NOAA TECHNICAL MEMORANDUM NMFS SEFC-146

APPLICATION OF A TRUNCATED POISSON MODEL TO SEAFOOD CONSUMPTION FREQUENCIES

Lysander Ng

September 1984

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Malcolm Baldrige, Secretary
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John V. Byrne, Administrator
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William G. Gordon, Assistant Administer for Fisheries

ABSTRACT

This report deals with the development and application of a model useful for describing and comparing frequency data. The model is a truncated Poisson probability density function with the zero-class unknown or censored. The development of the model is outlined and certain final derivations are explicitly presented. A numerical example is provided to clarify computation.

The model was fitted to data resulting from the seafood consumption survey carried out by the National Purchase Diary Inc.[1/] during the period 1973-74. The analysis resulted in a descriptive model of consumption frequencies by the American public on various fish species.

1/ Mention of commercial firms does not imply endorsement by the National Marine Fisheries Service, NOAA.

INTRODUCTION

It is of value to the fishing industry to understand the public's fish consumption pattern as related to frequency and quantity. The basic interest is in the marketability of various fishery products. Accordingly, issues such as economic feasibility, nutritional contribution, and safety of seafood need to be considered. As a step towards this, this report describes the consumption frequencies of various fish species by adopting a probabilistic model.

In this report, the basic system being modeled and the data requirements are described. The theoretical development of the model is outlined and certain final formulations are presented. A numerical example is provided to clarify computation. The model is applied to the data resulting from a seafood consumption survey of the American public. The data under consideration consist of the monthly consumption frequencies of individuals on various fish species. The model allows the consumption frequencies of each species to be described by a fitted probabilistic distribution. A table of the estimated parameters and their confidence limits of the distributions are provided for the various fish species being considered. The computer program written to perform the calculations is listed in the Appendix. Finally, the fitted models and the observed data are compared, and considerations for improving model accuracy are discussed.

Often, when the objective is to assess the frequency occurrence of events, it is difficult to discriminate between subsamples of the population in which no events occurred during the observational period and subsamples in which events could not have occurred. For example, a general population may be classified into two subpopulations: those who consume shrimp, and those who do not consume shrimp. It is possible for some shrimp-eaters not to consume shrimp while under observation. Therefore, if available data from the survey consist only of frequencies of consumption and not whether the individuals eat shrimp or not, the division between the two subpopulations becomes unclear. We can assume that those who have consumption frequencies other than zero to be shrimp-eaters, but it would not be possible to identify among those who have zero consumption frequency which of the two subpopulations they belong to.

The truncated Poisson model allows one to estimate the shrimp and non-shrimp eating proportions among the group with zero consumption frequency. The model assumes that the consumption frequency of the shrimp-eater subpopulation follows a Poisson distribution. For a review of the basic characteristics of the Poisson distribution, readers may wish to consult an elementary text on probability theory such as Ross (1976).

Since the shrimp-eaters, who had zero consumption frequency, are indistinquishable from non-eaters, the total number of shrimp-eaters within the sample is unknown. In other words, the zero-class of the eater subsample is unknown or missing. When certain values of a probability distribution are missing, we say that that distribution is truncated. Consequently, the model we are discussing here is a zero-class truncated Poisson model, or truncated Poisson in short.

The truncated Poisson model can be fitted to the shrimp-eater subsample which has non-zero consumption frequencies. Then, the fitted truncated Poisson model may be extended to include the zero-class, therefore, allowing one to estimate the actual size of the zero-class. From this estimation, we may evaluate the shrimp and non-shrimp eating proportions among the group with zero consumption frequency. Further, we may estimate the shrimp and non-shrimp eating proportions among the total general population.

MODEL

The chief body of theory used here is developed primarily in Blumenthal et al. (1978). The major theoretical results cogent to this report are explicitly presented here.

The probability density function of the Poisson random variable Y with parameter λ is given by

$$\text{Prob.}(Y=y) = e^{-\lambda} \cdot \frac{\lambda^y}{y!} , \quad y = 0,1,2,\ldots, \quad \text{and} \quad \lambda > 0 . \tag{1}$$

The truncated Poisson random variable X with parameter λ and the zero-class censored is given by

$$\text{Prob.}(X=x) = (e^{\lambda}-1)^{-1} \cdot \frac{\lambda^x}{x!} , \quad x = 1,2,3,\ldots, \quad \text{and} \quad \lambda > 0 . \tag{2}$$

Given n independent identically distributed random sample $x_1, x_2, x_3, \ldots, x_n$ from the truncated Poisson population as described by Statement (2), let n_0 be the size of zero-class, and N be the total sample size (i.e., $N = n_0 + n$).

The maximum likelihood estimator (M.L.E.) of λ, $\hat{\lambda}$, can be computed by the following equation:

$$\hat{\lambda} = \frac{1 - e^{-\hat{\lambda}}}{n} \cdot \sum_{i=1}^{n} x_i . \tag{3}$$

Let us define the following: $Q = e^{-\hat{\lambda}}$ and $P = 1 - Q$. Then, the M.L.E. $\hat{n}_0$ of the size of zero-class can be calculated by

$$\hat{n}_0 = nQ / P . \tag{4}$$

The M.L.E. $\hat{N}$ of the total sample size is then simply given by

$$\hat{N} = \hat{n}_0 + n. \tag{5}$$

Using the central limit theorem, the $(1-\alpha)100\%$ approximate asymptotic confidence interval (C.I.) for N is computed as

$$\hat{N} - Z_{1-\alpha/2}\cdot(\hat{N}Q\ /\ (P - \hat{\lambda}Q))^{\frac{1}{2}} < N < \hat{N} + Z_{1-\alpha/2}\cdot(\hat{N}Q\ /\ (P - \hat{\lambda}Q))^{\frac{1}{2}}\ , \tag{6}$$

where $Z_{1-\alpha/2}$ is the $100\times(1-\alpha/2)$ percentile of the standard normal variate.

The $(1-\alpha)100\%$ approximate asymptotic C.I. for n_0 is then

$$\hat{N} - n - Z_{1-\alpha/2}\cdot(\hat{N}Q\ /\ (P - \hat{\lambda}Q))^{\frac{1}{2}} < n_0 < \hat{N} - n + Z_{1-\alpha/2}\cdot(\hat{N}Q\ /\ (P - \hat{\lambda}Q))^{\frac{1}{2}}. \tag{7}$$

Using the fact that the statement

$$n_0 = \frac{ne^{-\lambda}}{1 - e^{-\lambda}} \tag{8}$$

is asymptotically true as the sample size increases, let us allow substitution for n_0 be made in Statement (7) with Statement (8). This can enable the determination of the approximate asymptotic C.I.'s for λ.

