NOAA Technical Report NOS 98 NGS 27

The 1981 Saugus to Palmdale, California, Leveling Refraction Test

Rockville, Md.
1983

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ABSTRACT

During May and June 1981 the U.S. Geological Survey and the National Geodetic Survey, of the National Ocean Service, NOAA, participated in a joint refraction test along a leveling route from Saugus to Palmdale, Calif. The purpose was to determine (1) the magnitude of the differences between heights determined using short- and long-sight distances along the same leveling route; (2) the ability of standard refraction models, used in conjunction with both measured and predicted temperature gradients, to explain the observed differences, and (3) the ability of the temperature model used by S. R. Holdahl to reproduce observed temperature differences. The survey used a Jenoptik NI 002 reversible-compensator leveling instrument and a pair of ½-centimeter-scale Kern leveling rods which had been previously calibrated at every graduation by the National Bureau of Standards. Air temperatures were observed near each instrument station at heights of 50, 150, and 250 cm above the ground. After applying all office corrections except for refraction, the sum of the differences between absolute values of short- and long-sight elevation differences accumulated to +51 mm over a distance of 50 km, involving a height difference of 611 m. Application of refraction corrections based on observed temperature differences and Holdahl's predicted temperature differences reduced the accumulated sum to +4.1 and +6.4 mm, respectively, when using the single-sight refraction equation of T. J. Kukkamaki.

BACKGROUND

The existence of atmospheric refraction effects in leveling, even when balanced-sight lengths are used, has been known since the 1930's (Kukkamaki 1938, 1939). However, refraction corrections were seldom applied to leveling data in the United States in subsequent years. In the past few years, there has been renewed interest in refraction errors associated with leveling observations. This interest results from the recognition that near-surface temperature gradients, and thus refraction errors, are often much greater in lower latitudes (e.g., in the United States) than in Finland where Kukkamaki worked, or in England where an extensive series of temperature measurements were observed by Best (1935) and subsequently used by Kukkamaki.

Recent micrometeorological studies have given support to the temperature model used by Kukkamaki in developing his refraction corrections (Deacon 1969; Webb 1964, 1965; Priestly 1959; Angus-Leppan and Webb 1971). Angus-Leppan (1979) has developed procedures for applying the micrometeorological studies directly to the problem of leveling refraction corrections.

In 1977 Holdahl (1981) carried out temperature measurements near Gorman, Calif. These measurements indicated an average temperature difference of −0.75°C between thermisters placed at 2.5 and 0.5 m above the ground during midday in December. This was followed by measurements in Hawaii (Holdahl 1980), which also indicated much larger temperature differences than were observed by Best in England. In retrospect, it is clear that many temperature measurements had previously been made by meteorologists in the United States showing that temperature gradients in the first 3 m above the ground are often quite large, and that the variations had the logarithmic form proposed by Kukkamaki.

At this point, the study of refraction correction took two directions. One direction was the undertaking of carefully controlled experiments to determine the adequacy of existing refraction models, using observed temperature variations. The other direction was the development of algorithms to predict vertical temperature variations where temperature differences were not observed, as is the case with most historic leveling data in the United States.

A test of the adequacy of refraction models under controlled conditions was considered important because leveling refraction models assume that, over the distance of a leveling setup, isotherms are everywhere equidistant from the ground surface. However, it is well known that heat is transferred upward from the ground by convection in the form of convection cells of small lateral dimensions. Thus the idea of isotherms being everywhere parallel to the ground is not strictly true.

Special tests were carried out by Whalen in 1978 and 1979 (Whalen 1980, 1981) at Gaithersburg, Md., and near Tucson, Ariz., consisting of repeated measurements over a set of lines 30 to 60 m long with temperature measurements made at the same time as the leveling observations. The tests clearly showed the existence of leveling refraction effects on the observations, and demonstrated that existing refraction models, when used with observed temperature differences, remove approximately 85 percent of refraction error. A test carried out in Turkey (Banger 1982), similar to that conducted by Whalen, has given similar indications of the refraction effect on leveling.

Because of the large amount of leveling performed in the United States in the past without adequate temperature measurements to compute refraction corrections, it was also important to develop algorithms to estimate vertical temperature differences. Holdahl (1980, 1981) developed algorithms applicable to the estimation of temperature variations for the 48 contiguous United States. Application of the algorithms to the leveling tests in Gaithersburg and Tucson, and to the estimation of the temperatures measured near Gorman, Calif., demonstrated that the Holdahl algorithm was sufficiently accurate to remove the major portion of the refraction effect (Holdahl 1980, 1981; Whalen 1981).

One of the most important aspects of the application of refraction corrections has been in crustal motion studies. Strange (1981), using estimates of temperature gradients obtained by doubling the temperature intervals of Best (1935) (the doubling being more appropriate for California), found that much of the southern California uplift proposed by Castle et al. (1976) and Castle (1978) was the result of the lack of proper correction for refraction. Holdahl (1982) has applied his improved method of estimating temperature variations to the computation of refraction corrections in southern California. His results verify those of Strange and indicate, in greater detail, the impact of refraction corrections on presumed aseismic movement in southern California.

Because of the significant impact of the application of refraction corrections on the evaluation of crustal motion in southern California, the U.S. Geological Survey proposed a refraction test to be carried out along a leveling line that was essential to the interpretation of the proposed southern California uplift (Castle 1978), i.e., the leveling line from Saugus to Palmdale. The leveling route chosen between Saugus and Palmdale (fig. 1) was the route of 1955 and 1961 levelings. Strange (1981) had estimated that the refraction correction for the 1955 leveling was 11.6 cm, and for the 1961 leveling was 8.0 cm. Holdahl (1982) obtained refraction corrections of 12.2 and 9.2 cm, respectively, for these two levelings.

The purpose of the Saugus-to-Palmdale test was threefold:

1. To measure the magnitude of the differences between heights determined using two different sight lengths along the same leveling line.
2. To determine if standard refraction models, in conjunction with measured vertical temperature differences, would explain the observed differences in heights.
3. To determine how well the temperature model used by Holdahl (1980, 1981) reproduces observed temperature differences.

SURVEY PROCEDURES

Specifications

First-order, class II specifications (FGCC 1980) were followed: the maximum allowable sight length was 60 m; the maximum difference in sight lengths per setup was 5 m with a maximum accumulation of 10 m per section; the low-minus-high-scale elevation difference per setup was 0.30 mm or less (except for long-sight distances); the algebraic sum of backward and forward runnings for each section was $4\sqrt{\text{km}}$ mm or less; and loop misclosures were $5\sqrt{\text{km}}$ mm or less. The specifications were relaxed to permit a low-minus-high-scale elevation difference per setup of 0.75 mm for long-sight distances (50-60 m), so observations with long-sight distances could continue throughout the observing day. The observing sequence at each instrument station was backsight low scale, backsight stadia, foresight low scale, foresight stadia (reverse compensator), foresight high scale, and backsight high scale (Whalen 1978).

