Supplementary Material

# Derivations of vertical oxygen variance budget

The Reynolds-averaged advection and diffusion equation for the dissolved oxygen (DO) conservation is given by

, (S1)

where *x*, *y* and *z* represent the horizontal and vertical coordinates. , *v* and *w* are horizontal and vertical components of the three-dimensional velocity vector. , and are eddy diffusivity in horizontal and vertical directions. is the primary photosynthetic production and is the sum of water column respiration and nitrification. Note that the air-sea oxygen exchange and the sediment oxygen demand at bottom cause the boundary oxygen fluxes, and . Consequently, the boundary conditions of Eq. S1 at surface and bottom are and , where *h* is depth of water column.

By decomposing the variables in Eq. S1 into a depth-averaged and a relative various component, i.e., , , , ,,, and dropping horizontal diffusion terms, a conservation equation for the depth averaged DO concentration may be derived:

. (S2)

Taking the difference between Eqs. S1 and S2 results in

. (S3)

Subtracting the depth averaged value of DO does not change the formula of boundary conditions for , i.e., and *.*

Multiplying Eq. S3 by , the vertical oxygen variance equation is obtained:

. (S4)

Given , Eq. S4 can be further simplified as

. (S5)

Furthermore, the boundary conditions for the vertical oxygen variance are and , where and are DO variation at surface and bottom respect to depth averaged value.

Note that and are depth independent, , and . Therefore, by vertically integrating Eq. S5 and applying the boundary conditions and zero transport of vertical oxygen variance thought surface and bottom, the budget for vertical oxygen variance is

. (S6)

# Scaling advection and dissipation during reoxygenation

The relative contribution to reoxygenation from advection and dissipation due to turbulent diffusion can be determined by scaling the 2D advection-diffusion equation in horizontal and vertical directions in the sub-pycnocline water column, i.e.,

where is the sub-pycnocline velocity in horizontal direction. is the vertical eddy diffusivity at the pycnocline. Given *L* is the hypoxic zone length scale in streamwise flow direction, is sub-pycnocline layer thickness, and is the oxygen concentration at surface, applying the scaling , , and , the non-dimensional advection-diffusion equation is

The scaling parameter is the ratio of time scales taken by advection and dissipation to individually remove hypoxia. The scaling of and rely on estuary dynamics and stratification.

Note that, the location of pycnocline relies on bottom mixing layer thickness, , which is assumed to be the sub-pycnocline layer thickness, i.e., . Meanwhile, wind stress also generates a surface boundary layer. Theoretically, bottom and surface boundary layer thickness () can be estimated from friction velocities (), i.e. (Kato and Phillips 1969; Chant et al. 2007; Geyer and MacCready 2014). Where, can be calculated from wind stress and reference density (=1027 kg m-3), i.e., . While can be computed from current velocity at 1 m above the bottom and bottom drag coefficient (= 0.0035), i.e., . *C* ≈ 0*.*6 is a constant related to the mixing efficiency; is the stratification above the boundary layer which was taken as in this work; and hours by following Chen and Sanford (2009). At peak wind speed, the sum of bottom and surface boundary exceeds the total water column (*h*) as . Under this condition, it is assumed .

Two types of scaling formulations for were tested in this work. Firstly, by following MacCready (2007) and Ralston et al. (2008), the eddy diffusivity can be scaled as , where is the eddy viscosity, (= 2 in this work) the Schmidt number, and is a tuning coefficient (0.03 in this work). This approach resulted in ~ 10-5 m2 s-1 and ~ 10-6 - 10-5 m2 s-1 at the pycnocline, which well-matched values of from model results. Furthermore, one can also assumed followed a parabolic shape of where = 0.41 is the Karman-constant and is the bottom roughness length (Burchard and Hetland 2010; Lange and Burchard 2019). By assuming the pycnocline acting as a wall, rather than using the whole water column *h*, it is reasonable to have the parabolic profile hold in bottom boundary layer . Therefore, by replacing with and let , then m2 s-1 at the top of bottom boundary layer (i.e., pycnocline) with m in this work. Consequently, both approaches provide similar scaling of and at pycnocline.

