NOAA Technical Memorandum NMFS F/NWC-90

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October 1985

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
National Marine Fisheries Service

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October 1985

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ABSTRACT

All correctly designed regulatory instruments, such as taxes and quotas, are equally effective in increasing consumer and producer surpluses in markets when externalities exist, when complete information is available, and when adjustments of the level of the instruments are costless. However, with the existence of uncertainty, it is well known that one instrument or another may produce a higher expected present value of. net social benefits than the others. In addition, costs of or constraints on adjusting the levels of the instruments also create differences in the relative performance of alternative instruments.

A combination discrete-time and continuous-time stochastic model of a fishery is constructed in this report. Under the assumption that the level of an instrument cannot be changed during the fishing season, analytical and numerical dynamic programming are employed to find the optimal levels of a per-unit tax and an instantaneous harvest rate quota, given the observed size of the fish stock at the beginning of the season. Numerical dynamic programming is applied to the Pacific coast pink shrimp fishery, and tax and quota systems are found to produce approximately equal net benefits in this fishery.

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CHAPTER 1

INTRODUCTION AND PRELIMINARY MODELS

INTRODUCTION

The economic efficiency of markets in which externalities are present can theoretically be improved by regulation. A wide variety of regulatory instruments is available for this purpose, including taxes, quotas, price controls, input controls, and combinations of two or more basic instruments. The question of which type of instrument is superior has been argued in the literature for some time.

Consensus has been reached on the point made by Weitzman (1974) that in the presence of perfect information, and in the absence of flexibility constraints, properly designed instruments of all types are ranked equally on the basis of economic efficiency (Laffont 1977; Dasgupta and Heal 1979; and Brown and Boontherawara 1982). There may be distributional, social, political, administrative, or enforcement considerations which swing the balance in favor of one or another, but in terms of maximizing the sum of consumer and producer surpluses, none can be shown to be superior.

This conclusion follows from the fact that when complete information is available, the economically efficient rate and distribution of production (or effluent discharge) are known. If there are no constraints on the levels at which regulatory instruments can be set or on the frequency with which they can be adjusted, any correctly designed instrument can be set at the level which elicits this rate.

Even in the presence of uncertainty, if it is feasible to implement contingency rules which specify the level of the instrument under all possible future states of the world, all instruments perform equally well. But such rules are complicated, and are not used.

The literature initiated by Weitzman, however, demonstrates that when relatively simpler instruments must be used in the face of uncertainty, one instrument or combination of instruments may outperform the others (Laffont 1977; Yohe 1978; and Dasgupta and Heal 1979)'. Which one performs best depends on the assumptions and parameter values of the model being used to describe a particular activity. The key feature leading to this result is the requirement that the level of the instrument must be set before all information is gathered, but the economic agents subject to regulation make their decisions about the rate at which to produce output after more information has become available. Therefore, the agents produce different rates of output under different instruments, whose levels were set on the basis of expected parameter values.

Weitzman considered a model-in which marginal benefit and marginal cost were linear, with known slopes. The uncertainty was in the vertical position of these curves, but means and variances of the intercepts were known. No instrument would be set, except with probability less than one, at the full-information optimum level, and therefore, some welfare loss was to be expected. Assuming that the chosen instrument would be set at the level which minimizes expected welfare loss, Weitzman determined that the difference in expected welfare loss under price controls and under quantity controls depends on' the values of the slopes, variances, and covariances, the number of separate production units, and the correlation between the marginal costs of the production units. In the simplest case, in which the costs of the production units are perfectly correlated and the cost and benefit disturbances are uncorrelated, the choice of instruments depends only on the relative slopes of the marginal cost and benefit curves. For example, when the marginal cost (supply) curve is less steep than the marginal benefit curve, quotas are preferred to controls on the price received by producers. The reason is that vertical perturbations of the marginal cost curve will result in relatively large over- or under-production if the price is fixed.

Inflexibility in, or the presence of significant costs of, setting and adjusting instrument-levels affects the relative performance of different instruments-independently of uncertainty. This fact, which is mentioned only in passing by Dasgupta and Heal (1979), will be demonstrated heuristically, later in this chapter.

Until recently, the voluminous literature on management of the, externality arising from the common property nature of many natural resources was largely concerned with determining the optimal level or time path of either the harvest rate or the effort rate (Gordon 1954; Plourde 1970; Brown 1974; Clark and Munro 1975; and Burt and Cummings 1977). In other words, the focus has been on how to set the level of one or two instruments. A subset of this literature deals with optimal management under uncertainty (Burt 1964; Reed 1979; Ludwig 1980; and Charles 1983b; also see Andersen and Sutinen 1981 for a survey of the literature on fisheries and uncertainty).

To date, few researchers have delved into the relative performance of alternative instruments in regulating fisheries. Three exceptions are Beddington and May (1977), Andersen (19820, and Koenig (1984). Beddington and May, who are biologists, compared the time required for a population to return to equililbrium after a disturbance under constant (inflexible) effort and constant harvest policies, and did not consider economic criteria. Andersen concluded that when the only uncertainty is in the exogenous ex-vessel price level and when fishermen are risk averse, price controls are superior to quotas and per-unit taxes.

Koenig made a more general extension of the analysis of optimal instrument choice under uncertainty to the dynamic setting of natural resource management. Using a linear-quadratic programming model, he applied an approach similar to Weitzman’s as to the choice of instrument in a fishery and-extended the analysis to other instruments in addition to quotas and controls on prices received by producers, including specific and ad valorem taxes and controls on pricespaid by consumers. He found that when the size of the fish stock can be accurately observed, a combination of specific and ad valorem taxes always outperforms the other instruments. If, however, this combination tax is not feasible, his conclusion is similar to Weitzman's, i.e., the choice of instrument depends on certain parameter values in the fishery being modelled. These parameters include the slope of the social marginal cost function, which is the sum of the private and external marginal costs. In this context, marginal external cost is the discounted future surplus foregone by society when current production is increased.

A feature of previous studies of optimal quota and effort control adjustment is the use of models and dynamic optimization methods which consider time to be either strictly, discrete or strictly continuous. Neither approach is completely satisfactory when used alone. Koenig's results, for example, are limited to fisheries in which no natural stock growth occurs during the fishing season. This structure can be realistically assumed only for fisheries with very short open seasons. Policy implications of continuous-time analyses, on the other hand, tend to be impractical in that they-require continuous monitoring of stock size and continuous adjustment of the chosen regulatory instrument. In reality, measurement of stock size and adjustment of instrument level are accomplished only periodically. That is, there are flexibility costs which are ignored in these models.

While these models may adequately approximate reality in some cases, there is potential benefit in developing an approach to setting instruments at their optimal levels and to choosing the optimal instrument, which incorporates both the continuous-time nature of fishing and stock dynamics in many fisheries and the discrete-time nature of regulatory behavior. The longer the season and the faster the rate of change in stock size, the greater the potential benefit will be. This is due to the fact that a long season and a fast stock size change rate both reduce the realism of discrete-time models and increase the welfare loss from insufficiently frequent adjustment of instrument level.

It has been suggested that a rapid. stock growth rate reduces the need for careful management of the stock because the stock can recover quickly from overfishing. In other words, nature "forgives" management errors. However, forgiveness is forthcoming only if errors are detected and corrected. If inappropriate models are used continually' for management, the welfare loss might be significant.

The objective of this research is to develop a simple combination discrete-time and continuous-time stochastic model, and to incorporate both uncertainty and inflexibility into the decision process for choosing the optimal type and level of regulatory-instrument.

In order to make the problem analytically tractable, it is necessary in both discrete-time and combination models to make linearity and capital exogeneity assumptions which put it in the form of a dynamic programming problem with a quadratic objective function and linear stock growth. The analytical approach demonstrates that' under these simple assumptions, no particular instrument can be presumed to be superior in all fisheries. However, the true forms of some of the basic functions in the models are clearly nonlinear, and the quantity of-capital in the fishery is usually endogenous. This approach is probably not useful, therefore, for actually managing a real fishery.

Alternatively, one can construct a more plausible model and employ' numerical dynamic programming on a computer. A problem with this approach is that the number of variables whose- ranges must be divided. into discrete intervals and iteratively looped through can be sufficiently large that the programs are very expensive to run. This is the so-called "curse of dimensionality."

As a compromise between using the unrealistic linear model and the computationally expensive nonlinear model, it may be worthwhile to perform numerical dynamic programming with linear functions, which are obtained by approximating the true nonlinear functions at appropriate points. The points around which Taylor series expansions were made would change in each period, so the approximation accuracy of the solutions might be acceptable. Some indication of the accuracy of this method could be gained by comparing its results in specific fisheries with those obtained by using a nonlinear model when available resources permit doing so. No such comparison is made here.

One of the unique implications of the combined discrete-time/ continuous-time model is that uncertainty about the stock growth rate is important; in both strictly continuous-time and strictly discrete-time models, growth rate uncertainty is irrelevant to the choice of instrument. However, the general conclusion that the optimal choice of the simple instrument depends on the fishery under consideration remains valid in the combination approach.

In the remainder of this first chapter, a heuristic demonstration of the fact that inability to change the level of the regulatory instrument can render one instrument superior to the others, even with complete information, is presented. A stochastic model is then presented which requires the assumption that the fishery system proceeds very rapidly to a steady state in every period. While the model has only very limited applicability because of this assumption, which is a very strong one, it serves to illustrate some principles of optimal regulatory instrument choice under uncertainty which cannot easily be seen in the more general models developed in later chapters. Moreover, it has the advantage of needing less information to make it operable than do the more general models.

Chapters 2 and 3 discard the steady-state assumption and present a more general model with linear marginal benefit, marginal cost, and stock growth rate functions, and with exogenous harvesting capital. Chapter 2 presents expressions for single-period net benefits which will be incorporated into the dynamic programming formulae of Chapter 3. Rules are derived for setting the instruments at approximately optimal levels when stock size at the beginning of a period does not depend on the quantity harvested during the previous period.

Chapter 3 presents the linear-quadratic multiple-period dynamic programming model. Rules for setting instruments at their approximately optimal levels when beginning-of-period stock size is dependent on fishing in the previous period are derived. Expressions for expected net present values under taxes and quotas are compared.

Chapter 4 presents a more plausible nonlinear model with endogenous harvesting capital and describes the numerical dynamic programming algorithm.

Chapter 5 presents estimates of the marginal benefit, marginal fishing cost, and stock growth functions in the west coast pinkshrimp, Pandalus jordani fishery. It also applies the numerical dynamic programming algorithm explained in the previous chapter to the pink shrimp fishery in an illustrative way, and presents the results, which show that the two instruments would produce approximately equal expected present values in this fishery.

Finally, the appendices show detailed derivations of some of the equations presented in each chapter.

DETERMINISTIC MODEL WITH CONSTANT INSTRUMENT LEVEL

When the frequency with which the regulating authority can change the level of the chosen instrument in a dynamic setting is constrained, one instrument may outperform the others, even in the deterministic case. An example is production from a natural resource stock in which extraction occurs continuously, at least over some time intervals, as does natural growth of the stock. Unless the system is initially already in the optimal steady state, the stock size will change, continuously over time, which means that the level of the regulating instrument must also change continuously over time if complete optimality is to be achieved. (A quota must control the extraction rate at every instant, and not just the cumulative quantity extracted each period.) If, however, the level of the instrument can be reset only at periodic intervals, different instruments will have different effects on the time paths of all variables in the system and will result in different expected present values of the stock.

Figure 1 illustrates the simplest case: perfectly elastic demand and marginal harvest cost. In this case; the optimal management program when the planning horizon is infinite calls for driving the stock as rapidly as possible to a steady-state level (Clark 1973), designated X*. This means that if the initial stock size is greater than X*, maximum possible fishing effort should be applied to the stock until it is reduced to X*, and if initial stock size is less than X*, there should be no fishing at all until the stock grows to X*. When the stock reaches X*, the optimal steady-state instantaneous harvest rate is H*. This harvest rate can be achieved by imposing

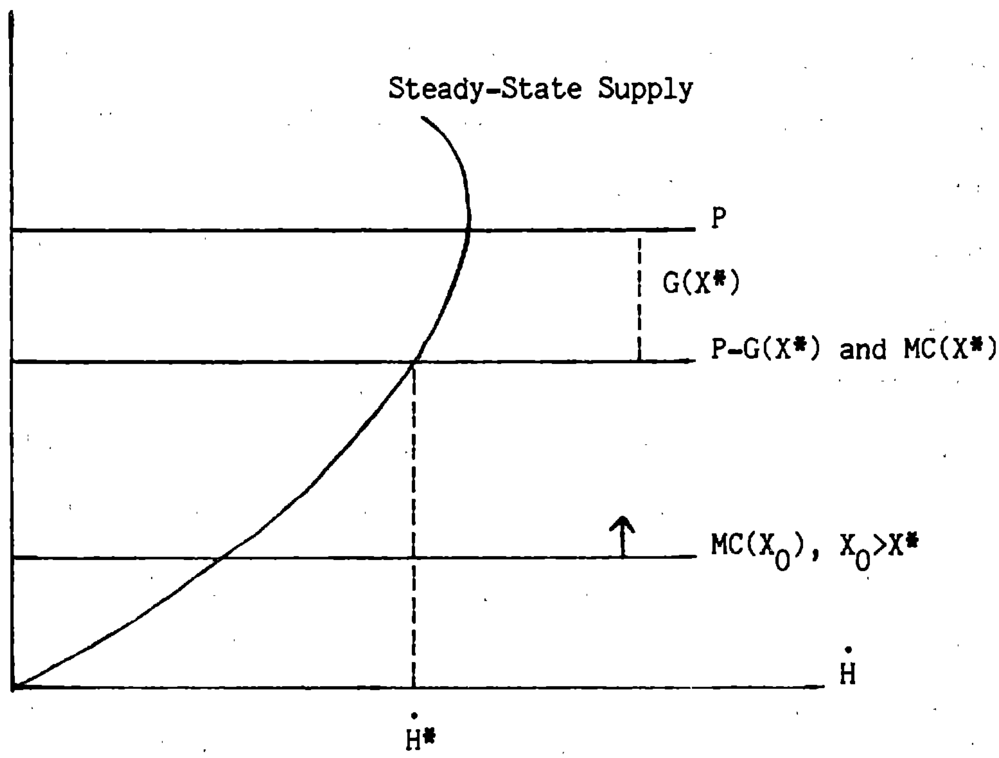

Figure 1 .--Deterministic fishery with perfectly elastic supply and demand.

instantaneous harvest rate quotas. (In this context, "quotas" means a system of individual firm quotas, because an aggregate industry quota alone would result in excessive effort rate and dissipation of rents.) This rate can also be achieved-by levying a per-unit tax equal to C(X*), the marginal value of stock, or by employing some other instrument. The first order equilibrium and optimum condition describing an interior optimal steady state is P-G(X*) = MC(X*), where P is the constant price of harvested fish, and MC is the instantaneous marginal harvest cost, which is constant with respect to harvest rate, but which rises with falling stock size (Herfindahl and Kneese 1974).

Ideally, the regulating authority would vary the tax or quotas through time until the stock size reaches X*. If, as depicted in Figure 1, initial stock size, X_0, is larger than X*, the optimal quota program consists of no quotas at all until X* is reached, and then an industry quota equal to H*. Similarly, a tax set equal to G(X(t)) would vary as stock size declined until the steady state was attained.

If one assumes that the level of the chosen instrument must be set once and for all, and can never be reset, a constant industry quota would not be able to drive the system along the fully optimal time path. It will take longer for the system to reach steady state under quotas. Morever, since the optimal level of the fixed quota is not H*, the final steady-state values of all variables, including X, will differ from those of the fully optimal solution. However, a fixed tax equal to G(X*) will drive the system along the fully optimal time path. As long as stock size is greater than X*, marginal harvest cost will be lower than price minus the tax, and fishermen will voluntarily commit all, available effort to the fishery. When stock size equals X*, the marginal cost curve coincides with P-T, and H* is the equilibrium harvest rate. This argument for the superiority of a fixed tax over a fixed industry quota follows symmetrical lines when initial stock size is less than X*.

STOCHASTIC STEADY-STATE MODEL

A continuous-time dynamic model to which Pontryagin's maximum principle is applied is used to identify the relevant marginal benefit and cost functions of fish production. Three restrictions are placed on the continuous-time approach: 1) stochastic disturbances change values only at the beginning of each time period, 2) instrument levels are changed only at the beginning of each period, and 3) the optimal and actual paths of all variables are assumed to reach the steady state very quickly. More will be said about the latter restriction below.

First, a deterministic model is employed. (Uncertainty is 'incorporated in a later section.) The objective is to maximize the present value of the stream of net social benefits, defined as consumer and producer surpluses, over time:

$$\max \int_0^{\infty} e^{-rt}\{ B(\dot{H}_t) - C(X_t,\dot{H}_t) \}dt \ ,$$

subject to the constraint that net stock growth rate equals natural growth rate minus harvest rate:

$$\dot{X}_t = F(X_t) - \dot{H}_t \ .$$

In addition, the maximization is also -subject to the initial stock and non-negativity constraints.

In the above formulae,

X_t is the size of fish stock at time t,

H_t is the instantaneous rate of harvest,

$B(H_t)$ is the instantaneous total consumption benefit rate,

$C(X_t,H_t)$ is the instantaneous total harvest cost rate,

r is the (constant) discount rate, and

$F(X_t)$ is the natural growth (surplus production) function,

A dot over a symbol indicates the derivative with respect to time of a cumulative quantity or stock variable.

The Hamiltonian expression is:

$$e^{-rt}\{ B(\dot{H}_t) - C(X_t,\dot{H}_t) + G_t[F(X_t) - \dot{H}_t] \} ,$$

where G_t is the co-state variable. The maximizing conditions are:

(1.01) $B_H(\dot{H}_t) - C_H(X_t,\dot{H}_t) - G_t = 0$,

(1.02) $-C_X(X_t,\dot{H}_t) + G_tF_X(X_t) = -\dot{G}_t + G_tr$, and

(1.03) $\dot{X}_t = F(X_t) - \dot{H}_t$,

where $B_H()$ and $C_H()$ are the partial derivatives of total benefit and total cost, respectively, with respect to harvest rate (i.e., marginal benefit and marginal cost), and $C_X()$ and $F_X()$ are partial derivatives with respect to stock size.

Assuming that the optimum time path rapidly leads to or approaches a steady state, the analysis is confined to steady-state situations, so set $G_t = X_t = 0$. The three equations can now be solved for optimal steady-state values of H_t, X_t, and G_t.

Rearranging 1.01 and 1.02 yields

(1.04) $G_t = B_H() - C_H()$, and

(1.05) $G_t = \dfrac{C_X()}{F_X() - r}$.

Recalling that G_t is defined as the marginal value of stock at time t, assuming that an optimum harvesting strategy will be followed from then on, one sees that equation 1.04 means the control variable, H_t, should be selected so as to set the difference between marginal benefit and marginal harvest cost equal to marginal stock value, given the values of X_t and G_t inherited from the previous instant.

Stock size, X, is not .a control variable. However, since primary interest is in the steady state to which the system is driven, one can treat the problem as one of choosing the optimal steady-state stock size. Seen from this perspective, equation 1.04 means that X should be chosen so as to set the marginal value of stock equal to the marginal cost of obtaining it. The term $B_H() - C_H()$ is marginal stock cost, as it is the rate at which profit must be foregone while the harvesting rate is temporarily reduced by one unit-per period so that stock size can be allowed to grow at the rate of one unit per period. Making use of the steady-state condition, H = F(X), one finds that marginal steady-state stock cost, MSC, is solely a function of X:

(1.06) $MSC(X) = B_H(F(X)) - C_H(X,F(X))$.

Equation 1.05 is derived by differentiating an expression‘ defining the present value of the net benefit stream when the optimal time path for the control variable is assumed to be followed at every instant. In solving the problem of which steady-state stock level to choose, however, one can use the more easily understood concept of the steady-state marginal stock value: the derivative with respect to X of the present value of the (constant) consumer and producer surplus stream to be yielded by any level of stock, if that level of stock were to be held constant forever. This steady-state marginal stock benefit is:

$$\frac{d[B(\bar{H}) - C(X,\bar{H})]}{r\,dX} .$$

Carrying out the differentiation and making use of the steady-state constraint, $\dot{H} = F(X)$, one finds that this is also a function solely of X, designated MSB:

(1.07) $MSB(X) = \frac{1}{r}\{ [B_H(F(X)) - C_H(X,F(X))]F_X(X) - C_X(X,F(X)) \}$.

It is easily shown that at the optimal level of X (chosen by equating MSB(X) with MSC(X)), MSB = G, where G is given by equation 1.05.