If besides the x_i's, $i = 1,2,3,\ldots,n$, we observed k zeroes as a result of taking samples from the general population, we may estimate the proportion of observations which are members of the population of interest, C. The M.L.E. of C, $\hat{C}$, is computed by

$$\hat{C} = \frac{\hat{N}}{k + n}\ . \tag{9}$$

Finally, the $(1-\alpha)100\%$ approximate C.I. for C is given by

$$\{\hat{N}-Z_{1-\alpha/2}(\hat{N}Q/(P-\hat{\lambda}Q))^{\frac{1}{2}}\}(k+n)^{-1} < C < \{\hat{N}+Z_{1-\alpha/2}(\hat{N}Q/(P-\hat{\lambda}Q))^{\frac{1}{2}}\}(k+n)^{-1}\ . \tag{10}$$

NUMERICAL EXAMPLE

Suppose that $k + n = 90$ independent observations are collected from the general population. Further suppose that the resulting sample have the following frequency distribution:

Frequency	Number Oberved
0	40
1	20
2	24
3	4
5	1
9	1

Using Statement (3), we have

$$\hat{\lambda} = \frac{1 - e^{-\hat{\lambda}}}{50} \times (20 + 24 \times 2 + 4 \times 3 + 5 + 9) .$$

Using the Newton-Raphson method, $\hat{\lambda}$ is computed to be 1.43. The Newton-Raphson procedure is an iterative technique for solving complicated equations. Readers may refer to Appendix A of Lee (1980) for detailed explications.

Using Statements (4) and (5), we have

$$\hat{N} = \frac{50\ e^{-1.43}}{1 - e^{-1.43}} + 50 = 65.73 .$$

From Statement (6), the 95% C.I. for N is computed to be

$$53.71 < N < 77.74 .$$

The 95% C.I. for λ can be approximated using Statements (7) and (8), and the Newton-Raphson method. It is computed to be

$$1.03 < \lambda < 2.67 .$$

The proportion of the 90 observations being members of the population of interest (i.e., members belonging to the complete Poisson sample) is estimated to be 65.73 / 90 = 0.73 using Statement (9). The 95% C.I. can be evaluated by way of Statement (10) to be

$$0.60 < C < 0.86 .$$

The Appendix provides a listing of the program written to accomplish the above computations. The program is implemented on a desktop computer with a BASIC interpreter.

APPLICATION

The truncated Poisson model was fitted to consumption frequencies of various species of fish. The data are results of the seafood consumption survey undertaken by the National Purchase Diary Inc. during 1973-74. The survey included 25,947 subjects, whose monthly fish consumption rates were recorded.

Table 1 gives the computed M.L.E. estimates and the 95% confidence intervals for λ and C of consumption frequencies of various species of fish. The computations were performed by the BASIC computer program listed in the Appendix.

Since λ is the theoretical average (average frequency) of the Poisson distribution, high λ value would correspond to high consumption frequency by the subpopulation of eaters. Since C is defined as the fraction of the population who are eaters, high C value would correspond to high popularity. Thus, the combination of λ and C can be indicative of the general population's consumption frequency rates. Tuna, for example, has the highest estimates of both λ and C among all the species considered, with $\lambda = 2.6$ and $C = 0.7$. Of course, tuna is a very popular seafood. The two estimated parameters of tuna indicate that 70% of the general U.S. population are tuna-eaters, and, of these tuna-eaters, the average consumption frequency is 2.6 times per month.

As a step towards ascertaining the applicability of the model, one may compare values of the actual data with those fitted by the model. Figures 1 through 7 give comparisons between model and data for seven of the popular species, respectively, clam, flounder, marine perch, pollock, salmon, shrimp, and tuna.

The square symbols connected by the dotted lines represent the actual observations. The data being considered are those with consumption frequencies of one or above. Each square symbol gives the fraction of the observations which belong to its corresponding frequency. For example, from Figure 1, of all the observations with monthly clam consumption frequencies of one or above, approximately 60% have the frequency of one.

The diamond shape symbols connected by the dash lines represent the estimated probabilities of the fitted truncated Poisson models. The 95% C.I. of each of these estimates are represented by vertical line segments through the point estimates.

Although the actual data and the fitted estimates tend to follow similar trends, there are some consistent disparities. The fitted models tend to overestimate the probabilities of consumption frequencies of two and three times per month.

DISCUSSION

The truncated Poisson model as applied to seafood consumption data has resulted generally in reasonably good fits except for some small but consistent disparities. Because the differences between the model and the data are consistent, it is quite possible that there exists a remedy which would improve the model's accuracy when applied to consumption frequencies of all fish species. As such, other truncated models such as the binomial model proposed by Blumenthal and Dahiya (1981) may be systematically considered.

Although the truncated Poisson model has slightly fallen short of providing us a perfect description of seafood consumption frequencies, there is a good theoretical justification for adopting it. The Poisson density arises when the variable of interest is the number of times an event occurs during a specified period of time (Ross, 1976). Clearly, seafood consumption frequency could follow the Poisson distribution on this basis.

There can be many reasons for the slight but consistent discrepancies between the data and the model. It could be due to the diversity of the population being studied. The truncated Poisson model assumes that the general population is made up of two subpopulations: a 1-C fraction having zero frequency with probability of one, and a C fraction being distributed according to a Poisson density. However, the actual population is likely made up of several subpopulations following the Poisson distribution with different λ values, in addition to the subpopulation of non-eaters. In other words, the actual eater subpopulation may follow a mixture of Poisson distributions. Additional research according to this hypothesis may be conducted with good possibility of fruitful results.

Meanwhile, the model may be further applied to subpopulations stratified according to combinations of demographic variables. As such, comparisons and trends among subpopulations may be inferred. Other consumption surveys taken at different periods may also be studied to allow assessment of temporal trends.

Because it is probabilistic, the truncated Poisson model is valuable analytically. It can help resolve questions which are probabilistic in nature. For example, the probability that a shrimp-eater will eat shrimp two or more times within the period of a month can be estimated.

Further, the potentially applicable scope of the truncated Poisson model is not restricted to seafood consumption. For example, it can be extended to depict catch rates of fish per boat-trip when quarries were specific but not recorded. Blumenthal et al. (1978) gave an example of the model's application to occurrences of disease in communities when the exposure possibilities to causes are unknown.

Since the truncated Poisson model is a relatively new development as compared to other statistical methodologies such as analysis of variance, applied researchers in general are not yet familiar with the model's usage. Because of the potential applicable scope and analytical value of the model, however, its popularity should increase.

REFERENCES

Blumenthal, S., Dahiya, R.C., and Gross, A.J. (1978), "Estimating Sample Size from an Incomplete Poisson Sample," Journal of American Statistical Association, Volume 73, pp. 182-187.