Data Recording

The following information was recorded at each instrument station:

1. Month, day, hour, and minute.
2. Air temperatures, measured at 50, 150, and 250 cm above the ground.
3. Wind speed, to the nearest 1 mph.
4. Sun code of 0, 1, or 2, corresponding to overcast, partly cloudy, or clear.
5. Ground surface code of A, B, C, D, E, or F, corresponding to asphalt, gravel or sand, concrete, dirt, sparse vegetation, or dense vegetation.
6. Soil moisture code of 0, 1, or 2, corresponding to dry, moist, or visibly wet.

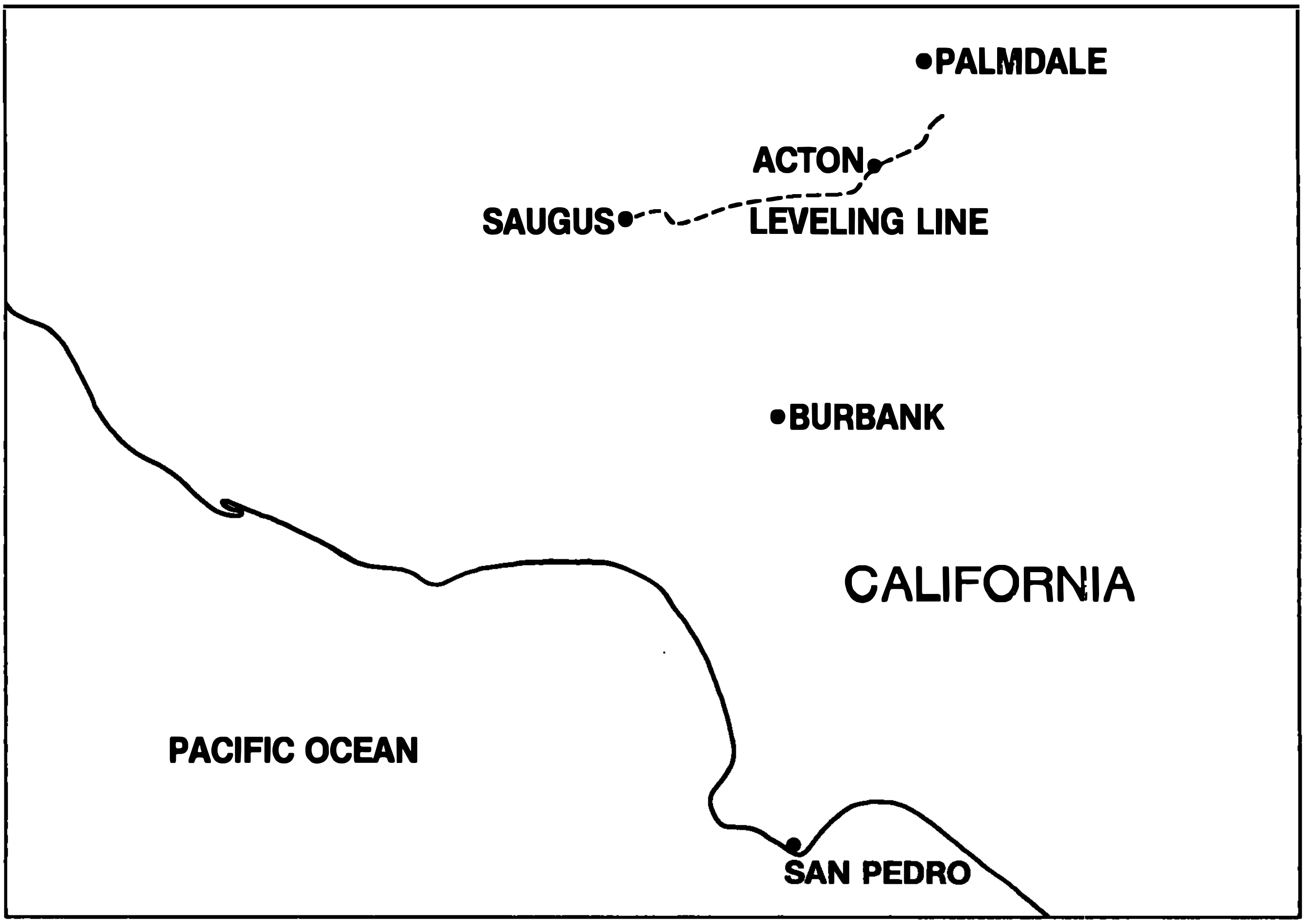

Figure 1.—Saugus to Palmdale survey route.

The leveling observations were recorded on an HP-97 programmable calculator using a Teledyne Geotronics, Inc. program which had been validated by NGS. The calculator applied first-order, class II tolerances to the data for sight length, difference in sight length, and low-minus-high-scale elevation difference per setup, and rejected observations when tolerances were exceeded. Observations which met the tolerance tests were recorded by the HP-97.

Equipment

A Jenoptik NI 002 geodetic leveling instrument (fig. 2) furnished by Teledyne Geotronics, Inc. was used for the project. A pair of NGS unmatched Kern ½-centimeter-scale leveling rods (fig. 3) was used for the survey. These rods had been calibrated at every graduation by the National Bureau of Standards before the survey. Small turning plates ("turtles") weighing approximately 2 kg were used for turning points.

NGS provided three air-temperature sensors mounted 50, 150, and 250 cm above the bottom of a pole (fig. 4). Each temperature sensor consisted of a thermistor with a metal cap, mounted within a tubular metal shield, which was mounted within a second tubular metal shield. The metal cap gave the thermistor an averaging time of approximately 1 minute. The outer shield was polished, and a small fan was attached to each sensor to draw outside air past the thermistors and expel it from the back of the tube. The fans of the three sensors were powered by a common pack of batteries which provided the same voltage to all three fans. A Doric T-meter (fig. 5) was used with a rotatable switch to display temperatures from each sensor to 0.1°F.

A portable Airguide anemometer measured wind speed. The minimum wind speed which could be measured with this device was 5 mph. Wind speeds of less than 5 mph were estimated to the nearest 1 mph.

DATA PROCESSING

Weather Data

Weather data (temperature differences; wind speed; and sun, ground, and moisture codes), which had been recorded at each setup on coding sheets in the field during the survey, were keypunched at NGS Headquarters, transmitted to a computer, and stored on a disk file. Copies of the weather data were listed and sent to the other participants in the refraction experiment.