Figure 2 depicts the solution to the problem of choosing the optimal steady-state stock size, designated X*. The quantity T* is the optimal per-unit tax to charge fishermen if the fleet is characterized by competitive conditions so that the marginal harvest cost function, $C_H(X,H)$, is the industry fish supply curve for given levels of X. Since the fishing fleet will harvest at the rate at which marginal harvest cost equals ex-vessel price less the tax, $MSC(X) = B_H(X) - C_H(X)$ is a tax response function. When per-unit tax T is specified, the fleet will harvest, in a way which will drive the stock to the steady-state level implied by T = HSC(X). The corresponding steady-state harvest rate, H*, is found by examining the growth function, F(X), shown in Figure 2 as having the well known "dome" shape. The quantity X_{MSH} is the stock size which produces maximum sustainable harvest.

In this deterministic model, the tax, quota, and other instruments are equally efficient. The correct output rate, H*, and the correct. tax, T*, which will elicit H* (when stock size reaches X*) are known.

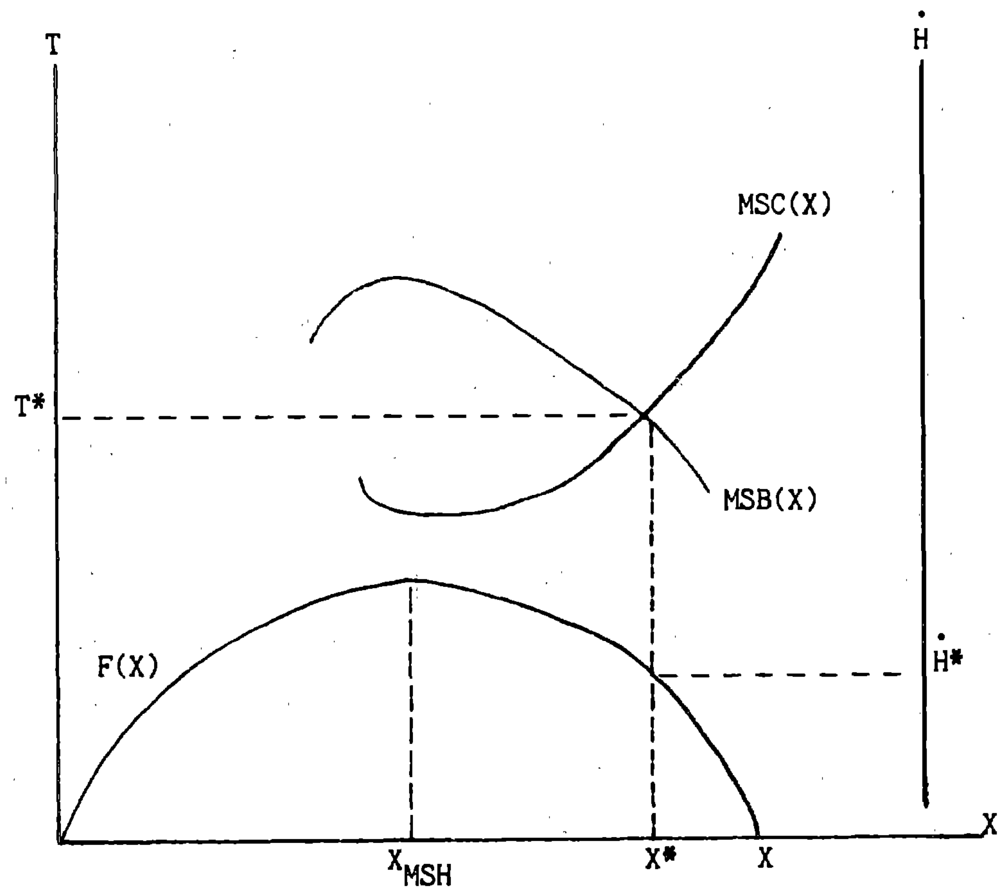

Figure 2.--Optimal steady state.

Either instrument will drive the stock to X*, the correct level, as long as initial stock size is to the right of X_{MsH}.

A problem in both deterministic and stochastic steady-state models is the inability of a quota to drive the stock to a desired equilibrium level on the left side of the stock growth dome. Such an equilibrium is unstable, as is shown in Figure 3. The quantity X* is the desired equilibrium stock level and H*, the corresponding harvest rate. If harvest is controlled by a quota set at H* and if initial stock is greater than X*, the rate of natural growth of the stock, F(X), is greater than H*, and stock grows to X'. If initial stock level is less than X*, then F(X) is less than H*, so the stock is driven to the level which it would reach in the absence of-regulation.

This means that if the desired stock level is on the left side of the dome, a constant tax is superior to a constant quota, rather than equally as efficient. In the stochastic case, the simplest way to deal with the problem is to refuse to consider using a quota unless the expected value of optimal equilibrium stock size resulting from the use of a quota is “safely” to the right of X_{MsH}. Of course, “safely” would be arbitrarily defined. Another alternative is to account explicitly for the probability of failure to reach desired stock level through use of a quota and the attendant welfare loss.

Returning to general discussion of the deterministic model, one notes that a change in one or more of the parameters of the system, whether or not expected, results in a shifting of both MSB(X) and MSC(X), and a new optimal steady-state stock size. The new value of X* does not depend on whether or not the new parameter values are known with certainty before the change occurs, although the course of the

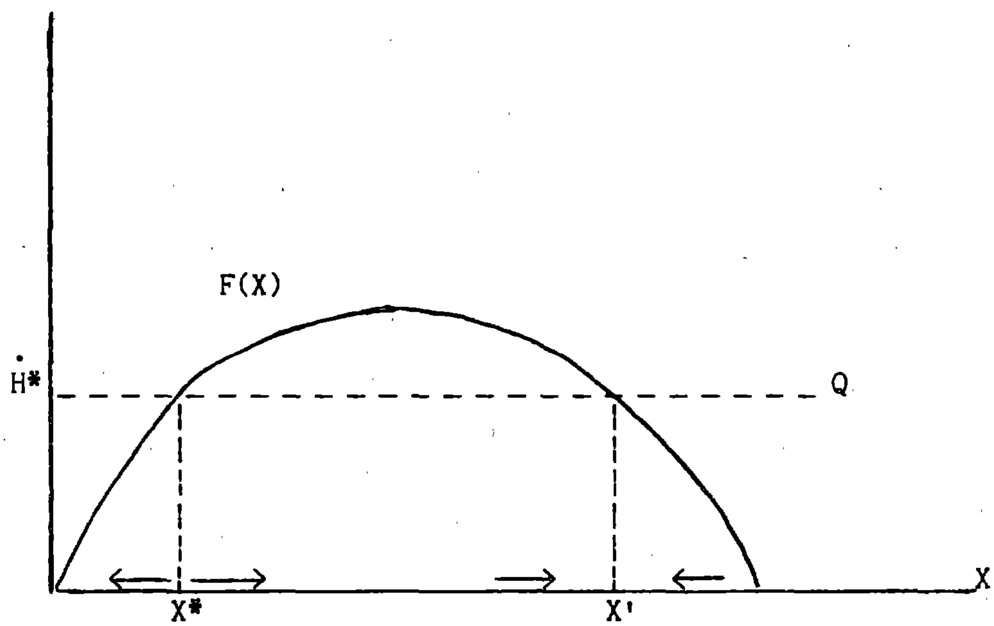

Figure 3.--Instability of quota equilibrium on left side of yield dome.

optimal transition path between the old X* and the new one does (Clark and Munro 1975). In order to make the model stochastic, introduction of random disturbances into some of the parameters, which change value only at the beginning of each time period, is necessary. The disturbances are thus the only discrete-time elements of an otherwise continuous-time model. Assuming for the moment that the values taken by the disturbances each period are known, one has a model in which the parameters change every period, resulting in a full-information optimal time path which leads to a new steady-state X* and corresponding optimal tax and harvest rate quota. Such a path is illustrated in Figure 4. It is crucial to assume that the amount of time spent in steady state during each period is great relative to the amount of time spent in transition from the previous period's steady state. This allows focusing on the choice of regulatory instrument on the basis of performance during the steady states.

The basic equations are given more specific form:

$$B(\dot{H}_t,u) = (b_0 + u)\dot{H}_t - \frac{b_1}{2}\dot{H}_t^2 \text{ (total benefit)}, \tag{1.08}$$

$$C(X_t,\dot{H}_t,v) = (c_0 + v - c_2X_t)\dot{H}_t + \frac{c_1}{2}\dot{H}_t^2 \text{ (total fishing cost)}, \tag{1.09}$$

and

$$\dot{X}_t = F(X_t,w) - \dot{H}_t = w + f_0 - f_1X_t - \dot{H}_t \text{ (stock growth)}, \tag{1.10}$$

where all the subscripted lower case Roman letters are known parameters and u, v, and w are stochastic disturbances. Parameters b_1, c_1, and c_2 are assumed to be non-negative. The disturbances are assumed to be independently distributed, with expected values of zero and known variances.

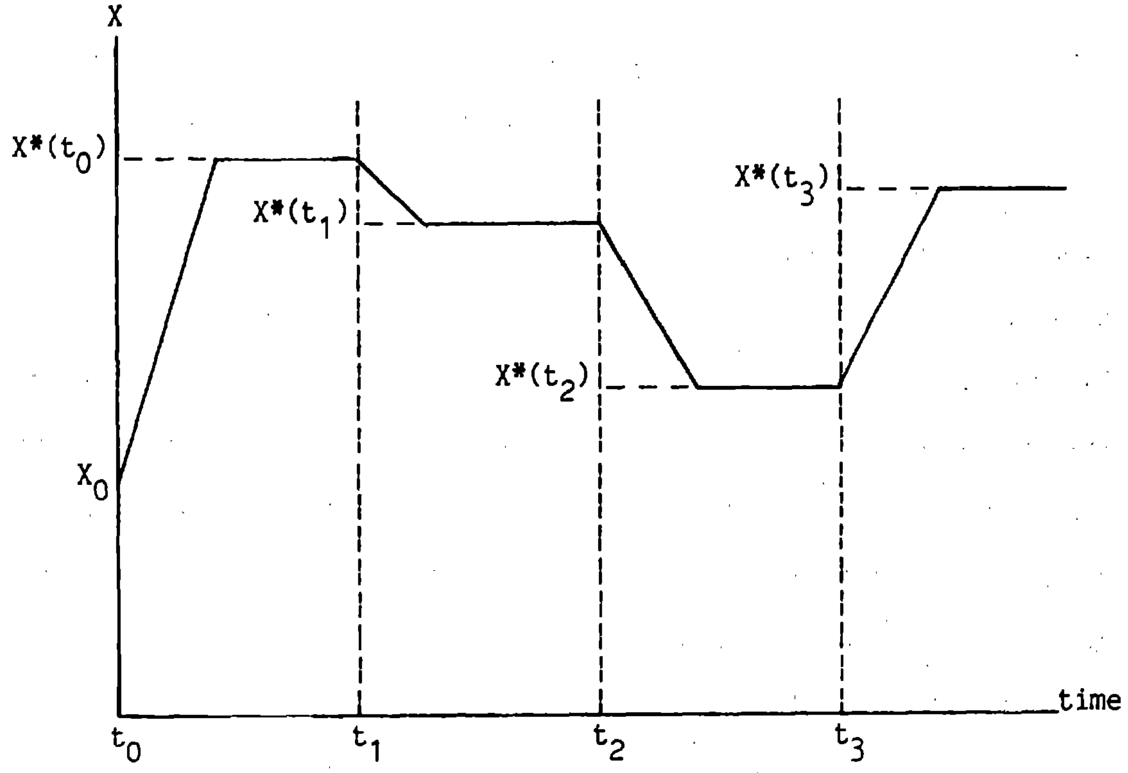

Figure 4.--Example of full-information optimal time path of stock size.

The problem facing the regulators is that they must decide which instrument to use, and at what level to set it, before they know the values of u, v, and w, all of which are determinants of the full-information optimum levels. This uncertainty is depicted in Figure 5. For diagrammatic simplicity, the distributions of the disturbances are assumed to consist of only two possible values for each of u, v, and w, with each value having a-probability of occurring of 0.5. There are two possible states of the world.- If state 1 occurs, MSB_1 and MSC_1 are the true positions of marginal-stock benefit and cost because disturbance values are u_1, v_1, and w_1. The position of the growth function in state 1, when $w = w_1$, is $F_1()$. Analogous statements apply to MSB_2, MSC_2, and $F_2()$. The quantities X_i*, T_i*, and H_i* are, respectively, the full-information optimal values of stock size, specific tax, and, harvest rate, if state i occurs. The quantity T" is the level of the tax which would actually be imposed, resulting in a stock level of either X_1" or X_2". The quantity Q^1 is level of the, quota which would actually be imposed, resulting in either X_1' or X_2'. The triangles shaded with vertical lines represent welfare loss which would be incurred in each state of the world if T" is imposed, and the triangles shaded with horizontal lines are welfare loss under a quota of Q'.

The regulators face two decisions: which instrument to employ, and at what level to set the chosen instrument. The latter is easily dispensed with, for if growth and marginal cost and benefit are linear with parallel shifting, setting the chosen instrument at the expected value of its optimal level will result in the minimum expected value of welfare loss.

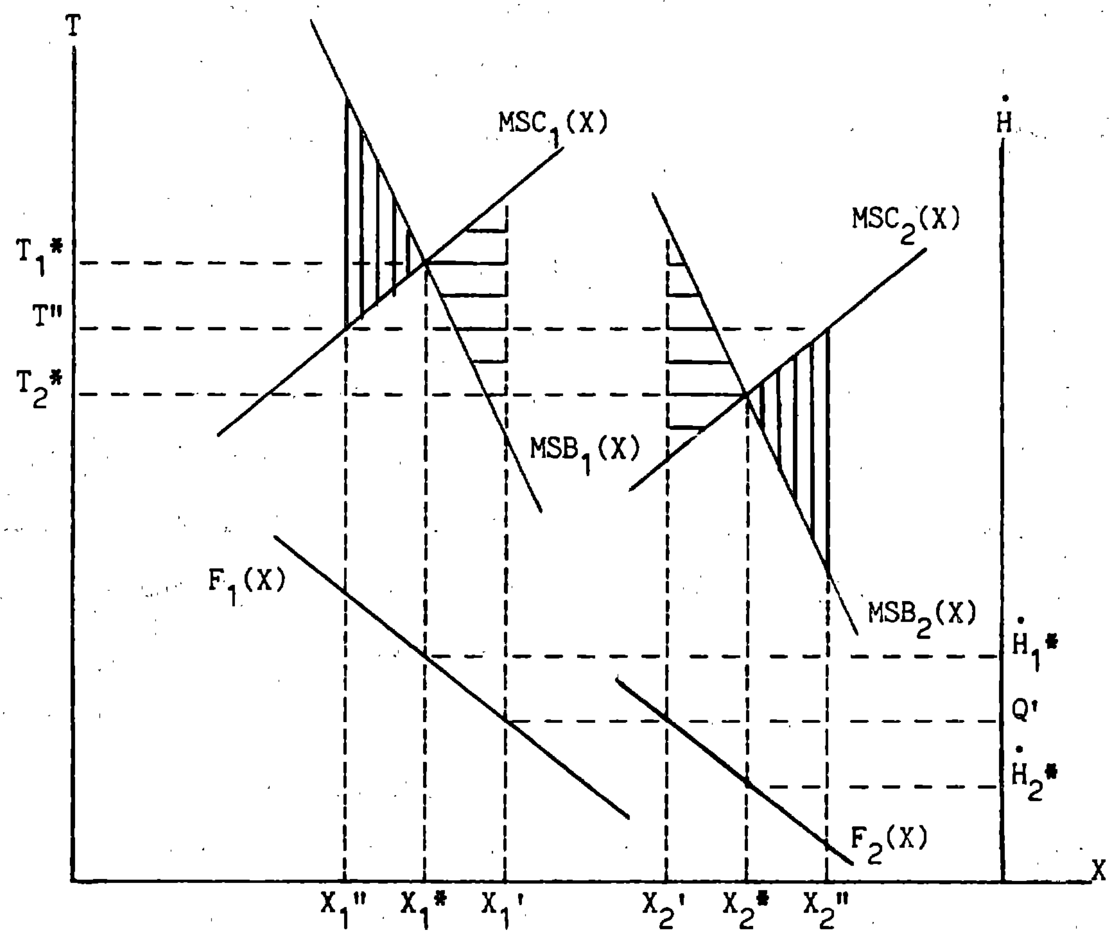

Figure 5.--Expected welfare losses under tax and quota.

Given that the instrument selected will be set at its optimal level, the question of which instrument gives the lowest expected welfare loss is raised. That question is answered by developing explicit expressions for the expected welfare loss with each instrument. The expected welfare loss with the quota, WL', is

$$(1.11)\quad E[WL'] = E[\frac{(X' - X^*)(MSC(X') - MSB(X'))}{2}] ,$$

where E[] is the expectation operator. This is simply the expected value of the area of the welfare loss triangle exemplified in Figure 5. Similarly,., the expected welfare loss with the specific tax is:

$$(1.12)\quad E[WL''] = E[\frac{(X'' - X^*)(MSC(X'') - MSB(X''))}{2}] .$$

The disturbances in equations 1.11 and 1.12 have been suppressed in the interest of simplicity.

Before proceeding, one must write explicit expressions for MSB and MSC incorporating equations 1.08, 1.09 and 1.10 into 1.06 and 1.07. Begin with MSC:

$$(1.13)\quad MSC(X,u,v,w)$$

$$= u - v - (b_1+c_1)w + [b_0-c_0-(b_1+c_1)f_0] + [c_2 + (b_1+c_1)f_1]X .$$

This can be simplified by writing:

$$(1.14)\quad MSC(X,x) = x + A + BX ,$$

where $x = u - v - (b_1+c_1)w$,

$A = b_0 - c_0 - (b_1+c_1)f_0$, and

$B = c_2 + (b_1+c_1)f_1$.

Following the same process with MSB yields:

$$(1.15)\quad MSB(X,u,v,w) = -\{\tfrac{1}{r}f_1(u-v) + [c_2 + (b_1+c_1)f_1]w - f_1[b_0 - c_0 - (b_1+c_1)f_0] + c_2f_0 - f_1[2c_2 + (b_1+c_1)f_1]X\}\ ,\ \text{or}$$

$$(1.16)\quad MSB(X,n) = n + C + DX\ ,$$

where $n = \frac{1}{r}[-f_1(u-v) + Bw]$,

$$C = \frac{1}{r}[-f_1A + c_2f_0]\ ,$$

$$D = \frac{1}{r}[-f_1(c_2 + B)]\ .$$

Some of the terms in equations 1.11 and 1.12 can now be described more fully.

The full-information optimal stock level, X*, is found by equating MSC with MSB:

$$(1.17)\quad MSC(X^*,x) = MSB(X^*,n) \longrightarrow x + A + BX^* = n + C + DX^*\ .$$

Solving for X* gives

$$(1.18)\quad X^* = \bar{X} + \frac{n-x}{B-D}\ ,$$

where $\bar{X} = \frac{C-A}{B-D}$.

The tax, T", is set at the expected value of the full-information optimal tax, T* :

$$(1.19)\quad T'' = E[T^*] = E[MSC(X^*,x)] = A + B\bar{X}\ .$$

The harvest rate quota, Q', is set at the expected value of the full-information optimal harvest rate, $\dot{H}^*$:

$$(1.20)\quad Q' = E[\dot{H}^*] = E[F(X^*,w)] = f_0 - f_1\bar{X}\ .$$

The level to which stock will be driven when the tax is imposed, X", is found by equating MSC with T":

$$(1.21)\quad MSC(X'',x) = T''.$$

Solving for X"' gives:

(1.22) $X'' = \bar{X} - \frac{x}{B}$.

Finally, X', the level to which stock is driven when the quota is imposed, is found by writing the steady state condition:,

(1.23) $F(X',w) = Q'$,

and solving for X', which gives

(1.24) $X' = \bar{X} + \frac{w}{f_1}$.

Substituting equations 1.14, 1.16, 1.18, and 1.24 into equation 1.11, multiplying through, and taking expectations gives

(1.25) $E[WL'] = \frac{1}{f_1}(d_{xw}^2 - d_{nw}^2) + \frac{B-D}{2f_1^2} d_w^2 + \frac{1}{2(B-D)}(d_x^2 + d_n^2 - 2d_{xn}^2)$,

where d^2 with subscripts is the variance or covariance of the indicated variables. Substituting equations 1.14, 1.16, 1.18, 1.19, and 1.22 into equation 1.12, multiplying through, and taking expectations gives

(1.26) $E[WL''] = \frac{D^2}{2B^2(B-D)} d_x^2 - \frac{D}{B(B-D)} d_{xn}^2 + \frac{1}{2(B-D)} d_n^2$.

