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North American Regional Comparison of Absolute Gravimeters (NACAG18)

Derek van Westrum

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Table Mountain Geophysical Observatory (TMGO)
Boulder, Colorado

Pilot laboratory: NOAA-NGS

1. Introduction

A North American Comparison of Absolute Gravimeters (NACAG18) took place at the Table Mountain Geophysical Observatory (TMGO) near Boulder, Colorado between September 18 and September 21, 2018. Dr. Derek van Westrum of NOAA-NGS, Boulder, Colorado served as the site host and coordinator.

While not affiliated with the Consultative Committee on Mass and Related Quantities (CCM), the 2018 comparison was organized following the procedures of the 2016 comparison at TMGO: SIM.M.G-K1, and the technical protocol (again, based on the 2016 comparison) was approved by all participants.

The list of participants and a description of the site (including vertical gravity gradients, observed tidal parameters, and superconducting gravimeter signal observed during the comparison) are presented, followed by a discussion of the analysis. The results of the data analysis, and the degrees of equivalence (DoE) of the gravimeters are also presented. For the final solution of the Reference Values (RVs), the contributions of absolute gravity data inconsistent at the 95% confidence level are investigated. Overall, the results and uncertainties indicate an excellent agreement between the gravimeters, with a standard deviation of the gravimeters' biases better than 1.4 µGal when all instruments are included, and 0.4 µGal when an outlier is removed.

2. List of participants

Table 1 lists the participants in the comparison. All gravimeters were of the FG5(X) type with two bulk interferometer systems (FG5-107 and FG5-205). No instruments present were associated with a National Metrology Institute (NMI) or a Designated Institute (DI).

Table 1. Participants in the comparison. NGS is the National Geodetic Survey, NGA is the National Geospatial Intelligence Agency, and LSU is the Louisiana State University.

#	Country	Institution	Gravimeter	NMI or DI	Operator(s)
1	USA	NOAA-NGS	FG5X-102	NO	Derek van Westrum
2	USA	Micro-g LaCoste	FG5X-302	NO	Brian Ellis Colt Edwards
3	USA	NGA	FG5-107	NO	David Wheeler Jessica Freeman
4	USA	NGA	FG5-205	NO	David Wheeler Jessica Freeman
5	USA	LSU	FG5X-258	NO	Larry Dunaway Jon Cliburn

3. Site description and relative gravity measurements

A schematic of the TMGO facility, renovated in 2010, is shown in Figure 1. Five of the 10 available piers (AK is permanently reserved for the SG) were employed during NACAG18: AG, AH, AI, AJ, and AT. Each pier is constructed of approximately 1m^3 of concrete, isolated from the building's foundation. The building itself is located on top of a remote mesa, far from any cultural noise sources.

