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U.S. DEPARTMENT OF COMMERCE
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National Ocean Survey
Rockville, Maryland

LECTURE NOTES FOR USE AT WORKSHOPS ON SURVEYING INSTRUMENTATION AND COORDINATE COMPUTATION

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Office of National Geodetic Survey

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TABLE OF CONTENTS

Page

Theodolites..... 1
by Carl F. Kelley

Basic Principle - Electro-Optical Distance Measuring Instruments..... 32
by George B. Lesley

Electronic Distance Measuring Instruments..... 60
by Raymond W. Tomlinson

Rectangular Coordinate Systems..... 80
by Joseph F. Dracup

Corrections to Measured Lengths and Computation of Grid Azimuths..... 105
by Joseph F. Dracup

Computations and Adjustments..... 120
by Joseph F. Dracup

Bibliographies..... 213

NOTE: Sketch and table numbers are not numbered consecutively. Consequently, references to sketches and tables in the text of a particular report are limited to that report only.

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THEODOLITES

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Introduction

Many surveyors avoid using or even becoming familiar with theodolites because they fear the name, theodolite, and its inference to control surveys. This fear is unjustified as the definitions below will illustrate.

THEODOLITE: A precise surveying instrument consisting of an alidade with telescope, mounted on an accurately graduated circle, equipped with necessary levels and reading devices. Sometimes the alidade carries a graduated vertical circle. The two classes of theodolites are direction instruments and repeating instruments.

TRANSIT: A surveying instrument composed of a horizontal circle graduated in circular measure and an alidade with a telescope which can be reversed in its supports without being lifted therefrom.

Based on these definitions it can be stated that a theodolite is nothing more than a glorified transit. However, such instruments provide a system for obtaining reliable horizontal and vertical directions with a minimum of effort.

This text is designed to familiarize the reader with theodolites in general and point out major principles in their operation. As most theodolites are similar, to avoid repetition small differences will not be explained. However, to maintain continuity all aspects of one type theodolite, the Wild T-2, will be discussed with other types being referenced whenever necessary. Specific information on the instrument of your choice may be found in the manual which will accompany your instrument. Read this manual prior to operating the instrument.

Unpacking the Instrument

Theodolites are normally transmitted in a shipping case which, including instrument, has a total weight of approximately 35 pounds. The removal of the instrument, which is in a carrying case, from the shipping container will present no problem. It is accomplished utilizing the carrying strap or handle on the carrying case. As with all instruments be sure to place the case containing the instrument on a firm level surface when setting it down.

The cover, lid, or door of the carrying case is then removed or opened as the case may be. The cover for the T-2 case is connected to the base of the case by two catches. By pulling outward on the top of these catches, the top is freed and may be lifted off the instrument and base.

The instrument is connected to the base of the carrying case during transportation to prevent damage. The T-2 is loosened from the base by freeing three sliding levers that hold it in place and sliding them away from the instrument. Instruments in wooden carrying cases are normally held in place by rotating wooden blocks and slid into the case rather than set on the base. The T-2 is always placed in the case with the telescope in the vertical position eye piece down and clamped although this procedure may differ with other instruments. The T-2 is then grasped by both Y axes, near the base, and removed from the case base and placed on the tripod. The instrument should be held in a near vertical position during this operation and care taken to assure that it is secured to the tripod thus preventing its being knocked off.

The reverse procedure is followed when replacing the instrument in the carrying case, which is where it should be when transported from station to station. In no case should the theodolite be left attached to the tripod, and

the two units carried in a horizontal position as the instrument especially the spindle could be damaged.

Tripod Setup, Leveling Instrument

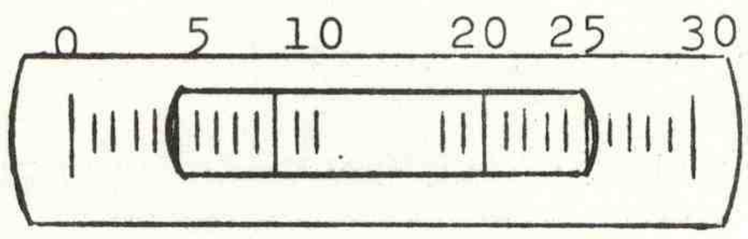
Prior to unpacking the instrument, the tripod should be set over the point with the plate approximately level. This is accomplished by the use of a plumb bob which is inserted in the fastening plug. When performing this orientation, be sure the plug is in the center of the tripod to allow for final centering when the instrument is placed on the tripod. The legs of the tripod are firmly dug into the ground (for stabilizing remove any sod which may be around the legs of the instrument) by stepping on the foot pieces. If good observations are to be obtained, you must have a stable setup. In some cases it is necessary to anchor or stake the tripod legs, place a walking platform around the tripod and so forth to insure stability.

After placing the instrument on the tripod, final centering is performed. This is often performed with a plumb bob, but when the instrument is equipped with an optical plummet, it may be used. Some tripods such as the Kern are equipped with a centering rod or forced centering device. The methods for utilizing the forced centering rod and checking the optical plummet will be outlined in the instrument's handbook.

Using the circular level or bull's-eye bubble, bring the instrument into approximate level. Using the horizontal

or plate level, the instrument is now leveled exactly. Rotate the instrument so that the level vial is parallel to any two of the three leveling screws. Bring the bubble to center, using these two screws as shown in Figure 1. Turn the instrument 90° and center the bubble, using the third screw. Return to the first position to see that the bubble is still centered. Now, turn the instrument 180°. If the bubble is centered, the instrument is now level. If not, (Figure 2) the level is out of adjustment.

T-3



T-2

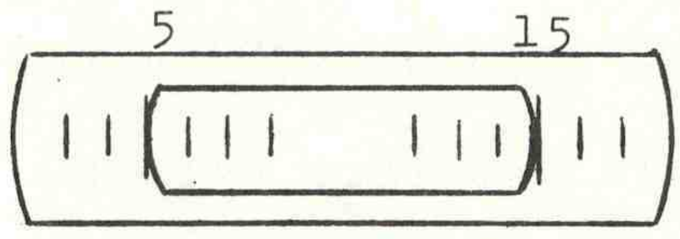
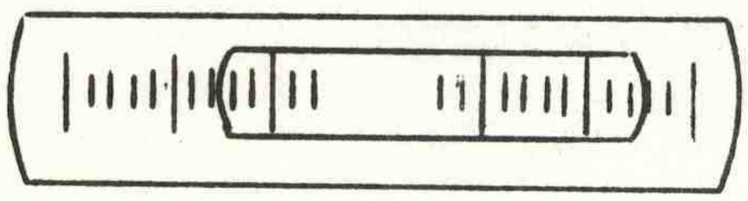


Figure 1



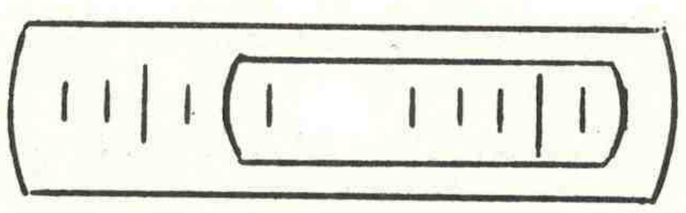
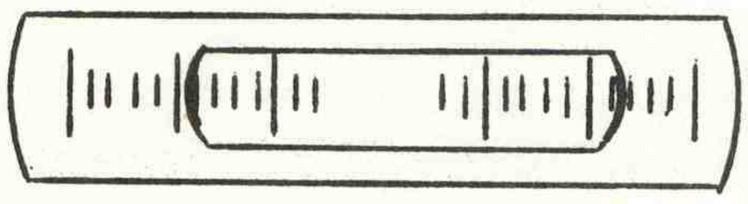


Figure 2



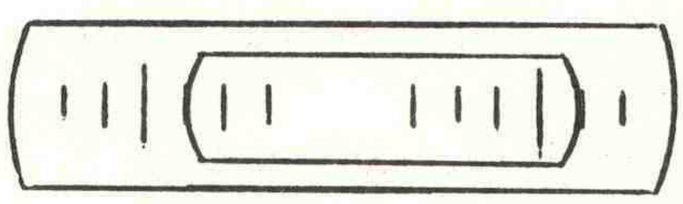


Figure 3

Unless the bubble is out a large amount, it should not be adjusted as it adds to the time required to level the instrument and has a tendency to wear the threads of the

adjusting screws and nuts. Instead, after the normal leveling procedure has been completed, rotate the alidade so that the bubble axis is in line with one of the foot screws. Note, by the graduation, the position of the lower end of the bubble (toward the vertical circle), then rotate to 180°. Note again the position of the same (lower) end of the bubble (Figures 1 & 2). The mean of these two readings is the so-called "reversing position" of the bubble. Assume, for example, that the first reading is 4.6 and the second reading after rotating 180° is 7.2. The reversing point is 5.9. Now turn the alidade 90° so that the bubble axis is parallel to the other two foot screws. Bring the bubble to its reversing position by rotating these foot screws equally in opposite directions (Figure 3). This procedure may be repeated to refine the determination of the reversing position. Once the reversing position is determined, all that needs to be done to level the theodolite is to use the foot screws, only, to bring the level to its reversing position in each of the two directions of the alidade 90° apart.

Normally, only one end of the bubble need be considered since it is not likely to change significantly in length during the period of observations. By employing this method, the level of the theodolite may be checked and corrected very simply without resorting to adjusting the vial itself.

The key to this method is that once the reversing position of the bubble is well determined and the theodolite is leveled as described, then the vertical axis on which the alidade rotates is truly vertical and the bubble will remain in the same position, whether centered or not, in any direction the alidade is rotated. The reversing position of the bubble may drift slightly from day to day or even during a period of observations; thus, it should be checked before each observing period and occasionally during the period. It should seldom be necessary to readjust the bubble vial.

When it is necessary to readjust the vial, the instrument should be leveled as indicated above and the bubble brought to center, using the level adjusting screw nuts only. If the means of adjusting the level vial is a screw which opposes a spring, the last turn in bringing the bubble to center should be made against the spring.

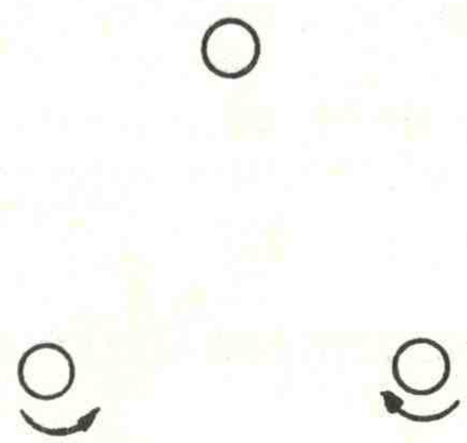


Figure 4

The bubble travels in the direction of the left thumb when you are facing the instrument.

Prior to starting observations a final check should be made with the instrument in level to assure that it has remained centered over the point.

Parallax and Focusing

Parallax is the apparent movement of the cross-hairs over the image of the target when the eye is moved from side to side. The following procedures will eliminate parallax:

1. Using the black milled ring on the eye-piece, make the crosswires very sharp and black while sighting on a moderately bright background approximately 300 meters away or on the sky.
2. Sight on an object approximately 100 meters away and focus it sharply with the focusing ring.
3. Move the head from side to side to see if parallax exists.
4. If it does, use the milled ring to eliminate the parallax. It will be necessary to correct the focus each time the milled ring is moved.
5. Note the setting on the milled ring so that it may be reset after dark when this test is not possible.

For focusing of the reading microscope, the milled occular tube is rotated while the graduation become sharply defined.

Pointings

The definition of the term "pointing" is critical as the operation is involved in performing this act. To obtain.

relatively rapid and accurate readings, the observer should point and not aim at his target(s). Aiming, or excessive time expended while pointing on an object, will cause eye strain and give the object the appearance of moving. The first effort or when the target first appears to be properly aligned with the cross hairs is normally the most reliable.

When moving the target into the cross hairs with the slow motion screw, it is recommended that the final movement be made in a clockwise direction. This procedure will eliminate small errors caused by slack in the slow motion screen.

All pointing should be made by placing the object near the point where the horizontal and vertical wires cross and preferably at their intersection. Small deviations from this point are allowable, but all pointings should be made with the object in the same relationship with the cross hairs to eliminate errors due to possible tilt of the hairs.

The following figures illustrate typical cross hair arrangements and indicate the recommended location of the target with a dot. These locations are for horizontal directions.

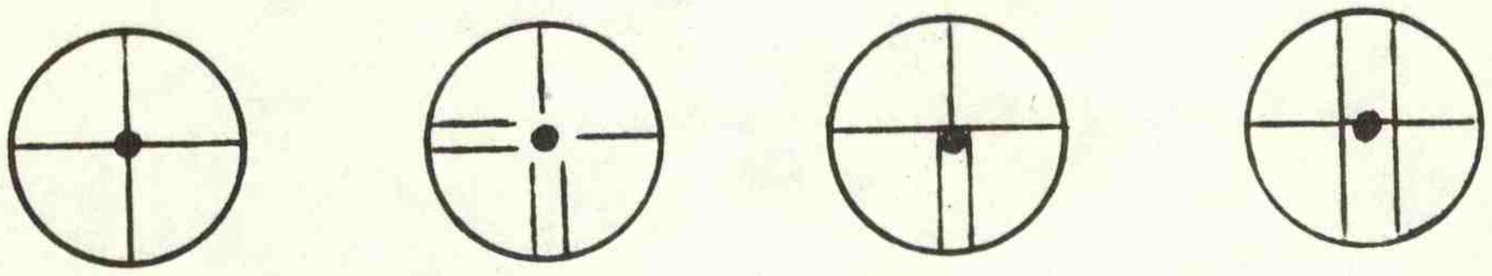
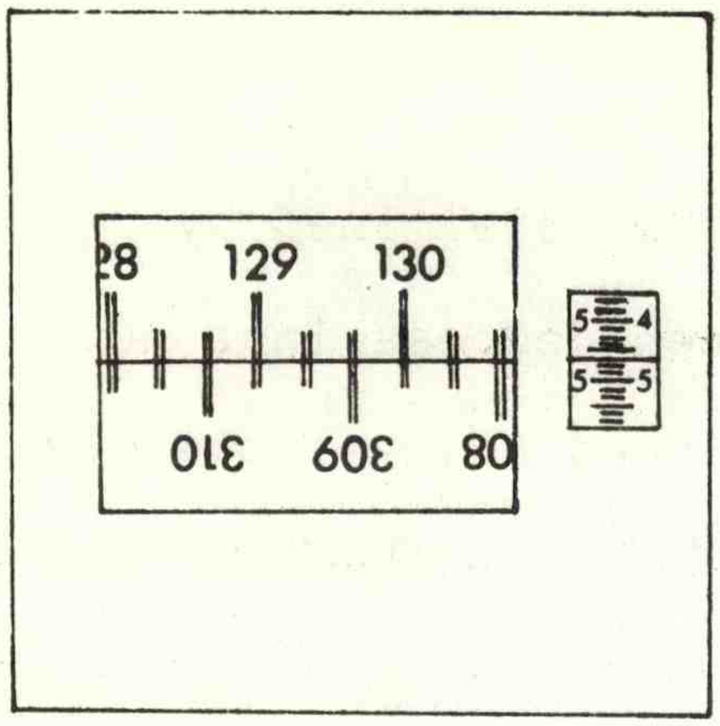


Figure 5

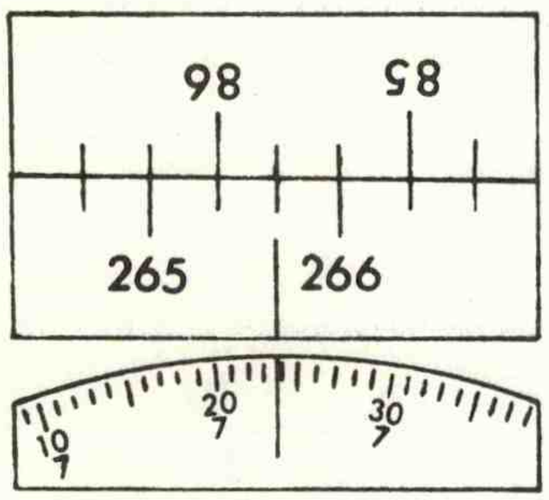
The horizontal and vertical circles are normally read using a microscope which adjoins the telescope. In some instruments such as the T-2, it is necessary to rotate a knob allowing you to view either the horizontal or vertical circle. By turning this knob clockwise to the stop, the horizontal circle is visible, and likewise by turning it counterclockwise to the stop, the vertical circle may be viewed. In some instruments both circles are visible at the same time.

The illumination of the circles should be as bright and equally illuminated as possible. During the measurement of an angle, the illumination should not be changed. In the reading microscope appear the images of the diametrically opposite portions of the circle, separated by a fine horizontal line, and below that is the scale image of the micrometer drum (this system varies with different instruments). Circles for the Wild T-2, DKM 2A and THEO 010 are shown in Figure 6. As this figure illustrates the DKM 2A provides a digitized reading double circle system. The 10 minute multiples are bracketed by the index lines for every setting of the micrometer.



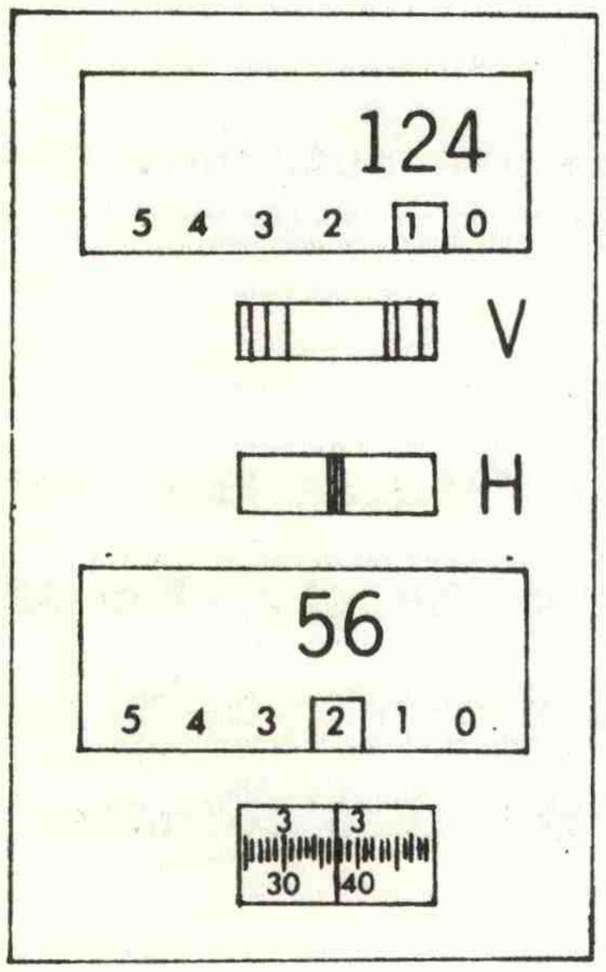
JENA THEO 010

129° 25' 47"5



WILD T-2

265° 47' 23"6



KERN DKM 2-A

56° 23' 37"0

Figure 6

Horizontal Circle

The graduation of the circle may be determined by dividing the number of divisions between degrees into 60. Reading of this circle may be accomplished using the index marker or by counting the number of divisions between degree readings which are 180° opposite and multiplying by 1/2 the value of each graduation.

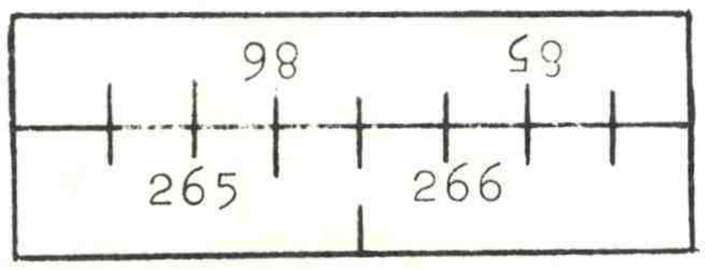


Figure 7

Each division equals 20 minutes.

There are four divisions between 265° and 85°.

$4 \times \frac{20}{2} = 40$ minutes.

Readings therefore indicate 265° 40'.

The additional minutes and seconds are obtained from the micrometer circle.

Vertical Circle

The same procedure may be used for reading the vertical circle. Using the procedure of counting the division is often more reliable than using the index marker as it may be in error. The circles must be brought into coincidence before these readings are made.

Electrical Illumination

Most modern theodolites are equipped with electrical illumination in addition to mirrors which may be used for obtaining light from natural sources. It is recommended that electrical illumination be used for all observations as it provides a constant and bright illumination for all plate readings. You may also vary the intensity of your light by use of a rheostat.

A diffused light bulb is essential in properly illuminating the horizontal circle. You may find that frosted bulbs do not accomplish this to a desirable degree. For this reason many bulb tubes are equipped with colored lens to further diffuse the light. The bulbs may be modified by painting them white or placing a piece of paper over the end of the light tube when the colored filament is not available.

Horizontal Plate Circle Settings

When observing horizontal angles, it is essential that different plate settings be used for each position. These positions should be selected in such a manner that the entire range of the circle and micrometer are utilized. This is necessary to minimize any errors that may be present in the circle and micrometer. The following tables for 4, 8, 12, and 16 positions will accomplish this objective and are therefore recommended.

CIRCLE SETTINGS

(1) Four Positions of Circle

	5-Minute Micrometer Drum	10-Minute Micrometer Drum	Wild T-3 Micrometer Circle Readings
1	0° 00' 40"	0° 00' 10"	0° 00' 10 units
2	45 01 50	45 02 40	45 00 25 units
3	90 03 10	90 05 10	90 00 35 units
4	135 04 20	135 07 40	135 00 50 units

(2) Eights Positions of Circle

1	0 00 40	0 00 10	0 00 10 units
2	22 01 50	22 01 25	22 00 25 units
3	45 03 10	45 02 40	45 00 35 units
4	67 04 20	67 03 55	67 00 50 units
5	90 00 40	90 05 10	90 00 10 units
6	112 01 50	112 06 25	112 00 25 units
7	135 03 10	135 07 40	135 00 35 units
8	157 04 20	157 08 55	157 00 50 units

(3) Twelve Positions of Circle

	5-Minute Micrometer Drum	10-Minute Micrometer Drum	Wild T-3 Circle	Micrometer Readings
1	0° 00' 40"	0° 00' 10"	0° 00'	10 units
2	15 01 50	15 01 50	15 00	25 units
3	30 03 10	30 03 30	30 00	35 units
4	45 04 20	45 05 10	45 00	50 units
5	60 00 40	60 06 50	60 00	10 units
6	75 01 50	75 08 30	75 00	25 units
7	90 03 10	90 00 10	90 00	35 units
8	105 04 20	105 01 50	105 00	50 units
9	120 00 40	120 03 30	120 00	10 units
10	135 01 50	135 05 10	135 00	25 units
11	150 03 10	150 06 50	150 00	35 units
12	165 04 20	165 08 30	165 00	50 units

(4) Sixteen Positions of Circle

	5-Minute Micrometer Drum	10-Minute Micrometer Drum	Wild T-3 Circle	Micrometer Readings
1	0° 00' 40"	0° 00' 10"	0° 00"	10 units
2	11 01 50	11 01 25	11 00	25 units
3	22 03 10	22 02 40	22 00	35 units
4	33 04 20	33 03 55	33 00	50 units
5	45 00 40	45 05 10	45 00	10 units
6	56 01 50	56 06 25	56 00	25 units
7	67 03 10	67 07 40	67 00	35 units
8	78 04 20	78 08 55	78 00	50 units
9	90 00 40	90 00 10	90 00	10 units
10	101 01 50	101 01 25	101 00	25 units
11	112 03 10	112 02 40	112 00	35 units
12	123 04 20	123 03 55	123 00	50 units
13	135 00 40	135 05 10	135 00	10 units
14	146 01 50	146 06 25	146 00	25 units
15	157 03 10	157 07 40	157 00	35 units
16	168 04 20	168 08 55	168 00	50 units

The Kern DKM-3 theodolite is an example of an instrument where the micrometer drum has a range of 5 minutes.

The Wild T-2 and the Kern DKM-2 theodolites are examples of instruments whose micrometer drums have a range of 10 minutes.

Recording

The following formats illustrate the recommended form for recording horizontal and vertical observational data:

Horizontal Directions

Station: BALDY Instrument: Wild T-2 Date: 7/31/68

Position	Observed	D or R	°	'	"	*"	Mean	Mean D & R	Direction "
1	MARBLE	D	0	00	15	16	15.5		
			180	00	10	12	11.0	13.2	00.0
	VALE	D	79	47	46	48	47.0		
			259	47	40	43	41.5	44.2	31.0
	RUSH	D	104	09	22	23	22.5		
			284	09	18	19	18.5	20.5	07.3
2	MARBLE	R	191	01	28	30	29.0		
			11	01	33	34	33.5	31.2	00.0
	VALE	R	270	49	02	04	03.0		
			90	49	06	07	06.5	04.8	33.6
	RUSH	R	295	10	35	36	35.5		
			115	10	40	40	40.0	37.8	06.6

*To insure against blunders, the micrometer should be brought into coincidence twice and both the readings recorded. The spread between the readings should not exceed 3" or 3 units for a Wild T-3.

Vertical Directions

Station: BALDY Instrument: Wild T-2 Date 7/31/68

Position	Observed	Tel D or R	°	'	"	"	Mean	Mean D & R	Direction
1	MARBLE	D	87	10	07.0	05.0	06.0		
		R	272	50	03.5	04.5	04.0	02.0	87 10 01.0
		D	87	10	02.5	02.5	02.5		
		R	272	50	10.0	08.0	09.0	53.5	87 09 56.7
		D	87	10	09.0	07.5	08.2		
		R	272	50	08.5	09.0	08.8	59.4	87 09 59.7
									87 09 59.1
2	VALE	D	90	19	10.0	12.0	11.0		
		R	269	41	08.0	06.0	07.0	04.0	90 19 02.0
		D	90	19	11.0	12.0	11.5		
		R	269	40	58.0	59.0	58.5	03.0	01.5
		D	90	19	08.0	10.0	09.0		
		R	269	41	06.0	08.0	07.0	02.0	01.0
									90 19 02.2

Observing Procedures

Observations should be made following procedures which remove the collimation error of the instrument and range over the complete circle and micrometers. Recommended circle settings for various instruments and number of positions were given earlier.

Although other observational practices could be developed that would satisfy the above requirements, the procedures given in US C&GS Special Publication No. 247 are as simple as most and once the pattern is developed become almost automatic.

To illustrate this technique, let us say several signals are to be observed from a point. One signal, usually the one furthermost to the left is selected as the initial. The circle and micrometer are set for a particular position. It is not necessary to set the seconds (or units for the Wild T-3) on the micrometer exactly as specified, but every effort should be made to have the setting within 10" (or 5 units for the T-3). Each signal is then observed in a clockwise order and the results recorded. At the last target, the telescope is reversed (plunged) and the procedure repeated in a reverse (counterclockwise) order. The observed seconds are meaned and the initial direction is subtracted from each observation, thus referencing the measurements to an initial of 0° 00' 00".00. This completes one position. To continue the observations, it is not necessary to have the telescope in the direct position. The next circle setting as given in Table A may be increased by 180° if the telescope is in reverse.

If a first-order theodolite is used, the rejection limit is 4" from the mean for first-order surveys and 5" from the mean for second and third-order surveys for limited surveys where it would be more likely a Kern DKM-2 or its equivalent would be utilized and for these instruments a rejection limit of 5" from the mean is acceptable for all orders of accuracy. An observation which falls outside the specified tolerance should be rejected and reobserved using the same circle setting.

Observations of Astronomic Azimuths

In order to obtain reliable astronomic azimuth values, it is necessary that the theodolite be equipped with a good level vial. Therefore, it is recommended that a striding level should be used with second-order theodolites such as the Wild T-2. The vial should be graduated from left to right with zero at the vertical circle end of the vial. A strip of adhesive tape may be used for this purpose and the white background will make the numbers easy to read.

If the value of the level vial is not available with the instrument, it may be determined in the field following the procedure explained later.

Special care must be employed to maintain the vertical axis of the instrument as nearly vertical as possible throughout the azimuth observations. This requires keeping the bubble at/or very close to its "reversing point", or the point at which the bubble remains stationary when the alidade is rotated. This condition is achieved entirely by adjustment of the footscrews and is independent of the level adjustment. The latter adjustment merely controls the position of the bubble reversing point and is not critical to the observations as long as the reversing point is fairly well centered. If necessary, the bubble can be brought close to its reversing point at the end of each position by a slight "touching up" of the footscrews,

especially when the vial is broadside to the direction of the star. Bubble readings will be recorded immediately after each pointing on the star, and also on the mark if the line departs more than 1 degree from the horizontal. Careful control of the vertical axis as outlined above will render unnecessary the recording of bubble readings of inclined sights of less than 5 degrees.

Level Readings When Using Plate Level

	W		E
D	08.0		21.1
R	22.2		08.9
	14.1	+1.9	12.2

Level Readings Using Stride Level

W		E	
08.0		21.1)	
		)	made prior to star
22.2		08.9)	
14.1	+1.9	12.2	
08.2		21.3	
22.0		08.7	
13.8	+1.2	12.6	
	+3.1 ÷ 2 = 1.6		

Additional equipment needed to observe a second-order azimuth are a portable short wave battery powered radio receiver with a frequency range of from about 1 to 30 megacycles per second. Time comparisons should be made at the station so that the chronometer will not be moved after comparisons.

A standard chronometer or watch with a good rate is necessary for use in azimuth observations. Chronometers are very delicate instruments and should be handled and packed with care. Information from "Apparent Places of Fundamental Stars" for the current year is needed for azimuth observations and computations.

Time will be recorded in the following manner:

Object Observed				Telescope
Polaris				
	7^h	38^m	$47^s.0$	Direct
	7	41	40.0	Reverse
Mean	7	40	13.5	
Diff.		2	53.0	

Determination of Level Value

The value of plate or striding level vials for Wild T-2, Wild T-3, and Kern DKM-2 theodolites may be determined by differential vertical angles using the following method. For maximum accuracy and greater simplicity a second theodolite, a transit works equally well, is recommended as the target for the level determination. This will require illuminating the target crosswires, making sure the instrument is focused near infinity. This can be accomplished by focusing the instrument on a sufficiently distant physical feature.

1. Select an area for the instrument support stand which will provide maximum stability for the level determination.

2. If a second instrument is not available, select a clearly defined target for the vertical observations at least 0.25 mile distant. The observations should be made when there is a minimum change in refraction (normally between 1200 Noon and 4 P.M.)

3. When attaching the tribrach to the observing stand, make certain the grooves are oriented such that two legs of the theodolite will be parallel to the direction of the target and the instrument is shaded. (Figure 8.)

4. The instrument should be leveled paying close attention to the cross leveling; that position which places the level vial perpendicular to the target direction. No additional cross-leveling is made during a set, but may be performed between sets.

5. With the theodolite pointed on target, set the horizontal plate to read 0° 00', thus facilitating the placing of the level vial parallel to the target line-of-sight by simply rotating the instrument to read 90° or 270° on the circle. The altitude level slow motion screw is not used for the determination and should be tightened to put the level out of action.

6. Placing the instrument at a right angle to the target line-of-sight, move the plate or striding

bubble near one end of the vial by turning both parallel foot screws an equal amount. Read and record both ends of the bubble, allowing approximately one minute for the bubble to come to rest.

7. Repoint on the target, reading and recording the vertical angle. Since we are interested only in angular tilt (difference in vertical angles) required to move the level bubble a given distance, telescope reversal is not required.

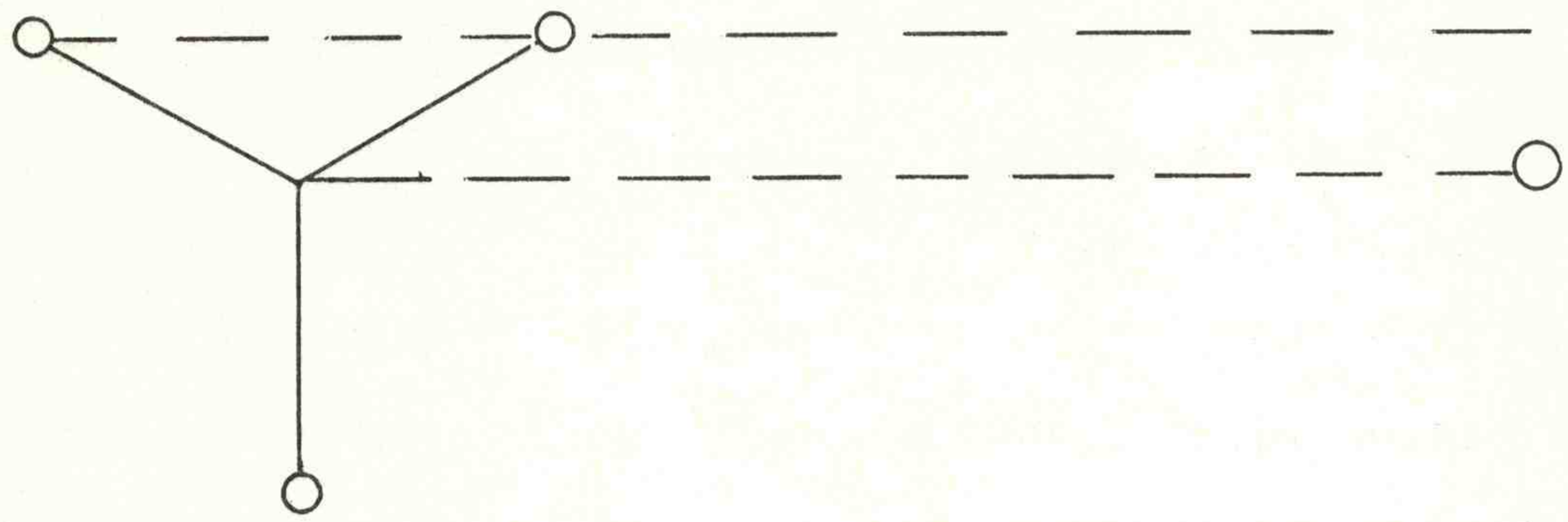
8. Repeat step six, moving the bubble near the opposite end of the vial.

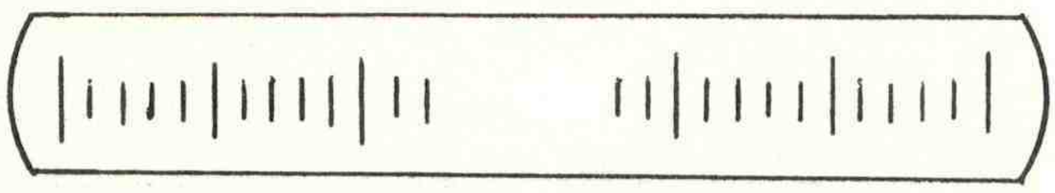
9. Repeat step seven. This completes one determination of the vial.

10. In order to assess the uniformity of the graduated vial and obtain the best mean value, it is necessary to determine a value from each graduation. The following abstracts exemplify the method used, noting that the mean level differences (column 3) decrease approximately one division with each successive determination.

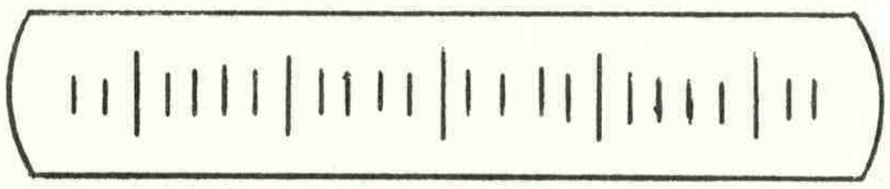
11. The number of determinations per set will be governed by the length of the bubble and the number of vial graduations, however, these abstracts indicate the optimum number obtainable.

12. The value of the level is obtained from a minimum of two sets. The range between the determinations within a set shall not exceed two seconds. The sets shall not differ by more than one second.

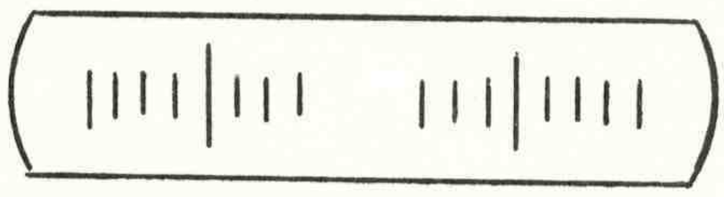




Wild T-3 Plate Bubble



Wild T-2 Stride Level



Wild T-2 Plate Bubble

Value of One Division of Bubble
Wild T-2 XXXXXXX
Striding Level

Set 1 of 2

	Level L	Level R	Mean Diff.	Vertical Angle	Mean Seconds	Diff. Seconds	Seconds value per Division
1.	09.7	22.8		270 04 39-4	40.0		
2.	17.0	30.1	7.30	04 05-06	05.5	34.5/7.30=04.73	
3.	11.0	24.1	6.00	04 32-30	31.0	25.5/6.00=04.25	
4.	15.9	29.1	4.95	04 07-06	06.5	24.5/4.95=04.95	
5.	12.0	25.1	3.95	04 24-24	24.0	17.5/3.95=04.43	
6.	14.9	28.1	2.95	04 10-09	09.5	14.5/2.95=04.92	
7.	13.0	26.2	1.90	04 19-20	19.5	10.0/1.90=05.26	
8.	13.9	27.1	0.90	04 14-14	14.0	5.5/0.90=06.11	

mean of seven 04.95

Value of One Division of Bubble
Wild T-3 XXXXXXX

Set 1 of 2

	level L	level R	mean diff.	Vertical Angle	Seconds sum	Double Diff.	Seconds value per division
1.	27.2	07.7		89 20 50.2-50.5	100.7		
2.	22.3	03.2	4.70	20 42.0-42.3	84.3	*32.8/4.70=06".98	
3.	25.9	06.7	3.55	20 48.8-48.8	97.6	26.6/3.55=07.49	
4.	23.2	04.0	2.70	20 43.7-43.9	87.6	20.0/2.70=07.41	
5.	25.0	05.8	1.80	20 47.3-47.3	94.6	14.0/1.80=07.78	
6.	24.0	04.9	0.95	20 45.3-45.4	90.7	07.8/0.95=08.21	

mean of five 07".57

* Double difference of sum of seconds

e.g. 100.7
- 84.3
16.4 x 2 = 32.8

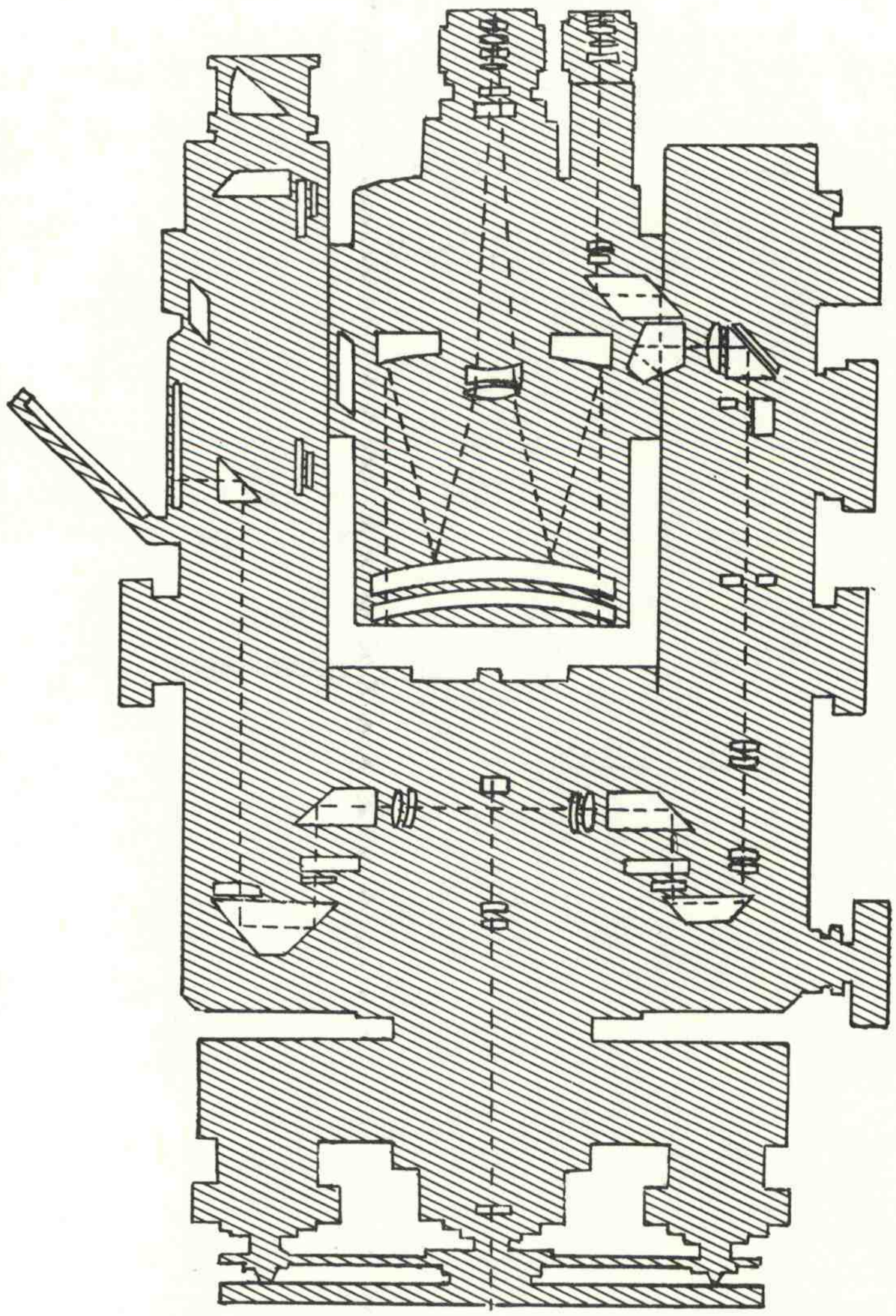
Value of One Division of Bubble
Kern DKM 2 XXXXXX
Striding Level

	Level L	Level R	Mean Diff.	Vertical Angle	Seconds	Seconds	Set 1 of 2 Seconds Value per Division
1.	04.4	17.2		88 02 48-49	48.5		
2.	07.7	20.4	3.25*	02 06-06	06.0	42.5/3.25	=13.08
3.	05.0	17.9	2.6	02 40-41	40.5	34.5/2.60	=12.27
4.	06.8	19.6	1.75	02 18-19	18.5	22.0/1.75	=12.57
						mean of three	12.97

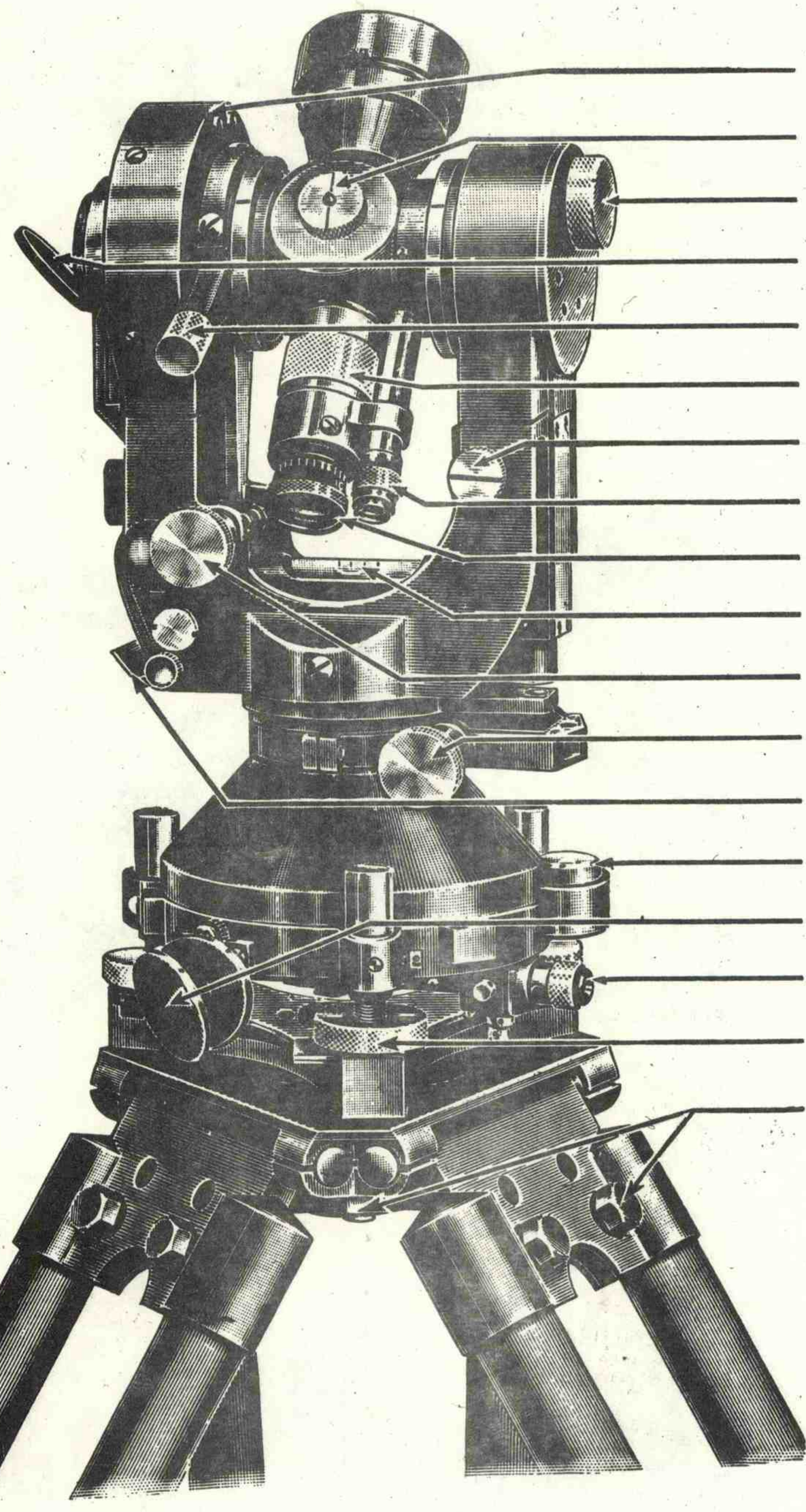
* This value is the mean of the differences between level readings for this set and the previous set.

e.g.