Blumenthal, S., and Dahiya, R.C. (1981), "Estimating the Binomial Parameter n," Journal of American Statistical Association, Volume 76, pp. 903-909.

Lee, E.T. (1980), Statistical Methods for Survival Data Analysis, Lifetime Learning Publications, Belmont, California.

Ross, S.M. (1976), A First Course in Probability, Macmillan, New York.

Table 1. The maximum likelihood estimates and the 95% approximate confidence intervals of the parameters λ and C for the various fish species.

SPECIES	λ M.L.E.	λ 95% C.I.	C M.L.E.	C 95% C.I.
Abalone	0.86	0.00,1.70	0.0034	0.0024,0.0044
Anchovy	0.56	0.00,1.05	0.0067	0.0044,0.0090
Bass	1.20	1.09,1.34	0.0453	0.0428,0.0477
Bluefish	0.70	0.00,0.92	0.0182	0.0153,0.0211
Bluegills	1.47	1.25,1.80	0.0132	0.0121,0.0142
Bonito	1.91	1.57,2.55	0.0066	0.0061,0.0071
Buffalofish	2.27	1.71,4.66	0.0026	0.0023,0.0028
Butterfish	0.69	0.00,1.79	0.0030	0.0018,0.0042
Carp	1.13	0.83,1.91	0.0036	0.0029,0.0044
Catfish,freshwater	1.31	1.19,1.45	0.0463	0.0441,0.0485
Catfish,marine	1.04	0.76,1.73	0.0041	0.0032,0.0050
Clam	1.16	1.09,1.24	0.1256	0.1213,0.1300
Cod	1.39	1.30,1.51	0.0759	0.0733,0.0785
Crab,king	0.68	0.00,1.00	0.0102	0.0079,0.0124
Crab,other	0.89	0.81,0.98	0.0762	0.0716,0.0807
Crappie	1.37	1.15,1.71	0.0118	0.0108,0.0129
Croaker	1.14	0.85,1.80	0.0044	0.0036,0.0052
Dolphin	0.54	0.00,2.00	0.0031	0.0015,0.0048
Drum	0.54	0.00,1.17	0.0054	0.0032,0.0075
Flounder	1.22	1.16,1.29	0.1804	0.1756,0.1852
Grouper	1.05	0.77,1.76	0.0040	0.0032,0.0049
Haddock	1.07	0.99,1.17	0.0842	0.0803,0.0880
Hake	1.26	1.10,1.48	0.0210	0.0194,0.0226
Halibut	0.83	0.73,0.97	0.0391	0.0356,0.0426
Herring	1.32	1.22,1.44	0.0657	0.0631,0.0683
Kingfish	1.17	0.78,3.06	0.0018	0.0013,0.0024
Lobster,northern	0.49	0.00,0.58	0.0675	0.0591,0.0758
Lobster,spiny	0.56	0.00,0.71	0.0310	0.0261,0.0359
Mackerel,jack	2.09	1.25, ∞	0.0006	0.0004,0.0007
Mackerel,other	1.48	1.33,1.67	0.0306	0.0291,0.0322
Mullet	1.47	1.15,2.14	0.0049	0.0042,0.0055
Oyster	1.07	0.99,1.18	0.0725	0.0690,0.0761
Perch,freshwater	1.38	1.17,1.69	0.0138	0.0126,0.0149
Perch,marine	1.38	1.31,1.47	0.1294	0.1260,0.1329
Pike	1.61	1.41,1.89	0.0188	0.0178,0.0199
Pollock	1.85	1.73,2.00	0.0669	0.0652,0.0686
Pompano	0.19	0.00, ∞	0.0022	0.0000,0.0062
Rockfish	0.72	0.00,1.24	0.0057	0.0041,0.0073
Sablefish	1.40	0.70, ∞	0.0004	0.0002,0.0005
Salmon	1.29	1.22,1.37	0.1296	0.1258,0.1334
Scallops	0.40	0.00,0.50	0.0612	0.0512,0.0711
Scup	0.50	0.00,1.18	0.0054	0.0031,0.0077
Shark	0.61	0.00, ∞	0.0003	0.0000,0.0007
Shrimp	1.25	1.20,1.30	0.3141	0.3080,0.3203
Smelt	1.43	1.24,1.72	0.0164	0.0153,0.0176
Snapper	0.66	0.00,0.79	0.0388	0.0342,0.0434
Snook	0.61	0.00, ∞	0.0010	0.0002,0.0018
Spot	1.49	1.16,2.21	0.0045	0.0039,0.0051
Squid and Octopus	1.03	0.71,2.09	0.0027	0.0020,0.0034
Sunfish	1.79	1.34,3.14	0.0028	0.0024,0.0031
Swordfish	0.05	0.00, ∞	0.0334	0.0000,0.0980
Tilefish	0.18	0.00, ∞	0.0026	0.0000,0.0075
Trout,freshwater	1.11	1.01,1.23	0.0550	0.0521,0.0580
Trout,marine	0.87	0.71,1.13	0.0147	0.0126,0.0167
Tuna	2.60	2.55,2.66	0.6999	0.6966,0.7031
Whitefish	1.14	1.00,1.32	0.0278	0.0258,0.0299
Finfish,other	0.88	0.81,0.96	0.0988	0.0935,0.1040
Shellfish,other	0.48	0.00,1.17	0.0055	0.0030,0.0079
Unspecified	1.37	1.31,1.43	0.2548	0.2499,0.2597

