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GEOCON 1.0 Technical Report: Coordinate Transformation between the High Accuracy Reference Network and the NAD83 (NSRS 2007)

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Coordinate Transformation between the High Accuracy Reference Network and the NAD 83(NSRS2007)

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ABSTRACT

GEOCON performs three-dimensional coordinate transformations between the NAD 83(HARN) coordinates and the NAD 83(NSRS2007) coordinates. GEOCON also issues information about the quality of the transformation at each point, and notifications regarding poor quality results.

GEOCON employs high resolution grids (1’ by 1’) to obtain unprecedented fidelity in modeling coordinate differences. Frequently, one may see that the reported quality is extremely high (e.g. 1 cm or better), and could be considered comparable to a geodetic readjustment of survey measurements. Nonetheless, the National Geodetic Survey considers actual readjustment of survey measurements, and not coordinate transformations, as “best practice”.

PREFACE

This document (“The Technical Report”) is a companion to the GEOCON Operating Instructions and the GEOCON User Guide. I assume the reader is familiar with the material in those two documents. However, I will replicate that material in this document, as needed, to provide a smooth reading experience.

This document will also have a strong focus on computational details. But, it is not a step-by-step cookbook.

This report is augmented by a number of image files denoted as Electronic Support Material (ESM). These supplemental images are more numerous, and provide a comprehensive view of the results. The images are in PDF format, and are in high resolution. As such, they may be magnified and inspected in any PDF viewer to see detail. It is anticipated that the ESM will be disseminated in conjunction with this report’s distribution.

INTRODUCTION

With the advent of the Global Positioning System (GPS) came a revolution in the ease and accuracy of geodetic surveying. This was a challenge for the National Geodetic Survey (NGS). Suddenly a technology was available whose accuracy greatly exceeded the current geodetic network, and allowed connections beyond line of sight limitations.

GPS spawned a 15 year NGS effort that established a nationwide GPS backbone to the National Spatial Reference System (NSRS).

Naturally, the user community wanted immediate access to such a fiducial network. And this demand greatly exceeded NGS survey capabilities. NGS decided to gradually update the NAD 83(86) coordinate set (and establish new ellipsoidal heights) by performing GPS survey adjustments in groups of one or more states. While the set of coordinates had a number of names, they were generally known as the High Accuracy Reference Network (HARN) (Bodnar 1990, Milbert and Milbert 1994).

When NGS formulated the strategy of gradual coordinate updates, it was fully expected that a national readjustment of all the GPS survey data would be performed at the completion of the campaigns. This was achieved in February 2007 with the publication of the NAD 83(NSRS2007) National Readjustment (Milbert 2008, Pursell and Potterfield, 2008).

NGS had published transformation software to support all preceding coordinate sets. As the coordinate sets became progressively more accurate, the successive coordinate shifts decreased in magnitude. Concern was raised that a NSRS2007-HARN transformation would be largely comprised of measurement noise. However, the increasing use of GIS systems, with their provisions for coordinate transformations, and the user desire for consistent coordinates led to demand for an official NSRS2007-HARN transformation.

The GEOCON software is the NGS response to the user needs. GEOCON performs three-dimensional coordinate transformations between the NAD 83(HARN) coordinates and the NAD 83(NSRS2007) coordinates (Milbert 2012a, Milbert 2012b). Also, as a new development, GEOCON also issues transformation quality estimates that should be used to enlarge the network accuracy of transformed data sets.

SECTIONS

1. IERS Terminology

This report uses terminology that is adopted by the International Earth Rotation and Reference Systems Service (IERS). The IERS was established in 1987 by the International Union of Geodesy and Geophysics (IUGG) and the International Astronomical Union (IAU). The IERS is the key organization for establishing global geodetic references. For example, the Continuously Operating Reference Station (CORS) coordinates are based on the latest IERS reference frame.

In particular, a sharp distinction is made between a *reference system* and a *reference frame*. Briefly, a reference system is the set of theories, models, and adopted constants used to define coordinates. In contrast, a reference frame is the materialization of a reference system into a coordinate set.

These distinctions are detailed in the Appendix of this report. I encourage readers to review this material.

2. Construction of the Master Coordinate File

The Master Coordinate File (MCF) contains records for all NGS database points holding both a NSRS2007 and a HARN coordinate. Each record contains NSRS2007 and HARN three dimensional coordinates, and the shifts of NSRS2007 minus HARN.

Retrieval was more difficult than expected, since the data did not have direct codes for HARN coordinates. HARN points needed to be detected indirectly by means of coordinate dates. Additionally, HARN points could have different dates for horizontal and vertical components. Further, a collection of publication codes needed to be re-evaluated to allow retrieval. Finally, a set of points that support Federal Aviation Administration (FAA) requirements were maintained in a separate table with restricted access rights. It was decided not to include these latter points.

It should be noted that points observed and adjusted after the NSRS2007 cut-off date were adjusted in the NSRS2007 coordinate, and assigned that datum tag. This led to an increase in the expected number of NSRS2007 points. Similarly, points were adjusted in the HARN coordinate up to the NSRS2007 cut-off date. After cut-off additional survey work was collected into a backlog. It was decided to issue both HARN and NSRS2007 coordinate coordinates by means of two distinct adjustments for these backlog points. This led to an increase in the expected number of HARN points.

The November 22, 2011 database retrieval contained 79232 points in NSRS2007, and 70390 points in the HARN. After cross-matching the data, and imposing certain data requirements, 69540 common points were written to the MCF. The points for the conterminous U. S. are displayed in Figure 2.1.

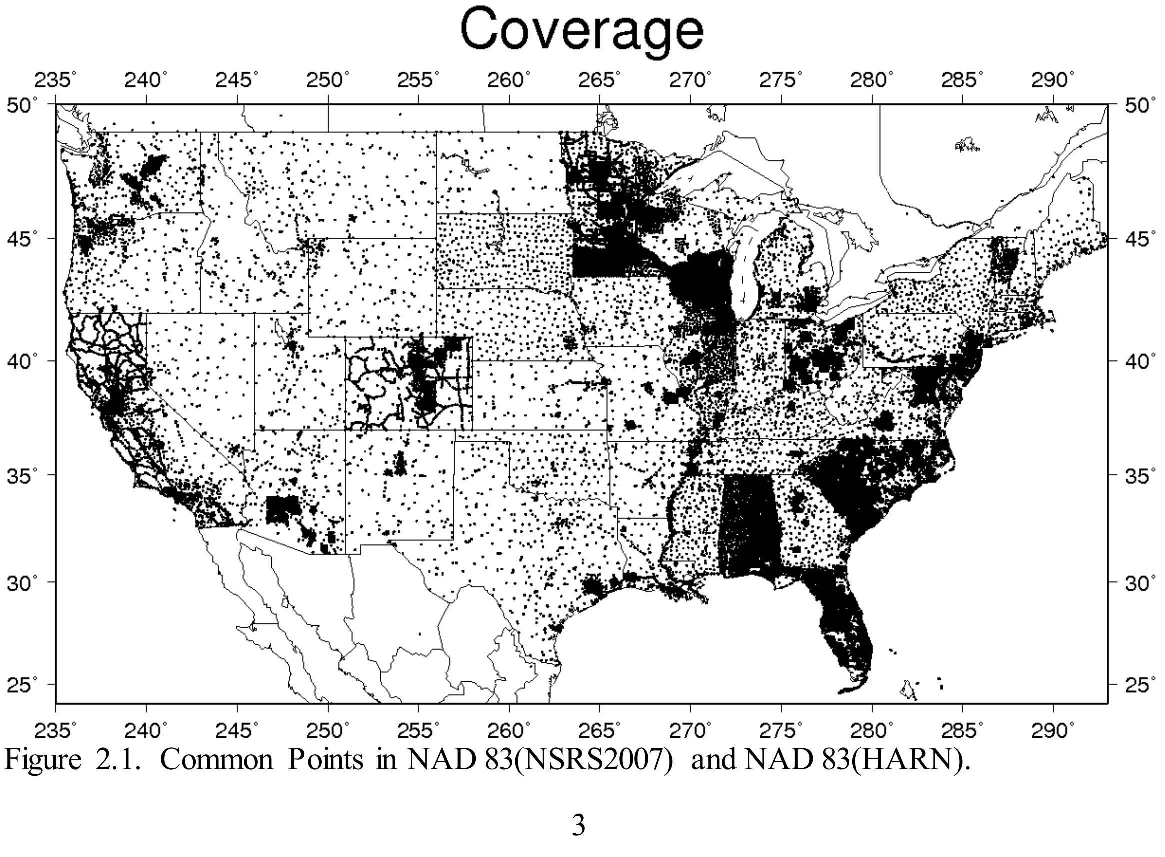

Figure 2.1. Common Points in NAD 83(NSRS2007) and NAD 83(HARN).

High resolution images of the conterminous U. S., as well as Alaska and Puerto Rico/Virgin Islands are available in the supporting ESM.

Inspection of Figure 2.1 shows quite good coverage across the conterminous U. S. As expected, data are not found in the Great Lakes or the oceans. Only a handful of point may be found in Canada and Mexico.

To support NGS, a series of horizontal and vertical vector plots of coordinate shifts were made to illustrate the character of the differences. One example of this series of vector plots is Figure 2.2. High resolution versions of this image and the remainder of the horizontal and vertical series may be found in the ESM.