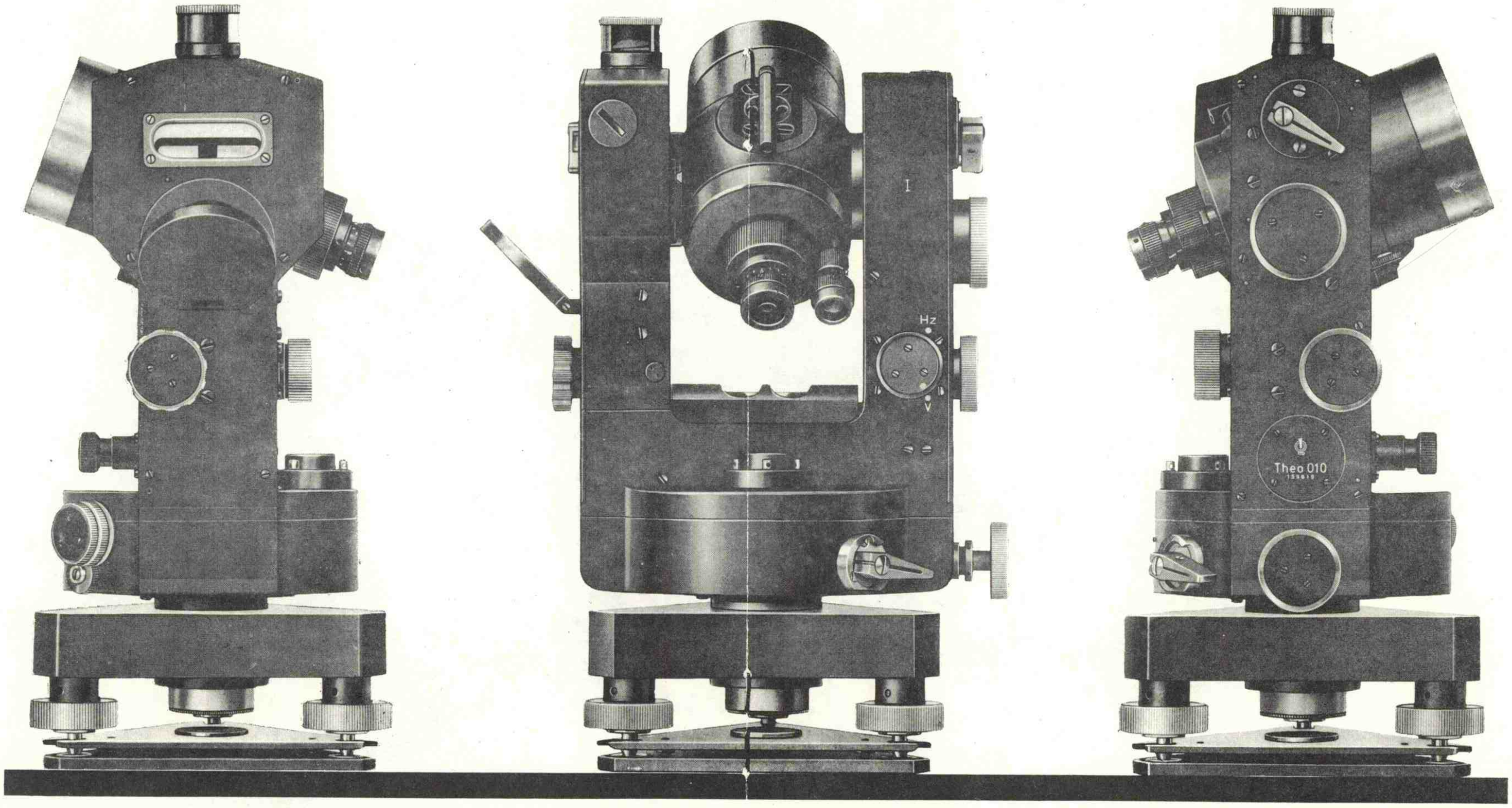
	L	R	
1.	04.4	17.2	$\frac{3.3 + 3.2}{2} = 3.25$
2.	07.7	20.4	
	3.3	3.2	



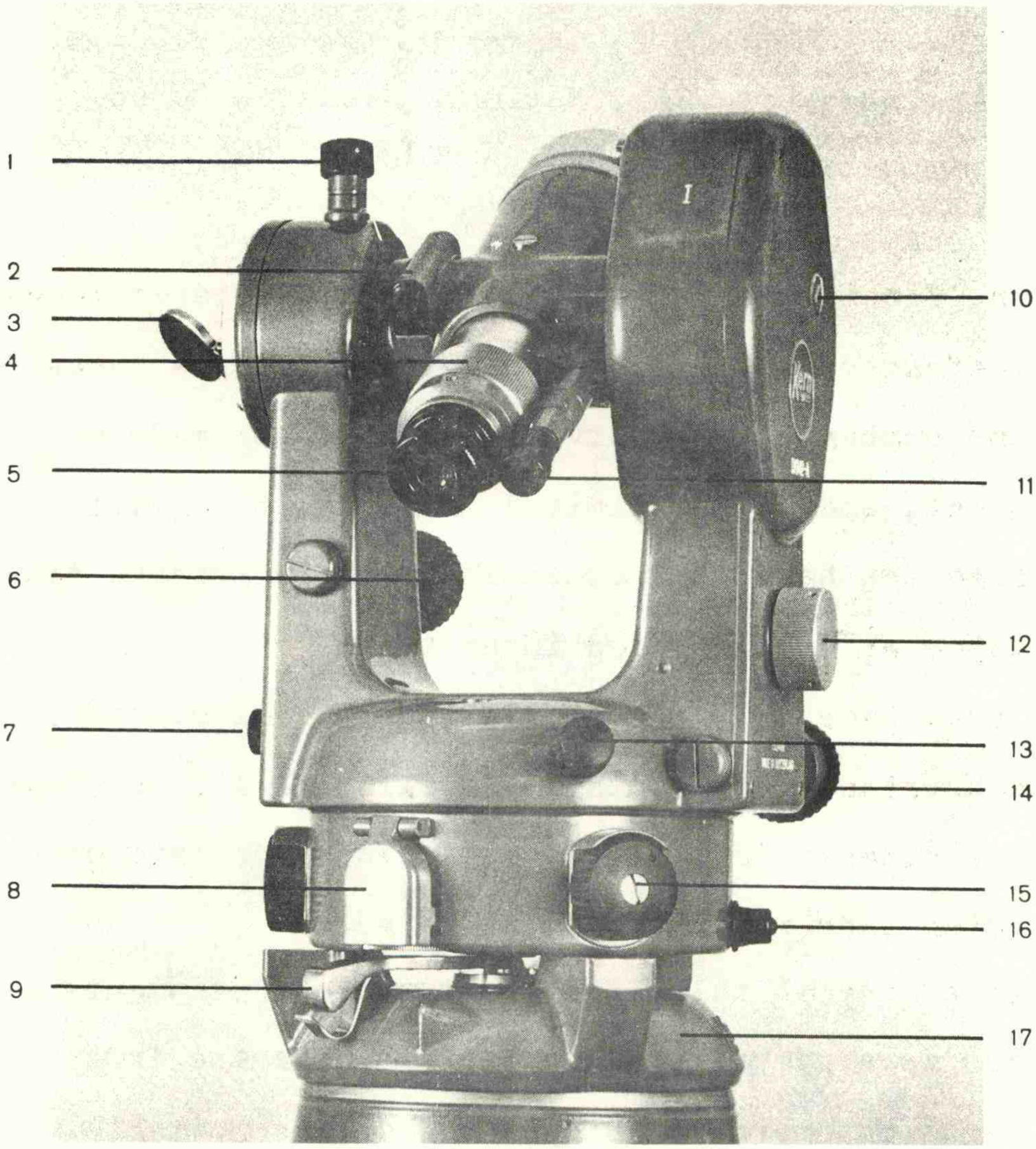
Schematic diagram of the THEO 010 Theodolite



Universal-Theodolite WILD T 2



JENA THEO 010



1	Horizontal axis clamp	10	Index error adjusting screw
2	Finder collimator	11	Circle reading eyepiece
3	Illuminating mirror	12	Micrometer knob
4	Focusing ring	13	Vertical axis clamp
5	Telescope eyepiece	14	Horizontal slow-motion screw
6	Vertical slow-motion screw	15	Leveling knob
7	Optical plummet	16	Terminal for electric illumination
8	Coarse-fine circle orienting drive	17	Centering tripod head
9	Locking lever		

BASIC PRINCIPLE

ELECTRO-OPTICAL DISTANCE MEASURING INSTRUMENTS

By George B. Lesley
NOAA, National Geodetic Survey
Corbin, Virginia 22446

The first precise electro-optical distance measuring instrument was introduced in 1948. It was large and cumbersome, observations had to be made at night, and it was limited in range, but simplified the hard task of establishing a precise distance with calibrated invar tapes.

Today several types of electro-optical distance measuring instruments are available. All of these instruments fall into one of two major categories being long range and short range.

As a general rule, the short range instruments are more compact and portable, while the long range counterpart is larger and less portable. Virtually all the new instruments manufactured in the past five years, both long and short range, have transistorized circuitry and 12 VDC power sources. Most of these instruments fall into one of two categories as far as their light source is concerned, that being infrared and red laser

light. The short range instruments generally fall in the infrared category and the long range in the red laser category. Several of the short range instruments feature direct distance readout while most of the long range instruments require reduction computations for distance determination. The red laser and the infrared has an advantage over the previously used tungsten light in that a filter can be used at the instrument end of a line being measured which will pass only the transmitted light. This system makes daylight measurements possible because direct sunlight and other reflected stray light, which represents noise, is blocked from entry into the instrument.

Infrared is often considered to be light. Technically, light is that part of the spectrum that is visible to the eye and infrared is that part of the spectrum that is between visible light and radio waves. This article will be on Geodimeter models 6A and 8 which use visible light.

Geodimeter Models 6A and 8 use practically the same measuring principle. The primary difference between the Model 8 and the Model 6 series is that

the Model 8 uses a KDP Cell modulator and a laser light whereas the Model 6 uses a Kerr Cell modulator and a tungsten or mercury vapor light. Both models use a phase resolver to resolve the proportional part of the quarter-wavelength and the same modulating frequencies.

The Geodimeter uses the following principles:

Velocity of light, VL = 299,792,500 m/s

Modulation Frequencies, F1 = 29,970,000 Hz
F2 = 30,044,922 Hz
F3 = 31,468,500 Hz
F4 = 31,465,500 Hz

Receiver Frequencies, F1 = 29,968,500 Hz
F2 = 30,043,422 Hz
F3 = 31,467,000 Hz
F4 = 31,467,000 Hz

Refractive Index, RI = 1.0003086

The unit lengths are computed as follows:

$$U1 = \frac{299,792,500}{4(29,970,000)(1.0003086)} = 2.500\ 000$$

$$U2 = \frac{299,792,500}{4(30,044,922)(1.0003086)} = 2.493\ 766$$

$$U3 = \frac{299,792,500}{4(31,468,500)(1.0003086)} = 2.380\ 952$$

$$U4 = \frac{299,792,500}{4(31,465,500)(1.0003086)} = 2.381\ 179$$

The refractive index, RI, is uncorrected for temperature, pressure, humidity, and color of light. The formulas for the correction are as follows:

$$6{,}328A^{\circ}\ \text{Laser Light} = 1 + \frac{0.00010\ 7925 \times P}{273.2 + t} - \frac{1.5026e10^{-5}}{273.2 + t}$$

$$5{,}500A^{\circ}\ \text{Light} = 1 + \frac{0.00010\ 9129 \times P}{273.2 + t} - \frac{1.5026e10^{-5}}{273.2 + t}$$

P = Pressure in millimeters of mercury

t = Temperature in degrees centigrade

e = Vapor pressure in millimeters of mercury

The last term in the formula, correction for relative humidity, is seldom greater than one part per million and is usually taken from a graph.

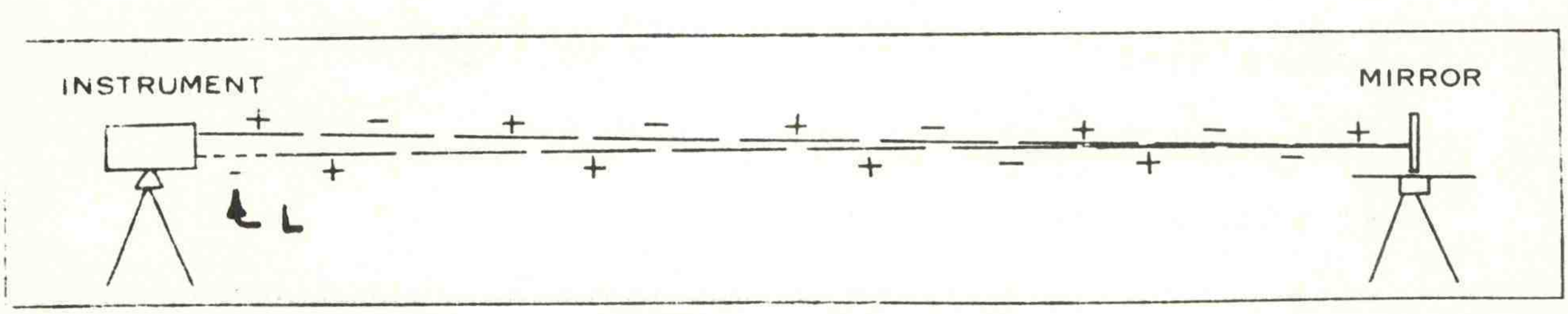


Figure 1

Measuring method, showing unit lengths for one measurement

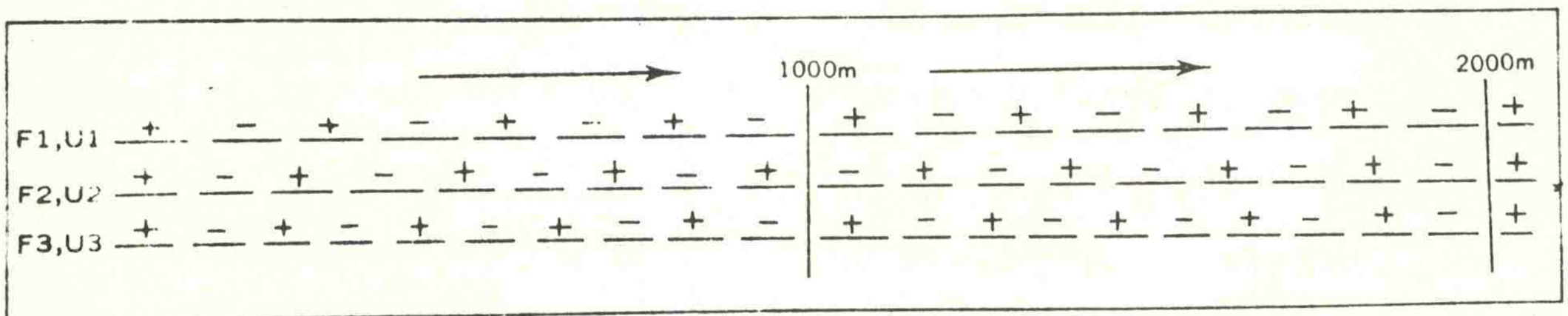


Figure 2

Measuring method using three different frequencies

Distances are determined by methods shown in figures 1 and 2. In figure 1, unit lengths for one measuring frequency are shown; however, the instrument uses three different frequencies, as shown in figure 2. The instrument transmits pieces of light, and the lengths of the pieces for each measuring frequency are determined by computing U1, U2 and U3. The instrument measures the last part of a unit length (Fig.1), which is usually less than 2.5 meters. In 1000 meters there are 400 unit lengths for F1, 401 for F2 and 420 for F3. When F1 has transmitted 400 unit lengths, it is lagging F2 by one full unit length. When F1 has transmitted 21 unit lengths, it is lagging F3 by one full unit length.

At 1000 meters, all unit lengths will coincide; but F1 and F3 will have an even number of unit lengths and F2 will have an uneven number of them. At 2000 meters, all unit lengths will coincide, and all frequencies will have an even number of unit lengths. The unit lengths will start through another cycle and repeat the same procedure for the next 2000 meters. Therefore, any distance measured will have to be known to within 2000 meters or less when using three modulating frequencies.

In order to resolve the distance to 50,000 meters or less the fourth frequency is added and only F3 and F4 are used to determine how many 2000 meter increments there are. The unit lengths will coincide the first time at 25,000 meters but F3 will have an even number of unit lengths and F4 will have an uneven number of unit lengths. At 50,000 meters F3 and F4 will coincide again and both of them will have an even number of unit lengths.

COMPUTATIONS

A recording and computation form is shown in figure 3. The observations are with a Model 8 but the same principle is used with the Model 6A. The form should be fairly selfexplanatory and every step of the computation will not be discussed, however, a few steps will be explained.

One complete measurement consists of readings on all four frequencies and each frequency consists of four phases on interior Calibrate "C" and four phases on the Mirror "R". The sign of the first phase reading is recorded for each set of four phases to show what part of the unit length is being measured. It is not important that the values on phases 1-2 or 3-4 are

in close agreement but the difference should not exceed 35 to 40 divisions. The difference between the sum of 2-3 and 1-4 should be in close agreement and should not exceed 20 to 25 divisions.

The values L1, L2' and L3' are computed by subtracting the calibrate value from the mirror value (R-C), however, the signs of "R" and "C" must be the same before subtracting. If one unit length is added, the sign will change and if two unit lengths are added the sign will remain the same. If "C" is greater than "R" and the signs are opposite, add 2.500 meters to "R" before subtracting "R-C". If "C" is greater than "R" and the signs are the same, add 5.000 meters to "R" before subtracting "R-C". If L2' or L3' is smaller than L1, add 5.000 meters to L2 or L3 in order to have a positive value for "A" and "B".

Observations are made with frequency 4 to determine the number of 2000 meter increments in the distance measured and should not be used in the mean with D1, D2 and D3 because there is no oscillator for F4. An additional capacitor is switched into the F3 circuit in order to lower the transmitting frequency 3000 Hz for F4.

In order to obtain L4, the reflector value is subtracted

ESSA FORM 76-33 (9-69) — U.S. DEPARTMENT OF COMMERCE, ENVIRONMENTAL SCIENCE SERVICES ADMINISTRATION

GEODIMETER MODEL 8 OBSERVATIONS

STATE	LOCATION	DATE
Virginia	Culpeper	13 July 1970
GEOD. NO.	**REFLEX NO.**	**PROJECT NO.**
80056	3 Prism AGA Type	
STATION	**STATION**	**OBSERVER**
Mitchell N. Base 1965	TO Mitchell S. Base 1965	C. C. Glover

	GEOD.	MIRROR	GEOD.	MIRROR	MEAN
TIME	2020		2024		
TEMP.	23.9	24.2	23.4	24.2	23.9 °C
PRESS.	1180	1165	1180	1165	1172 ft
RI. HU.	69.5/77.0				68%

GEOD. ECC. = - 0.950
GEOD. CONST. = + 0.211
REFLEX CONST. = - 0.027
REFLEX ECC. = 0.000
ATMOS. CORRECTION = + 0.319
35.2 ppm SUM = - 0.447 M

HT. GEOD. = 1.30 m

HT. MIRROR = 13.72 m

PHASE	*F1, F2, F3: If the signs are different, add 2,500 meters to "R" before subtracting "R–C".*						*F4: If the signs are different, add 2,500 meters to "C" before subtracting "C–R".*	
	FREQUENCY 1		FREQUENCY 2		FREQUENCY 3		FREQUENCY 4	
	C	R	R	C	C	R	R	C
1	+ .333	- .306	+ .326	+ .323	+ .377	+ .375	+ .475	+ .233
2	.334	.305	.329	.326	.380	.378	.470	.238
3	.639	.611	.631	.633	.686	.684	.776	.541
4	.638	.607	.633	.632	.688	.687	.775	.542
SUM 2 & 3	.973	.916	.960	.959	1.066	1.062	1.246	.779
SUM 1 & 4	.971	.913	.959	.955	1.065	1.062	1.250	.775
TOTAL SUM	1.944	1.829	1.919	1.914	2.131	2.124	2.496	1.554

L1 = (R–C) = 2.385

L2' = (R–C) = 5.005
L2 = 0.9975 × L2' = 4.992
–L1 = 2.385
A = 2.608
A × 400 = 1043.

L3' = (R–C) = 4.993
L3 = 0.9524 × L3' = 4.755
–L1 = 2.385
B = 2.370
B × 21 = 49.8

L4 = (C–R) = 4.058
L3' (R–Conly) = 4.993
G = (L3'–L4) = .935
n = 10000 × G–D' 8300
(nearest 2000) = 8000

A X 400 MINUS B X 21
(nearest hundreds) = 1000.
(B x 21 nearest 5) = 50.
D' = SUM = 1050.

F1	F2	F3
D' = 1050.	D' = 1050.	D' = 1050.
+L1 = 2.385	+L2 = 4.992	+L3 = 4.755
D1 = 1052.385	1054.992	1054.755
	–K2 = 2.618	–K3 = 2.381
	D2 = 1052.374	D3 = 1052.374

MEAN D1, D2, D3, = 1052.378
n = 8000.000
D = 9052.378
SUM CORR'NS. = - 0.447
D (Slope dist.) = 9051.931

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Figure 3

39

from the calibrate value (C-R). This procedure differs from the first three frequencies because on the first three frequencies the transmittimg frequencies are higher than the receiving frequencies but on F4 the transmitting frequency is lower than the receiving frequency.

The phase resolver rotates through one complete phase every 2.500 meters which is the unit legth for F1 (see figure 1), therefore there is no correction to L1.

In order to obtain L2 and L3 (see figure 1) the resolver length is multiplied by 0.9975 for L2 and 0.9524 for L3.

2.500 x 0.9975 = 2.494 which is the unit length for F2 and 2.500 x 0.9524 = 2.381 which is the unit length for F3 and F4.

L4 is not reduced to its proper unit length because the unit length, in this computation, for F4 is the same as F3. If we use the uncorrected value for both frequencies, the difference "G" would be the same as if they were corrected. The term (R-C only) means that if two unit lengths were added to L3' because it was smaller than L1, those two unit lengths should not be used when computing for "G" because only F3 and F4 should be considered when computing for the multiple of 2000 meter pieces.

GEODIMETER CORRECTIONS

The corrections include the geodimeter and mirror eccentricity (see figure 4), geodimeter zero constant, reflector zero constant, and atmospheric correction.

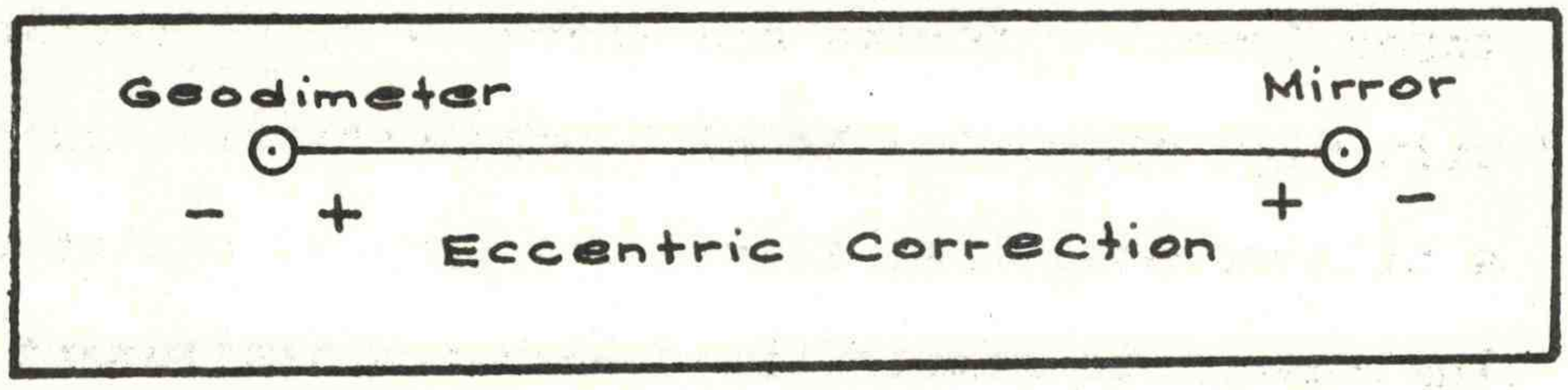


Figure 4

GEODIMETER ZERO CONSTANT

The electrical center of the geodimeter is not over the plumb line when the instrument is plumbed over the mark. It would be very difficult (if not impossible) to measure the eccentric distance with a tape because the light path passes over several optics, so the zero constant is established by making a measurement with the instrument over a precise taped distance and the difference between the taped distance and the geodimeter distance is the geodimeter zero constant. The tape distance for the Model 6 series should be 49.860 meters and it should be 50.210

meters for the Model 8. When those distances are used the values for "R" and "C" are practically the same on F1 and F3 and any small error in the phase resolver will be cancelled.

GEODIMETER TILT CORRECTION

The bases of the Model 6 Geodimeters are leveled with three leveling screws and there is no correction for tilt with them. The Model 8 Geodimeter is equipped with a tilting head that allows the optical center of the instrument to remain fixed when the instrument is rotated vertically. The head is equipped with a scale graduated in centimeters and a bubble. The scale is to measure the offset of the instrument over the plumb line due to an unlevel tripod or stand (see figure 5). The slope of the line being measured has nothing to do with the tilt correction. If the tripod or stand is level, there will be no tilt correction. Figure 6 shows a scale reading of +0.009 m but the optical center of the instrument is over the plumb line and there is no tilt correction. The tilt correction is determined as follows: Point the instrument in the general direction of the mirror, level the bubble with the vertical slow motion screw and read the scale. The tilt correction is recorded on the geodimeter eccentricity line.

Figure 5

Geodimeter with tilt correction

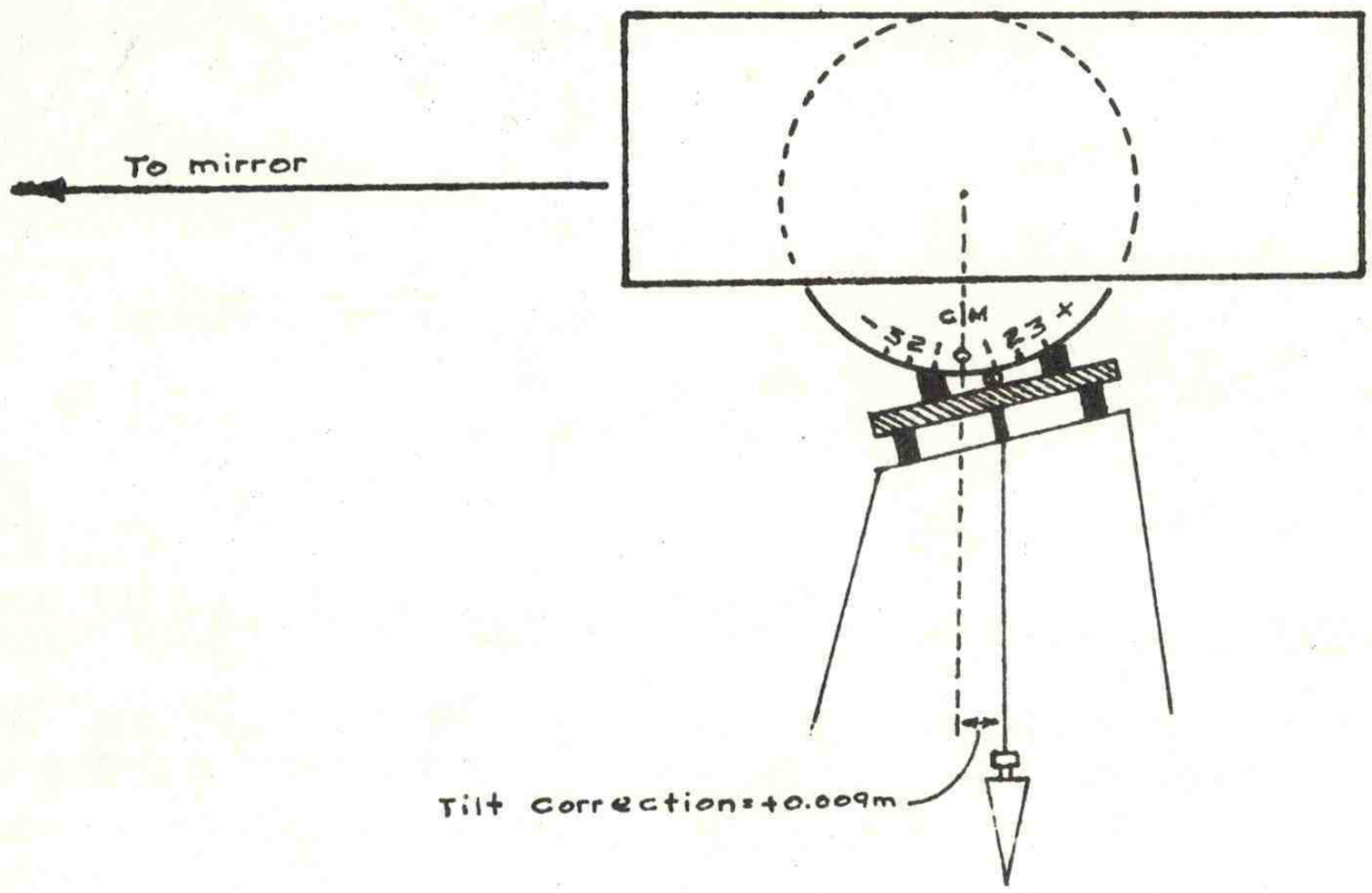
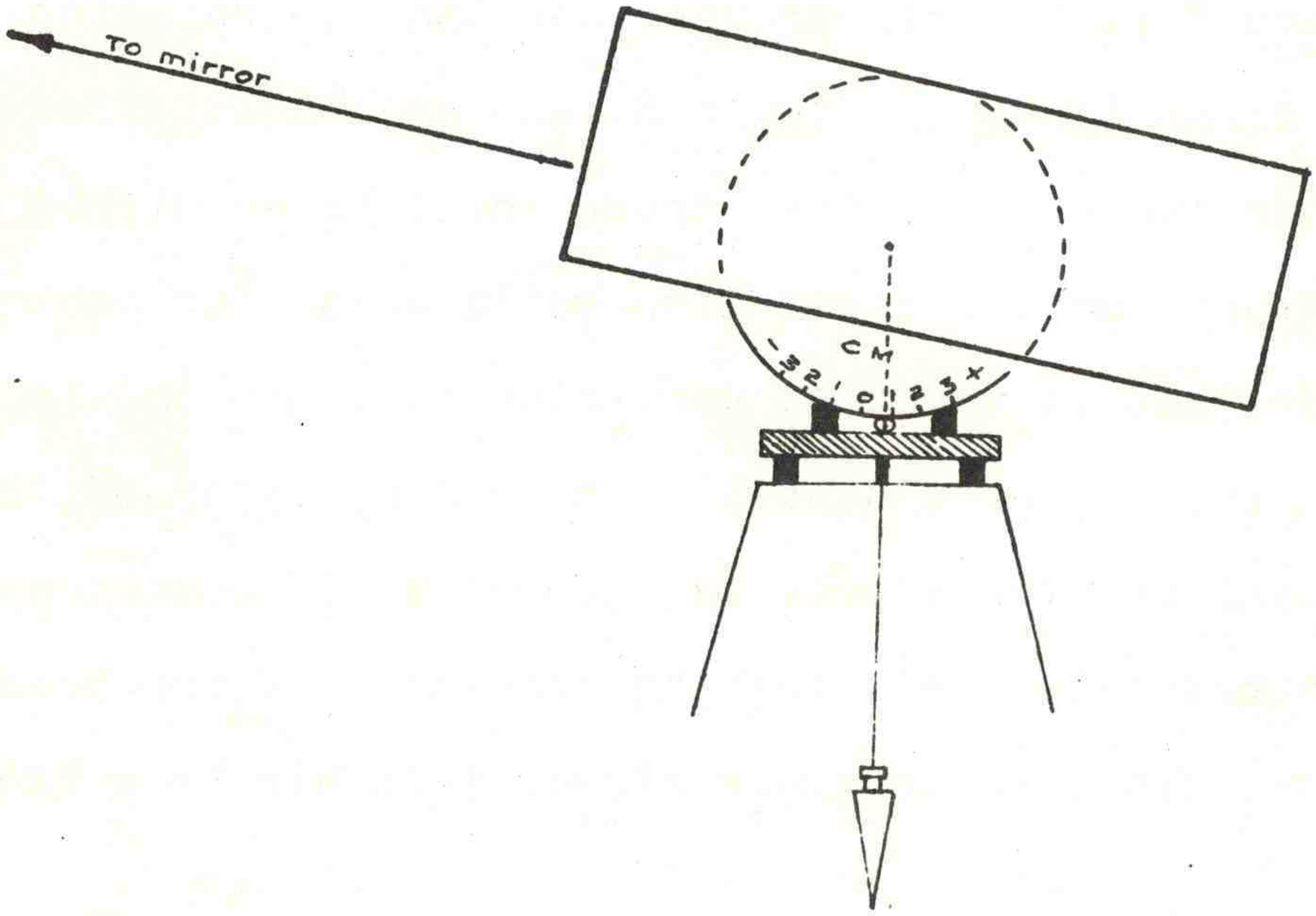


Figure 6

Geodimeter without tilt correction



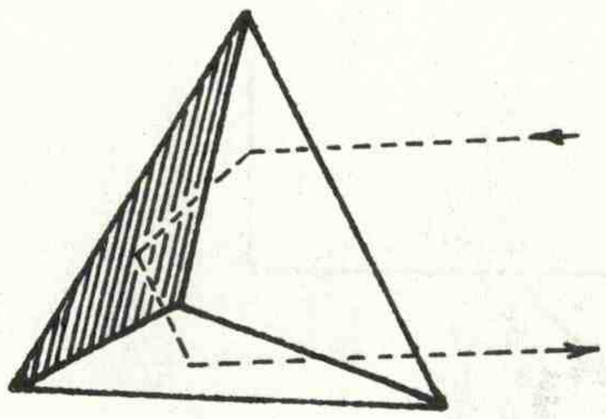


Figure 7

Light path through retroreflector

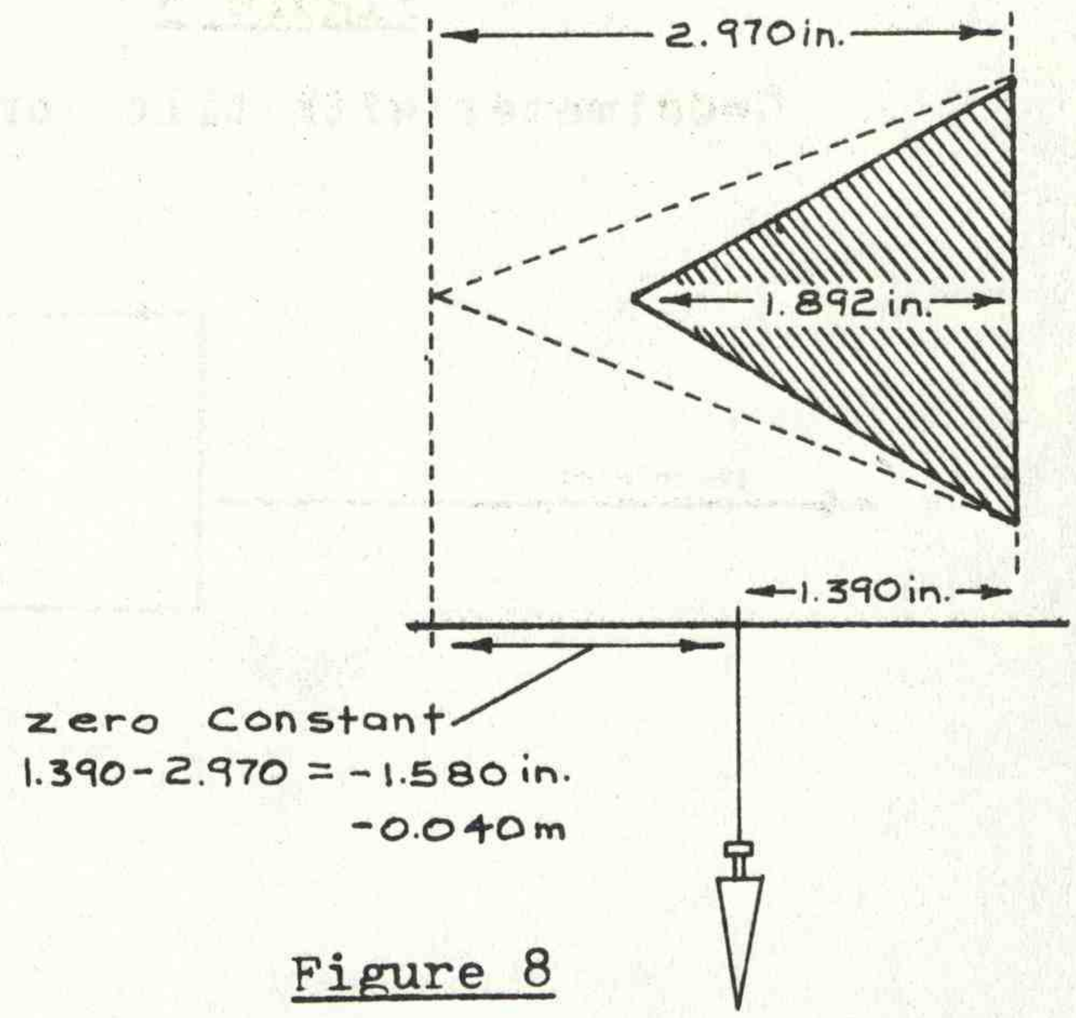


Figure 8

Retroreflector zero constant computation

MIRROR ZERO CONSTANT

The mirror zero constant is the distance the retro-reflector is eccentric to the plumb line. Figure 7 is a sketch showing the actual lightpath making three reflective turns through a retroreflector.

Figure 8 is the mirror zero constant computation. If the plumb line, in figure 8, was at the intersection of the two dashed lines there would be no mirror zero constant but the plumb line would be so far behind the prism that it would be very clumsy to use in the field. The plumb line is usually set in the center of the mirror housing and the mirror zero constant is computed. The lightpath will be twice the thickness of the prism and it travels through glass slower than air by a factor

of 0.57. The geodimeter resolves one half the total distance the lightpath travels, so the 1.892 inches will not be doubled. The computation in figure 8 is: 1.390 - (1.892 x 1.57) = -1.580 inches or-0.040 meter.

When using the Model 8 Geodimeter on distances of two miles or less, a diffused mirror is usually used. The diffused mirrors are made of plastic and the average refractive index for plastic is 1.47. The same principle is used for computing the zero constant for a diffused mirror as it is for a retroreflector.

ATMOSPHERIC CORRECTION

The correction for temperature and pressure for the observations on figure 3 are:

mean pressure = 1172 feet = 754.7 mm (see Table 1)

mean temperature = 23.9°C

$$1 + \frac{0.000107925 \times 754.7}{273.2 + 23.9} = 1.0002742 \quad (274.2 \text{ PPM})$$

The correction for relative humidity is taken from the graph on Table 2 and it is -0.8 PPM. The total value is:

Temperature and pressure	= 274.2 PPM
Relative humidity	= - 0.8 PPM
	273.4 PPM

The standard unit lengths for the geodimeter are computed using 308.6 PPM for atmosphere. Atmosphere correction for the observations on figure 3 are 308.6 - 273.4 = +35.2 Parts-Per-Million. In the lower right portion of figure 3, the computed distance (D) up to this point is 9052.378 meters. The atmosphere correction is 0.009052 x 35.2 = +0.319 meter.

ERRORS

There are several ways to introduce errors when measuring a distance with a geodimeter. Following is a list of a few possible errors that will be discussed:

1. Frequency count in error
2. Transient time in instrument
3. Barometric pressure
4. Air temperature
5. Relative humidity
6. Wrong sign on first phase reading
7. Wrong phase resolver reading

FREQUENCY COUNT

The Model 6A and 8 Geodimeters have two crystal ovens in them. One oven is in the transmitting section and the other one is in the receiving section. An error of 30 Hz in the transmitting frequency will introduce an

error of one part in one million of the distance measured. The receiving frequency will introduce no error in the distance but will cause spreads between the phase values. There are several reliable frequency counters on the market today and much thought and consideration should be given in selecting one of reputable dependability.

In some older model geodimeters, the oven temperature would cycle through several degrees of temperature causing the modulating frequency to vary. The frequency change is proportional to the crystal oven temperature change. On the newer model geodimeters, the crystal oven temperature is very stable allowing the modulating frequency to be very stable, however, the frequency count should be checked on a reliable frequency counter occasionally. If observations are made in bright sunlight 65°F or above it is possible for the crystal ovens to overheat and to be on the safe side the instrument should be shaded.

TRANSIENT TIME IN INSTRUMENT

The geodimeter and mirror should be pointed properly when measuring a distance, however, if they are not, it should introduce no error in the distance if there is no transient time in the instrument and it is

operating properly. Figure 9 is an example of how to check the instrument transient time.

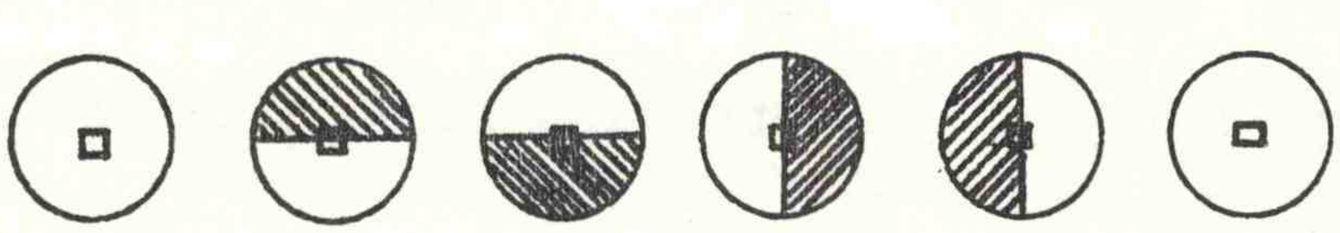


Figure 9

Phase 2	227	228	228	222	227	226
Phase 3	562	557	559	557	558	560
Mean	394	392	394	390	392	393

The circles in figure 9 represent the optical receiving tube on the geodimeter. The shaded areas represent that half of the tube covered up. The transient time check is as follows: Place a diffused mirror approximately 50 meters away from the instrument. All observations are made on the diffused mirror on F1. The mean values for 2, 3, 4, and 5 should not differ from the mean of 1 and 6 more than 7 or 8 units.