∞ - Infinity

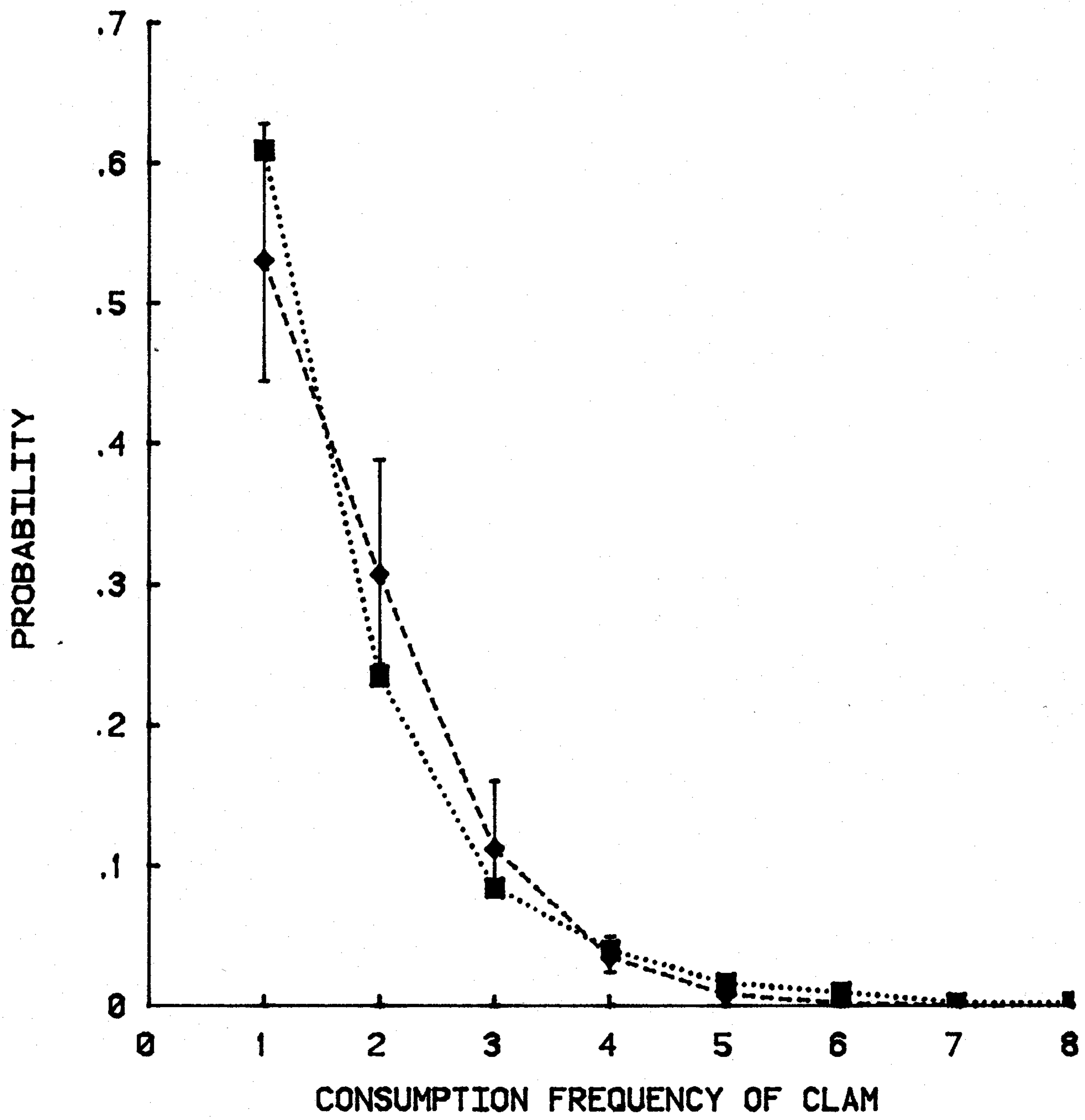

Figure 1. Comparisons between the observed data (square symbols) and the fitted truncated Poisson model (diamond shape symbols) with 95% C.I. on monthly non-zero clam consumption frequencies.

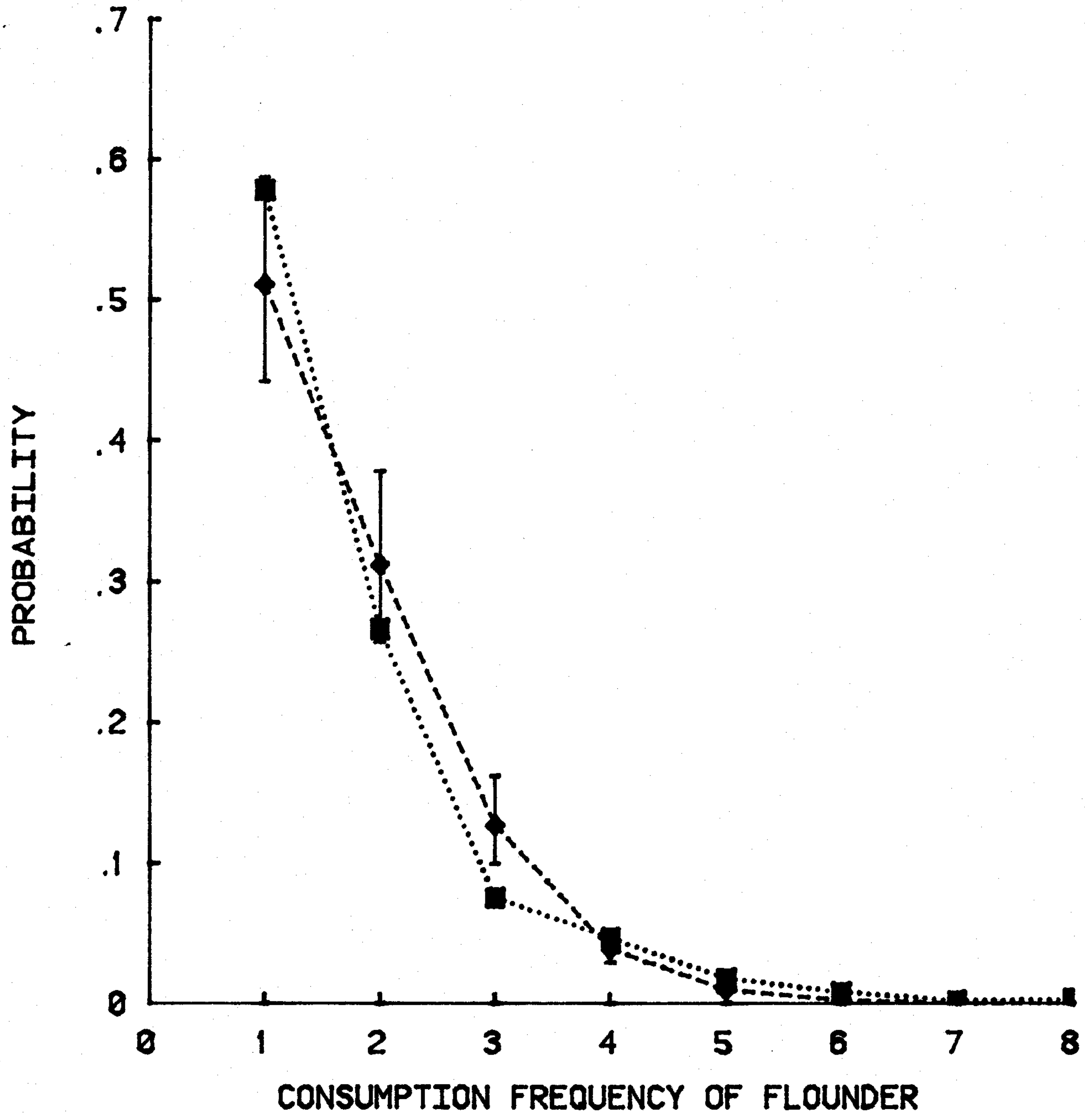

Figure 2. Comparisons between the observed data (square symbols) and the fitted truncated Poisson model (diamond shape symbols) with 95% C.I. on monthly non-zero flounder consumption frequencies.