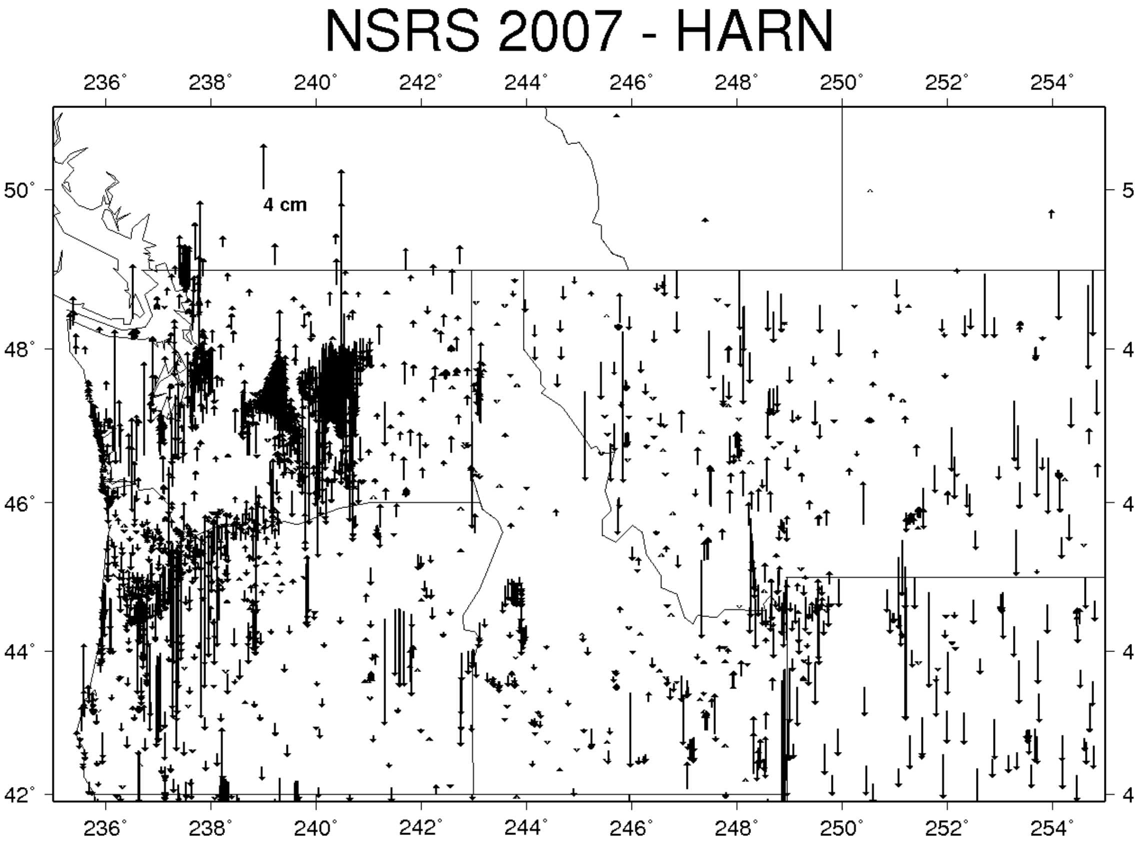

Figure 2.2. Vertical Shift Vectors, NW CONUS.

Figure 2.2 displays regions of common shifts, some larger than others. Also seen are occasional anomalous shifts ("spikes"). These anomalous shifts are legitimate coordinate differences between NSRS2007 and the HARN. They are plotting differences in the NGS database. And, other surveying, mapping, or GIS work may be founded on these anomalous values. To follow the philosophy of modeling all coordinate differences (Milbert 2012b) these anomalies must be represented to the user.

An important point in the difference between the NSRS2007 and the HARN is the fact that they have different reference dates. NSRS2007 has a reference of 2007.0, whereas the HARN coordinates have a variety of dates. Section 14 of the GEOCON

Operating Instructions (Milbert 2012a) lists a number of datum tag dates. In addition, the list does not fully represent the 15 year interval, since existing datum tags were often reused for later resurvey work.

The distinction in epochs between the NSRS2007 and HARN is important due to the presence of horizontal crustal motion. This is a significant systematic effect along the West Coast, which had a nominal HARN epoch of 2002.0. Aside from anomalous points, the horizontal shifts in California often exceed 10 cm in the time interval. This systematic effect is an additional reason to provide a new coordinate transformation model. The horizontal shifts for the Northeast conterminous U. S. are plotted in Figure 2.3 to illustrate this point.

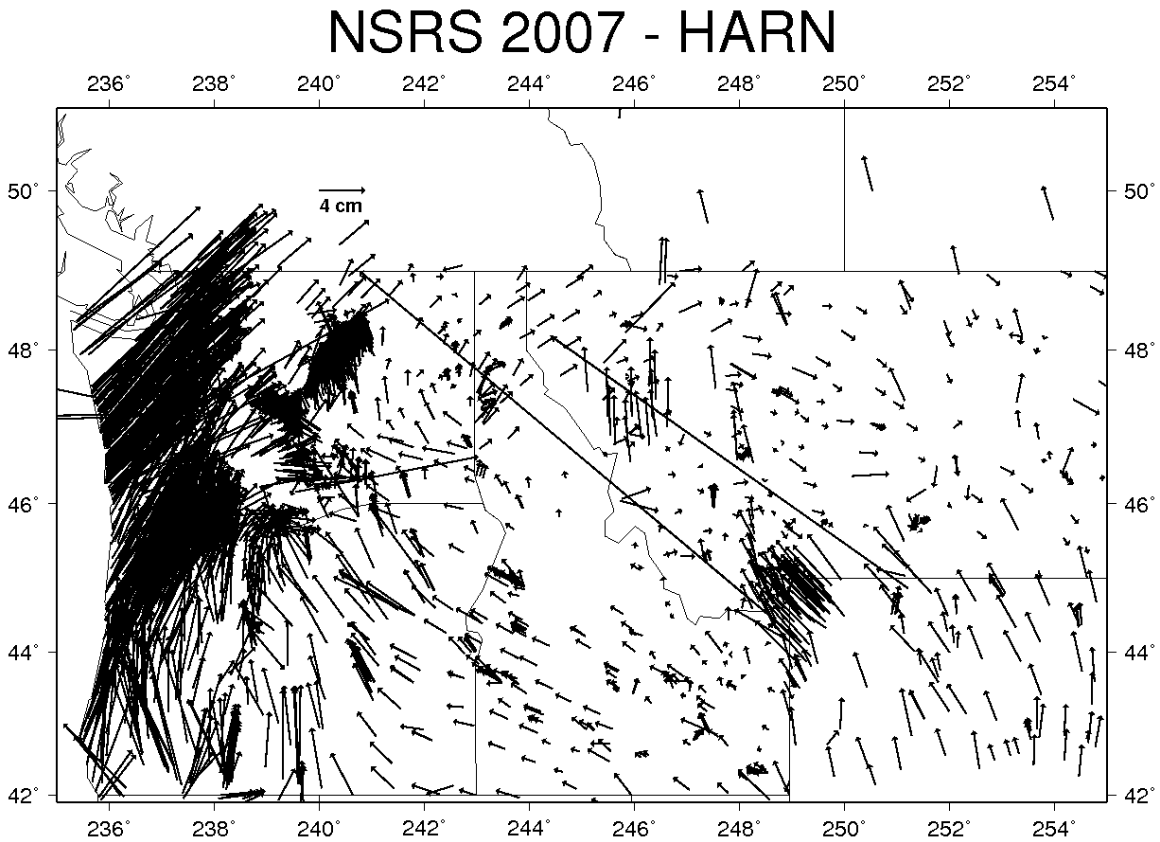

Figure 2.3. Horizontal Shift Vectors, NW CONUS.

As described earlier, the general impression of Figure 2.3 is variable shifts, with occasional anomalous coordinate differences. In addition, one sees the systematic horizontal shifts due to plate tectonics along the West Coast.

3. Treatment of Data Voids

To perform coordinate transformation, we require a model that honors the input data points. And, experience has shown great success in use of gridded models in transformation, such as in NADCON (Dewhurst 1990), VERTCON 2.0 (Milbert and

Holdahl, 1994), and VDatum (Parker, Milbert, and Gill, 2003). These methods fit splines to the data to establish modeled transformations on a regular grid.

However, this method can lead to large excursions in void areas on the boundaries, such as the oceans surrounding the U. S. Such behavior is an extrapolation of the coastal values, and may or may not represent valid predictions of the model. Since the NSRS2007 and the HARN share a generally common geocenter, alignment, and ellipsoid, we may expect those coordinate sets to coincide in void, boundary regions.

For this reason, it was decided to include synthetic data, near-zero, in the regions of the Atlantic and Pacific Oceans, and in the Great Lakes. This entails the generation of a land-water mask for the regions of the conterminous U. S. and Alaska. A regular data spacing of 3' by 3' was chosen. The GLOBE30 data set (GLOBE Task Team *et al.* 1999) was selected as the source, since it had high resolution and specially flagged ocean values.

The source elevation data were downloaded and assembled into the grids for the desired regions. Next, the land-water relation was mapped from the 30" by 30" to a 3' by 3' grid. Note that the mapping was not a decimation operation. If land was detected at any of the 30" grid intersections surrounding a 3' grid intersection, then the 3' intersection was marked as land.

It was desired that the coordinate transformation model would function slightly offshore. Therefore the land settings were allowed to "bleed" into adjacent water settings. Each cycle was done in two steps. First, water intersections adjacent to land were set to be changed to land throughout the land-water grid. Then, the flagged grid intersections were actually changed into land. This two step process prevented a land value from propagating beyond one 3' intersection in a single cycle. As a refinement, the land value was allowed to propagate diagonally only on even numbered cycles. This process was executed for 3 cycles. This provided an approximate 9 mile border where the transformation could penetrate into the ocean and blend into the synthetic values held near-zero.

Land-water masks for the Great Lakes were taken from earlier work in gridding hydraulic correctors in support of datum transformation between the North American Vertical Datum 1988 (NAVD 88) and the International Great Lakes Datum 1985 (IGLD 85). Those older 3' by 3' masks were created with a Generic Mapping Tools (GMT) function, `grdlandmask`, from a GMT shoreline data set (Wessel and Smith, 1995). As with the ocean masks, near-zero coordinate shifts were imposed in the Great Lakes.

The Puerto Rico/Virgin Island transformation involved a set of islands. In this case it was easy to generate a series of synthetic near-zero values in a border zone along the perimeter of the region.