The next step is to measure a distance with the instrument of about 300 meters, using standard observing procedures, on a retroreflector and on a diffused mirror. The two computed distances should agree very closely and the difference should not exceed 6 millimeters. If

the instrument passes the two above tests, the receiver is functioning properly. If it did not pass the above tests, then the pointing of the instrument is critical and it should go back to the manufacturer for maintenance.

BAROMETRIC PRESSURE

If precise distances are to be established, a good quality altimeter or barometer should be used and they should be checked for precision at the local airport, however, it is possible to check them on the station site if the elevation of the station is known. Following is the procedure to check an altimeter:

For every 0.10 inch the barometer increases the altimeter will decrease approximately 90 feet and vice versa. Barometric pressure announced on the local weather report is always in inches of mercury reduced to sea level but you are very seldom at sea level so you will have to compute the difference. If the local weather station announces the barometric pressure is 30.10 inches and your station elevation is 550.50 feet the altimeter should read:

From Table 1, 30.10 in.	= 819	ft.
Station elevation	= 550.50	ft.
Altimeter should read	1,369.50	ft.

If the altimeter is checked on the station site without

the tables and we used a conversion factor of 90 feet per 0.10 inch of mercury, the computation would be:

29.90 in. - 30.10 in. = -0.20 in. = -180 ft.

550 ft. - 180 ft. = 370 ft.

Most altimeters, by manufacturer's design, read 1000 feet too high, so we will add the 370 feet to the 1000 feet equals 1,370 feet. An error of 90 feet or 0.10 inch will make an error of one part in one million of the distance measured.

AIR TEMPERATURE

When using electro-optical distance measuring instruments, the air temperature is the most uncertain correction. According to several tests performed by the National Geodetic Survey, the air temperature is fairly constant along the line being measured when the wind is about 3 miles-per-hour or above, however, when it is calm it would be difficult to obtain the correct mean temperature of the line by measuring temperatures at the two end terminals. When distances are measured from a tripod or a 4-foot stand, the most accurate method of measuring air temperature is with thermistors at least 25 feet above the ground. If thermistors are not used the thermometer should be as high above the ground as possible and in the wind.

If distances are measured in the daytime, the thermometer or thermistor should be well shaded from the sun and in the wind.

RELATIVE HUMIDITY

The correction for relative humidity for light waves is very small and very seldom exceeds one part in one million. It has been found that the average correction is 0.4 PPM and this is normally used unless the highest precision is required. If the humidity is taken, it is only necessary to take it at one end of the line.

WRONG SIGN ON FIRST PHASE READING

It is fairly common to read the first sign of the first phase wrong. If the distance is computed with a wrong sign, you wouldn't even be in the ball park. It is very easy to detect a wrong sign in the calibrate column because they should always be the same. They can be plus or they can be minus and all four frequencies should be the same and they will not change from day to day. On most geodimeters, the sign of the first phase is plus, but if the wires are reversed on the null meter the signs will reverse and the sign for F1 in figure 3 would be C = - and R = + but the distance would still compute properly. It is very difficult (if not impossible) to detect a wrong sign in the mirror (R)

column when computing the distance. The observer can eliminate the problem by going back to the reflector position and double checking the sign of the first phase position after he has completed the sheet.

WRONG PHASE RESOLVER READING

In some cases, the phase resolver is read in error or the value is recorded wrong. One way to detect the error is to use a mirror bar (see figure 10) and take one complete sheet with the mirror plumb over the mark on the zero point, one sheet with the mirror at +0.400 m and one sheet with the mirror at -0.400 m. When the mirror is at +0.400 m, each phase reading on the mirror should be about 100 units lower than the one on the zero point and when the mirror is at -0.400 m each phase reading should be about 100 units higher than the ones on the zero point. By using the mirror bar as shown in figure 10, and measuring distances to all three points, any small error in the phase resolver will be cancelled.

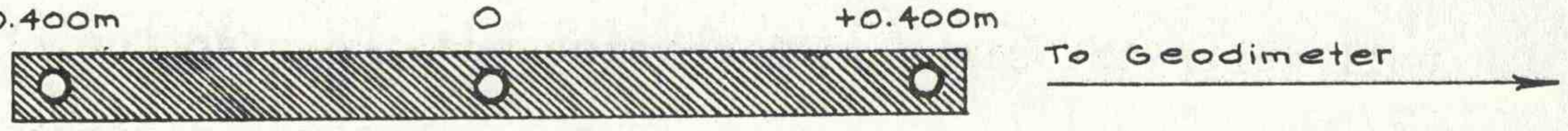


Figure 10

LASER SAFETY

The author is not an expert on laser safety. It is not intended for this article to govern the rules and regulations of laser safety, however, several years of experience with laser electro-optical distance measuring instruments indicate a few safety precautions should be observed.

NOAA, National Geodetic Survey (formerly ESSA) has the safety standards for the Model 4L Geodimeter in the Safety Handbook. Most of the standards do not apply to the Model 8 because the intensity of light, line of polarization of the light, and the eyepieces are different. A few laser safety procedures with the Model8, which may be helpful, will be discussed.

The lasers in the Model 8 Geodimeter are the helium-neon continuous gas lasers. The power output of this type laser ranges from 5.0 milliwatts to 6.5 milliwatts. These lasers operate at 6,328 Angstroms (red light) and the exit beam diameter is 0.6 millimeter.

The laser beam is transmitted through a prism that turns the light beam 180° through a quarter-wave plate, a KDP Cell, a polarizer, and a beam expander that collimates the beam to a diameter of 20 millimeters.

When the beam has been expanded to 20 millimeters, the

power output will range from 1.3 milliwatts to 1.7 milliwatts of light which is almost parallel.

When working or operating in heavily populated areas, or where there is danger to the public or someone coming into visual contact with the light at a distance of less than 6,000 meters, the geodimeter should be roughly put on target before the laser is turned on.

When putting the geodimeter on target, the instrument is centered on the mirror by sighting through a 42 power eyepiece. This is probably the most dangerous part of the system and precautions should be taken to protect the eye. On short distances, two miles or less, diffused mirrors should be used. If diffused mirrors are not available, the retroreflector should be covered with a thin piece of plastic to diffuse the return light.

The laser geodimeter should never be left unattended unless the laser is turned off or the instrument is on interior calibrate.

Personnel should never observe the laser light with binoculars, theodolites or any magnifying lens unless some filtering system is used to dim the light.

Laser danger signs should be displayed in all areas where the laser geodimeter is being used.

The geodimeter observer and the mirror tender should see that no one comes into visual contact with the laser beam. The hazard zone should be defined as an area of 5 meters in radius along the line of sight from the instrument out to 6,000 meters when the laser is on.

Technical specifications furnished by AGA Corporation are:

Model	Light Source	Range day	Range night	Accuracy
6A & 6B	Tungsten lamp	up to 5 km	15 km	5mm + 1mm/km
	Mercury vapor	up to 10 km	25 km	5mm + 1mm/km
8	Laser	45 km	60 km	5mm + 1mm/km

The above maximum distances with the Model 6 series are probably based on the most ideal conditions. Past experience indicates those distances could not be obtained under normal working conditions.

Several lines exceeding 60 kilometers have been measured under ideal conditions at night with the Model 8, however, the daylight range would be about 30% less than that at night.

According to numerous tests, the above stated accuracies can be expected if the geodimeters are operating properly.

TABLE 1

Barometric Pressure Conversion Tables*

in	ft	mm
31.00	0	787
		786
30.90	1 00	785
		784
		783
30.80	2 00	782
		781
30.70	3 00	780
		779
		778
30.60	4 00	777
		776
30.50	5 00	775
		774

in	ft	mm
	500	773
30.40		772
	6 00	771
		770
30.30		769
	7 00	768
30.20		767
		766
30.10	8 00	765
		764
		763
30.00	9 00	762
		761
29.90	10 00	760

in	ft	mm
29.90	1000	759
		758
29.80	11 00	757
		756
		755
29.70	12 00	754
		753
29.60	13 00	752
		751
		750
29.50		749
	14 00	748
29.40		747
	15 00	746

in	ft	mm
	1500	745
29.30		744
	16 00	743
		742
29.20		741
	17 00	740
29.10		739
	18 00	738
		737
29.00		736
	19 00	735
28.90		734
	20 00	733

in	ft	mm
	2000	732
28.80		731
	21 00	730
		729
28.70		728
	22 00	727
28.60		726
	23 00	725
28.50		724
	24 00	723
28.40		722
	25 00	721
28.30		720
		719

* NOTE: Zero on above tables corresponds to minus 1000 ft. on Smithsonian Meteorological Table 51. (i.e. 1000 has been added to tabular values in feet) 56

TABLE 1 (continued)

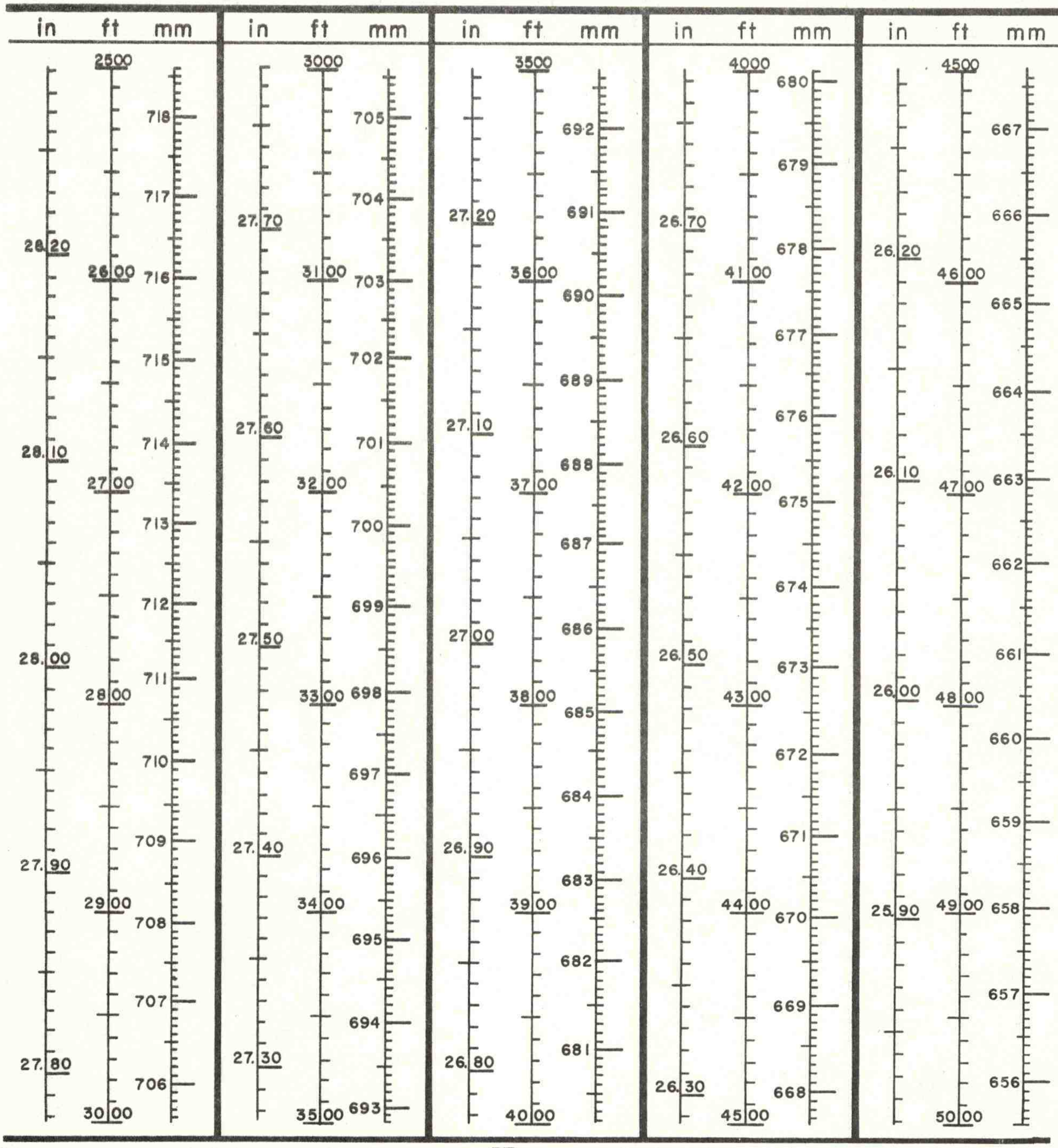
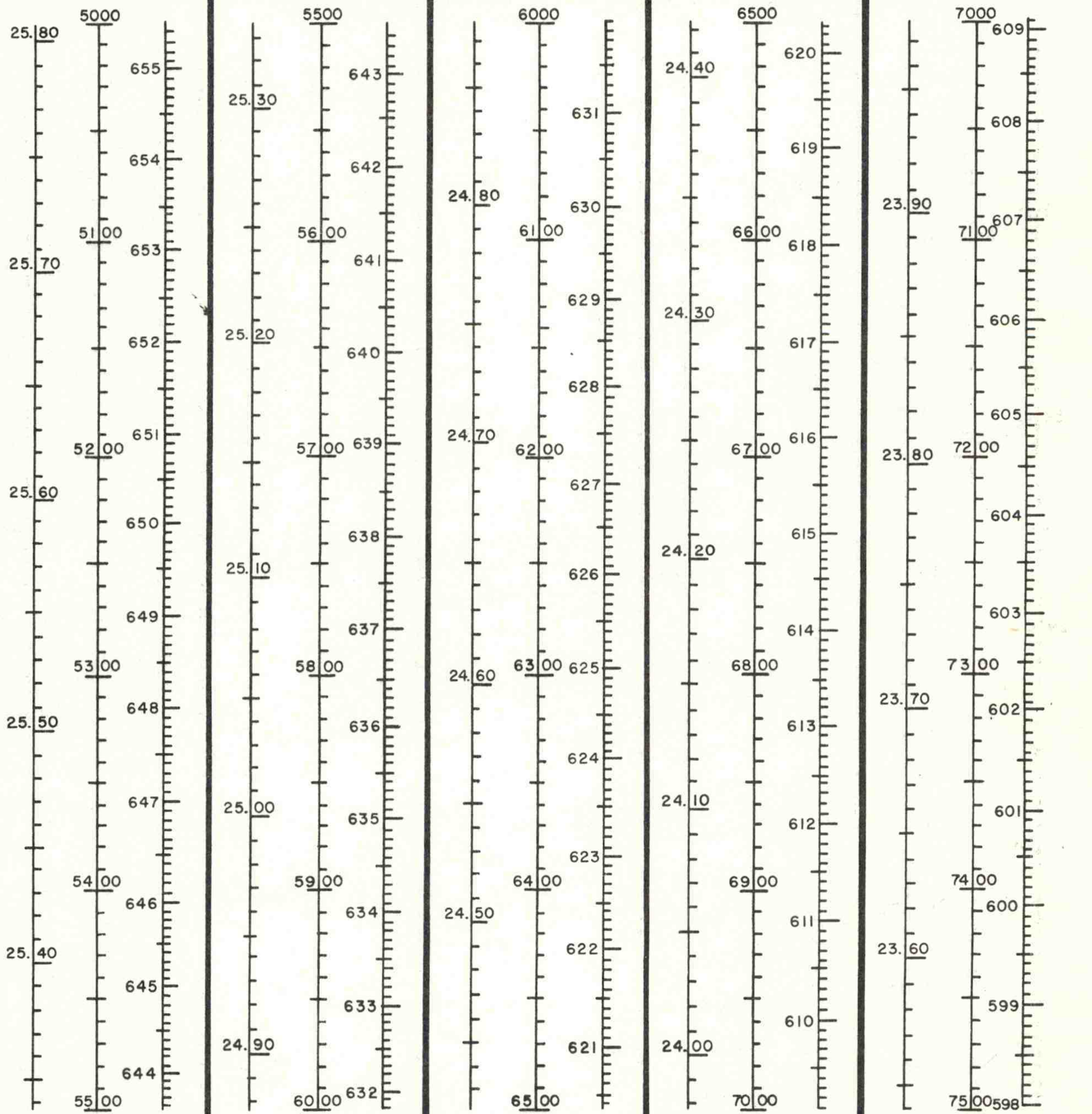
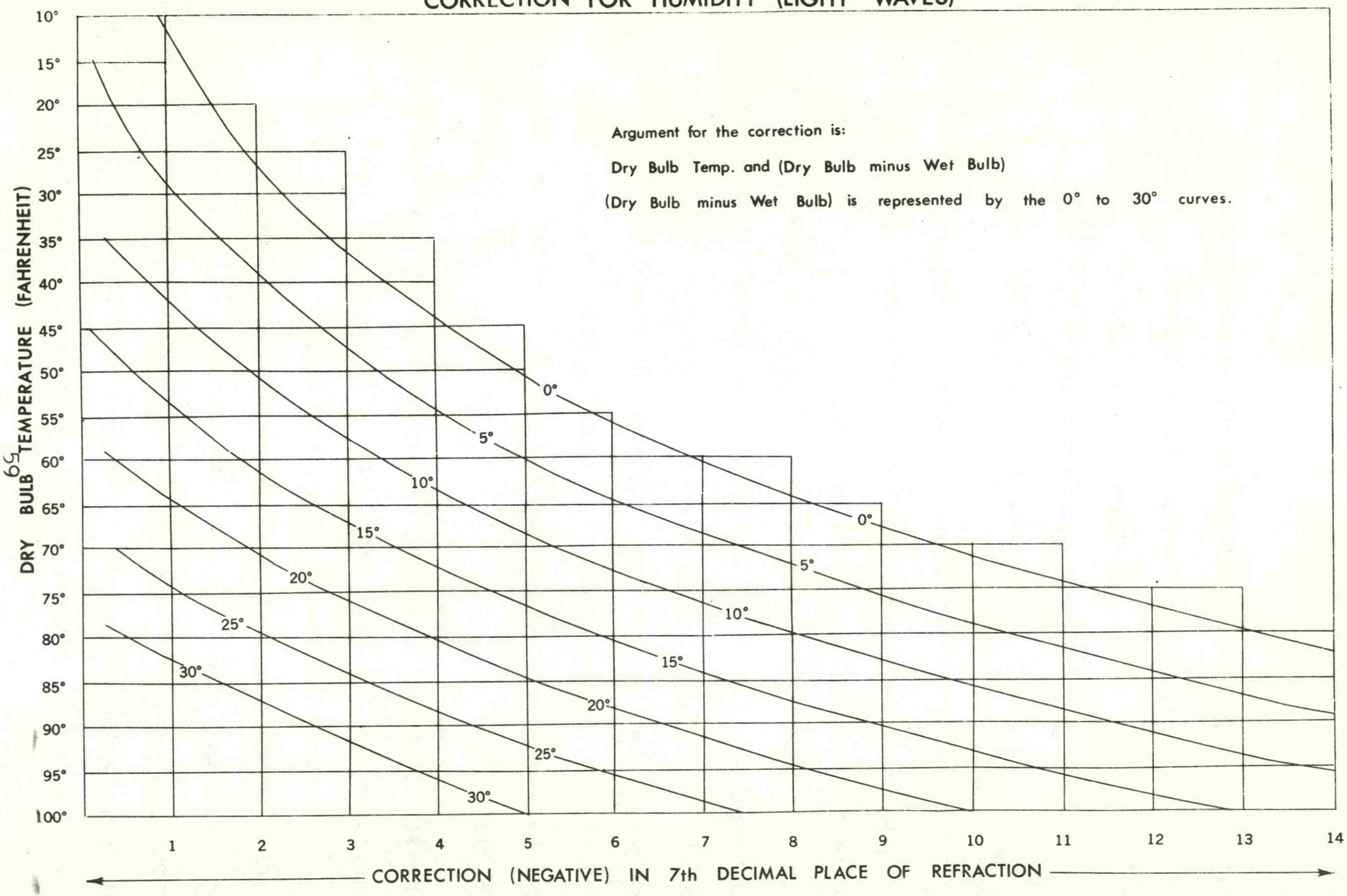


TABLE 1 (continued)



CORRECTION FOR HUMIDITY (LIGHT WAVES)



Argument for the correction is:

Dry Bulb Temp. and (Dry Bulb minus Wet Bulb)

(Dry Bulb minus Wet Bulb) is represented by the 0° to 30° curves.

TABLE 2

ELECTRONIC DISTANCE MEASURING INSTRUMENTS

by Raymond W. Tomlinson

NOAA, National Ocean Survey

National Geodetic Survey

Rockville, Maryland 20852

Two basic types of Electronic Distance Measuring Instruments are currently being used for obtaining accurate distance measurements. These types are lightwave and microwave (infrared being considered as lightwave as the computations are based on the same formulas). Numerous instruments are available and statistical data are shown on Table I with servicing and company information addresses shown on Table II. The theory of operation and detailed servicing has been neglected since most users will not be performing their own service.

The selection of the proper Electronic Distance Measuring Instruments is a major problem. Consideration must be given to the accuracies desired, operational and maintenance problems, personnel, power requirements, size, weight, durability, and numerous other things. The first problem to solve is whether to select Microwave or Lightwave equipment. Items to be considered are length of lines, terrain, climate, usability and versatility of equipment. In other words, will this

equipment permit an increase in the number of projects that could be undertaken, thereby,increasing the gross business and profits? Only the user can make these decisions as often the price for a complete system is very nearly the same.

The prime concern, regardless of the system purchased, is operational personnel. Their advice should be considered prior to purchase since often they will be responsible for the system paying for itself or continually be inoperative.

As most are well aware, the surveyor purchasing a new electronic measuring instrument a few years ago was considered foolish and wasting good money. Today most, if not all the larger projects, which require precision length control, are awarded to the surveyor with up-to-date equipment. If a business is faltering, one should consider a change and plan a program to include Electronic Distance Measuring Instruments. The Model T era has passed and those who fail to keep abreast of the latest technology, will seldom receive the prime contracts.

Electronic Distance Measuring Instruments have really come into its own for control surveys in the past 10 to 20 years. No longer do we have to pull out the standardized tapes and perform days of laborious work to measure base lines.

Many people feel that we have only to solve the refractive index problem and then Electronic Distance Measuring Instruments can measure to the accuracy standard of 1 part in 1 million. There is not total agreement in this regard, because there are numerous additional problems -- instruments delivered without adequate training, trouble shooting problems, proper alignment and/or adjustments, frequency checks, obtaining correct meteorological data, and misuse and abuse of equipment. All of these and others lead to blunders and/or unnecessary errors.

Brochures distributed for the various Electronic Distance Measuring Instrument firms have impressive claims regarding accuracies and probably these accuracies can be obtained under certain atmospheric conditions and properly tuned instruments. In the general run of the mill control survey projects, the surveyor will seldom have good atmospheric conditions and often uses improperly tuned instruments. There is little that can be done to change the atmospheric conditions, but better meteorological data can be obtained and the instruments can be kept properly tuned.

MICROWAVE DISTANCE MEASURING INSTRUMENTS

Microwave instruments were first used in the radar method of aircraft detection, and navigation developed during World War II. The first use of microwaves by surveyors for distance measurement on the ground became a reality with the develop-

ment of the Tellurometer in 1957. Since that date, other microwave systems have been developed, but all are essentially the same. The Cubic Corporation introduced their version of microwaves under the name of Microdist in 1960. Shortly thereafter, the name was changed to Electrotape DM-20, which is still in production today. The Wild-Heerbrugg Company introduced the Distomat DI-50 in 1966, which has been replaced with the DI-60. Other microwave systems have since been developed for positioning ships, offshore points, etc., but none have the accuracy for precise geodetic surveys.

Microwave instruments have an advantage over lightwaves for distance measurement because they can be used day and night, can penetrate fog, clouds and so forth. They are less accurate due to the difficulty in determining the humidity corrections, which have little effect on lightwave instruments.

SUGGESTED PROCEDURES FOR CHECKOUT AND CALIBRATION

Upon receiving EDMI certain checks should be made to insure that proper measurements will be obtained. Some of the major items are:

1. Training of Operating Personnel
   a. Trained in proper operating procedures, which should be received from the manufacturer.
   b. Training in precise geodetic measurements should be obtained from experienced geodetic personnel.

2. Selection of Related Equipment

   a. The EDMI cost from $4,000 to $10,000 and will not give accurate measurements unless the related equipment selected is also reliable.

   b. Cheap thermometers, barometers, tripods, etc., will give nothing but poor results and trouble. When 5 to 10 thousand dollars is expended for the EDMI, the additional cost of reliable meteorological equipment and good tripods is rather insignificant.

   c. Select stable tripods, accurate optical plummets , calibrated barometers, physchrometers, thermometers, and a proven power supply.

   d. Check barometers, psychrometers, and thermometers with a standard at least each month for precision and more often when subject to abuse or misuse. These can be checked at the nearest airport.

3. Checking Instrument Constant

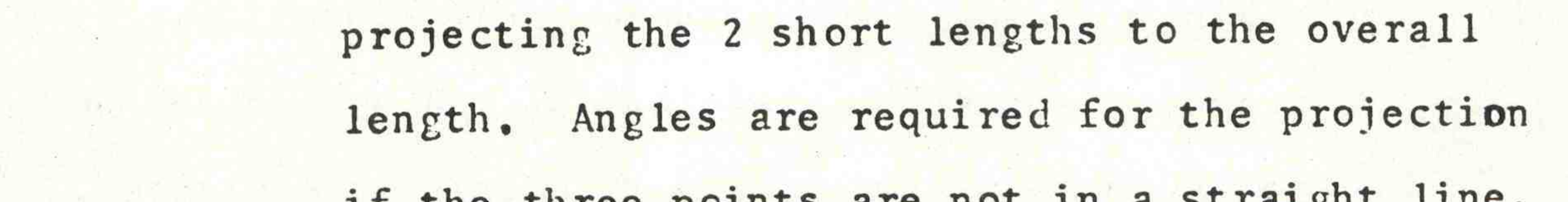
   a. Microwave and lightwave equipment should be checked over precision base lines at the start and end of each project. Measure this line on at least two different days and compute the results using meteorological data, difference in elevation, mirror constants, etc., but do

not apply any instrument constant and the difference between the EDMI measured distance and base line measurement should be the instrument constant. Microwave measurements should be made over a line length in excess of one mile and preferably about 5 miles with each terminal well elevated to prevent ground swing or grazing lines.

b. If a taped base line is not available, a line of about 5 miles in length should be selected (depending on range of your instrument). This line should be measured in two parts and then overall. The check is obtained by adding or projecting the 2 short lengths to the overall length. Angles are required for the projection if the three points are not in a straight line.

A _____ B _ _ _ _ _ _ _ _ _ _ C

AC − AB − BC = Instrument Constant



c' = c cos A

a' = a cos C

b − c' − a' = Instrument Constant

SYSTEMATIC AND ACCIDENTAL ERRORS

Systematic and accidental errors or blunders can often be eliminated through proper training of personnel. An employee who has been properly trained will know to look for misreadings and avoid the following:

1. Centering error - not using proper procedures to plumb over the stations. Tripods that are loose or damaged.
2. Improper pointing - all EDMI should be pointed to obtain maximum signal return.
3. Improper voltage - full charged batteries and/or quality generators are a must for all EDMI.
4. Improper readings - one of the major blunders is taking readings with improper null and nulling with improper signs. Misreadings of 100 are common and yet so unnecessary if a little care is exercised.
5. Improper meteorological data - using cheap thermometers, barometers, psychrometers, etc. Readings observed with thermometers in direct sunlight, not wetting the wet bulb sock, improper fan ventilation, and most of all, not waiting until the thermometers have reached equilibrium before taking the readings.

EFFECTS OF ATMOSPHERE AND INTERFERENCE

One of the largest sources of error in electronic distance measuring is due to the uncertainty in the refractive index.

In order to obtain the best possible value of the refractive index, atmospheric parameters should be taken at terminals of the line and at points along the line. For lines under 5 miles in length, an average of the atmospheric parameters taken at terminals of the lines (if taken properly) should give the refractive index correction accurate to within 1 cm. for lightwave instruments and 1-3 cm. for microwave instruments.

The relative humidity has a very small effect on the results from instruments using lightwaves, seldom more than a fraction of one part per million of the distance measured. The humidity effect in the refraction computation involving microwave instruments is much greater than that for lightwaves. For microwave instruments, a measurement under normal conditions, ($t = 20^{o}C$, $p = 760$ mm Hg and $e = 10$ mm Hg), an error of $1^{o}C$ between the dry and wet bulb temperatures will produce an error of 7 parts per million in the distance. The effect of this $1^{o}C$ error increases considerably as the temperature increases, rising to about 17 p.p.m. at a temperature of $45^{o}C$.

Numerous things cause interference in the measurement of distances with lightwave or microwave instruments. The following should be eliminated if possible:

Reflection and ground swing from improper line clearance

Reflection over water

Line of sight broken during measurement

Measuring in rain, blowing sand, foggy, etc.

Radio communication on same frequency

Measuring through uncleared line of sight

Measuring while near microwave towers

Measuring through or underneath power lines

Excessive heat waves along line

Loose power connections

SUGGESTED FIELD PROCEDURES TO DETECT ERRORS IN THE FIELD

Numerous points could be listed here, but it is assumed personnel have been trained in the procedures for plumbing and instrument operational procedures. In other words, the following field procedures are suggested for fully trained observers who have the basic knowledge for obtaining the accuracy desired.

Lightwave Instruments - Sources of error relate to the following: Alignment of optics, frequency checks, calibration, zero constant, proper voltage, and meteorological data.

Alignment of optics is very critical to achieve accurate measurements. This includes alignment of the Kerr Cell, KDP Cell, Photomultiplier for transit time, etc.

Frequency checks for the three frequencies shall be taken every two or three months for lower accuracy work and each

week for high precision work. This check is necessary due to frequency drift, which must be monitored to obtain precision length measurements.

A trained observer can minimize calibration curve errors by watching the spread of the phase readings and the spread of the three frequencies on D1, D2, and D3. The spread between any of the four phase readings should not exceed 20 units on the earlier Model nor 50 units for the later Model instruments. The spread between the means of phase 1-4 and 2-3 should not exceed 4 divisions. For precision work, the phase limits should not exceed 10 and 25 units, respectfully, nor 2 divisions between the mean of phase 1-4 and 2-3. When the frequencies contain a spread of more than 6 cm between D1, D2, and D3, the instrument should be recalibrated as it becomes very difficult to resolve the proper 5 and 100 meter intervals of the AX400 and BX21.

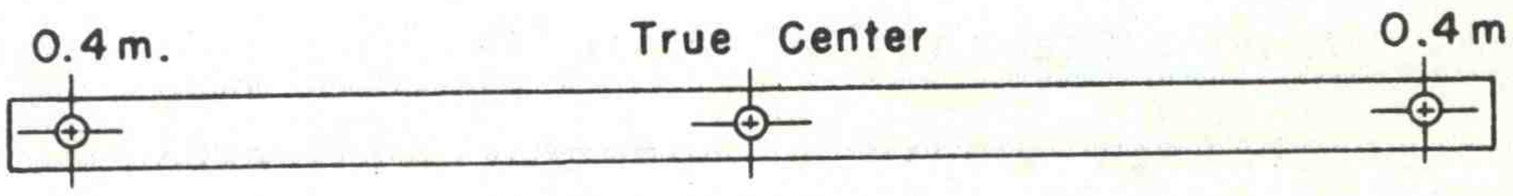
Calibration curves and zero constants should be checked if the optics are realigned or the photomultiplier moved or replaced.

Proper voltage is a must for accurate measurements and quality generators and fully charged batteries will enhance the quality and production.

Humidity is not a major factor with lightwaves, and for most projects a correction of +0.4 ppm of the distance measured

can be used, but good dry bulb readings and altimeter readings should be obtained. Thermometers should be placed in a shaded area 6 to 8 feet above ground level for lower order work and 20 feet or more for high precision projects.

A complete set of measurements for first- and second-order work should consist of three complete measurements. One each with the prism over true center, set-up and set-back 0.4 meter. The set-ups and set-backs are easily accomplished by fabricating a mirror bar (Fig. 2) with holes drilled at +0.4 and -0.4 meter from true center.



Mirror Bar

Figure 2

For high precision work, spur lines, open end traverses, etc., 4 complete measurements should be made at +0.4, -0.4 and two from true center, one of which should be on high calibration and one on low calibration. These set-ups and set-backs permit covering more of the delay line, and where possible, high and low calibration readings should be taken for high precision work. The spread between these measurements on one night should not exceed 2 cm. If two

nights' measurements are used for high precision work, the spread between the mean of the two nights should not exceed 2.5 cm for lines less than 8 kilometers in length and 1.7 cm + 1 part per million for lines longer than 8 kilometers.

Microwave Instruments - Microwave instruments are electronically sound, have well balanced circuits with precision crystal ovens. The major drawbacks are obtaining representative humidity corrections for the longer lines, reflections and ground swing on the shorter lines. Reflection and ground swing can give ambiguous readings, but this has been greatly reduced in some of the more recent models due to the higher frequency employed and one and one-half degree beam width.

Reflections when measuring over water can be a problem unless the terminal points are elevated for proper line clearance. When measuring from a tripod on a shore point, it is sometimes helpful to place the master unit behind or partially blocked by a sand dune. This will chop off the bottom portion of the radio wave and allow the top portion the necessary clearance across the water. Some skeptics will argue that this will give erroneous readings, but several experiences have not shown an appreciable loss of accuracy.

These instruments, like any other Electronic Distance Measuring Instruments, Theodolite, etc., should receive

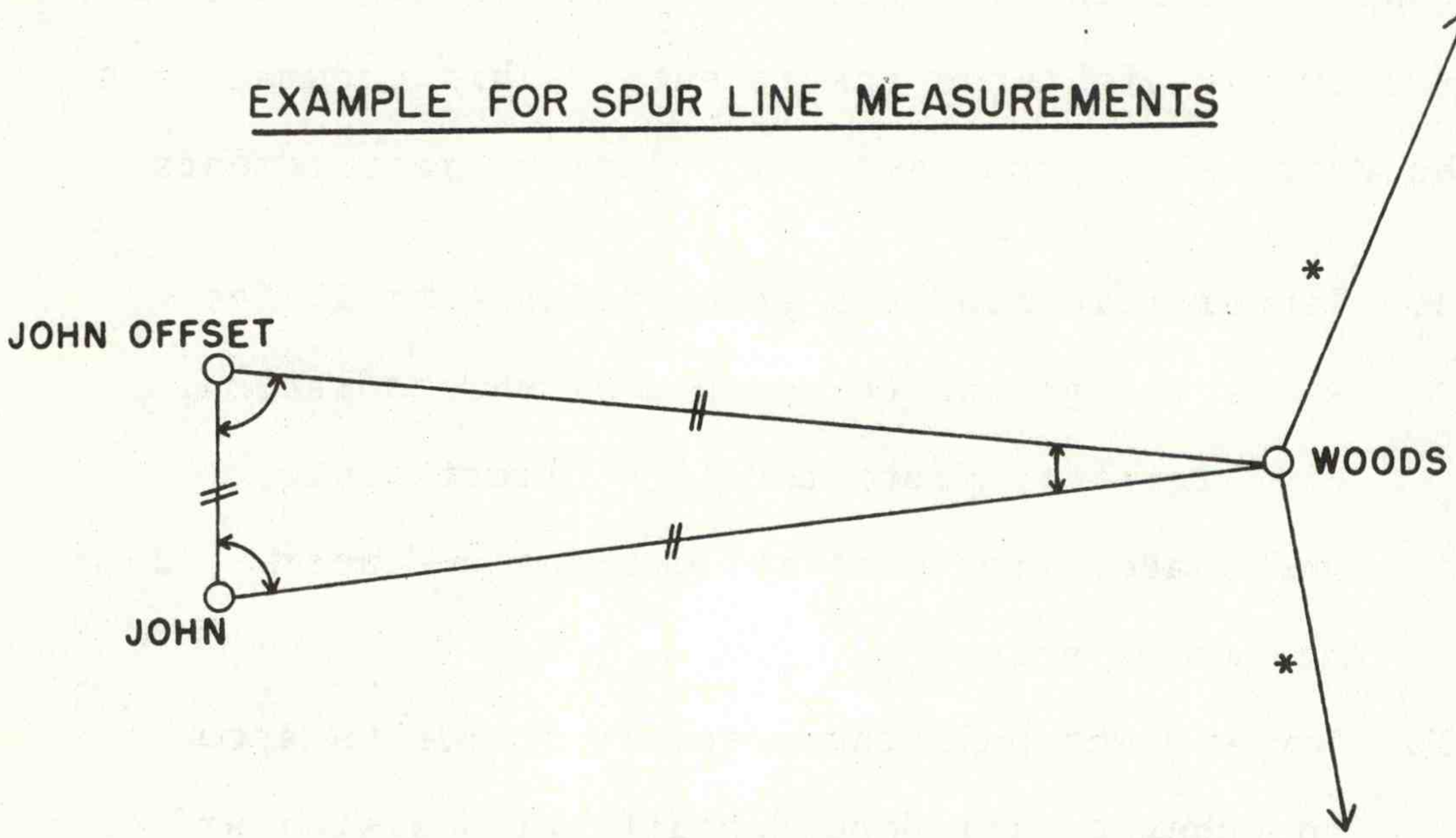
preventative maintenance. Microwave instruments are too frequently sent to the manufacturer for repairs. If the company was contacted by telephone and the malfunction explained, the majority of the repairs could be performed in the field.

A complete measurement with Microwave equipment should consist of a measurement from both ends of the line with a minimum of ten fine readings and course readings at the beginning and end of the measurements. Observers should take these measurements independent of each other as it has been proven that one observer making the setting and then switching to the other observer for a reading results in incorrect measurements.

The two measurements should agree in the field by less than 0.1 meter for lines under 10,000 meters. Lines in excess of 10,000 meters should agree within 1:100,000. The spread of the fine readings should not exceed 4 cm. Observers should avoid stations near high voltage lines, microwave towers, and grazing lines. When any of these factors are encountered, notes should be entered in the field records. Batteries should be well charged and lines should not be measured when the battery reading is less than specified in manufacturer's manual.

When measuring to spur lines or open end traverse points, the following method should be used to prevent angular and distance blunders.

EXAMPLE FOR SPUR LINE MEASUREMENTS



*Main line of traverse or progress of project

Observe horizontal angles at all three points.

Measure JOHN-WOODS and JOHN OFFSET-WOODS with EDM Instruments.

Measure JOHN to JOHN OFFSET with standardized tape.

The closure of the triangle (in seconds) multiplied by the offset distance (in meters) should not exceed 400.

METEOROLOGICAL DATA

One of the largest sources of error in electronic distance measuring is due to the uncertainty in the refractive index. In order to obtain the best possible value of the refractive index, meteorological data must be taken very accurately at

the terminals of the line and at points along the line for more precision. For lines under 5 miles in length, an average of the atmospheric parameters taken at the terminals of the lines should give the refractive index correction accurate within 1 cm for lightwave instruments. This accuracy can only be obtained if you meet the following requirements:

1. Temperature readings to be taken 6 to 10 feet above the ground to reduce ground radiation. Psychrometers protected from direct sunlight and shaded from heat reflections and nearby illuminated objects.
2. Dry and wet bulb thermometers should be accurate and checked for dependability with a standard each week.
3. Psychrometers should be slightly tilted and pointed into the wind to increase the air velocity around the thermometer bulbs and to insure adequate ventilation. It is very important to have a motor sufficiently powered to keep air moving past the bulbs at a rapid pace. Poor circulation past the wet bulb can be a source of serious error.
4. When wetting the wet bulb sock, care should be taken to prevent water drops from forming a bridge between the wet bulb and the shield as this reduces the proper circulation.

5. Barometers should be systematically calibrated with a mercury standard or to station barometers that have been calibrated.
6. Barometers should be placed in a shaded area and protected from gusty wind conditions. These instruments should be handled with care and suitable cushioned carrying cases used for transporting them. Care must be exercised in shipment as some are damaged during air shipments if they are not locked or sealed in a pressurized container.

Regardless of the precision thermometers, barometers, etc., on hand, the data obtained is worthless unless proper procedures are used. This comes right back to training, as trained observers know that correct meteorological data are just as important as having the instrument centered over the station.

Reading the dry and wet bulbs should be done when the thermometers have reached equilibrium, not before or after, as such readings will be erroneous. For microwave instruments an error of 1°C between dry and wet bulb readings under normal conditions (t=20°C, P=760 mm Hg and e=10 mm Hg) will produce an error of 7 parts per million in distance. This 1°C error at a temperature of 45°C will produce an error of 17 parts per million. These errors are actually blunders

and are often due to lack of personnel training. Everyone wishes to obtain the maximum accuracies from his equipment and to do this, we must spend the time and money for training our personnel.

Regardless of the type or accuracy of instrument used in a survey, it is of little value unless it is operated by qualified observing personnel. Recently the National Geodetic Survey received traverse data obtained by a state highway department using theodolites and a Model 4D Geodimeter. It was requested that the data be evaluated to locate blunders as the loops failed to close by three to five hundred feet in X and Y. Upon reviewing the Geodimeter field records, it was noted there were excessive spreads between the D1, D2, and D3, and it was very difficult to resolve the nearest multiple of 5 meters and 100 meters, because the AX400 and BX21 values often fell halfway and thus were ambiguous. Two employees from the state highway department along with their 4D instrument came to Rockville for three days of training. The Geodimeter was checked and found out of alignment and over 200 cycles away from the operating frequency. The operators stated the instrument had been in continuous use for six years, had received only one day of instruction, and were told "they could measure accurately any time they could null the instrument." They did not

know how to calibrate the instrument or make the necessary alignment adjustments. A similar situation was encountered with another state highway department, which employed Tellurometers and inexperienced operators were obtaining readings that were the reverse of the proper readings. Normally, a company will give one or two days of training and instruction when their representative delivers the instruments, but to obtain the maximum from your instruments and protect your investment, some companies have a five-day training course for $100 to $250, plus employees expenses. This type of training does pay dividends in the long run!

REFRACTIVE INDEX COMPUTATIONS

The value for the velocity of light used in the reduction of electronic distance measurements, 299,792.5 km/sec in vacuo, was adopted by the International Association of Geodesy in 1957. In the development of any type of EDM equipment, some measuring frequency and a unit length has been adopted. If the unit lengths are used as constants in the reductions, a constant value for the refractive index formula is obtained from,

$$C = \frac{V}{4UF}$$, where, V = velocity of light
U = unit length
F = measuring frequency

Using frequency 1 for the Geodimeter, where F = 29,970,000 c/s and U = 2.5 meters, C is then 1.000 30864, or 308.64×10^{-6}. This value of C is referred to as the preset, or assumed, index value. Index values for DI-10 = 1.0002819, H P-3800= 1.0002783, MA-100 = 1.000274, and 76 = 1.0003086. In any case, the assumed value should be consistent with the instrument frequency, unit length, and the adopted value for the velocity of light.

For Instruments Using Light Waves and Infrared

The Barrell and Sears formula for group velocity of the refractive index of light in the atmosphere at 0 degrees centigrade, 760 mm of Mercury pressure, and 0.03% Carbon Dioxide is:

$$n_g = 1 + \left(287.604 + \frac{4.8864}{\lambda^2} + \frac{0.068}{\lambda^4}\right) 10^{-6}$$

where λ is the wavelength of light expressed in microns.

Due to variations in temperature, pressure, and humidity, the refractive index changes to a new value n_a,

$$n_a = 1 + \frac{n_g - 1}{1 + \frac{t}{273.2}} \cdot \frac{p}{760} - \frac{5.5 \text{ e } 10^{-8}}{1 + \frac{t}{273.2}}$$

p is the pressure in mm of mercury, t is degrees Centigrade, and e is vapor pressure in mm or mercury.