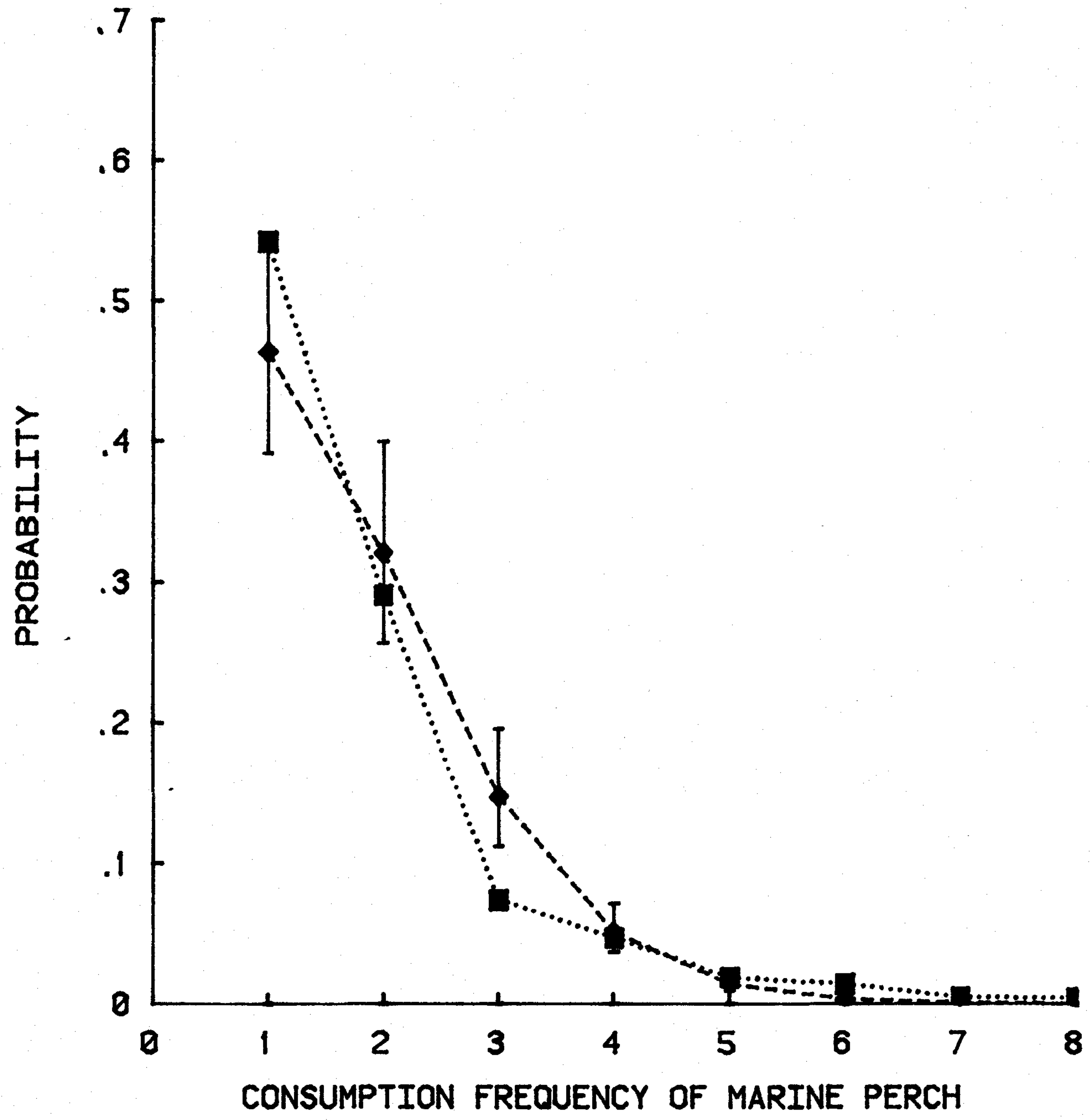

Figure 3. Comparisons between the observed data (square symbols) and the fitted truncated Poisson model (diamond shape symbols) with 95% C.I. on monthly non-zero marine perch consumption frequencies.