# Supplementary Table

Table S1. Parameters and parameter values of the biological model.

|  |  |  |  |
| --- | --- | --- | --- |
| **Symbol** | **Description** | **Value** | **Unit** |
| *Asw* | Light attenuation due to sea water | 0.04 | m-1 |
| *Achl* | Light attenuation by chlorophyll | 0.02486 | (mgChl m2)-1 |
| *n* | Nitrification rate | 0.05 | d-1 |
| *I0* | Threshold for light-inhibition of nitrification | 0.0095 | W m-2 |
| *kI* | Half-saturation radiation for nitrification inhibition | 0.1 | W m-2 |
| *µ0* | Phytoplankton growth rate at 0 oC | 0.59 | d-1 |
| *α* | Initial slope of P-I curve | 0.025 | (W m-2 d)-1 |
| *mp* | Phytoplankton mortality | 0.15 | d-1 |
| *kI-NH4* | NH4 inhibition parameter | 1.5 | mmolN-1 |
| *kNO3* | Half-saturation for phytoplankton NO3 uptake | 0.5 | mmolN m-3 |
| *kNH4* | Half-saturation for phytoplankton NH4 uptake | 0.5 | mmolN m-3 |
| *θphy-CN* | Phytoplankton Carbon to Nitrogen ratio | 6.625 | molC molN-1 |
| *θchl-C* | Maximum chlorophyll to Carbon ratio | 0.0535 | mgChl mgC-1 |
| *τ* | Phytoplankton and suspended detritus aggregation rate | 0.005 | d-1 |
| *wphy* | Phytoplankton vertical sinking velocity | 0.1 | m d-1 |
| *µzoo* | Zooplankton maximum growth rate | 0.6 | d-1 |
| *mz* | Zooplankton mortality rate | 0.025 | d-1 |
| *θzoo-CN* | Zooplankton Carbon to Nitrogen ratio | 6.625 | molC molN-1 |
| *lE* | Zooplankton specific excretion rate | 0.1 | d-1 |
| *lBM* | Zooplankton Basal metabolism | 0.1 | d-1 |
| *β* | Zooplankton Nitrogen assimilation efficiency | 0.75 | --- |
| *rLDeN* | Large detritus remineralization rate N-fraction | 0.01 | d-1 |
| *rLDeC* | Large detritus remineralization rate C-fraction | 0.01 | d-1 |
| *wLDeN* | Large detritus vertical sinking velocity | 1.0 | m d-1 |
| *rSDeN* | Small detritus remineralization rate N-fraction | 0.03 | d-1 |
| *rSDeC* | Small detritus remineralization rate C-fraction | 0.03 | d-1 |
| *wSDeN* | Small detritus vertical sinking velocity | 0.1 | m d-1 |

Table S2. Error estimates for model-data comparison for 2019.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Variables | Station | Total | | | Subtidal | |
| ME | MAE | Skill | MAE | Skill |
| Salinity (PSU) | Dauphin Island (DI) | 2.13 | 4.76 | 0.86 | 4.00 | 0.90 |
| Bon Secour (BS) | 1.94 | 4.01 | 0.87 | 3.94 | 0.87 |
| Meaher Park (MP) | 0.06 | 0.73 | 0.89 | 0.71 | 0.90 |
| Temperature (oC) | Dauphin Island (DI) | 1.35 | 1.48 | 0.98 | 1.39 | 0.99 |
| Bon Secour (BS) | 1.41 | 1.49 | 0.97 | 1.47 | 0.98 |
| Meaher Park (MP) | 0.37 | 1.67 | 0.97 | 1.48 | 0.98 |

# Supplementary Reference

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