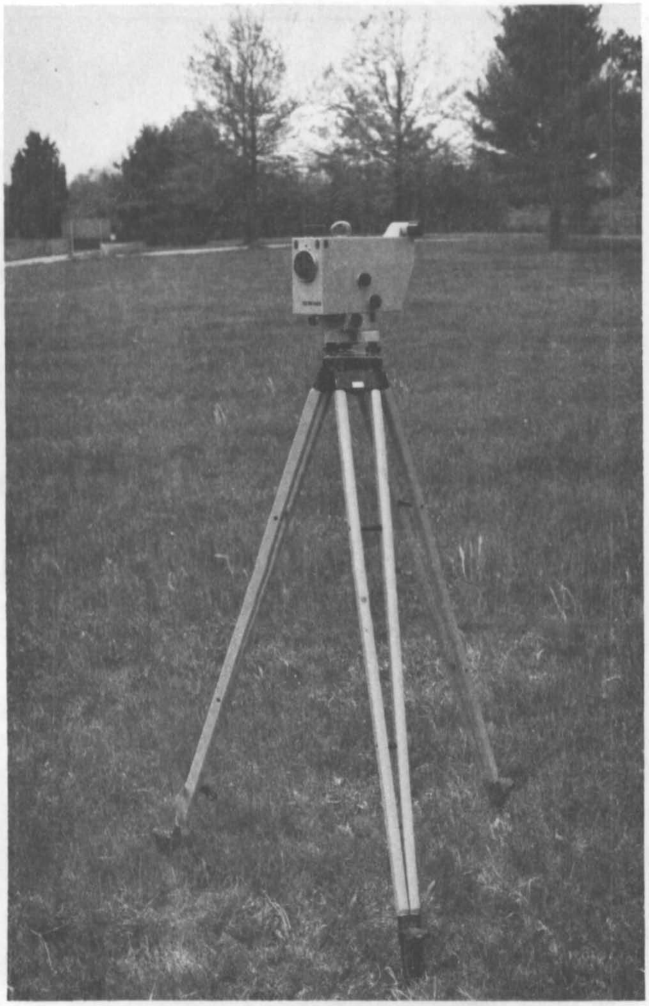

Figure 2.—Jenoptik NI 002 leveling instrument.

Leveling Observations

Leveling observatons, which had been recorded and printed by the HP-97 calculator in the field, were rerecorded on NGS Monroe recording system cassettes (Whalen and Balazs 1976). The observations were transferred to a TI 742 terminal cassette, transmitted by telephone to the computer, and stored on a disk file. The temperature observations from the 2.5 and 0.5 m sensors were taken from the weather data file and merged with the leveling observations file.

Corrections To Leveling Observations

Corrections to leveling observations were applied for the following errors: collimation, Invar-band scale, Invar-band thermal expansion, and astronomic effects (Earth tide). For details on these corrections, see Balazs and Young (1982). Refraction equations are given in the appendix.

The average height of the leveling instrument was estimated for the survey by averaging the backsight and foresight low-scale rod readings for a sample of 56 setups distributed throughout the survey. The average height of the instrument was determined to be 160 ±3 cm. The temperature sensors agreed with each other before the test to ±0.1°C, when checked by the NGS Operations Branch, Instrumentation and Equipment Section. When checked against standard thermometers after the test, the lower (0.5 m) sensor consistently read 0.1°C too low. A linear change was assumed and one-half of the change (0.05°C) was added to the bottom sensor temperatures, before computing mean temperature differences or refraction corrections. The average air-temperature differences for the survey were −0.46 ±0.02°C (standard error) between the sensors located at 2.5 and 1.5 m above the surface, and −0.87 ±0.04°C (standard error) between the sensors located at 1.5 and 0.5 m above the surface.

Figure 3.—Kern ½-centimeter leveling rod.

Separate solutions were obtained with the temperatures and refraction equations shown in table 1. Results of the solutions are summed for the survey and shown in table 2 with solution numbers in parentheses. In table 2, DDH= $|dh_s| - |dh_l|$, where the vertical bars show absolute values, dh is a section height difference, and subscripts s and l show whether short or long sight distances were used. SDDH is the sum of DDH values along the leveling line. DDH is the difference between absolute values of height differences determined for each section using short and long sight distances. All refraction equations were based on Kukkamaki's studies (1938, 1939). The first solution used the temperatures observed at each instrument station. The A-factor (designated gamma by Kukkamaki) was

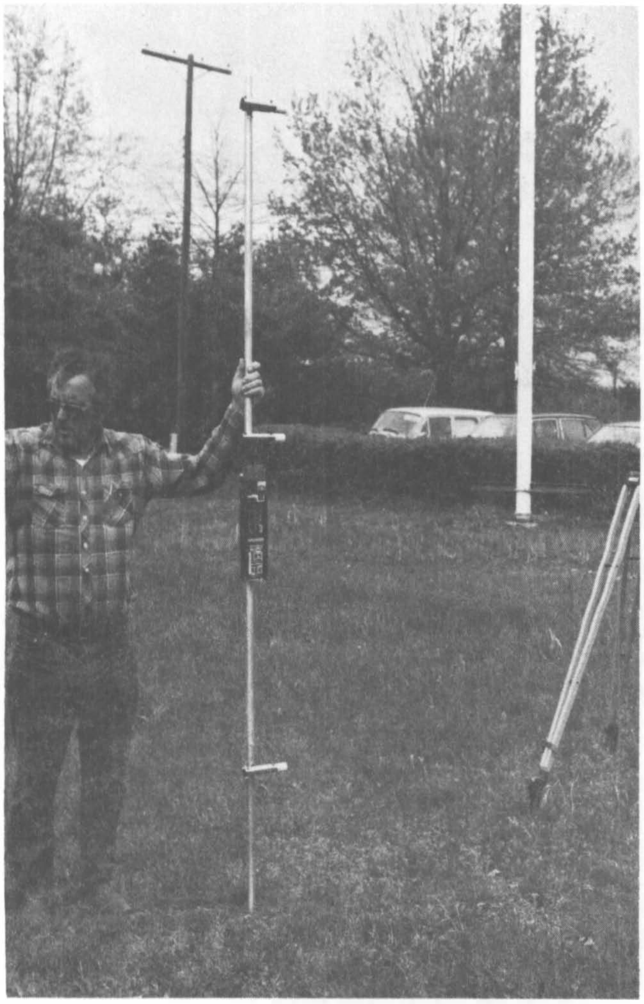

Figure 4.—Air-temperature sensors.

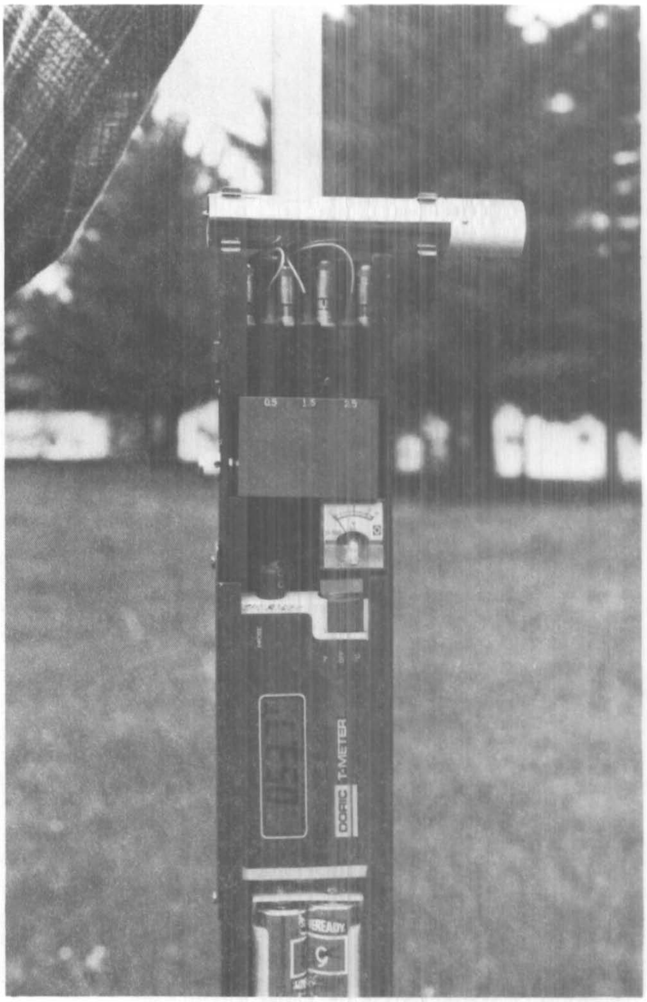

Figure 5.—Doric T-meter.