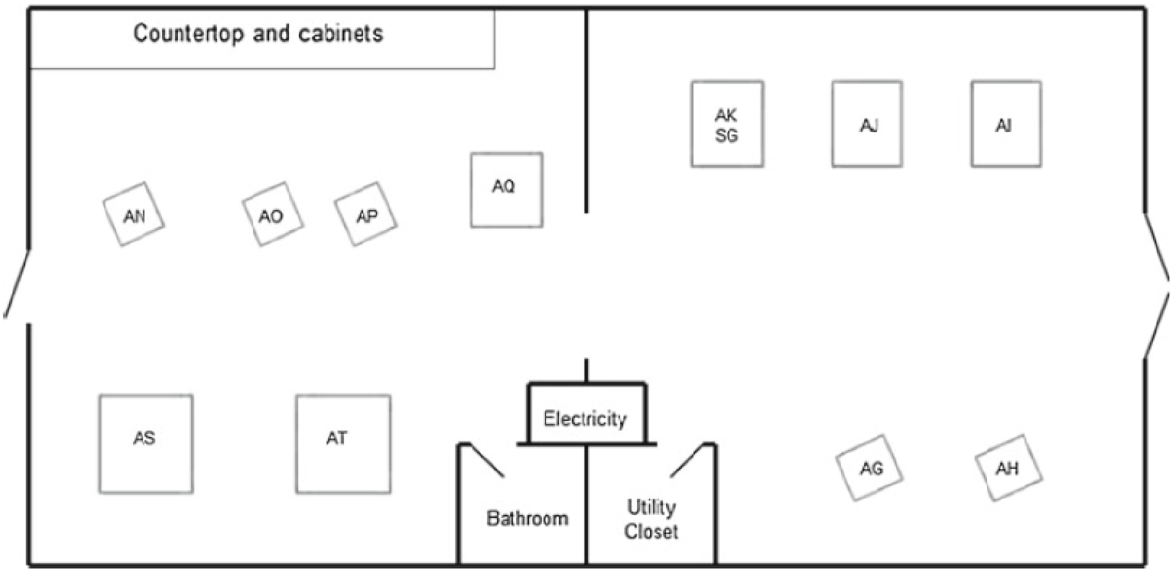

Figure 1. A schematic of the TMGO gravity piers. AK is permanently occupied by the SG, and only piers AG, AH, AI, AJ, and AT were used in the comparison.

In the summer of 2017, a Scintrex CG6 (#001) was used to remeasure relative gravity values on a fixed tripod at three heights above each pier at TMGO. Three heights allows for the determination of any nonlinearity in the gradient. A given measurement consists of three or more "laps" of each gravimeter up the three-tiered tripod. Each tier occupation consists of approximately 100 one second gravity samples. After first rejecting any sample outliers (>3σ, k=1), the instrument drift and any significant tares are removed. A weighted least-squares analysis provides a quadratic fit to all measured gravity differences, resulting in the following function of gravity with height:

$$g(z) = az^2 + bz + c. \qquad (1)$$

Note that when determining the difference in gravity value at two different heights, the constant term, *c*, cancels. Uncertainties of the parameters *a* and *b* are derived from the variance-covariance matrix. Uncertainties in the height measurements (tier locations) are considered negligible. The parameters are listed (for all piers for completeness) in Table 2.

Finally, note that the gradient value is not expected to change at TMGO over the time frame discussed. TMGO is located on a broad, flat, undisturbed expanse of land, and even unmodeled environmental factors like water table fluctuations will not cause a change in the value of the gradient.

Table 2. Vertical gravity gradients at the TMGO piers (only piers AG, AH, AI, AJ, and AT were used for the comparison).

Site	a /µGal m^{-2}	σ_a /µGal m^{-2}	b /µGal m^{-1}	σ_b /µGal m^{-1}	σ_{ab} /µGal^2 m^{-3}
AG	3	1	-322	3	3
AH	1.5	1	-319	3	3
AI	1	1	-322	3	3
AJ	-1	1	-317	3	3
AN	4	1	-323	3	3
AO	-2	1	-315	3	3
AP	4	1	-325	3	3
AQ	0	1	-316.5	3	3
AS	3	1	-324	3	3
AT	1	1	-320	3	3

The gravity difference between height z_1 and z_2 is given by:

$$\Delta g(z_1 - z_2) = g(z_2) - g(z_1) = a \times (z_2^2 - z_1^2) + b \times (z_2 - z_1) \qquad (2)$$

and the associated uncertainty

$$\sigma_{\Delta g}^2 = (z_2^2 - z_1^2)^2 \times \sigma_a^2 + (z_2 - z_1)^2 \times \sigma_b^2 + 2 \times (z_2^2 - z_1^2) \times (z_2 - z_1) \times \sigma_{ab} \qquad (3)$$

The participants reported gravity results at the “effective measurement height,” h_{eff}, of their respective gravimeters where the gravity value is least sensitive to the effect of the gradient [1] [2]. The gradient formula for that respective pier was then used to transfer the value to a final, common height of 125 cm (an approximate average of the effective measurement height for FG5 and FG5-X gravimeters).

The observed tidal parameters – used by all participants to correct for earth tides and ocean loading simultaneously – were provided by Olivier Francis, based on an analysis of SG CT-024. They are listed in Table 3 [3].

Table 3. Observed tidal parameters for TMGO from SG CT-024 via Olivier Francis.