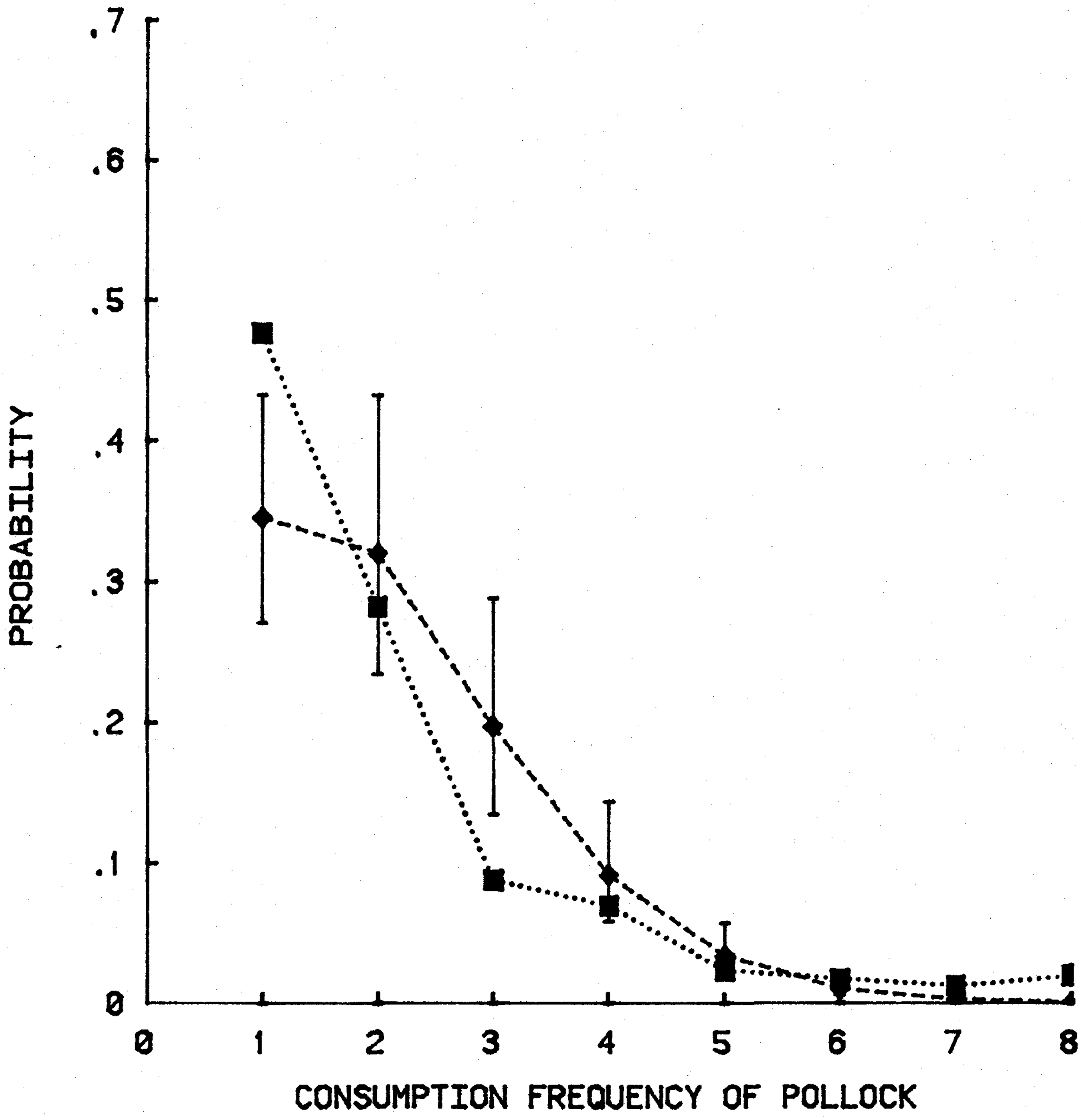

Figure 4. Comparisons between the observed data (square symbols) and the fitted truncated Poisson model (diamond shape symbols) with 95% C.I. on monthly non-zero pollock consumption frequencies.

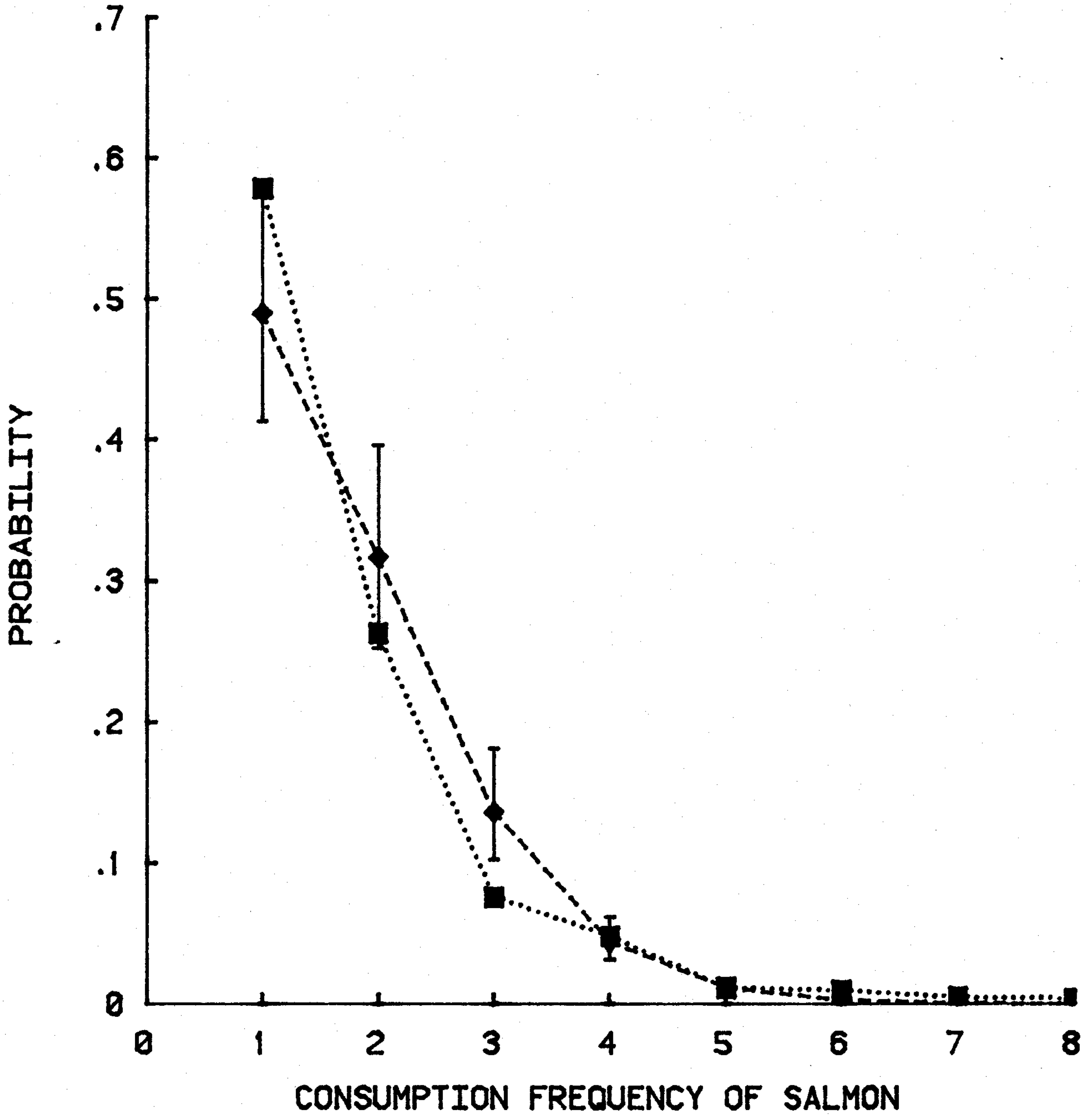

Figure 5. Comparisons between the observed data (square symbols) and the fitted truncated Poisson model (diamond shape symbols) with 95% C.I. on monthly non-zero salmon consumption frequencies.

