Appendix S1: SUPPLEMENTAL MATERIAL

Article Information

Ecosphere

Title: Calibrating and Adjusting Counts of Harbor Seals in a Tidewater Glacier Fjord to Estimate Abundance and Trends from 1992-2017

Jamie N. Womble[1], Jay M. Ver Hoef[2], Scott M. Gende[1], Elizabeth A. Mathews[3,4]

Fitting Count Models

Let $Y_{j,k,\ell}(i, t_j, r)$ be the count of pups ($\ell = u$) or nonpups ($\ell = n$) for survey type k, in year i on day t of season j, and the rth daily replicate. Note that $Y_{m,k,u}(i, t_j, r)$ exists for neither $k = g$ nor $k = a$. Write the lognormal probability density as $[Y_{j,k,\ell}(i, t_j, r)|\mu_{j,k,\ell}(i, t_p), \nu]$ and the joint probability density as

$$[\boldsymbol{y}|\boldsymbol{\mu}, \nu], \tag{S.1}$$

where $\boldsymbol{\mu}$ is a vector of means,

$$\mu_{j,k,\ell}(i, t_j) = \exp(\eta + \xi_j + \tau_k + \xi\tau_{j,k} + \xi\pi_{j,\ell} + N_i + Z_j(i, t_j)),$$

and ν is a coefficient of variation parameter common to all $\boldsymbol{y}$.

Equation (S.1) is a conditionally independent "data model" (Cressie et al. 2009), whose mean vector depends on multivariate normal model with autocorrelation, denoted $[\boldsymbol{N}|\phi, \delta_N]$, $[\boldsymbol{Z}_p|\omega_p, \delta_p]$, and $[\boldsymbol{Z}_m|\omega_m, \delta_m]$. Then, the posterior distribution of the parameters and latent variables is written

$$\begin{aligned}[\eta, \boldsymbol{N}, \xi_p, \tau_g, \xi\tau_{p,g}, \xi\pi_{p,u}, \boldsymbol{Z}_p, \boldsymbol{Z}_m, \phi, \omega_p, \omega_m, \delta_N, \delta_p, \delta_m, \nu|\boldsymbol{y}] \propto \\ [\boldsymbol{y}|\boldsymbol{\mu}, \nu][\boldsymbol{N}|\phi, \delta_N][\boldsymbol{Z}_p|\omega_p, \delta_p][\boldsymbol{Z}_m|\omega_m, \delta_m]\times \\ [\eta, \xi_p, \tau_g, \xi\tau_{p,g}, \xi\pi_{p,u}, \phi, \omega_p, \omega_m, \delta_N, \delta_p, \delta_m, \nu]\end{aligned} \tag{S.2}$$

where $[\eta, \xi_p, \tau_g, \xi\tau_{p,g}, \xi\pi_{p,u}, \phi, \omega_p, \omega_m, \delta_N, \delta_p, \delta_m, \sigma^2]$ is the prior distribution on all of the parameters. We used vague, flat priors on all parameters, where priors ranged from $-\infty$ to ∞ for η, ξ_p, τ_g, $\xi\tau_{p,g}$, and $\xi\pi_{p,u}$, from -1 to 1 for ϕ, ω_p, and ω_m, and from 0 to 20 on the standard deviation parameters δ_N, δ_p, δ_m, and from 0 to 10 for ν.

The model was fitted with Markov Chain Monte Carlo (MCMC) methods using the Metropolis algorithm (Metropolis et al. 1953), which was custom coded in R (R Core Team 2017). We used a burn-in of 4,000 iterations, and then kept each 20th sample from the next 20,000 iterations, yielding 1,000 samples from the posterior distribution (S.2). The chains were monitored for stationary behavior and the proposal distributions tuned so that acceptance rates were from 0.2 to 0.5.

Fitting Haul-out Models

Let $H_{i,t}$ be a random variable for haul-out for animal i for the tth hour of the summer. Write the beta probability density as $[H_{i,t}|\boldsymbol{\theta}, \gamma]$, and the joint probability density as

$$[\boldsymbol{h}|\boldsymbol{\theta}, \gamma], \tag{S.3}$$

where $\boldsymbol{\theta}$ is a vector of means,

$$\theta_{i,t} = \frac{\exp(\beta_0 + \beta_1 t + \beta_2 t^2 + T_i(t) + A_i)}{1 + \exp(\beta_0 + \beta_1 t + \beta_2 t^2 + T_i(t) + A_i)},$$

and γ is a variance parameter common to all $\boldsymbol{h}$.

Equation (S.3) is a conditionally independent "data model", whose mean vector depends on multivariate normal model with temporal autocorrelation, denoted as $[\mathbf{T}_i|\alpha, \zeta]$ for the ith animal, and so the joint probability density is $\prod_i [\mathbf{T}_i|\alpha, \zeta]$, and normal distribution for animal random effects with mean $\mathbf{0}$ and with variance κ, $[\mathbf{A}|\kappa]$. Then, the posterior distribution of the parameters and latent variables is written

$$\begin{gathered}[\gamma, \beta_0, \beta_1, \beta_2, \boldsymbol{T}, \boldsymbol{A}, \alpha, \zeta, \kappa | \boldsymbol{h}] \propto \\ [\boldsymbol{h}|\boldsymbol{\theta}, \gamma] \prod_i [\mathbf{T}_i|\alpha, \zeta][\mathbf{A}|\kappa] \times \\ [\gamma, \beta_0, \beta_1, \beta_2, \alpha, \zeta, \kappa]\end{gathered} \tag{S.4}$$

where $[\gamma, \beta_0, \beta_1, \beta_2, \alpha, \zeta, \kappa]$ is the prior distribution on all of the parameters. We used vague, flat priors on all parameters, where priors ranged from $-\infty$ to ∞ for β_0, β_1, and β_2, from -1 to 1 for α, and from 0 to 20 on the standard deviation parameters γ, ζ, and κ.

The model was fitted with Markov Chain Monte Carlo (MCMC) methods using the Metropolis algorithm (Metropolis et al. 1953), which was custom coded in `R` (R Core Team 2017). We used a burn-in of 10,000 iterations, and then kept each 100th sample from the next 100,000 iterations, yielding 1,000 samples from the posterior distribution (S.4). The chains were monitored for stationary behavior and the proposal distributions tuned so that acceptance rates were from 0.2 to 0.5.

Literature Cited

Metropolis, N., A.W. Rosenbluth, M.N. Rosenbluth, A.H. Teller, and E. Teller, 1953. Equation of State Calculations by fast computing machines. The Journal of Chemical Physics 21 (6): 1087-1092.

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