Wave	Start Frequency (cpd)	Stop Frequency (cpd)	Amplitude Factor	Phase Lag (degrees)
DC	0.000000	0.000001	1.00000	0.0000
long	0.000002	0.249951	1.16000	0.0000
Q1	0.721500	0.906315	1.16052	1.1570
O1	0.921941	0.940487	1.16468	1.1775
NO1	0.958085	0.974188	1.15951	1.0326
P1	0.989049	0.998028	1.16539	1.1041
S1	0.999853	1.000147	1.49457	15.9599
K1	1.001825	1.003651	1.15452	1.1761
PSI1	1.005329	1.005623	1.30377	1.3908
PHI1	1.007595	1.011099	1.20411	0.6319
J1	1.013689	1.044800	1.18028	1.1094
OO1	1.064841	1.216397	1.18279	0.3491
2N2	1.719381	1.872142	1.16806	-0.4567
N2	1.888387	1.906462	1.15681	-0.2398
M2	1.923766	1.942754	1.15945	0.1973
L2	1.958233	1.976926	1.16297	0.3812
S2	1.991787	2.002885	1.17172	-0.5305
K2	2.003032	2.182843	1.17348	-0.4844
M3	2.753244	3.081253	1.07285	-0.2409
M4	3.381379	4.347615	1.03900	0.0000

4. Superconducting gravity meter measurements

A GWR superconducting gravity meter, CT-024, was operated continuously throughout the comparison on pier AK, situated near the center of the absolute piers. Figure 2 displays the results after the earth tide, ocean loading, and atmospheric pressure effects have been removed (The yellow curve is the raw data, and the alternating black and red splines indicate the periods of absolute gravity measurements). The resulting change in gravity, ~4 µGal peak-to-peak, throughout the comparison was presumably observed by all gravity meters. However, the exceptionally large change in gravity during the third day of measurements seems to be an instrument artifact. There was no precipitation at that time, nor did any of the absolute gravimeters (AGs) observe such a large change in gravity during their observations (starting on September 20 and crossing into the 21st). Finally, if we attempt to correct the AG results with the SG signal, the agreement between the AGs gets worse (the standard deviation of the instrument biases will be presented below, but as an example, σ = 0.38 µGal without the SG correction, and σ = 0.52 µGal with the SG correction). This can only be true if the SG signal is somehow corrupted. For these reasons, the SG signal is neglected in the analysis, and the assumption is made that any true change in the local gravity was well within the ~2 µGal systematic uncertainty of the AGs.

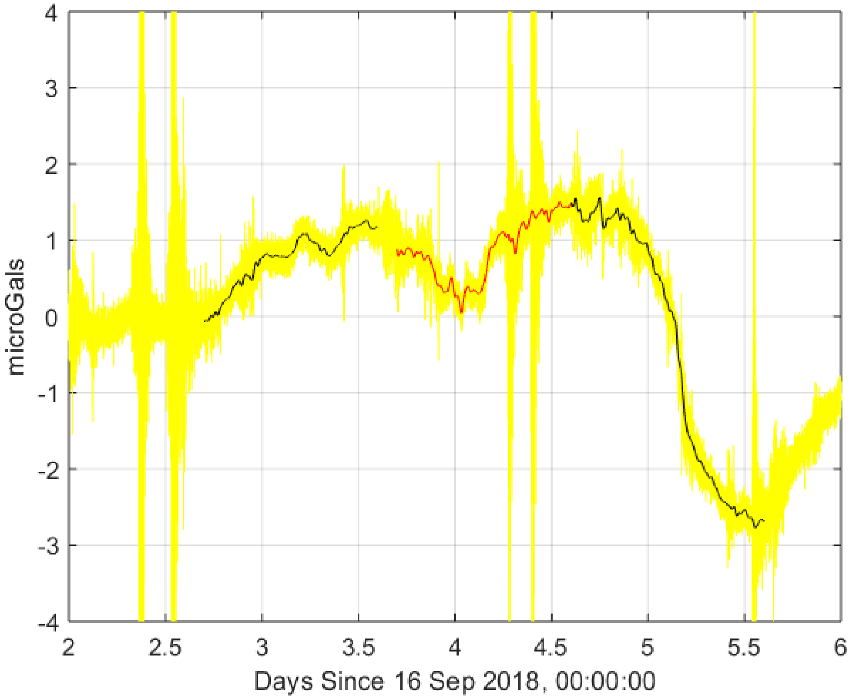

Figure 2. The output of the superconducting gravity meter, CT-024 with earth tides and barometric pressure effects removed (yellow line). A maximum peak-to-peak signal of about 4.0 µGal is observed during the entire duration of the comparison. The alternating black and red splines indicate the times of the absolute gravity observations (days 1 through 3). The large decrease in gravity at September 21 is thought to be an instrument artefact. See text.