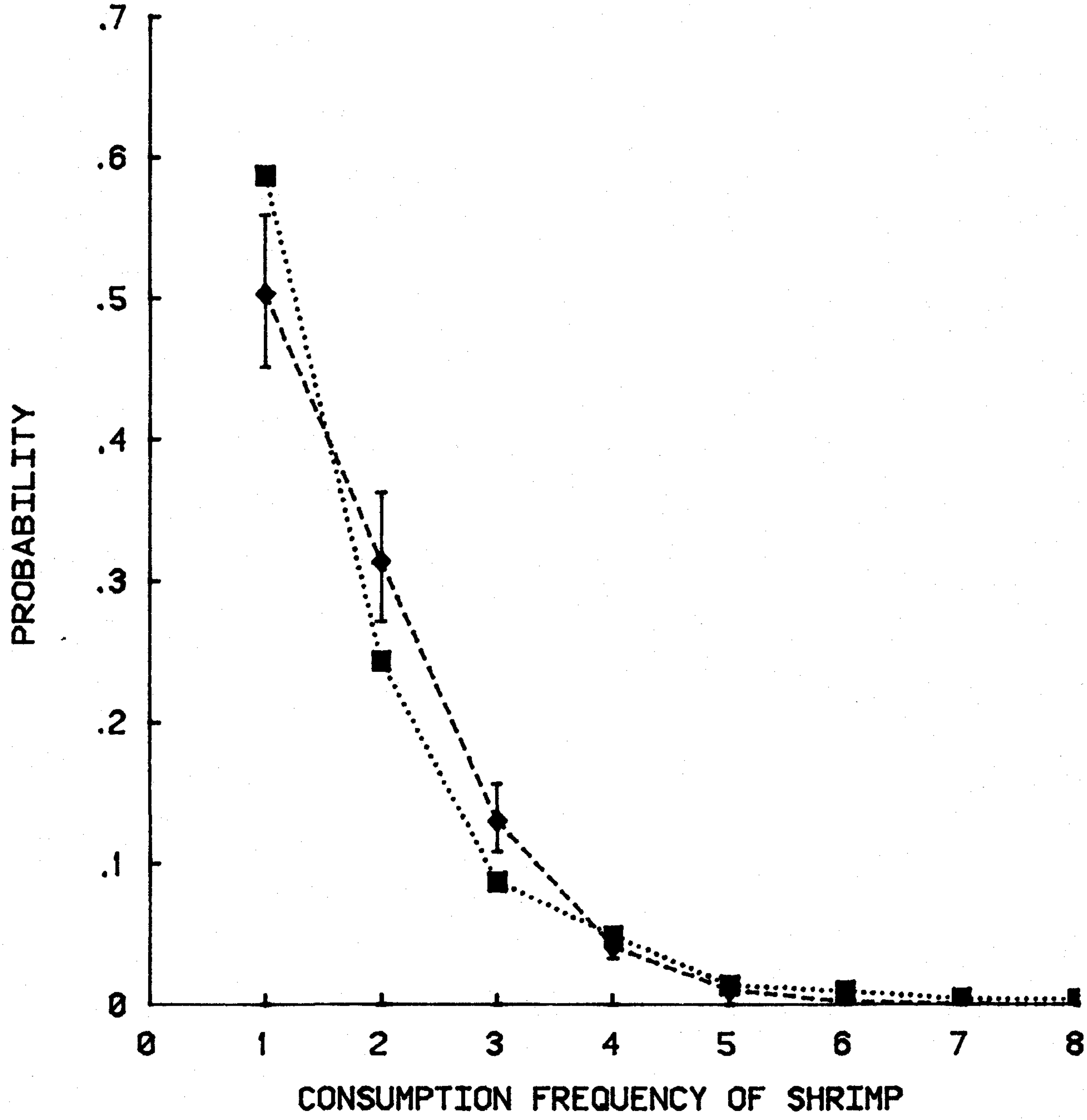

Figure 6. Comparisons between the observed data (square symbols) and the fitted truncated Poisson model (diamond shape symbols) with 95% C.I. on monthly non-zero shrimp consumption frequencies.

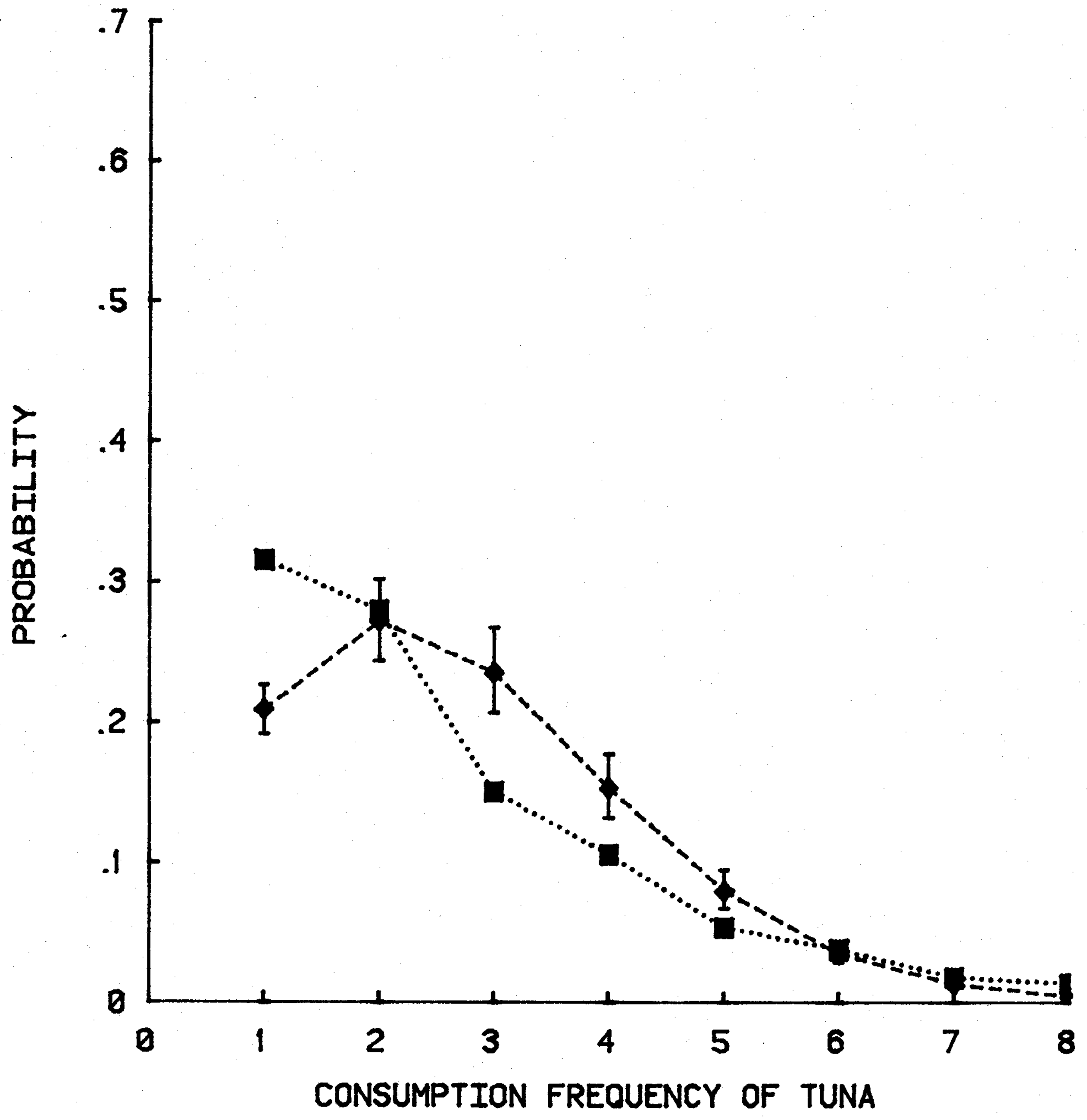

Figure 7. Comparisons between the observed data (square symbols) and the fitted truncated Poisson model (diamond shape symbols) with 95% C.I. on monthly non-zero tuna consumption frequencies.