For computational purposes, the formula above reduces to,

$$n_a = 1 + \frac{0.359474\ (n_g - 1)\ p}{273.2 + t} - \frac{1.5026\ e\ 10^{-5}}{273.2 + t}$$

If $N \times 10^{-6} = 0.359474\ (n_g - 1)$, then values of N for various values of λ are,

λ		N
0.5500	mercury vapor lamp	109.460
0.5650	standard lamp	109.129
0.6328	red laser light	107.925
0.8750	infrared	105.72
0.9000	infrared	105.59
0.9050	infrared	105.57
0.9100	infrared	105.54
0.9200	infrared	105.496
0.9300	infrared	105.450

The basic formula for computing the refractive index correction in parts per million of the distance, is used in the form,

$$\text{corr'n (ppm)} = I - \frac{N\ p}{273.2 + t} + \frac{15\ e}{273.2 + t}$$

where,

I = assumed index
N = constant for given value of λ
p = pressure in millimeters
t = Centigrade temperature
e = vapor pressure in mm.

Examples:

DI-10 Measured = 950.000 meters
I = 281.9
N = 105.72
p = 752.9
t = 26.0°C

$$\frac{N\ p}{273.2 + t} = \frac{105.72 \times 752.9}{273.2 + 26.0} = 266.0$$

NOTE: An average value of 0.4 ppm is used for the humidity correction, and the term involving e is not used.

Corr'n in meters = 950.000×10^{-6} (281.9 - 266.0)
= 0.000950 x (+15.9) = 0.015
Slope distance = 950.000 + 0.015 = 950.015 meters

H-P 3800 Measured 1199.9890 meters
I = 278.7
N = 105.46
P = 754.9
t = 266°C

$$\frac{N\ p}{273.2 + t} = \frac{105.54 \times 754.9}{273.2 + 26.6} = 265.8$$

Corr'n in meters = 1199.9890×10^{-6} (278.7 - 265.8)
= 0.001200 x (12.9) = +0.0155
Slope distance = 1199.9890 + 0.0155 = 1200.0045 meters

MA-100 Measured 1650.0203 meters
I = 274.4
N = 105.450
p = 758.2
t = 28.0°C
Inst. Constant = - 0.0144
Prism Constant = - 0.0270

$$\frac{N\ p}{273.2 + t} = \frac{105.450 \times 758.2}{273.2 + 280} = 265.4$$

Corr'n in meters = 1650.0203×10^{-6} (274.4 - 265.4)
= 0.001650 x (+9.0) = 0.0148
= 1650.0203 + 0.0148 - 0.0144 - 0.0270 =
1649.9937 meters

RANGER Measured 1649.823 meters
I = 310.4
N = 107.925
p = 758.2
t = 19.5°C
Inst. Constant = 0.165
Prism Constant = - 0.028

$$\frac{N\ p}{273.2 + t} = \frac{107.925 \times 758.2}{273.2 + 19.5} = 279.6$$

Corr'n = 1649.823 x 10^{-6} (310.4 - 279.6)
= 0.001650 x (+30.6) = + 0.0505
Slope distances = 1649.823 + 0.051 + 0.165 - 0.027 =
1650.012 meters

FOR INSTRUMENTS USING MICRO-WAVES

The Essen and Froome formula for the refractive index of the atmosphere using electromagnetic waves was provisionally adopted by the International Association of Geodesy in 1960. This formula is,

$$(n_r - 1)10^6 = \frac{103.49}{T} \cdot (p - e) + \frac{86.26}{T}\left(1 + \frac{5748}{T}\right) e$$

in which,

T = temperature in degrees Kelvin
p = pressure in millimeters
e = vapor pressure in mm

A modified form of this formula is,

$$(n_r - 1)10^6 = \frac{103.46\ p}{273.2 + t} + \frac{490,814.24}{(273.2 + t)^2}\ e$$

where, t = Centigrade temperature

If the temperatures are in Farenheit, atmospheric and vapor pressures in inches of mercury, the above formula is,

$$(n_r - 1)10^6 = \frac{4730\ p}{459.7 + t} + \frac{40,394,200}{(459.7 + t)^2}\ e$$

or

$$I - (kP_m + 1.805k^2\ (e' - rP_m\ (t - t')))$$

where:

I = preset, or assumed Index
P_m = mean pressure in inches of Hg.
t^m = mean dry-bulb temp. in °F.
t' = mean wet-bulb temp. in °F.
k = 4730/(459.688 +t)
e' = saturation vapor pressure in inches.
r = 0.000367 (1 + (t' - 32)/1571)

The terms k, e' and r are tabulated in Tellurometer Manual, Publication 62-1, Revised edition. t is used as argument for k. t' is used as argument for e' and r. (Table on page 77g)

Example:

MRA - 3 - measured = 11620.019 meters
I_n = 325 °
t^n = 50.9°F
t' = 48.1 F

Sheet

l k = 9.264 e' = 0.337 r = 0.000371

k P k^2 e' r P t-t'

325.0- 9.264x29.09 + 1.805x85.8217 0.337-(0.000371x29.09x2.8)

325.0- 269.49 + (154.908 x 0.307) =

325.0- 269.49 + 47.56) = 8.0 ppm.

NOAA FORM 76-61
(6-71)

U. S. DEPARTMENT OF COMMERCE
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION

TELLUROMETER OBSERVATIONS
MODEL MRA 3 - MK-2

STATE	LOCALITY	DATE
NEW YORK	City	1/15/72
STATION	**INST. NUMBER**	**HEIGHT OF INST.**
SCM	1012	1.62 m.
TO STATION	**INST. NUMBER**	**HEIGHT OF INST.**
EMPIRE	1014	1.70 m.
PROJECT NUMBER	**CHIEF OF PARTY**	**OBSERVER**
W12	A.B.C.	C.B.D.

METEOROLOGICAL DATA				B	C	D	E	A_F	A_R	MEAN A
TIME	T_{DB}	T_{WB}	P	10	14	61	20	0.04	0.15	
MASTER	52.0	48.0	29.08	1	1	6	2	0.02	0.10	
REMOTE	50.0	48.0	29.10					0.02	0.15	
MASTER	51.5	48.0	29.08					9.99	0.00	
REMOTE	50.0	48.5	29.10					9.96	0.00	
SUM	203.5	192.5	116.36					9.89	9.97	
MEAN	50.9	48.1	29.09					9.95	0.00	
								9.93	0.06	
								9.99	0.04	
				10	15	61	20	0.02	0.10	
				1	1	6	2	SUM		
								MEAN		0.019

ASSUMED INDEX 325

FIELD INDEX _____

ASSUMED - FIELD ( )

DISTANCE X 10^{-6} X (8) ⟶

DISTANCE 11620.019

METEOROLOGICAL CORRECTION 0.093

FIXED CORRECTION _____

SLOPE DISTANCE 11620.113

NOAA FORM 76-61 (6-71) SUPERSEDES C&GS FORM 5135 WHICH MAY BE USED.

77f

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l	e'	r	k	l	e'	r	k	l	e'	r	k
		.000				.000				.000	
-40	0.004	350	11.270	15	0.081	363	9.964	70	0.739	376	8.930
-39	.004		11.243	16	.085		9.943	71	.765		8.913
-38	.004	351	11.217	17	.089		9.922	72	.791		8.896
-37	.005		11.190	18	.093	364	9.902	73	.818	377	8.879
-36	.005		11.164	19	.098		9.881	74	.846		8.863
-35	0.005		11.137	20	0.103		9.861	75	0.875		8.846
-34	.005	352	11.111	21	.108		9.840	76	.905		8.830
-33	.006		11.085	22	.113	365	9.819	77	.935	378	8.813
-32	.006		11.060	23	.119		9.799	78	.967		8.797
-31	.007		11.033	24	.124		9.779	79	.999		8.780
-30	0.007	353	11.008	25	0.131		9.759	80	1.032		8.764
-29	.007		10.982	26	.137	366	9.739	81	1.067		8.748
-28	.008		10.957	27	.143		9.719	82	1.102	379	8.732
-27	.008		10.931	28	.150		9.699	83	1.138		8.716
-26	.009		10.906	29	.157		9.679	84	1.175		8.700
-25	0.009	354	10.881	30	0.165	367	9.659	85	1.214		8.684
-24	.010		10.856	31	.172		9.639	86	1.253	380	8.668
-23	.011		10.831	32	.180		9.620	87	1.294		8.652
-22	.011		10.806	33	.188		9.600	88	1.335		8.636
-21	.012	355	10.782	34	.195		9.581	89	1.378		8.620
-20	0.013		10.757	35	0.203	368	9.561	90	1.422	381	8.605
-19	.013		10.733	36	.212		9.542	91	1.467		8.589
-18	.014		10.709	37	.220		9.523	92	1.514		8.574
-17	.015	356	10.684	38	.229		9.504	93	1.561		8.557
-16	.016		10.660	39	.238	369	9.485	94	1.610		8.543
-15	0.017		10.636	40	0.248		9.466	95	1.661	382	8.527
-14	.018		10.613	41	.257		9.447	96	1.712		8.512
-13	.019		10.589	42	.268		9.428	97	1.766		8.496
-12	.020	357	10.565	43	.278	370	9.409	98	1.820		8.481
-11	.021		10.542	44	.289		9.391	99	1.876	383	8.466
-10	0.022		10.518	45	0.300		9.372	100	1.933		8.451
- 9	.023		10.495	46	.312		9.353	101	1.992		8.436
- 8	.025	358	10.472	47	.324	371	9.335	102	2.053		8.421
- 7	.026		10.448	48	.336		9.317	103	2.115	384	8.406
- 6	.027		10.425	49	.349		9.298	104	2.179		8.391
- 5	0.029		10.402	50	0.362		9.280	105	2.244		8.376
- 4	.030	359	10.380	51	.376		9.262	106	2.311		8.361
- 3	.032		10.357	52	.390	372	9.244	107	2.380	385	8.347
- 2	.034		10.334	53	.405		9.226	108	2.450		8.332
- 1	.036		10.312	54	.420		9.208	109	2.523		8.317
0	0.038	360	10.289	55	0.436		9.190	110	2.597		8.303
+ 1	.040		10.267	56	.452	373	9.172	111	2.673		8.288
2	.042		10.245	57	.468		9.154	112	2.751	386	8.274
3	.044		10.223	58	.486		9.137	113	2.831		8.259
4	.046		10.200	59	.503		9.119	114	2.913		8.245
5	0.049	361	10.179	60	0.522	374	9.101	115	2.996		8.230
6	.051		10.157	61	.540		9.084	116	3.082	387	8.216
7	.054		10.135	62	.560		9.067	117	3.170		8.202
8	.057		10.113	63	.580		9.049	118	3.261		8.188
9	.060	362	10.092	64	.601		9.032	119	3.353		8.174
10	0.063		10.070	65	0.622	375	9.015	120	3.448	388	8.160
11	.066		10.049	66	.644		8.998				
12	.069		10.028	67	.667		8.980				
13	.073	363	10.006	68	.690		8.963				
14	.077		9.985	69	.714	376	8.946				
15	.081	363	9.964	70	.739	376	8.930				

TABLE 1

STATISTICAL DATA FOR ELECTRONIC DISTANCE MEASURING INSTRUMENTS

INSTRUMENTS	MANUFACTURER	EMMISION SOURCE	RANGE	ACCURACY	APPROX. PRICE
Microwave					
DI-60	Wild	Microwave	20 m. to 50 km.	± 1 cm + 3 PPM	$18,000
Electrotype	Cubic Corporation	Microwave	50 m. to 50 km.	± 1 cm ± 1 P/300,000	11,500
MRA-101	Tellurometer	Microwave	100 m. to 50 km.	± 1.5 cm ± 3 PPM	9,000
MRA-3	Tellurometer	Microwave	75 m. to 65 km.	± 1.5 cm ± 3 PPM	9,500
MRA-4	Tellurometer	Microwave	25 m. to 50 km.	± 3 mm ± 3 PPM	20,000
CA-1000	Tellurometer	Microwave	150 to 100,000 ft.	± 0.05 ft.	4,000
Lightwave					
6A	AGA Corporation	Tungsten	15 m. to 15 km.	5 mm ± 1 PPM	8,000
		Mercury Vapor	15 m. to 25 km.	5 mm ± 1 PPM	10,000
6B	AGA Corporation	Tungsten	15 m. to 15 km.	5 mm ± 1 PPM	7,000
		Mercury Vapor	15 m. to 25 km.	5 mm ± 1 PPM	9,000
8	AGA Corporation	Laser	15 m. to 60 km.	6 mm ± 1 PPM	17,500
700	AGA Corporation	Laser	Up to about 3 km	+ 5 mm + 1 PPM	12-15,000
76	AGA Corporation	Laser	0 to 3,000 m.	± 1 cm + 1 PPM	4,100
Geodolite	Spectra-Physics	Laser	Up to 80 km	± 1 mm or o PPM	49,500
Ranger	Laser Sys. & Elec.	Laser	0 to more than 6 km	± 5 mm + 1 PPM	8,000
Rangemaster	Laser Sys. & Elec.	Laser	0 to 60 km	± 5 mm + 1 PPM	17,000
Infrared					
Akkuranger	Scintrex Limited	Infrared	0 to 1,350 m.	± 1 cm + 1 P/100,000	3,900
Cubitape	Cubic Corporation	Infrared	1 to 2,000 m.	5 mm + 1 P/100,000	4,000
DI-10	Wild	Infrared	1 to 2,000 m.	± 1 cm	8,500
H-P 3800	Hewlett-Packard	Infrared	0 to 3,000 m.	± 5 mm + 1 P/100,000	4,200
MA-100	Tellurometer	Infrared	0 to 2,000 m.	± 2 to 5 mm.	7,000
Micro-Ranger	Laser Sys. & Elec.	Infrared	1 to 3,000 m.	± 5 m + 2 PPM	

TABLE II

INSTRUMENT SERVICE OR INQUIRIES CONSULT THE FOLLOWING:

Firm Names	Instruments Manufactured & Serviced
AGA Corporation 550 County Avenue Secaucus, New Jersey 07094	Geodimeters, Models 4B, 4D, 6, 6A, 6B, 8, 76, & 700
Cubic Corporation P. O. Box 769 San Diego, California 92112	Autotape, Electrotape DM-20, & Cubitape DM-60
Hewlett-Packard Loveland Divisions P. O. Box 301 Loveland, Colorado 80537	HP3800A and HP3800B
Laser Systems & Electronics Box 248 Tullahoma, Tennessee 37388	Ranger, Ranger II, Rangemaster, & Micro-Ranger
Spectra-Physics 1250 West Middlefield Road Mountain View, California 94040	Geodolite 3G
Tellurometer, Inc. 170 Finn Court Farmingdale, Long Island, New York 11735	MRA-1, MRA-2, MRA-3, MRA-101, MRA-4, MRB-2, MA-100, CA-1000
Wild Heerbrugg Instruments, Inc. 465 Smith Street Farmingdale, Long Island, New York 11735	Distomat DI-10, Distomat DI-50, & Distomat DI-60
Scintrex Limited 222 Snidercroft Road Concord, Ontario, Canada	Akkuranger MkII

RECTANGULAR COORDINATE SYSTEMS

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Introduction

Rectangular coordinate systems are probably as old as the surveying profession. The Romans employed such systems in the layouts of their cities and military encampments, and they have been in more or less general use since that time. In the United States, the public land surveys were defined in rectangular coordinates, and as early as 1846, Simeon Borden, who initiated triangulation surveys in Massachusetts, proposed that the state be divided into five tangent plane systems. Coordinates based on a tangent plane system can have a direct relationship to national network geographic coordinates, but the projection is not conformal except at the origin and is limited to a rather small area. The State coordinates systems, on the other hand, are conformal and cover a much larger area. In addition to these grid systems, Universal Transverse Mercator coordinates are sometimes used in this country. Although other coordinate systems are employed in various

countries, the following discussions will deal only with those noted above.

Coordinate Systems

Tangent Plane Coordinates - Prior to the development of the State coordinate system, several cities, including New York, employed this map projection which can be described as an approximate azimuthal equidistant projection. Although some cities continue to use this system, its use today should be restricted only to those areas to which the national network has not been extended or for special purpose engineering surveys such as those required for bridges, tunnels, etc., where a temporary system of plane coordinates of high accuracy and limited extent is required. It is very important to note here, that in no case should this plane coordinate system be extended more than twenty or twenty-five miles from the origin.

As noted in the Introduction, this projection is not conformal, and perhaps a definition of this term is in order. In a Conformal projection the scale at any particular point is constant in all directions, and small figures on the surface of a sphere retain their original forms on the map. Although this condition is met for the point selected as the origin, provided the system is confined to the recommended area size, the scale at the other points is not the

same in all directions, being zero in the direction of the origin and varying between points.

To develop this grid system, a basic framework of triangulation, traverse, and/or trilateration is established and geographic positions computed for the points involved. Inasmuch as this is a purely local system, the geographic position of only one point in the net is required to initiate the computation of the other points. This position can be obtained by any available source, even by scaling from a map. Orientation is usually more critical, the accuracy of which would depend upon various circumstances and generally should be obtained by astronomical observations or connections to the national net.

Once the basic framework is adjusted, one point is selected as the origin and plane coordinates are determined using the formulae given below. Generally, it is best to select some point near the center of the system as the origin and to assign coordinate values of sufficient numerical size to insure that negative coordinates do not occur. In some cases, however, the following sample computation, for example, the origin may be defined as having 0,0 coordinates and the coordinates for the other points referred to the quadrant in which they appear.

The computation of local coordinates from geographic positions, and vice versa, can readily be made with the functions listed in Special Publication No. 241, "Natural Tables for the Computation of Geodetic Positions." (p. 84A)

As an example, assume that triangulation station BOGART is the origin, and its plane coordinates are 0, 0. The plane coordinates of station ROSSVILLE in relation to BOGART are required. The following data are given:

		Latitude		Longitude
BOGART	Φ_o	40° 36' 07".281	λ_o	74° 06' 58".125
ROSSVILLE	Φ_1	40° 32' 39".250	λ_1	74° 13' 07".843
			$\Delta\lambda$	0° 06' 09".718
			or	369".718 (west)

The formulas are:

$$x = \Delta\lambda''/H,\ y = y_1 - y_o + V\ (x/10{,}000)^2$$

From the tables in Sp. Pub. No. 241, take out y_o and y_1 (meridional arcs) for Φ_o and Φ_1, H for Φ_1 and V for y_o.*

H (8 dec. pl.)	= 0.0424 9577	y_o	= 4,496,166.595
V (app.) (4 dec. pl.)	= 6.7099	y_1	= 4,489,749.774
corr. (-.642) (.0213)	= -.0137		
V (corrected)	= 6.6962	$y_1 - y_o$	= -6,416.821
		$V(x/10{,}000)^2$	= +5.068
$x = \Delta\lambda''/H$	= 8,700.113 meters	y	= -6,411.753 meters
or	28,543.62 ft. (west)	or	21,035.89 ft. (south)

* Note: V should be taken out for y_1 but, in making many computations based on one origin, it is easier to use

one basic value of V and correct it by multiplying ΔV by $(y_1 - y_o)$, pointed off four decimal places, and applying the result algebraically.

As an example of the inverse computation: assume that the local plane coordinates, x and y, of ROSSVILLE and the geographic position origin, BOGART, are known. Required is the geographic position of ROSSVILLE. The distance from BOGART to ROSSVILLE may be taken as $\sqrt{x^2 + y^2} = 35,457.68$ ft. or 10,807.522 meters. The tangent of the azimuth from BOGART to ROSSVILLE is x/y or 1.356 9010. The azimuth is then 53° 36' 38".8. Using this length and azimuth and the geographic position of BOGART, compute the geographic position of ROSSVILLE, as explained on page 84, Sp. Pub. No. 241, for computation over short lines. (pp. 84A and 84B)

It would always be recommended that the computations and adjustment of the primary network be carried out at the sea level surface, and then if actual ground distances are required in the operations, to raise the coordinates to the average elevation of the project by multiplying the coordinates by a factor. For most work this factor would be, if the elevations are in feet, $\frac{20,906,000 + h}{20,906,000}$, or $\frac{6,372,000 + h}{6,372,000}$ for elevations in meters, with "h" being the elevation in feet or meters, respectively. A further

Lat.	Meridional arcs (meters)	Difference per second	H	Diff. per second	Sin ϕ	Diff. per second	y_1 for V and K	V	ΔV	K
° ′										
40 00	4,429,318.908	30.842450	0.0421 56492	170.88	0.642 78761	371.35	4,420,000	6.54918	2091	0.0959
01	4,431,169.455	30.842533	0.0421 66745	171.03	0.643 01042	371.25	4,430,000	6.57009	2096	0.0960
02	4,433,020.007	30.842633	0.0421 77007	171.17	0.643 23317	371.15	4,440,000	6.59105	2102	0.0961
03	4,434,870.565	30.842717	0.0421 87277	171.32	0.643 45586	371.08	4,450,000	6.61207	2108	0.0962
04	4,436,721.128	30.842817	0.0421 97556	171.45	0.643 67851	370.98	4,460,000	6.63315	2113	0.0963
40 05	4,438,571.697	30.842900	0.0422 07843	171.60	0.643 90110	370.88	4,470,000	6.65428	2119	0.0964
06	4,440,422.271	30.843000	0.0422 18139	171.75	0.644 12363	370.80	4,480,000	6.67547	2125	0.0966
07	4,442,272.851	30.843083	0.0422 28444	171.88	0.644 34611	370.70	4,490,000	6.69672	2130	0.0967
08	4,444,123.436	30.843167	0.0422 38757	172.03	0.644 56853	370.62	4,500,000	6.71802	2135	0.0968
09	4,445,974.026	30.843267	0.0422 49079	172.17	0.644 79090	370.53	4,510,000	6.73937	2142	0.0969
40 10	4,447,824.622	30.843350	0.0422 59409	172.32	0.645 01322	370.43	4,520,000	6.76079	2147	0.0970
11	4,449,675.223	30.843433	0.0422 69748	172.47	0.645 23548	370.35	4,530,000	6.78226	2154	0.0972
12	4,451,525.829	30.843533	0.0422 80096	172.60	0.645 45769	370.25	4,540,000	6.80380	2158	0.0973
13	4,453,376.441	30.843617	0.0422 90452	172.75	0.645 67984	370.17	4,550,000	6.82538	2165	0.0974
14	4,455,227.058	30.843717	0.0423 00817	172.90	0.645 90194	370.07				
40 15	4,457,077.681	30.843800	0.0423 11191	173.05	0.646 12398	369.98				
16	4,458,928.309	30.843900	0.0423 21574	173.18	0.646 34597	369.88				
17	4,460,778.943	30.843983	0.0423 31965	173.33	0.646 56790	369.80				
18	4,462,629.582	30.844067	0.0423 42365	173.48	0.646 78978	369.70				
19	4,464,480.226	30.844167	0.0423 52774	173.62	0.647 01160	369.62				
40 20	4,466,330.876	30.844250	0.0423 63191	173.77	0.647 23337	369.53				
21	4,468,181.531	30.844350	0.0423 73617	173.92	0.647 45509	369.43				
22	4,470,032.192	30.844433	0.0423 84052	174.05	0.647 67675	369.33				
23	4,471,882.858	30.844517	0.0423 94495	174.20	0.647 89835	369.25				
24	4,473,733.529	30.844617	0.0424 04947	174.35	0.648 11990	369.17				
40 25	4,475,584.206	30.844700	0.0424 15408	174.50	0.648 34140	369.07				
26	4,477,434.888	30.844800	0.0424 25878	174.65	0.648 56284	368.97				
27	4,479,285.576	30.844883	0.0424 36357	174.78	0.648 78422	368.88				
28	4,481,136.269	30.844967	0.0424 46844	174.93	0.649 00555	368.80				
29	4,482,986.967	30.845067	0.0424 57340	175.08	0.649 22683	368.70				
40 30	4,484,837.671	30.845150	0.0424 67845	175.23	0.649 44805	368.60				
31	4,486,688.380	30.845250	0.0424 78359	175.38	0.649 66921	368.52				
32	4,488,539.095	30.845333	0.0424 88882	175.53	0.649 89032	368.43				
33	4,490,389.815	30.845417	0.0424 99414	175.67	0.650 11138	368.33				
34	4,492,240.540	30.845517	0.0425 09954	175.82	0.650 33238	368.25				
40 35	4,494,091.271	30.845600	0.0425 20503	175.97	0.650 55333	368.15				
36	4,495,942.007	30.845700	0.0425 31061	176.12	0.650 77422	368.05				
37	4,497,792.749	30.845783	0.0425 41628	176.27	0.650 99505	367.97				
38	4,499,643.496	30.845883	0.0425 52204	176.40	0.651 21583	367.88				
39	4,501,494.249	30.845967	0.0425 62788	176.55	0.651 43656	367.78				
40 40	4,503,345.007	30.846067	0.0425 73381	176.70	0.651 65723	367.68				
41	4,505,195.771	30.846150	0.0425 83983	176.85	0.651 87784	367.60				
42	4,507,046.540	30.846233	0.0425 94594	177.00	0.652 09840	367.52				
43	4,508,897.314	30.846333	0.0426 05214	177.17	0.652 31891	367.42				
44	4,510,748.094	30.846417	0.0426 15844	177.30	0.652 53936	367.32				
40 45	4,512,598.879	30.846517	0.0426 26482	177.45	0.652 75975	367.23				
46	4,514,449.670	30.846600	0.0426 37129	177.60	0.652 98009	367.15				
47	4,516,300.466	30.846683	0.0426 47785	177.75	0.653 20038	367.03				
48	4,518,151.267	30.846783	0.0426 58450	177.90	0.653 42060	366.97				
49	4,520,002.074	30.846867	0.0426 69124	178.05	0.653 64078	366.87				
40 50	4,521,852.886	30.846967	0.0426 79807	178.20	0.653 86090	366.77				
51	4,523,703.704	30.847050	0.0426 90499	178.35	0.654 08096	366.68				
52	4,525,554.527	30.847133	0.0427 01200	178.48	0.654 30097	366.58				
53	4,527,405.355	30.847233	0.0427 11909	178.65	0.654 52092	366.48				
54	4,529,256.189	30.847317	0.0427 22628	178.80	0.654 74081	366.40				
40 55	4,531,107.028	30.847417	0.0427 33356	178.95	0.654 96065	366.32				
56	4,532,957.873	30.847500	0.0427 44093	179.10	0.655 18044	366.22				
57	4,534,808.723	30.847600	0.0427 54839	179.23	0.655 40017	366.13				
58	4,536,659.579	30.847683	0.0427 65593	179.40	0.655 61985	366.03				
59	4,538,510.440	30.847767	0.0427 76357	179.55	0.655 83947	365.93				
41 00	4,540,361.306		0.0427 87130		0.656 05903					

Correction for H

Seconds	Cor.
0 & 60	0
10 & 50	−1
20 & 40	−1
30 & 30	−1

Correction for V

Decimal	Cor.
0 & 10	0
1 & 9	0
2 & 8	0
3 & 7	−1
4 & 6	−1
5 & 5	−1

CORRECTION FACTORS
(Corrections to unity in 7th decimal place.)

Correction for x	$-\frac{1}{2}fb$
Correction for y	$+fa$
Arc-sine correction	$+V(Va)/15$

Lat.	f	F
° ′		
40 00	8.2035	0.739×10^{-12}
40 30	8.2025	0.736
41 00	8.2016	0.732

POSITION COMPUTATION OVER SHORT LINES

In order to show the comparative ease with which a computation over short lines is accomplished, we include here such an example. For lines under 8,000 meters in length, none of the terms involving the use of f or b has a significant effect. As both lines here are well under this 8,000-meter limit, none of the corrective terms is needed. Seven decimal places are sufficient for sin α and cos α. Both x' and y' may be taken the same as x and y, respectively. V need be taken out only to the same number of significant figures as a or perhaps one more. In the example shown, V may be interpolated by inspection to the number of figures needed. The arc-sine correction is not needed, and $\Delta\lambda''$ is the product of x and H. Five-place sines are sufficient for the computation of $\Delta\alpha''$.

POSITION COMPUTATION, FIRST-ORDER TRIANGULATION

(For calculating machine computation)

							°	′	″
α	3		to 2						
3d∠			&				−		
α	3		to 1				53	36	38.8
$\Delta\alpha$							−	04	00.5
							180	00	00.00
α'	1		to 3				233	32	38.3

	°	′	″			°	′	″
ϕ	40	36	07.281	3 Bogart	λ	74	06	58.125
				s= 10,807.522	$\Delta\lambda$	+	06	09.7181
ϕ'	40	32	39.2500	1 Rossville	λ'	74	13	07.8431

$\Delta\phi$	(log s =) $b=(y/10{,}000)^2$
sin α +0.8050054	x cor. $=-\frac{1}{2}fb$
cos α +0.5932675	x' (same as x)
$x=s \sin \alpha$ +8700.114	H 0.042495771
$y=-s \cos \alpha$ −6411.752	Hx'=(approx. $\Delta\lambda''$)
$a=(x'/10{,}000)^2$ 0.7569	Arc−sin cor $=+\frac{V(Va)}{15}$
y cor. $=+fa$	$\Delta\lambda''$ +369.7181
y_0 4,496,166.595	sin ϕm. 0.65042
y' − 6,411.752	sin ϕ'
y_1 4,489,754.843	$1+\cos \Delta\phi$
Va − 5.068	$\frac{\sin \phi+\sin \phi'}{1+\cos \Delta\phi}$
y_2 4,489,749.775	$-\Delta\alpha''$ (approx.)
V 6.696	$+F(\Delta\lambda'')^3$
K $(Va/1{,}000)^2+$	$-\Delta\alpha''$ +240.47

discussion on this subject will be made later in the section concerned with project datum coordinates.

State Plane Coordinates - In 1933, at the request of a North Carolina Highway Department engineer, Dr. Oscar S. Adams of the United States Coast and Geodetic Survey undertook a feasibility study for a plane coordinate system utilizing the geodetic data derived from the national network. As a result of this study, coordinate systems were developed for each state.

Several factors entered into the selection of a particular system or systems for each State and the partitioning or zoning required. It was decided for reasons that will be given later, that the Lambert conformal map projection with two standard parallels would be used for those states which extended primarily east and west, and the transverse Mercator projection for those states whose larger dimensions were north and south. In addition, it was considered desirable that the zone boundaries followed county lines, and with one exception this condition was satisfied. A standard of 1:10,000 was set as the maximum scale reduction, thus limiting the zone widths to 158 miles. Also considered were the scale reductions in the vicinity of metropolitan areas, and with few exceptions the zone designs are such that scale reductions in these regions are smaller than the standard.

Care was taken to insure sufficient overlap and to avoid negative coordinate values; but in two instances small islands lying some distance off the coast were overlooked with the result that points on these islands have negative coordinates.

In preparing the tables for the transverse Mercator systems, a small corrective term which would come in only at the extremities of a particularily wide zone was neglected. Except in northern Nevada, where the Central zone does not extend and the East and West zones meet, the neglecting of this term is of no consequence. At worst it could introduce errors of 0.1 to 0.2 feet in relating geodetic distances to grid values and might affect the grid azimuths by a few hundredths of a second. Since this would occur only when the points involved are widely separated, the error introduced is of little or no significance. When the transverse Mercator systems for Alaska Zones 2-9 were developed, this corrective term was included.

Prior to 1939 Los Angeles County was included as part of California Zone 5, but many surveys had been computed on a local plane system which consisted of three zones designated as A, B, and C. The origins of these zones were all at latitude 34° 08' with the central meridians for A, B, and C being at longitude 118° 58', 118° 20', and 117° 42', respectively. Although these coordinate systems were considered to be computed on the polyconic projection, the computations were made using the formulae for Tangent Plane Coordinates as described on pp. 81-85. It was contended that for the small areas involved, the differences are insignificant and for all intents and purposes this is true. While this grid system had been rather extensively

employed, the need for three zones (required to maintain the irreconcilable scale error at better than 1:100,000) made for an awkward situation. Consequently, zone 7 was developed covering all of Los Angeles County with a maximum computable scale reduction of 1:87,300. The origin of zone B which was located at latitude 34° 08' and longitude 118° 20' with previously assigned values for X = 1,186,692.58 and Y = 1,160,926.74 was selected as the origin for zone 7, and to distinguish between the systems, 3,000,000.00 feet was added to these origin coordinates.

As finally developed, nine states contain but one zone; in two states, Florida and New York, both plane coordinate systems were employed; and by accepting a slightly larger scale reduction standard (as was done in several instances), a state as large as Texas was covered by five zones. In later years, plane coordinate systems were developed for Alaska and Hawaii and most possessions. Alaska employs both projections, and in addition, an oblique Mercator projection for southeast Alaska, and is divided into ten zones. Guam utilizes a tangent plane system.

Recently, Michigan proposed that the state be placed on a Lambert system rather than the original transverse Mercator projection. In addition, it was requested that the system be referenced at an elevation of 800 feet (the average elevation of the state) rather than sea level as

are all other state systems. Although it was understandable, with the large population centers spread east to west across southern Michigan, that the Lambert projection would be more satisfactory, there was some reluctance to reference the system at other than sea level. However, the proposal was accepted and plane coordinates for all national network stations in Michigan are now available on both systems.

With the discovery of oil off the Gulf coasts of Louisiana and Texas, the south zone for Louisiana was extended to cover the original discovery area. However, negative coordinates resulted in some instances. As the oil leases moved further south into the Gulf of Mexico, an additional extension to this zone could not be satisfactorily made and it was necessary to develop a new grid zone. The solution was rather simple. The south zone of Texas covered much of the latitude range required and by simply shifting the central meridian from 98° 30' to 91° 20' and extending the tables to cover the entire area, the Louisiana offshore zone was created. In this regard, it may be of interest to know that the State systems can be shifted anyplace in the world, north or south of the equator without additional computations, by shifting the central meridian to that locality providing the area is within the latitude range of the tables and the basic

surveys are computed on the Clarke Spheroid of 1866. Actually, an area could be covered by using a system for one State zone for a part, a second State zone for another part, etc. The latitude bands for the Lambert projection extend around the world, and the latitude range of the transverse Mercator systems can be satisfactorily extended to within a few degrees of the poles. As a case in point, the UTM coordinates which will be described later are nothing more than the transverse Mercator system with the equator as the origin.

Tables are available for the direct and inverse computation of coordinates for all systems using desk calculators. With the exception of Alaska, the computational format for each projection is identical. For Alaska the 2-1/2 minute intersection tables are employed using an interpolation technique which is fully explained in the publications.

Coordinates for the tangent plane system used on Guam are computed using Special Publication No. 241, as described previously.

Although each State coordinate projection table contain examples of the direct and inverse computations, sample computations on the Lambert and transverse Mercator system are given here for handy reference (Figures 1, 2, 3, 4). The formulae are straightforward and no detailed

$X = R \sin\theta + C$ $\quad Y = R_b - R\cos\theta$ $\quad$ State-Zone Kansas-North

$C =$ 2,000,000.00 $\quad R_b =$ ~~25,979,068~~.57 $\quad$ Grid Az. = Geod. Az. $- \theta$

Station	Latitude / Longitude	R / θ	sin θ / cos θ	X / Y
1 ROBBINS	38 58 52.096	25,743,138.15	+0.01326 78253	2,341,555.46
	96 47 54.567	+0 45 36.7657	0.99991 19786	238,196.37
	Grid azimuth to azimuth mark			229° 36' 42"
2 COOPER	39 58 41.957	25,379,925.63	-0.00651 51579	1,834,645.78
	98 35 23.954	-0 22 23.8573	0.99997 87762	599,681.60
	Grid azimuth to azimuth mark			256° 58' 07"

Figure 1 - Plane Coordinates on Lambert Projection

STATE—ZONE Kansas - North $\quad l =$ 0.63271 48646

Station ROBBINS

C	-2,000,000.00	R_b	25,979,068.57
x	2,341,555.46	y	- 238,196.37
$x' = x - C$	+ 341,555.46	$R_b - y$	25,740,872.20
$\tan\theta = x' \div (R_b - y)$	+0.01326 89933	θ	+2736.7657
θ	+0° 45' 36".7657	$\Delta\lambda = \theta \div l$	+4325."433
$\cos\theta$	0.99991 19786	$\Delta\lambda$	+1° 12' 05".433
$R = (R_b - y) \div \cos\theta$	25,743,138.15	Central Meridian	98° 00' 00".000
ϕ	38° 58' 52".096	$\lambda =$ C. M. $- \Delta\lambda$	96° 47' 54".567

Station COOPER

C	-2,000,000.00	R_b	25,979,068.57
x	1,834,645.78	y	- 599,681.60
$x' = x - C$	- 165,354.22	$R_b - y$	25,379,386.97
$\tan\theta = x' \div (R_b - y)$	-0.00651 52961	θ	-1343."8573
θ	-0° 22' 23".8573	$\Delta\lambda = \theta \div l$	-2123."954
$\cos\theta$	0.99997 87762	$\Delta\lambda$	-0° 35' 23".954
$R = (R_b - y) \div \cos\theta$	25,379,925.63	Central Meridian	98° 00' 00".000
ϕ	39° 58' 41".957	$\lambda =$ C. M. $- \Delta\lambda$	98° 35' 23".954

Figure 2 - Geodetic Positions from Lambert Coordinates

State	Zone
Idaho	East
Central meridian	
112° 10' 00".000	

Station	WALKER		PINHEAD	
ϕ	43° 48' 07".616		43° 35' 26".260	
λ	111 42 29.824		112 22 35.516	
$\Delta\lambda$ = Central mer. $-\lambda$	+ 0° 27' 30".176		- 0° 12' 35".516	
$\Delta\lambda''$	+ 1,650".176		- 755".516	
$\left(\frac{\Delta\lambda''}{100}\right)^2$	272.308		57.080	
H	73.336360		73.594594	
V	1.230701		1.230273	
a | b	-0.357	+1.175	-0.418	+0.595
$x' = H \cdot \Delta\lambda \pm ab$	+ 121,017.48		- 55,601.64	
$V\left(\frac{\Delta\lambda''}{100}\right)^2 \pm c$	335.07		70.21	
Tabular y	778,234.67		701,147.74	
x	621,017.48		444,398.36	
y	778,569.74		701,217.95	
$\Delta\alpha''$	+ 1,142".21		- 520".93	
$\Delta\alpha$	+ 0° 19' 02".2		- 0° 08' 40".9	
Geod. Az. to Az. Mk.	53 26 16.7		200 33 42.8	
Grid. Az. to Az. Mk.	53 07 14		200 42 24	

$x = x' + 500,000$

$y = \text{Tab. } y + V\left(\frac{\Delta\lambda''}{100}\right)^2 \pm c$

$\Delta\alpha'' = \Delta\lambda'' \sin\phi + g$

Grid Az. = Geod. Az. $-\Delta\alpha$

H and V = Tab. H and Tab. V.

When ab is $\frac{-,\ \text{decrease}}{+,\ \text{increase}}$ $H \cdot \Delta\lambda$ numerically

g increases $\Delta\lambda'' \cdot \sin\phi$ numerically

Figure 3 - Plane Coordinates on Transverse Mercator Projection

STATE – ZONE Idaho - East

Station WALKER

X	621,017.48	Y	778,569.74
C	– 500,000.00	$P(\frac{X'}{10,000})^2 + d$	– 335.08
X'	+ 121,017.48	Y_o	778,234.66
P	2.28775	Approx. $\Delta\lambda = X' \div H$	+ 1,650"
d	+ 0.03	$\Delta\lambda = (X' \mp ab) \div H$	+ 1,650."176
H	73.336360	$\Delta\lambda$	+ 0° 27' 30."176
a | b	- 0.357 | + 1.175	Central Meridian	112 10 00.000
ϕ	43° 48' 07."616	λ = C.M. – $\Delta\lambda$	111° 42' 29."824

Station PINHEAD

X	444,398.36	Y	701,217.95
C	– 500,000.00	$P(\frac{X'}{10,000})^2 + d$	– 70.21
X'	- 55,601.64	Y_o	701,147.74
P	2.27087	Approx. $\Delta\lambda = X' \div H$	- 756"
d	+ 0.01	$\Delta\lambda = (X' \mp ab) \div H$	- 755."516
H	73.594594	$\Delta\lambda$	- 0° 12' 35."516
a | b	- 0.418 | + 0.595	Central Meridian	112 10 00.000
ϕ	43° 35' 26."260	λ = C.M. – $\Delta\lambda$	112° 22' 35."516

When ab is $\frac{+, \text{decrease}}{-, \text{increase}}$ X' numerically

Figure 4 - Geodetic Positions from Transverse Mercator Coordinates

detailed instructions are presented.* Tables for the computations on the transverse Mercator projection contain all data necessary, but an additional table containing sines, cosines, and tangents to ten decimal places is required for the Lambert projection. A table containing this information from 0° to 6° (Special Publication No. 246) is available from the Government Printing Office at a moderate cost. This range (0° to 6°) is more than sufficient for the computations within the area limitations as employed for the system. * See Addendum, p. 104.

For those who wish to program the computations on electronic computers, there is available from the GPO Publication 62-4, "Plane Coordinates by Automatic Data Processing." This publications contains the constants required for the computations involved in all the systems devised to date by the Coast and Geodetic Survey

(now known as the National Ocean Survey) and complete descriptive examples. Computations can be carried out beyond the range of the published tables even to computing negative values; but it must be remembered that should these data be used to control surveys, the scale factors increase rapidly beyond the published range and equally important the second term corrections also significantly increase in size. Although the programs are not included because of the variety

and capacity of available computers, no information has been received of any great programming difficulties. As a matter of fact, these formulae have been programmed very successfully on a number of the small programmable desk type computers.

None of the conformal map projections used in the national grid systems can be truly illustrated graphically and the proofs can only be demonstrated mathematically. For those who may be interested, the mathematical details are given in several publications dealing with map projections, one of which (Special Publication No. 251) is given in the list of references. In general, the primary concern of most practicing surveyors is the scale reduction and this can be shown graphically, in a broad sense, as illustrated in Figures 5, 6, 7. Of lesser concern at this time, but of increasing importance as more accurate surveys are made, is the second term correction. This correction will be considered in some detail in the chapter on computations.

Figure 5 illustrates the Lambert projection with two standard parallels. This projection can be viewed as a cone, cutting or being secant to the sphere (circle) represeting the earth at these parallels. The axis of the cone coincides with the polar axis of the reference spheroid. To simplify the explanation of the scale reduction, consider chord distances passing through the polar axis at right angles

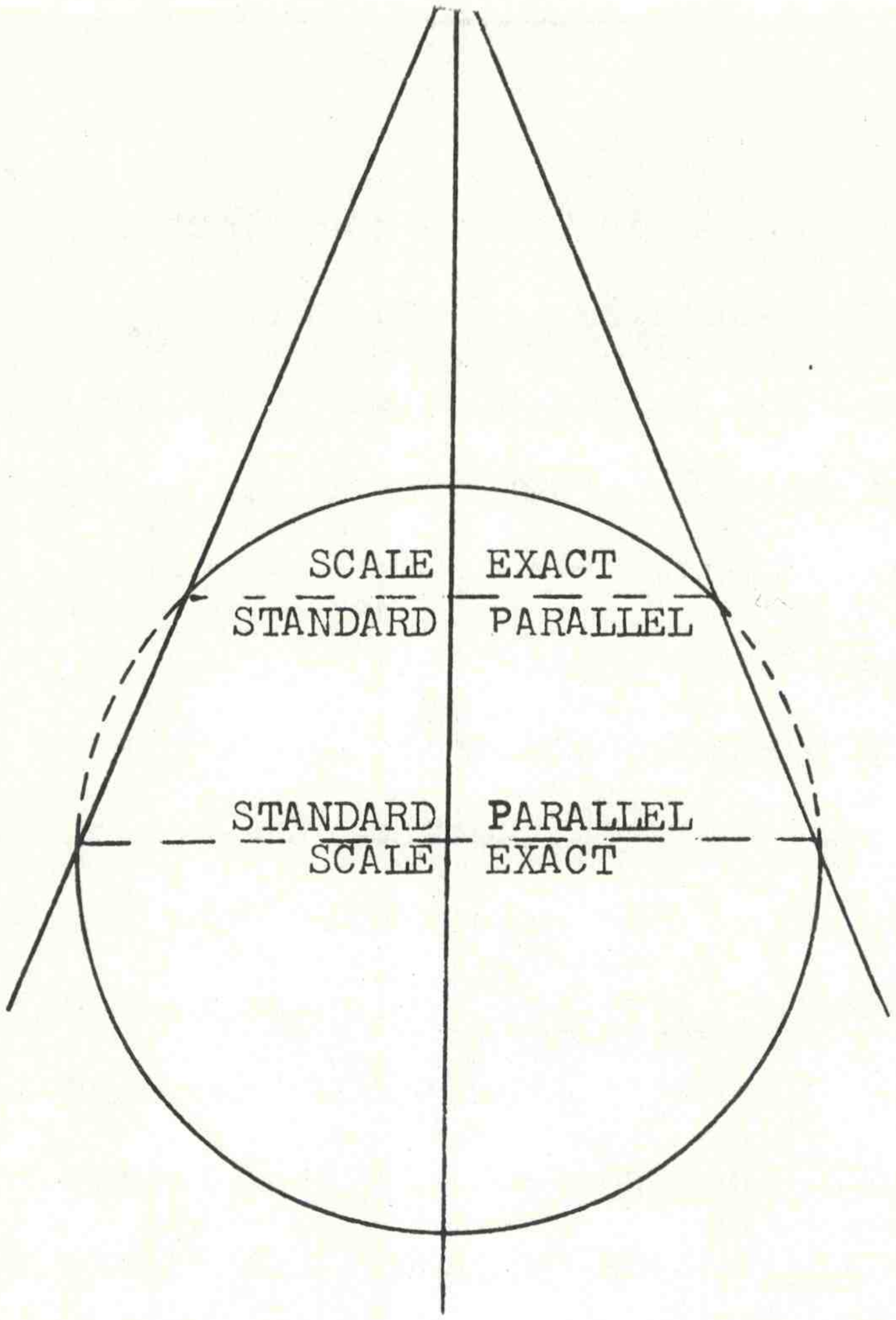


Figure 5 - Lambert Projection - Cone Secant to Sphere

as being geodetic and the corresponding distances on the cone as grid values. As can be seen, geodetic distances are longer than the corresponding grid distances when located between the standard parallels, with the opposite being true for distances north and south of these parallels. Along the standard parallels, geodetic and grid distances are identical. In other words, since any measured distance is a geodetic quantity, these distances should be made smaller by applying a scale factor before using in a

computation involving Lambert plane coordinates for points within the standard parallels. Similarily, the distances should be increased for points outside these parallels. The maximum negative scale reduction is along the central parallel or y_o of the projection which is approximately midway between the standard parallels.