5. Absolute gravity measurements

Each participant was free to measure gravity over a duration of their choosing within a window of approximately 24 hours (before being required to move to the next scheduled pier). Table 4 lists the measurements. The schedule was designed to maximize the overlap of gravimeters, minimize the number of occupied piers, and prohibit reoccupations of the same pier by a single gravimeter [4]. Every gravimeter overlapped with every other gravimeter at least two times and no more than three times.

Table 4. Nominal pier occupation schedule for the comparison. Days 1-3 were 18-20 September 2018.

Pier\Day	1	2	3
AG	FG5X-302	FG5-205	FG5-107
AH	FG5X-102	FG5X-302	FG5-205
AI	FG5-205	FG5-107	FG5X-258
AJ	FG5-107	FG5X-258	FG5X-102
AT	FG5X-258	FG5X-102	FG5X-302

Gravity was determined above each pier benchmark at h_{eff} unique to each gravimeter (and in principle, each gravimeter set up). Each raw gravity value is corrected for:

- Earth tides and ocean loading via a common set of observed tidal parameters [3]
- Barometric pressure changes from the nominal value of 826.74 mBar using a common admittance factor of -0.3 µGal/mbar [5]
- Polar Motion using common values obtained from the Earth Rotation and Reference Systems Service (IERS) [6]
- Self attraction of the gravimeter itself [7] [8]
- Diffraction correction due to the finite width of the laser beam [9]

These raw gravity values were then transferred to a common height of 125 cm using the quadratic gradient parameters for each pier listed in Table 2. The gravimeter results are listed in Table 5.

Table 5. Absolute gravity observations g_{ij}. u_{ij} is the standard measurement uncertainty (k=1). See text for discussion. A common value of 979 622 000 µGal has been subtracted from each value. The days are UTC dates in September 2018.

Gravimeter	Pier	Gravity @ h_{eff} (µGal)	h_{eff} (m)	diffC (µGal)	SAC (µGal)	Start Date	Start Time	Stop Date	Stop Time	Gravity g_{ij} (µGal)	Uncert u_{ij} (µGal)
FG5X-102	AH	742.4	1.287	1.05	-1.2	18	22:09	19	14:09	753.9	1.84
FG5X-102	AT	741.2	1.283	1.05	-1.2	19	21:52	20	13:52	751.5	1.81
FG5X-102	AI	750.2	1.286	1.05	-1.2	20	19:50	21	13:50	761.6	1.81
FG5X-302	AG	745.1	1.279	1.05	-1.2	18	22:09	19	14:09	754.0	1.81
FG5X-302	AH	744.8	1.277	1.05	-1.2	19	20:23	20	14:23	753.1	1.82
FG5X-302	AT	743.4	1.274	1.05	-1.2	20	19:50	21	13:50	751	1.82
FG5-107	AI	769.7	1.224	1.05	-1.7	18	17:15	19	14:15	760.7	1.87
FG5-107	AJ	769.6	1.232	1.05	-1.7	19	21:12	20	15:12	763.2	1.87
FG5-107	AG	760.9	1.231	1.05	-1.7	20	19:43	21	14:43	754.1	1.85
FG5-205	AJ	772.1	1.228	1.05	-1.7	18	22:58	19	15:06	764.4	1.91
FG5-205	AG	765.6	1.228	1.05	-1.7	19	17:34	20	14:34	758.1	1.91
FG5-205	AH	765.1	1.228	1.05	-1.7	20	17:01	21	13:01	757.5	1.92
FG5X-258	AT	744.9	1.270	1.05	-1.2	18	0:06	19	14:06	751.2	1.82
FG5X-258	AI	753.3	1.272	1.05	-1.2	19	20:17	20	14:17	760.3	1.82
FG5X-258	AJ	755.1	1.274	1.05	-1.2	20	19:43	21	14:43	762.7	1.82