In general the GEOCON coordinate transformations should not be used offshore, in the middle of the Great Lakes, or in foreign countries. GEOCON will report a transformation near-zero when far offshore, for example. There will be a gradual transition to zero across an approximate 9 mile buffer zone along the coast. GEOCON is a simple demonstration program. Future versions of GEOCON could incorporate land-water and data void masks as files, and take more sophisticated actions with input data in void areas.

4. Coordinate Difference Preprocessing

As detailed in Section 11, a spline method was selected to generate the coordinate transformation grids. The technique supports irregularly spaced data, but requires the input to be thinned to the minimum grid resolution. However, the station spacing in the NSRS can frequently be smaller than any grid spacing (Milbert 2008, pg. 12). This means we must honor the data for multiple coordinate differences near a grid node.

A modified median procedure was applied prior to gridding. In the case of a cluster of exactly 2 points, a search was performed in the 1' to 2' ring surrounding the central 1'x1' cell. If the search found a point, it was used as a tiebreaker to select the winning median point in the central cell. If the 1' to 2' ring was insufficient, then the 2' to 3' ring was searched for a tiebreaker. If, after two ring searches no tiebreaker was found, then the central cell median was selected at random. It was found that that the ring search procedure was able to reduce 8177 pairs to 1534 pairs. And, by using a random selection, the possibility of the median being influenced by an abnormal point is additionally halved.

The philosophy in choosing a median procedure is rooted in the likely practice of geospatial professionals. In the presence of a cluster of control points, connections should be made to a sufficient number to confirm a valid connection to the network. Depending upon the accuracy of the positioning measurements, significant control point discrepancies may be identified. And, ties to suspect control points would be discarded. This is standard practice in surveying.

5. Gridding by Splines in Tension

To satisfy the need to honor the coordinate difference data in the model, we select a gridding method of splines in tension (Smith and Wessel, 1990). This models the physical behavior of a thin, flexible plate that passes through the defining points. However, such a model, by itself, is subject to overshoots and undershoots when data differences occur near gaps in irregularly spaced data. By mathematically applying tension at the edges of a grid, it is possible to suppress the oscillations, and generate representative intermediate values. The tension parameter is a normalized quantity that ranges from 0 to 1, where 0 represents no tension and 1 represents infinite edge tension. The gridding software is "`surface`" in the Generic Mapping Tools (GMT) package (Wessel and Smith, 1995).

To establish reasonable gridding parameters, a series of tests was performed with a preliminary data set of 65944 coordinate difference triplets in the conterminous U. S. In each test, model grids were computed. Then the input coordinate differences were predicted with biquadratic interpolation from the model grids. Differences between the measurement and prediction were tabulated in Tables 5.1 through 5.3.

Table 5.1 – Percentiles of Prediction Error, 2’ x 2’, T=0.4

Percentile	Latitude (0.00001 arc sec)	Longitude (0.00001 arc sec)	Height (cm)
68%	0	0	0.1
90%	10	20	0.4
95%	30	40	0.8
99%	100	130	2.7
99.9%	480	580	8.4

Table 5.2 – Percentiles of Prediction Error, 1’ x 1’, T=0.4

Percentile	Latitude (0.00001 arc sec)	Longitude (0.00001 arc sec)	Height (cm)
68%	0	0	0.1
90%	10	10	0.5
95%	20	25	1.0
99%	75	95	3.3
99.9%	380	450	10.6

Table 5.3 – Percentiles of Prediction Error, 2’ x 2’, T=0.25

Percentile	Latitude (0.00001 arc sec)	Longitude (0.00001 arc sec)	Height (cm)
68%	5	5	0.2
90%	15	20	0.8
95%	25	35	1.4
99%	95	130	4.2
99.9%	460	565	12.8

Comparison of Table 5.1 to 5.2 isolates the effect of grid spacing. A noticeable improvement in horizontal prediction was attained with the 1’ grid. A slight degradation in height prediction is seen. However the horizontal improvement exceeds the vertical degradation.

Comparison of Table 5.1 and 5.3 isolates the tension parameter. A tension of 0.25 is suggested by Wessel and Smith (1995) for potential field data. They also suggest a tension in excess of 0.35 for steep data, such as topography. In this test the horizontal results are not clear. But a definite degradation is seen in height with the lower tension value. Based on these results, a 1’ x 1’ grid with a tension parameter of 0.4 was selected for GEOCON.

It must be understood that fitting a surface to underlying coordinate difference data implies that a surveyor or GIS practitioner will have followed certain procedures in the field. In particular, it is expected that they will have connected to the network using multiple points in a region, and not by a simple, single mark tie. If, however, a practitioner only performs a single point tie, then all of the geospatial data should be transformed by the unique coordinate differences of the source control point.

Note that for the case of nearby anomalous, but valid, coordinate differences, the surface will interpolate between anomalous values, and the regular values of the surrounding neighbors. This is ideal if the coordinate data are connected to both the anomalous and regular values. If the practitioner has only connected to regular values, or only connected to anomalous values, then the surface will not represent the users coordinate differences. This point is further illustrated by the case studies in Sections 10 through 12 of the GEOCON User Guide (Milbert 2012b).

6. Results from the Transformation Grids

It is instructive to consider some graphics of the coordinate transformation from the NAD 83(HARN) to the NAD 83(NSRS2007). Only selected images are reproduced here. Many more high resolution images are available in the ESM. Figure 6.1 displays the horizontal coordinate differences, and Figure 6.2 plots the vertical coordinate differences. These figures may also be compared to Figures 9.1 and 9.2 in the GEOCON User Guide (Milbert 2012b).

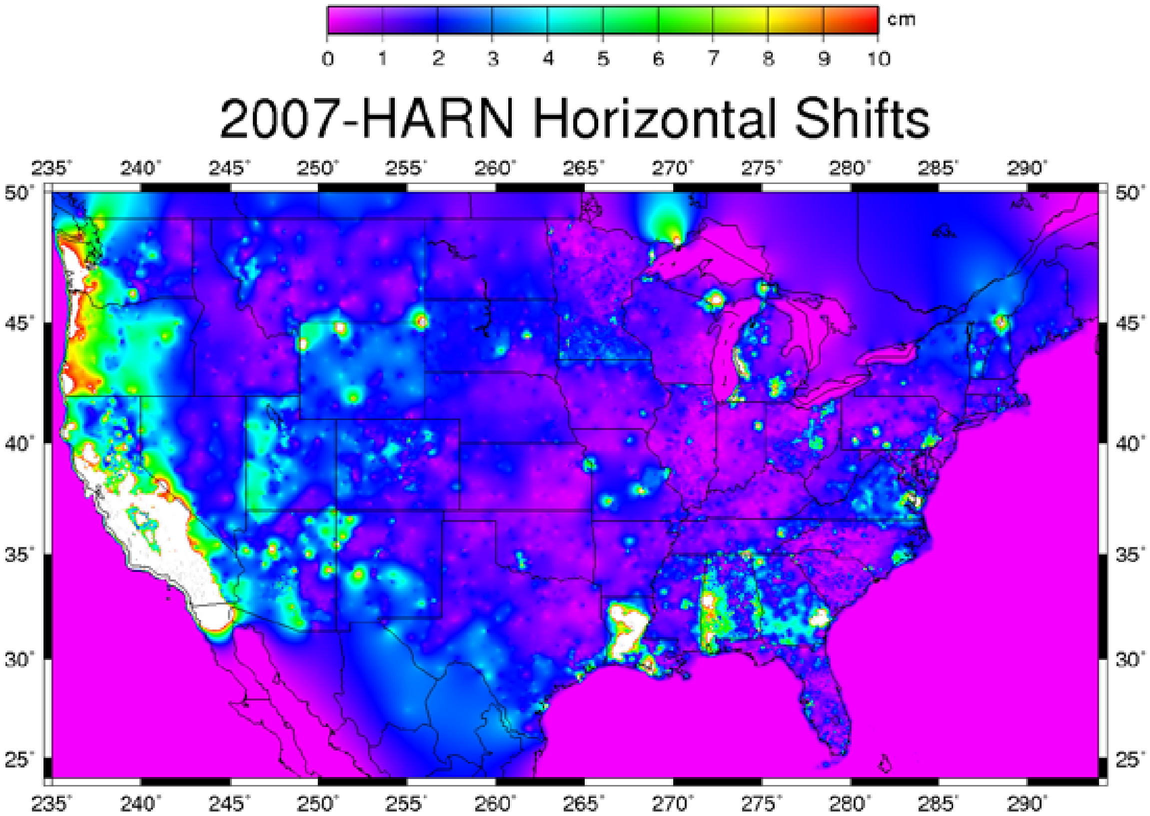

Figure 6.1. Horizontal Coordinate Differences.

