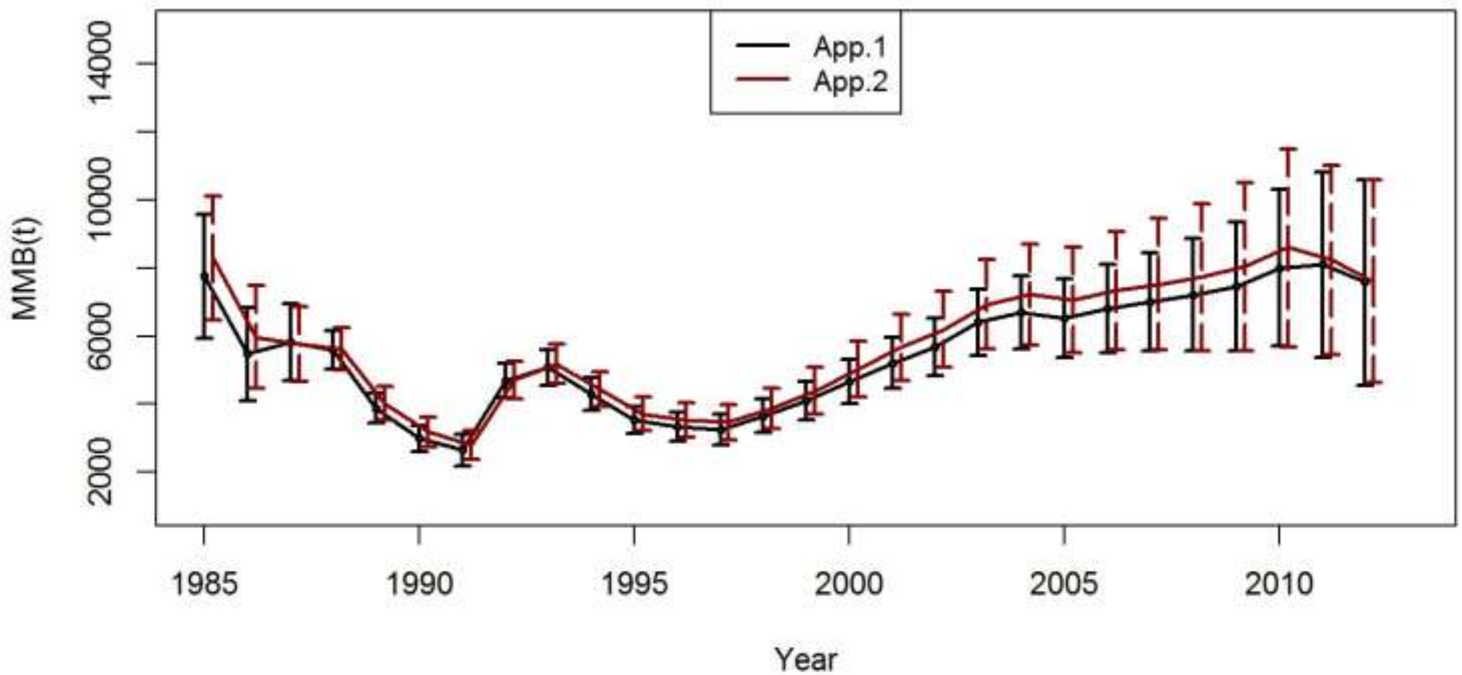
Figure 2. Trends (and associated 95% confidence intervals) in golden king crab mature male biomass (MMB) for approaches 1 (App.1) and 2 (App.2) for the eastern Aleutian Islands golden king crab, 1985/86 – 2012/13. Mature male crabs are ≥ 121 mm CL.

Figure 3. Predicted (line) and observed (bar) retained catch length frequency distributions for approaches 1 (green dashed line) and 2 (red line) for the eastern Aleutian Islands golden king crab, 1985 – 2012. 1985 refers to the 1985 to 1986 fishing season and length-class 1 refers to the midpoint of the first length-class, 103 mm CL.



App.1
App.2
Recaptures (no.)
0
100
200
300
400
100
120
140
160
Carapace Length (mm)
App.1 95
App.2 95
App.1 05
App.2 05
CPUE Index
0.0
0.5
1.0
1.5
2.0
2.5
1995
2000
2005
2010
Year

Figure 1



App.1
App.2
MMB(t)
2000
6000
10000
14000
1985
1990
1995
2000
2005
2010
Year

Figure 2.