6. Comparison Analysis Methods

Following Koo and Clare [10], the gravity values for each gravimeter on each pier are expressed as

$$g_{ij} = g_j + \delta_i + \varepsilon_{ij} \tag{4}$$

where g_{ij} is the gravity value measured on pier j by gravimeter i as given in Table 5, g_j is the true (unknown) gravity value at pier j, δ_i is the true (unknown) bias of gravimeter i, and ε_{ij} is the measurement error. A variance-weighted least squares analysis (LSA) is performed to give the best estimates of the pier gravity Reference Values g_j and the gravimeter biases δ_i. Since we do not consider correlations among the gravity measurements, the input (or measurement) covariance matrix $\boldsymbol{V}$ is diagonal with diagonal elements u_{ij}^2 where u_{ij} is given in Table 5. Thus the corresponding weight matrix $\boldsymbol{W} = \boldsymbol{V}^{-1}$ is also diagonal with diagonal elements $w_{ij} = 1/u_{ij}^2$. The set of equations is ill defined with an infinite number of solutions, so an additional constraint on the weighted sum of the biases is imposed:

$$\sum_{i=1}^{n} \overline{w}_i \ \delta_i = d \tag{5}$$

where d can be employed as a "linking converter" between Key Comparisons. In the case of NACAG18, no metrological institutes participated, so we take d to be zero. The normalized variance weights $\overline{w}_i$ are

$$\overline{w}_i = \frac{w_i}{\sum_{i=1}^{n} w_i} \quad , \tag{6}$$

where $w_i = 1/u_i^2$ and u_i is the root mean square of u_{ij} given in Table 5 for gravimeter i.

7. Results

7.1 Initial results

For the initial solution, all measurements presented by the operators as given in Table 5 were included in the LSA. The gravimeter biases (δ) and pier Reference Values (g) are presented in Table 6 and Figure 3. All the gravimeters are in excellent equivalence with a standard deviation of the biases of 1.43 µGal.

Table 6. Initial biases (δ) of all instruments and pier Reference Values (g) at a height of 125 cm of the comparison using all the reported absolute measurements. The constant value 979 622 000.0 µGal is subtracted from the Reference Values, U is the expanded standard uncertainty at 95% confidence (k=2).

Gravimeter	δ/µGal	U/µGal
FG5X-102	-0.17	0.82
FG5X-302	-1.06	0.82
FG5-205	2.53	0.86
FG5-107	-0.44	0.84
FG5X-258	-0.64	0.82

Pier	g/µGal	U/µGal
AG	755.05	0.92
AH	754.38	0.92
AJ	762.97	0.92
AI	761.29	0.90
AT	751.86	0.90

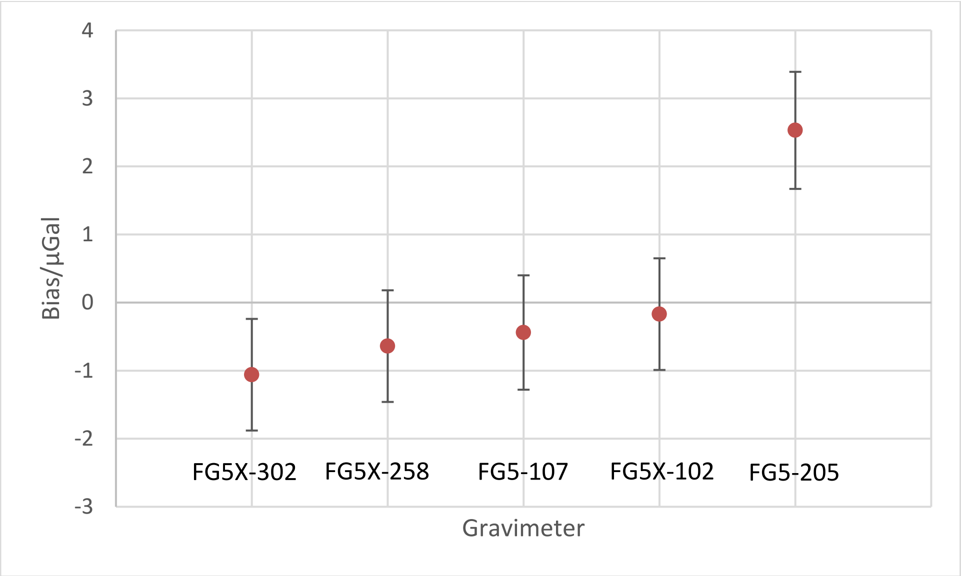

Figure 3. Initial biases of the gravimeters from the initial weighted LSA.. The error bars are the expanded standard uncertainties (k=2) from the weighted LSA and are listed in Table 6.