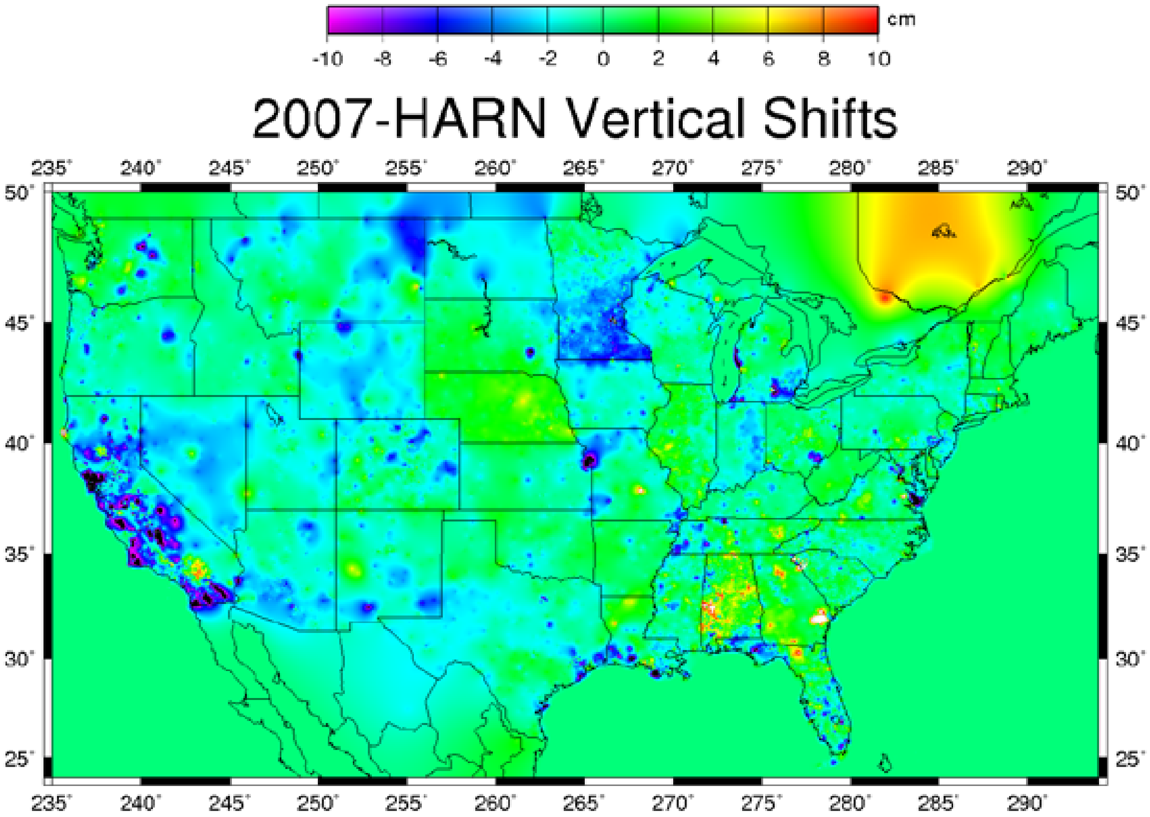

Figure 6.2. Vertical Coordinate Differences.

The white areas in Figure 6.1 show where the horizontal shifts exceed the color scale of 10 cm. This serves to highlight the tectonically active areas along the West Coast. It is also noted that Louisiana takes some large horizontal shifts. Results in Canada and Mexico are to be disregarded. Differences in the oceans and Great Lakes are set to near zero as described in Section 5. One may discern occasional anomalous shifts that appear as bumps in Figure 6.1.

Both white and black locations may be seen in Figure 6.2. White areas are where the vertical shift is greater than the color scale, and black areas are where the shift is smaller than the color scale. Of note are vertical shifts in portions of California, southern Minnesota, and Alabama. Isolated anomalous shifts are noticeable, too.

To determine if NGS needed to perform the National Readjustment, one can consider the signal (coordinate shift) against the noise (coordinate accuracy). If the shifts significantly exceed the coordinate precision, then coordinates were improved by the adjustment. For the NSRS2007 National Readjustment the horizontal coordinates achieved 1 cm network accuracy and the vertical coordinates achieved 2 cm network accuracy (Milbert 2008). To explore this, I plot the coordinate shifts with color limit scales set to the appropriate network accuracies. Figure 6.3 displays horizontal coordinate differences, and Figure 6.4 plots the vertical coordinate differences.

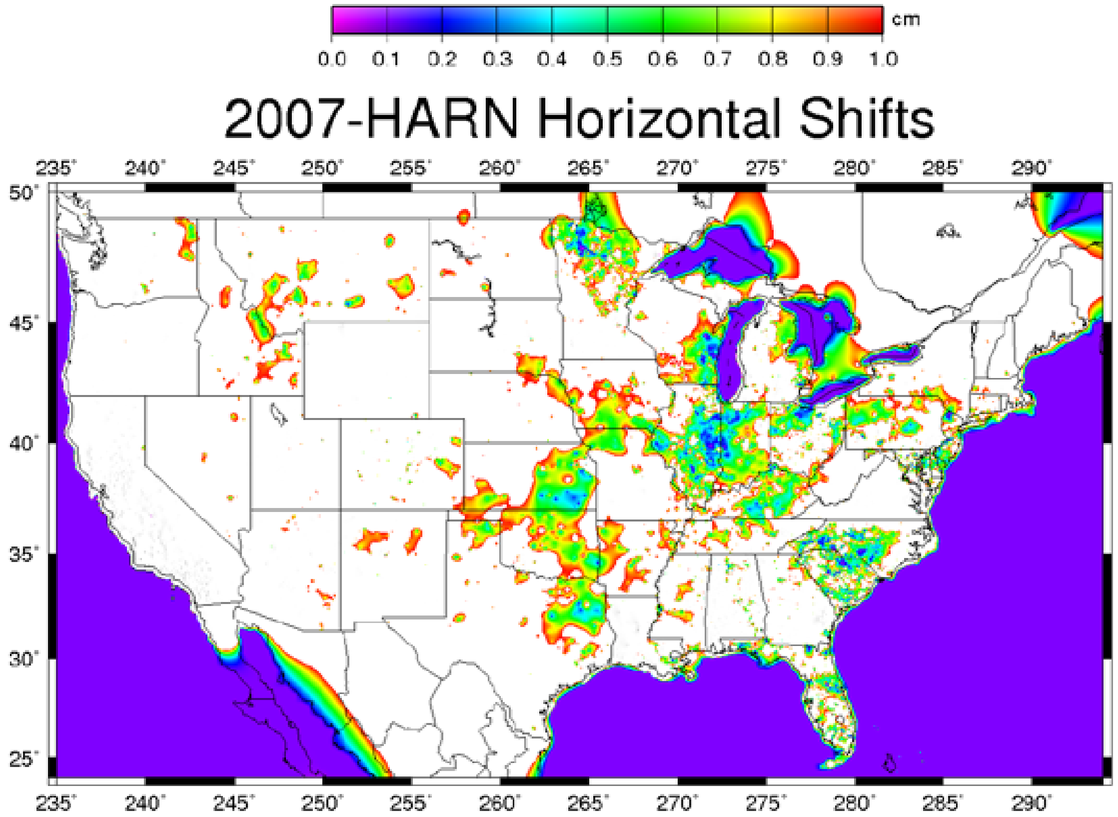

Figure 6.3. Horizontal Coordinate Differences at the Network Accuracy Limit.

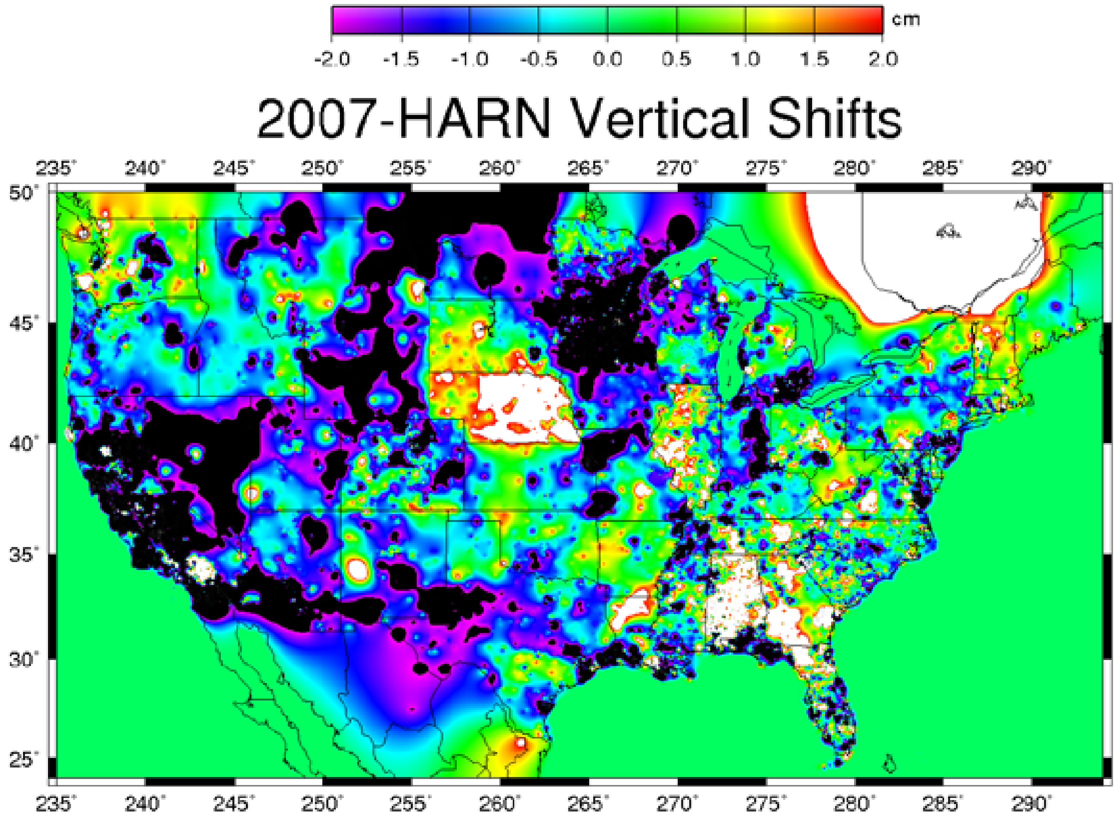

Figure 6.4. Vertical Coordinate Differences at the Network Accuracy Limit.

Figure 6.3 is mostly white. This means the horizontal coordinate shifts exceeded the 95% network accuracy throughout most of the conterminous U. S. This figure demonstrates the NSRS2007 National Readjustment was necessary to get the best results from the GPS surveys.

Figure 6.4 contains large patches of black and white. Either black or white indicates a vertical shift exceeding the 95% network accuracy limits. Of interest is that one may find locations where black areas are adjacent to white areas, illustrating opposing vertical shifts. We see that, compared to Figure 6.3, less of the country takes large vertical shifts when compared to the vertical network accuracy. This is less surprising when we recall that the HARN readjustments went through two cycles (the second one being the HARN FBN/CBN). The ellipsoidal heights were updated in the second cycle even when the horizontal coordinates or the HARN datum tag was not updated. Refer to Section 14 of the GEOCON Operating Instructions (Milbert 2012a) for more detail on the HARN coordinates.