Subtracting equation 1.26 from equation 1.25 and simplifying gives (following Weitzman) the "coefficient of comparative advantage of specific taxes over quantities", the CCA:

(1.27) $CCA = \frac{B+D}{2B^2} d_x^2 - \frac{1}{B} d_{xn}^2 + \frac{B-D}{2f_1^2} d_w^2 + \frac{1}{f_1}(d_{xw}^2 - d_{nw}^2)$.

Recalling that B and D are the slopes of marginal steady-state stock cost and benefit, respectively (see equations 1.14 and 1.16), and that x and n are their disturbances, it can be easily shown that the first two terms of equation 1.27 are identical to Weitzman's coefficient 'of comparative advantage of prices over quantities (when marginal benefit and marginal cost are correlated). This is because levying a specific tax on harvested fish is equivalent to controlling the price received by fish stock owners (the public) for holding stock. The remaining tens in equation 1.27 represent the effect of uncertainty about the stock growth function and therefore about the relationship between harvest rate and equilibrium stock level.

Now in order to write the CCA in terms of the original parameters of the model., substitute from equations 1.14 and 1.16 into equation 1.27 to obtain:

$$(1.28)\quad CCA = \{\frac{rc_2 + (b_1+c_1)(f_1+r)f_1}{2r[c_2 + (b_1+c_1)f_1]^2}\}(d_u^2 + d_v^2)$$

$$+ \{\frac{(b_1+c_1)^2(r-f_1)}{2r[c_2 + (b_1+c_1)f_1]} - \frac{(b_1+c_1)^2 f_1 c_2}{2r[c_2 + (b_1+c_1)f_1]^2}$$

$$+ \frac{2(b_1+c_1)(f_1-r) + c_2}{2r} + \frac{(r-f_1)[c_2 + (b_1+c_1)f_1]}{2rf_1^2}\}d_w^2.$$

The first term in brackets [] is positive, but the second term in brackets []. cannot be signed without the parameter values of a specific fishery, even when ex-vessel price and marginal harvest cost are constant, i.e., when $b_1 = c_1 = 0$. Therefore, the sign of the CCA, and hence, the ranking of the tax and quota instruments, depends on the fishery in question. Note, however, that if the growth function is not stochastic ($d_w^2 = 0$), the CCA is clearly positive, and the tax is always preferred.. This is not surprising, because x and n are negatively correlated when w is known to be zero, and as Weitzman shows, negative correlation between marginal benefit and marginal cost increases the relative attractiveness of price regulation, other things being equal.

This is because fluctuations of the marginal benefit and Cost curves in opposite directions cause greater variation in the full-information optimal output rate, and. a fixed price allows the realized output rate to fluctuate in the same direction, thereby remaining relatively close to the full-information optimum.

CHAPTER. 2

LINEAR MODEL: SINGLE-PERIOD NET BENEFIT

INTRODUCTION

In this and the following chapters, the steady-state assumption is discarded. In this chapter, expressions for net benefit accumulated during the course of each single period under tax and quota systems in a linear model are derived. These expressions will be incorporated into the dynamic programming analysis of optimal instrument choice in the next chapter. Rules for finding approximately optimal levels of each instrument when the stock size at the beginning of a period is independent of harvest in the previous period are also obtained in this chapter, and expected net benefit expressions for each system are compared. Neither instrument can be shown to be consistently superior.

The model used in the analysis is constructed first under the assumption that the level of capital is exogenously fixed. Then the model is reconstructed under the alternative assumption that capital is endogenous, with fishermen choosing the level of capital each year on the basis of expected profit over the coming season. Expressions for predicted fleet size, optimal tax level, and expected present value of' the single-period fishery are not presented for tax regulation because they are impossible to derive when endogenous capital enters the marginal harvest cost curve in the particular manner chosen in this chapter. This specification of the marginal-cost-with-capital curve was chosen because, for quota regulation, it allows retention of the' linear-quadratic dynamic. programming formulation needed for the analysis in Chapter 3. While it is possible to alter the model in a way that permits derivation of the single-period expressions for tax regulation, there is no way to construct the model which will permit retention of the linear-quadratic formulation for tax regulation when capital is endogenous.

SOME NOTATION

"The following notation list revises and expands that of the previous chapter:

r is the (constant) discount rate.

k is the instrument index: $k = 1$ for quota, 2 for tax, and so on for any other instruments which might be considered.

i is the period index. It is time measured discretely. $i = 1,\ldots,\infty$.

t is time measured continuously. t runs from $i-1$ to i during period i.

Z_{ki} is the level of instrument k during period i.

$X_t = X_k(X_{i-1}, Z_{ki}, t, u_i, v_i, w_i)$ is stock size at instant t. It is a function of stock size at the beginning of the period (when $t = i-1$), which is X_{i-1}, and of the level of instrument k in period i. It is a stochastic function of the growth disturbance, w_i, (with w_i assuming a new unknown value at the beginning of each period), and of the benefit and cost disturbances, u_i and v_i, through their effect on the equilibrium harvest rate. The subscript k indicates that the form of the function is different for each type of instrument.

X_0 is the stock size at t = 0, i.e., initial stock size in period 1.

H_t is the equilibrium instantaneous rate of harvesting at instant t. At each instant, H_t is determined by equating marginal benefit (demand) and marginal harvest cost (supply) plus tax, or else it is fixed by quota. Optimal quotas are assumed to be binding, although occasionally equilibrium harvest rate may be lower than the aggregate fleet quota. This may occur when demand is unexpectedly low, harvest cost is unexpectedly high, and stock growth rate is unexpectedly low. It is especially likely at the end of the season, when stock size may become low.

$B(H_t,u_i)$ is the instantaneous rate of accrual, of total consumption benefit during period i, a function of instantaneous harvest rate (the entire harvest is assumed to be purchased, with no dead loss, and with net additions to inventory assumed to confer the same benefit as consumption). A stochastic disturbance, u_i, shifts the function at the beginning of each period.

$C(X_t,H_t,v_i)$ is the instantaneous rate of total harvest cost incurrence during period i. It is a function of both harvest rate and stock size, and is shifted each period by a stochastic disturbance, v_i.

$R_k(X_{i-1},Z_{ki},u_i,v_i,w_i)$ is the present value of net benefit which accrues over the course of period i (the single-period net benefit function), and is equal to

$$\int_{i-1}^{i} e^{-rt}\{ B(\dot{H}_t,u_i) - C(X_t,\dot{H}_t,v_i) \}dt .$$

It is a function of beginning-of-season stock size, instrument level, and growth disturbance because of the effect each has on stock size and equilibrium harvest rate.

MODEL

The three basic functions of the model are repeated here from Chapter 1:

(2.01) $$B(\dot{H}_t, u_i) = (b_0 + u_i)\dot{H}_t - \frac{b_1}{2}\dot{H}_t^2,$$

(2.02) $$C(X_t, \dot{H}_t, v_i) = (c_0 + v_i - c_2X_t)\dot{H}_t + \frac{c_1}{2}\dot{H}_t^2,$$

(2.03) $$\dot{X}_t = F(X_t) - \dot{H}_t = f_0 - f_1X_t - \dot{H}_t ,$$

where F(X) is the natural stock growth function, u_i and v_i are stochastic disturbances, and subscripted lowercase Roman letters are fixed, known. parameters. Parameters b_1, c_1, and c_2 are assumed to be non-negative. All the other parameters may be positive, negative, or zero.

No stochastic disturbance is shown in the stock growth function because it is used only to derive the stock size functions

$$X_t = X_k(X_{i-1}, Z_{ki}, t, u_i, v_i, w_i) .$$

The disturbance in the stock size functions is assumed to have the same distribution for all the functions $X_k()$, i.e., for all regulatory instruments. All disturbances in the system are assumed to have expected values of zero, to be serially uncorrelated and uncorrelated with each other, and to have known variances.

QUOTA REGULATION

Under a quota system (k = 1), the level of the instrument, Z_{1i}, is Q_i. For notational convenience, the following discussion will refer to the first period, in which i = 1 and t goes from 0 to 1, but all expressions are identical in every period. Moreover, the period subscript i = 1 will be omitted from Q and from the disturbances.

The expected present value of instantaneous net benefit accumulated during the first period (discounted to time t = 0) is

(2.04) $E[R_1(X_0,Q,u,v,w)]$

$$= E[\int_0^z e^{-rt}\{ (b_0+u)Q - \frac{b_1}{2}Q^2 - (c_0+v-c_2X_t)Q - \frac{c_1}{2}Q^2\}dt] ,$$

where z is the time at which the fishing season closes, perhaps by decree, e.g., for the protection of gravid females, or perhaps by natural event, such as the onset of winter weather or the annual departure of the fish. The parameter z can take any value between zero and one. The open season and the period both begin at time t = 0. Instantaneous harvest rate, H_t, is fixed at the level of the aggregate quota, Q.

Equation 2.03 is used to derive $X_1(X_0,Q,t,u,v,w)$. The stock growth function during the fishing season is

(2.05) $\dot{X} = f_0 - f_1X - Q$.

Solving this differential equation (see Appendix B for details of this derivation), and adding the stochastic disturbance, w, yields

(2.06) $$X_t = \frac{f_0 - Q}{f_1}(1-e^{-f_1t}) + X_0e^{-f_1t} + w .$$

Substituting this expression for X_t into equation 2.04, and rearranging, integrating, and taking expectations gives

$$(2.07) \quad E[R_1(X_0,Q)] = (W+YX_0)Q - LQ^2,$$

$$\text{where } W = \{b_0 - c_0 + \frac{f_0}{f_1}c_2\} \frac{e^{-rz}-1}{-r} - \{\frac{f_0}{f_1}c_2\} \frac{e^{-(f_1+r)z}-1}{-(f_1+r)},$$

$$Y = c_2 \frac{e^{-(f_1+r)z}-1}{-(f_1+r)},$$

$$\text{and} \quad L = \{\frac{c_2}{f_1}\} \left(\frac{e^{-rz}-1}{-r} - \frac{e^{-(f_1+r)z}-1}{-(f_1+r)}\right) + \{\frac{b_1+c_1}{2}\} \frac{e^{-rz}-1}{-r}.$$

In some fisheries, fishing during the current period may have no effect on the size of the fish stock at the beginning of the following period. This could occur if the number of eggs laid each year at spawning time is independent of stock size. Fishing always, however, affects the stock size at each instant during the open season, as described by equation 2.06. Then the objective of the regulating authority, after observing the stock size at the beginning of each period, is to set the quota at Q*, the level which maximizes $E[R_1(X_0,Q)]$. (Risk neutrality on the part of the authority is assumed, since the variance of $R_1(X_o,Q)$ is proportional to Q.) To find Q*, one differentiates $E[R_1()]$ with respect to Q and sets it equal to zero:

$$\frac{\partial}{\partial Q} E[R_1(X_0,Q^*)] = W + YX_0 - 2LQ^* = 0 .$$

The term -2L is the slope of the cumulative net marginal benefit of harvest curve, given the assumption of constant harvest rate throughout the season. Similarly, $W + YX_o$ is the interceptof the cumulative net marginal benefit curve. Setting equation 2.08 equal to zero is equivalent to equating cumulative marginal benefit of harvest with cumulative marginal cost.

Solving for Q* gives

(2.09) $$Q^* = \frac{W + YX_0}{2L}.$$

Substituting Q* into equation 2.07 gives the maximum possible expected present value to be obtained by an optimal quota during the current period:

(2.10) $$E[R_1(X_0,Q^*)] = \frac{(W + YX_0)^2}{4L}.$$

The second order condition for a maximum to exist is $-2L < 0$, or $L > 0$. On examination of the expression for L in equation 2.07, it can be seen that this condition does hold for all values of f_1, the negative of the slope of the stock growth function, given the assumption of non-negative values for all other parameters, and provided that the effect of stock size on fishing cost, c_2, is not zero.

The term Y is unambiguously non-negative. However, W could take either sign, giving rise to the possibility that the optimal quota, could be negative; This would be the case when fishing is prohibitively expensive at small stock sizes (i.e., b_0-c_0 is highly negative), and when initial stock size is low. A negative quota might actually be achieved in some fisheries by stocking.

Variances of the stochastic disturbances do not appear in the decision rule for setting the quota, and the disturbances themselves have been replaced by their expected values (namely zero). If quota regulation is chosen, the fishery can be managed as if the disturbances were known to be equal to zero. The assumption of risk neutrality is required for this conclusion.

Also, when stock size has no effect on fishing costs ($c_2 = 0$), the optimal quota is equal to the harvest rate that would result if the fleet were allowed to fish without regulation. The optimal quota in this case is

$$\frac{b_0 - c_0}{b_1 + c_1},$$

provided that natural stock growth can be fast enough to produce this yield rate until the end of the planning horizon without extinction of the stock (Levhari, et al. 1981). This is also the expected harvest rate obtained by equating marginal benefit (demand) with marginal harvest cost (supply) when, $c_2 = 0$, provided that $b_0 - c_0$ is positive, that b_1 and c_1 are not both zero, and that, if $c_1 = 0$, storage costs are not zero.

TAX REGULATION

Under a-per-unit tax system ($k = 2$), the level of the instrument in period i, Z_{2i}, is T_i. The discussion in this section will again refer to the first period for notational convenience, and the period subscript, $i = 1$, will again be omitted.

The expected present value of net benefits accumulated in the first period is

$$(2.11)\quad E[R_2(X_0,T)]$$

$$= E[\int_0^z e^{-rt}\{ (b_0+u)\dot{H}_t - \frac{b_1}{2}\dot{H}_t^2 - (c_0+v-c_2X_t)\dot{H}_t - \frac{c_1}{2}\dot{H}_t^2 \}dt],$$

where X_t is a function of T-and H_t is a function of X_t and T, as is shown below.

Instantaneous harvest rate is determined by equating the demand function with the marginal harvest cost function plus tax and solving **for** equilibrium harvest rate:

$$(2.12)\quad \dot{H}_t = \frac{b_0 - c_0 + u - v + c_2X_t - T}{b_1 + c_1} .$$

Inserting equation 2.12 into the stock growth function gives

$$(2.13)\quad \dot{X} = \frac{A}{-(b_1+c_1)} + \frac{x}{-(b_1+c_1)} + \frac{B}{-(b_1+c_1)}X_t - \frac{T}{-(b_1+c_1)} ,$$

where $A = b_0 - c_0 - (b_1+c_1)f_0$,

$B = c_2 + (b_1+c_1)f_1$, and

$x = u - v$.

Solving this differential equation for X_t and adding the growth disturbance, w, gives

$$(2.14)\quad X_t = \frac{A - T}{-B}(1 - e^{-F}t) + X_0\, e^{-F}t - \frac{1}{B}(1 - e^{-F}t)x + w ,$$

where $F = \dfrac{B}{b_1+c_1}$.

Inserting the above expressions for $\dot{H}_t$ and X_t in equation 2.11 and rearranging, integrating, and taking expectations yields

$$(2.15)\quad E[R_2(X_0,T)] = I + JX_0 + RX_0^{\,2} + (W'+Y'X_0)T - L'T^2 ,$$

where a prime (') after a character not designating a function or a variable indicates that the character represents a term in a tax system expression having a counterpart in a quota system expression. (Absence of a prime does not necessarily indicate that the character represents a quota system term.) The following additional notation is used in equation 2.15:

$$I = \frac{1}{2(b_1+c_1)}\Big[\ \{ (b_0-c_0-\frac{A}{B}c_2)^2 + (d_u^{\ 2} + d_v^{\ 2})(1-\frac{c_2}{B})^2 + c_2^{\ 2}d_w^{\ 2} \} \frac{e^{-rz}-1}{-r}$$

$$+ \ 2c_2\{ (b_0-c_0-\frac{A}{B}c_2)\frac{A}{B} + \frac{1}{B}(d_u^{\ 2} + d_v^{\ 2})(1-\frac{c_2}{B}) \} \frac{e^{-(F+r)z}-1}{-(F+r)}$$

$$+ \ c_2^{\ 2}\{ (\frac{A}{B})^2 + (\frac{1}{B})^2(d_u^{\ 2} + d_v^{\ 2}) \} \frac{e^{-(2F+r)z}-1}{-(2F+r)} \ \Big] ,$$

$$J = \frac{1}{2(b_1+c_1)}[2c_2\{b_0-c_0+\frac{A}{B}c_2\}\frac{e^{-(F+r)z}-1}{-(F+r)} + c_2^{\ 2}\{2\frac{A}{B}\}\frac{e^{-(2F+r)z}-1}{-(2F+r)}] ,$$

$$R = \frac{c_2^{\ 2}}{2(b_1+c_1)} \ \frac{e^{-(2F+r)z}-1}{-(2F+r)} ,$$

$$W' = \frac{1}{b_1+c_1}\frac{c_2}{B}[\ \{b_0-c_0-\frac{A}{B}c_2\}\frac{e^{-rz}-1}{-r} - \{b_0-c_0-2\frac{A}{B}c_2\}\frac{e^{-(F+r)z}-1}{-(F+r)}$$

$$- \ \{\frac{A}{B}c_2\}\frac{e^{-(2F+r)z}-1}{-(2F+r)} \ ] ,$$

$$Y' = \frac{1}{b_1+c_1}\{\frac{c_2^{\ 2}}{B}\}(\frac{e^{-(F+r)z}-1}{-(F+r)} - \frac{e^{-(2F+r)z}-1}{-(2F+r)}) , \text{ and}$$

$$L' = \frac{1}{2(b_1+c_1)}[\{1-(\frac{c_2}{B})^2\}\frac{e^{-rz}-1}{-r} + \{(\frac{c_2}{B})^2\}(2\frac{e^{-(F+r)z}-1}{-(F+r)} - \frac{e^{-(2F+r)z}-1}{-(2F+r)})].$$

If the size of the stock at the beginning of the next period is independent of fishing during the current period, the maximization problem has a one-period planning horizon. The objective of the regulating authority is to select the level of T which maximizes $E[R_2(X_0,T)]$; this is accomplished by differentiating E[R()] with respect to T and setting the derivative equal to zero:

$$(2.16)\quad \frac{\partial}{\partial T}E[R_2(X_0,T^*)] = W' + Y'X_0 - 2L'T^* = 0 \ .$$

Solving for the optimal tax, T*, gives

$$(2.17)\quad T^* = \frac{W' + Y'X_0}{2L'} \ .$$

Substituting equation 2.17 into equation 2.15 gives the maximum possible cumulative net benefits:

$$(2.18)\quad E[R_2(X_0,T^*)] = I + JX_0 + RX_0^2 + \frac{(W'+Y'X_0)^2}{4L'} \ .$$

The second order condition for a maximum in T to exist is $-2L' < 0$, which in turn requires that L' be positive. Examination of the expression for L' above shows that L' is unambiguously positive as long as C_2, the effect of stock size on costs, is greater than zero, and as long as f_1, the negative of the slope of the stock growth function, is greater than or equal to $-c_2/(b_1+C_1)$. This condition assures that F is non-negative. When $f_1 < -c_2/(b_2+c_1)$, however, the sign of L' can not be determined, i.e., it is impossible to solve for the negative value of f_1, if one exists, at which L' becomes negative.

The term W' appears to be impossible to sign, but it could never be sufficiently negative to imply a negative optimal tax, a subsidy to fishermen. This is because unregulated equilibrium effort and harvest rates are always greater than optimal effort and harvest rates, and must always be restrained rather than encouraged. If, however, the effect of changing stock size on fishing costs is negligible ($c_2 = 0$), both W' and Y' are zero, and therefore, so is the optimal tax. No regulation is necessary if the stock is capable of growing rapidly enough to support the untaxed market equilibrium harvest rate. continuously until the end of the planning interval without extinction of the stock.

The variances of the stochastic disturbances do not appear in the expression for the optimal tax. This means that, as in the case of a quota system, the stochastic fishery can be managed simply as if the disturbances were known to be equal to their expected values, as long as risk neutrality is assumed.

TAX VERSUS QUOTA REGULATION WITH A ONE-PERIOD PLANNING HORIZON

Continuing. for the time being the assumption that stock size at the beginning of the next period is independent of fishing in the current period, one can then make the choice between tax and quota systems. This is done by comparing the two on the basis of expected present value of net benefits to be accrued over the course of the upcoming season, given the size of the fish stock at the beginning of the season. The “coefficient of comparative advantage of tax over quota," the CCA, can be defined as

$$(2.19)\quad CCA = E[R_2(X_0,T^*)] - E[R_1(X_0,Q^*)]$$

$$= I + JX_0 + RX_0^2 + \frac{(W' + Y'X_0)^2}{4L'} - \frac{(W + YX_0)^2}{4L} .$$

In general, there can be no presumption about the sign of the CCA, because the sign depends on X_0 (although it is possible that the sign could be the same for all X_0 in the relevant range). The sign of $I + JX_o + RX_0$ must be positive, since this expression can be shown to be the integral over time of a discounted squared quantity. Thus, if there were conditions under which it could be shown that $(W'+Y'X_0)^2/4L'$ is always larger than $(W+YX_0)^2/4L$, then the superiority of the tax would be guaranteed under those conditions. However, no such conditions can be readily identified.