7.2 Consistency of input data

The consistency of the input data is investigated by calculating the ratio of the difference between measured values of gravity and the reference value to the uncertainty of the difference, called the compatibility index E_{ij},

$$E_{ij} = \frac{(g_{ij} - g_j)}{\sqrt{U_{ij}^2 + U_j^2}} \quad , \tag{7}$$

where U_j is the expanded uncertainty (k=2) of the reference value g_j from the weighted LSA in Table 6. An absolute value of E_{ij} larger than 1 indicates inconsistency at the 95% confidence level for the given test. Values of E_{ij} are given in Table 7. None of the input data has an E_{ij} value larger than 1, though two of the values for FG5-205 are close and significantly larger than all others. Indeed a noticeable offset is apparent for FG5-205 in Figure 3. An inspection of the system by Micro-g LaCoste after the comparison indicated large wear on the falling object contacts, causing a significant offset in the results. Therefore the results of FG5-205 are omitted in the final analysis without further consideration.

Table 7. Consistency of input data: Comparison of measured gravity values g_{ij} (along with standard uncertainties U_{ij}) with reference values g_j (along with standard uncertainties U_j) by means of E_{ij}. The constant value 979 622 000.0 µGal has been subtracted from the gravity measurements. E_{ij} values indicating a possible problem are in yellow.

Gravimeter	Pier	g_{ij} /µGal	U_{ij} /µGal	g_j /µGal	U_j /µGal	g_{ij}-g_i /µGal	E_{ij}
FG5X-102	AH	753.9	3.68	754.38	0.92	-0.48	-0.13
FG5X-102	AT	751.5	3.62	751.86	0.89	-0.36	-0.10
FG5X-102	AI	761.6	3.62	761.29	0.90	0.31	0.08
FG5X-302	AG	754	3.62	755.05	0.92	-1.05	-0.28
FG5X-302	AH	753.1	3.64	754.38	0.92	-1.28	-0.34
FG5X-302	AT	751	3.64	751.86	0.89	-0.86	-0.23
FG5-205	AJ	764.4	3.82	762.97	0.92	1.43	0.36
FG5-205	AG	758.1	3.82	755.05	0.92	3.05	0.78
FG5-205	AH	757.5	3.84	754.38	0.92	3.12	0.79
FG5-107	AI	760.7	3.74	761.29	0.90	-0.59	-0.15
FG5-107	AJ	763.2	3.74	762.97	0.92	0.23	0.06
FG5-107	AG	754.1	3.72	755.05	0.92	-0.95	-0.25
FG5X-258	AT	751.2	3.64	751.86	0.89	-0.66	-0.18
FG5X-258	AI	760.3	3.64	761.29	0.90	-0.99	-0.26
FG5X-258	AJ	762.7	3.64	762.97	0.92	-0.27	-0.07

7.4 Final results

A final LSA was performed excluding the measurements of FG5-205 and with d = 0 µGal to obtain the best estimates for the reference values (RVs), given in Table 8. Following Jiang et al.

[11], the DoEs are the weighted average difference between the measurements of a gravimeter *i* and the RVs at given site *j*,

$$D_i = \left[\sum w_{ij}\,(g_{ij} - g_j)\right] \;/\; \sum w_{ij} \quad , \tag{8}$$

where the weights $w_{ij} = 1/U_{Dij}^2$ and U_{Dij} is the expanded uncertainty of the difference g_{ij} - g_j. The differences and expanded uncertainties are shown in Table 9. U_{Di} is the expanded uncertainty of the weighted differences. All the gravimeters are in excellent equivalence with a standard deviation of the biases of 0.38 µGal.

Table 8. NACAG pier Reference Values (RVs). The constant value 979 622 000.0 µGal is subtracted from the RVs. U is the expanded standard uncertainty at 95% confidence computed as the root mean square of the expanded standard uncertainty from the LSA.

Official Key Comparison Results		
Pier	KCRV/µGal	*U*/µGal
AG	754.17	0.42
AH	753.33	0.43
AJ	763.12	0.44
AI	760.80	0.33
AT	751.22	0.33

Table 9. DoEs determined according to Eq. 9. g_{ij} are the measured gravity values transferred to 125 cm with expanded uncertainty U_{ij}. g_j are the RVs with associated expanded (k=2) uncertainties U_j given in Table 8. U_{Dij} is the expanded uncertainty of differences g_{ij}-g_j. D_i is the final DoE computed according to Eq. 7 along with the expanded uncertainty U_{Di}. The constant value 979 622 000.0 µGal was subtracted from the gravity measurements.