7. Representation of Anomalous Coordinate Shifts

The discussion at the end of Section 7 emphasized that the gridded transformation best represents data sets that have been connected throughout their extent to the parent geodetic control network. By contrast, data sets connected to control with only regular coordinate differences transform less well when in the vicinity of unconnected points with anomalous coordinate differences. And, in a pathological case, one may have a single-mark tie to an isolated, anomalous control point. It is desirable to alert users to the presence of anomalous shifts.

In a related issue, prior to computing the coordinate transformation grids, the data were preprocessed by a modified median procedure. This means that in the cases of clusters, an anomalous coordinate difference would be dropped prior to gridding. There would be no expression in the transformation grid for such points. Yet, it is possible that a data set could be connected to one of these anomalous points. It is important to alert users that such anomalous points existed, even when they are not evident by visual inspection of the transformation grid.

It was decided to satisfy both requirements by using a statistical resampling procedure known as cross-validation (Efron and Tibshirani, 1998). In its simplest form, cross-validation consists of cutting a data set in half. Call the first half the *training set*, and build the model from the first half. Then compare the second half of the data, called the *validation* or *testing set*, to the model predictions. Similarly, one may exchange the two data halves, and repeat the process.

In a more elaborate form, one can compute K-fold cross-validation. The data set is partitioned into K subsets. In sequence, each subset is designated as a testing set, and is temporarily withheld from the data set. The model is computed from each reduced data set, and differences are computed between the temporarily withheld data and the model prediction. The process is sequenced K times until a prediction error is established for each data point. Larger values of K allow the training data set to approach the size and resolution of the original data set.

While cross-validation errors (measured shift – predicted shift) are typically aggregated to tune model parameters, the individual errors do a very good job in quantifying anomalous shifts. They also show propagation of uncertainty in cases where an anomaly and a few neighbors are isolated. And, the cross-validation procedure issues errors even when a point is present in a cluster.

Therefore, grids of cross-validation error were selected to represent transformation model quality and warn the user about anomalous coordinate shifts. To address the situations of an anomalous point in a cluster, and to provide a conservative quality value, the worst case cross-validation error is gridded. The worst case is selected by choosing the error that is furthest in magnitude from the median error.

It was realized that in choosing the worst case cross-validation error, that cluster cases would arise where a large error value would be issued where a modest coordinate shift is predicted. This is an outcome of providing the most likely transformation surface and a conservative quality estimate. To help explain these cases, notification messages are issued GEOCON. These messages are informational, and list anomalous points within 5 km that are members of clusters. Median drops are considered anomalous and placed into an information file if they depart from the gridded value by 6 cm in latitude or longitude, or by 7 cm in height.

8. Cross Validation Computations

For GEOCON, 69-fold cross-validation was performed. The master data set was 69540 point pairs. So each withheld testing set was a little over 1000 points. This means 69 training grids, derived from a little under 68540 point pairs, were input for each component (latitude, longitude, and height), for the regions of CONUS, Alaska, and Puerto Rico/Virgin Islands.

A representative schematic of the data flow for a single component in the conterminous United States is displayed in Figure 8.1. This procedure is implemented as a Windows™ Console batch file for each component in each region.

```
                   work07.txt

                   PREPDATA3

                   alldat.txt

========================================== (top of loop)

       --<-----<-- EXTRACT2
       |
      \ /          thindat.txt
       |
       |           MYMEDIAN
       |
       |           tempmedian.txt
  luckydat.txt
       |           SURFACE
       |
       |           temp.grd
       |
       |           GRD2XYZ + XYZ2B
      \ /
       |           thindat.b
       |
       -->------>- BIQUADRA

                   fitsnn.txt  (nn -- serial counter)

======================================= (bottom of loop)

                   CONCATENATE FILES
```

Figure 8.1. Data Flow for Cross-validation Computations

Figure 8.1 portrays computer programs in all capital letters, and the key input and output files are shown in all lower case letters. The master loop is delimited by the equal signs. Since 69-fold cross-validation is performed, the loop is executed 69 times. The batch file allows for a selectable number of loops. The path on the left shows that the testing set, `luckydat.txt,` is also generated as output from `extract2`, is held, and is also input to program `biquadra`.

The file, `work07.txt`, is the complete working file, holding 69540 point pairs. To obtain the effect of sequential sampling with replacement, the working file is shuffled, and then successive pieces of the shuffled file are withheld (`luckydat.txt`) for validation. The complete shuffled file, `alldat.txt` , is retained throughout the cycles.

Program `prepdata3` reads `work.txt`, shuffles it, adds land-water mask constraints (not shown in Figure 8.1) at the end of the file, and produces `alldat.txt`. The processing loop typically executes 69 times. Program `extract2` copies `alldat.txt` into `thindat.txt`, while withholding the portion of the sorted point pairs appropriate to the current cycle number in the procedure. For each loop cycle, `thindat.txt` is the training set, and `luckydat.txt` is the testing set. These sets are re-written each cycle.

The training set, `thindat.txt`, is then gridded exactly as the complete, original data set. Program `mymedian` performs the modified median filter de-clustering procedure described in Section 4. Program `surface` is a GMT procedure that grids by splines in tension (Section 5). Program `grd2xyz` is a GMT procedure that converts a GMT grid file into a list representation. And, `xyz2b` converts that list into a standard binary grid file described in Section 12 of the GEOCON Operating Instructions (Milbert 2012a). Each one of the binary grid files in `thindat.b` provides a replica of the original transformation grid. Each one differs, of course, due to withholding the data in `luckydat.txt`.

Program `biquadra` performs biquadratic interpolation with the training set binary grid, `thindat.b`. The interpolation provides a prediction of the coordinate difference. The interpolation is performed for each point in the testing set, `luckydat.txt`. Then, `biquadra` outputs the differences, in the sense actual coordinate difference – gridded coordinate difference, to a file `fitsnn.txt` (where `nn` denotes a cycle number 01 through 69). The `fitsnn.txt` files contain the raw cross-validation (prediction) errors.

Finally, `concatenate` expresses the procedure of joining all the `fits01.txt` through `fits69.txt` files into a combined cross-validation error file, `xvalz.txt` (where `z` denotes a code representing the coordinate component and region). This operation is implemented as a `copy` command in the Console and by a `cat` command in Unix.

Note that program `mymedian` is used to compute the cross-validation errors. More exactly, `mymedian` is used to grid the training data, which, in turn, is used to compute the cross-validation errors. This is exactly correct. The procedures after `extract2`, down to and including `biquadra2`, must match the standard gridding procedure for the full data set. Note that program `antimedian` is used to grid the *output* of the process described in Figure 8.1. This insures that conservative, worst-case error estimates are stored in the GEOCON error grids.

9. Results from the Cross Validation

The cross-validation results are also presented in the GEOCON User Guide (Milbert 2012b). The results are repeated here for continuity and convenience.

The two-tailed percentiles of the distributions of the cross-validation for the conterminous U.S. are collected in Table 9.1. Approximately 68490 points were validated.

Table 9.1 – Percentiles of Cross-Validation Error, CONUS

Percentile	Latitude (0.00001 arc sec)	Longitude (0.00001 arc sec)	Height (cm)
50%	3.1	3.8	0.2
68%	7.2	9.0	0.5
90%	31.2	40.9	1.9
95%	62.5	82.0	3.5
99%	324.0	405.6	8.9
99.9%	895.7	1003.6	23.6

It is seen that the 95% limits are remarkably good. We have 95% bounds of +/- 1.9 cm in latitude, +/- 2.0 cm in longitude, and +/- 3.5 cm in height. Note that the distribution is not Gaussian. It is very peaked (leptokurtic), with long tails. The 68% bounds are almost 10 times smaller than the 95% bounds. In general, the quality of the coordinate transformation is remarkably good. In fact, at the 90% level it is comparable to the network accuracy of NSRS2007.

The two-tailed percentiles of the distributions of the cross-validation for Alaska are collected in Table 9.2. Approximately 770 points were validated.

Table 9.2 – Percentiles of Cross-Validation Error, Alaska

Percentile	Latitude (0.00001 arc sec)	Longitude (0.00001 arc sec)	Height (cm)
50%	28.5	61.3	1.4
68%	67.8	162.7	3.9
90%	246.7	534.4	13.7
95%	323.7	735.3	23.3
99%	1091.3	1338.0	95.6
99.9%	1739.5	2255.1	8602.7

It is seen that the 95% limits are much poorer than for the conterminous U. S. We now have 95% bounds of +/- 10.0 cm in latitude, +/- 10.8 cm in longitude, and +/- 23.3 cm in height. Even so, these are sufficient quality to transform many types of geospatial data. Note that because of the small sample size (770), when the limits are established at the 99.9% boundary, height outliers are seen to appear.

The two-tailed percentiles of the distributions of the cross-validation for Puerto Rico/Virgin Islands are collected in Table 9.3. Approximately 145 points were validated.

Table 9.3 – Percentiles of Cross-Validation Error, Puerto Rico/Virgin Islands

Percentile	Latitude (0.00001 arc sec)	Longitude (0.00001 arc sec)	Height (cm)
50%	5.3	7.9	0.4
68%	8.7	15.6	0.7
90%	37.1	82.3	2.3
95%	64.3	169.0	3.1
99%	112.0	400.6	5.2
99.9%	120.3	434.7	9.9

Here the 95% limits fall between those of the conterminous U. S. and Alaska. The 95% bounds are +/- 2.0 cm in latitude, +/- 5.2 cm in longitude, and 3.1 cm in height. Note that the relatively small values at the 99% and 99.5% limits can not be given much interpretation, since the sample size is so small (145).

In addition to the percentiles, grids of the cross-validation error were computed. Recall from Section 7 that the worst case cross-validation error was gridded. Only some selected images are reproduced here. Many more high resolution images are available in the ESM,

Figures 9.1, 9.2, and 9.3 plot the worst case cross-validation errors in latitude, longitude, and ellipsoidal height. These portray the transformation quality grids.