Expression 2.19 is quadratic in X_0. Written in standard quadratic form, it is

$$(2.20)\ CCA = (I + \frac{W'^2}{4L'} - \frac{W^2}{4L}) + (J + \frac{W'Y'}{2L'} - \frac{WY}{2L})X_0 + (R + \frac{Y'^2}{4L'} - \frac{Y^2}{4L})X_0^2.$$

If all three coefficients of this quadratic were of the same sign, the CCA would also have the same sign for all positive X_0. However, signs cannot be determined without specific values for the parameters of the model.

QUOTA WITH CAPITAL ENDOGENOUS

All analysis thus far has treated capital as being exogenously fixed. Now an alternative approach will be taken, with the level of, capital being selected by the fleet at the beginning of each season, using knowledge of the stock size at the beginning of the season and of the level of the quota to be set by the authority for the upcoming season. Capital is then assumed to be fixed during the season.

This approach requires the assumption that the level of capital can be varied instantaneously at the beginning of the season, as soon as initial stock size and the quota level are known. Charles (1983a) described an alternative model in which the decision on investment for the current season is made in the previous period, based- on knowledge of escapement at the end of the open fishing season. Both the model discussed here and the one constructed by Charles require the assumption that investors consider only their expected profits for the immediate season, and not for seasons farther in the future. For simplicity, processing capital is assumed to be exogenously fixed, as before.

Before proceeding, one should consider the production functions of harvesting and processing. In general terms, the output of processed fish is a function of two inputs, unprocessed fish and a composite input similar to fishing effort, which will be-called processing effort. The processing production function is assumed to embody a fixed ratio between unprocessed fish and processed fish. Thus, the derived marginal benefit accruing to processors who buy fish from fishermen is found by simply subtracting the marginal cost of producing processing effort from the marginal benefit accruing to buyers of processed fish (who are assumed to be the final consumers>.

One can assume that harvesting and processing capital are fixed during the season, and can only be varied between seasons.. Hence, due to the short run, fixity of capital, short run marginal cost is positively sloped, and short term rents, or producer surplus, can accrue to the fixed capital in both harvesting and processing sectors, even if long run marginal and average costs are constant.

Figure 6 illustrates these concepts. The diagram represents the situation at a single instant in time. The line labeled MB is marginal benefit to consumers, and MC_p is instantaneous marginal processing cost, exclusive of the cost of purchasing unprocessed fish from fishermen. Thus, MB - MC_p is derived demand by processors for

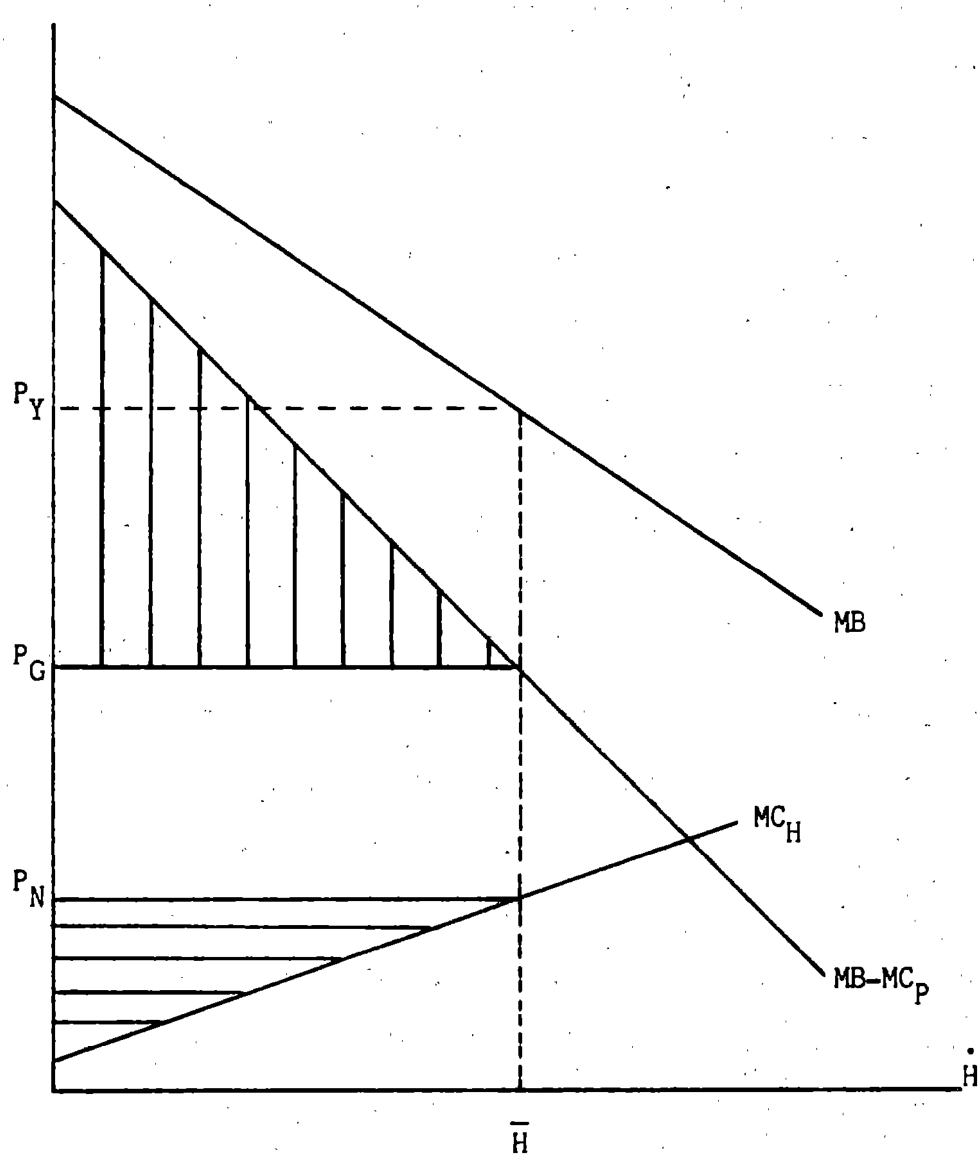

Figure 6.--Consumer surplus, produce? surplus, and rent to fishery.

unprocessed fish. It is more steeply sloped than the marginal benefit curve because MC_p rises with harvest rate. The position of the marginal processing cost curve, and therefore the distance between MB and $MB - MC_p$, is determined by the quantity of processing ‘capital which is fixed during the season. (One should keep in mind that MC_p is marginal variable cost.)

The position of marginal (variable) harvesting cost (MC_H) at each instant is determined by the stock size at that instant and by the quantity of fishing capital, which is fixed during the season.

The quantity H is the instantaneous harvest rate that results from regulation, either by quota or by tax. The resulting charges are the the processors' output price, P_y; the gross ex-vessel price, PC (the amount paid by processors for unprocessed fish);’ and the net ex-vessel price, P_N (the amount received by fishermen after paying the tax or buying quota rights to catch another pound of fish).

The area shaded with vertical lines represents the instantaneous accrual rate of the sum of consumer surplus and processors’ gross producer surplus, or gross rents to fixed processing capital. The term “gross” applies to the producer surplus because fixed cost has not been subtracted from it. Consumer surplus can also be measured separately as the area between the MB curve and the processor price, P_y: The area shaded with horizontal lines represents instantaneous rate -of gross producer surplus accrual to fishermen, or gross rents to fixed harvesting capital. The unshaded rectangle between gross ex-vessel pride and net ex-vessel price is rent to the fishery, which may be transferred to the regulating authority in the form of revenue from taxes or the sale of quotas, or may accrue to the owners of quotas, if quotas are granted free of charge to selected fishermen.

Under a system in which shares of the aggregate instantaneous harvest rate quota are allocated to specific firms, each firm will choose-its level of capital for the upcoming season so as to minimize the expected present value of the cost of harvesting at the assigned rate all season. The fleet will, choose the level of capital which minimizes

$$(2.21)\quad E\left[\int_0^z e^{-mt}\{\, VC(X_t,K,Q,v) + FC(K)\,\}dt\right],$$

where VC is total variable harvesting cost, FC is fixed harvesting cost, and m is the interest rate.

Total variable harvesting cost can have the form

$$(2.22)\quad VC(X_t,K,\dot{H}_t,v) = (c_0+v-c_2X_t)\dot{H}_t + \frac{c_1}{2K}\dot{H}_t^{\,2}.$$

This is similar to equation 2.02, with the exception that the slope parameter, c_1, is now divided by the quantity of capital, K, available during the season. Fixed cost, FC(K), is the instantaneous cost rate of holding capital, and is equal to (m+d)K, where d is the depreciation rate, and the units of capital are defined so that the purchase price of capital is one dollar.

If equation 2.22 and (m+d)K are substituted into equation 2.21 (with Q substituted for H_t), then differentiating with respect to K and setting the result equal to zero yields

$$(2.23)\quad \int_0^z e^{-mt}\{\, \frac{c_1}{2K}Q^2 - (m+d)K\,\}dt = 0\ .$$

Integrating gives

(2.24) $$\frac{c_1}{2K}Q^2\,\frac{e^{-mz}-1}{-m} + (m+d)K\,\frac{e^{-mz}-1}{-m} = 0 .$$

Solving for equilibrium level of harvesting capital yields

(2.25) $$K = \left(\frac{c_1}{2(m+d)}\right)^{1/2} Q .$$

The (fixed) cost of holding capital is assumed- to be borne only during the fishing season, and not during the closed season. This assumption makes the algebra simpler, but can be easily discarded by multiplying the expression for equilibrium capital level by $(e^{-mz}-1)\ /\ (e^{m}-1)$, which gives equilibrium K when capital must be held for a full year.

Having determined the equilibrium level of capital chosen by fishermen in response to the announcement of the upcoming season's aggregate quota level, one is now in a position to derive the optimal rule for setting the quota level. The first step in this process is to derive an expression for single-period net benefit. This is defined-as total consumer benefit minus harvesting and processing costs:

(2.26) $$E[R_1(X_0,Q)] = E\left[\int_0^z e^{-rt}\{\, B(Q,u) - VC(X_t,K,Q,v) - FC(K)\,\}dt\right] .$$

Substituting equation 2.01 for B(), equation 2.22 for VC(), and (m+d)K for FC() gives

(2.27) $E[R_1()] =$

$$E\left[\int_0^z e^{-rt}\{\,(b_0+u)Q - \frac{b_1}{2}Q^2 - (c_0+v-c_2X_t)Q - \frac{c_1}{2K}Q^2 - (m+d)K\,\}dt\ \right].$$

Substituting equation 2.25 for equilibrium K and taking expectations gives

$$(2.28)\quad E[R_1()] = \int_0^z e^{-rt}\{ (b_0-c_0-(2c_1(m+d))^{1/2} + c_2X_t)Q - \frac{b_1}{2}Q^2 \}dt \ .$$

Finally, integrating and simplifying yields

$$(2.29)\quad E[R_1(X_0,Q)] = (W''+YX_0)Q - L''Q^2,$$

where $W'' = \{b_0-c_0-(2c_1(m+d))^{1/2} + \frac{f_0}{f_1}c_2\} \frac{e^{-rz}-1}{-r} - \{\frac{f_0}{f_1}c_2\} \frac{e^{-(f_1+r)z}-1}{-(f_1+r)}$,

$$Y = c_2 \frac{e^{-(f_1+r)z}-1}{-(f_1+r)} \text{ , and}$$

$$L'' = \{\frac{c_2}{f_1}\} (\frac{e^{-rz}-1}{-r} - \frac{e^{-(f_1+r)z}-1}{-(f_1+r)}) + \{\frac{b_1}{2}\}(\frac{e^{-rz}-1}{-r}) \ .$$

The optimal level of the aggregate quota when fishing in the current period does not affect stock size in the following period is found by differentiating $E[R_1(X_0,Q)]$ with respect to Q, setting the result equal to zero, and solving for Q* :

$$Q^* = \frac{W'' + YX_0}{2L''} \ .$$

These expressions are very similar to the corresponding expressions derived earlier in this chapter for the model with capital assumed to be exogenously fixed. The expression designated L” is identical to L except that the second term in L" does not contain c_1. The expression designated W' is identical to W, except that the first term in W does not contain $-(2c_1(m+d))^{1/2}$. Thus, both W" and L” are smaller than their counterparts in the exogenous capital-version of the model, and it is impossible to say whether the quota would be set too high or too low if capital were incorrectly assumed to be exogenous.

CHAPTER 3

LINEAR MODEL: ANALYTICAL DYNAMIC PROGRAMMING

INTRODUCTION

In this chapter, the assumption that stock size at the beginning of any period is independent of the quantity harvested during the previous period is discarded. Dynamic programming is employed to derive rules for setting quota and tax instruments at their optimal levels; and the, expected present values of net benefits produced by each instrument over multiple periods, are- compared.

DYNAMIC PROGRAMMING FORMULATION

The following discussion draws extensively on Koenig's exposition (1984). One can assume that the fishery managing authority has committed to a regulatory strategy at the beginning of the first (current) period which will be adhered to from now until the terminal period, period i. (The-value of i will be set later at infinity.) The objective of the authority is to select the optimal instrument and the time path of the chosen instrument which maximizes the expected present value of the stream of single-period net benefits produced:

$$(3.01) \quad E[\sum_{i=o}^{\bar{i}} p^i R_{ki}(X_{i-1}, Z_{ki})] ,$$

where i is the terminal period, p is the nonstochastic discount factor, and all other notation is as described in the previous chapter.

The second of these two tasks must be performed first, and must be performed for every instrument under consideration, because only after optimal time paths for every instrument have been found can the expected present value of net benefits generated by each be computed and compared.

When the fishery system is stochastic, the optimal time paths of a regulatory instrument and other system variables cannot be specified fully in advance. Given that the level of the instrument in each period need not be set until the beginning of the period, a rule is derived which specifies the instrument level on the basis of information available at that time. Some of that information becomes available only at the beginning of the period, e.g., beginning-of-period stock size.

There is a distinction which should be made clear. The chosen instrument will be reset each period, using knowledge of the stock size at the beginning of the period. However, the choice between instruments is to be made only once, at the beginning of the first period. Which instrument is chosen will depend on stock size at the beginning of the first period.

Following standard dynamic programming procedure, it isassumed that the rule for setting the instrument at the optimal level in all periods, beginning with period i, has already been determined. The expression $V_{ki}(X_{i-1})$ can represent the expectation, as of period 1, of the maximum possible present value, as of period i, of net benefits that can be obtained by setting instrument k optimally in, every period from period i onward. This is a function of the size of the stock at the beginning of period i, X_{i-1}. The function V(X) can be written in the following recursive relationship in which the instrument subscript k has been suppressed:

$$(3.02) \quad V_i(X_{i-1}) = \max_{Z_i} E[\ R_i(X_{i-1}, Z_i) + pV_{i+1}(X_i)\ ] .$$

This equation reduces the complex problem of determining the entire optimal time path of the control variable, Z, to the easier problem of determining the optimal value of Z during period i only. It expresses the expected present value of benefits to be obtained by beginning an optimal management program during the period i as the maximum of the sum of benefits in period i plus the expected present value of benefits to be obtained by beginning an optimal program during the next period, discounted to period i. In other words, one must find the optimal value of Z_i, knowing that the setting of Z will affect the stock size available at the beginning of the following period, given that future Z's will be chosen optimally for whatever period i+1 initial stock size is.

In general, the functional form of $V_{i+1}(X_i)$ is unknown, but in the special case of a single-period net benefit function that is quadratic in both control and stock variables, and of a stock size function which is linear in both types of variables, the functional form is known to be quadratic (Chow 1975). Thus, for each instrument,

$$(3.03) \quad V_{i+1}(X_i) = s_0(i+1,\bar{i}) + s_1(i+1,\bar{i})X_i + s_2(i+1,\bar{i})X_i^2,$$

where s_0, s_1, and s_2 are known parameters.

Since any stock remaining after the end of the terminal period will have no value, $s_0(\bar{i}+1,\bar{i}) = s_1(\bar{i}+1,\bar{i}) = s_2(\bar{i}+1,\bar{i}) = 0$.

Moreover, it follows from equation 3.02 that

$$(3.04)\quad s_0(i) + s_1(i)\, X_{i-1} + s_2(i)\, X_{i-1}^{\,2}$$
$$= \max_{Z_i} E[\, R_i(X_{i-1}, Z_i) + p\{\, s_0(i+1) + s_1(i+1)\, X_i + s_2(i+1)\, X_i^{\,2}\}\,]$$

where i has been omitted from s_0 s_1, and s_2- This equation permits the derivation of relationships between s(i) and s(i+l), as will be demonstrated below.

QUOTA REGULATION

In many fisheries, fishing does not proceed year round, but is subject to a closed season. Fishing may be halted each year by regulations designed, for example, to protect gravid females, or by natural events such as the annual departure of the fish or the onset of winter weather. The period can be divided into two seasons: the first season open for fishing, and the second season closed. The open season, of length z, begins at the same time the period begins, when t = i-1, and ends at t = i-l + z; while the closed season, of length l-z, begins at t = i-l + z and ends when the period ends, at t = i. Then the size of the stock at the end of the fishing season in the first period is found by substituting z for t in equation 2.06, derived in the previous chapter:

$$(3.05)\quad X_z = \frac{f_0 - Q}{f_1}(1 - e^{-f_1 z}) + X_0 e^{-f_1 z} + w\,.$$

The size of the stock at the end of the closed season, which is also the end of period 1 (the beginning of period 2) is found by substituting X_z for X_0 and l-z for t in equation 2.06 and setting

$Q = 0$. After substituting equation 3.05 for X_z and rearranging, 'one gets

(3.06) $X_1 = M - NQ + PX_0 + w$,

where $M = \frac{f_0}{f_1}(1 - e^{-f_1})$,

$N = \frac{e^{-f_1}(e^{f_1 z} - 1)}{f_1}$, and

$P = e^{-f_1}$.

The quantities M, N, and P are non-negative for all values of f_1.

Substituting expression 2.07 derived in the previous chapter for $E[R_{11}(X_0,Q)]$ and expression 3.06 for X_1 in equation 3.04, yields

(3.07)

$$V_{11}(X_0) = \max_Q \{ (W+YX_0)Q - LQ^2$$

$$+ p\, E[\, s_0(2) + s_1(2)(M-NQ+PX_0+w) + s_2(2)(M-NQ+PX_0+w)^2] \}.$$

Rearranging and taking expectations gives

(3.08) $$V_{11}(X_0) = \max_Q \{ S + (E+UX_0)Q - VQ^2 + ZX_0 + ps_2(2)P^2X_0^{\,2} \} ,$$

where $S = p[s_0(2) + s_1(2)M + s_2(2)(M^2 + d_w^{\,2})]$,

$E = W - p[s_1(2)N + 2s_2(2)MN]$,

$U = Y - 2ps_2(2)NP$,

$V = L - ps_2(2)N^2$ (not the function $V_{ki}()$), and

$Z = p[s_1(2)P + 2s_2(2)MP]$,

and where W, Y, and L are as defined in the previous chapter.

Differentiating the terms in brackets [] with respect to Q, setting the resulting derivative equal to zero, and solving for the optimal quota, Q*, gives

$$(3.09)\quad Q^* = \frac{E + UX_0}{2V}.$$

Substituting Q* into equation 3.08 gives

$$(3.10)\quad V_{11}(X_0) = S + \frac{(E+UX_0)^2}{4V} + ZX_0 + ps_2(2)P^2X_0{}^2.$$

Collecting terms containing X_0 and $X_0{}^2$ and recalling the relationship between $V_{11}(X_0)$ and $V_{12}(X_1)$ in equation 3.04 result in the relationships between s(1) and s(2), which are also the general relationships between s(i) and s(i+1):

$$(3.11)\quad s_0(i) = ps_0(i+1) + p[s_1(i+1)M + s_2(i+1)(M^2 + dw^2)] + \frac{E^2}{4V},$$

$$(3.12)\quad s_1(i) = \{\frac{-pNU}{2V} + pP\}\, s_1(i+1) + 2ps_2(i+1)MP + \frac{[W - 2ps_2(i+1)MN]U}{2V}, \text{ and}$$

$$(3.13)\quad s_2(i) = ps_2(i+1)P^2 + \frac{[Y - 2ps_2(i+1)NP]^2}{4[L - ps_2(i+1)N^2]}.$$

If the maximization problem in equation 3.08 is to be well defined, the second order condition $-2V < 0$, or $V > 0$ (where the symbol “V" without subscript or parentheses is defined as above, and does not refer to the function $V_i(X)$), must be met. Making use of this condition and of the fact that $s_2(i+l) = 0$, one can see from equation 3.13 that s_2 is non-negative for all $i < i$. If $V < 0$, then $s_2(i+1) \geq L/pN^2$, which is sufficient to quarantee non-negative s_2, since p, L, and N are all non-negative.