The Lambert projection was selected for those states extending mostly east and west because the scale varies with the latitude and is not effected by longitude differences.

The transverse Mercator projection is a so called "developed cylinder" type as shown in a very simple form by Figures 6 and 7. The spheres (circles) represent the earth and the rectangles the cylinder. In Figure 6, the cylinder is tangent to the sphere at the poles and the scale is exact along any meridian selected as the central meridian. Grid meridians are parallel to this meridian and do not intersect at the poles as do geodetic meridians. As a result, the scale varies in this projection with the distance between these meridians and the central meridian. This condition may be seen by visualizing the chord distances passing through the line representing the equator at right angles as geodetic values and the corresponding distances on the cylinder as grid distances. Obviously, the grid distances are longer with the exception of those along the central meridian. This projection, of course, is perfectly

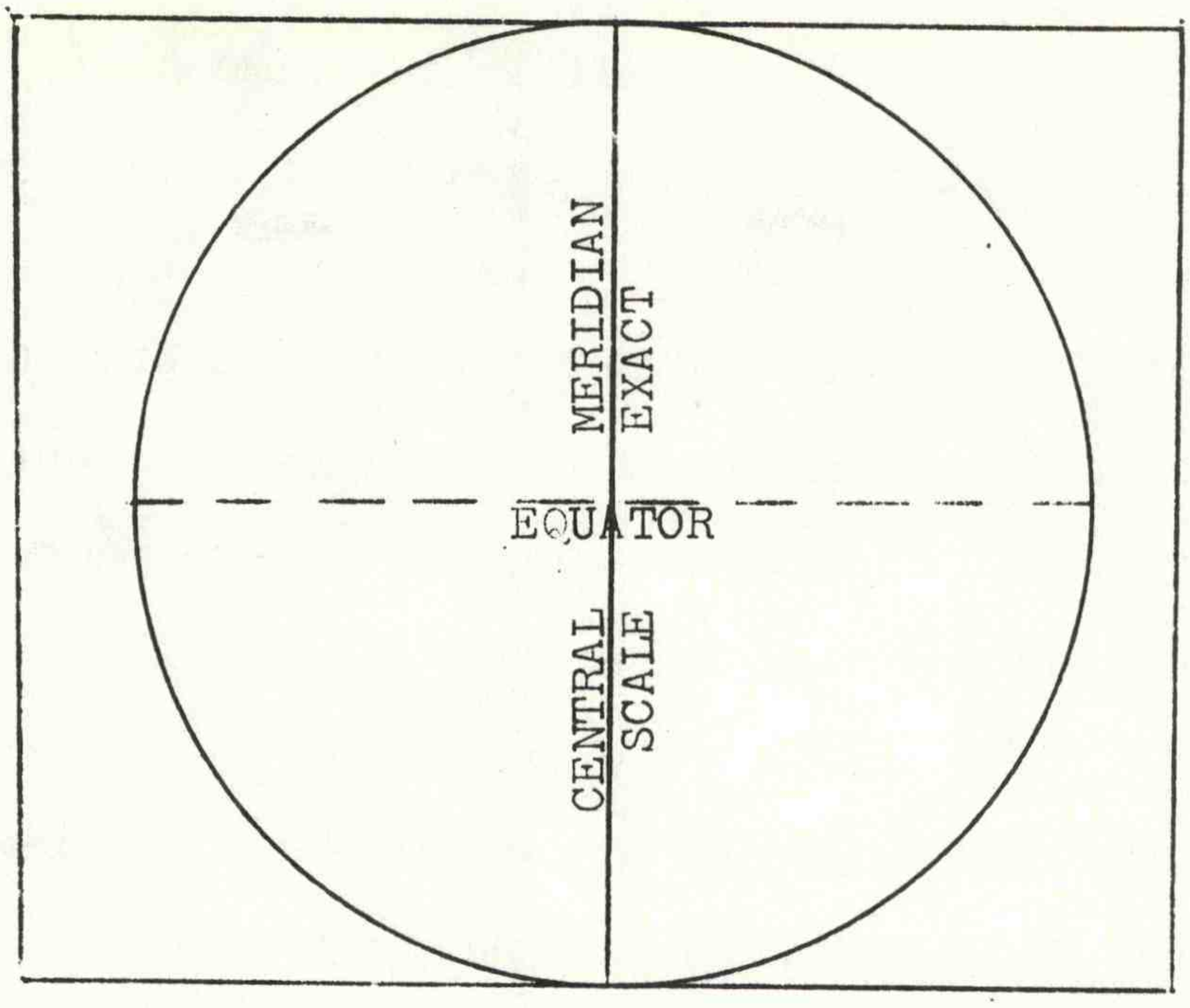


Figure 6 - Transverse Mercator Projection, Cylinder Tangent to Sphere at Poles

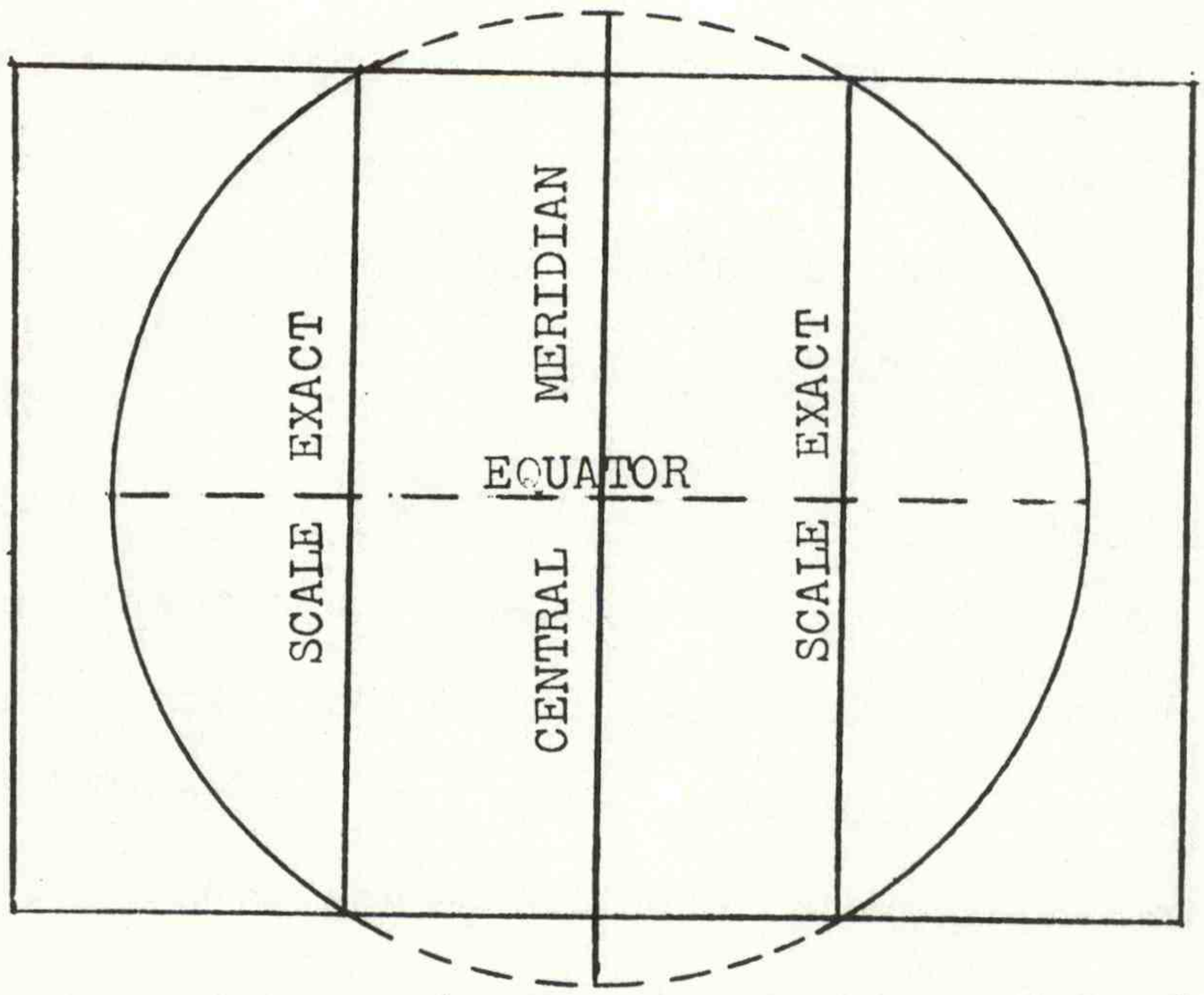


Figure 7 - Transverse Mercator Projection, Cylinder Secant to Sphere

valid; however, a greater area within the same scale standard can be utilized by applying a scale reduction on the central meridian thus creating a secant type condition similar to that used for the Lambert projection. Incidentally, one standard parallel, or the cone tangent to the sphere, could have been employed for the Lambert projection but for the same reason noted above this was not done.

Figure 7 illustrates the transverse Mercator projection as used in the State coordinate systems. By placing a scale reduction on the central meridian, the scale is exact along the meridians where the cylinder intersects the sphere. Between these meridians, grid distances are smaller than geodetic distances, and beyond these meridians the opposite condition exists.

As noted previously, the scale varies east and west of the central meridian, and for this reason the transverse Mercator projection was selected for those states extending primarily north and south.

Universal Transverse Mercator Coordinates - This grid system has been adopted by the Defense Department for world-wide military mapping purposes, but on occasion it has been employed by civilian agencies. The system has the equator as the origin and extends in 6° bands eastward from the 180° meridian, thus establishing 60 zones within the area between

latitudes 80° S and 84° N. The central meridian is always the middle meridian of a zone. For example, zone 17 is between 78° and 84° west longitude, and the central meridian is 81° longitude. The scale reduction along the central meridian is 1:2,500 and exact 180,000 meters (about 112 miles) east and west of the central meridians.

The actual map projection is for all intents and purposes the same as utilized in the State coordinate systems, but with a considerably larger scale reduction on the central meridian, and as a consequence of the wider zones significantly larger second term corrections at the extremities. Also, the coordinates are in meters while in the State systems these values are in feet.

Sample computations are shown in Figures 8 and 9. The formulas are easy to follow in a relatively simple straight-forward computation, and no detailed instructions will be given here.* The Roman numerials and alphabetical notations refer to the tabulated functions.

* See Addendum, p. 104.

SPHEROID	CENTRAL MERIDIAN	ZONE	UNIT	FE
Clarke of 1866	99° 00'	14	Meter	500,000 m.

Station	BASSETT (Texas)			Station	LOLITA (Texas)		
ϕ	30°04'48."992	λ	101°43'18."013	ϕ	28°50'04."095	λ	96°32'35."983
(I)	3327500.243	$\Delta\lambda$	- 2 43 18.013	(I)	3189471.376	$\Delta\lambda$	+ 2 27 24.017
(II)	3252.483	p^2	0.9600106	(II)	3168.101	p^2	0.7821664
(III)	2.247	p^4	0.92162	(III)	2.259	p^4	0.61178
A_6	+ 0.001			A_6	+ 0.001		
(IV)	267699.763	p	0.9798013	(IV)	270990.581	p	0.8844017
(V)	52.581	p^3	0.940620	(V)	57.201	p^3	0.691749
B_5	- 0.006			B_5	- 0.002		
N	3330624.73	E	237657.97	N	3191950.74	E	739704.10

Figure 8 - U.T.M. Grid Coordinates From Geographic Coordinates

The computations are made using the following formulae:

North of Equator $N = (I) + (II)p^2 + (III)p^4 + A_6$

South of Equator $N = 10{,}000{,}000 - \boxed{(I) + (II)p^2 + (III)p^4 + A_6}$

$$E' = (IV)p + (V)p^3 + B_5$$

E' is added to produce E if the point is east of the central meridian, subtracted if it is west.

The Roman numerials, A_6, and B_5 refer to functions appearing in the tables: $\Delta\lambda$ is the central meridian minus the longitude of the point; and "p" is (0.0001) ($\Delta\lambda$").

SPHEROID	CENTRAL MERIDIAN	ZONE	UNIT	FE
Clarke of 1866	99° 00'	14	Meter	500,000 m.

Station	BASSETT (Texas)		
N	3330624.73	E	237657.97
ϕ'	30°06'30".5044		
(VII)	1476.219	q^2	0.06882334
(VIII)	18.12	q^4	0.0047367
D_6	- 0.001		
(IX)	37365.884	q	0.26234203
(X)	256.583	q^3	0.0180553
E_5	+ 0.0042		
$\Delta\phi$	- 01'41".5124	$\Delta\lambda$	+ 2 43 18.0134
ϕ	30°04'48".9920	λ	101°43'18".0134

Station	LOLITA (Texas)		
N	3191950.74	E	739704.10
ϕ'	28°51'24".6631		
(VII)	1403.181	q^2	0.05745806
(VIII)	16.95	q^4	0.0033014
D_6	0		
(IX)	36909.550	q	0.23970410
(X)	243.648	q^3	0.0137729
E_5	+ 0.0021		
$\Delta\phi$	- 01'20".5681	$\Delta\lambda$	- 2 27 24.0168
ϕ	28°50'04".0950	λ	96°32'35".9832

Figure 9 - Geographic Coordinates From U.T.M. Grid Coordinates

The computations are made using the following formulae:

Latitude $(\Phi) = \Phi' - (VII)q^2 + (VIII)q^4 - D_6$

Longitude Difference $(\Delta''\lambda) = (IX)q - (X)q^3 + E_5$

Longitude (λ) = Central Meridian $\pm \Delta\lambda$

When E is less than 500,000, $\Delta\lambda$ is added to the Central Meridian to produce λ, and subtracted when it is more.

The Roman numerials, Φ', D_6 and E_5 refer to functions appearing in the tables; and "q" is (0.000001)(E').

In addition to the publications concerned with computations on the Clarke Spheroid of 1866 as given in the References, there are tables available in the TM 5-241 series for the Australian National, Bessel, Clarke of 1880, Everest, and International spheroid and the extension of the tables to 84° north latitude for several of these spheroids.

Surveys computed on this grid system are carried out following the same procedures as when employing State plane coordinates. All distances must be in meters, of course. As noted previously, the scale reductions and second-term corrections may be much larger than those for the State systems, and an inspection, at the very least, should be made to ascertain whether it is necessary to apply these corrections.

Some consideration is being given to adopting a similar grid system as a replacement for the State systems when a new adjustment of the national geodetic network is made. In order to maintain a maximum scale reduction equivalent to, or as is more likely smaller than the present standard used in the State systems, the zone widths will be considerably less than that used in the UTM system. As a point of interest, the new adjustment will probably be initiated in this decade and hopefully completed in the early 1980's.

Project Datum Coordinates - Occasionally for some engineering and special projects, it is pertinent that the grid system be related to actual horizontal ground distances. As noted earlier, tangent plane coordinates can be satisfactorily used in such situations, however, this would involve the more difficult geodetic computations in establishing the basic framework. For some areas, even as large as a moderately sized county, State plane coordinates may be raised to the average elevation of the project, and by applying the average scale factor the resulting distances are essentially at ground level. There are three considerations that must be fully taken into account and understood before anyone employs such grid systems, however. One, the scale factor: since the average scale factor is used, the maximum deviation from the true scale will be at the fringes of the project, however, should the area be large, significant errors may be introduced elsewhere as well. An investigation should always be made to assure that these errors are acceptable within the survey requirements. Two, elevations: where there are large differences of elevations, this type of grid system can prove troublesome. It must be remembered that for each 21 feet variance from the average elevation, an error of about one part in a million is introduced. An error of this magnitude is insignificant, of course. However, should the difference be about 400 feet, the error would

approach 1:50,000, and in many instances this would be unacceptable. It must always be kept in mind that the scale and elevation factors are used in combination and the combined error must be considered in determining whether an acceptable accuracy results. Three, documentation: to aid in reducing misinterpretations and costly mistakes, all computations, maps, drawings, etc., must carry explanatory notations in sufficient detail.

In general, to obtain the best results, the basic net computations should be carried out at the sea level reference and the resulting coordinates divided by the combined average elevation-scale factor. It has been suggested by some that rather than divide the "X" coordinate by the combined factor, that the X' value be used and then the constant for the zone added. Either method is entirely satisfactory.

Project datum coordinates would seldom be recommended. Often the sole purpose is to eliminate the sea level and scale factor computations. In all candidness, the amount of time saved in such instances is rather small, and the possibility of problems developing at a later date is quite large.

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* Addendum: Step-by-step computations for the conversion of latitudes and longitudes to State plane coordinates and UTM coordinates and the inverse solutions are included in the paper entitled, "The National Geodetic Survey, Its Products and Their Application to Local Surveying Needs" which is included with the Workshop published material.

CORRECTIONS TO MEASURED LENGTHS AND COMPUTATION OF GRID AZIMUTHS

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For distance measurements, the corrections involving the instrument and mirror constants and eccentricities are generally straightforward and no further comment will be offered except to express a word of caution - be sure of the signs and that all corrections are included. Corrections for weather conditions or the meteorological effect, on the other hand, are often not full understood; the data are incorrectly obtained and equally often incorrectly applied. The graphs furnished by the manufacturer to determine this correction will easily provide the necessary corrections and are recommended for general use, but one must remember that paper shrinks and periodically precise computations should be made and compared with the graphical results. Corrective terms, if significant, can then be applied to the corrections obtained from the graphs.

Plastic computers supplied by some manufacturers are fine when first put into use, but as time goes on, the components loosen and erroneous data may result. Too often the field equipment may be in excellent operating condition, but

the end results may be something less than desired, simply because the office computing devices are not kept in the same condition.

Computations made on electronic computers, properly programmed, will give results that are correct for the data entered--but often these results do not reflect the accuracies of the observations primarily because less than adequate care was taken in obtaining the meteorological data. Tabulated below are the errors to lengths resulting from meteorological observations:

	Electro-Optical	Microwave
Temperature		
Dry Bulb 1°C	1 PPM	1 PPM
Difference in Dry and Wet Bulbs 1°C	*	1 PPM to 17 PPM
Barometer 0.1 inch or Altimeter 100 feet	1 PPM	1 PPM

* The correction for humidity may be ignored or taken at 0.4 PPM.

As may be seen, the humidity correction for microwave equipment can be quite significant and it is imperative that accurate dry and wet bulb temperatures be obtained at both ends of the lines. For measurements made over steep lines, the barometric or altimeter observations are critical. As an example, a difference of elevation of 1000 ft. introduces an error of about 5 PPM if these observations are made at one end only. To many, these possible errors may seem

small and irrelevant, yet carelessness leads to more carelessness, and inaccuracies of 1 to 2 PPM soon evolve into much larger errors.

As final comments regarding this subject, it is rather ludicrous to pay thousands of dollars for distancing equipment and then buy the cheapest necessary accessories such as thermometers and barometers. Equally ludicrous is the fact that many surveyors actually believe they are saving money by ignoring or estimating the fundamental corrections to electronic distance measurements. Nothing can be further from the truth, the time spent in obtaining the meteorological observations is measured in but fractions of a minute.

In addition to the meteorological corrections, the measured distances should be corrected to the horizontal (slope or inclination correction), and generally to sea level if the measurements are to be used in State plane coordinate computations; and occasionally when the lines are long (10 miles or more) for the arc correction. Forms such as that shown on the next page are quite handy. Note that all forms given in the Workshop courses are not cluttered and seldom are in combination with other processes involved in the total computation. Long experience has shown that forms designed for specific tasks with sufficient space for the necessary computations are much

Form 603-5A

CORRECTION TO BE APPLIED TO GEODIMETER SLOPE DISTANCE

GEODIMETER STATION _____

MIRROR STATION _____

Elevation of Geodimeter Station _____M h1 _____

Height of Geodimeter _____M h2 _____

h1 _____M h1-h2 _____

Elevation of Mirror Station _____M $(h1-h2)^2$ _____

Height of Mirror _____M h1 + h2 _____

h2 _____M D* _____

D^2 _____

P** _____

2P _____

Mean latitude of base line _____° '

Azimuth of base line _____° '

Sea Level $\frac{(h1 + h2) D}{2P}$ - _____M

Slope $D - \sqrt{D^2 - (h_1 - h_2)^2}$ - _____M

Arc 1.027 D3 X 10-15 + _____M

Sum of Corrections - _____M

*Geodimeter slope distance in meters

**Radii of curvature of the earth's surface
From: Pages 58 through 65, Geodimeter Manual, Publication 62-2

Arc Correction Table

D km	Cor. m	D km	Cor. m	D km	Cor. m	D km	Cor. m	D km	Cor. m	D km	Cor. m
1	.000	11	.001	21	.010	31	.031	41	.071	51	.136
2	.000	12	.002	22	.011	32	.034	42	.076	52	.144
3	.000	13	.002	23	.012	33	.037	43	.082	53	.153
4	.000	14	.003	24	.014	34	.040	44	.088	54	.162
5	.000	15	.003	25	.016	35	.044	45	.094	55	.171
6	.000	16	.004	26	.018	36	.048	46	.100	56	.180
7	.000	17	.005	27	.020	37	.052	47	.107	57	.190
8	.000	18	.006	28	.022	38	.056	48	.114	58	.200
9	.001	19	.007	29	.025	39	.061	49	.121	59	.211
10	.001	20	.008	30	.028	40	.066	50	.128	60	.222

better than those which use all available space to squeeze relevant and irrelevant data on a single sheet.

The primary discussion here will be directed towards the slope correction. Corrections to sea level are only briefly mentioned since a fuller discussion will be found in the section on Computations and Adjustment.

The slope corrections may be determined from vertical angle observations or from differences of elevations. For the more precise surveys, differences of elevations are preferred providing these values reflect the difference of elevation between the actual heights of the instruments and/or reflectors. Note on form 603-5A the heights of the instruments and mirror are added to the elevations of the respective ground points prior to making the reduction computations. To many surveyors whose operations are carried out using tripods, all more or less the same height, this additional refinement may seem unnecessary, and generally this is true; however in some instances rather small differences in the heights of the tripods can significantly effect the reduced distances. For examples: a difference of one foot in the tripod heights on a 100-foot line differing in elevation by 10 feet, will produce an error in the reduced length by 0.1 ft. or 1:1,000. Similarly, an error of 2 feet in the heights where the elevation difference is 200 feet on a line, 5000 feet in length will introduce an error of 0.08 ft. in the horizontal distance. The important issue here--why not employ

correct practices regardless of the desired accuracy of a survey? The time required to do so is exceedingly small.

Whenever the cosines of the vertical angles between points are used to obtain horizontal distances, two important considerations must always be kept in mind.

One, are there differences in the tripod heights as discussed above? Two, have the observations been corrected for curvature and refraction? This correction is negligible for short lines or/has no effect where small differences in elevations occur, but can be very significant in some instances as will be shown in the following examples. These examples cover most of the situations encountered in reducing slope distances to the horizontal.

(1) Elevations: A = 280.6 ft. B = 518.4 ft. Slope distance A to B = 7456.35 ft. Difference of elevation A-B = 227.8 ft.

The exact formula is $S = \sqrt{D^2 - (h)^2}$, where "S" is the horizontal distance; "D" the slope distance; and "h" the difference in elevation. Therefore $S = \sqrt{(7456.35)^2 - (227.8)^2}$ = 7453.168 ft.

(2) Vertical angles observed. The theoretical observation at A to B = + 1° 19' 54".4 and the cosine is 0.99957774 with the resulting reduced distance being 7453.201 ft. But this differs from the exact reduction by 0.033 ft. and is due to ignoring the correction to

the observed vertical angles for curvature and refraction. This correction K (in seconds) = 0.004231 x D when "D" is in feet or 0.01388 x D when D is in meters. Since the constant includes, inaddition to the correction for curvature, the coefficient of refraction which varies according to meteorological conditions, the formula used is approximate (an average value of 0.071 for the coefficient of refraction was used);

but it is sufficiently accurate for most purposes. The correction is added to angles of increasing elevation and subtracted from depression angles. In this case K" = 7456 x 0.004231 = 31".5, and the corrected vertical angle A to B = + 1° 39' 54".4 + 31".5 or + 1° 40' 25".9. The cosine of this angle = 0.99957329 and multiplying this value by the slope distance the horizontal distance is 7453.168, which checks the exact reduction.

The computation of "K" is not necessary if reciprocal vertical angles are observed since the mean of the observations A-B and B-A eliminates the correction for curvature and refraction. But as a check on the observations, the value should be computed. The difference between the vertical angles is 2K $\pm$ N where N should seldom exceed 20" if the observations are made on the same day and at approximately the same time. The theoretical observation B to A =-1° 40' 57".4 and the mean observed value is 1° 40' 25".9 checking the corrected observations A-B and B-A.

(3) Occasionally the vertical angle observations are made at different heights of instruments (t) and signals (o) than the length measurements. These varying heights present no problem if elevations for the monuments are first determined (see publication G-56 in list of references) but the observations must be corrected if the cosine formula is to be used.

Let us assume that at A the theodolite (t) was 5-feet higher than the distancing measuring equipment and at B the signal (o) was 15 feet higher. The formula to correct the observations is C (in seconds) = $-\frac{(t-o) \times 206265}{D}$ or in this case C = $-\frac{(-10) \times 206265}{7456}$ = +276".6.

The theoretical observed vertical angle A-B in this case is + 1° 44' 30".9 and the corrected angle is:

	+ 1°	44'	30".9
*C = -		04	36.6
K = +			31.5
	+ 1°	40'	25".8

which checks the corrected angle in (2) above by 0".1.

* Applied with opposite sign for angles of increasing elevation.

In the most accurate surveys the sea level correction (form 603-5A) should be made using a value of "P" determined for the mean latitude of the points involved and the mean azimuth (geodetic) between these points. Tables

are available for "P", however, for the average length of line involved in most local surveys, the sea level reduction described in the section on Computations and Adjustment is sufficiently accurate. But in those cases where the lines are long in areas of higher elevation, the more precise computation should be considered.

Grid Azimuths

Grid azimuths may be determined directly from the plane coordinates of the points involved or through the reduction of geodetic azimuths. The formulae follow:

(1) Using plane coordinates:

$$\text{Tan } \alpha_{A-B} = \frac{X_A - X_B}{Y_A - Y_B} \quad \text{or} \quad \text{Cot } \alpha_{A-B} = \frac{Y_A - Y_B}{X_A - X_B}$$

where A and B are the points involved and "α" is the grid azimuth of the line. The signs of $X_A - X_B$ and $Y_A - Y_B$ correspond to the quadrant in which the azimuth appears with an azimuth of 0° 00' 00" due south, 90° 00' 00" due west, 180° 00' 00" due north, and 270° 00' 00" due east. Geodetic azimuths in the United States are referenced clockwise from the south with the quadrants as given above, however, many surveyors perfer to orient surveys from the north and this is generally acceptable, but in the following discussions the azimuths follow the geodetic practice.

It is generally considered best when using the formulae given above, because of the number of significant figures involved, to utilize the value for Tan α or Cot α which is less than one. When the azimuth is in the first and third quadrant, the angle corresponding to the tangent or cotangent should be determined and added to 0° or 180°, respectively. Azimuths in the second and fourth quadrants are obtained by determining the angle for the cofunction and adding this value to 90° or 270°, respectively. The cofunction of the tangent is the cotangent and vice versa.

(2) When geodetic azimuths are available, the corresponding grid azimuth = Geod. Az_{A-B} - $\Delta\alpha$ (or θ) + second term correction. $\Delta\alpha$ is the mapping angle for the transverse Mercator system and θ for the Lambert system. The second term corrections are described in the section on Computations and Adjustment.

$\Delta\alpha'' = \text{Sin } \Phi \ (CM - \lambda)'' + g$ where Φ and λ are the latitude and longitude respectively of the point, CM is the longitude of the central meridian for a zone and values for "g" are given in a tabular form in the projection tables. The "g" term may be ignored when $(CM - \lambda)''$ is less than 2500". $\Delta\alpha''$ may also be computed using the formula $\Delta\alpha'' = MX' - e$, where M is a function of the Y coordinate of a point. Tables for M and e are given in the projection tables. As an example, the table for Arizona is given on p. 118.

The θ angle may be computed using the longitude for a point and interpolating from the table given with the projection tables. As an example, a partial table for Utah south zone corresponding to the longitude range for the numerical example that follows is given on p. 119. The θ angle may also be computed by the formula $\text{Tan } \theta = \frac{X'}{R_b - Y}$, where R_b is a constant for the zone involved.

Numerical examples for the transverse Mercator and Lambert systems follow:

				Arizona Central Zone
(1)	A	ϕ = 33° 19' 11".1287		X = 482,449.72
		λ = 111 58 26.8321		Y = 843,845.64
	B	ϕ = 33 15 56.1137		X = 506,105.19
		λ = 111 53 48.0940		Y = 824,132.48

$X_A - X_B = -23,655.47$ $\quad$ $Y_A - Y_B = +19,713.16$

$\text{Cot } \alpha_{A-B} = \frac{+19,713.16}{-23,655.47} = 0.83334468$. Azimuth is in fourth quadrant, interpolate for tan 0.83334468 = 39° 48' 21".5 and add to 270° or Grid Azimuth A-B = 309° 48' 21".5.

When geodetic azimuths are available, the grid azimuths may be derived as follows: Using the formula $\Delta\alpha = \text{Sin } \phi \text{ (CM-}\lambda\text{)}''$

$\Delta\alpha$ at A = 0.549311 (111° 55' 00".0000 - 111° 58' 26".8321)" = -113".6

$\Delta\alpha$ at B = 0.548521 (111° 55' 00".0000 - 111° 53' 48".0940)" = + 39.4

Geod Az A-B	309°	46'	27".9	Geod Az B-A	129°	49'	00".9
- $\Delta\alpha$ at A	+ 0	01	53.6	- $\Delta\alpha$ at B	-		39.4
Grid Az A-B	309	48	21.5	Grid Az B-A	129	48	21.5

The second term correction is less than 0".1 and was neglected in this example.

Utah South Zone

(2) A Φ = 38° 21' 47.76185 X = 2,235,545.34
λ = 110 40 42.75141 Y = 618,804.51

B Φ = 38 16 43.30415 X = 2,259,464.19
λ = 110 35 46.18097 Y = 588,225.29

$X_A - X_B = -23,918.85$ $Y_A - Y_B = +30,579.22$

Tan $\alpha_{A-B} = \frac{-23,918.85}{+30,579.22} = 0.78219294$. Azimuth is in fourth quadrant, interpolate for cot 0.78219294 = 51° 58' 03.8 and add to 270° or Grid Azimuth A-B = 321° 58' 03.8.

When geodetic azimuths are available, the grid azimuths may be derived as follows:

θ at A (from tables) + 0° 30' 11.9

θ at B (from tables) + 0 33 13.6

For those who wish to compute the θ angles using the coordinates, R_b = 27,432,812.88 for Utah south zone.

Geod Az A-B	322°	28'	14.6		Geod Az B-A	142°	31'	18.5
- θ at A	- 0	30	11.9		- θ at B	- 0	33	13.6
*	+		01.1		*	-		01.1
Grid Az A-B	321	58	03.8		Grid Az B-A	141	58	03.8

* Second term corrections. See explanation with Computation and Adjustment Section. Utah south zone Y_o = 406,857.53

TRANSVERSE MERCATOR PROJECTION

TABLE FOR g

$$\Delta\alpha'' = \sin\phi\,(\Delta\lambda'') + g$$

Latitude	$\Delta\lambda''$						
	0"	1000"	2000"	3000"	4000"	5000"	6000"
24°	0".00	0".00	0".02	0".07	0".17	0".33	0".58
25	0	0	0.02	0.07	0.17	0.34	0.59
26°	0.00	0.00	0.02	0.08	0.18	0.35	0.60
27	0	0	0.02	0.08	0.18	0.35	0.61
28	0	0	0.02	0.08	0.18	0.36	0.62
29	0	0	0.02	0.08	0.19	0.37	0.63
30	0	0	0.02	0.08	0.19	0.37	0.64
31°	0.00	0.00	0.02	0.08	0.19	0.37	0.64
32	0	0	0.02	0.08	0.19	0.38	0.65
33	0	0	0.02	0.08	0.19	0.38	0.65
34	0	0	0.02	0.08	0.19	0.38	0.65
35	0	0	0.02	0.08	0.19	0.38	0.65
36°	0.00	0.00	0.02	0.08	0.19	0.38	0.65
37	0	0	0.02	0.08	0.19	0.38	0.65
38	0	0	0.02	0.08	0.19	0.38	0.65
39	0	0	0.02	0.08	0.19	0.37	0.64
40	0	0	0.02	0.08	0.19	0.37	0.64
41°	0.00	0.00	0.02	0.08	0.19	0.37	0.63
42	0	0	0.02	0.08	0.18	0.36	0.63
43	0	0	0.02	0.08	0.18	0.36	0.62
44	0	0	0.02	0.08	0.18	0.35	0.61
45	0	0	0.02	0.08	0.18	0.35	0.60
46°	0.00	0.00	0.02	0.07	0.17	0.34	0.59
47	0	0	0.02	0.07	0.17	0.33	0.58
48	0	0	0.02	0.07	0.17	0.33	0.56
49	0	0	0.02	0.07	0.16	0.32	0.55
50	0.00	0.00	0.02	0.07	0.16	0.31	0.54

$$g = \left[\frac{C\,(\sin 1'')\,\cos^3\phi}{2A^2} + F\right](\Delta\lambda'')^3$$

A, C and F are position factors.

TRANSVERSE MERCATOR PROJECTION

Arizona

$$\Delta\alpha = Mx' - e$$

y	East and central zones		West zone	
	M	ΔM	M	ΔM
0	0.005 9177	644	0.005 9175	644
100,000	0.005 9821	648	0.005 9819	648
200,000	0.006 0469	652	0.006 0467	652
300,000	0.006 1121	656	0.006 1119	655
400,000	0.006 1777	659	0.006 1774	660
500,000	0.006 2436	664	0.006 2434	663
600,000	0.006 3100	667	0.006 3097	668
700,000	0.006 3767	672	0.006 3765	672
800,000	0.006 4439	676	0.006 4437	675
900,000	0.006 5115	680	0.006 5112	680
1,000,000	0.006 5795	684	0.006 5792	685
1,100,000	0.006 6479	689	0.006 6477	689
1,200,000	0.006 7168	693	0.006 7166	693
1,300,000	0.006 7861	698	0.006 7859	698
1,400,000	0.006 8559	703	0.006 8557	702
1,500,000	0.006 9262	707	0.006 9259	707
1,600,000	0.006 9969	712	0.006 9966	712
1,700,000	0.007 0681	717	0.007 0678	717
1,800,000	0.007 1398	722	0.007 1395	722
1,900,000	0.007 2120	727	0.007 2117	727
2,000,000	0.007 2847	732	0.007 2844	732
2,100,000	0.007 3579	737	0.007 3576	737
2,200,000	0.007 4316	742	0.007 4313	742
2,300,000	0.007 5058		0.007 5055	

e

y \ x'	200,000	300,000	400,000	500,000
0	0.0	0.1	0.3	0.6
1,000,000	0.0	0.1	0.4	0.8
2,000,000	0.0	0.2	0.5	1.0

LAMBERT PROJECTION FOR UTAH (SOUTH)

Table II (Cont'd).

1" of long. = 0".61268734 of θ

Long.	θ	Long.	θ	Long.	θ
110° 26'	+0° 39' 12".7194	111° 01'	+0° 17' 46".0760	111° 36'	−0° 03' 40".5674
27	+0 38 35.9582	02	+0 17 09.3147	37	−0 04 17.3287
28	+0 37 59.1969	03	+0 16 32.5535	38	−0 04 54.0899
29	+0 37 22.4357	04	+0 15 55.7923	39	−0 05 30.8512
30	+0 36 45.6744	05	+0 15 19.0310	40	−0 06 07.6124
110° 31'	+0 36 08.9132	111° 06'	+0 14 42.2698	111° 41'	−0 06 44.3737
32	+0 35 32.1520	07	+0 14 05.5085	42	−0 07 21.1349
33	+0 34 55.3907	08	+0 13 28.7473	43	−0 07 57.8961
34	+0 34 18.6295	09	+0 12 51.9861	44	−0 08 34.6574
35	+0 33 41.8682	10	+0 12 15.2248	45	−0 09 11.4186
110° 36'	+0 33 05.1070	111° 11'	+0 11 38.4636	111° 46'	−0 09 48.1799
37	+0 32 28.3458	12	+0 11 01.7023	47	−0 10 24.9411
38	+0 31 51.5845	13	+0 10 24.9411	48	−0 11 01.7023
39	+0 31 14.8233	14	+0 09 48.1799	49	−0 11 38.4636
40	+0 30 38.0620	15	+0 09 11.4186	50	−0 12 15.2248
110° 41'	+0 30 01.3008	111° 16'	+0 08 34.6574	111° 51'	−0 12 51.9861
42	+0 29 24.5395	17	+0 07 57.8961	52	−0 13 28.7473
43	+0 28 47.7783	18	+0 07 21.1349	53	−0 14 05.5085
44	+0 28 11.0171	19	+0 06 44.3737	54	−0 14 42.2698
45	+0 27 34.2558	20	+0 06 07.6124	55	−0 15 19.0310
110° 46'	+0 26 57.4946	111° 21'	+0 05 30.8512	111° 56'	−0 15 55.7923
47	+0 26 20.7334	22	+0 04 54.0899	57	−0 16 32.5535
48	+0 25 43.9721	23	+0 04 17.3287	58	−0 17 09.3147
49	+0 25 07.2109	24	+0 03 40.5674	59	−0 17 46.0760
50	+0 24 30.4496	25	+0 03 03.8062	112° 00	−0 18 22.8372
110° 51'	+0 23 53.6884	111° 26'	+0 02 27.0450	112° 01'	−0 18 59.5985
52	+0 23 16.9271	27	+0 01 50.2837	02	−0 19 36.3597
53	+0 22 40.1659	28	+0 01 13.5225	03	−0 20 13.1210
54	+0 22 03.4047	29	+0 00 36.7612	04	−0 20 49.8822
55	+0 21 26.6434	30	0 00 00.0000	05	−0 21 26.6434
110° 56'	+0 20 49.8822	111° 31'	−0 00 36.7612	112° 06'	−0 22 03.4047
57	+0 20 13.1210	32	−0 01 13.5225	07	−0 22 40.1659
58	+0 19 36.3597	33	−0 01 50.2837	08	−0 23 16.9271
59	+0 18 59.5985	34	−0 02 27.0450	09	−0 23 53.6884
111° 00	+0 18 22.8372	35	−0 03 03.8062	10	−0 24 30.4496

COMPUTATIONS AND ADJUSTMENTS

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ERRATA

Lecture Notes for Use At Workshops on Surveying Instrumentation and Coordinate Computation

page 110 - (2) Fourth line from bottom, + 1° 19' 54".4 should be +1° 39 54".4.

page 163 - Second sentence should read, "When employing condition equations involving a mixture of angle and distance measurements, the formation of the equations is probably simpler when the weights are expressed in the same measure."

page 192 - Length A = 0.9154 ft., not 0.29 ft.; p = 1.1933, not 1.1891.

The changes in "A" and subsequently to "p" were due to a mistake in the original computation, and rather than change the value for "kp" which is used in further computations, it was necessary to revise the numerical value for "A" in the following paragraph as well as to modify the first statement. In addition, "A" would not generally be given to more than three decimal places as was necessary here.

*"For observation equations, the weights for various observational data, that is directions, lengths, and azimuths, need not be based on precisions expressed in the same measure such as seconds, as was used in the previous example where condition equations were employed. Nor was it necessary in that example, but the coefficients involved with the length measurements would be numerically different. For the measured length (p. 198) used in this survey, the "A" value was determined using 0.5 ft. + 1:90,000 of the length as the criterion."

COMPUTATIONS AND ADJUSTMENTS

Traverse computations, whether made on a local system or using State plane coordinates, are for all intents and purposes identical. It is true that to obtain the best results on the State plane coordinate systems, lengths should be corrected by a scale factor and reduced to sea level, and in some instances second term corrections should be applied to the observed angles prior to computation. These are rather simple corrections to derive and apply. However, the scale and sea level factors, in particular, have long been given as the principal reasons for not employing the State coordinate systems. It is understandable that the distances shown on a plat should be those at the elevation plane of the measurements. These values, however, can easily be obtained after a survey has been computed and adjusted on the State system. As a matter of fact, the scale and sea level factors can often be ignored and ground distances used in computing third order (1:5,000) or lesser accurate surveys in those areas where elevations are 2000 feet or less. In some instances, the effects of the scale and sea level factors may counterbalance one another to a great extent, and distances at elevations of as much as 4000 feet may be used as measured.

In the following description of the computation and adjustment of a simulated traverse survey in southern

Wisconsin, the explanation of each operation is concise. More complete details will be found in publications given in the list of references. A sketch of the traverse is shown by Figure (1). The configuration violates many of the rules for a good network design but this was done purposely in order to emphasize the differences in adjustment techniques. The dotted curved lines will be used to show the relationship between observed and grid angles.