Table 1.—References to temperatures and refraction equations

Solution	*Temperatures*	*A*	*c*	Z_o *(cm)*	*Appendix eq. no.*	*Kukkamaki's references (yr., eq. nos.)*
1	Observed	48	0.161	160	11	1939, eqs. 5,6
2	Observed, section means		0.161	160	1,2	1938, eqs. 10,11,12
3	Holdahl's predicted		0.161	160	1,2	1938, eqs. 10,11,12
4	−1.336 °C, all sections		0.161	160	1,2	1938, eqs. 10,11,12
5	Observed, section means		−⅓	150	8,9,10	1938, eqs. 10,14
6	Holdahl's predicted		−⅓	150	8,9,10	1938, eqs. 10,14

Table 2.—Cumulative values of DDH (SDDH)

			Uncorrected	*Refraction—corrected SDDH*					
Z_o *(cm) =*				*160*	*160*	*160*	*160*	*150*	*150*
c=				*.161*	*.161*	*.161*	*.161*	*−⅓*	*−⅓*
Section Number	*Distance km*	*Elevation m*	*SDDH mm*	*(1) mm*	*(2) mm*	*(3) mm*	*(4) mm*	*(5) mm*	*(6) mm*
1	0.9	377.6	0.1	0.1	0.1	0.0	−0.4	0.1	0.1
2	1.9	388.7	0.3	−0.2	−0.1	0.0	−1.1	−0.1	0.1
3	2.6	397.0	0.8	−0.4	−0.2	0.1	−1.5	−0.2	0.2
4	3.5	410.1	1.1	−0.3	−0.1	0.3	−2.5	−0.1	0.4
5	4.2	419.3	1.3	−0.6	−0.3	−0.3	−3.0	−0.2	0.1

Table 2.—Continued

			Uncorrected	*Refraction—corrected SDDH*					
Z_o (cm) =				*160*	*160*	*160*	*160*	*150*	*150*
c=				*.161*	*.161*	*.161*	*.161*	*−⅓*	*−⅓*
Section Number	*Distance km*	*Elevation m*	*SDDH mm*	*(1) mm*	*(2) mm*	*(3) mm*	*(4) mm*	*(5) mm*	*(6) mm*
6	5.4	426.9	−0.9	−3.1	−2.9	−2.7	−5.4	−3.0	−2.7
7	6.2	438.9	−2.3	−5.2	−4.9	−5.1	−7.7	−5.5	−5.7
8	6.8	435.5	−1.0	−4.2	−3.8	−4.1	−6.7	−4.4	−4.7
9	7.2	433.9	1.1	−2.0	−1.4	−1.7	−4.3	−2.4	−2.7
10	7.9	438.7	0.5	−2.9	−2.2	−2.7	−5.4	−3.2	−3.7
11	9.4	461.0	2.3	−3.1	−2.4	−4.0	−6.2	−4.4	−6.3
12	10.3	460.2	2.5	−2.8	−2.2	−3.7	−5.9	−4.2	−6.1
13	11.0	475.0	3.0	−3.1	−2.6	−4.5	−6.5	−4.2	−6.6
14	11.8	478.2	3.0	−4.0	−3.2	−5.2	−7.1	−4.3	−6.8
15	12.7	491.6	1.3	−6.6	−5.6	−7.8	−9.9	−6.4	−8.9
16	13.2	498.5	1.9	−5.9	−5.0	−7.2	−9.2	−5.9	−8.4
17	14.0	497.8	2.0	−5.7	−4.7	−6.7	−8.6	−5.9	−8.3
18	14.7	499.1	3.2	−5.0	−3.9	−6.1	−7.9	−4.7	−7.2
19	15.4	511.9	5.1	−4.0	−3.0	−5.0	−6.8	−3.5	−5.9
20	16.3	514.0	5.8	−3.4	−2.4	−4.4	−6.3	−2.9	−5.3
21	16.8	518.8	5.6	−3.5	−2.5	−4.7	−6.7	−3.0	−5.7
22	17.6	528.9	7.3	−3.2	−2.0	−3.9	−5.8	−2.5	−4.9
23	18.5	542.0	7.6	−4.2	−2.9	−4.7	−6.6	−3.4	−5.8
24	18.8	550.7	8.1	−3.7	−2.2	−3.9	−6.0	−2.8	−5.0
25	19.4	559.3	8.4	−3.6	−2.0	−4.3	−6.4	−2.6	−5.5
26	20.0	569.3	9.7	−2.9	−1.3	−3.2	−5.7	−2.0	−4.5
27	21.2	591.0	10.6	−3.3	−1.4	−3.7	−6.5	−2.3	−5.2
28	22.3	605.7	12.0	−3.2	−1.4	−4.2	−6.8	−2.6	−6.2
29	23.1	617.2	14.5	−2.3	−0.3	−2.8	−5.3	−2.0	−5.1
30	24.0	627.7	14.6	−2.4	−0.4	−3.6	−6.4	−2.1	−6.1
31	24.8	642.8	15.4	−2.5	−0.5	−4.1	−7.0	−2.9	−7.6
32	25.6	658.3	18.9	−1.3	1.3	−2.0	−4.7	−1.8	−6.1
33	26.2	664.9	20.3	−0.9	1.9	−1.3	−3.9	−1.3	−5.5
34	26.9	676.4	20.7	−1.3	1.6	−1.7	−4.4	−1.7	−6.0
35	27.6	683.5	21.7	−0.7	2.2	−1.2	−4.1	−1.1	−5.4
36	28.8	697.5	23.0	−1.9	1.0	−2.1	−4.6	−1.1	−5.3
37	29.9	713.9	22.3	−3.2	−0.3	−3.6	−6.3	−2.5	−7.0
38	30.8	729.8	25.9	−2.0	1.4	−1.5	−4.0	−0.7	−4.9
39	31.9	748.5	29.1	−1.9	2.2	0.7	−2.2	−0.1	−2.7
40	32.1	752.4	29.0	−2.4	1.8	0.3	−2.7	−0.7	−3.3
41	33.1	769.3	29.7	−2.9	1.3	−0.6	−3.8	−1.6	−4.7
42	34.0	785.8	31.6	−3.2	1.5	0.0	−3.1	−2.3	−4.8
43	35.1	799.8	32.9	−3.1	1.9	0.3	−3.1	−2.4	−5.0
44	36.1	814.1	37.5	−0.6	4.8	3.6	0.4	0.3	−1.9
45	36.7	822.4	39.4	0.4	6.0	4.8	1.7	1.2	−1.0
46	37.5	829.0	40.6	0.9	6.7	5.5	2.4	2.0	−0.2
47	38.4	844.4	39.9	0.1	5.9	4.3	0.9	1.0	−1.7
48	39.1	850.3	40.8	−0.1	5.9	4.8	1.4	1.2	−1.1
49	40.0	864.6	40.7	−0.8	5.3	4.1	0.4	0.6	−1.9
50	40.6	875.3	41.0	−1.7	4.4	3.4	−0.2	−0.1	−2.4
51	41.5	898.8	44.6	−3.0	4.5	5.6	2.1	0.7	0.2
52	42.3	913.9	46.2	−2.9	4.7	6.2	2.7	0.1	0.2
53	43.6	940.9	47.0	−3.1	4.5	5.7	1.5	−0.3	−0.6
54	45.1	975.1	49.2	−4.1	3.9	5.9	1.2	−1.6	−0.9
55	45.8	983.5	51.6	−2.1	6.0	8.0	3.4	0.1	0.8
56	46.9	975.8	52.0	−2.5	5.8	8.0	3.4	−0.4	0.6
57	47.8	935.0	49.9	−4.7	3.7	5.8	1.1	−2.7	−1.7
58	48.7	906.5	52.4	−3.2	5.4	7.7	3.2	−1.4	−0.1
59	48.7	906.9	52.4	−3.2	5.4	7.8	3.2	−1.4	0.0
60	49.5	909.3	51.0	−4.8	4.1	6.4	1.9	−2.8	−1.5