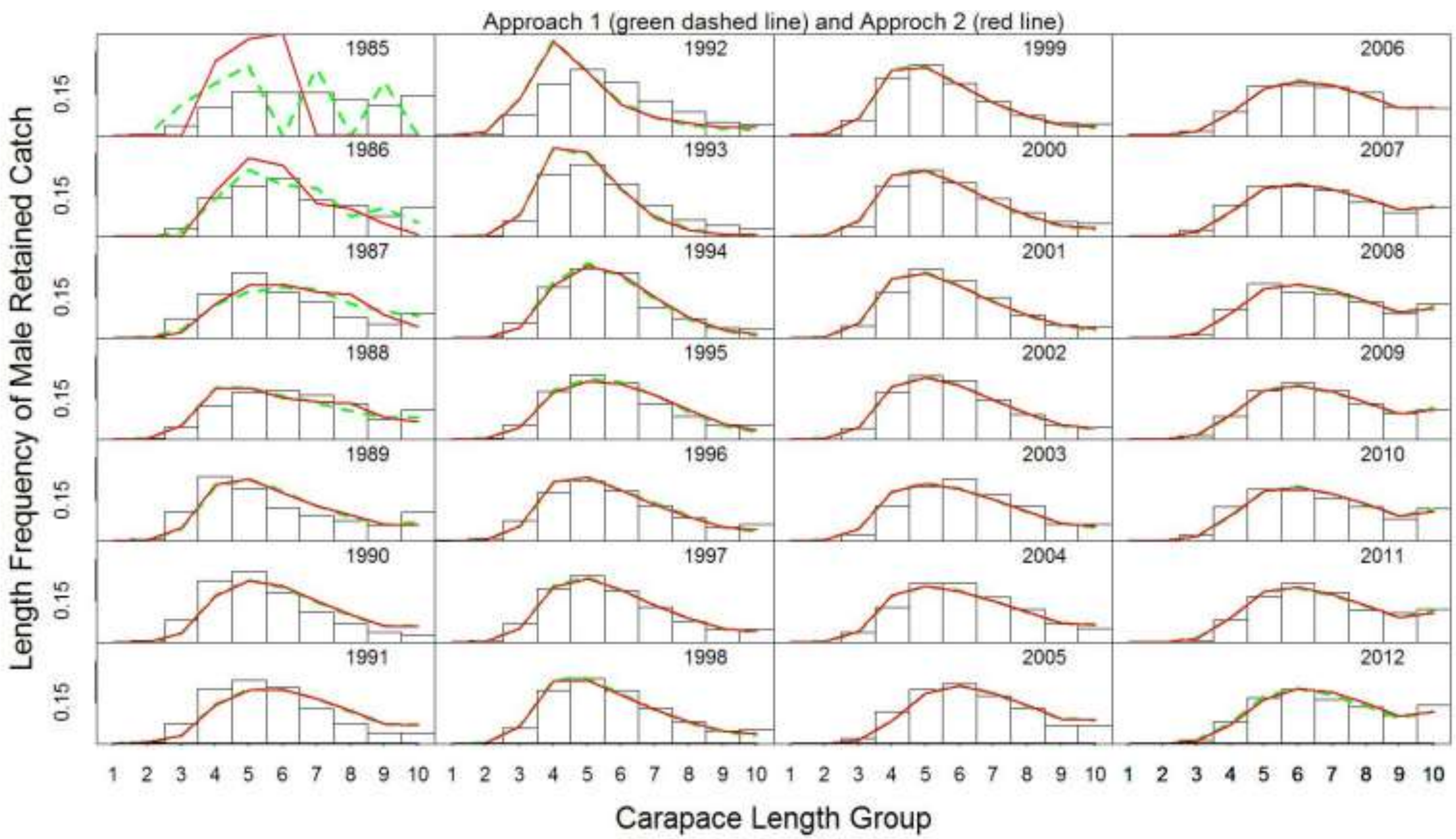

Approach 1 (green dashed line) and Approch 2 (red line)
1985
1992
1999
2006
1986
1993
2000
2007
1987
1994
2001
2008
1988
1995
2002
2009
1989
1996
2003
2010
1990
1997
2004
2011
1991
1998
2005
2012
0.15
Length Frequency of Male Retained Catch
1 2 3 4 5 6 7 8 9 10
Carapace Length Group

Figure 3.