Gravimeter	Pier	g_{ij} /µGal	U_{ij} /µGal	g_j /µGal	U_j /µGal	g_{ij}-g_i /µGal	U_{Dij} /µGal	D_i /µGal	U_{Di} /µGal
FG5X-102	AH	753.9	3.68	753.33	0.43	0.57	3.64	0.55	0.30
FG5X-102	AT	751.5	3.62	751.22	0.33	0.28	3.64		
FG5X-102	AI	761.6	3.62	760.80	0.33	0.80	3.64		
FG5X-302	AG	754	3.62	754.17	0.42	-0.17	3.63	-0.21	0.32
FG5X-302	AH	753.1	3.64	753.33	0.43	-0.23	3.63		
FG5X-302	AT	751	3.64	751.22	0.33	-0.22	3.63		
FG5-107	AI	760.7	3.74	760.80	0.33	-0.10	3.73	-0.03	0.33
FG5-107	AJ	763.2	3.74	763.12	0.44	0.08	3.73		
FG5-107	AG	754.1	3.7	754.17	0.42	-0.07	3.73		
FG5X-258	AT	751.2	3.64	751.22	0.33	-0.02	3.64	-0.31	0.30
FG5X-258	AI	760.3	3.64	760.80	0.33	-0.50	3.64		
FG5X-258	AJ	762.7	3.64	763.12	0.44	-0.42	3.64		

Table 10. DoEs (according to Eq. 9) of each instrument. The uncertainty U_{DoE} is the expanded uncertainty at 95% confidence of the weighted differences.

Comparison Results		
Gravimeter	DoE/µGal	U_{DoE}/µGal
FG5X-102	0.55	0.30
FG5X-302	-0.21	0.32
FG5-107	-0.03	0.33
FG5X-258	-0.31	0.30

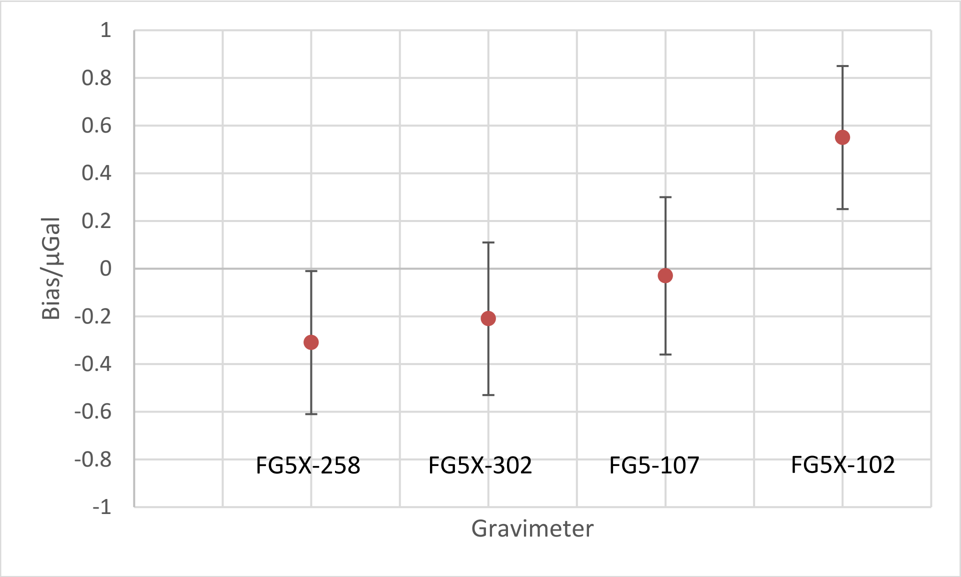

Figure 4. Degrees of Equivalence (DoE) of the gravimeters calculated from the difference between the gravimeter measurements and the RVs. The error bars represent the expanded uncertainties (U_{DoE}) of the DoE at 95% confidence.

8. Conclusions

Five absolute gravimeters were compared during the regional North American Comparison of Absolute Gravimeters (NACAG18). The raw agreement was within the instruments' uncertainties, with a standard deviation in the biases of 1.4 µGal. However, one instrument was later found to be in need of repair and was excluded in the final estimate of the reference values. The final results and uncertainties indicate an excellent agreement between the gravimeters, with a standard deviation of the gravimeters' biases of 0.4 µGal.

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