Mean observed Δ T in degrees Celsius = −1.336 ± 0.053

Mean Holdahl Δ T in degrees Celsius = −1.290 ± 0.027

Estimated mean Δ T in degrees Celsius = −1.273 ± 0.236

(1) = simplified balanced-sight equation, observed temperature differences applied at setup level.

(2) = single-sight equation, section means of observed temperature differences applied at setup level.

(3) = single-sight equation, Holdahl's predicted temperature differences applied at setup level.

(4) = single-sight equation, mean of observed temperature differences for survey: −1.336°C.

(5) = balanced-sight equation, section mean observed temperature differences applied at section level.

(6) = blanaced sight equation, Holdahl's predicted temperature differences applied at section level.

based on a c-value of 0.161 which was computed from the mean temperature differences for the survey (−0.46 and −0.87°C) and on the mean height of instrument (Z_0) of 160 cm. Solutions 2, 3, and 4 used the single-sight equations.

Equation elements for backsight and foresight, excluding d, Δt and ($z^c_2 - z^c_1$), were differenced at each setup and summed for each section. The sums were multiplied by d and divided by ($z^c_2 - z^c_1$) which were computed for each section. z_2 is the height of the upper temperature sensor (2.5 m) and z_1 is the height of the lower temperature sensor (0.5 m), above the surface. The resulting partial corrections (PC) for each section were multiplied by the temperature differences used with solutions 2, 3, and 4 to obtain the refraction corrections. Solutions 2 and 3 provide a comparison of results based on means of observed temperature differences and Holdahl's predicted temperature differences, using the single-sight equation. The mean of the observed temperature differences, −1.336°C, was used for all sections in solution 4.

Solutions 5 and 6 use section means of observed temperature differences and Holdahl's predicted temperature differences, respectively, with the balanced-sight equation and rod readings estimated from mean slopes between bench marks. The estimated mean temperature difference shown at the bottom of table 2 was estimated using the equation:

$$\Delta t' = \Sigma(|dh_s| - |dh_l|)/\Sigma(|PC_s| - |PC_l|)$$

where the vertical bars show an absolute value; subscripts s and l denote sections run with short- and long-sight distances, respectively, dh is the elevation difference for a section with all corrections except refraction applied, PC is the partial correction for refraction mentioned previously, and summations are made for all sections for which the temperature differences are to be estimated. The estimate of mean temperature difference was made to determine if the mean temperature difference could be recovered from the leveling observations when sight distances for forward and backward runnings of the sections are unbalanced, as was the case for this survey.

TEST RESULTS

As best illustrated in equation 11 (appendix, eq. 11), the refraction error would be expected to be greater for long-sight lengths than for short-sight lengths. The effect of refraction error is to make the determination of a section height difference smaller in magnitude than its true value. Thus, if a refraction correction is not applied, the magnitude of a section height difference determined using long sight lengths, $|dh_l|$, would be expected to be systematically smaller than that determined using short sight lengths, $|dh_s|$.

Figure 6a shows a bar graph of the distribution of the values of DDH = $|dh_s| - |dh_l|$. As expected (if refraction effects have the general form indicated by equation 11), the quantity DDH is predominantly positive. In fact, 47 sections had a positive DDH, 11 sections were negative, and 2 sections were 0. One would not expect all sections to have a positive DDH, since random observing error is of the same order of magnitude as refraction error and would, on occasion, be expected to dominate.

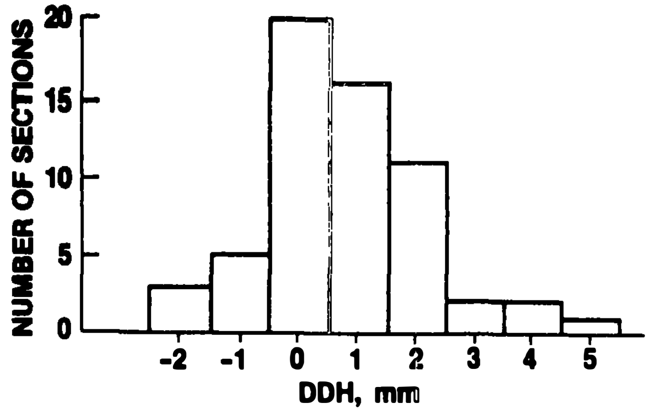

Figure 6a.—Distribution of differences between long and short sight-length section determinations using data uncorrected for refraction.

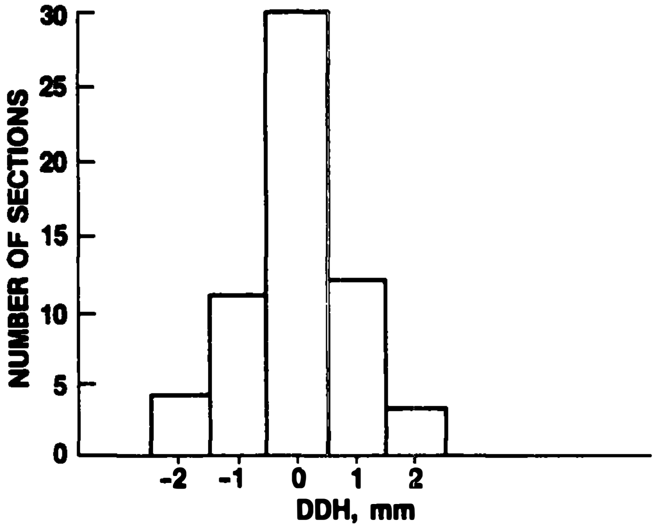

Figure 6b.—Distribution of differences between long and short sight-length section determinations using refraction-corrected data.