Figure 7 displays the behavior of equation equation 3.13. The value of $s_2(i)$ can be determined at any i by beginning at i+l, when s_2 is known to be zero, and working backward in time, recursively applying equation 3.13.

Since the terminal date would usually be specified as infinity, it is essential to determine what happens to s_2 as i grows large, or equivalently, what happens as the time interval traversed backward from i grows large. The process can be viewed in Figure 7 as one in which s_2,(i) begins at the point where equation 3.13 intersects the vertical axis, and moves upward along the curve as i retrogresses from i toward the present. Whether or not s_2 converges to a stable steady-state value, s_2, depends on whether or not the curve twice intersects the 45° identity line representing the function $s_2(i) = s_2(i+l)$ and having a slope of one.

Intersection is guaranteed if the point of tangency between a ray from the origin and equation 3.13 is below the identity line. This point is found by equating $s2(i)/s_2(i+l)$ with $ds_2(i)/ds_2(i+l)$, and solving for the value of $s_2(i+l)$ at which the-tangency occurs. The solution is $s_2(i+l) = Y/2pNP$. The slope of both the ray and s,(i) at this point is pP^2, and the necessary and sufficient condition for the existence of a stable s_2 is that it be less than one;

The interseason discount factor, p, is defined as e^{-r}, rather than l/(l+r), so as to be consistent with the intraseason discount factor, and so as to make algebraic operations easier. Therefore,

(3.14) $pP^2 = e^{-r} e^{-2f_1} < 1$

implies that $f_i > -r/2$, where f_1 is the negative of the slope of F(X),

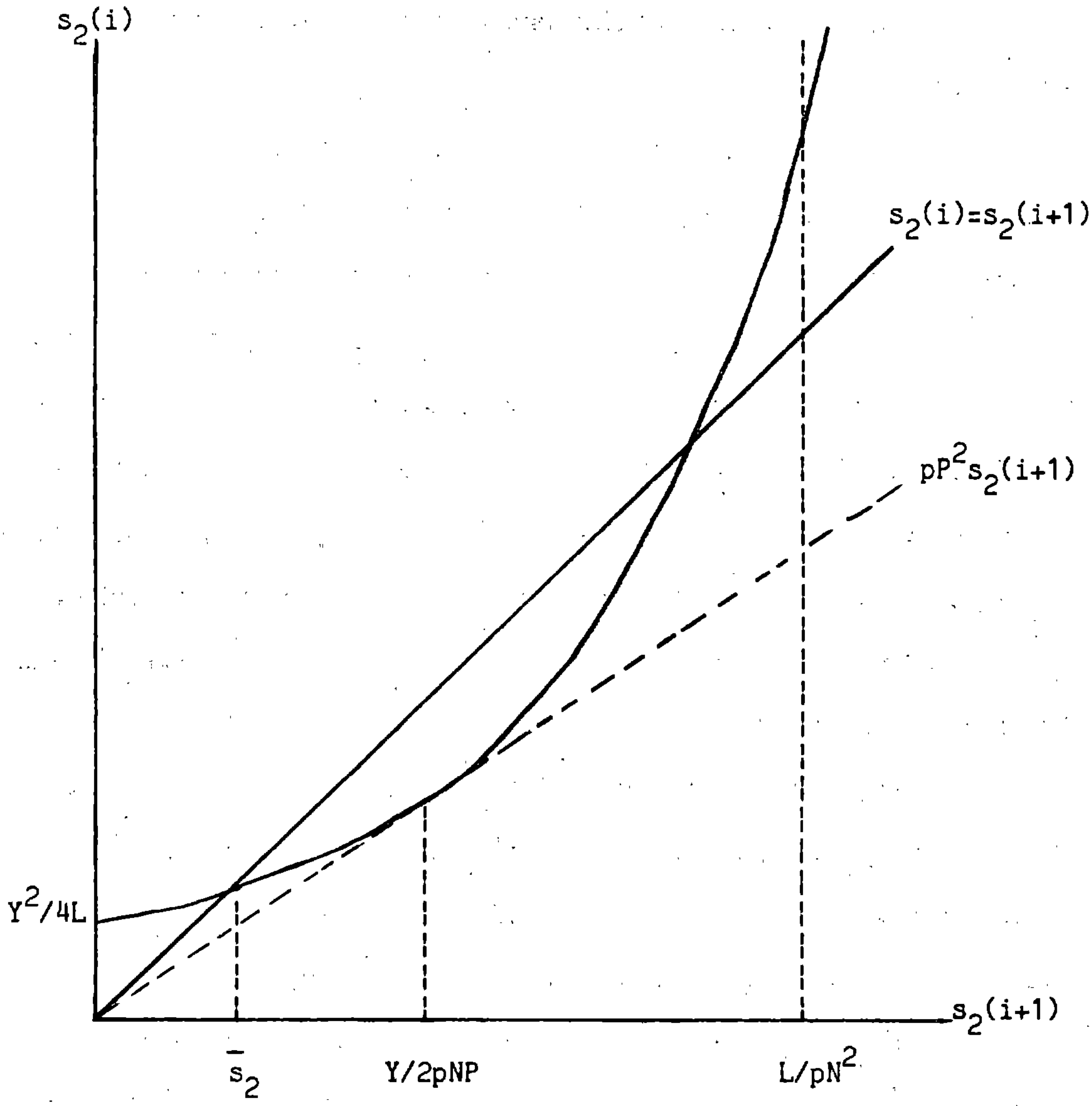

Figure 7.--Phase diagram for equation 3.13.

the natural stock growth function. This-is similar to the condition necessary for convergence of s_2 in Koenig's discrete-time model, which is

$$(1-f_1)^2/(1+r) < 1 ,$$

where f_1 is the negative of the slope of the natural growth function

$$X_{i+1} - X_i = f_0 - f_1 X_i .$$

Condition 3.14 means that when the slope of the growth function (-f,) is any more than slightly positive, no stable steady-state value of s_2 exists. As the terminal date grows more distant from the present, s_2 increases without bound, eventually exceeding L/pN^2. An analytical rule for settling a quota at the optimal level cannot be obtained by the method described in this chapter, because no maximum exists in Q. This inconvenient result can be attributed to the use of linear approximation to a dome-shaped growth curve. If the slope of the growth function truly were a positive constant the stock could grow without bound, and if it did so at a sufficiently rapid rate (f_1 larger than half the interest rate), it would pay to refrain from harvesting for an indefinite period of time, or better yet, to add fish to the stock as rapidly as possible. The expected net present value of the stock, $V(X_0)$, is a quadratic function of stock size, with the coefficient of X (s_2) being positive. Therefore, the value of the stock must eventually begin to rise as stock size increases. Moreover, the value rises at an an ever-increasing absolute rate. With stock size itself growing at an ever-increasing absolute rate over time, the combined effect is a relative rate of growth in value of the stock over time which exceeds the interest rate, thus making abstinence from fishing optimal..

When stock size is low, i.e., on the left side of the growth dome, and linear approximation to the dome results in failure to meet the necessary condition for convergence of s_2, useful analytical results cannot be obtained, even by a method, such as approximation of the present value function, in which convergence is assumed. Numerical methods applied to a model with a nonlinear growth function must be used.

If condition 3.14 is not met, it may still be possible to solve. equation 3.13 for a steady-state value of s_2, but this value could only be negative. Since $s_2(i+l)$ has been shown to be positive for all i, s_2 could never actually attain such a steady state.

It is obvious from Figure 7 that if convergence does occur then s_2 meets the condition

(3.15) $Y^2/4L \leq \bar{s}_2 < Y/2pNP$.

Steady-state values of s_0, s_1, and s_2 are found by, replacing both s(i) and s(i+l) with s in equations 3.11, 3.12, and 3.13, respectively, and solving for s. Since equation 3.13 is quadratic in s, there are, two solutions, but Figure 7 makes it clear that only the lower one is stable, and is the one to which s_2 will actually tend. The stable steady-state value of s_2 is

(3.16) $$\bar{s}_2 = \frac{L(1-pP^2) + pNPY - [\ (L(1-pP^2) + pNPY)^2 - pN^2Y^2\ ]^{1/2}}{2pN^2} .$$

Equation 3.12 shows that $s_1(i)$ is linear in $s_1(i+1)$. Assuming that s_2 converges, s_1 will converge to a stable steady state only if the absolute value of the slope of equation 3.12 is less than 'one, that is, if

$$(3.17)\quad \left|\frac{-pNU}{2V} + pP\right| < 1 .$$

Since U and V must be positive if s_2 converges, -pNU/2V is negative. If it is so strongly negative that it is less than -pP (making the slope of equation 3.12 negative), condition 3.17 implies that pP > -1 + pNU/2V. The term pP is defined as $e^{-(f_1+r)}$, and is positive for all values of f_1. Also, pNU/2V < 1 when f_1 is positive or somewhat negative. A glance at the definitions of U and V and at Figure 7 shows that U < V, and pN can be shown to be less than one for all $f_1 > 0$. Hence, a sufficient condition for convergence of s_1 when -pNU/2V is less than -pP is that f_1 is positive or only slightly negative. If -pNU/2V is greater than -pP (the slope of equation 3.12 is positive), condition 3.17 implies that pP < 1 + pNU/2V. Since pP is less than one for all $f_1 > -r$, this condition would be violated only if f_1 is less than some value less than -r. Hence, if there exists a value of f_1 which is sufficiently negative to prevent convergence of s_1, it is probable that the condition for convergence of s_2 (namely, $f_1 > -r/2$) is also violated.

However, if a situation should arise in which s_2 converges, but s_1 does not, the implication for setting the quota is the same as when s_2 fails to converge. The optimal quota level is

$$Q^* = \frac{E + UX_0}{2V} .$$

The term $-ps_1N$ is contained in E, and becomes negatively infinite when s_1 fails to converge, thereby overwhelming the finite positive terms in Q*. Thus, the quota should be set as low as is physically possible;

i.e., it should be zero, at most. Here again, this conclusion should be rejected, and an alternative analytical method or numerical methods should be employed.

The steady-state value of s_1 is

$$(3.18)\quad \bar{s}_1 = \frac{(W-2p\bar{s}_2MN)U + 4Vp\bar{s}_2MP}{2V(1-pP) + UpN} .$$

Equation 3.11 shows $s_0(i)$ to be linear in $s_0(i+1)$. Provided that s_1 and s_2 converge, s_0 converges to a stable value as long as $|p| < 1$, which is guaranteed when the discount rate is positive. The steady-state value of s_0 is

$$(3.19)\quad \bar{s}_0 = \frac{p[\bar{s}_1M + \bar{s}_2(M^2 + dw^2)]}{1-p} + \frac{[W - p(\bar{s}_1N-2\bar{s}_2MN)]^2}{4(1-p)V} .$$

There can be no guarantee that $s_1 > 0$ or $s_o > 0$, although if s_1 is non-negative, then s_o must also be non-negative.

Again, as in the single-period optimization case, the variances of the disturbances do not appear in equation 3.09, the expression for Q*. The quota fishery can be managed as if there were no uncertainty. Also, while U and V must be positive (condition 3.15 assures that U is positive) if the problem is to be well defined, E may be sufficiently negative to require a negative optimal quota.

The definitions of E, U, and V reveal that the expression for optimal quota level in the multiple-period case, which is

$$\frac{E + UX_0}{2V} ,$$

is similar to the expression for optimal quota. in the single-period case, which is

$$\frac{W + YX_0}{2L} .$$

The terms E, U, and V contain W, Y, and L, respectively. The difference is that new terms have been subtracted from W, Y, and L in the multiple-period expression. All of these new terms are positive, except ps_1N, which is one of two terms subtracted from W, and whose sign is unknown, due to the presence of s_1. Since the denominator is smaller in the multiple-period expression for Q* than in the single period case, and the numerator may actually be larger if ps_1N is strongly negative, it seems impossible to conclude that the optimal quota is smaller when current harvest affects stock size in the following period as one would expect.

However, it also appears that unless ps_1N is strongly negative, the optimal quota in the multiple-period case is more likely to be negative than in the single-period case. This is because although V is positive, it is smaller than L, and because it is no longer necessary that W be negative for this result to occur.

TAX REGULATION

Stock size at the end of the open season in the first period when the tax is set at T is found by substituting z for t in equation 2.14, derived in the previous chapter:

$$(3.20) \quad X_z = \frac{A + x - T}{-B}(1-e^{-Fz}) + X_0 e^{-Fz} + w ,$$

where the period subscript i=1 has been again omitted. Stock size at the end of the closed season (the beginning of the next period) is found by substituting X_z for X_0 and l-z for t in equation 2.14 and setting Q equal to zero (no harvest). After substituting equation 3.20 for X_z, and rearranging, we have

$$(3.21)\quad X_1 = M' + N'T + P'X_0 - N'x + w ,$$

where $M' = \frac{f_0}{f_1}(1-e^{-f_1(1-z)}) - \frac{A}{B}(e^{-f_1(1-z)} - e^{-(Gz + f_1)})$,

$N' = \frac{1}{B}(e^{-f_1(1-z)} - e^{-(Gz + f_1)})$,

$P' = e^{-(Gz + f_1)}$, and

$G = \frac{c_2}{b_1+c_1}$,

where a prime after a symbol indicates that the symbol refers to a term in a tax system expression which has a counterpart in the corresponding quota system expression. All of the above defined terms are non-negative, with the possible exception of M', which could be negative only if fishing were highly profitable at very low stock levels (b_0-c_0 highly positive, which would make A sufficiently positive).

Substituting expression 2.15 from the previous chapter for $E[R_{21}(X_0,T)]$ and expression 3.21 for X_1 in expression 3.04 gives

$$(3.22)\quad V_{21}(X_0) = \max_T \{ I + JX_0 + RX_0^2 + (W'+Y'X_0)T - L'T^2$$

$$+ p\, E[s_0'(2) + s_1'(2)(M'+N'T+P'X_0-N'x+w)$$

$$+ s_2'(2)(M'+N'T+P'X_0-N'x+w)^2] \} .$$

Rearranging and taking expectations yields

(3.23) $V_{21}(X_0)$

$$= \max_T \{ S' + (E'+U'X_0)T - V'T^2 + Z'X_0 + [R + p\, s_2'(2)\, P'^2]X_0^2 \} ,$$

where

$$S' = I + p[s_0'(2) + s_1'(2)M' + s_2'(2)(M'^2 + N'^2 d_u^2 + N'^2 d_v^2 + d_w^2)] ,$$

$$E' = W' + p[s_1'(2)N' + 2s_2'(2)M'N'] ,$$

$$U' = Y' + 2ps_2'(2)N'P',$$

$$V' = L' - ps_2'(2)N'^2,$$

$$Z' = J + p[s_1'(2)P' + 2s_2'(2)M'P'] ,$$

and I, J, R, W', Y', and L' are as defined in the previous chapter.

To find the optimal tax, one should differentiate the expression in {} with respect to T, set equal to zero, and solve for T*:

(3.24) $$T^* = \frac{E' + U'X_0}{2V'} .$$

Substitution of this expression for T* into equation 3.23 gives

(3.25) $$V_{21}(X_0) = S' + \frac{(E'+U'X_0)^2}{4V'} + Z'X_0 + [R + ps_2'(2)P'^2]X_0^2.$$

After collecting terms containing X_0 and X_o^2 and recalling the relationship between $V_{21}(X_o)$ and $V_{22}(X_1)$ in equation 3.04, one can derive the relationships between s'(l) and s'(2), which are also the general relationships between s'(i) and s'(i+l):

(3.26) $$s_0'(i) = ps_0'(i+1) + I + p[s_1'(i+1)M' + s_2'(i+1)(M'^2 + N'^2 d_u^2 + N'^2 d_v^2 + d_w^2)] + \frac{E'^2}{4V'},$$

(3.27) $$s_1'(i) = \{\frac{pN'U'}{2V'} + pP'\} s_1'(i+1) + J + 2ps_2'(i+1)M'P' + \frac{[W' + 2ps_2'(i+1)M'N']U'}{2V'},$$

and

(3.28) $$s_2'(i) = R + ps_2'(i+1)P'^2 + \frac{[Y' + 2ps_2'(i+1)N'P']^2}{4[L' - ps_2'(i+1)N']^2}.$$

If the maximization problem in equation 3.23 is to be well defined, the second order condition -2V' < 0, or V' > 0, must be met. Making use of this condition and of the fact that $s_2'(i+1) = 0$, it can be seen from equation 3.28 that s_2' is non-negative for all i ??i. (The term R must be non-negative for this result to hold unambiguously, and examination of the definition of R in the previous chapter shows this to be the-case.) If V' ??0, then $s_2'(i+l)$??L'/pN^2, which is. sufficient to assure non-negative s_2', provided that L' is non-negative. The term L' is guaranteed to be non-negative when $f_1 \geq -c_2/(b_1+c_1)$, and may be non-negative for other values of f_1.

If L' is negative, the maximization problem cannot be well defined, because the second order condition V' > 0 implies that s_2 is non-negative, but if $s_2 > 0$ and L', which is a component of V', is negative, V' cannot be positive.

Figure 8 displays the behavior of equation 3.28. The necessary and sufficient condition for convergence to a stable steady-state value, s_2', is difficult to derive, requiring solution of a third

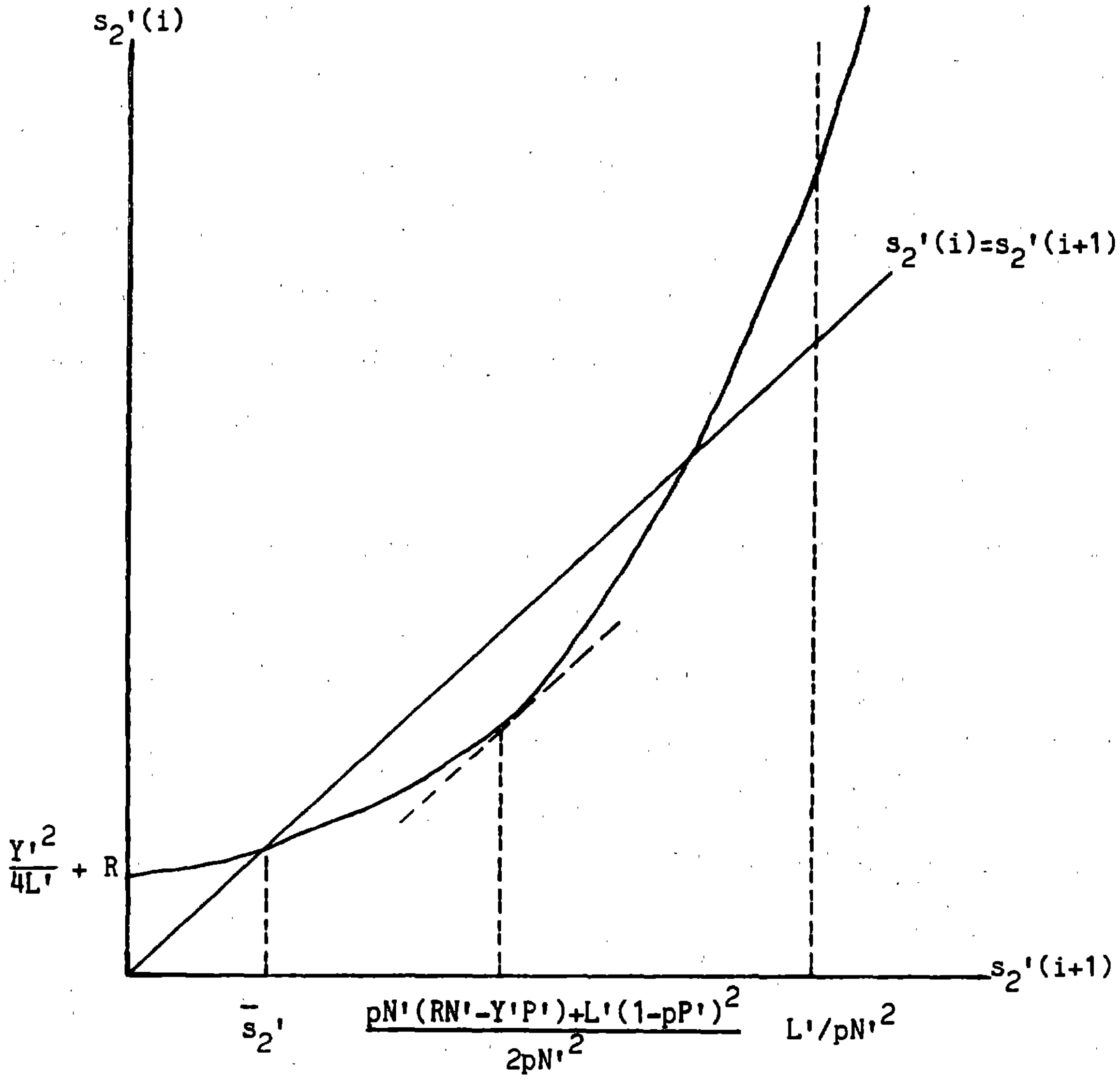

Figure 8. Phase diagram for equation 3.28.

degree polynomial in $s_2'(i+1)$. However, insight can be gained by observing that the point of tangency between equation 3.28 and a ray from the origin takes place at the point where the slope of a ray to the curve is at a minimum. The slope of this ray is

$$(3.29)\quad \frac{s_2'(i)}{s_2'(i+1)} = \frac{U'^2}{4V's_2'(i+1)} + \frac{R}{s_2'(i+1)} + pP'^2.$$

The minimum value of this expression must be less than one for convergence. Since the first two of the three terms on the right side are non-negative, it is clear that the sum of all three terms could not be less than one if the third term, pP'^2, were greater than one. Thus, it is necessary, but not sufficient, that $pP'^2 < 1$, which implies $f_1 > -r/2$. The sufficient condition is that f_1 be greater, than some value greater than -r/2, which is more limiting than the sufficient condition for convergence of s_2 in the quota case.