APPENDIX

The computer listing of a program used for modeling with the truncated Poisson density is provided in the following pages. The program was written and implemented on a desktop computer with a BASIC interpreter. The input includes the number of observations at each frequency. The output includes the maximum likelihood estimates and the 95% approximate confidence intervals for the Poisson parameter λ, the zero-class sample size n_0, the total sample size N, and the Poisson population fraction of observations C.

```
100 REM AUTHOR:LYSANDER NG. AFFILIATION:NMFS-CHARLESTON. DATE:5/84
110 REM THIS PROGRAM IS WRITTEN TO MODEL FREQUENCY DATA.
120 REM THE MODEL ASSUMES THAT THE RANDOM VARIABLE FOLLOWS A POISSON
130 REM DISTRIBUTION WITH THE ZERO-CLASS CENSORED, THAT IS, A TRUNCATED
140 REM POISSON.
150 REM INPUT TO THE PROGRAM IS THE NUMBER OF SAMPLES AT EACH FREQUENCY.
160 REM OUTPUT IS THE MAXIMUM LIKELIHOOD ESTIMATES OF LAMBDA, SIZE OF
170 REM ZERO-CLASS, SIZE OF COMPLETE SAMPLE.
180 REM THE 95% ASYMPTOTIC CONFIDENCE LIMITS FOR THESE PARAMETERS ARE
190 REM ALSO PROVIDED.
200 REM IF THE  ZERO-CLASS PLUS ADDITIONAL SAMPLE WHICH IS NOT MEMBER OF
210 REM THE POPULATION IS KNOWN, THEN THE MLE OF PROBABILITY OF BEING
220 REM A MEMBER OF THE POPULATION IS PROVIDED AS WELL AS THE 95% C.I.
230 INIT
240 DIM X(200)
250 N=0
260 W=0
270 L=0
280 PRINT "BEGIN TO ENTER THE NUMBER OF SAMPLES AT EACH FREQUENCY."
290 PRINT "DISCONTINUE ENTRY BY ENTER -1."
300 N=N+1
310 PRINT "THE NUMBER OF SAMPLES WITH FREQUENCY OF ";N;" IS ";
320 INPUT X(N)
330 IF X(N)=-1 THEN 370
340 L=L+X(N)*N
350 W=W+X(N)
360 GO TO 300
370 N9=N-1
380 N=W
390 PRINT "ENTER THE SAMPLE NUMBER OF SUM OF ZERO-CLASS AND";
400 PRINT " COMPLEMENTAL POPULATION."
410 PRINT "IF NOTHING IS KNOWN OF THE ZERO-CLASS, ENTER 0."
420 INPUT N7
430 PRINT "ENTER TITLE OF RUN."
```

```
440 INPUT A$
450 PRINT "ENTER OUTPUT DEVICE CODE. 1 FOR PLOTTER. 32 FOR SCREEN."
460 INPUT O
470 REM THE FOLLOWING LINES GIVE THE MLE OF LAMBDA BY WAY OF THE
480 REM NEWTON-RAPHSON ITERATIVE METHOD.
490 L1=L/N
500 L=L1
510 Q=EXP(-L)
520 L2=L1*(1-Q)-L
530 IF L2=>-1.0E-5 AND L2<=1.0E-5 THEN 570
540 L=L1*((L+1)*Q-1)/(L1*Q-1)
550 GO TO 510
560 REM THE FOLLOWING LINES GIVE THE MLE OF ZERO-CLASS
570 Q=EXP(-L)
580 P=1-Q
590 N0=N*Q/P
600 REM THE FOLLOWING LINE GIVES THE MLE OF TOTAL SAMPLE SIZE
610 N1=N0+N
620 REM UPPER AND LOWER LIMITS OF ASYMPTOTIC 95% CONFIDENCE INTERVAL
630 REM OF TOTAL SAMPLE SIZE.
640 N2=1.96*SQR(N1*Q/(P-L*Q))
650 N3=N1-N2
660 N4=N1+N2
670 REM UPPER AND LOWER 95% C.I. LIMITS OF ZERO-CLASS
680 N5=N0-N2
690 N6=N0+N2
700 REM UPPER AND LOWER 95% C.I. LIMITS OF LAMBDA
710 L3=N5/N
720 L4=N6/N
730 L5=0.5
740 L7=L5/(1-L5)-L3
750 IF L7=>-1.0E-5 AND L7<=1.0E-5 THEN 780
760 L5=L3*(1-2*L5)+L5^2*(1+L3)
770 GO TO 740
```

```
780 L6=0.5
790 L7=L6/(1-L6)-L4
800 IF L7=>-1.0E-5 AND L7<=1.0E-5 THEN 830
810 L6=L4*(1-2*L6)+L6^2*(1+L4)
820 GO TO 790
830 L5=-LOG(L5)
840 L6=-LOG(L6)
850 IF N7=0 THEN 930
860 REM MLE OF PROBABILITY OF BEING MEMBER OF POPULATION
870 N7=N7+N
880 C1=N1/N7
890 REM 95% C.I. OF THE PROBABILITY
900 C2=N3/N7
910 C3=N4/N7
920 REM OUTPUT
930 PRINT @O:" "
940 PRINT @O:" "
950 PRINT @O:A$
960 PRINT @O:" "
970 IF O=1 THEN 990
980 PAGE
990 PRINT @O:"MLE FOR LAMBDA IS ";L
1000 PRINT @O:"95% C.I. FOR LAMBDA IS ( ";L6;", ";L5;" )"
1010 PRINT @O:" "
1020 PRINT @O:"MLE FOR ZERO-CLASS SIZE IS ";N0
1030 PRINT @O:"95% C.I. FOR ZERO-CLASS SIZE IS ( ";N5;", ";N6;" )"
1040 PRINT @O:" "
1050 PRINT @O:"MLE FOR TOTAL SAMPLE SIZE IS ";N1
1060 PRINT @O:"95% C.I. FOR TOTAL SAMPLE SIZE IS ( ";N3;", ";N4;" )"
1070 PRINT @O:" "
1080 IF N7=0 THEN 1110
1090 PRINT @O:"MLE FOR PROB. OF MEMBERSHIP IS ";C1
1100 PRINT @O:"95% C.I. FOR PROB. OF MEMBERSHIP IS ( ";C2;", ";C3;" )"
1110 END
```