Figure 7 illustrates the systematic nature of DDH in another way, with a plot of the sums of DDH, designated SDDH, as a function of distance along the profile. Also shown in figure 7 is a plot of terrain elevation along the profile and SDDH after correcting for the refraction effect using the observed temperature difference at each setup and the single-sight equation (table 1). The quantity SDDH increases systematically with distance along the profile using data without refraction corrections. The increase is not uniform, with the rate of increase during the first 12 km along the profile being less than the rate during the remainder of the profile. This is partially due to the fact that cloudy conditions (and possibly the type of ground cover) caused temperature differences for this

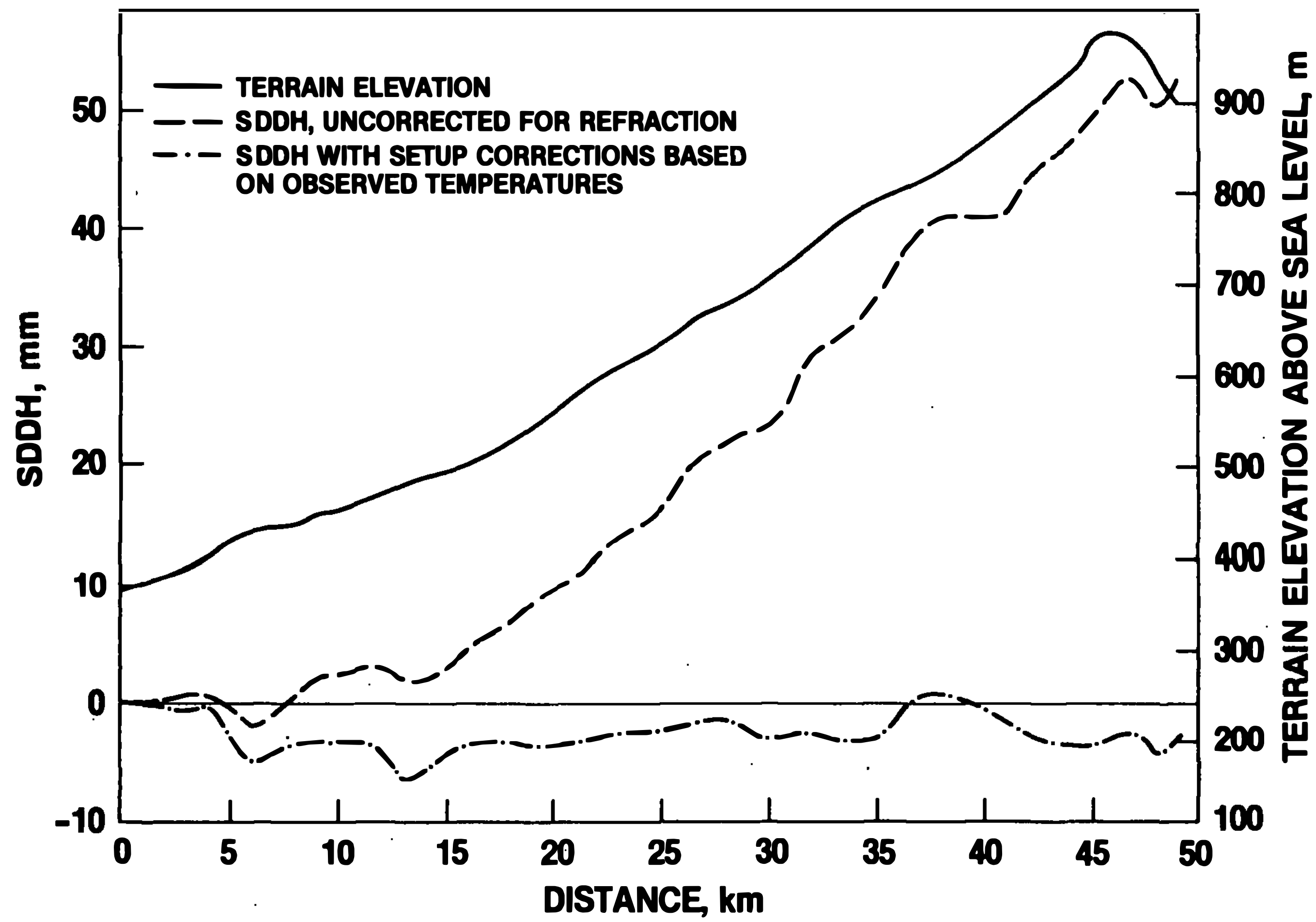

Figure 7.—Effect of refraction corrections for balanced-sight equation, with computed c.

part of the line to be smaller than over parts of the line surveyed later. The refraction effect portion of the SDDH signal may have been partially canceled by observing errors.

Qualitatively the results indicated in figures 6a and 7 are compatible with the existence of significant refraction effects. The next question to be examined is how well the existing refraction models can account for the results shown in figures 6a and 7 using observed temperature information and the Kukkamaki balanced-sight and single-sight equations. (See appendix.) Kukkamaki's refraction-correction model uses observed temperatures to provide two parameters. These are dt, the temperature difference between 2.5 and 0.5 m above the ground, and c, the exponent in the equation used to model the temperature variations:

$$t=a+bz^c$$

where t is temperature, z is the height above the ground, and a, b, and c are constants determined from the observations.

In this analysis it was decided to determine a mean value of c for the entire survey. This was done by using the mean values of the temperature differences $t_{1.5}-t_{0.5}=-0.87^\circ C$ and $t_{2.5}-t_{1.5}=-0.46^\circ C$. The value of c determined in this way was +0.161. This value of c was used directly in equation 1 to compute the refraction corrections (solutions 2, 3, and 4 in table 2), and was substituted into equation 12, with a value of $Z_0=1.6$ m, to obtain a value of $A=48$ for use in equation 11, the balanced-sight equation (solution 1 of table 2).

The initial computation of refraction-correction values was obtained using the balanced-sight equation, eq. 11 (appendix), applied to each setup, using the measured Δt, Δh, and L values for that setup, and $A=48$. The cumulative correction for the long-sight-length leveling over the line was 8.2 cm; for the short-sight-length leveling it was 2.7 cm. Figure 7 presents SDDH values before and after application of the refraction corrections obtained from the table 2 solutions. The remaining difference between the two levelings is less than 5 mm. This is well within the expected random error of leveling even if one were comparing two first-order, class I (double-run) levelings.

Another way of visualizing the effect of applying refraction corrections is to construct a graph, such as shown in figure 6a, using refraction-corrected data. Such a graph is shown in figure 6b. This graph emphasizes the random nature of the remaining differences between the long- and short-sight-length levelings after the application of refraction corrections. There are 31 sections with positive