If convergence of s_2' fails, there will be no maximum of the present value in T, and the optimum tax is one which is high enough to prevent all fishing.

If convergence does occur, falls into the following range:

$$(3.30)\quad \frac{Y'^2}{4L'} + R < \bar{s}_2' < \frac{pN'(RN'-Y'P') + L'(1-pP'^2)}{2pN'^2}.$$

The right-most term is the value $s_2f(i+1)$ at which the slope of equation 3.28 is one and, therefore, overstates the maximum possible value of s_2'. It proved to be too difficult to solve for the value of $s_2'(i+1)$ at which a ray from the origin is tangent to the curve.

The-actual value of the stable steady-state s_2' is

$$(3.31)\quad \bar{s}_2' = \frac{L'(1-pP')^2 - p(Y'N'P'-RN'^2)}{2pN'^2} - \frac{[(L'(1-pP'^2) - p(Y'N'P'-RN'^2)) - pN'^2(Y'^2+4RL')]^{1/2}}{2pN'^2} .$$

Equation 3.27 shows that $s_1'(i)$ is linear in $s_1'(i+1)$. The slope of the equation is positive when the maximization problem is well defined ($V' > 0$). Assuming that s_2' converges, s_1' will converge to a stable steady-state value, $\bar{s}_1'$, if and only if the slope is less than one:

$$(3.32)\quad \frac{pN'U'}{2V'} + pP' < 1 .$$

This implies that $N'U'/2V' < (1-pP')/p$, which cannot be true unless $1-pP' > 0$, although this is not sufficient. The term $1-pP'$ will be positive only if

$$(3.33)\quad f_1 > -(r + \frac{c_2 z}{b_1+c_1}) .$$

Once again, when f_1 is sufficiently negative, convergence is prevented, and even if s_2' does converge, failure of s_1' to converge means the optimal tax is one that is high enough to stop all fishing for an indefinite period while the stock is allowed to grow.

The actual steady-state value of s_1' is

$$(3.34)\quad \bar{s}_1' = \frac{(W'+2p\bar{s}_2'M'N')U' + 2V'(J+2p\bar{s}_2'M'P')}{2V'(1-pP') - pN'U'} .$$

Equation 3.26 shows that $s_0'(i)$ is linear in $s_0'(i+1)$, with a slope of p. Provided that s_2' and s_1' converge, s_0 will converge if p < 1, which will occur if the discount rate r is positive. The actual steady-state

$$(3.35)\quad \bar{s}_0' = \frac{I + p[\bar{s}_1'M' + \bar{s}_2'(M'^2 + N'^2 d_u^2 + N'^2 d_v^2 + d_w^2)]}{1-p} + \frac{[W' + p(\bar{s}_1'N' + 2\bar{s}_2'M'N')]^2}{4V'(1-p)} .$$

There can be no guarantee that either s_1' or s_o' is non-negative, but if s_1 is non-negative, then s_o' is also non-negative,.

As before, the variances of the disturbances do not appear in the expression for the optimal tax, equation 3.24. The tax regulated fishery may be managed as if there were no uncertainty. Also, while U' and V' are positive in a well defined problem, E' may be negative, although one would not expect it to be sufficently negative to require a negative tax. Optimal management always calls for restraint of the unregulated effort level, except when stock size has no effect on costs.

Comparison of the expressions for optimal tax level in the single-period case, which is

$$\frac{W' + Y'X_0}{2L'} ,$$

and in the multiple-period case, which is

$$\frac{E' + U'X_0}{2V'} ,$$

reveals similarities, because E' contains W', U' contains Y', and V' contains L'. The difference is that in the multiple-period expression, a new, non-negative term has been subtracted from the denominator, and new terms have been added to the numerator, most of which are non-negative. Therefore, unless the one new numerator term which may be negative, $ps_1'N'$, is strongly negative, the optimal tax is higher in the multiple-period case than in the single-period case, as expected.

TAX REGULATION VERSUS QUOTA REGULATION

The coefficient of comparative advantage (CCA) of tax versus quota is the difference between the expected present value obtainable under tax regulation and expected present value obtainable under quota regulation:

$$(3.36)\quad CCA = V_{21}(X_0) - V_{11}(X_0)\ .$$

Writing the V(Xo)'s in their quadratic forms and combining terms gives

$$(3.37)\quad CCA = (\bar{s}_0'-\bar{s}_0) + (\bar{s}_1'-\bar{s}_1)X_0 + (\bar{s}_2'-\bar{s}_2)X_0^2.$$

Unless all three of the coefficients of this quadratic expression have the same sign,. the' sign of the CCA depends on X_0 (although it is possible that the sign could be the same for all X_0 in the relevant range). Since no presumption can be made about the sign of any of the three coefficients, the choice between instruments must be made on a fishery-by-fishery basis.

There are at least two interesting points ofcomparison with Koenig's model and results to be made. First, s_o and s_o' are both functions of the variance of the stock growth disturbance, whereas the growth disturbance is irrelevant to the choice of instrument in Koenig's analysis. The reason for this result of Koenig's strictly discrete-time approach is that the growth disturbance does not enter the expressions for single-period (current) benefits and costs, but enters only into future net benefits foregone by affecting the size of the stock at the beginning of the next period. Thus, it enters only into the marginal external cost function. In applying Weitzman's analysis of instrument choice, one can Subtract marginal external cost from marginal benefit to obtain a partial net marginal benefit function. The slope of this net marginal benefit function can then be used in place of gross marginal benefit in computing expected welfare losses with Weitzman's formulae. And, as Weitzman makes clear, unless the disturbance in marginal benefit is correlated with the disturbance in marginal private cost, the variance of the former has no bearing on the comparison of instruments, unless consumers, rather than producers, decide how much is to be produced.

However, in the present combination discrete-time/continuous-time model, in which fishing and natural stock growth occur. simultaneously over the course of the fishing season, the growth rate disturbance enters both marginal external cost through its effect on stock size, at the beginning of the next period and current marginal fishing cost. Thus, marginal external cost is correlated with marginal private cost.

Second, Koenig's model is more helpful in that it neatly lends itself to interpretation in terms of the same intuitive concepts needed to understand Weitzman's analysis: slope and variance of marginal benefit and harvest cost curves and welfare loss triangles. This is achieved by designing the fishery in such a way that marginal harvest cost each period depends only on the, cumulative catch during the period. Optimal management requires knowing only 'how cumulative catch is affected by the different instruments;

In the present model, marginal harvest cost each period depends on the time path followed by the instantaneous harvest rate during the period. This time path will be different under different instruments. Therefore, marginal cost cannot be expressed in conventional form as a function of a single variable, cumulative catch. Consequently, intuitively satisfying explanations of the model's results are difficult to produce.

CHAPTER 4

NONLINEAR MODEL WITH ENDOGENOUS HARVESTING CAPITAL: NUMERICAL DYNAMIC PROGRAMMING

INTRODUCTION

The linear specification of the marginal benefit, marginal cost, and growth functions, and the treatment of capital as an exogenously fixed parameter, are necessary for derivation of analytical results. However, these assumptions are unrealistic for several reasons. Two of them are among the most important. First, the linear appearance of stock size in the marginal harvest cost function implies that harvesting could take place even when stock size is zero. Second, linearity of all three functions implies a maximum present value from optimal management, V(X), which is quadratic in stock size. This means either 1) that as stock size multiplies, present value increases at an ever-growing rate, thus defying the law of diminishing returns; or 2) that present value eventually declines and becomes negative, which is also counter-intuitive.

If the model is constructed and estimated in more plausible non-linear form, comparison of regulatory instruments can only be accomplished through numerical dynamic programming. This chapter describes such a nonlinear model and discusses numerical dynamic programing with it.

STOCK GROWTH FUNCTION

The growth function, F(X), is most commonly estimated in "generalized stock production model". form:

(4.01) $$\dot{X}_t = F(X_t) = f_1 X_t - f_2 X_t^n ,$$

where f_1, f_2, and n are parameters, and n is usually assumed to be greater than one (Abramson and Tomlinson, 1972). This form has the so-called "dome" shape, with the the highest point on the dome representing maximum sustainable yield.

MARGINAL COST

A more reasonable marginal harvest cost can be derived from harvest production and effort production, functions. The instantaneous, harvest production. function is

(4.02) $$\dot{H}_t = qX_t\dot{E}_t ,$$

where q is a parameter called the "coefficient of catchability," and E_t is the instantaneous rate of effort application to the fish stock.

Effort in a trawl fishery may be measured in units of time spent towing the trawl net through the water. Equation 4.02 is a common specification in the biology literature, and is not unreasonable for effort and harvest rates below the levels at which the fishing grounds become congested with vessels, or at which the entire stock is taken instantaneously. Assuming that the stock is of uniform density, harvest could plausibly be proportional to effort at a given stock size.

Production of effort, in turn, is assumed to be a function of three inputs: capital, measured by the number of boats fishing for the species in question; labor, measured in man-hours spent on board (whether or not the net is being towed); and fuel, measured in ‘quantity consumed both in fishing and in running to and from port. The amount of running time per unit of fishing time is fixed to the fisherman, i.e., it is not a choice variable, but it is affected by the weather, the time of day (light or dark), and the level of maintenance that has been performed on the vessel and its equipment. This assumption explains how the instantaneous marginal cost derived below when the number of vessels in the fleet is temporarily fixed can be upward sloping even if the parameter values in the production function imply constant long run marginal cost. Increased effort requires ‘existing vessels to run to the grounds and fish at times when they otherwise would have remained in port performing maintenance operations, among other things.

It is further assumed that the number of crew members on a boat is fixed, and that the quantity of fuel consumed in operating the vessel for a unit of time is fixed; Thus, the ratio of fuel to labor is also fixed.

The effort production function is assumed to have the Cobb-Douglass form in capital in season i, K_i, and instantaneous fuel and labor consumption (a composite input), F_t

$$\dot{E}_t = e_0 K_i^{e_1} \dot{F}_t^{e_2} , \quad (4.03)$$

where e_0, e_1, and e_2 are parameters.

For a given number of boats, which is assumed to be fixed for the duration of the season, the total variable cost of effort is $P_F F_t$, where P_F is the price of a fuel-labor unit. Rearranging the effort production function (equation 4.03) to express F_t as a function of effort and capital (and suppressing the rate of change dot and the subscripts) gives

(4.04) $F = (E/(e_0 K^{e_1}))^{1/e_2}$.

Substituting into $P_F F$ gives total variable cost as

(4.05) $P_F E^{1/e_2} / (e_0 K^{e_1})^{1/e_2}$.

Marginal cost of effort is then

(4.06) $P_F E^{(1/e_2)-1} / (e_2 (e_0 K^{e_1})^{1/e_2})$.

For a given stock size at instant t, the instantaneous marginal harvest cost is obtained by dividing the marginal cost of effort by the marginal product of effort, qX :

(4.07) $$MC = \frac{P_F E^{(1/e_2)-1}}{qX(e_0 K^{e_1})^{-1/e_2}} .$$

Rearranging the harvest production function (equation 4.02) to express E as a function of stock size and harvest rate, and substituting into equation 4.07 gives short run marginal harvest cost as a function of stock size, fleet size, and instantaneous harvest rate:

(4.08) $$MC(X,K,\dot{H}) = \frac{c_0 \ \dot{H}^{c_1}}{X^{c_2} \ K^{c_3}} ,$$

where $c_0 = P_F / (e_2 (e_0 q)^{1/e_2})$,

$c_1 = (1/e_2)-1$,

$c_2 = 1/e_2$, and

$c_3 = e_1/e_2$.

MARGINAL BENEFIT

Marginal benefit at the ex-vessel level is derived from the marginal benefit accruing to processors from their sales of processed fish to buyers at higher levels. The derivation takes into account the cost of processing the fish purchased from fishermen.

One can assume that processors are output price takers, and that the processing production function is characterized by fixed ratios of output of processed fish, Y, to input of raw fish, H (C. Carter, Oregon Dept. of Fish and Wildlife, 521 S. W. Mill St., Portland, OR 97201, pers. commun.), and of output to input of a composite factor called "processing effort E:

(4.09) $Y = \min(y_1H, y_2E)$,

where y_1 and y_2 are fixed proportions. Marginal benefit' is therefore obtained by subtracting marginal cost of processing effort from the output price (converted to dollars per pound of raw fish).

The production of processing effort is assumed to be described by a Cobb-Douglass function of processing capital, K; and labor:

(4.10) $E = n_0 K^{n_1} L^{n_2}$,

where n_o, n_1, and n_2 are parameters. Processing capital is assumed to be exogenously fixed, so short run total variable cost is P_LL, where P_L is the wage rate. Rearranging the effort production function gives

(4.11) $L = (E/(n_0K^{n_1}))^{1/n_2}$.

Substitituting this into P_LL gives total variable cost as a function of processing effort:

(4.12) $P_LE^{1/n_2} / (n_0K^{n_1})^{1/n_2}$.

From the processing production function (‘equation 4.09), one gets

(4.13) $E = \frac{y_1}{y_2} H$.

Substituting’ this into equation 4.12 gives total-variable cost as a function of harvest rate:

Finally, one candifferentiate equation 4.14 with respect to H to obtain marginal processing cost (exclusive of the price of raw fish):

(4.15) $MC_P(K,H) = b_0\ H^{b_1}$,

where $b_0 = P_L y_1^{1/n_2} / (n_2(n_0 y_2)^{1/n_2} K^{n_1/n_2})$, and

$b_1 = (1/n_2)-1$.

Marginal benefit is thus

(4.16) $MB(\dot{H}) = P_Y - b_0 \dot{H}^{b_1}$,

where P_Y is the price of processed fish.

ENDOGENOUS HARVESTING CAPITAL

Harvesting capital is assumed to be completely fixed during the fishing season, and perfectly variable between seasons. Fishermen choose the number of vessels in their fleet for the upcoming season on the basis of the expected present value of their profits during the season, given knowledge of the stock size at the beginning of the season and the level of the regulatory instrument that will be in effect during the season. Obviously, these assumptions imply that boats are added or removed instantaneously at the beginning of the season, and that fishermen are not concerned about the future beyond the current year. For an alternative treatment of fleet investment, see Charles (1983a).

As does any industry characterized by many firms in perfect competition, the fleet behaves as if it were trying to maximize the sum of consumer and producer surpluses (discounted by the interest rate, m), minus total tax expense (which is zero if a quota is used),. That is, it chooses the fleet size, K, which maximizes

$$(4.17)\quad E\left[\int_0^z e^{-mt}\{ B(\dot{H}_t,u) - VC(X_t,K,\dot{H}_t,v) - \dot{H}_t T \}dt - \int_0^1 e^{-mt}\{ FC(K) \}dt \right],$$

where B() is instantaneous total benefit, VC() is instantaneous total variable cost, and FC() is instantaneous fixed cost, or cost of holding capital. (One should recall that z is the fishing season closing date.) Total variable cost is the indefinite integral of marginal cost with respect to harvest rate, and fixed cost is defined as

$$(4.18)\quad FC(K) = (m+d)\ P_C\ K,$$

where d is the depreciation rate, and P_C is the price of a unit of capital.

It was shown in Chapter 2 that X_t and H_t are ultimately functions of beginning-of-season stock size, X_0, and the current level of the instrument,. Q or T, as well as of time.

While the unit of measurement of capital is the vessel,‘ it is not necessarily true that the relevant price of capital, P_C, is the price of the entire vessel. If the vessels used in the fishery in question are also used in other fisheries, then P_c is less than the purchase cost of the vessel. The important consideration is whether the decision to build, import to the region, or keep in the region a boat which is equipped to fish for the species in question and others was made with this fishery explicitly in mind. If the decision was made without thinking of this fishery, then only the the price of adding equipment specialized to this fishery is included in P_C.

The decision to build or import a vessel is based on the expected profit stream from all fisheries in which it is intended for use. The price of the vessel is a fixed cost for the entire collection of fisheries. The approach taken here to determining the value of P_C is to adjust it until the predicted equilibrium fleet size under a tax level set equal to zero is approximately the observed fleet size for a comparable estimated beginning-of-period stock size.

When vessels participate in more than one fishery and when there are significant output market and ecological interactions between species, it may be worthwhile to manage all the affected fisheries jointly. On the other hand, the net effect of the various types of interactions may be negligible.

An additional consideration in cost analysis is whether the boats are used for other fisheries only in the off-season or could be used in alternative fisheries at the same time the open season for this species is under way. In the latter case, the potential alternative profit availableto the vessel is part of the opportunity cost of fishing in this fishery. If it is possible to switch between fisheries at any time, alternative profit rate is part of variable cost; if the choice of fishery is irrevocable during the season, the expected present value of alternative season profits is an additional fixed cost, and another term should be added to the expression for FC above.

With endogenous capital now incorporated into the model, the single-period net benefit function is

$$(4.19)\quad R() = \int_{0}^{z} e^{-rt}\{\ B(\dot{H}_t,u) - VC(X_t,K,\dot{H}_t,v)\ \}dt - \int_{0}^{1} e^{-rt}\{\ FC(K)\ \}dt.$$

NUMERICAL DYNAMIC PROGRAMMING

Introduction of either nonlinear specifications of the three basic functions or of endogenous capital renders analytical dynamic programming, illustrated in Chapter 3, impossible. However, numerical computer methods are available. Numerical dynamic programming comprises a family of efficient algorithms for searching a set of feasible time paths of control variables to find the one which maximizes or minimizes a series of objective functions. Just as in analytical dynamic programming, when the model is stochastic, the entire optimal time path of the control cannot be determined in advance. Instead, a rule for setting the control at its optimal level at each instant or time period, given the value of the state variable(s) in that instant or period, is derived. The principle ‘behind dynamic programming is explained in the first section of Chapter 3. Dreyfus and Law (1977) provide a well written, more detailed introduction to the method, while Swierzbinski (l981) is helpful on the subject of actually implementing the method on a computer.

Numerical dynamic programming requires the approximation of the infinite number of values that can be assumed by continuous variables with a finite number of discrete values. In two-dimensional terminology, each combination of state and control variable levels is called a grid point. In each of a specified number of time periods,' and for each of the selected stock sizes, the algorithm searches the range of selected instrument levels to find the level which maximizes the value of the stock.

The value of the stock in a particular period consists of current single-period benefits plus the present value of the stock at the beginning of the following period. It is necessary to begin the algorithm at the last, or terminal, period, when the value of the stock in the following period is known. (This stock value is assumed to be zero if the terminal period is far enough in the future, typically 100 ye&s, to adequately approximate infinity.)

Once the value of the stock at each stock size in the terminal period has been determined; these values are used to determine the following-period stock values for the period just prior to the terminal period. This process continues as the algorithm works backward in time to the first period;

The first step in computing the current single period net benefits produced by each of the grid point instrument levels for a given stock size is to find the equilibrium fleet size chosen by fishermen, given stock size and instrument level. This is achieved by searching over a range of selected fleet sizes to find the one which maximizes the expected present value of net benefits minus tax revenue, equation 4.17. For each of the trial fleet sizes, the algorithm must integrate the instantaneous benefit and cost rates over time from the beginning of the season to the closing date. This is accomplished numerically by dividing the season into small, discrete time intervals and assuming that the benefit and cost functions are constant over each time interval. The area under the net benefit curve is then the sum of the areas of the columns created by division into discrete intervals.