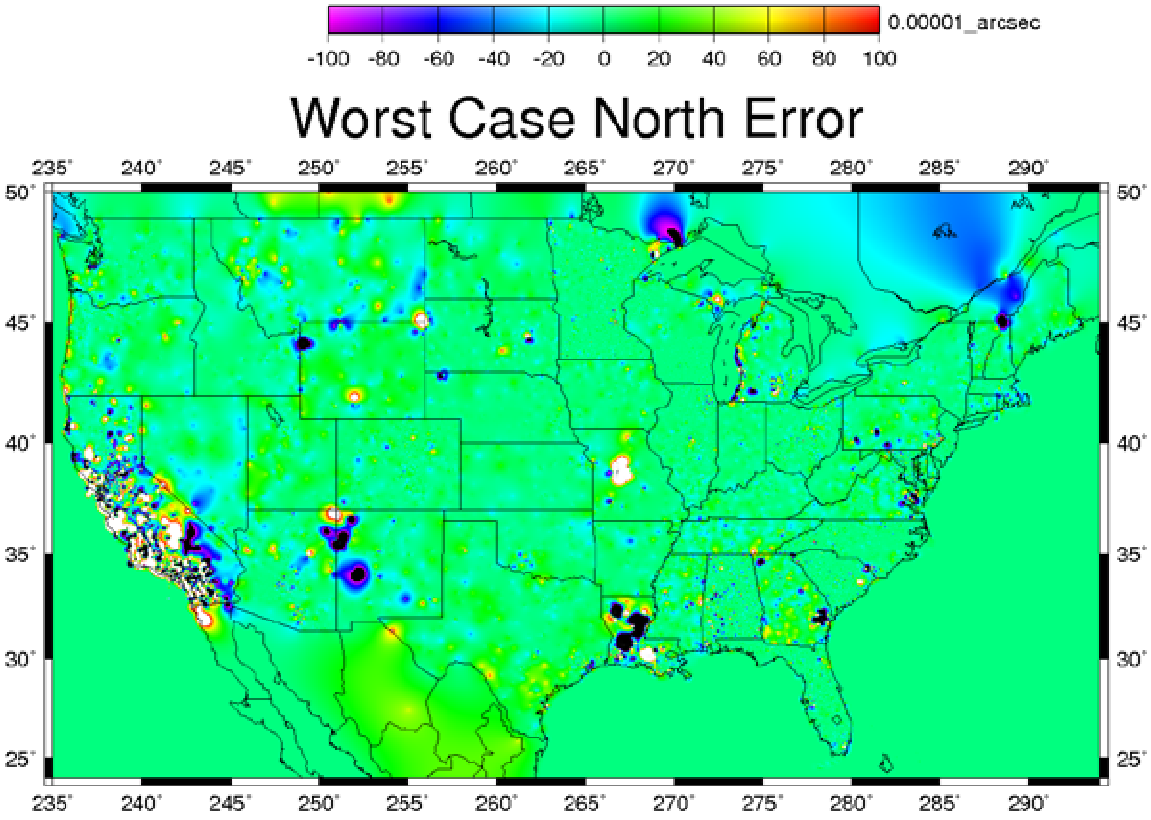

Figure 9.1. Worst Case Cross Validation Error, Latitude.

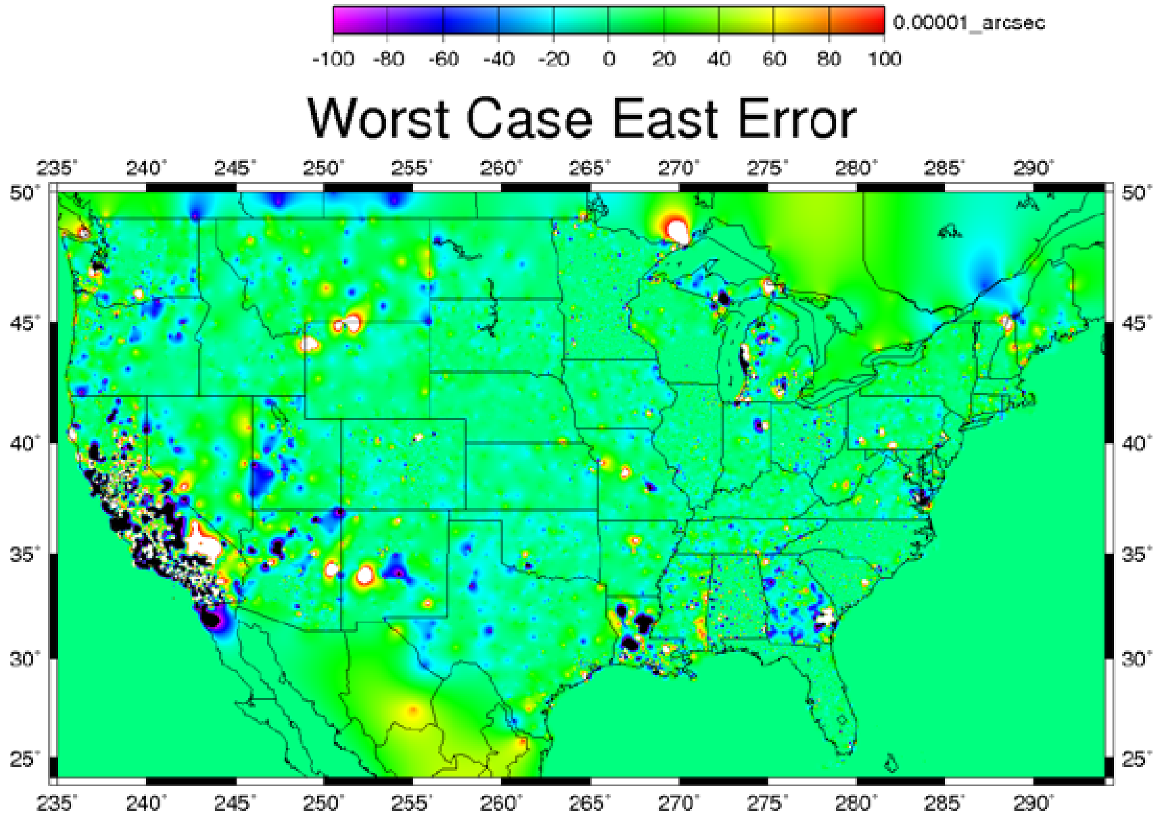

Figure 9.2. Worst Case Cross-Validation Error, Longitude.

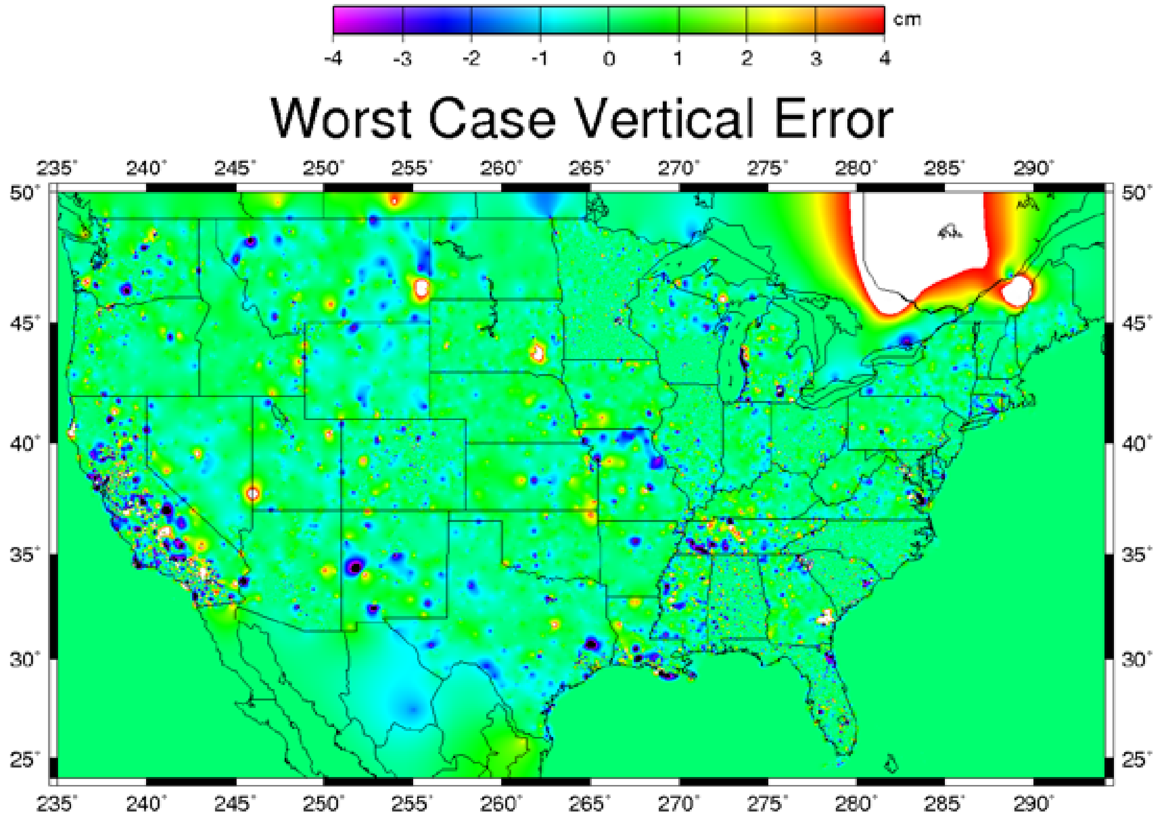

Figure 9.3. Worst Case Cross-Validation Error, Height.

The error grids should be inspected in conjunction with the percentiles of Table 9.1. It is seen that the coordinate shifts can be predicted quite well. The abnormal cases are quite sporadic. California and Louisiana have the most troublesome horizontal coordinate shifts. However, the vertical shifts in Louisiana are seen to be well modeled in GEOCON.