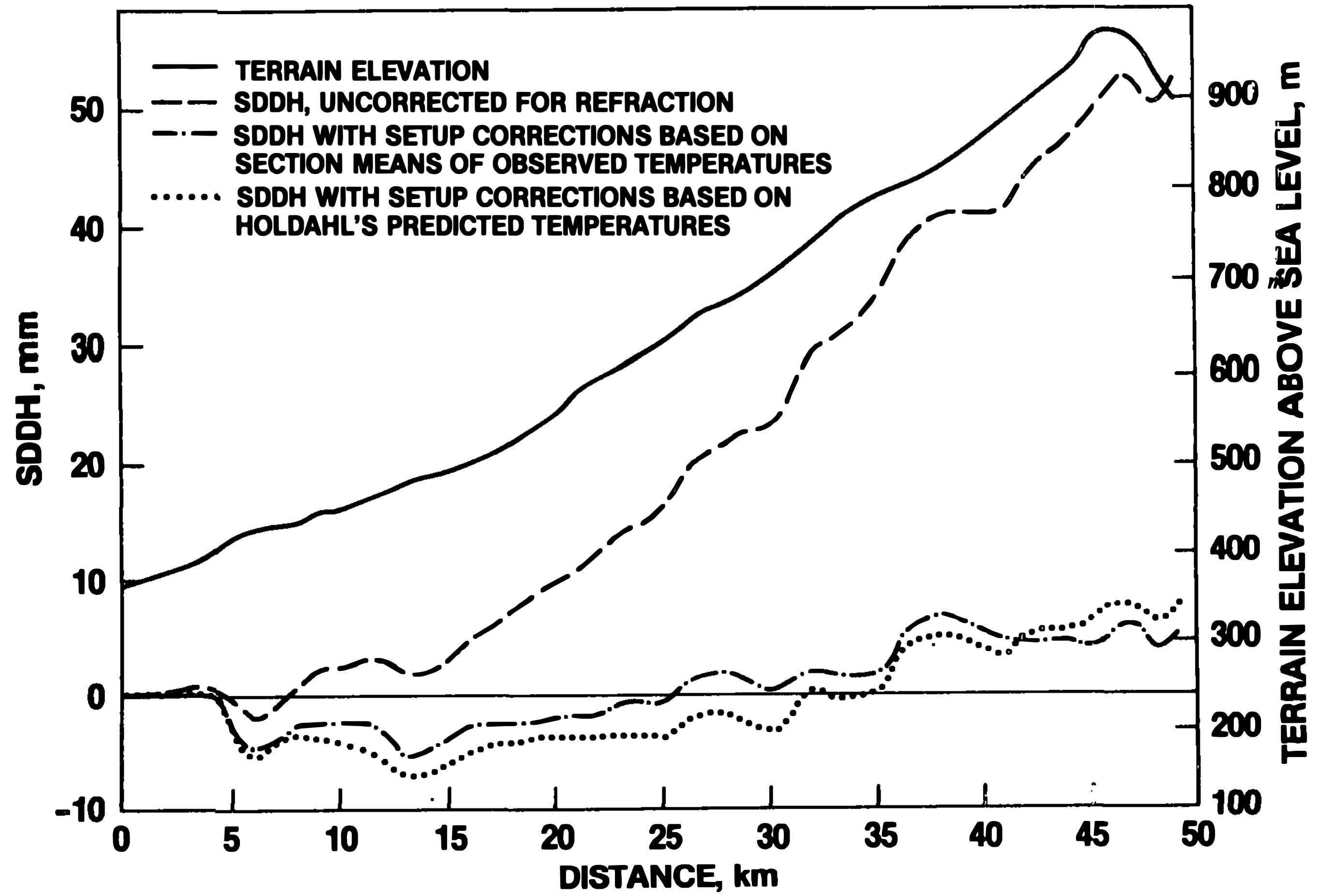

Figure 8.—Effect of refraction corrections for single-sight equation, with computed c.

residuals, 28 with negative residuals, and 2 with residuals of 0. Another characteristic of the section residuals of DDH after application of refraction correction is the small magnitude. Only 9 of the 60 section residuals are greater than 1.2 mm, and none is greater than 2.5 mm.

Figure 8 presents results based on single-sight equations 1 and 2 (appendix) from table 2, solutions 2 and 3, which used meaned differences of observed temperatures for each section and Holdahl's predicted temperature differences, respectively. In both solutions, refraction corrections are applied for each setup, using the sums of the partial refraction corrections for each section mentioned in this report. There is excellent agreement between the two solutions based on observed and predicted temperatures. Both reduce SDDH to within 8 mm.

Figure 9 is a plot of results based on balanced-sight equation 8 (appendix) and solutions 5 and 6, table 2, which use section means of observed temperature differences and Holdahl's predicted temperature differences, respectively. Both solutions use reconstructed rod readings based on the mean slope between consecutive bench marks when computing refraction corrections. The figure shows how well Holdahl's predicted temperatures and the balanced-sight equation correct for refraction errors along this leveling line without knowledge of rod readings or observed temperatures. Estimating rod readings from the mean slope between consecutive bench marks works particularly well for leveling along railroad tracks because the ground slope is usually very constant between bench marks. Results based on predicted and observed temperatures are in close agreement, and SDDH is reduced to within 9 mm.

CONCLUSION

Observations of elevation differences along the leveling profile between Saugus and Palmdale were made using both long- and short-sight lengths. The difference in accumulated observed elevation differences between the two levelings increased systematically along the profile. Application of refraction corrections using the Kukkamaki balanced-sight equation and observed temperature differences from each instrument station removed most of the systematic component of the differences. The remaining differences were well within what would be expected due to random leveling error. One can therefore conclude that the balanced-sight equation, used with observed temperatures, adequately models leveling refraction effects along the Saugus to Palmdale leveling line. The results indicate that for best results in modeling refraction, three temperatures should be measured to allow computation of c in the equation $t=a+bz^c$. However, it should be noted that realistic estimates of c can only be obtained using the mean values of a large number of measurements of temperature differences.

The Holdahl procedure for modeling temperature variations also produced temperature differences which removed the bulk of refraction error. This demonstrates the general validity of the Holdahl model for estimating temperatures to be used in computing refraction corrections for historic leveling data for which temperature differences were not observed.