At each point in time during the integration, equilibrium instantaneous harvest rate must be found by solving the equality of marginal benefit and marginal cost plus tax. This is done numerically by searching over a range of selected harvest rates to find the solution. Alternatively, an efficient iterative program for maximization of total net benefits over harvest rate may be used. The functional forms of MB and MC postulated above guarantee that there is no danger of multiple maxima. Of course, in the quota program, if equilibrium harvest rate exceeds the quota level, harvest rate is set equal to the quota.

Since both instantaneous variable cost and equilibrium harvest depend on the stock size at instant t, integration of the stock growth differential equation in order to compute the stock size at each time point is performed simultaneously with the integration of the net benefit function.

The entire simultaneous integration procedure must be repeated for each combination of the possible values of the stochastic disturbances so that expected values of current and future net benefits can be calculated. The distributions of these disturbances (which might be assumed to be normal) are divided into discrete intervals by selecting a limited number of possible disturbance values, and approximating the probability density function as constant around each disturbance value for half the distance between disturbance values. The probability of each disturbance value is then the area of its column under the probability density function. If the disturbances are independently distributed, the joint probability of each combination of values for the three disturbances is simply the product of their marginal probabilities.

After the equilibrium fleet size for a possible instrument level is computed, the evaluation of benefits produced by that instrument level can proceed. The expected present value of single-period net benefits minus tax revenue has already been computed during the search for equilibrium fleet size, and after adding tax revenue to this quantity again, all that. remains is to compute the expected present value-of following-period stock size.

Following-period stock size resulting from the trial tax level for each combination of disturbance values is computed by integrating the growthdifferential equation over time from zero to one, solving for equilibrium harvest rate at each time point. This has already been done for the fishing season ($t = 0 \rightarrow z$) during the search for equilibrium fleet size, but the integration must still be carried out for the closed season.

The final step is to compute the present value of each of the stochastic following-period stock sizes by referring to the array of present values already computed for the following time period during the previous iteration of the algorithm’s outermost loop. These present values were computed for each of the grid point stock sizes. Since the following-period stock sizes computed by integration of the growth equation will rarely equal any of the grid point stock levels, it is necessary to estimate their present values by linear interpolation of the present values of the grid point stock sizes.

CHAPTER 5

APPLICATION TO THE PINK SHRIMP (Pandalus jordani) FISHERY

INTRODUCTION

The parameters of the bioeconomic model developed in previous chapters were statistically estimated for the fishery based on pink shrimp, Pandalus jordani, off the U.S. Pacific coast. Then numerical dynamic programming was applied to the statistical model in order to. compare the expected present values of the stock under both optimal tax and optimal' quota regulation. This chapter first describes the biometric.-and-econometric methods used and presents the estimates. Next, it describes the dynamic programming algorithm and reports the computed present values, along with optimal tax and quota levels and predicted fleet sizes.

Superscript numerals in the text which are not associated with mathematical expressions refer to the notes at the end of this chapter.

ESTIMATION OF PARAMETERS

The three equations of the model outlined in Chapter 4 are the marginal benefit (MB)., marginal cost (MC), and stock growth functions:

(5.01) $$MB(\dot{H},u) = P_Y - b_0 \dot{H}^{b_1} + u ,$$

(5.02) $$MC(X,K,\dot{H},v) = \frac{c_0 \dot{H}^{c_1}}{X^{c_2} K^{c_3}} + v , \text{ and}$$

(5.03) $$\dot{X} = f_1 X - f_2 X^n - \dot{H} ,$$

where H is harvest, X is stock size, P_Y is price of processed shrimp, and K is the number of boats fishing, during the period. A dot over a symbol means instantaneous time rate. of change,, the subscripted lower case letters and n are parameters to be estimated, and u and v are stochastic disturbances. The continuous time subscript t has been omitted from the rate variables, and the period subscript i has been omitted from K and from the disturbances. The constants b_0 and c_o are composites of exogenous shift variables which will be named below.

No disturbance is shown in the stock growth function because it is used only to derive the stock size function

$$(5.04) \qquad X_t = X_k(X_{i-1}, Z_{ki}, t) + w_i ,$$

where X_{i-1} , is the stock size at the beginning of period i, Z_{ki} is the level of regulatory instrument k during period i, t is (continuous) time, and w_i is a stochastic disturbance. The disturbance is assumed to have the same distribution regardless of which instrument is chosen. If a disturbance had been written into the stock -growth function, equation 5.03, it would have implied disturbances which entered the stock size' function in complicated and varying ways, depending on the instrument chosen and the value of n. Estimation of the variance of the growth function disturbance would be quite difficult.

The stock growth function cannot be estimated directly in the form specified by equation 5.03 because there are no data on the rate of change of stock size. Instead, a transformation is made, beginning with assumption of a harvest production function of the form

$$(5.05) \qquad \dot{H}_t = q \, \dot{E}_t \, X_t ,$$

where q is the catchability coefficient, and E is instantaneous rate of fishing effort. No data on instantaneous harvest and effort rates exist, but periodic cumulative figures do exist. With substitution of. the expected value of stock size plus disturbance, $E[X_t] + w_i$, for actual stock size, X_t, cumulative harvest in period i is

$$(5.06) \qquad H_i = \int_{i-1}^{i} q \dot{E}_t \, (E[X_t] + w_i) \, dt \; .$$

$\dot{E}_t$ is assumed to be constant throughout period i (as is w), allowing equation 5.06 to be written as

$$(5.07) \qquad H_i = qE_i \left(\int_{i-1}^{i} E[X_t]dt + w_i \right) .$$

Multiplying qE_i through gives

$$(5.08) \qquad H_i = qE_i \int_{i-1}^{i} E[X_t]dt + qE_i w_i \; .$$

Equation 5.05 is substituted into the stock growth function (equation 5.03) to obtain

$$(5.09) \qquad \dot{X}_t = f_0 X_t - f_1 X_t^{\,n} - qX_t \dot{E}_t \; ,$$

and the solution to this differential equation (the stock size function) is substituted for $E[X_t]$ in equation 5.08. The resulting equation can be estimated by a nonlinear least squares procedure on harvest and- effort data.[1] (See Rivard and Bledsoe 1978 for a more complete discussion.)

A computer program (PARFIT) written at the University of Washington Center for Quantitative Science. in Forestry, Fisheries, and Wildlife (Rivard 1977) was used to solve equation 5.09 and to integrate and estimate equation 5.08 by means of an iterative least squares method applied to semiannual[2] catch and effort data for pink shrimp, Pandalus jordani. Account was taken of the unequal variances of the disturbances in equation 5.08 by modifying PARFIT so that it divided the residuals by qE_i, before computing the sum of squares. The data were obtained from the Pacific Marine Fisheries Commission's Data Series, and covered the period 1957-81, providing 50 observations. The results of the estimation are

$$\dot{X} = 2.097\ X - 0.3443\text{x}10^{-10}\ X^{2.304} - 0.5908\text{x}10^{-5}\ X\dot{E}$$

$$\text{estimated variance of } w = 7.856\text{x}10^{15}.$$

The R^2, F, and t statistics are not presented because PARFIT produces estimates of the parameters and statistics of equation 5.08, not of the stock growth equation shown. The parameter estimates shown above were derived from the estimated parameters of euation 5.08 (Rivard, and Bledsoe 1978), all of which were significant at the 1% level, both individually and jointly (the R^2 and F statistics for equation 5.08 are 0.116 and 33.97, respectively). The estimated variance is of the stock size function, not the stock growth function. The estimated growth function is a near-parabola, a dome with a maximum sustainable yield of 116.6 million pounds per year (in year-round fishing), and a maximum sustainable stock size of 186.4 million pounds. These parameter estimates, if accurate, describe a stock capable of quite rapid growth. They do not appear, however, to be excessively out of line with some estimates obtained for other fast growing species. Fox (1972) presents estimates of f_0 for Alaska pink shrimp ranging as high as 1.66 and Francis (1974) uses estimates of f_0 for yellowfin tuna, thunnus. albacares, ranging up to 1.9.

Ex-vessel price and instantaneous harvest rate in the pink shrimp fishery are assumed to be jointly determined at every instant by equilibrium in a demand (marginal benefit) and supply (marginal cost) equation system; This assumption seems a reasonable approximation because: l) quotas have not been used, except in California, which has contributed only a small proportion of the catch; 2) the negotiated price agreed to. by organizations representing fishermen and processors has apparently been lower than the actual price most of the time (C. Carter, Oregon Department of Fish and Wildlife, pers. commun.); and 3) effort on shrimp is rapidly adjustable, either by varying the length and frequency of fishing trips or by switching to alternative fisheries (e.g., groundfish or crabs). This permits equilibrium to be quickly reached as conditions change (C. Carter, Oregon Department of Fish and Wildlife, pers. commun.).

The marginal benefit and marginal harvest cost functions were ‘estimated simultaneously using a nonlinear two-stage least squares method developed by Kelejian (1971) and Amemiya (1974). This method involves regressing the endogenous variables of the system on low degree polynomials of the exogenous variables (to approximate unknown nonlinear reduced form equations),. then iteratively minimizing the sum of squared differences between observed and predicted values of the endogenous variables. The method yields consistent but generally not asymptotically efficient estimators when the model is nonlinear in both variables and parameters.

An alternative estimation procedure is accomplished by linearizing the equations by taking the logarithm-of each side. Then the standard’ two-stage least squares method is applied, with the coefficients constrained to reflect relationships implied in the derivation of the marginal benefit and marginal cost functions from the harvest and processing production functions, e.g., $c_1 = (1/e_2)-1$. When serial correlation is indicated, estimation is redone using the maximum likelihood method.

This method also yields consistent estimators, but in this. particular model it probably does notachieve asymptotic efficiency. The reason is that the price of the processed product, P_y, enters the original marginal benefit equation additively (with its coefficient known to be equal to one), and before logarithms are taken, P_y must be moved to the left side of the equation, from which it is subtracted. The exogenous variable P_y affects-the predicted value of the endogenous variables, but having been incorporated into the dependent variable of the MB equation, it does not appear as an argument in the reduced form equations of the log-linear model. Hence, the predicted values of the endogenous variables obtained in the first-stage regression are biased. estimates of the true expected values of the endogenous variables, and therefore may not be the best instruments to use for the endogenous variables in the second stage.

The log-linear forms of MB and MC were estimated in this study as described in the preceding paragraphs, but despite constraining the exponents of the harvesting and processing effort production functions to sum to one (the linear homogeneity assumption), the implied value of e_1, the exponent of harvesting capital, was negative, and e_2, the exponent of the fuel-labor input, was greater than one. This can happen because the constraint does not force the individual exponents to lie between zero and one, only to sum to one.

Neither the nonlinear two-stage method of Kelejian and Amemiya or the log-linear two-stage least squares method produce estimators which are known to be unbiased and efficient in finite samples. Therefore, the estimates obtained by the former method were. selected for presentation and for use in the dynamic programming solely because they conformed more’ closely to expectations about signs and magnitudes.

It is assumed that buyers and sellers equate marginal. benefit and marginal cost, respectively, to the ex-vessel price of pink shrimp. Thus, price data give an accurate measure of marginal benefit and marginal cost.

At the ex-vessel level, marginal benefit from shrimp accrues to processors, and isderived from marginal benefit at the wholesale and higher market levels. The arguments in the marginal benefit function include, in addition to the-rate at which shrimp are purchased from fishermen (quantity), the price of processed pink shrimp, the quantity of fixed capital extant, variables contributing to processing cost, and processor inventories.

Since Pacific. coast pink shrimp production is a minor (less than 10%) part of the total production and importation of shrimp in the United States, it was assumed that the price received by pink shrimp processors for their processed product is unaffected by the quantity they produce. Hence, the price of processed pink shrimp can be treated as an exogenous variable.

Unfortunately, no data on pink shrimp processor inventories could be found, nor could data on processing input prices be found. The results of estimating the marginal benefit function with the data that were available are

$$PEX = PWH - \frac{\underset{(0.05912)}{0.1105\times10^{-6}}\ \underset{(0.8097)}{HAR^{0.8857}}}{\underset{(1.001)}{PLT^{0.8614}}}$$

estimated variance of u = 0.0130

Durbin-Watson statistic = 0.5255

where PEX = ex-vessel price of pink shrimp,

HAR= harvest rate,

PWH= wholesale price of processed pink shrimp, reduced by a processed weight/round weight conversion factor of 0.22,

PLT = Number of plants processing pink shrimp, and

the numbers in parentheses are the t-statistics for the associated parameter estimates. The t-statistics are measures of the reliability of the estimates. The Durbin-Watson statistic is a measure of the degree of serial correlation in the disturbances.

All coefficients have the appropriate signs, and the exponents have theoretically reasonable magnitudes. The implied degree of homogeneity of the processing effort production function derived in Chapter 4 is 0.9871, which in turn implies very nearly constant long run average cost.

The t-statistics are all rather low, and none of the variables appear significant if the t-statistics are used for hypothesis testing. The Durbin-Watson statistic suggests serial correlation. However, the econometrics program, TSP, used to estimate the equations has no capability to treat this problem in nonlinear models. Moreover, the computed t-statistics do not have-the distributions necessary for use in standard tests when the estimator is not efficient, and should not be so used.

In addition to the harvest rate, stock size, and number of vessels fishing for shrimp, marginal harvest cost is a function of the prices of fishing inputs, such as diesel fuel, and the profitability of alternative fisheries. Data on profitability of alternative fisheries, on prices of inputs other than fuel, and on stock size were not readily available, but estimates of stock size were obtained by dividing monthly catch per unit of effort by the estimated catchability coefficient.

Stock size is determined by time path of previous harvest rates, but is exogenously fixed to the system at instant t. Fleet size is endogenous in the sense that it is determined by beginning-of-period stock size and instrument level and may even be stochastic, but it is not correlated with any of the current disturbances of the system. It is therefore also treated as an exogenous variable for estimation purposes.

The estimated marginal cost function is

$$PEX = \frac{\underset{(0.03106)}{4.333\times10^{8}}\ PFU\ \underset{(0.2279)}{HAR^{0.9183}}}{\underset{(0.3879)}{STO^{1.713}}\ \underset{(0.2073)}{BOT^{0.9337}}}$$

estimated variance of v = 4.754

Durbin-Watson statistic = 0.1396

where STO = estimated stock size,

PFU = price of diesel fuel, and

BOT = fleet size.

Again all coefficients have the appropriate signs, and the exponents have reasonable magnitudes, implying an effort production function which is homogeneous of degree 1.008. The Durbin-Watson and t-statistics are low, but hypothesis testing in this case is not reliable.

Ideally, for the model constructed in this study, data on flow variables would measure instantaneous rates and data on state variables would measure levels. at points in time. In reality, however, average rates and levels must be used, preferably measured over the shortest intervals possible. The assumption is then made that levels and instantaneous rates were constant over the interval. In the case of the pink shrimp model, weekly, monthly, semiannual,, and annual data were used, depending on the variable.

One observation per year was taken on all variables because the theoretical model is based on assuming disturbances which remain constantthroughout each year. The econometric method employed here, on the other hand, requires assuming that the disturbances change at observation. If observations taken at more than one point in each year were used in the regressions, difficult adjustments for autocorrelation in the disturbances would be called for.

Data for variables collected weekly were measured over a week near the beginning of June, which is well into the April-through-October shrimp fishing season. Data for variables collected monthly were measured over the entire month of June.

The period of the annual time series was 1968-82, giving 15 observations. Sources of the data:

PEX: 1976-82 data are early June ex-vessel prices taken from a series on prices at Oregon ports reported weekly Fishery Market News ("the pink sheet") published by the National Marine Fisheries Service Northwest Regional Office. Prices for 1968-75 are estimated season average prices collected from various state government sources and reported in the Draft Fishery Management Plan for the Pink Shrimp Fishery produced by the Pacific Fishery Management Council in 1981. Use of these season averages was necessary because the weekly series did not begin until 1976, For the years 1976-78, the price changed very little during the course of each season, a fact which gives hope that the same was true in earlier years. If so, the annual average prices would- be approximately the same as the early June prices.

HAR: June landings coastwide, taken from monthly. data reported by the Pacific Marine Fisheries Commission (PMFC), and multiplied by 12 to convert to annual equivalent.

STO: Estimated stock size, generated by dividing June catch per unit of effort (as reported by PMFC) by an estimate of the catchability coefficient obtained from the procedure employed in estimating the' stock growth function. The harvest production function, $H = qXE$, yields this estimator of stock size. The simple correlation between harvest and harvest per unit of effort is 0.0705.

An alternative method of estimating stock size is to solve the estimated stock growth function, which is a differential equation expressing the rate of growth as a function of stock size, then use the solution to generate a time series. of stock sizes. However, this procedure requires an estimate of initial stock size, which must be obtained by dividing catch per unit of effort in the first period by the estimated catchability coefficient. Moreover, the procedure would yield increasingly less accurate estimates as the time series progressed farther from the starting time because of the cumulative effect of the disturbances on actual stock growth.

PLT: Number of plants processing pink shrimp each year. From Pacific Packers Report, published annually by National Fisherman. These data contain some apparent errors, and are of doubtful accuracy. Also, the number of plants is not a good measure of the quantity of capital specialized to shrimp processing.

BOT: Number of boats landing pink shrimp each year. From Draft Fisher Management Plan for the Pink shrimp fishery, issued by the Pacific Fishery Management Council in 1981. The quantity of capital specialized to shrimp fishing and the proportion of the value of the entire vessel allocated to shrimp fishing are both assumed to be constant over time and across vessels.

PWH: Annual unit value (total value divided by total quantity) to Pacific coast processors of cooked, peeled, and frozen shrimp. From Processed-Fishery Products Annual summary, issued by NMFS. The use of these annual averages is appropriate because frozen shrimp can be stored for months and released to the market on a schedule which smooths price fluctuations. The price should not vary much over the course of a- year. Furthermore, processor demand for landed shrimp should depend on the expected price of their product over the next few months rather than the immediate instantaneous price, and the annual average price may correlate with expected price more accurately than does instantaneous price.

PFU: Data for 1976-82 are wholesale prices of diesel fuel to commercial consumers in the Pacific region. Data for 1974-75 are wholesale prices of diesel fuel to commercial consumers in the entire United States. Data for 1968-73 are wholesale prices of diesel fuel in Los Angeles. All are taken from June issues of Producer Prices and Price Indexes published by the Bureau of Labor Statistics.

ADDITIONAL PARAMETER VALUES AND GRID RANGES

Exogenous variables in the marginal benefit and cost functions are assumed to remain constant at their 1982 levels throughout the planning horizon.

The interest rate, m, is 0.05.

The discount rate, r, is 0.05.

The depreciation rate, d, is 0.4.

The price of harvesting capital used in shrimp fishing, P_C, is $75,000.

The length of the fishing season, z, is 7 months (0.5833 years).

With the exception of season length, which is set by regulation in Oregon and by winter weather in Washington, the above parameter values are ones which were found by trial and error to result in a reasonable prediction of fleet size with the tax set at zero and all other parameters set at their 1982 values. (Stock size was set at thirty million pounds, the value obtained by dividing June 1982 catch per unit of effort by estimated catchability coefficient.) The predicted fleet size under these conditions is 200 vessels, compared with an observed fleet of 226 vessels in 1982.

Grid variable characteristics are given in Table 1.

DYNAMIC PROGRAMMING RESULTS

Tables 2 and 3 present the numerical dynamic programming results, which show that tax and quota regulation would produce approximately the same expected present value of net benefits in the pink shrimp fishery. Tax regulation benefits are slightly higher at most initial stock sizes, but the difference is insignificant, and may be due to approximation error.

The optimal quota levels are expressed as annualized instantaneous harvest rates. The cumulative harvest for the open season if the quota harvest rate were binding at every instant during the season is found by multiplying the quota by the length of the season (7/12 in this case), and is shown in the third column of Table 2. However, since the quota is only an upper bound on the instantaneous harvest rate, equilibrium harvest rate is sometimes less than the quota. Thus, the expected cumulative harvest for the season, in the fourth column, is always less than the fixed rate total.

Table 1.--Grid variable characteristics.

Variable	Number of Levels	Range	Increment	Units
Stock size	10	30 - 300	30	million lbs.
Quota	11	0 - 300	30	million lbs.
Tax	11	0 - 0.10	0.01	dollar
Capital	11	0 - 1000	100	vessels
Growth disturbance	2	-1 - +1	2	standard deviations
Time	13	0 - 1.0	0.0833	year
Harvest rate	37	0 - 360	10	million lbs.

Grid size 10x11x11x2x13x37 = 1,164,020

Table 2.--Optimal quota level, cumulative harvests, fleet size, and expected present value of fishery under optimal quota management.