Yo ———————— - ———————— — ———————— -- ——

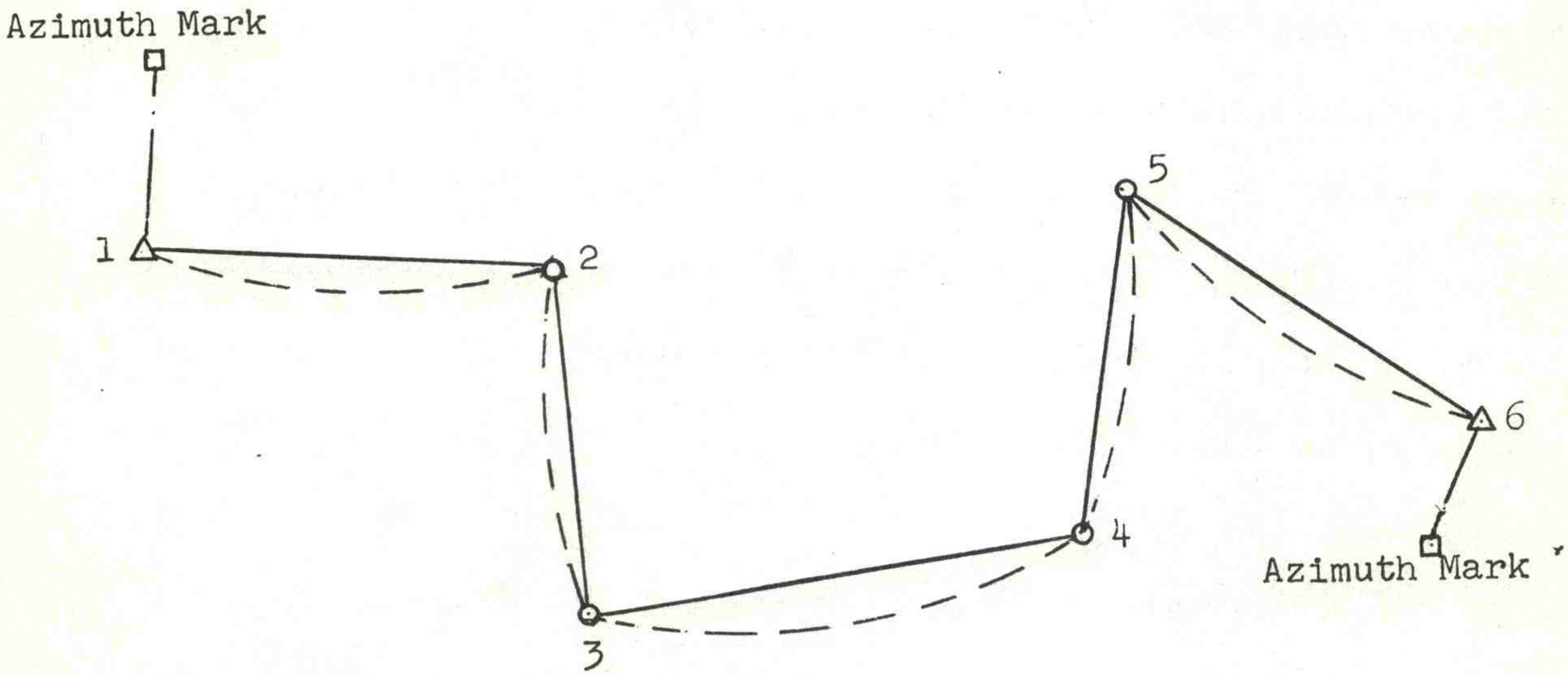


Figure 1

Second Term Corrections: Any observed angle is a geodetic quantity. In those cases where a survey has been made to a high quality and the computations are carried out on a plane, these observations should be corrected by the second term, or T-t correction, in order to reduce them to grid angles. The basic formulae are given in each of the State projection tables. However, the following approximate formula is sufficiently accurate for most purposes: the correction at each end of a line = $2.36\ (\Delta X)(\Delta Y) \times 10^{-10}$. For the transverse Mercator system, ΔX is the difference between the X value on the central meridian (usually 500,000 feet) and the mean X value of the two points involved. ΔY is the difference in the Y coordinates for the two points. For the Lambert projection, ΔX is the difference in the X coordinates of the two points and ΔY is the difference between the y_o value (given with the tables) and the mean Y value for the points involved.

Approximate coordinates are required to make the computations, but these values given to the nearest one thousand feet are adequate. For the Wisconsin south zone, the y_o = 510,702.41. The corrections need to be computed for one end of the line only as the value at the opposite end, using the approximate formula, is identical. No corrections are necessary to the observations to the azimuth marks in this example because the distance from the stations

to these marks are usually quite short and the correction would be zero. If more distant points were used, corrections should, of course, be applied. The computations of the second term corrections for the traverse example are tabulated in Table 1.

From	To	* ΔX	* ΔY	2.36(ΔX)(ΔY)
1	2	0.158	3.095	1".1
2	3	0.008	3.162	0.1
3	4	0.160	3.211	1.2
4	5	0.008	3.148	0.1
5	6	0.114	3.127	0.8

Table 1

* Coordinate values were available to hundreds of feet and were used, although as noted earlier coordinates to the nearest thousand feet are sufficient.

Although the signs of the corrections to the observed angles can be obtained from the computation, or through the azimuth of the lines, it is much easier to determine these signs in the following manner. Visualize the observations (geodetic) as being a curved line concave towards the central meridian for surveys computed on the transverse Mercator projection and towards the y_o (central parallel) for the Lambert system. The dotted curved lines in Figure (1) represent the observed values, and the straight lines the grid quantities. Since the angles were

measured clockwise in the direction of the traverse west to east, it can be seen the geodetic angles are larger than their corresponding grid angles in every case. The correction to each angle, in this example, is simply the sum of the corrections for the two lines involved at each point as follows:

Point	Observed Angle	Corr'n	Grid Angle
1	90° 44' 18".3	-1".1	17".2
2	265 15 55.2	-1.2	54.0
3	82 48 26.9	-1.3	25.6
4	105 03 08.6	-1.3	07.3
5	304 33 46.2	-0.9	45.3
6	245 17 39.5	-0.8	38.7

Table 2

Although taken individually, the above corrections are quite small; they are accumulative and would effect the azimuth closure by 6".6. Whenever a long traverse is located in an area which is near the limits of a zone, the second term corrections can, on occasion, be rather large, and it is always prudent to compute the correction between the terminal points. The accumulated corrections in a traverse between these points cannot be less than twice the correction for one end. For example, the second-term correction between points 1 and 6 is 3".3. The value

at the opposite end of the line is the same and the total correction is therefore 6".6, which is equal, in this case, to the sum of the second-term corrections at all the points in the traverse.

Scale and Sea Level Corrections: Any length measurement made on the surface of the earth is a geodetic distance at the mean elevation of the points involved. The plane coordinates which have been determined from the geographic position of points in the national network, with one exception (the Lambert system for Michigan), are based on a sea-level reference. Accordingly, to obtain the best results for surveys made at some other elevation surface, the lengths should be reduced to the datum reference plane. In addition, geodetic lengths should be corrected for the scale differential between the actual length and the same length as represented on a map projection. As mentioned previously, these corrections may be neglected in some instances for lower order surveys, but generally should be applied where accuracies of 1:10,000 or better are anticipated.

For the transverse Mercator projection, the scale is constant along the central meridian and varies as the X values extend east and west from this meridian. The scale for the Lambert system is constant along any parallel and varies in a north-south direction. These values are tabulated in the State projection tables. The scale factors are a function of the X' values for the transverse Mercator

system and the latitude of the points for the Lambert projection.

While in theory each length should be corrected for the mean scale factor between the points, in actual practice, for limited surveys, a mean scale factor computed from the positional data for the terminal control points will usually suffice. For this sample computation the mean latitude is about 42° 32!5, and by interpolating between the scale factors, as given in the Wisconsin south zone tables, for 32' and 33' a value of 1.0000442 was determined. Although each of the measured distances could be multiplied by this factor, with the results being grid distances at the elevation plane of the measurements, it is easier to compute a combined scale and sea level factor and make the reductions in one step. This operation will be explained shortly.

A distance measured between two points at an elevation above sea-level will be longer than the corresponding distance between the same two points should these points be plumbed to the sea level surface. This may be easily seen by referring to Figure (2) below. The following formula, while an approximation, may be used to determine the difference between ground level and sea level lengths for most surveys: $\frac{Sxh}{20,906,000}$, where "S" is the measured distance, "h" is the mean elevation of the line, and 20,906,000 is

LAMBERT PROJECTION FOR WISCONSIN (SOUTH)

Table I.

Lat.	R (feet)	y' y value on central meridian (feet)	Tabular difference for 1 sec. of lat. (feet)	Scale in units of 7th place of logs	Scale expressed as a ratio
42° 00'	22,672,134.66	0	101.24717	+990.9	1.0002282
01	22,666,059.83	6,074.83	101.24700	+960.7	1.0002212
02	22,659,985.01	12,149.65	101.24650	+930.8	1.0002143
03	22,653,910.22	18,224.44	101.24617	+901.2	1.0002075
04	22,647,835.45	24,299.21	101.24567	+872.1	1.0002008
05	22,641,760.71	30,373.95	101.24533	+843.3	1.0001942
42° 06'	22,635,685.99	36,448.67	101.24500	+814.8	1.0001876
07	22,629,611.29	42,523.37	101.24467	+786.7	1.0001811
08	22,623,536.61	48,598.05	101.24433	+759.0	1.0001748
09	22,617,461.95	54,672.71	101.24400	+731.6	1.0001685
10	22,611,387.31	60,747.35	101.24367	+704.6	1.0001622
42° 11'	22,605,312.69	66,821.97	101.24333	+677.9	1.0001561
12	22,599,238.09	72,896.57	101.24300	+651.6	1.0001500
13	22,593,163.51	78,971.15	101.24267	+625.7	1.0001441
14	22,587,088.95	85,045.71	101.24250	+600.1	1.0001382
15	22,581,014.40	91,120.26	101.24217	+574.9	1.0001324
42° 16'	22,574,939.87	97,194.79	101.24183	+550.0	1.0001266
17	22,568,865.36	103,269.30	101.24167	+525.5	1.0001210
18	22,562,790.86	109,343.80	101.24133	+501.4	1.0001155
19	22,556,716.38	115,418.28	101.24100	+477.6	1.0001100
20	22,550,641.92	121,492.74	101.24083	+454.2	1.0001046
42° 21'	22,544,567.47	127,567.19	101.24067	+431.1	1.0000993
22	22,538,493.03	133,641.63	101.24033	+408.4	1.0000940
23	22,532,418.61	139,716.05	101.24017	+386.0	1.0000889
24	22,526,344.20	145,790.46	101.24000	+364.0	1.0000838
25	22,520,269.80	151,864.86	101.23983	+342.4	1.0000788
42° 26'	22,514,195.41	157,939.25	101.23950	+321.1	1.0000739
27	22,508,121.04	164,013.62	101.23933	+300.2	1.0000691
28	22,502,046.68	170,087.98	101.23917	+279.7	1.0000644
29	22,495,972.33	176,162.33	101.23917	+259.5	1.0000598
30	22,489,897.98	182,236.68	101.23883	+239.7	1.0000552
42° 31'	22,483,823.65	188,311.01	101.23867	+220.2	1.0000507
32	22,477,749.33	194,385.33	101.23850	+201.1	1.0000463
33	22,471,675.02	200,459.64	101.23850	+182.3	1.0000420
34	22,465,600.71	206,533.95	101.23833	+164.0	1.0000378
35	22,459,526.41	212,608.25	101.23817	+145.9	1.0000336

the mean radius (R) of the Earth for the United States. All values are in feet. In the more exact formula, "S" would be the sea level distance and "R" would be computed for the mean latitude and azimuth of each line.

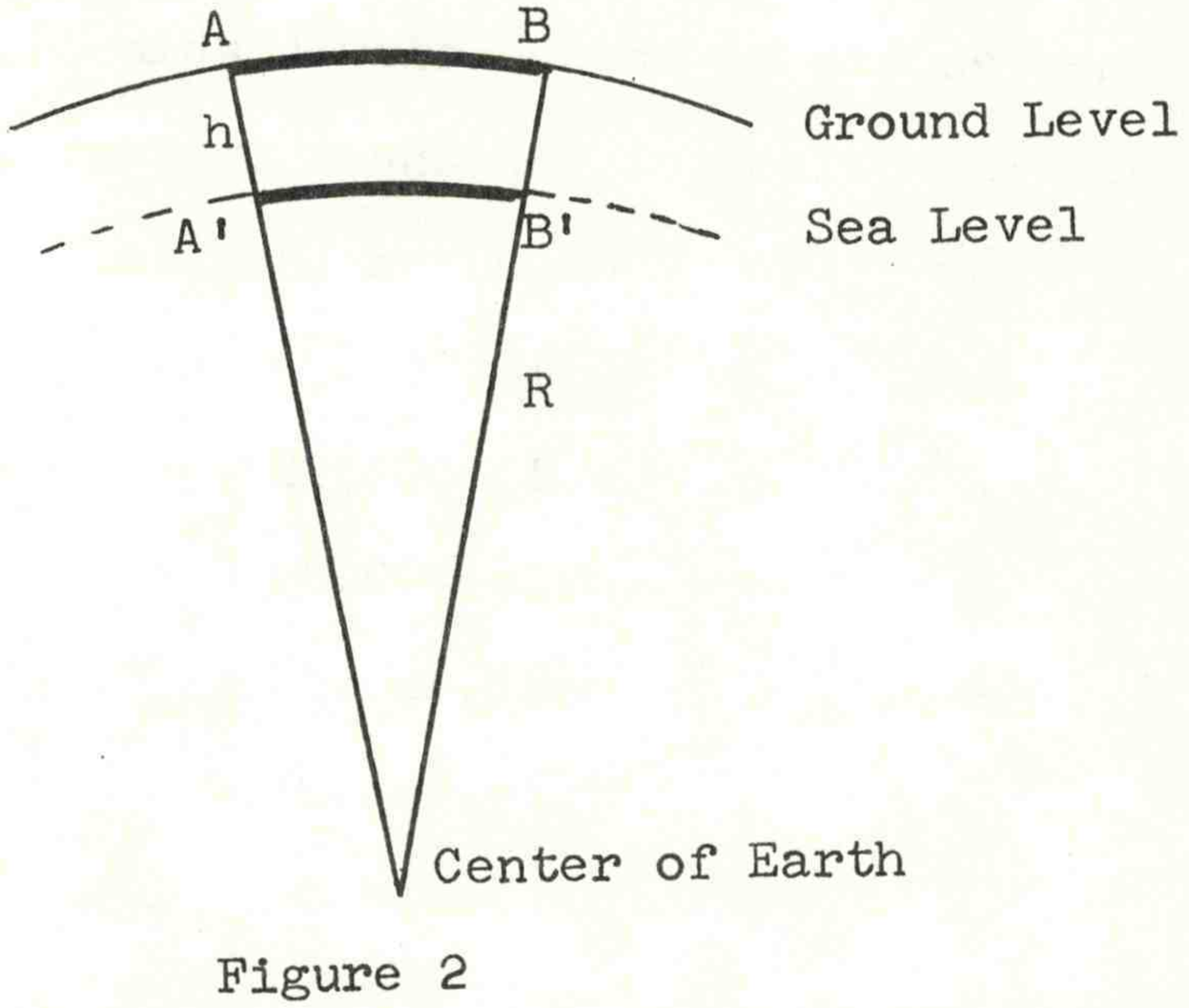


Figure 2

The sea level reduction formula may be put in the more convenient factor form, $1. - \frac{h}{20,906,000}$. Generally a mean elevation for an entire traverse can usually be used without any great loss in accuracy. For example, if the maximum deviation from a mean elevation is 418 feet, all distances would be correct within 1:50,000 in as far as the sea level correction is concerned.

In the traverse serving as the example, the average elevation was assumed to be 750 feet above sea level. Using the reduction formula given last, the sea level factor was computed as follows, $1. - \frac{750}{20,906,000} = 0.9999641$.

This factor and the scale factor, computed previously, were combined by multiplication (1.0000442 x 0.9999641) and the resulting combined factor = 1.0000083 was obtained. Each of the measured lengths was multiplied by this factor, with the results as shown below (table 3). The measured lengths are horizontal distances in feet.

From	To	Measured Lengths	Grid Lengths
1	2	15,765.94	15,766.07
2	3	13,004.22	13,004.33
3	4	16,292.89	16,293.03
4	5	11,486.93	11,487.03
5	6	14,655.27	14,655.39

Table 3

Computation of Grid Azimuths: As shown in table 4, the preliminary grid azimuths are computed using the fixed grid azimuth between Station 1 and its Azimuth Mark and the grid angles. The closure is obtained by subtracting the preliminary grid azimuth Station 6 to Azimuth Mark from the fixed value. The closure is divided by the number of angles (-10".8/6 = -1".8), and each grid angle is corrected by this amount. A recomputation is then made to obtain azimuths corrected for closure.

STATION		PRELIMINARY AZIMUTH			CORRECTION FOR CLOSURE		CORRECTED AZIMUTH		
FROM	TO	°	′	″	′	″	°	′	″
1	Azimuth Mk.	180	20	31.2*		*	180	20	31.2
∠		90	44	17.2		-1.8	90	44	15.4
1	2	271	04	48.4			271	04	46.6
2	1	91	04	48.4			91	04	46.6
∠		265	15	54.0		-1.8	265	15	52.2
2	3	356	20	42.4			356	20	38.8
3	2	176	20	42.4			176	20	38.8
∠		82	48	25.6		-1.8	82	48	23.8
3	4	259	09	08.0			259	09	02.6
4	3	79	09	08.0			79	09	02.6
∠		105	03	07.3		-1.8	105	03	05.5
4	5	184	12	15.3			184	12	08.1
5	4	4	12	15.3			4	12	08.1
∠		304	33	45.3		-1.8	304	33	43.5
5	6	308	46	00.6			308	45	51.6
6	5	128	46	00.6			128	45	51.6
∠		245	17	38.7		-1.8	245	17	36.9
6	Azimuth Mk.	14	03	39.3		*	14	03	28.5
		Closure = -10.8							
∠	* Fixed Grid Azimuths								
		Table 4							

On occasion where lines significantly shorter than the average are involved, an unequal distribution of the azimuth closure might be considered. The angles which include the short lines would receive larger corrections than those where the distances are more nearly equal. Depending upon various circumstances, the corrections to the angles involving the short lines could be larger by perhaps as much as a factor of ten. Some caution must be exercised in making

this decision, however, since a continuous tilt will be introduced in the traverse unless counterbalanced elsewhere.

Azimuths rather than bearings are preferred because angles can be added or subtracted without considering the quadrant involved. It should also be noted that azimuths determined in the national network are referenced in a clockwise manner with 0° 00' 00" being due south.

The azimuths corrected for closure will be used to compute the plane coordinates of the traverse points which will be adjusted by the Compass and Transit Rules. In the Least Squares adjustments, the coordinates are computed using the uncorrected preliminary azimuths and an azimuth equation is included in the simultaneous solution with the position equations.

Adjustment Methods: Traverses may be adjusted by the Compass and Transit Rules and by the Method of Least Squares. All other methods are either a combination or modified versions of these procedures. More often than not, these other methods are employed to obtain some desired result. The sample problem will be adjusted by the three methods noted above and a comparison of the results made. The angle observations and distance measurements were given equal weight in the adjustments. A weighted least squares computation is not difficult to process, but the application of weights to Compass

and Transit Rule adjustments, while feasible, might produce rather arbitrary results because of the independent manner in which the closures are adjusted.

The Compass Rule adjustment is shown in Table 5. This method is usually employed when the angles and distances have been considered to be of equal accuracy. Unfortunately the adjusted results do not always justify this consideration. Such is the case in this particular traverse, as may be seen in the comparison of the results (Table 13).

It is assumed that all surveyors are familiar with the computations shown in Table 5 and no detailed explanation will be given. The closure in X is +3.43 ft., and in Y -3.31 ft., or a total closure of 4.77 ft. or 1:14,900. The lengths are the basis for the adjustment and the two factors determined are as follows: "X" correction = +3.43/71205.85 = +0.4817 per 10,000 ft., and "Y" correction = -3.31/71025.85 = -0.4648 per 10,000 ft. The corrections to the coordinates are accumulated values. For example, the correction to the preliminary "X" coordinate for point 4 = +0.4817 (1.5766 + 1.3004 + 1.6293) = +2.17 ft.

The Transit Rule adjustment method is often suggested for use in those cases where the angles are thought to be more accurate than the distances. It is contended that this method will produce smaller corrections to the angles. This contention is, of course, not completely true in all cases

FORM 738
(8-28-58)

U.S. DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY

* Preliminary Coordinates
** Accumulative Corrections for Closure
*** Adjusted Coordinates

COMPASS RULE

TRAVERSE COMPUTATION BY LATITUDES AND DEPARTURES

STATION	PLANE AZIMUTH	GRID DISTANCE	SIN AZIMUTH / COS AZIMUTH	DEPARTURE	LATITUDE	GRID COORDINATES	
						x	y
1	° ' "	Feet		Feet	Feet Fixed	Feet 2,197,895.36	Feet 201,334.92
			0.99982248	+ 15763.27			
	271 04 46.6	15766.07			*	2,213,658.63	201,037.86
			0.01884165		- 297.06	** + 0.76	- 0.73
2					***	2,213,659.39	201,037.13
L34			0.06376401	+ 829.21			
	356 20 38.8	13004.33				2,214,487.84	188,059.99
			0.99796501		-12977.87	+ 1.39	- 1.34
3						2,214,489.23	188,058.65
			0.98212573	+16001.80			
	259 09 02.6	16293.03				2,230,489.64	191,126.76
			0.18822607		+ 3066.77	+ 2.17	- 2.09
4						2,230,491.81	191,124.67
			0.07327736	+ 841.74			
	184 12 08.1	11487.03				2,231,331.38	202,582.91
			0.99731160		+11456.15	+ 2.72	- 2.63
5						2,231,334.10	202,580.28
			0.77972788	+11427.22			
	308 45 51.6	14655.39				2,242,758.60	193,406.90
			0.62611855		- 9176.01	+ 3.43	- 3.31
6					Fixed	2,242,762.03	193,403.59

since the angles will usually be distorted should a large closure exist perpendicular to the general direction of the survey. In this particular example, however, the premise held, even though there is a rather large closure in "Y" in a line running essentially west to east. But the smaller corrections to the angles are more a result of the geometry of the traverse, with two long lines perpendicular to the direction of the survey, then anything else.

The preliminary computations required to make the Transit Rule adjustment are identical to those for the Compass Rule adjustment, and the complete computation is shown in Table 6. For the Transit Rule adjustment, the Departures and Latitudes are used for distributing the closures. The Departures are summed without regard to sign, and the "X" correction factor is determined by dividing the "X" closure by this sum. Similarly the "Y" correction factor is found from the sum of the Latitudes and "Y" closure. The following factors were obtained: "X" correction = +3.43/44863.24 = +0.7645 per 10,000 ft., and "Y" correction = -3.31/36973.86 = -0.8952 per 10,000 ft. As was the case with the Compass Rule adjustment, the corrections shown are accumulated values. For example, the correction to the preliminary "Y" coordinate for point 4 = -0.8952 (0.0297 + 1.2978 + 0.3067) = -1.46 ft.

FORM 738
(3-28-58)

U.S. DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY

* Preliminary Coordinates
** Accumulated Corrections for Closure
*** Adjusted Coordinates

TRANSIT RULE

TRAVERSE COMPUTATION BY LATITUDES AND DEPARTURES

STATION	PLANE AZIMUTH	GRID DISTANCE	SIN AZIMUTH / COS AZIMUTH	DEPARTURE	LATITUDE	GRID COORDINATES	
						x	y
1	° ' "	Feet		Feet	Feet Fixed	Feet 2,197,895.36	Feet 201,334.92
			0.99982248	+ 15763.27			
	271 04 46.6	15766.07			*	2,213,658.63	201,037.86
			0.01884165		- 297.06	** + 1.21	- 0.03
2					***	2,213,659.84	201,037.83
			0.06376401	+ 829.21			
1-6	356 20 38.8	13004.33				2,214,487.84	188,059.99
			0.99796501		-12977.87	+ 1.27	- 1.19
3						2,214,489.11	188,058.80
			0.98212573	+ 16001.80			
	259 09 02.6	16293.03				2,230,489.64	191,126.76
			0.18822607		+ 3066.77	+ 2.49	- 1.46
4						2,230,492.13	191,125.30
			0.07327736	+ 841.74			
	184 12 08.1	11487.03				2,231,331.38	202,582.91
			0.99731160		+11456.15	+ 2.56	- 2.49
5						2,231,333.94	202,580.42
			0.77972788	+ 11427.22			
	308 45 51.6	14655.39				2,242,758.60	193,406.90
			0.62611855		- 9176.01	+ 3.43	-3.31
6					Fixed	2,242,762.03	193,403.59

Table 6

USCOMM-DC 5270

The Compass and Transit Rules are the most commonly used techniques for balancing traverses. In the past, when very few local surveys were accomplished to even second-order accuracy, their use was acceptable and justified. But to apply these methods to the adjustment of higher grade surveys, as is done daily, makes little sense in this age of electronic computers. The best method, Least Squares, for adjusting any survey data has long been available but seldom used except by geodetic organizations. There is nothing mystic about Least Squares, nor is great mathematical knowledge required to carry out the computations. Once the technique is learned, the additional time required to adjust a simple traverse run between two control points would seldom be more than one-half day if the computations are made on a desk calculator and only a very few minutes if a computer is available. The adjustment method which will be described in the following pages was developed by Buford K. Meade, Chief, Triangulation Branch, National Ocean Survey (Coast and Geodetic Survey), and was presented in a paper entitled, "The Practical Use of the Oregon State Plane Coordinate System". This paper was given at the Surveying and Mapping Conference, Oregon State University, Corvallis, Oregon, March 1964. Copies are available from the author.

There are two Least Squares methods which may be used to adjust traverse and the results will be identical regardless of whether the computations are carried out on the reference spheroid; or, when all corrections are applied, on the plane. One method employs condition equations and the other observation equations.

Three condition equations are required for a simple traverse run between two control points. Any number of new points may be included without increasing the number of equations to be solved. For each additional control point used, the number of equations is increased by three if each control point includes an azimuth tie and two if no azimuth tie is made.

The observation equation method requires a minimum of two normal equations to be solved for each new point. Obviously this method would not be recommended because in a simple traverse, as noted above, containing 20 new points, 40 equations would need to be solved while only 3 condition equations would be required to obtain the same results. In addition, the computation of the observation equations take a considerably greater effort than the formation of the condition equations. In the example here, only the condition method on a plane will be described.

The preliminary computations are identical to those made for the Compass and Transit Rule adjustment except

FORM 738
(8-28-58)

U.S. DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY

LEAST SQUARES
PRELIMINARY COMPUTATION

TRAVERSE COMPUTATION BY LATITUDES AND DEPARTURES

STATION	PLANE AZIMUTH	GRID DISTANCE	SIN AZIMUTH / COS AZIMUTH	DEPARTURE	LATITUDE	GRID COORDINATES	
						x	y
1	° ' "	Feet		Feet	Feet	2,197,895.36 Feet	201,334.92 Feet
			0.99982231	+15763.269			
	271 04 48.4	15766.07					
			0.01885038		- 297.196		
2							
			0.06374659	+ 828.982			
	356 20 42.4	13004.33					
			0.99796611		-12977.881		
3							
			0.98213066	+16001.884			
	259 09 08.0	16293.03					
			0.18820036		+ 3066.354		
4							
			0.07331217	+ 842.139			
	184 12 15.3	11487.03					
			0.99730904		+11456.119		
5							
			0.77970056	+11426.816			
	308 46 00.6	14655.39				2,242,758.450	193,405.806
			0.62615257		- 9176.510	+ 3.580	- 2.216
6						2,242,762.03	193,403.59

USCOMM-DC 5270

Table 7

that the azimuth closure is not distributed and the preliminary azimuths uncorrected for the azimuth closure (Table 4) are used with the grid lengths to obtain the preliminary Departures and Latitudes. In this computation it is not necessary to tabulated the preliminary coordinate for the new stations. These computations are shown in Table 7. The closures in X and Y are slightly different than those previously obtained because the uncorrected grid azimuths are used.

The three basic conditions are for the closures in azimuth, X and Y are as follows:

(1) $0 = (A_n - A_f) + \Sigma[(V_a)]$

(2) $0 = +206265\ (X_n - X_f) + \Sigma\ \overline{(Y_n - Y_c)\overline{V_a}} + \Sigma\ \overline{(X_k - X_i)\overline{V_s}}$

(3) $0 = +206265\ (Y_n - Y_f) - \Sigma\ \overline{(X_n - X_c)\overline{V_a}} + \Sigma\ \overline{(Y_k - Y_i)\overline{V_s}}$

where, V_a and V_s represent the corrections to be applied to the angles and distances,

A_n and A_f are the computed and fixed azimuths at the terminal point,

i and k represent the initial and computed coordinates for consecutive points,

X_n and Y_n are the computed coordinates of the terminal point,

X_f and Y_f are the fixed coordinates of the terminal point,

X_c and Y_c are the computed coordinates.

The azimuth equation for the example is as follows:

(1) $0 = +10.8 \quad +1.00\,(1) \quad +1.00\,(2) \quad +1.00\,(3) \quad +1.00\,(4)$
$+1.00\,(5) \quad +1.00\,(6)$

The first item in the equation is the $(A_n - A_f)$ value and is usually referred to as the constant term. This value was obtained from the data appearing on Table 4 and was computed thusly:

$(A_n = 14^\circ\ 03'\ 39''.3) - (A_f = 14^\circ\ 03'\ 28''.5) = +10''.8$.

The angle corrections or unknowns which will be determined from the solution of the equations are identified by the numbers in parenthesis. These numbers correspond to those assigned the traverse points (Figure 1). The coefficients for the angle corrections in the azimuth equations are always assigned the value 1.00.

The formation of the equations for the closures in X and Y follows:

(2) $0 = +206265\,(X_n - X_f) + \boxed{\overline{(Y_n - Y_1)(1) + (Y_n - Y_2)(2) +}}$
$(Y_n - Y_3)(3) + (Y_n - Y_4)(4) + \boxed{(Y_n - Y_5)(5)}$
$+ \boxed{\overline{(X_2 - X_1)(1\text{-}2)} + (X_3 - X_2)(2\text{-}3) +}$
$(X_4 - X_3)(3\text{-}4) + (X_5 - X_4)(4\text{-}5) + \boxed{(X_n - X_5)(5\text{-}6)}$

In computing the coefficients involving the Y coordinates, the computed Latitude values may be substituted for the coordinates. $(Y_n - Y_1)$ is the algebraic sum of the Latitudes; $(Y_n - Y_2)$ = the algebraic sum of the Latitudes $(Y_n - Y_1)$ minus the Latitude $_{1\text{-}2}$. The other coefficients for the angle corrections maybe obtained in a similar manner. The coefficients for the

corrections to the lengths are the computed Departures between the points indicated. The constant or closure term 206265 $(X_n - X_f)$ = 206265X (-3.580). It is not necessary to carry the elements involved in the computation of the coefficients to more than values in feet. The complete X correction equation for the example follows:

(2) 0 = -738428.700 -7929 (1) -7632 (2) +5346 (3)
+2280 (4) -9177 (5) +15763 (1-2)
+829 (2-3) +16002 (3-4) +842 (4-5)
+11427 (5-6)

In order to reduce the size of the numbers used in a computation on a desk calculator, the equation should be divided by 2000.

$$(3)\quad 0 = +206265\,(Y_n - Y_f) - \boxed{(X_n - X_1)(1) + (X_n - X_2)(2) + (X_n - X_3)(3) + (X_n - X_4)(4) + (X_n - X_5)(5)} + \boxed{(Y_2 - Y_1)(1\text{-}2) + (Y_3 - Y_2)(2\text{-}3) + (Y_4 - Y_3)(3\text{-}4) + (Y_5 - Y_4)(4\text{-}5) + (Y_n - Y_5)(5\text{-}6)}$$

The coefficients computed using the X coordinates may be determined using the same procedure as described previously for the Y coordinates. Similarly, the length coefficients are the computed Latitudes between the points indicated. The constant or closure term 206265 $(Y_n - Y_f)$ = 206265X (+2.216). The complete Y correction equations for the example follows:

(3) 0 = +457083.240 -44863 (1) -29100 (2) -28271 (3)
-12269 (4) -11427 (5) -297 (1-2)
-12978 (2-3) +3066 (3-4) +11456 (4-5)
-9177 (5-6)

The equation should be divided by 5000 for the same reason as noted previously.

The equations are now tabulated vertically as shown in Table 8. This tabulation is known as the Correlate equations and will be used to form the Normal equations, and in the computation of the corrections to the angles and lengths.

CORRELATE EQUATIONS

Corr'n No.	Equations 1	2	3	Corrections V's	Adopt
1	+1.00	-3.964	-8.973	+ 4".340	+ 4".34
2	+1.00	-3.816	-5.820	- 2.466	- 2.46
3	+1.00	+2.673	-5.654	+ 8.851	+ 8.85
4	+1.00	+1.140	-2.454	- 1.089	- 1.09
5	+1.00	-4.588	-2.285	-11.788	-11.79
6	+1.00			- 8.648	- 8.65
					ds in ft.
1-2		+7.882	-0.059	+14.332	+ 1.095
2-3		+0.414	-2.596	+ 6.570	+ 0.414
3-4		+8.001	+0.613	+13.039	+ 1.030
4-5		+0.421	+2.291	- 4.381	- 0.244
5-6		+5.714	-1.835	+14.411	+ 1.024

Table 8

The Normal Equations are formed using the Correlate Equations in the following manner and tabulated as shown

in Table 9. Step 1 - square each term in equation 1 and tabulate the result.

Step 2 - multiply each term in equation 1 by its corresponding term in equation 2 and tabulate the result.

Step 3 - multiply each term in equation 1 by its corresponding term in equation 3 and tabulate the result. The values in the "η" column are the constant terms of the equations.

Step 4 - square each term in equation 2 and tabulate the result.

Step 5 - multiply each term in equation 2 by its corresponding term in equation 3 and tabulate the result.

Step 6 - square the terms in equation 3 and tabulate the result. All multiplications are made algebraically and summed in each step.

NORMAL EQUATIONS

1	2	3	η
+6	-8.555	-25.186	+10.8
	+218.909783	+44.195122	-369.214350
		+173.332558	+91.416648

Table 9

The Forward Solution is shown in Table 10, and an explanation follows:

Step 1 - arrange the Normal Equations from Table 9 as shown.

Step 2 - divide the terms for equations 2 and 3 and η,

on the first line, by the term on the far left and change the sign: -8.555/+6 =+1.425833; -25.186/+6 =+4.197667; and +10.8/+6 = -1.800000.

In the following steps the actual multiplications will be shown. The same pattern would be followed in all other adjustments.

Step 3 - (-8.555) x (+1.425833) + (+218.909783) = +206.711782.

Step 4 - (-8.555) x (+4.197667)+ (+44.195122) = +8.284081.

Step 5 - (-8.555) x (-1.800000) + (-369.214350) = -353.815350. The resulting quantities are known as the reduced normal terms.

Step 6 - divide the reduced normal terms for equation 3 and n by the reduced normal term for 2 and change the signs: +8.284081/+206.711782 = -0.040076; and -353.815350/+206.711782 = +1.711636.

Step 7 - (-25.186) x (+4.197667) + (+8.284081) x (-0.040076) + (+173.332558) = +67.278124

Step 8 - (-25.186) x (-1.800000) + (+8.284081) x (+1.711636) + (+91.416648) = +150.930779.

Step 9 - divide the reduced normal term for n by the reduced normal term for equation 3 and change the sign: +150.930779/+67.278124 = -2.243386.

The cross multiplications made in Steps 4, 5, and 8 can be made in either direction. For example Step 4 could

be made in the following manner: (+1.425833) x (-25.186) + (+44.195122) = +8.284092. The results are slightly different because of the significant figures involved. These small differences will not effect the final result. As a matter of fact, 4 decimal places are sufficient for the normal and reduced normal terms, and 5 decimal places for the divided quantities. Also, two decimal places will usually be sufficient for the Correlate equations.

FORWARD SOLUTION

1	2	3	n
+6	-8.555	-25.186	+10.8
	+1.425833	+4.197667	-1.800000
	+218.909783	+44.195122	-369.214350
	+206.711782	+8.284081	-353.815350
		- .040076	+1.711636
		+173.332558	+91.416648
		+67.278124	+150.930779
			-2.243386

Table 10

The results of the Back Solution are shown in Table 11. These quantities are identified as "C"'s and were computed as follows:

Step 1 - C_3 = Step 9 of the Forward Solution = -2.243386.

Step 2 - C_2 = (+1.711636) + (-0.040076) x C_3 = +1.801542.

Step 3 - C_1 = (-1.800000) + (+4.197667) x C_3 + (+1.425833) x C_2 = -8.648289.

Generally 4 or 5 decimal places for the C's will be all that is required.

$C_1 = -8.648289$

$C_2 = +1.801542$

$C_3 = -2.243386$

Table 11

The C's are substituted in the Correlate equations (Table 8) as described below, and the corrections to the angles and lengths determined. Each correction is the sum of the multiplications of the C's by their corresponding coefficients. For examples: the angle correction for station 2 = +1.00 x (C_1) + (-3.816) x C_2 + (-5.820) x C_3 = -2".466; the length correction for the distance 3-4 = +8.001 x C_2 + (+0.613) x C_3 = 13".039.

The computed length corrections are in angular measure and must be converted to feet before being applied to the measured grid distances by the following formula:

$$\text{ds in feet} = \frac{V''_s \times (\text{distance in feet})}{206265}$$

. For example, the correction in feet to the length 1-2 = $\frac{+14.332 \times 15766}{206265}$ = +1.095 ft.

The corrections to the angles are adopted to hundredths of seconds to fit the azimuth closure and applied to the angles in the preliminary azimuth computation (Table 4). The final adjusted azimuths are determined using the corrected angles. These values are not shown in Table 4, but will be found in the computation of the adjusted coordinates (Table 12). The corrections to the lengths are applied and are shown also in Table 12.

FORM 738
(3-28-58)

U.S. DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY

LEAST SQUARES
FINAL COMPUTATION

TRAVERSE COMPUTATION BY LATITUDES AND DEPARTURES

STATION	PLANE AZIMUTH	GRID DISTANCE	SIN AZIMUTH / COS AZIMUTH	DEPARTURE	LATITUDE	GRID COORDINATES	
						x	y
1	° ' "	Feet		Feet	Feet	Feet	Feet
					Fixed	2,197,895.36	201,334.92
			0.99982192	+15764.357			
	271 04 52.74	15767.165					
			0.01887142		- 297.549		
2						2,213,659.72	201,037.37
			0.06373749	+ 828.890			
	356 20 44.28	13004.744					
			0.99796670		-12978.301		
3						2,214,488.61	188,059.07
			0.98214044	+16003.055			
	259 09 18.73	16294.060					
			0.18814927		+ 3065.715		
4						2,230,491.66	191,124.78
			0.07335878	+ 842.657			
	184 12 24.94	11486.786					
			0.99730561		+11455.836		
5						2,231,334.32	202,580.62
			0.77970708	+11427.710			
	308 45 58.45	14656.414					
			0.62614444		- 9177.032		
6					Fixed	2,242,762.03	193,403.59

Table 12

USCOMM-DC 5270

The final coordinates are computed using standard practices. Note the corrected lengths and computed Departures and Latitudes are shown to 3 decimal places. In general usage, these computations may be made to 2 decimal places, but the computed coordinates for the terminal point could differ from the fixed values by several hundredths of feet because of round-offs. For the example the check was 0.001 ft. in X and Y.

A comparison of the results is tabulated in Table 13. As might be expected, a review of the total survey corrections (angles and lengths) as expressed by the averages values shows quite conclusively that the results obtained from the Least Squares solution are superior to those resulting from the other methods. In this particular traverse, the smallest angular corrections were obtained from the Transit Rule computation, and the smallest length corrections from the Compass Rule adjustment. The opposite is true when the maximum corrections in angle and length are considered.

There are significant differences in the adjusted plane coordinates with the maximum of 0.70 ft. being between the "Y" coordinates for point 2, as computed by the Compass and Transit Rules. For many mapping projects these differences are not too important but could present problems should these points be used to control other surveys of equal accuracy.

COMPARISONS

Point		Plane Coordinates A-C	Plane Coordinates A-T	Angles A-O	Angles C-O	Angles T-O
1		FIXED	FIXED	+ 4".3	+ 7".6	- 1".7
2	ΔX	+0.33	-0.12	- 2.5	-20.6	- 1.7
	ΔY	+0.24	-0.46			
3	ΔX	-0.62	-0.50	+ 8.9	+18.8	+ 4.3
	ΔY	+0.42	+0.27			
4	ΔX	-0.15	-0.47	- 1.1	- 2.4	- 5.5
	ΔY	+0.11	-0.52			
5	ΔX	+0.22	+0.38	-11.8	-11.2	- 3.1
	ΔY	+0.34	+0.20			
6		FIXED	FIXED	- 8.6	- 3.0	- 3.1
			Average	6.2	10.6	3.2

DISTANCES

From	To	A-O ft.	C-O ft.	T-O ft.
1	- 2	+1.10	+0.77	+1.21
2	- 3	+0.41	+0.65	+1.17
3	- 4	+1.03	+0.62	+1.14
4	- 5	-0.24	-0.50	-1.02
5	- 6	+1.02	+0.98	+1.19
	Average	0.76	0.70	1.15
	*	19,500	20,200	12,300

A = Least Squares
C = Compass Rule
T = Transit Rule
O = Observed or measured
* = Average 1: part in correction

Table 13

In the final analysis, some may not be overly impressed with the concluded superiority of the method of Least Squares. An example could have been set up to make this more positive, but this was not the intent. The real value of the method of Least Squares is better shown in a complicated network and/or where the observed quantities are considered to be of different weight.

ADJUSTMENTS USING WEIGHTS

Weights: With the requirements for higher accuracy surveys ever increasing, more surveyors are employing one-second theodolites and distance measuring equipment. Although these devices, properly used, can produce almost any required accuracy, they will seldom, if ever, produce results of exact equal reliability, and in many instances the variances are of sufficient size to justify the use of equating factors or weights in the adjustment of the data.

As noted previously, no great mathematical ability is required to employ a least squares adjustment, and this condition is unchanged when weights are introduced into the adjustment. Weights may be defined in a broad sense as simply the relative worth of various measurements. Usually these values are obtained from a statistical evaluation of the data, but any good approximation of the relative accuracies of the various survey components is acceptable.

If an analysis is made of the observed data, the Weight $(p) = \frac{1}{(SE)^2}$, where SE is the Standard Error of the mean and is defined by the formula $SE = \sqrt{\frac{\Sigma(v)^2}{n(n-1)}}$. In this formula $\Sigma(v^2)$ is the sum of the squares of the residuals (v's) and "n" is the number of observations. The residuals are the differences between the observed quantities and the mean values. Several examples follow.

(1) Two observations are made with a 5" rejection limit. The maximum SE in this case would be $\pm$ 5" as may be seen by the following computation. Only the seconds of the observations have been tabulated.

		v	v^2
Obs. no. 1	11".3	+5".0	25
Obs. no. 2	21.3	-5.0	25
Mean obs.	16.3		

$$SE = \pm \sqrt{\frac{50}{2(2-1)}} = \pm 5"$$

(2) Assume four observations are made and each observation differs from the mean by exactly 4"; the

$$SE = \pm \sqrt{\frac{64}{4(4-1)}} \text{ or } \pm 2".3$$

(3) A distance is measured in feet on three occasions with the following results:

		v	v^2
Meas. no. 1	3259.87	-0.13	0.0169
Meas. no. 2	3259.65	+0.09	0.0081
Meas. no. 3	3259.70	+0.04	0.0016
Mean	3259.74		

$$SE = \pm \sqrt{\frac{0.0266}{3(3-1)}} = \pm 0.07 \text{ ft.}$$

Some caution must be exercised in accepting such values in all cases as indicating the true relative worth of the observations. For examples: (1) assume a number of traverse angles were observed twice and in no case were there differences in excess of 1" - the maximum SE would be $\pm$ 0.5; (2) the corresponding distances were also measured twice and the differences did not exceed 0.1 ft. - the maximum SE being $\pm$ 0.05 ft. In the first example the SE is most likely unreliable, however, in the second the SE could be considered reasonable, depending, of course, on the length of the line and the equipment used.

In higher quality surveys, it is presupposed that the specifications are such that the resulting SE's are meaningful, however, even in these cases a considered judgement must occasionally be made. As an example, if the lines are short, say about 1000 feet, centering errors of 0.003 ft. in the instrument and signals can produce a maximum error in an angle of about 3", yet the SE's may be the same or even smaller than that obtained over much longer lines. It must also be kept in mind, whenever steep slopes are involved, that relatively small errors in the differences of elevation can produce errors in the reduced distances much larger than that which might be indicated by the SE's. Obviously in these examples, weights determined from the SE's may not be good measures of relative worth.

Tabulated below are average Standard Error values for the number of horizontal observations as indicated and a 5" rejection limit from the mean. Also included are similar values for electronic distancing equipment, but it must be stressed that such results can only be obtained when precise procedures are followed and the atmospheric data reflects the average conditions along the lines. In those cases where these conditions are not rigorously met, SE's 2 to 5 times larger are probably closer to the true value. No estimates are given for the new short range equipment, however, preliminary evaluations give good indication that the advertised accuracy can generally be achieved.