In closing this section, it should be recognized that both the GEOCON transformation model and the cross-validation error model are validated by the Case Studies found in the GEOCON User Guide (Milbert 2012b). Briefly, synthetic GPS data conforming to both benign and anomalous coordinate pairs were adjusted into control points sets with both NSRS2007 and HARN coordinates. In Case Study 1, adjusted GPS data constrained to both NSRS2007 and HARN matched transformed values to a few millimeters. In Case Study 2, where abnormal shifts were present, adjusted synthetic data matched the transformation within a few millimeters to 2 centimeters. Even in the 2 centimeter case, the mismatch was less than the transformation error predicted by cross-validation. And, in Case Study 3, where abnormal shifts are clustered with normal shifts, good transformations are obtained, even though large predicted errors and notifications are issued by GEOCON.

10. Materialization of NAD 83

In addition to the production of the GEOCON coordinate transformation, it was desired that an assessment be made of the most appropriate means for defining the relationship between NAD 83(NSRS2007) and NAD 83(CORS96). In order to consider such a relationship, it is necessary to examine how one materializes NAD 83 reference coordinate frames.

To begin, it is necessary to briefly recall the IERS terminology in Section 1 and the Appendix. A reference system is the theory that defines coordinates. And, a reference frame is the materialization of a reference system into a coordinate set. We must further distinguish the *axial set* of the reference frame as those elements which involve geocenter origin, axial orientation, and scale.

NAD 83 coordinates are obtained by a 14 parameter transformation from International Terrestrial Reference Frame (ITRF) coordinates. The transformation maps positions and velocities from one coordinate set into the other. Further, an important physical element is the stable parts of the North American Plate. Ideally, control points that are assigned coordinates are imbedded in a stable part of the North American Plate.

Figure 10.1 is an idealized sketch of the relative locations of a numerical coordinate (e.g. 40° North, 100° West, 10 meters Up). These are locations as viewed by an external observer in an arbitrary reference frame. The figure is two dimensional, however it represents a three dimensional situation. Further, the figure is static, however it represents a dynamic situation. Each of the coordinates depicted in the figure also have velocities relative to the observer's reference frame (O); and, in the most general case, velocities relative to one another.

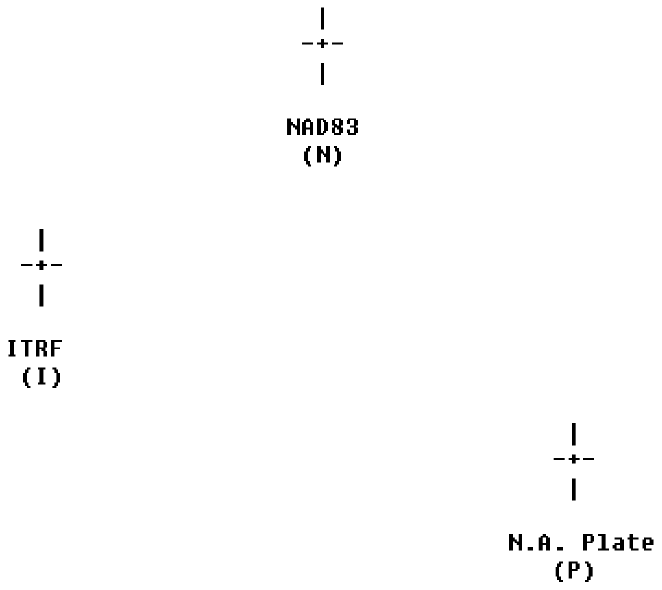

Figure 10.1. Schematic of Coordinates in an Observer's Reference Frame (O).

NAD 83 (N), when viewed as an *ideal reference system* (c.f. Appendix), should have zero relative motion to the North American Plate (P). A plate-fixed system is a natural design element of NAD 83. And, when the ITRF96–to-NAD 83 transformation was defined in Craymer *et al.* (2000), care was taken to accommodate plate motion and obtain a plate-fixed system. Without a plate-fixed system, all points in NAD 83 will have non-zero velocities. And, a control point, rigidly fixed in the stable North American Plate will have NAD 83 coordinates that change in time.

The general mechanism for having a fixed-plate system can be seen in Figure 10.1. One desires the transformation, P-N (NAD 83-to-N.A. Plate) be zero. One begins with the plate model, P-I (ITRF-to-N.A. Plate). By simple algebra, one sees the velocity transformation, N-I (ITRF-to-NAD 83) must equal the plate motion velocity to maintain a fixed-plate system. This is demonstrated in Craymer *et al.* (*ibid*). Once a notional N-I transformation is established, one may update the transformation to serve subsequent versions of the ITRF that may have different velocities. In essence, this is simple conversion of that older plate model to refer to the newer ITRF.

A few remarks can be made about this general system. First, errors in a plate velocity model, P-I, will map directly into the derived N-I transformation. Secondly, the transformations in question should be framed as 14 parameter transformations, since the versions of ITRF are related to one another by 14 parameters. Finally, if one chooses to implement a different plate velocity model, P'-I', given in some desired ITRF frame, then one must alter the N-I transformation to match the P'-I' velocities. This will maintain the plate-fixed property of NAD 83.

11. Assessment of NAD 83(NSRS2007) and NAD 83(CORS96)

A cursory examination would suggest the axial sets of NSRS2007 and CORS96 are identical. After all, 673 CORS were held fixed in all three dimensions at the 10 micrometer level. (The remaining 15 CORS could readjust at greater levels.) However, consider the axial set of CORS96 . CORS96 is defined as a 14 parameter similarity transformation from 12 defining points in ITRF00 with a reference epoch of 1997.0 (later updated to 2002.0), where 7 parameters are constant, and 7 parameters are time dependent. This informs us that the origin, orientation, and scale of CORS96 changes in time relative to the origin, orientation, and scale of ITRF00. However, it is also true that the ITRF00 changes in time relative to the ITRF96. One naturally wonders if the plate-fixed condition of the ITRF96–to-NAD 83 transformation was maintained in NAD 83(CORS96).

Now consider the treatment of the CORS coordinates used as control for the NSRS2007 National Readjustment. Briefly recapitulating Pursell and Potterfield (2008),

1. In California, the CORS coordinates at the 2007.0 epoch were obtained from the California Spatial Reference Center (CSRC) on 1/18/2007 from modeled position time series up to 12/28/2006. These were transformed from ITRF2005 instead of ITRF2000.

2. In Arizona, Nevada, Oregon, and Washington, the CORS coordinates at the 2007.0 epoch were obtained by applying the HTDP 2.9 models.

3. In Alaska, the CORS coordinates at the 2003.0 epoch were fixed. Note that the GPS vectors themselves were transformed to 2007.0 with HTDP 2.9.

4. For the remainder of the country, the CORS coordinates at the 2002.0 epoch were fixed.

There are problematic issues in this list that bear on the relationship of NSRS2007 and CORS96. Foremost among the items is the variety of epochs that represent NSRS2007. And, the key element is that the majority of the NAD 83(CORS96) coordinates were held fixed at 2002.0 values. This was not an oversight. It was a necessity, since at the time many CORS did not have reliable velocities. Further, it was a necessity founded on the principle that NAD 83 is a plate-fixed system. The application of HTDP modeling in areas of known crustal motion corrected for the principal non-zero velocities relative to the stable North American Plate. The list above is an operational procedure that expresses reliance on a plate-fixed reference system.

This immediately calls into question if, in fact, NAD 83 is plate-fixed or not? And, if NAD 83 is no longer plate-fixed, then when did NAD 83 stop being plate-fixed, and by how much is NAD83 not plate-fixed? This question is not academic. It is based upon recent comparisons of NAD 83(NA2011) and NAD 83(NSRS2007) coordinates. And these comparisons show a systematic 2 centimeter eastward shift across the majority of the conterminous United States.

It is known that NAD 83 was plate-fixed for ITRF96 transformations (Craymer *et al.*, 2000). That transformation conforms to the procedure described in Section 10. Granted, the underlying NUVEL-1A plate motion model may not be perfect, but it is an open question regarding NUVEL-1A performance in North America.

The first publication of systematic non-zero NAD 83 velocities in stable North America is found in Pearson et al. (2010, pg. 85). This reference describes the June 2008 release of HTDP 3.0. This version of HTDP incorporated a different plate motion model by Altamimi *et al.* (2007). However, no indication is found that the N-I (ITRF-to-NAD 83) transformation is re-derived according to the methodology of Craymer *et al.* (2000) and as described in Section 10. Hence, it is no surprise that non-zero NAD 83 velocities in stable North America were found.

Further, Pearson et al. (2010, pg. 85) refer to motion of ITRF2005 relative to ITRF96 as a second source of non-zero NAD 83 velocities. This is not a valid source, since any such motion should be incorporated into the transformation for the new ITRF frame. As an example, consider both ITRF97 and ITRF00, which also possess velocities relative to ITRF96. The Section 10 procedure was correctly followed for the ITRF97-to-NAD 83 and ITRF00-to-NAD 83 transformations, where those transformations were generated as compositions of earlier ITRF transformations. Soler and Snay (2004, pg. 53) report zero NAD 83 velocities in stable North America.

If we hypothesize that zero NAD 83 velocities in the stable North American plate were present in the interval 2002.0 to 2007.0, then HTDP 2.9 would be satisfactory. One can examine the CORS – HTDP differences in the displacements over 2002.0 to 2007.0 for those CORS that have non-zero velocities. This was performed by Milbert (2008, pg. 125-126) in his Figure 37.1. That figure is reproduced below as Figure 11.1.