Initial stock size (mil.lbs.)	Optimal quota (mil.lbs.)	Fixed rate harvest (mil.lbs.)	Expected harvest (mil.lbs.)	Fleet size (vessels)	Expected pres. val. ($ mil.)
30	30	18	11	100	473
60	90	53	33	200	479
90	150	88	56	400	486
120	210	123	73	500	494
150	210	123	88	600	501
180	240	140	101	700	508
210	270	158	113	800	514
240	300	175	120	800	518
270	300	175	127	900	522
300	300	175	126	800	524

Table 3.--Optimal tax level, expected cumulative harvest, fleet size, and expected present value of fishery under optimal tax management.

Initial stock size (mil.lbs.)	Optimal tax ($)	Expected harvest (mil.lbs.)	Fleet size (vessels)	Expected pres. val. ($ mil.)
30	.08	13	100	474
60	.08	37	300	481
90	.04	54	400	488
120	.03	71	500	496
150	.02	87	600	503
180	.01	101	700	509
210	0	113	800	515
240	0	120	800	520
270	0	127	900	523
300	0	126	800	525

The reason the optimal tax declines as initial stock size rises is that stock size enters the marginal harvest cost function in its denominator. Thus, the marginal effect of stock size on harvesting cost declines as stock size rises, and the size of the externality also declines.

It should be noted that the optimal tax level of zero shown in Table 3 for initial stock sizes greater than 180 million pounds does not imply that no restraint of fishing activity is required. Rather, zero is the closest grid point tax level to the true optimal tax level, which is small, but positive. The quota levels shown in Table 2 for these same stock sizes are probably almost, but not quite, high enough to be nonbinding at all times during the fishing season, even when stock growth is unexpectedly rapid and stock size grows unexpectedly large.

It should also be noted that the stock would almost never remain at the initial, or beginning-of-period, stock size as the season progresses. Morever, the system may never reach a steady state during the season. In addition, there is no reason to expect that beginning-of-period stock size will be constant from one period to the next.

The complexity of the combined discrete- and continuous-time model precludes an intuitive explanation of the result that tax and quota regulation seem to be equally efficient in this fishery. However, it is interesting to note that when the quota is assumed to be binding at all times,, as in Weitzman’s analysis, the efficiency of quota regulation is considerably weakened. This is because harvest rate is less flexible under a binding quota than under an upper limit quota or a tax. As Weitzman shows, output flexibility does not guarantee instrument superiority. However, in this model, it sometimes happens, that equilibrium harvest rate is less than the quota., Since the full-information optimal harvest rate at each instant is always less than the equilibrium rate, the welfare loss at times when regulation causes the actual harvest rate to be higher than the equilibrium rate is greater than when the fishery is unregulated. The larger-is the variance of the stochastic disturbance in the growth rate, the more likely are very slow growth rates, and the longer is the part of a season (which begins with a low initial stock size) during which equilibrium harvest rate is less than the quota. Thus, under the binding quota assumption, higher growth rate uncertainty decreases the relative efficiency of the quota.

A noteworthy feature of the numerical dynamic programming exercise is the amount of time required for the programs to run on a computer. They were so expensive that it was necessary to shorten them by reducing the number of stochastic equations in the model from three to one (the stock size equation) and assuming that the remaining disturbance had only two possible values instead of a normal distribution, by running for only 50 time periods instead of 100, and by reducing the number of grid point levels of all variables.

The same constraints on research time and computer resources which forced these reductions prevented assessment of the loss in approximation accuracy they imposed. A single run of one of the shortened programs required almost 7 hours of processing time on the Northwest and Alaska Fisheries Center’s Burroughs B7800. Total computer use charges accumulated during the course of debugging and calibrating the programs exceeded $30,000. Sensitivity analyses were prohibitively expensive and were not performed.

The results displayed in Tables 2 and 3 do not mean that managers of the pink shrimp fishery should be indifferent about the best instrument for regulating the fishery., In the first place, the econometric model is highly simplified, for reasons both of convenience and of data unavailability. In the second place, as was made clear in the introduction, the only criterion for ranking instruments in this analysis is expected present value of consumer and producer surpluses. No consideration was given to administrative and enforcement costs, political and social acceptability, or other factors.

CHAPTER 5

NOTES

1. The independent variable, effort, is treated as though it were exogenous. In fact, effort is determined by the solution of the three-equation system comprising marginal benefit, marginal harvest cost, and stock growth. It might be worthwhile to regress effort on the exogenous variables of the system, and then use predicted values of effort in the estimation of the stock growth function. However, this. was not done because of the inconsistency described in note 2 and because of the complicated transformations required.

2. Since the estimation procedure requires the assumption of a (probably) different disturbance value at each observation, use of semiannual data is inconsistent with the assumption made in this study that disturbances change value only once a year. Annual data could have been used, but that would have resulted in a serious violation of another assumption made in the estimation procedure, namely, that the effort rate is constant during the period of each observation. Use of semiannual data (monthly data was available) was a compromise in satisfying the assumptions of annual disturbance changes and constant effort rate during observation period.

Each year was divided into a 6-month open season and a 6-month closed season. The actual shrimp fishing season is about 7 months long, but the last month, October, is usually a light harvest month, and its catch was assigned to the closed season. This was necessary because PAFFIT requires observation periods of equal length, and no time gaps between observations are permitted.

ACKNOWLEDGMENTS

The author wishes to express sincere appreciation to Professors Joseph Swierzbinski, Evan Koenig, Robert Mendelsohn, and Gardner Brown of the University of Washington, Seattle, for their guidance while this research was being conducted. Thanks are also due to Dr. Richard Marasco and C. Macgill Lynde of the Northwest and Alaska Fisheries Center (NWAFC), Seattle, for their assistance, and to the NWAFC for financial support.

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APPENDIX A

DERIVATION OF SELECTED EQUATIONS IN CHARTER 1

MSB = G (Page 15)

It is easily shown that at the optimum point, MSB, defined by equation 1.07, is equal to G, defined by equation 1.05. Repeating equation 1.05, one has,

(A.01) $G = \frac{C_X}{F_X - r}$.

Multiplying both sides by the denominator of the right side gives

(A.02) $G(F_X - r) = C_X$.

Multiplying G through the left side and transferring $G\ F_X$ to the right side gives

(A.03) $- G\ r = C_X - G\ F_X$.

Dividing both sides by -r gives

(A.04) $G = \frac{1}{r} \{ G\ F_X - C_X \}$.

Finally, one substitutes the optimizing condition $G = B_H - C_H$, equation 1.04, for G in the right side to obtain

(A.05) $G = \frac{1}{r}\{ [B_H - C_H]F_X - C_X \}$,

the right side of which is MSB.

CCA (Equation 1.28)

Equation 1.28 is derived from equation 1.27 by substituting the definitions for B and D from equation 1.14 and equation 1.16 into equation 1.27 and by breaking $d_x{}^2$, $d_{xn}{}^2$, $d_{xw}{}^2$, and $d_{nw}{}^2$ into the variances of u, v, and w. The latter step is accomplished as follows:

$$\text{(A.06)}\quad d_x^{\,2} = \mathrm{Var}(x) = \mathrm{Var}(\ u-v-(b_1+c_1)w\ ) = d_u^{\,2} + d_v^{\,2} + (b_1+c_1)^2 d_w^{\,2}.$$

$$d_{xn}^{\,2} = \mathrm{Cov}(x,n) = \mathrm{Cov}(\ u-v-(b_1+c_1)w,\ \tfrac{1}{r}[-f_1(u-v)+Bw]\ )$$

$$= E[\ (u-v-(b_1+c_1)w)\ \tfrac{1}{r}[-f_1(b_1+c_1)+Bw]\ ]$$

(One should recall that E[u] = E[v] = E[w] = 0.)

$$= \tfrac{1}{r}E[\ -f_1(u-v)^2 - (b_1+c_1)Bw^2\ ]$$

(One should recall that E[uv] = E[uw] = E[vw] = 0.)

$$= -\ \tfrac{1}{r}[\ f_1(d_u^{\,2} + d_v^{\,2}) - (b_1+c_1)B\ d_w^{\,2}]\ .$$

$$d_{xw}^{\,2} = \mathrm{Cov}(x,w) = \mathrm{Cov}(\ u-v-(b_1+c_1)w,\ w\ )$$

$$= E[\ (u-v-(b_1+c_1)w)w\ ] = -(b_1+c_1)\ E[w^2] = -(b_1+c_1)\ d_w^{\,2}.$$

$$d_{nw}^{\,2} = \mathrm{Cov}(n,w) = \mathrm{Cov}(\ \tfrac{1}{r}[-f_1(u-v) + Bw,\ w\ )$$

$$= \frac{B}{r}\ E[w^2] = \frac{B}{r}\ d_w^{\,2}.$$

APPENDIX B

DERIVATION OF SELECTED EQUATIONS IN CHAPTER 2

QUOTA

Stock Size Function (Equation 2.05)

Equation 2.03 is used to derive $X_{ki}(X_{i-1}, Z_{ki}, t)$. For k = 1 (quota), harvest rate is fixed during each period by the quota. The stock growth function is

(B.01) $$\dot{X} = f_0 - f_1 X - Q_i ,$$

where $Q_i = Z_{1i}$ is the level of the industry quota in period i. Now the elapsed time required for the stock to grow in period 1 from its size at the beginning of the period, X_0, to X_t is t :

(B.02) $$t = \int_{X_0}^{X_t} (1/\dot{X})dX = \int_{X_0}^{X_t} (f_0 - f_1 X - Q)^{-1} dX .$$

Performing the integration gives

(B.03) $$t = \frac{1}{-f_1}\ln(f_0 - f_1 X_t - Q) - \frac{1}{-f_1}\ln(f_0 - f_1 X_0 - Q) .$$

Taking the antilogarithm of both sides gives

(B.04) $$e^t = \left(\frac{f_0 - f_1 X_t - Q}{f_0 - f_1 X_0 - Q}\right)^{-1/f_1} .$$

Raising both sides to the $-f_1$ power and multiplying by the denominator of the right side gives

(B.05) $$(f_0 - f_1 X_0 - Q)e^{-f_1 t} = f_0 - f_1 X_t - Q .$$

Finally, collecting terms with f_0-Q, and solving for X_t (and adding a stochastic disturbance, w), one derives

$$\text{(B.06)}\quad X_t = \left(\frac{f_0 - Q}{f_1}\right)(1 - e^{-f_1 t}) + X_0\, e^{-f_1 t} + w\,.$$

Single-Period Net Benefit Function (Equation 2.07)

Single-period net benefits are defined as in equation 2.04, which is repeated here as the first step in the derivation:

(B.07) $E[R_1(X_0,Q)]$

$$= E\left[\int_0^z e^{-rt}\left\{ (b_0+u)Q - \frac{b_1}{2}Q^2 - (c_0+v-c_2X_t)Q - \frac{c_1}{2}Q^2 \right\} dt\right]$$

$$= E\left[\int_0^z e^{-rt}\left\{ (b_0-c_0+u-v+c_2X_t)Q - \frac{b_1+c_1}{2}Q^2 \right\} dt\right]$$

$$= E\left[\int_0^z e^{-rt}\left\{(b_0-c_0+u-v+c_2\left[\frac{f_0-Q}{f_1}(1-e^{-f_1 t}) + X_0 e^{-f_1 t} + w\right])Q - \frac{b_1+c_1}{2}Q^2\right\}dt\right]$$

$$= \int_0^z e^{-rt}\Big\{ (b_0-c_0+\frac{f_0}{f_1}c_2 - \frac{f_0}{f_1}c_2\, e^{-f_1 t} + c_2X_0\, e^{-f_1 t})Q$$

$$- \left(\frac{c_2}{f_1} - \frac{c_2}{f_1}e^{-f_1 t}\right)Q^2 - \frac{b_1+c_1}{2}Q^2 \Big\}dt$$

$$= \int_0^z \Big\{ \left[(b_0-c_0+\frac{f_0}{f_1}c_2)e^{-rt} + c_2(X_0 - \frac{f_0}{f_1})e^{-(f_1+r)t}\right]Q$$

$$- \left[\frac{c_2}{f_1}e^{-rt} - \frac{c_2}{f_1}e^{-(f_1+r)t} + \frac{b_1+c_1}{2}e^{-rt}\right]Q^2 \Big\}dt$$

$$= \Big[\{b_0-c_0+\frac{f_0}{f_1}c_2\}\frac{e^{-rz}-1}{-r} - \{\frac{f_0}{f_1}c_2\}\frac{e^{-(f_1+r)z}-1}{-(f_1+r)} + \{c_2X_0\}\frac{e^{-(f_1+r)z}-1}{-(f_1+r)}$$

$$- \Big[\{\frac{c_2}{f_1}\}\left(\frac{e^{-rz}-1}{-r} - \frac{e^{-(f_1+r)z}-1}{-(f_1+r)}\right) + \{\frac{b_1+c_1}{2}\}\frac{e^{-rz}-1}{-r}\Big]Q^2.$$

This last expressioncan be written

$$(W+YX_0)Q - LQ^2,$$

where $$W = \{b_0 - c_0 + \frac{f_0}{f_1} c_2\} \frac{e^{-rz}-1}{-r} - \{\frac{f_0}{f_1} c_2\} \frac{e^{-(f_1+r)z}-1}{-(f_1+r)},$$

$$Y = \{c_2\} \frac{e^{-(f_1+r)z}-1}{-(f_1+r)}, \text{ and}$$

$$L = \{\frac{c_2}{f_1}\} (\frac{e^{-rz}-1}{-r} - \frac{e^{-(f_1+r)z}-1}{-(f_1+r)}) + \{\frac{b_1+c_1}{2}\} \frac{e^{-rz}-1}{-r}.$$

TAX

Equilibrium Harvest Rate (Equation 2.12)

When a per-unit tax is employed (k = 2), the instantaneous harvest rate is a function of X_t and the current tax, T_i. The relationship is found by equating the demand function with the marginal harvest cost function plus tax:

(B.08) $b_0 + u - b_1\dot{H}_t = c_0 + v - c_2X_t + c_1\dot{H}_t + T_i$,

and solving for equilibrium harvest rate:

(B.09) $$\dot{H}_t = \frac{b_0 - c_0 + u - v + c_2X_t - T_i}{b_1 + c_1}.$$

Stock Size Function (Equation 2.14)

Inserting equation B.09 into the stock growth function gives

(B.10) $$\dot{X} = f_0 - f_1X - \frac{b_0 - c_0 + u - v + c_2X - T_i}{b_1 + c_1}.$$

After collecting terms, the stock growth function is

$$\text{(B.11)} \quad \dot{X} = f_0 - \frac{b_0-c_0}{b_1+c_1} - \frac{u - v}{b_1+c_1} - (f_1 + \frac{c_2}{b_1+c_1})X + \frac{T}{b_1+c_1} \ .$$

This can be written in the following form:

$$\text{(B.12)} \quad \dot{X} = \frac{A}{-(b_1+c_1)} + \frac{x}{-(b_1+c_1)} + \frac{B}{-(b_1+c_1)}X - \frac{T_i}{-(b_1+c_1)} \ ,$$

where $A = b_0 - c_0 - (b_1+c_1)f_0$,

$B = c_2 + (b_1+c_1)f_1$, and

$x = u - v$.

Following the same procedure to derive the stock size function as with the quota, the elapsed time required for stock size to grow from X_0 to X_t is

$$\text{(B.13)} \quad t = \int_{X_0}^{X_t} (1/\dot{X})dX = \int_{X_0}^{X_t} \frac{-(b_1+c_1)}{A + x + BX - T}dX \ .$$

Performing the integration gives

$$\text{(B.14)} \quad t = \frac{1}{-F}\ln(A + x + BX_t - T) - \frac{1}{-F}\ln(A + x + BX_0 - T) \ ,$$

where $F = \dfrac{B}{b_1+c_1}$.

Taking the antilogarithm of both sides, one has

$$\text{(B.15)} \quad e^t = \left(\frac{A + x + BX_t - T}{A + x + BX_0 - T}\right)^{-1/F} \ .$$

Raising both sides to the power $-F$ and multiplying by the denominator of the right side yields

$$\text{(B.16)} \quad (A + x + BX_0 - T)e^{-Ft} = A + x + BX_t - T \ .$$

Collecting terms with A + x - T , solving for X_t, and adding the growth disturbance, w, gives

(B.17) $$X_t = \frac{A + x - T}{-B}(1 - e^{-Ft}) + X_0\, e^{-Ft} + w\ .$$

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APPENDIX C

DERIVATION OF SELECTED EQUATIONS IN CHAPTER 3

QUOTA

Stock Size Function (Equation 3.06)

In many fisheries, fishing does not proceed year round, but is subject to a closed season. Fishing may be halted each year by regulations designed, for example, to protect gravid females, or by natural events such as the annual departure of the fish or the onset of winter weather. The period can be divided into two seasons: the first season open for fishing, and the second season closed. The open season, of length z, begins at time t = i-l and ends at t= i-l + z; while the closed season, of length l-z, begins at t = i-l + z and ends at t = i. Then the size of the stock at the end of the first period (the beginning of the second period) can be found by substituting X_l for X_t in equation B.06, X_z (stock size at the beginning of the closed season) for X_0, and l-z for t, one recalls that there is no harvest between t = z and t = 1:

$$\text{(C.01)}\quad X_1 = \frac{f_0}{f_1}(1-e^{-f_1(1-z)}) + X_z + e^{-f_1(1-z)}) + w\ .$$

In turn, X_z can be expressed by substituting z for t in equation B.06:

$$\text{(C.02)}\quad X_z = \frac{f_0-Q}{f_1}(1-e^{-f\ z}) + X_0\ e^{-f_1 z}\ .$$

Substituting this expression for X_z in equation C.01 gives

$$(C.03)\quad X_1 = \frac{f_0}{f_1}(1-e^{-f_1(1-z)}) + \frac{f_0-Q}{f_1}(1-e^{-f_1 z})e^{-f_1(1-z)} + X_0\, e^{-f_1} + w\ .$$

Rearranging the second of the three terms on the right side of this expression results in

$$(C.04)\quad X_1 = \frac{f_0}{f_1}(1-e^{-f_1(1-z)}) + \frac{f_0}{f_1}(e^{-f_1(1-z)} - e^{-f_1}) - \frac{Q}{f_1}(e^{-f_1(1-z)} - e^{f_1})$$

$$+ X_0\, e^{-f_1} + w\ .$$

Finally, collecting terms with f_0/f_1 gives

$$(C.05)\quad X_1 = \frac{f_0}{f_1}(1-e^{-f_1}) - \frac{Q}{f_1}(e^{-f_1(1-z)} - e^{-f_1}) + X_0\, e^{-f_1} + w\ ,\ \text{or}$$

$$(C.06)\quad X_1 = M - NQ + PX_0 + w\ ,$$

where $M = \frac{f_0}{f_1}(1-e^{-f_1})$,

$N = (e^{-f_1(1-z)} - e^{-f_1}) / f_1$,

$P = e^{-f_1}$.

TAX

Stock Size Function (Equation 3.21)

Finding stock size at the end of period 1 when there is a closed season beginning at t = z requires use of-equation B.06 as follows: Expressing X_1 as a function of stock size at the beginning of the closed season X_{il+z} gives

$$(C.07)\quad X_1 = \frac{f_0}{f_1}(1-e^{-f_1(1-z)}) + X_z\, e^{-f_1(1-z)} + w\ .$$

In turn, X_z is a function of X_0, again using equation B.06:

(C.08) $$X_z = \frac{A + x - T}{-B}(1-e^{-Fz}) + X_0\, e^{-Fz} \,.$$

Substitution of equation C.08 into equation C.07 and multiplying the term $e^{-f_1(1-z)}$ through the second of the two terms in equation C.07 gives

(C.09) $$X_1 = \frac{f_0}{f_1}(1-e^{-f_1(1-z)}) + \frac{A + x - T}{-B}(1-e^{-Fz})e^{-f_1(1-z)}$$
$$+ X_0\, e^{-Fz}\, e^{-f_1(1-z)} + w \,.$$

Multiplying $e^{-f_1(1-z)}$ through the second of the three terms in equation C.09 and consolidating terms containing e gives:

(C.10) $$X_1 = \frac{f_0}{f_1}(1-e^{-f_1(1-z)}) + \frac{A + x - T}{-B}(e^{-f_1(1-z)} - e^{-Gz-f_1})$$
$$+ X_0\, e^{-Gz-f_1} + w \,,$$

where $G = \frac{c_2}{b_1+c_1}$.

Finally, breaking the second of the three terms in equation C.10 into three parts and combining the first of the three parts with the first term of equation C.10 yields

(C.11) $$X_1 = M' + N'T + P'X_0 - N'x + w \,,$$

where $M' = \frac{f_0}{f_1}(1-e^{-f_1(1-z)}) + \frac{A}{-B}(e^{-f_1(1-z)} - e^{-Gz-f_1})$,

$N' = (e^{-f_1(1-z)} - e^{-Gz-f_1}) / B$,

$P' = e^{-Gz-f_1}$.