	Distancing Equipment	
Horizontal Observations	Electro-Optical	Micro-wave
4 Positions 1".5	1 cm. (0.03 ft.) + one part per million of the length	1.5 cm. (0.05 ft.) + three parts per million of the length
8 Positions 0.8		
16 Positions 0.5		

In theory, each observation and measurement should be weighted according to its SE; however, in practice an average weight for each observational component of a survey may be used providing the following conditions are met:

(1) All observations and measurements are secured using prescribed specifications. If varying procedures and acceptance limits are employed, individual weights would be in order.

(2) No significant differences in the length of the lines. In general, the accuracy of horizontal angles observed over lines, one mile or longer in length, would be more or less the same. The accuracy would only be impaired slightly at one half mile, but would decline rapidly as the distance shortens beyond this range. A similar situation exists with respect to distances measured electronically, but as the distances shorten, the differences in relative accuracy between angles and lengths increases rapidly. This is mostly due to the fact that distance accuracies are defined as a constant plus a variable such as 0.05 ft. + 1:300,000. The following table gives some indication of these differences.

Distance (feet)	A	1: Part In	1: Part In in seconds	B
100	0.05	2,000	103"	60"
500	0.05	10,000	21	15
1,000	0.05	20,000	10	10
5,000	0.07	70,000	3	5
10,000	0.08	125,000	2	5
20,000	0.12	167,000	1	5

A - Distance accuracy based on 0.05 ft. + 1:300,000 of distance.

B - Estimated accuracy of horizontal angles observed using four positions and a five-second rejection limit.

NOTE: The values given in columns A and B refer to estimated accuracy and not precision as expressed by the Standard Error. The distance accuracies (A) do not refer to any particular equipment and in some instances may be smaller than can be obtained unless precise procedures are employed. The estimated angle accuracies (B), on the other hand, are probably on the high side.

Number of Condition Equations: In a traverse, not involving loops, the number of condition equations is equal to $3(N-1) + M - U$, where N is the number of fixed control points, M is the number of observed azimuths, and U is the number of fixed control points without azimuth control. For complicated traverse networks, the number of equations may be more easily determined by the following formula: $CE = 3(L - S) + M - U$, where L is the number of lengths, S is the number of new points, and M and U are the same as noted above. This formula, of course, is applicable for all traverses. One word of caution, however, to obtain the proper number of equations at least one point

must be fixed or considered fixed. As an example of a point being considered fixed, take a traverse network consisting of one loop involving only four new points, four measured distances, and one observed azimuth CE = 3(4-3) + 1 - 1 = 3. Since the point considered fixed does not have a fixed azimuth, the "U" term of the equation must be included also.

When azimuth control is available at the control points, the equations are in groups of three - one for the azimuth closure, one for the X closure, and one for the Y closure. To avoid any confusion, the azimuth equations should follow the same routes as the X and Y equations. In those cases where there is no azimuth control at a fixed point, only the X and Y equations are necessary, of course. When observed azimuths are included, one additional equation is introduced for each of these azimuths and any route may be taken except that a fixed azimuth must be included, if available, in at least one of the equations where more than one observed azimuth is employed.

In a complicated network, the route selections for the equations may seem difficult to ascertain, but if three simple rules are followed no problems should be encountered.

(1) All fixed points, including the azimuths at these points, must be used at least once in the formation of the equations. But fixed points without azimuth control should not be used as a terminal point more than once.

(2) All observed azimuths and measured lengths must be used at least once in the formation of the equations.

(3) Each set of equations must contain some terms not appearing in any other set. This restriction does not apply to routes required specifically for observed azimuths.

Figure (3) is an example of a rather complicated network.

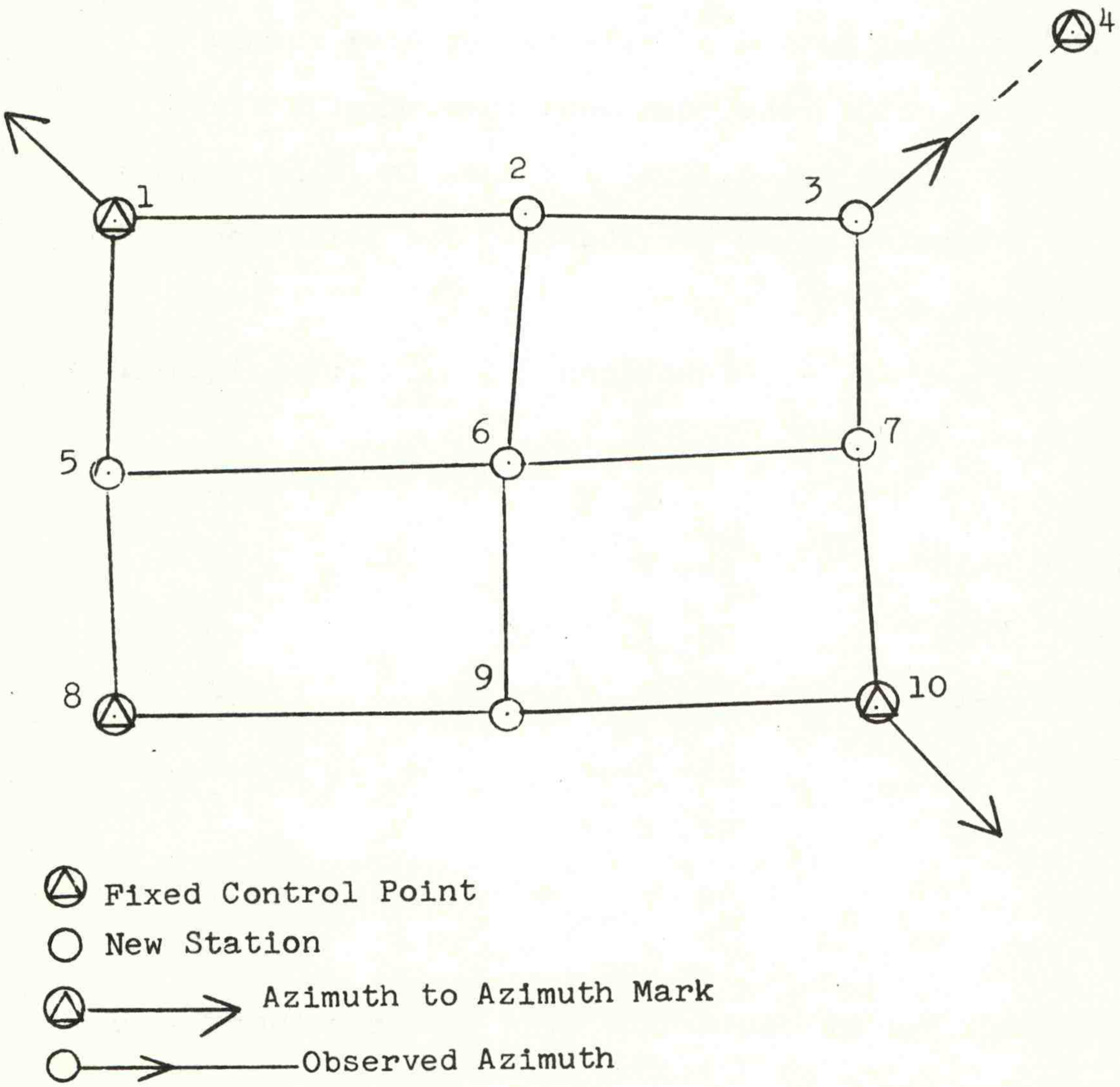


Figure 3

In this network (Figure 3), there are four fixed control points (but only two have azimuth control), six new stations, thirteen measured distances, and one observed azimuth. The number of condition equations is 3(13-6) + 1-2 = 20. It should be noted that only the fixed and junction points are shown in this example, any number of new points between these junction points may be introduced without changing the number of equations provided there are no multiple connections between these points.

The "buildup" method of selecting routes should be made. In this method the equations involving the fixed control would be selected first and then the other routes following the rules given previously. The selection would be as follows:

Route	Equations	Total Equations
1-2-3-4	X, Y	2
1-5-8	X, Y	2
1-5-8-9-10	Az, X, Y	3
1-2-3-7-10	Az, X, Y	3
1-2-6-9-10	Az, X, Y	3
1-2-6-7-10	Az, X, Y	3
1-5-6-9-10	Az, X, Y	3
1-2-3	Az	1
		20

Numerous other routes could have been selected that would fulfill the rules.

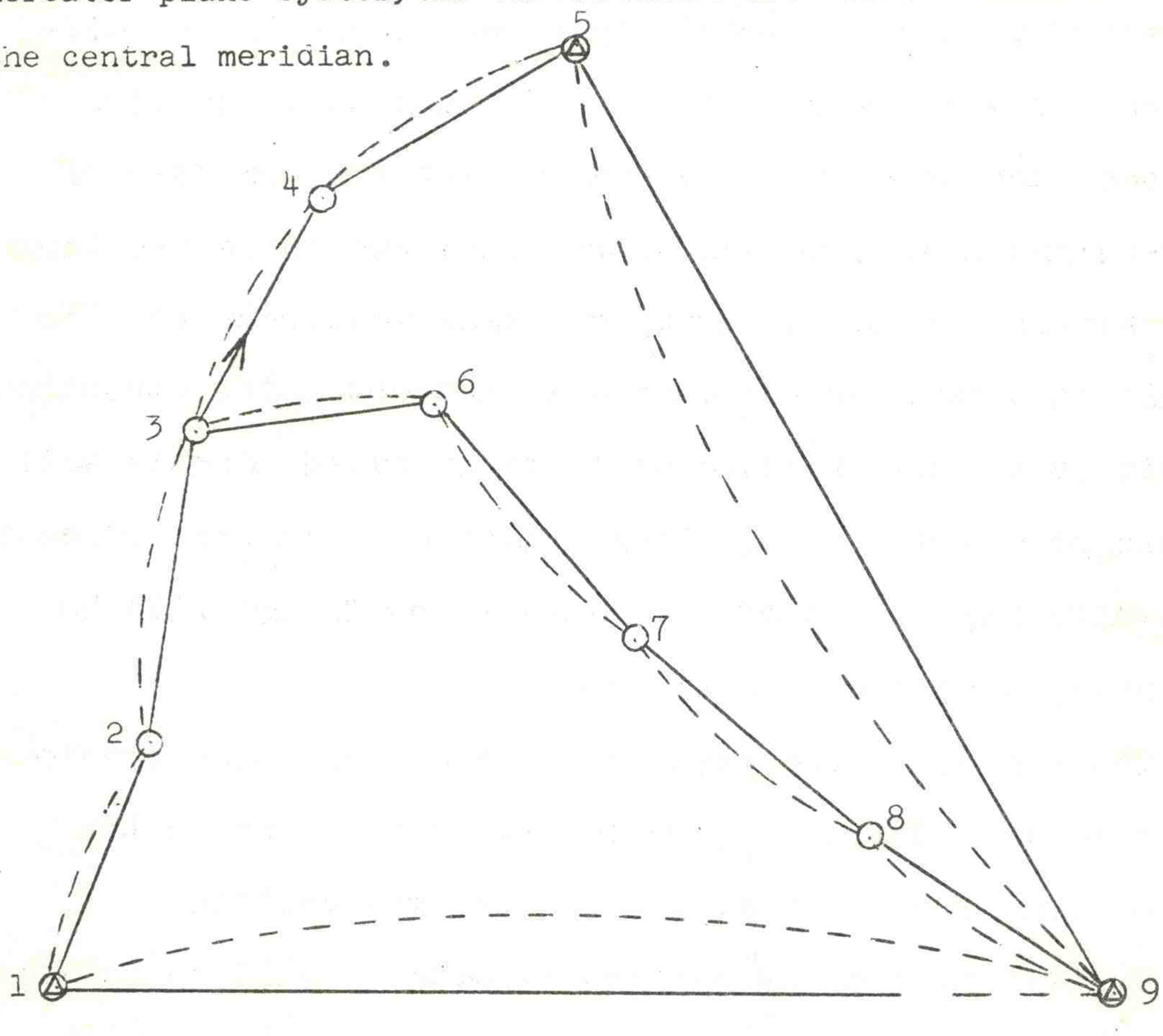
Preliminary computations are then made over the selected routes to obtain the closures and preliminary coordinates necessary to form the equations.

If the directions rather than angles are employed in a traverse adjustment, the number of equations is not increased but the number of terms in each equation is increased by about one-half since each angle is the difference between two directions. The use of directions has some advantages when several angles are composed of observations over long and short lines and there are large uncertainties in the accuracy of these measurements. No example is given for a traverse adjustment using directions in this paper, but a document to be released shortly will contain programs for adjusting traverse which uses directions. The sample adjustment of a triangulation figure will be made using directions, however.

The number of observation equations has been previously discussed and additional comments will be found in the section concerned with adjustment of triangulation.

Two examples of a weighted adjustment will be presented. The first, by condition equations, is recommended for traverse and the second, using the method of variation of coordinates which employs observation equations, for triangulation. Both computations will be made on the plane, using State plane coordinates, but the procedures are applicable to other grid systems.

Condition Equations: This example is a simulated traverse in western Georgia composed of two traverse lines forming a "T" configuration with fixed control on the north, east, and south. In addition, an azimuth was considered as being observed at the junction of the two lines.(3-4). Figure 4 is a sketch of the survey and the curved lines represent the geodetic angle observations and the straight lines the grid values. Since Georgia uses the transverse Mercator plane system, the curved lines are concave towards the central meridian.



Fixed Control Points
New Stations
Observed Azimuth

Figure 4

The first step is to determine the Weights for the various quantities. When employing condition equations involving a mixture of angle and distance measurements, the weights must be in the same measure. This does not present any problem in determining the weights for the angles and azimuths since the SE's or estimated accuracies would be in seconds, but this would not be the case for the lengths. Inasmuch as there is no great variances in the distances (Table 17), a mean value of 14,900 ft. was used with the SE assumed to be 0.1 ft. + 1:100,000 of the distance or 0.25 ft. Using the formula $SE'' = \frac{SE_L \times 206265}{Distance}$ or $SE'' = \frac{0.25 \times 206265}{14,900}$, the value was found to be 3".5.

The SE's for the angles and azimuths are given below.

The computation of the weights is as follows:

	SE	$p = \frac{1}{(SE)^2}$	kp*
Angles	2".0	0.2500	1.00
Length	3.5	0.0816	0.33
Azimuth	3.0	0.1111	0.44

* In this tables the values in the second column are the weights "p", but to simplify future computations they should be multiplied by a factor "k". This factor should be such that the weight to be applied to the majority of the elements involved is 1.00. For this example a factor of 4 served the purpose.

Using the formulae presented earlier, the Number of Condition Equations in this survey are CE = 3(2) + 1 = 7 or 3(8-6) + 1 = 7. The routes selected for the equations involving the control points were 1-3-5 and 1-3-9. The route for the azimuth equation containing the observed azimuth is 1-3. Other routes could have been selected, of course.

The Second Term Corrections are now computed as described on pp. 123-124 remembering that the survey is to be computed using transverse Mercator coordinates and ΔX is the mean X' of the two points involved and ΔY is the difference in the Y's. Since the observations at the terminal stations are over longer lines than usually associated with azimuth marks, corrections must be determined for these lines. The computations are shown in Table 14.

From	To	ΔX	ΔY	$2.36(\Delta X)(\Delta Y)$
1	2	3.791	0.135	1".2
1	5	3.691	0.510	4.4
1	9	3.541	0.025	0.2
2	3	3.763	0.155	1.4
3	4	3.728	0.126	1.1
3	6	3.696	0.025	0.2
4	5	3.634	0.093	0.8
5	9	3.419	0.484	3.9
6	7	3.578	0.126	1.1
7	8	3.460	0.090	0.7
8	9	3.333	0.074	0.6

Table 14

The second term corrections are applied to the Observed Angles to obtain the Grid Angles (Table 15) as explained on pp. 124-125. In this case the total corrections are not all of the same sign nor are the individual corrections added algebraically and a difference approach must be taken than previously. As examples: the correction geodetic to grid for the line 1-9 is +0".2, and 1-2 it is +1".2. If directions were observed, the clockwise angle at 1 from 9 to 2 would be direction 1-2 minus direction 1-9, therefore, the correction would be +1".2 - (+0".2) or 1".0. Similarly, the clockwise angle at 3 between 2 and 6 would be corrected by the correction 3-6 minus the correction 3-2 or +0".2 - (-1".4) = +1".6.

Point	Observed Angles	Corr'n	Grid Angle	Wt. = p
9				
1	291°12'59".4	+1".0	60".4	1.0
2	165 33 10.8	+2.6	13.4	1.0
3	200 25 53.4	+2.5	55.9	1.0
4	210 04 40.6	+1.9	42.5	1.0
5	273 42 00.9	-3.1	57.8	1.0
9				
2				
3	254 19 50.7	+1.6	52.3	1.0
6	239 53 37.9	-0.9	37.0	1.0
7	167 24 12.2	-1.8	10.4	1.0
8	174 19 21.4	-1.3	20.1	1.0
9	28 15 17.1	+3.3	20.4	1.0
5				

Table 15

The computation of the Scale and Sea Level Factors and the reduction of the Horizontal Measured Distances is shown in Tables 16 and 17. The discussion appearing on pp. 126-130 is applicable here except that the survey is being computed on the transverse Mercator grid system and individual corrections to each length have been determined. In interpolating for the scale factor from the tables, X' values to the nearest one hundred feet will normally be sufficiently accurate. As a case in point, a difference of one in the sixth decimal place of the scale factor will produce only a one part per million change in the length.

Station	Elevation (feet)	Elevation Factor	X'	Scale Factor	Combined Factor
1	1974	0.9999056	381356	1.0000665	0.9999721
2	1532	0.9999267	376825	1.0000626	0.9999893
3	1085	0.9999481	375714	1.0000616	1.0000097
4	764	0.9999635	369943	1.0000568	1.0000203
5	336	0.9999839	356807	1.0000458	1.0000297
6	1482	0.9999291	363404	1.0000512	0.9999803
7	1237	0.9999408	352234	1.0000421	0.9999829
8	1892	0.9999095	339705	1.0000322	0.9999417
9	2185	0.9998955	326924	1.0000224	0.9999179

Table 16

TRANSVERSE MERCATOR PROJECTION FOR GEORGIA (EAST & WEST)

x' (feet)	Scale in units of 7th place of logs	Scale expressed as a ratio	x' (feet)	Scale in units of 7th place of logs	Scale expressed as a ratio
0	-434.3	0.9999000	175,000	-282.0	0.9999351
5,000	-434.2	0.9999000	180,000	-273.2	0.9999371
10,000	-433.8	0.9999001	185,000	-264.1	0.9999392
15,000	-433.2	0.9999003	190,000	-254.8	0.9999413
20,000	-432.3	0.9999005	195,000	-245.2	0.9999435
25,000	-431.2	0.9999007	200,000	-235.4	0.9999458
30,000	-429.8	0.9999010	205,000	-225.3	0.9999481
35,000	-428.2	0.9999014	210,000	-215.0	0.9999505
40,000	-426.3	0.9999018	215,000	-204.4	0.9999529
45,000	-424.2	0.9999023	220,000	-193.6	0.9999554
50,000	-421.9	0.9999029	225,000	-182.5	0.9999580
55,000	-419.3	0.9999035	230,000	-171.2	0.9999606
60,000	-416.4	0.9999041	235,000	-159.7	0.9999632
65,000	-413.3	0.9999048	240,000	-147.9	0.9999659
70,000	-409.9	0.9999056	245,000	-135.8	0.9999687
75,000	-406.3	0.9999064	250,000	-123.5	0.9999716
80,000	-402.5	0.9999073	255,000	-110.9	0.9999745
85,000	-398.4	0.9999083	260,000	- 98.1	0.9999774
90,000	-394.0	0.9999093	265,000	- 85.1	0.9999804
95,000	-389.4	0.9999103	270,000	- 71.8	0.9999835
100,000	-384.6	0.9999114	275,000	- 58.2	0.9999866
105,000	-379.5	0.9999126	280,000	- 44.4	0.9999898
110,000	-374.1	0.9999139	285,000	- 30.4	0.9999930
115,000	-368.5	0.9999151	290,000	- 16.1	0.9999963
120,000	-362.7	0.9999165	295,000	- 1.5	0.9999997
125,000	-356.6	0.9999179	300,000	+ 13.3	1.0000031
130,000	-350.3	0.9999193	305,000	+ 28.3	1.0000065
135,000	-343.7	0.9999209	310,000	+ 43.6	1.0000100
140,000	-336.8	0.9999224	315,000	+ 59.1	1.0000136
145,000	-329.7	0.9999241	320,000	+ 74.9	1.0000172
150,000	-322.4	0.9999258	325,000	+ 91.0	1.0000210
155,000	-314.8	0.9999275	330,000	+107.3	1.0000247
160,000	-307.0	0.9999293	335,000	+123.8	1.0000285
165,000	-298.9	0.9999312	340,000	+140.6	1.0000324
170,000	-290.6	0.9999331	345,000	+157.6	1.0000363

TRANSVERSE MERCATOR PROJECTION FOR GEORGIA (EAST & WEST)

x' (feet)	Scale in units of 7th place of logs	Scale expressed as a ratio
350,000	+174.9	1.0000403
355,000	+192.4	1.0000443
360,000	+210.2	1.0000484
365,000	+228.2	1.0000525
370,000	+246.5	1.0000568
375,000	+265.0	1.0000610
380,000	+283.8	1.0000653
385,000	+302.8	1.0000697
390,000	+322.1	1.0000742
395,000	+341.6	1.0000787
400,000	+361.4	1.0000832
405,000	+381.4	1.0000878
410,000	+401.6	1.0000925
415,000	+422.1	1.0000972
420,000	+442.9	1.0001020
425,000	+463.9	1.0001068
430,000	+485.2	1.0001117
435,000	+506.7	1.0001167
440,000	+528.5	1.0001217
445,000	+550.5	1.0001268
450,000	+572.7	1.0001319
455,000	+595.2	1.0001370
460,000	+618.0	1.0001423
465,000	+641.0	1.0001476
470,000	+664.2	1.0001529
475,000	+687.7	1.0001583
480,000	+711.4	1.0001638
485,000	+735.4	1.0001693
490,000	+759.7	1.0001749
495,000	+784.2	1.0001806
500,000	+808.9	1.0001863
505,000	+833.9	1.0001920
510,000	+859.1	1.0001978
515,000	+884.6	1.0002037
520,000	+910.4	1.0002096

From	To	Measured Distance	Mean Combined Factor	Grid Distance	Wt. = p
1	2	14239.02	0.9999807	14,238.75	0.33
2	3	15520.26	0.9999995	15,520.25	0.33
3	4	13895.25	1.0000150	13,895.46	0.33
3	6	12564.12	0.9999950	12,564.06	0.33
4	5	16110.52	1.0000250	16,110.92	0.33
6	7	16803.42	0.9999816	16,803.11	0.33
7	8	15435.40	0.9999623	15,434.82	0.33
8	9	14767.31	0.9999298	14,766.27	0.33

Table 17

The computation of the Azimuth Closures is shown in Tables 18, 19, 20. The computed azimuths will be used in the computation of the X and Y Closures as shown in Tables 21 and 22. The fixed positions, fixed azimuths, and observed azimuth 3-4 are given in the tables. More particulars are given on pp. 130-132 and 138-140.

Note that the angle designations are tabulated on the azimuth computations. This is necessary because at the junction station, point 3, a distinction between the angles used in the computations must be made. Should directions, rather than angles be used, the numbering system employed would take this into account.

For obvious reasons, when determining the X and Y closures by hand computations, a minimum effort should be expended. The computations shown in Tables 21 and 22

STATION		PRELIMINARY AZIMUTH		
FROM	TO	°	′	″
1 **	9 *	267	20	13.6
∠ 1		291	13	00.4
1	2	198	33	14.0
2	1	18	33	14.0
∠ 2		165	33	13.4
2	3	184	06	27.4
3	2	4	06	27.4
∠ 3a		200	25	55.9
3	4	204	32	23.3
4	3	24	32	23.3
∠ 4		210	04	42.5
4	5	234	37	05.8
5	4	54	37	05.8
∠ 5		273	41	47.8
5	9	328	19	03.6
* Fixed Azimuth	*	328	18	53.0
∠ ** Angle Designation				+ 10.6
	Table 18			

STATION		PRELIMINARY AZIMUTH		
FROM	TO	°	′	″
1 **	9 *	267	20	13.6
∠ 1		291	13	00.4
1	2	198	33	14.0
2	1	18	33	14.0
∠ 2		165	33	13.4
2	3	184	06	27.4
3	2	4	06	27.4
∠ 3b		254	19	52.3
3	6	258	26	19.7
6	3	78	26	19.7
∠ 6		239	53	37.0
6	7	318	19	56.7
7	6	138	19	56.7
∠ 7		167	24	10.4
7	8	305	44	07.1
8	7	125	44	07.1
∠ 8		174	19	20.1
8	9	300	03	27.2
9	8	120	03	27.2
∠ 9		28	15	20.4
9	5	148	18	47.6
* Fixed Azimuth	*	148	18	53.0
∠ ** Angle Designation				- 5.4
	Table 19			

STATION		PRELIMINARY AZIMUTH		
FROM	TO	°	′	″
1 **	9 *	267	20	13.6
∠ 1		291	13	00.4
1	2	198	33	14.0
2	1	18	33	14.0
∠ 2		165	33	13.4
2	3	184	06	27.4
3	2	4	06	27.4
∠ 3a		200	25	55.9
3	4	204	32	23.3
* Fixed Az.	Observed Az.	204	32	16.4
∠ ** Angle Designation				+ 06.9
	Table 20			

reflect this reasoning. Also, the sines and cosines of the azimuths were omitted because the computations were made on a desk size programmable computer with built-in trigonometric functions.

The Formation of the Condition Equations is shown in Table 23. These equations, with one exception, are computed using the basic equations as given on p. 140 and explained on pp. 140-143. The one exception being that the azimuth equation involving the observed azimuth is in the form $0 = \left[A_n - (A_o + K)\right] + \Sigma \left[(V_a)\right]$, where A_o is the observed azimuth (i-k), and K is the correction to be determined in the adjustment for this azimuth. The other terms of the equation are as identified on p. 140. If for some reason the observed azimuth is to be held fixed, the "K" would be

PRELIMINARY COORDINATE COMPUTATION

STATION	PLANE AZIMUTH	GRID DISTANCE	LOG DEPARTURE / LOG SIN AZIMUTH / LOG DISTANCE / LOG COS AZIMUTH / LOG LATITUDE	DEPARTURE	LATITUDE	GRID COORDINATES	
						x	y
1	° ' "	Feet		Feet	Feet	Feet	Feet
				+ 4530.720	Fixed	118,643.92	1,468,415.31
	198 33 14.0	14238.75					
					+ 13498.688		
2							
				+ 1111.715			
	184 06 27.4	15520.25					
					+ 15480.383		
3							
				+ 5771.136			
	204 32 23.3	13895.46					
					+ 12640.324		
4							
				+13135.435			
	234 37 05.8	16110.92				143,192.926	1,519,363.268
					+ 9328.563	+ 2.486	- 2.492
5					Fixed	143,190.44	1,519,365.76

USCOMM-DC 8270

Table 21

PRELIMINARY COORDINATE COMPUTATION

STATION	PLANE AZIMUTH	GRID DISTANCE	LOG DEPARTURE / LOG SIN AZIMUTH / LOG DISTANCE / LOG COS AZIMUTH / LOG LATITUDE	DEPARTURE	LATITUDE	GRID COORDINATES	
						x	y
3	° ' "	Feet		Feet (From Table 21)	Feet	124,286.355	1,497,394.381
				+ 12309.150			
	258 26 19.7	12564.06					
					+ 2518.019		
6							
				+ 11170.839			
	318 19 56.7	16803.11					
					-12552.166		
7							
				+ 12528.811			
	305 44 07.1	15434.82					
					- 9014.575		
8							
				+ 12780.542			
	300 03 27.2	14766.27				173,075.697	1,470,949.682
					- 7395.977	- 1.613	+ 2.702
9					Fixed	173,077.31	1,470,946.98

USCOMM-DC 5270

Table 22

FORMATION OF CONDITION EQUATIONS

1) $0 = +10.6 + 1.00(1) + 1.00(2) + 1.00(3a) + 1.00(4) + 1.00(5)$

2) $0 = -5.4 + 1.00(1) + 1.00(2) + 1.00(3b) + 1.00(6) + 1.00(7)$
$+ 1.00(8) + 1.00(9)$

3) $0 = +6.9 - 1.00(3-4) + 1.00(1) + 1.00(2) + 1.00(3a)$

4) $0 = +2.486 \times 206265 + (Y_5 - Y_1)(1) + (Y_5 - Y_2)(2)$
$\div 10{,}000 + (Y_5 - Y_3)(3a) + (Y_5 - Y_4)(4)$
$+ (X_2 - X_1)(1-2) + (X_3 - X_2)(2-3)$
$+ (X_4 - X_3)(3-4) + (X_5 - X_4)(4-5)$
$0 = +51.2775 + 5.095(1) + 3.745(2) + 2.197(3a) + 0.933(4)$
$+ 0.453(1-2) + 0.111(2-3) + 0.577(3-4)$
$+ 1.314(4-5)$

5) $0 = -2.492 \times 206265 - (X_5 - X_1)(1) - (X_5 - X_2)(2)$
$\div 10{,}000 - (X_5 - X_3)(3a) - (X_5 - X_4)(4)$
$+ (Y_2 - Y_1)(1-2) + (Y_3 - Y_2)(2-3)$
$+ (Y_4 - Y_3)(3-4) + (Y_5 - Y_4)(4-5)$
$0 = -51{,}4012 - 2.455(1) - 2.002(2) - 1.891(3a) - 1.314(4)$
$+ 1.350(1-2) + 1.548(2-3) + 1.264(3-4)$
$+ 0.933(4-5)$

6) $0 = -1.613 \times 206265 + (Y_9 - Y_1)(1) + (Y_9 - Y_2)(2) + (Y_9 - Y_3)(3b)$
$\div 10{,}000 + (Y_9 - Y_6)(6) + (Y_9 - Y_7)(7) + (Y_9 - Y_8)(8)$
$+ (X_2 - X_1)(1-2) + (X_3 - X_2)(2-3)$
$+ (X_6 - X_3)(3-6) + (X_7 - X_6)(6-7)$
$+ (X_8 - X_7)(7-8) + (X_9 - X_8)(8-9)$

Table 23

$$0 = -33.2705 + 0.254(1) - 1.096(2) - 2.644(3b) - 2.896(6)$$
$$- 1.641(7) - 0.740(8) + 0.453(1-2)$$
$$+ 0.111(2-3) + 1.231(3-6) + 1.117(6-7)$$
$$+ 1.253(7-8) + 1.278(8-9)$$

7) $0 = +2.702 \times 206265 - (X_9 - X_1)(1) - (X_9 - X_2)(2)$
$\div 10{,}000$ $- (X_9 - X_3)(3b) - (X_9 - X_6)(6)$
$- (X_9 - X_7)(7) - (X_9 - X_8)(8)$
$+ (Y_2 - Y_1)(1-2) + (Y_3 - Y_2)(2-3)$
$+ (Y_6 - Y_3)(3-6) + (Y_7 - Y_6)(6-7)$
$+ (Y_8 - Y_7)(7-8) + (Y_9 - Y_8)(8-9)$

$$0 = +55.7328 - 5.443(1) - 4.990(2) - 4.879(3b) - 3.648(6)$$
$$- 2.531(7) - 1.278(8) + 1.350(1-2) + 1.548(2-3)$$
$$+ 0.252(3-6) - 1.255(6-7) - 0.901(7-8)$$
$$- 0.740(8-9)$$

(Table 23 - Continued)

be eliminated and the basic equation would be used. As may be seen from the above equation, the sign of the "K" term is minus with a coefficient of 1.00 generally assigned in the form -1.00(i-k) or in this example, -1.00(3-4) as shown in equation 3, Table 23.

These equations conform to the routes as selected on p.164. To save a step, the constant terms and coefficients for the X and Y equations were divided for the reasons given on pp. 142-143 prior to tabulating the values in Table 23

The equations are tabulated vertically (Correlate Equations) as shown in Table 24. This tabulation contains additional columns, a/p and Σ not shown in Table 8, p. 143.

The numbers appearing in the column labeled a/p are obtained by dividing some convenient number "a" by the weight "p" assigned the observations identified by the Correction Number column. Weights for the observations involved in this adjustment were determined earlier (p. 163). Although an examination of these weights showed that for practical purposes two choices for "a" would be acceptable (1.00 or 0.33), it was decided to use 1.00. In this manner the values a/p for the angles were 1.00; 3.03 for the lengths; and 2.27 for the observed azimuth. If an "a" of 0.33 had been used, a/p for the lengths would have been 1.00; the angles 0.33; and the azimuth 0.75. The a/p values are always considered positive in the various operations in which they are involved.

The "Σ" column is computed by summing the coefficients algebraically, the values for a/p not included. For examples: (1) the "Σ" for the line involving correction no. 1 = +1.00 + 1.00 + 1.00 + 5.095 - 2.455 + 0.254 - 5.443 = +0.451; (2) Σ for line 2-3 = +0.111 + 1.548 + 0.111 + 1.548 = +3.318. By introducing this column, the computations of the normal equations and the forward solution are self checking operations. This self checking feature is not restricted

CORRELATE EQUATIONS

Corr'n No.	a/p	Equations 1	2	3	4	5	6	7	Σ	Corrections V's	Adopt
1	1.00	+1.00	+1.00	+1.00	+5.095	-2.455	+0.254	-5.443	+0.451	-5".2746	-5".28
2	1.00	+1.00	+1.00	+1.00	+3.745	-2.002	-1.096	-4.990	-1.343	-2.1933	-2.19
3a	1.00	+1.00		+1.00	+2.197	-1.891			+2.306	-6.8493	-6.85
3b	1.00		+1.00				-2.644	-4.879	-6.523	+7.7305	+7.73
4	1.00	+1.00			+0.933	-1.314			+0.619	-2.7194	-2.72
5	1.00	+1.00							+1.000	+6.4366	+6.44
6	1.00		+1.00				-2.896	-3.648	-5.544	+3.4749	+3.48
7	1.00		+1.00				-1.641	-2.531	-3.172	+2.6511	+2.65
8	1.00		+1.00				-0.740	-1.278	-1.018	+0.6894	+0.69
9	1.00	+1.00							+1.000	-1.6779	-1.68
3-4	2.27			-1.00					-1.000	-7.4176	-7.42
											ds in ft.
1-2	3.03				+0.453	+1.350	+0.453	+1.350	+3.606	+1.9792	+0.137
2-3	3.03				+0.111	+1.548	+0.111	+1.548	+3.318	+4.6277	+0.348
3-4	3.03				+0.577	+1.264			+1.841	+9.0259	+0.608
4-5	3.03				+1.314	+0.933			+2.247	-3.9750	-0.310
3-6	3.03						+1.231	+0.252	+1.483	+5.3173	+0.324
6-7	3.03						+1.117	-1.255	-0.138	+18.4818	+1.506
7-8	3.03						+1.253	-0.901	+0.352	+16.0670	+1.202
8-9	3.03						+1.278	-0.740	+0.538	+14.7401	+1.055

Table 24

to weighted adjustments, it could have been used in the previous example given on pp. 143-146 but was omitted on order to reduce the length of the explanation given on those pages.

The "Corrections" columns are identical to those previously described (p.147) but the actual computation is somewhat different and will be described later.

The Normal Equations (Table 25) are formed in the same manner as described on pp. 143-144 except that each multiplication is also multiplied by its corresponding a/p. In addition, the check column "Σ" is introduced. A few examples involving a/p and Σ follow:

(1) An inspection reveals that the a/p values for the angles is 1.00 and therefore the formation of the normal equations for equations 1 and 2 are identical to the procedures explained previously. Σ's are obtained by treating the Σ terms in the correlate equations as if they were part of other equations and then the constant terms "n" are added to the result. For equation 1, Σ = (+1.00)(+0.451) + (+1.00)(-1.343) + (+1.00)(+2.306) + (+1.00)(+0.619) + (+1.00)(+1.000) + (+10.6) = +13.633.

For equation 2, Σ = (+1.00)(+0.451) + (+1.00)(-1.343) + (+1.00)(-6.523) + (+1.00)(-5.544) + (+1.00)(-3.172) + (+1.00)(-1.018) + (+1.00)(+1.000) + (-5.4) = -21.549.

NORMAL EQUATIONS

1	2	3	4	5	6	7	n	ε
+5.00	+2.00	+3.00	+11.970	- 7.662	- 0.842	- 10.433	+10.6	+ 13.633
	+7.00	+2.00	+ 8.840	- 4.457	- 8.763	- 22.769	- 5.4	- 21.549
		+5.27	+11.037	- 6.348	- 0.842	- 10.433	+ 6.9	+ 10.584
			+52.5808	-17.0880	- 2.1513	- 44.0460	+51.2775	+ 72.4200
				+35.5991	+ 3.9443	+ 36.1355	-51.4012	- 11.2773
					+38.6209	+ 25.4301	-33.2705	+ 22.1265
						+121.5448	+55.7328	+151.1622

Table 25

(2) The a/p column enters the computation with equation 3 but only effects the normal terms for 3 and Σ since the formation of the other normal terms are not involved with the coefficient 3-4. The normal terms for columns 3 and Σ were computed as follows:

$$(3) = (+1.00)^2 + (+1.00)^2 + (+1.00)^2 + (-1.00)^2(2.27) = +5.27$$

$$\Sigma = (+1.00)(+0.451) + (+1.00)(-1.343) + (+1.00)(+2.306) + (-1.00)(-1.000)(2.27) + (+6.9) = +10.584$$

(3) The formation of the **other** normal equations will also involve different values for a/p. In this sample adjustment, the formation is simplified because only two values for a/p are involved and the normal terms may be formed by summing two partial sets of multiplications as follows:

$$(4) = \left[(+5.095)^2 + (+3.745)^2 + (+2.197)^2 + (+0.933)^2\right] \times (1.00) + \left[(+0.453)^2 + (+0.111)^2 + (+0.577)^2 + (+1.314)^2\right] \times (3.03) = +52.5808$$

Let us assume the a/p values differ for each observed quantity. The formation of the normal term 4 on 5 would then involve the following multiplications. The values for a/p are as given in Table 24.

$$\begin{aligned}(4) \times (5) = {} & + (+5.095)(-2.455)(1.00) + (+3.745)(-2.002)(1.00) \\ & + (+2.197)(-1.891)(1.00) + (+0.933)(-1.314)(1.00) \\ & + (+0.453)(+1.350)(3.03) + (+0.111)(+1.548)(3.03) \\ & + (+0.577)(+1.264)(3.03) + (+1.314)(+0.933)(3.03) \\ = {} & -17.0880.\end{aligned}$$

The computation of the remaining normal equation terms are made in a similar fashion.

The normal equations are checked by summing all the terms vertically and horizontally for each equation including the "n" term but excluding the "Σ" term. This sum should equal the Σ term exactly if all decimal places are used. In this case only four decimals are retained and the sum of the normal terms may not equal the computed "Σ" term by as much as $\pm$0.0002. Where this occurred in this example, only the corrected values for Σ are shown. Several examples are given below:

(1) (1) = + 5.00(1) + 2.00(2) + 3.00(3) + 11.970(4) - 7.662(5) - 0.842(6) - 10.433(7) + 10.6(n) = +13.633 which checks the Σ value (+13.633)

(2) (4) = +11.970(1) + 8.840(2) + 11.037(3) + 52.5808(4) -17.0880(5) - 2.1513(6) - 44.0460(7) +51.2775(n) = +72.4200 which checks the Σ value (+72.4200)

NOTE: The numerical notations have no meaning except to indicate to the reader the location of the terms in the normal equations used in the example.

The computation of the Forward Solution is given in Table 26 and was performed using the same technique described on pp. 144-146. The "Σ" column is handled in an identical manner as the other columns. To employ the self checking

FORWARD SOLUTION

1	2	3	4	5	6	7	η	ξ
+5.00	+2.00	+3.00	+11.970	- 7.662	- 0.842	- 10.433	+10.6	+ 13.633
	- .400	- .600	- 2.394	+ 1.5324	+ .1684	+ 2.0866	- 2.120	- 2.7266
	+7.00	+2.00	+ 8.840	- 4.457	- 8.763	- 22.769	- 5.4	- 21.549
	+6.20	+0.80	+ 4.052	- 1.3922	- 8.4262	- 18.5958	- 9.64	- 27.0022
		- .12903	- .65355	+ .22455	+ 1.35906	+ 2.99932	+ 1.55484	+ 4.35519
		+5.27	+11.037	- 6.348	- 0.842	- 10.433	+ 6.9	+ 10.584
		+3.3668	+ 3.3322	- 1.5712	+ 0.7504	- 1.7737	+ 1.7839	+ 5.8884
			- .98972	+ .46667	- .22288	+ .52682	- .52985	- 1.74896
			+52.5808	-17.0880	- 2.1513	- 44.0460	+51.2775	+ 72.4200
			+17.9785	+ 3.7197	+ 4.6287	- 5.1607	+30.4357	+ 51.6019
				- .20690	- .25746	+ .28705	- 1.69289	- 2.87020
				+35.5991	+ 3.9443	+ 36.1355	-51.4012	- 11.2773
			+22.0424	+ 0.1545	+ 16.2123	-42.7870	- 4.3778	
				- .00701	- .73551	+ 1.94112	+ .19860	
				+38.6209	+ 25.4301	-33.2705	+ 22.1265	
				+25.6674	+ 0.0107	-52.5204	- 26.8423	
					- .00042	+ 2.04619	+ 1.04577	
					+121.5448	+55.7328	+151.1622	
					+ 29.6604	+90.1055	+119.7659	
						- 3.03791	- 4.03791	

Table 26

BACK SOLUTION

C's

1	+6.43657
2	-1.67793
3	+3.26765
4	-3.95301
5	+4.16118
6	+2.04747
7	-3.03791

Table 27

feature sum the normal terms for the first equation and the reduced normals for all other equations prior to dividing the terms. These sums should equal the computed Σvalues within $\pm$0.0002 generally, but in a large solution checks of $\pm$0.0010 may be satisfactory. The Σ should be changed to agree with the sum. The sum of the divided terms should check the Σ term by $\pm$0.00002 if five decimal places are used and within $\pm$0.000002 should the computations be carried to six decimal places. In summing the divided terms, -1.00000 must be included to account for the division of the diagonal term by the negative value of itself. Several examples are given below:

(1) Equation 1 - the terms are identical to those in the Normal equation which was previously checked. Divided terms sum = -1.0000(1) - 0.4000(2) - 0.6000(3) - 2.3940(4) + 1.5324(5) + 0.1684(6) + 2.0866(7) - 2.1200(η) = -2.7266 which checks the Σ value (-2.7266).

(2) Equation 3 - Reduced normal sum = + 3.3668(3) + 3.3322(4) - 1.5712(5) + 0.7504(6) - 1.7737(7) + 1.7839(η) = +5.8884 which checks the Σ value (+5.8884). Divided terms sum = - 1.00000(3) - 0.98972(4) + 0.46667(5) - 0.22288(6) + 0.52682(7) - 0.52985(η) = -1.74896 which checks the Σ value (-1.74896).

(3) Equation 6 - Reduced normals sum = + 25.6674(6) + 0.0107(7) - 52.5204(η) = -26.8423 which checked the Σ value (-26.8424) by -0.0001. The corrected Σ term is shown in the table. Divided terms sum = - 1.00000(6) - 0.00042(7) + 2.04619(η) = +1.04577 which checks the Σ value (+1.04577).

The numerical notations, as noted previously, are given only to indicate the location of the terms in the solution.

The results (C's) of the Back Solution are shown in Table 27. These values were computed using the identical method given on p.146. This computation is also made self checking by using the reduced normal terms as shown below. If "C's" to five decimal places are used, the sum should seldom differ from zero by more than $\pm$0.00002 of the diagonal term for the equation, but occasionally may approach $\pm$0.00003 of the diagonal term. Example (3) below is a case in point.

(1) Equation 7 $0 = + 90.1055 + (+29.6604) \times C_7 = -0.00013$

allowable = $29.6604 \times 0.00002 = \pm 0.00059$.

(2) Equation 5 $0 = - 42.7870 + (+16.2123) \times C_7 +$

$(+0.1545) \times C_6 + (+22.0424) \times C_5 = +0.00022$

allowable = $22.0424 \times 0.00002 = \pm 0.00044$

(3) Equation 2 $0 = -9.64 + (-18.5958) \times C_7 + (-8.4262) \times C_6 + (-1.3922) \times C_5 + (+4.052) \times C_4 + (+0.80) \times C_3 + (+6.20) \times C_2 = +0.00014$

allowable = $6.20 \times 0.00002 = \pm 0.00012$.