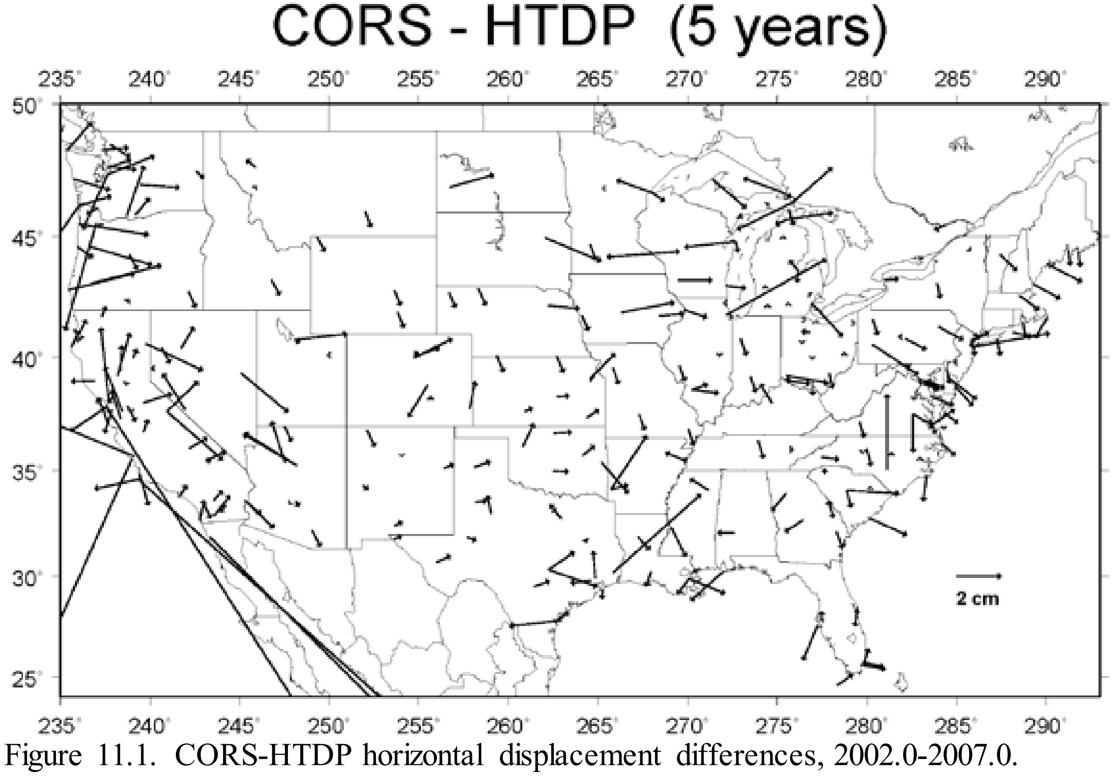

Figure 11.1. CORS-HTDP horizontal displacement differences, 2002.0-2007.0.

Figure 11.1 gives some suggestion of shifts to the southeast along the Eastern coast of the U.S. But, the figure does not reflect a systematic shift across every part of the country, as seen in comparisons of NAD 83(NA2011) and NAD 83(NSRS2007) coordinates. In fact, some of the vectors around the Great Lakes are actually two vectors, pointing in opposite directions.

An assessment of the relationship between NAD 83(NSRS2007) and NAD 83(CORS96) should address the 2002.0 versus 2007.0 epoch difference. However, that will require a detailed investigation of the genesis of the systematic non-zero NAD 83 velocities in the stable North American plate. That is beyond the scope of this report. However, it is a question that should be pursued.

Some general guidance for the near future can be provided. It is a fact that NAD 83(NA2011) is not plate-fixed. Therefore, adherence to the reference epoch of 2010.0 is of paramount importance to give an appearance of being plate-fixed. The good news is that the NAD 83(NA2011) adjustment was uniform in its adoption of a 2010.0 epoch for the GPS data and the CORS constraints. During the study, cases were found of the same point having multiple coordinates in the NGS Integrated Data Base (NGSIDB) and in the OPUS solution data base (eg. PID's `HG0637` and `EM0556`). Also, instances of CORS monuments having multiple coordinates and identifiers within the NGSIDB were found (e.g. PID's `AH7483/ DE6614` and `AI4508/ DK4143`). A harmonized representation of NGS data will serve the user community.

CONCLUSIONS

GEOCON successfully performs three-dimensional coordinate transformations between the NAD 83(HARN) coordinates and the NAD 83(NSRS2007) coordinates. The accuracy of the transformation is related to the small grid spacing and the use of a larger tension parameter (T=0.4). For the first time, detailed error estimates are provided with the coordinate transformations. Quality measures computed by 69-fold cross-validation identify anomalous coordinate shifts. And, an error grid generated by a "reverse median" procedure provides a worst-case description of the transformation error. Case studies confirm that manual coordinate transformation procedures are superior when geospatial data are not uniformly tied to all surrounding control points.

Assessment of the relationship between NAD 83(NSRS2007) and NAD 83(CORS96) is hampered by open questions regarding NAD 83 being fixed to the North American Plate. There are indications this is a fairly recent development. And, additional study is recommended.

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Dr. Dennis Milbert received his Bachelors degree in Physics from the University of Colorado, and his M.S. and Ph.D. degrees in Geodetic Science from The Ohio State University. He worked for over 29 years at the National Geodetic Survey (NGS) of the National Oceanic and Atmospheric Administration, where he was promoted to the position of Chief Geodesist. In his federal career he developed accuracy standards, adjustment software, gravity and geoid models, GPS kinematic surveys, and vertical datum transformations. Dr. Milbert served on numerous federal technical and policy working groups and he was an alternate representative to the Senior Steering Group of the Interagency GPS Executive Board. He served for eight years on the joint Editorial Board for Manuscripta Geodetica/Bulletin Geodesique and the Journal of Geodesy. Dr. Milbert is a recipient of the Kaarina and Weikko A. Heiskanen Award, the NOAA Administrator's Award, the Department of Commerce Bronze Medal, and two Department of Commerce Silver Medals. He is a member of the American Geophysical Union, the International Association of Geodesy, and the Institute of Navigation. Dr. Milbert retired from government service in 2004 and pursues research in various geodesy topics.

APPENDIX

A.1 -- Terminology

The IERS Conventions (Petit and Luzum (eds.), 2010) define:

Geocentric Terrestrial Reference System (GTRS): a system of geocentric space-time coordinates within the framework of General Relativity, co-rotating with the Earth, and related to the Geocentric Celestial Reference System by a spatial rotation which takes into account the Earth orientation parameters.

International Terrestrial Reference System (ITRS): according to IUGG 2007 Resolution 2, the ITRS is the specific GTRS for which the orientation is operationally maintained in continuity with past international agreements (BIH orientation). The co-rotation condition is defined as no residual rotation with regard to the Earth's surface, and the geocenter is understood as the center of mass of the whole Earth system, including oceans and atmosphere (IUGG 1991 Resolution 2).

International Terrestrial Reference Frame (ITRF): a realization of ITRS, through the realization of its origin, orientation axes and scale, and their time evolution.

To gain a better perspective on these definitions, it is useful to consider their history. The distinctions between reference system and reference frame had their origin in the nomenclature of Muller (1980) and Kovalevsky and Mueller (1981). The critical element is that the reference system provides the theory to obtain coordinates, whereas the reference frame is an actual materialization of coordinates. I shall review some of the concepts and nomenclature found in Muller (1980), Kovalevsky and Mueller (1981), Muller (1985), Moritz and Mueller (1988), and Mueller (1988). Note that in the most formal use, Mueller refers to *reference coordinate systems* and *reference coordinate frames*. However he will use *reference systems* and *coordinate systems* interchangeably, and *reference frames* and *coordinate frames* interchangeably.

Reference System: a general description of physical environment and physical theories used in definition of coordinates.

Conventional: an adjective indicating a choice in the resolution of ambiguities inherent in certain physical theories (e.g. choice of axial origin and alignment).

Conventional Reference System: a more specific description of physical environment and physical theories used in definition of coordinates.

(As a historical footnote; in 2007, the IERS adopted the GTRS to replace the Conventional Terrestrial Reference System (CTRS).)

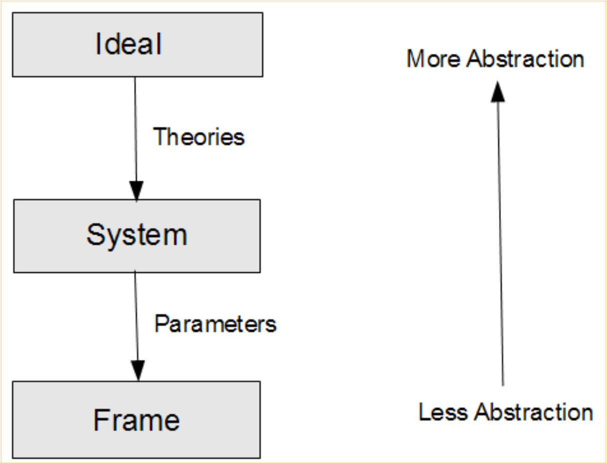
Reference Frame: the materialization a reference system. Provides a quantitative description of positions or motions on the Earth or of celestial bodies.

Conventional Reference Frame: the materialization of a *Conventional Reference System* by means of a set of conventionally chosen parameters.

Ideal: an adjective indicating greater abstraction; the conceptual definition only.

Ideal Reference System: the concept of a given reference system without the physical theories needed to construct that system or materialize it into a reference frame.

The notions of *Ideal Reference System*, *Reference System*, and *Reference Frame* form a hierarchy of abstraction. The addition of physical theories to an ideal system allows construction of a *Reference System*. The choice of parameters (e.g. a set of starting coordinates) allows the construction of a *Reference Frame*. This is depicted in the following figure.



Beginning with the ITRF2005, the ITRF was realized by time series of station positions and Earth orientation parameters (Petit and Luzum (eds.), 2010, p. 35-40). As such, the ITRF is composed of a global set of station coordinates and velocities.

The IERS Conventions (Petit and Luzum (eds.), 2010) also define:

datum: (plural datums) A geodetic reference frame. In surveying and geodesy, a datum is a set of reference points on the Earth's surface, and (often) an associated model of the shape of the Earth (reference ellipsoid) used to define a geographic coordinate system.

It is seen that a geodetic datum is a set of coordinates that are a materialization of a coordinate reference system

A.2 – References for the Appendix

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