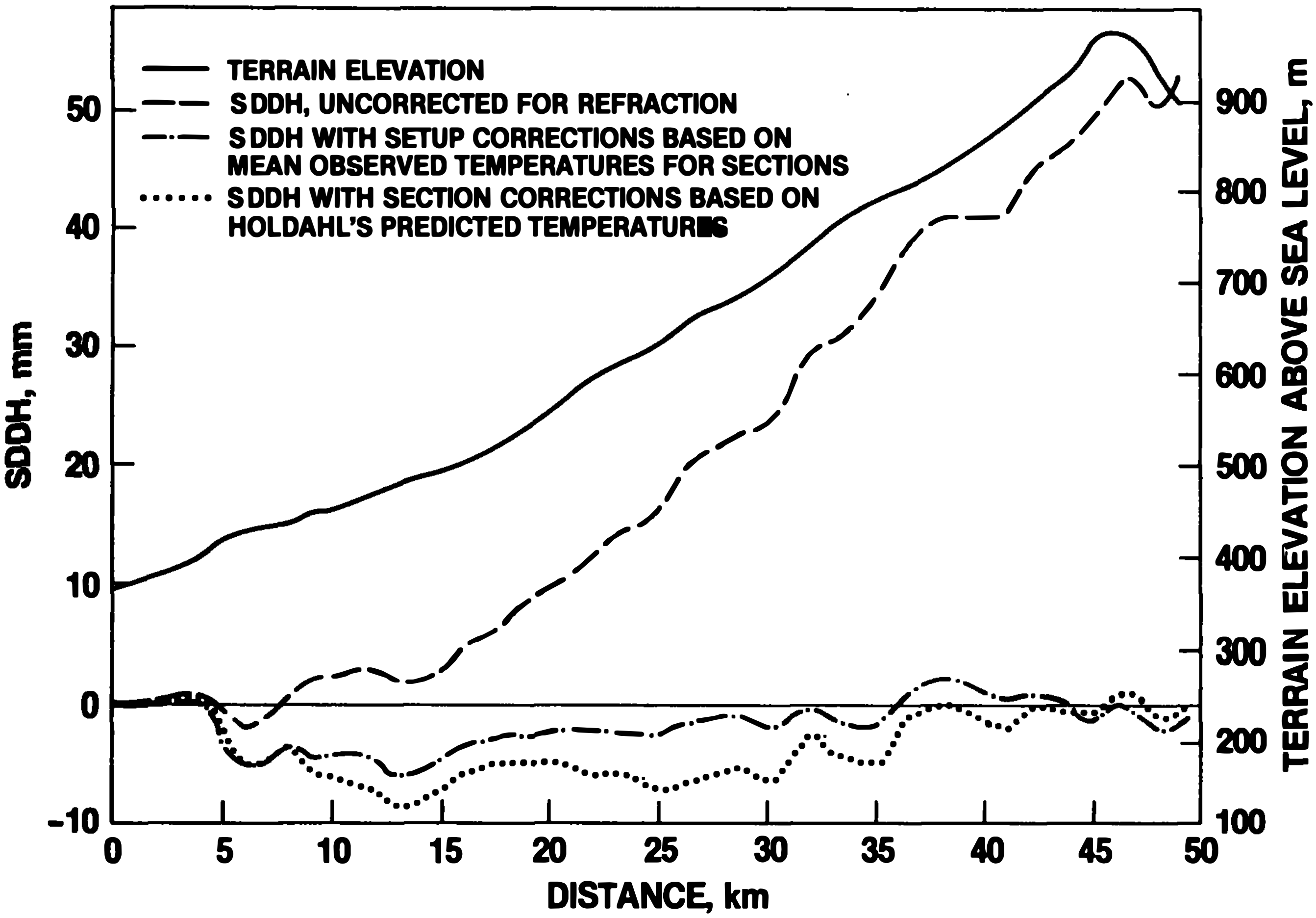

Figure 9.—Effect of refraction corrections for balanced-sight equation, with c = −⅓.

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APPENDIX—REFRACTION EQUATIONS

A refraction correction for leveling was developed by T. J. Kukkamaki in 1938. It was not applied historically to leveling in the United States because its magnitude was considered small, and extra equipment was required to measure temperature differences at two or more heights above the same point on the ground. Most experience with measuring these temperatures came from Finland and England where vertical temperature gradients are small. It was thought that by balancing sight lengths, keeping sight lengths reasonably short, and by not allowing the line of sight to be too close to the ground, refraction error would be insignificant.

Kukkamaki's single-sight refraction corrections are:

$$-R_f = d \cot^2 \epsilon_f \frac{\Delta t}{z^c_2 - z^c_1}$$

$$\left[\frac{1}{c+1} Z_f^{c+1} - Z_o^c Z_f + \frac{c}{c+1} Z_o^{c+1}\right] \qquad (1)$$

$$-R_b = d \cot^2 \epsilon_b \frac{\Delta t}{z^c_2 - z^c_1}$$

$$\left[\frac{1}{c+1} Z_b^{c+1} - Z_o^c Z_b + \frac{c}{c+1} Z_o^{c+1}\right] \qquad (2)$$

where $-R_f$ and $-R_b$ are the corrections to the foresight and backsight readings; c is the coefficient in Kukkamaki's temperature function, $t=a+bZ^c$; $\Delta t = t_2 - t_1$, the temperature difference between probe heights z_2 and z_1; Z_f and Z_b are the heights of the line of sight on the fore and back rods, respectively; Z_o is the instrument height; ϵ is the slope of the ground between the instrument and the rod; and

$$\cot \epsilon_f = L_f/(Z_o - Z_f) \text{ and } \cot \epsilon_b = L_b/(Z_o - Z_b), \qquad (3)$$

where L_f and L_b are the lengths of the foresight and backsight, respectively.

$$d = 10^{-6}(0.933 - 0.0064\,(T-293))P \qquad (4)$$

$$P = (1 - bH/T_o)^{g/Qb}, \qquad (5)$$

where

T is the temperature at the point considered, in degrees Kelvin;

H is the height above sea level, in meters;

T_o is the temperature at sea level, in degrees Kelvin;

g is mean gravity, 9.80665 m s^{-2};

Q is the gas constant at the point considered ($287 \text{ m}^2 \text{ s}^{-2} \text{ K}^{-1}$);

b is the lapse rate, 0.0065 K m^{-1};

P is the pressure at the point considered, in atmospheres (one atmosphere $= 1.013 \times 10^5$ newtons m^{-2}); and

$$T_o = T + bH. \qquad (6)$$

For new leveling the actual rod readings, Z_f and Z_b, are recorded on tape cassettes. The air temperature is recorded at the beginning and end of each section of leveling. Sighting distances, L_f and L_b, are calculated from the stadia and middle rod readings. The correction at a single setup is

$$R = R_f - R_b. \qquad (7)$$

For application of the refraction correction to old data when the original observed rod readings are not available, two assumptions must be made: (a) the ground slope is uniform between the rods, and (b) the foresight and backsight are of equal length. Mathematically this means $\epsilon_f = \epsilon_b = \epsilon$, $L_f = L_b = L$, and R simplifies to

$$R = \frac{-d \cot^2 \epsilon}{z^c_2 - z^c_1}$$

$$\left[\frac{1}{c+1}\left(Z_f^{c+1} - Z_b^{c+1}\right) - Z_o^c\,(Z_f - Z_b)\right] \Delta t, \qquad (8)$$

where rod readings Z_f and Z_b are reconstructed from the average slope, ϵ, and the average sight length, L,

where

$$L = S/2n,$$
$$\epsilon = \Delta h/S,$$
$$Z_f = Z_o - \epsilon L, \text{ and}$$
$$Z_b = Z_o + \epsilon L, \qquad (9)$$

where

n is the number of setups in the section, Δh is the difference of elevation for the section, and S is the section distance. The refraction correction for the entire section is

$$R' = nR. \qquad (10)$$

For balanced-sight formula (8), it is necessary to implement a test to avoid dividing by zero in evaluating the cotangent function. If $\Delta h < 0.0001$ m, R is set to zero.

Kukkamaki (1939) points out that leveling refraction varies with the measured difference in height in nearly linear fashion, on the condition that the line of sight does not come too near the Earth's surface. In fact, it deviates by at most 10 percent from linearly interpolated values between points $Z_2-Z_1=0$ and $Z_2-Z_1=200$ cm, when the target height is greater than 30 cm. This justifies replacing equation (8) with the interpolation equation for the refraction correction:

$$R=-8\times10^{-10}AL^2\Delta t\Delta h, \qquad (11)$$

where

$$A=5.95\left[\left((Z_0-100)^{c+1}-(Z_0+100)^{c+1}\right)/(c+1)+200\,Z_0^c\right]/(z^c_2-z^c_1). \qquad (12)$$

R, L, and Δh are in meters, Δt is in degrees Kelvin, and z_1, z_2, and Z_0 are in centimeters.

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