A further check on the "C's" may be obtained by multiplying each "C" by its corresponding "η" term (the original and not the reduced value) and summing the results. This sum should check the summation of the "η" column multiplications carried out in the same manner as used for the equations. In this sample computation, the sum of the C's multiplied by their η terms is -554.1864, and the sum of the η column multiplications is -554.1840. This check (0.0024) is considered satisfactory for the number of significant figures carried in the computations.

The "C's" are substituted in the Correlate Equations (Table 24) as described on p. 147 and the sum of these multiplications is then multiplied by the corresponding a/p to produce the Corrections to the Angles, Azimuths, and Lengths. Since a/p is 1.00 for the angles, the computations are identical to those previously described and no example will be given. The computations required for the azimuth correction and the correction to the length 1-2 follows:

Azimuth 3-4 = (-1.00)(+3.26765)(2.27) = -7".4176

Length 1-2 = [(+0.453)(-3.95301) + (+1.350)(+4.16118)
+ (+0.453)(+2.04747) + (+1.350)(-3.03791]
x (3.03) = +1".9792

The corrections to the angles and azimuth are adopted to fit the azimuth closures and the corrections to the length are converted to feet as described on p. 147 and the final coordinate computations are made using the adjusted values. These computations are shown in Tables 28 and 29. The check is exact to three decimal places in X and Y for the route 1-3-5 and 0.001 in X and Y for the route 1-3-9. The sines and cosines for the adjusted azimuths are not shown as the computations were made on a desk size programmable computer. Since this computer's built-in feature contains these trigonometrical functions to ten decimal places, anyone checking the Latitudes and Departures using eight place functions may find slight differences (±0.001 ft.) in the tabulated values.

FINAL COORDINATE COMPUTATIONS

STATION	PLANE AZIMUTH	GRID DISTANCE	LOG DEPARTURE / LOG SIN AZIMUTH / LOG DISTANCE / LOG COS AZIMUTH / LOG LATITUDE	DEPARTURE	LATITUDE	GRID COORDINATES	
						x	y
1	° ' "	Feet		Feet	Feet	Feet	Feet
				+ 4530.418	Fixed	118,643.92	1,468,415.31
	198 33 08.72	14238.887					
					+13498.934		
2						123,174.338	1,481,914.244
				+ 1111.179			
	184 06 19.93	15520.598					
					+15480.770		
3						124,285.517	1,497,395.014
				+ 5770.511			
	204 32 08.98	13896.068					
					+12641.278		
4						130,056.028	1,510,036.292
				+13134.412			
	234 36 48.76	16110.610					
					+ 9329.468		
5					Fixed	143,190.44	1,519,365.76

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Table 28

FINAL COORDINATE COMPUTATIONS

STATION	PLANE AZIMUTH	GRID DISTANCE	LOG DEPARTURE LOG SIN AZIMUTH LOG DISTANCE LOG COS AZIMUTH LOG LATITUDE	DEPARTURE	LATITUDE	GRID COORDINATES	
						x	y
	° ' "	Feet		Feet	Feet	Feet	Feet
3				From Table 28		124,285.517	1,497,395.014
				+12309.471			
	258 26 19.96	12564.384					
					+ 2518.068		
6						136,594.988	1,499,913.082
				+11171.612			
	318 20 00.44	16804.616					
					-12553.493		
7						147,766.601	1,487,359.589
				+12529.507			
	305 44 13.49	15436.022					
					- 9015.666		
8						160,296.108	1,478,343.923
				+12781.201			
	300 03 34.28	14767.325					
					- 7396.944		
9					Fixed	173,077.31	1,470,946.98

USCOMM-DC 5270

Table 29

VARIATION OF COORDINATES

To demonstrate the Variation of Coordinates method of adjustment, a simulated triangulation net along the Indiana-Ohio boundary consisting of a quadrilateral with one measured length and one observed azimuth will serve as the example. Thus all types of observation equations used in this adjustment method will be employed. Also, directions rather than angles will be used. The computations will be made on the Indiana east zone transverse Mercator grid system.

Many of the computational procedures are identical to those described in the two adjustment examples previously presented and no further comments will be made except in those instances where modifications or variations are such that additional explanations seem necessary.

A sketch of the survey is shown in Figure 5. Stations

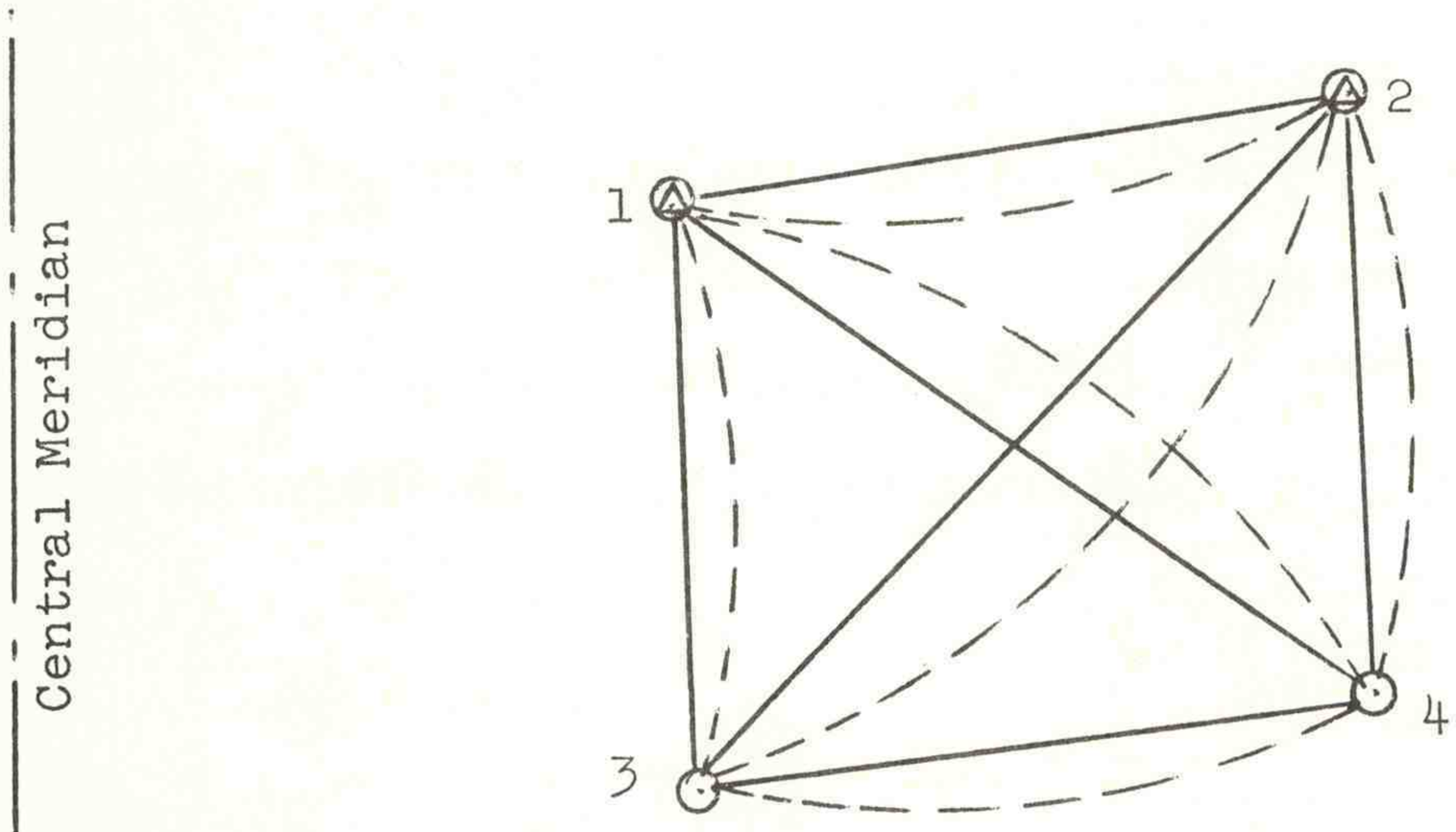


Figure 5

1 and 2 are considered fixed, 3 and 4 are new stations, and it is assumed the distance and azimuth between 3 and 4 are observed quantities.

The basic equations are given below:

Direction $d\alpha_{i-k} = N + Z_i + \frac{206265}{s^2} (\Delta Y dX_i - \Delta X dY_i - \Delta Y dX_k + \Delta X dY_k)$

or

$$N + Z_i + \frac{206265}{s} (\text{Cos } \alpha dX_i - \text{Sin } \alpha dY_i - \text{Cos } \alpha dX_k + \text{Sin } aDY_k)$$

Length $ds_{i-k} = N + \frac{1}{s} (\Delta X dX_i + \Delta Y dY_i - \Delta X dX_k - \Delta Y dY_k)$

or

$$N + \text{Sin } \alpha dX_i + \text{Cos } \alpha dY_i - \text{Sin } \alpha dX_k - \text{Cos } dY_k$$

Azimuth $dA_{i-k} = N + \frac{206265}{s^2} (\Delta Y dX_i - \Delta X dY_i - \Delta Y dX_k + \Delta X dY_k)$

or

$$N + \frac{206265}{s} (\text{Cos } \alpha dX_i - \text{Sin } \alpha dY_i - \text{Cos } \alpha dX_k + \text{Sin } \alpha dY_k)$$

"N" equals the computed quantity minus the observed value; "Z_i" is the orientation or correlation unknown for the directions at the "i" point and is assigned a coefficient of 1.00; "s" is the length of the line; "ΔX" = $X_i - X_k$; "ΔY" = $Y_i - Y_k$; "Sin α" and "Cos α" are the sine and cosine, respectively, of the azimuth of the line "i" to "k" and "dX_i", "dY_i", "dX_k", "dY_k" are the corrections to the assumed coordinates. Z_i, dX_i, dY_i, dX_k, and dY_k will be determined in the solution of the equations.

Observation equations are prepared for each observed direction, observed azimuth, and measured length. These equations resolve into three normal equations for each new occupied point and two normal equations for new intersected points. The solution of the normal equations produces corrections to the assumed coordinates ($dX_i dY_i$) for each new point and the orientation or correlation correction (Z_i) for the new occupied points. The solution (C's) of the Z equations are the corrections to the initial computed azimuths used to determine the "N" terms and also are responsible for the direction v's at each new point summing to zero (Table 34). In any Least Squares adjustment using directions, whether condition equations or observation equations are employed, the adjustment corrections (v's) at each new point must sum to zero. The results, of course, will be the same regardless of the adjustment method. In a weighted adjustment, the sum of the pv's equal zero, but in the example given here the weights are the same for all directions and this condition cannot be shown.

The "Z" equations may be eliminated prior to preparing the "Table for Formation of Normal Equations" (Table 34), but the procedure is rather complicated if corrections to directions are desired. Equations can be prepared in which corrections to angles, rather than directions, are determined. This method reduces the number of equations to be solved by one at each point, however, additional arithmetic exercises are required and little time is saved and it is not generally recommended. Examples of this adjustment method are given in Special Publication No. 193 (see references).

The computation of the Weights (p) follows:

	A	$p = \frac{1}{A^2}$	kp**
Directions	1".5	0.4444	1.00
Azimuth	3".0	0.1111	0.25
Length	0.29 ft.*	1.1891	2.68

"A" is the Standard Error or estimated accuracies of the observations.

* For observation equations it is not necessary that the weights be based on precisions in the same measure, such as seconds, which is required when condition equations are employed. For the measured length (p.198) used in this survey, the "A" value was determined using 0.1 ft. + 1:100,000 as the criterion.

** k = 2.25

Computation of the Second Term Corrections is shown in Table 30. The values at each end of the lines are given with the correct sign to reduce the opportunities for mistakes occurring when applying these quantities to the observed directions to obtain the Grid values. In addition, the correct signs will simplify the explanation of equating the grid corrections to the angles of a triangle with the spherical excess of the figure. It is for this purpose that the values are given to hundredths of seconds. The approximate nature of the formulae would preclude using anything more than tenths of seconds in practice, however.

The signs of the corrections are easily obtained from the sketch. At each point whenever the straight lines representing the grid directions are to the right (clockwise) of the curved lines representing the geodetic values, the signs are plus; to the left (counter-clockwise) the signs are minus.

From	To	ΔX	ΔY	2.36ΔXΔY
1	2	2.414	0.128	-0".73
1	3	2.267	0.268	+1.43
1	4	2.444	0.148	+0.85
2	1			+0.73
2	3	2.439	0.396	+2.28
2	4	2.616	0.276	+1.70
3	1			-1.43
3	2			-2.28
3	4	2.469	0.120	-0.70
4	1			-0.85
4	2			-1.70
4	3			+0.70

Table 30

To show the relationship of the second term corrections and the spherical excess of a figure, the triangle involving stations 1-2-3 will serve as an example. The numbering sequence is also identical to the numerical Identifiers printed on the Computation of Plane Triangles form (Figure 38).

In addition, these Identifiers are used to determine the order in which the directions are utilized to obtain the angles and this order relates to the signs of both grid and adjustment corrections as will be shown.

A	Station	Observed Angle	Grid Corr'n	Grid Angle
1	1	99° 48' 12".2	+2".1	14".3
2	2	33 01 40.9	-1.6	39.3
3	3	47 10 12.9	-0.9	12.0
	Sum	180° 00' 06".0	-0.4	05.6

A = Identifiers

The form shown in Table 38 is designed for triangles prepared in a clockwise direction, the standard procedure in all geodetic organizations and recommended for general use.

Using the Identifiers, the angles are always computed in the same manner as follows:

Angle at station 1 = direction 1-3 minus direction 1-2

Angle at station 2 = direction 2-1 minus direction 2-3

Angle at station 3 = direction 3-2 minus direction 3-1

In this particular example, the Identifiers and the station numbers are the same and as a result it is not necessary to substitute the actual angle identifications for the Identifiers in the grid correction computations that follows:

Angle	Designation	Grid Correction	Adopted
1	(1-3) minus (1-2)	+ 1.43 + 0.73 = + 2.16	+2".1
2	(2-1) minus (2-3)	+ 0.73 - 2.28 = - 1.55	-1.6
3	(3-2) minus (3-1)	- 2.28 + 1.43 = - 0.85	-0.9
		Sum = - 0.24	-0.4

Spherical Excess of Triangle = 0.23

The difference in the sum of the adopted corrections and the computed spherical excess of 0".2 is of no consequence. However, the spherical excesses of all triangles could be computed, adopted to fit the area of the figure, and then the grid corrections could be adopted to fit these values; but this effort would seldom if ever be justified for most local surveys.

A second example, for the triangle 4-1-2, is given below:

Identifiers	Designation	Grid Correction	Adopted
1	(4-2) minus (4-1)	- 1.70 + 0.85 = -0.85	-0".8
2	(1-4) minus (1-2)	+ 0.85 + 0.73 = +1.58	+1.6
3	(2-1) minus (2-4)	+ 0.73 - 1.70 = -0.97	-1.0
		Sum = -0.24	-0.2

Spherical Excess of Triangle = 0.24

The lists of directions are given in Table 31. Lists of directions are the summation of angles referenced to one station usually with an initial value of 0° 00' 00".0. Direction, in this instance, does not refer to bearings or azimuth. Observations made using the method of positions

and the technique employed by many geodetic groups will produce such lists with little or no office work required. The angles are obtained by subtracting one direction from another as shown below.

The angles are obtained using the procedures given in the description concerning the grid corrections. Two examples follow:

The observed geodetic angle at station 1 between stations 2 and 4 is 40° 32' 13".8; the corresponding grid angle is 40° 32' 14".7 minus 359° 59' 59".3 or 40° 32' 15".4. Similarly at station 1 between stations 4 and 3 the geodetic angle is 99° 48' 12".2 minus 40° 32' 13".8 or 59° 15' 58".4, and the grid angle is 99° 48' 13".6 minus 40° 32' 14".7 or 59° 15' 58".9.

The angle designations are (1-4) minus (1-2) and (1-3) minus (1-4). Since (1-2) is a fixed line, the designation is omitted when applying adjustment corrections, but would be used in obtaining the grid corrections.

LIST OF DIRECTIONS

Station = 1

To	Obs.	Geod.	Dir.	Grid Corr'n	Grid Dir.
2	0°	00'	00".0	-0.7	59".3
4	40	32	13.8	+0.9	14.7
3	99	48	12.2	+1.4	13.6

LIST OF DIRECTIONS (Continued)

Station = 2

To	Obs.	Geod.	Dir.	Grid Corr'n	Grid Dir.
4	0°	00'	00".0	+1.7	01".7
3	48	49	55.3	+2.3	57.6
1	81	51	36.2	+0.7	36.9

Station = 3

To	Obs.	Geod.	Dir.	Grid Corr'n	Grid Dir.
1	0°	00'	00".0	-1.4	58".6
2	47	10	12.9	-2.3	10.6
4	81	52	02.8	-0.7	02.1

Station = 4

To	Obs.	Geod.	Dir.	Grid Corr'n	Grid Dir.
3	0°	00'	00".0	+0.7	00".7
1	38	52	02.8	-0.9	01.9
2	96	28	15.3	-1.7	13.6

Table 31

Tabulated below are the fixed plane coordinates for stations 1 and 2 and the preliminary values for new stations 3 and 4. (Indiana East Zone)

(1) X = 724,211.02
Y = 833,160.26

(2) X = 758,634.25
Y = 845,965.49

(3) X = 729,230.10
Y = 806,329.05

(4) X = 764,640.90
Y = 818,338.03

The reductions of the observed azimuth and measured length to grid values follow:

From	To	Observed Azimuth	$-\Delta\alpha$	A	Grid Azimuth
3	4	251° 47' 10".14	-31' 14".3	-0.7	251° 15' 55".14

A = second term correction 3-4. $\Delta\alpha$ is computed from X' and Y for station 3 using data given in projection tables. For the Lambert projection, the θ angle is required and this may be computed by using the longitude scaled from a map, providing this can be done within 2"-3" of longitude or 200 to 300 feet and the data given in the projection tables. The precise computation may be made as follows: $\text{Tan } \theta = \frac{X'}{Rb - Y}$. Grid azimuth is given to hundredths of seconds but in practice should not be carried to more than tenths unless precise computation of $\Delta\alpha$ and "A" is made.

From	To	Measured Length	B	Grid Length
3	4	37388.373	0.9999741	37,387.405

B = Combined factor based on a mean elevation for stations 3 and 4 of 1100 ft./(0.9999474) and the mean X' for the stations of 246936 (1.0000267).

COMPUTATION OF ELEMENTS FOR FORMATION OF EQUATIONS

From	To	ΔX	ΔY	Tan or Cot*	Azimuth	$s^2 \times 10^{-8}$
1	2	-34423.23	-12805.23	0.37199385*	249°35'42".391	Fixed
1	3	- 5019.08	+26831.21	0.18706126	349 24 16.671	7.4510
1	4	-40429.88	+14822.23	0.36661573*	290 08 01.431	18.5427
2	3	+29404.15	+39636.44	0.74184639	36 34 11.059	24.3565
2	4	- 6006.65	+27627.46	0.21741593	347 44 01.965	7.9936
3	4	-35410.80	-12008.98	0.33913326*	251 15 59.375	13.9814

Table 32

TRANSVERSE MERCATOR PROJECTION

Indiana

Both zones

$$\Delta\alpha = Mx' - e$$

y	M	ΔM
0	0.007 5541	751
100,000	0.007 6292	757
200,000	0.007 7049	763
300,000	0.007 7812	768
400,000	0.007 8580	774
500,000	0.007 9354	780
600,000	0.008 0134	786
700,000	0.008 0920	793
800,000	0.008 1713	798
900,000	0.008 2511	805
1,000,000	0.008 3316	812
1,100,000	0.008 4128	818
1,200,000	0.008 4946	825
1,300,000	0.008 5771	831
1,400,000	0.008 6602	839
1,500,000	0.008 7441	846
1,600,000	0.008 8287	

e

y \ x'	100,000	200,000	300,000	400,000
0	0.0	0.0	0.2	0.5
500,000	0.0	0.0	0.2	0.6
1,000,000	0.0	0.1	0.2	0.6
1,500,000	0.0	0.1	0.3	0.7

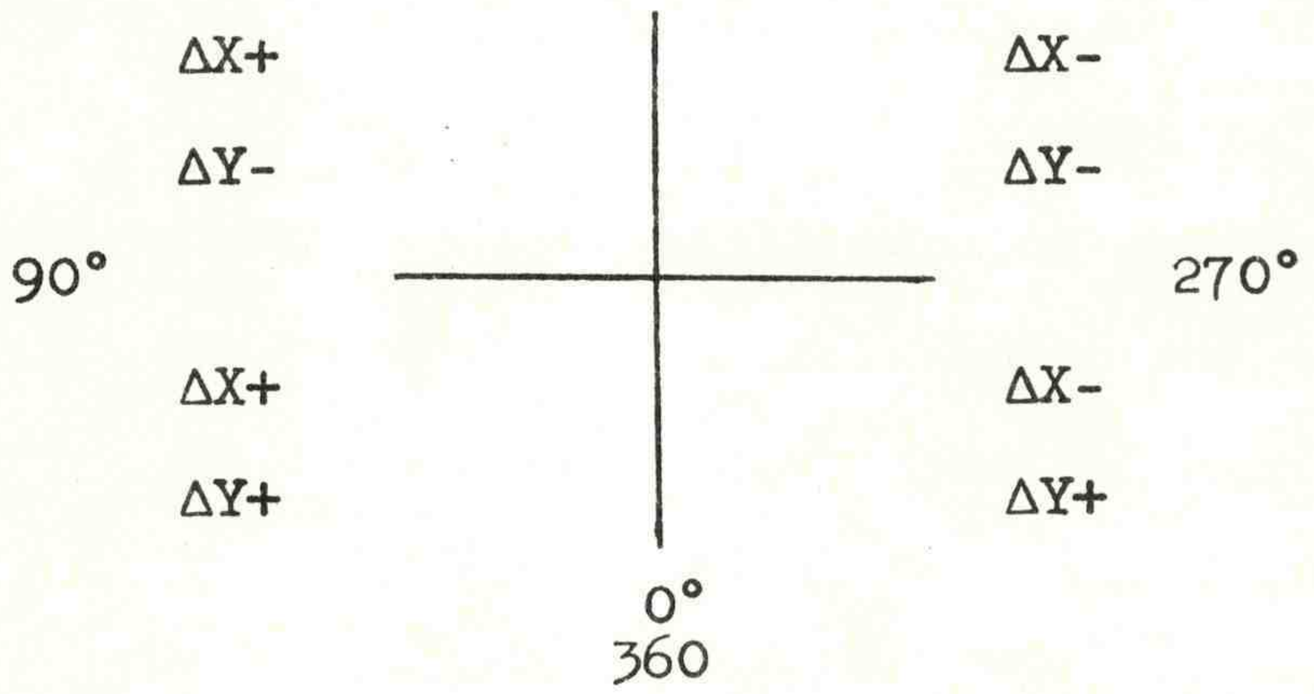
TRANSVERSE MERCATOR PROJECTION

INDIANA

Both zones

x' (feet)	Scale in units of 7th place of logs	Scale expressed as a ratio	x' (feet)	Scale in units of 7th place of logs	Scale expressed as a ratio
0	-144.8	0.9999667	175,000	+ 7.3	1.0000017
5,000	-144.7	0.9999667	180,000	+ 16.1	1.0000037
10,000	-144.3	0.9999668	185,000	+ 25.1	1.0000058
15,000	-143.7	0.9999669	190,000	+ 34.4	1.0000079
20,000	-142.8	0.9999671	195,000	+ 44.0	1.0000101
25,000	-141.7	0.9999674	200,000	+ 53.8	1.0000124
30,000	-140.3	0.9999677	205,000	+ 63.9	1.0000147
35,000	-138.7	0.9999681	210,000	+ 74.2	1.0000171
40,000	-136.9	0.9999685	215,000	+ 84.7	1.0000195
45,000	-134.8	0.9999690	220,000	+ 95.5	1.0000220
50,000	-132.4	0.9999695	225,000	+106.6	1.0000247
55,000	-129.8	0.9999701	230,000	+117.9	1.0000271
60,000	-126.9	0.9999708	235,000	+129.4	1.0000298
65,000	-123.8	0.9999715	240,000	+141.2	1.0000325
70,000	-120.5	0.9999723	245,000	+153.2	1.0000353
75,000	-116.9	0.9999731	250,000	+165.5	1.0000381
80,000	-113.0	0.9999740	255,000	+178.0	1.0000410
85,000	-108.9	0.9999749	260,000	+190.8	1.0000439
90,000	-104.6	0.9999759	265,000	+203.9	1.0000469
95,000	-100.0	0.9999770	270,000	+217.2	1.0000500
100,000	- 95.2	0.9999781	275,000	+230.7	1.0000531
105,000	- 90.1	0.9999793	280,000	+244.5	1.0000563
110,000	- 84.7	0.9999805	285,000	+258.5	1.0000595
115,000	- 79.1	0.9999818	290,000	+272.8	1.0000628
120,000	- 73.3	0.9999831	295,000	+287.3	1.0000662
125,000	- 67.2	0.9999845	300,000	+302.1	1.0000696
130,000	- 60.9	0.9999860	305,000	+317.1	1.0000730
135,000	- 54.3	0.9999875	310,000	+332.3	1.0000765
140,000	- 47.5	0.9999891	315,000	+347.8	1.0000801
145,000	- 40.4	0.9999907	320,000	+363.6	1.0000837
150,000	- 33.1	0.9999924	325,000	+379.6	1.0000874
155,000	- 25.5	0.9999941	330,000	+395.9	1.0000912
160,000	- 17.7	0.9999959	335,000	+412.4	1.0000950
165,000	- 9.6	0.9999978	340,000	+429.2	1.0000988
170,000	- 1.3	0.9999997	345,000	+446.2	1.0001027
			350,000	+463.4	1.0001067

The computation of the various quantities required for forming the equations is given in Table 32. ΔX and ΔY are the differences From station minus To station; Tan $A_{i-k} = \frac{\Delta X}{\Delta Y}$ or Cot $A_{i-k} = \frac{\Delta Y}{\Delta X}$, the use of the value which is less than one is generally considered best especially when small angles are involved; the Azimuth is derived from either the tangent or cotangent with the quadrant determined as shown below; $S^2 \times 10^{-8}$ is $\Delta X^2 + \Delta Y^2$ and pointing off twelve decimal places to the left.



To obtain the "N" terms (Table 33), the observed azimuths are computed by the following formula: $Az_{i-k} = Az_o - D_o + D_k$, where Az_o is the computed azimuth (from Table 32) to the initial station, or in the case where the observations are at a fixed control point, the azimuth to another fixed point. D_o is the observed grid direction corresponding to this azimuth. D_k is the observed grid direction to the point for which the observed azimuth is required. The "N" term for the observation equation to

the initial station is always zero since D_o and D_k are identical and there are no observation equations between fixed points. If the initial direction is 0° 00' 00".0, the subtraction of the D_o direction is of course omitted when computing the "N" terms. In this regard, the lists of grid directions can always be referred to an initial direction of 0° 00' 00".0 by subtracting the initial directions, in the case of new stations, from the other directions and for the fixed points, subtract the directions to a fixed point from the other quantities. The Computed $Azimuth_{i-k}$ is from Table 32, and N_{i-k} is this value minus the Observed $Azimuth_{i-k}$.

Also included in this table is the computation of the "N" terms for the observed azimuth and measured length. The computed azimuth is taken from Table 32 and the computed length is determined from the formula $\sqrt{\Delta X^2 + \Delta Y^2}$, using the ΔX and ΔY for the line from Table 32 also.

COMPUTATION OF N TERMS

From	To	Computed Az_{i-k}	Computation of Observed Az_{i-k}	N_{i-k}
1	2		Fixed Azimuth	
1	3		Az(1-2) 249° 35' 42".391 -D(1-2) -359 59 59.3 +D(1-3) + 99 48 13.6 Az(1-3) 349 23 56.691	+19".980
		349° 24' 16".671		
1	4		Az(1-2) 249 35 42.391 -D(1-2) -359 59 59.3 +D(1-4) + 40 32 14.7 Az(1-4) 290 07 57.791	+ 3.640
		290 08 01.431		

From	To	Computed Az_{i-k}	Computation of Observed Az_{i-k}			N_{i-k}
2	1		Fixed Azimuth			
2	3		Az(2-1)	69°	35'	42".391
			-D(2-1)	- 81	51	36.9
			+D(2-3)	+ 48.	49	57.6
		36° 34' 11".059	Az(2-3)	36	34	03.091 + 7".968
2	4		Az(2-1)	69	35	42.391
			-D(2-1)	- 81	51	36.9
			+D(2-4)	+ 0	00	01.7
		347 44 01.965	Az(2-4)	347	44	07.191 - 5.226
3	1	169 24 16.671	Az(3-1)	169	24	16.671 0.000
3	2		Az(3-1)	169	24	16.671
			-D(3-1)	-359	59	58.6
			+D(3-2)	+ 47	10	10.6
		216 34 11.059	Az(3-2)	216	34	28.671 -17.612
3	4		Az(3-1)	169	24	16.671
			-D(3-1)	-359	59	58.6
			+D(3-4)	+ 81	52	02.1
		251 15 59.375	Az(3-4)	251	16	20.171 -20.796
4	3	71 15 59.375		71	15	59.375 0.000
4	1		Az(4-3)	71	15	59.375
			-D(4-3)	- 0	00	00.7
			+D(4-1)	+ 38	52	01.9
		110 08 01.431	Az(4-1)	110	08	00.575 + 0.856
4	2		Az(4-3)	71	15	59.375
			-D(4-3)	- 0	00	00.7
			+D(4-2)	+ 96	28	13.6
		167 44 01.965	Az(4-2)	167	44	12.275 -10.310
3	4	251 15 59.375	Obs Az	251	15	55.14 + 4.235

From	To	Computed $Length_{i-k}$ (ft.)	Measured $Length_{i-k}$	N_{i-k}(ft.)
3	4	37,391.715	37,387.405	+ 4.310

Table 33

The Table for the Formation of Normal Equations (Table 34) is computed using the basic equations (p.190) and the data in Tables 32 and 33. Table 34 is used in the same manner as the Correlate Equations for condition equations to form the Normal Equations and the corrections to the observed quantities.

To form this table, the first step is to divide 206265 for the line by S^2/and then multiply the corresponding ΔX and ΔY to determine the coefficients for each direction observation equation. Only two multiplications are required for the direction observation equations over both ends of a line since the coefficients for dX_i and dX_k are identical numerically but opposite in sign, and this is also true for dY_i and dY_k. The signs of the direction coefficients may be obtained from the table below:

ΔX_{i-k}	ΔY_{i-k}	dX_i	dY_i	dX_k	dY_k
+	+	+	-	-	+
+	-	-	-	+	+
-	-	-	+	+	-
-	+	+	+	-	-

To simplify the determination of the decimal point, S^2 is given in the form of $S^2 \times 10^{-8}$; and to compensate for this decimal change, move the decimal point for 206265, ΔX and ΔY four places to the left. The values for $\frac{20.6265}{S^2 \times 10^{-8}}$ are as follows, $S^2 \times 10^{-8}$ from Table 32:

From	To		From	To	
1	3	2.7683	2	4	2.5804
1	4	1.1124	3	4	1.4753
2	3	0.8469			

A few examples of this computation follow:

(1) 1-3 = Since station 1 is fixed, there are no Z_i, dX_i, and Y_i quantities and only the "k" portion of the equation is used

= $0.00Z(1) + 0.00dX_1 + 0.00dY_1 - (2.7683)(2.6831)_3$ $(2.7683)(0.5019)_3$ or $- 7.428(dX_3) - 1.389(dY_3)$

For direction 3-1 the Z equation comes in and the equation is $+1.00(Z_3) - 7.428(dX_3) - 1.389(dY_3)$.

The "N" terms from Table 33 are tabulated with their respective equation.

(2) 3-4 = $+ 1.00(Z_3) - (1.4753)(1.2009)_3 + (1.4753)(3.5411)_3 + (1.4753)(1.2009)_4 - (1.4753)(3.5411)_4$

or $+ 1.00(Z_3) - 1.772(dX_3) + 5.224(dY_3) + 1.772(dX_4) - 5.224(dY_4)$

4-3 = $- 1.772(dX_3) + 5.224(dY_3) + 1.00\ (Z_4) + 1.772(dX_4) - 5.224(dY_4)$

Coefficients for the azimuth observation equation are identical to the direction equation coefficients for the line 3-4. The coefficients for the length observation equation are computed using the basic formula (p.190) and the data in Table 32.

TABLE FOR FORMATION OF NORMAL EQUATIONS

From	To	Wt.=p	Z_3	dX_3	dY_3	Z_4	dX_4	dY_4	N	ε	Corrections V's	Corrections Adopt
1	3	1.00		-7.428	-1.389				+19.980	+11.163	-2".233	-2".23
1	4	1.00					-1.649	-4.497	+ 3.640	- 2.506	+0.793	+0.80
2	3	1.00		-3.357	+2.490				+ 7.968	+ 7.101	+2.686	+2.69
2	4	1.00					-7.129	-1.550	- 5.226	-13.905	+0.428	+0.43
3	1	1.00	+1.00	-7.428	-1.389				0.000	- 7.817	+1.041	+1.04
3	2	1.00	+1.00	-3.357	+2.490				-17.612	-17.479	+0.360	+0.36
3	4	1.00	+1.00	-1.772	+5.224		+1.772	-5.224	-20.796	-19.796	-1.402	-1.40
4	1	1.00				+1.00	-1.649	-4.497	+ 0.856	- 4.290	+1.511	+1.51
4	2	1.00				+1.00	-7.129	-1.550	-10.310	-17.989	-1.153	-1.16
4	3	1.00		-1.772	+5.224	+1.00	+1.772	-5.224	0.000	+ 1.000	-0.358	-0.36
A 3	4	0.25		-1.772	+5.224		+1.772	-5.224	+ 4.235	+ 4.235	+0.375	+0.38
L 3	4	2.68		-0.947	-0.321		+0.947	+0.321	+ 4.310	+ 4.310	+0.623 ft.	

Table 34

The "N" terms are from Table 33 and the weights as determined earlier. Note that the weight (p) is used rather than a/p. "Σ" is the summation of each equation including "N" but excluding the weights.

The Normal Equations are shown in Table 35 and were computed as described previously on pp.178-181. The "n" term in computed using the "N" terms as though these values were another equation. For example, the "n" for dX_3 is:

(-7.428)(+19.980) + (-3.357)(+7.968) + (-7.428)(0.000)
+ (-3.357)(-17.612) + (-1.772)(-20.796) + (-1.772)(0.000)
+ (-1.772)(+4.235))0.25) + (-0.947)(+4.310)(2.68) = -92.0007.

The Forward and Back Solutions are given in Tables 36 and 37.

To obtain the Corrections to the Observed Quantities (V's), the "C" values are substituted in Table 34 and each sum is added to the "N" term for the equation involved as shown in the following examples:

1-3 = (-7.428)(+2.70519) + (-1.389)(+1.52565) + 19.980 = 2".233

3-4 = (+1.00)(+23.25472) + (-1.772)(+2.70519) + (+5.224)(+1.52565)
+ (+1.772)(-1.01139) + (-5.224)(+1.00394) - 20.796 =
-1".402

L3-4 = (-0.947)(+2.70519) + (-0.321)(+1.52565) + (+0.947)(-1.01139)
+ (+0.321)(+1.00394) + 4.310 = +0.623 ft. (Note correction is in feet.)

NORMAL EQUATIONS

Z_3	dX_3	dY_3	Z_4	dX_4	dY_4	n	E
+3.00	- 12.557	+ 6.325		+ 1.772	- 5.224	- 38.408	- 45.092
	+142.3576	-16.0962	-1.772	- 9.4684	+ 20.0134	- 92.0007	+ 30.4767
		+77.9378	+5.224	+ 20.0134	- 61.6790	-158.5810	-126.8560
			+3.000	- 7.006	- 11.271	- 9.454	- 21.279
				+116.5521	+ 16.9176	+ 79.3064	+218.0871
					+106.9300	+110.6775	+176.3645

Table 35

FORWARD SOLUTION

Z_3	dX_3	dY_3	Z_4	dX_4	dY_4	n	E
+3.00	- 12.557	+ 6.325		+ 1.772	- 5.224	- 38.408	- 45.092
	+ 4.18567	- 2.10833		- .59067	+ 1.74133	+ 12.80267	+ 15.03067
	+142.3576	-16.0962	-1.772	- 9.4684	+ 20.0134	- 92.0007	+ 30.4767
	+ 89.7981	+10.3781	-1.772	- 2.0514	- 1.8525	-252.7638	-158.2635
		- .11557	+ .01973	+ .02284	+ .02063	+ 2.81480	+ 1.76243
		+77.9378	+5.224	+ 20.0134	- 61.6790	-158.5810	-126.8560
		+63.4032	+5.4288	+ 16.5145	- 50.4510	- 48.3918	- 13.4963
			- .08562	- .26047	+ .79572	+ .76324	+ .21287
			+3.000	- 7.006	- 11.271	- 9.454	- 21.279
			+2.5002	- 8.4605	- 6.9878	- 10.2983	- 23.2464
				+ 3.38393	+ 2.79490	+ 4.11899	+ 9.29781
				+116.5521	+ 16.9176	+ 79.3064	+218.0871
				+ 82.5273	+ 9.4556	+ 73.9743	+165.9572
					- .11458	- .89636	- 2.01094
					+106.9300	+110.6775	+176.3645
					+ 37.0366	- 37.1826	- 0.1460
						+ 1.00394	+ .00394

Table 36

BACK SOLUTION

	C's
Z_3	+23.25472
dX_3	+ 2.70519
dY_3	+ 1.52565
Z_4	+ 3.50243
dX_4	- 1.01139
dY_4	+ 1.00394

Table 37

The Final Adjusted Coordinates are obtained by adding the dX and dY values (C's) to their respective preliminary coordinates.

(3)

X_p	729,230.10		Y_p	806,329.05	
	+ 2.705			+ 1.526	
X_F	729,232.805		Y_F	806,330.576	

(4)

X_p	764,640.90		Y_p	818,338.03	
	- 1.011			+ 1.004	
X_F	764,639.889		Y_F	818,339.034	

To check the adjustment, the observed angles in the triangles should be taken out using the lists of directions and tabulated as shown on Table 38. The corrections (V's) are then adopted to fit the triangle closures and the triangle sides computed using the Law of Sines. The angle corrections are determined in the following manner as previously described for the Grid corrections (pp. 193-195).

Triangle 4-3-1

Identifier

1 Corr'n angle 4 = V_{4-1} minus V_{4-3} = (+1".51)-(-0".36) = +1".87

2 Corr'n angle 3 = V_{3-4} minus V_{3-1} = (-1.40)-(+1.04) = -2.44

3 Corr'n angle 1 = V_{1-3} minus V_{1-4} = (-2.23)-(+0.80) = -3.03

Sum -3.60

The triangle computation is self checking as each side appears in two triangles. Note also that the computed side 3-4 = 37,388.028 is equal to the measured grid length plus the adjustment correction (37,387.405 + 0.623 = 37,388.028).

As a final check on the adjusted coordinates, the values are computed using the adjusted angles and lengths from Table 38. This computation is also self checking and is shown in Table 39. The form is self explanatory. Note that the computed coordinates check those obtained by applying the "C"'s to the preliminary values (p.209). Also that the adjusted azimuth 3-4 251° 15' 55".52 checks exactly the observed azimuth plus the adjustment correction (251° 15' 55".14 + 0".38 = 251° 15' 55".52).

COMPUTATION OF PLANE TRIANGLES

Form 25g
DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
(Ed. June 1950)

State: Indiana

$$s_{1\text{-}3} = \frac{s_{2\text{-}3}}{\sin(1)} \times \sin(2)$$

$$s_{1\text{-}2} = \frac{s_{2\text{-}3}}{\sin(1)} \times \sin(3)$$

	STATION	OBSERVED ANGLE	CORR'N	PLANE ANGLE	
	Identifiers ↓ 2-3				36727.819
	1 3	47 10 12.0	-0.68	11.32	0.73337177
	2 1	99 48 14.3	-2.23	12.07	0.98539793
	3 2	33 01 39.3	-2.69	36.61	0.54503179
	1-3		-5.6		49349.482
	1-2	05.6			27295.609
	2-3				27295.609
	1 4	38 52 01.2	+1.87	03.07	0.62752177
	2 3	81 52 03.5	-2.44	01.06	0.98994224
	3 1	59 15 58.9	-3.03	55.87	0.85954487
	1-3		-3.6		43059.982 +1
	1-2	03.6			37388.027 +1
	2-3				49349.482
	1 4	96 28 12.9	-0.80	12.10	0.99363094
	2 3	34 41 51.5	-1.76	49.74	0.56923863
	3 2	48 49 55.9	+2.26	58.16	0.75279212
	1-3		-0.3		28271.696
	1-2	00.3			37388.028
	2-3				36727.819
	1 4	57 36 11.7	-2.67	09.03	0.84435138
	2 1	40 32 15.4	+0.80	16.20	0.64995002
	3 2	81 51 35.2	-0.43	34.77	0.98992420
	1-3		-2.3		28271.697 -1
	1-2	02.3			43059.984 -1

M 2755-4

Table 38

Form 26g
DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
(Ed. June 1950)

PLANE COORDINATE COMPUTATION, TRIANGULATION
(For calculating machine computation)

			°	′	″				°	′	″
α	2	to 3	249	35	42.39	α	3	to 2	69	35	42.39
$2^d \angle$		&	+ 99	48	12.07	$3^d \angle$		&	- 33	01	36.61
α_1	2	to 1	349	23	54.46	α_2	3	to 1	36	34	05.78
			180	00	00.00				180	00	00.00
α_1'	1	to 2	169	23	54.46	α_2'	1	to 3	216	34	05.78
		First Angle of Triangle	47	10	11.32						

$\sin \alpha_1$ -0.18397775 $\cos \alpha_1$ +0.98293041 $\sin \alpha_2$ +0.59578022 $\cos \alpha_2$ +0.80314752

x_2	724,211.02	2 1	y_2	833,160.26	x_3	758,634.25	3 2	y_3	845,965.49
$-s \sin \alpha_1$	+ 5021.785	s = 27295.609	$-s \cos \alpha_1$	- 26829.684	$-s \sin \alpha_2$	- 29401.445	s = 49349.482	$-s \cos \alpha_2$	- 39634.914
x_1	729,232.805	1 3	y_1	806,330.576	x_1	729,232.805	1 3	y_1	806,330.576

			°	′	″				°	′	″
α	2	to 3	216	34	05.78	α	3	to 2	36	34	05.78
$2^d \angle$		&	+ 34	41	49.74	$3^d \angle$		&	- 48	49	58.16
α_1	2	to 1	251	15	55.52	α_2	3	to 1	347	44	07.62
			180	00	00.00				180	00	00.00
α_1'	1	to 2	71	15	55.52	α_2'	1	to 3	167	44	07.62
		First Angle of Triangle	96	28	12.10						

$\sin \alpha_1$ -0.94701662 $\cos \alpha_1$ -0.32118457 $\sin \alpha_2$ -0.21242583 $\cos \alpha_2$ +0.97717719

x_2	729,232.805	2 3	y_2	806,330.576	x_3	758,634.25	3 2	y_3	845,965.49
$-s \sin \alpha_1$	+ 35407.084	s = 37388.028	$-s \cos \alpha_1$	+ 12008.458	$-s \sin \alpha_2$	6005.638	s = 28271.696	$-s \cos \alpha_2$	- 27626.456
x_1	764,639.889	1 4	y_1	818,339.034	x_1	764,639.888	1 4	y_1	818,339.034

+1

M-2756-2

Table 39

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NOTE: On October 3, 1970, a governmental reorganization created the National Oceanic and Atmospheric Administration, NOAA, replacing the Environmental Science Services Administration, ESSA. The Coast and Geodetic Survey, which was a component of ESSA, has been renamed the National Ocean Survey (NOS) in NOAA. A major component of the NOS is the Office of the National Geodetic Survey (NGS).

GPO 930-611