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 parameters whose ratio is theorized to be constant across species. LHI then promise to allow prediction of many parameters from field-measurements of a few important traits. Using LHI in this way, however, neglects any residual patterns in parameters when making predictions. We therefore apply a multivariate model for eight variables (seven parameters and temperature) in all 32,000 marine fishes, and include taxonomic structure for residuals (with levels for class, order, family, genus, and species). We illustrate that this approach predicts variables probabilistically for taxa with many or few data. We then use this model to resolve three questions regarding life-history parameters in fishes. Specifically we show that: (1) on average there is a 1.24% decrease in the Brody growth coefficient for every 1% increase in maximum size; (2) the ratio of natural mortality rate and growth coefficient is not a LHI but instead varies systematically based on the timing of maturation, where movement along this life-history axis is predictably correlated with species taxonomy; and (3) three variables must be known per species to precisely predict remaining life-history variables. We distribute our predictive model as an R package to allow future life-history predictions for fishes to be conditioned on taxonomy and life-history data for fishes worldwide. This package also contains predictions (and predictive intervals) for mortality, maturity, size, and growth parameters for all described fishes. 24 temperature invali 32,000 marine fistes, and include taxonomic structure for residuals (with
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 Keywords: Life-history theory; natural mortality; individual growth; meta-analysis; phylogenetic regression; life-history invariant

Introduction

 Biological characteristics (e.g., individual growth rates, age at maturation, maximum body length and mass, and adult rates of natural mortality) greatly differ among different animal species. These characteristics (or more precisely, their value averaged among individuals for a species at a given location and time) are sometimes termed "life-history parameters", and these parameters play an important role for understanding and managing populations and ecosystems. In fishes, for example, these life-history parameters are important to determine sustainable harvest rates, estimate harvest rates from changes in average length, and project the likely impact of different management actions (Quinn and Deriso 1999). Unfortunately, some or all of these life-history parameters are unknown or poorly measured for most harvested species. Ideally, these parameters would be estimated at the same time as trends in population abundance and fishery harvest rates, e.g., by using a data-integrated stock assessment model to jointly analyze data from surveys and fisheries (Maunder and Punt

 in a stock assessment model given available data or resources (Magnusson and Hilborn 2007).

 As one example, the average adult rate of natural mortality is perhaps the most difficult- to-estimate life history parameter. Estimating natural mortality using mark-recapture sampling, cohort analysis, multispecies models of predation rates, or data-integrated stock assessment models are each expensive, resource-intensive, and often laden with modeling assumptions. As a consequence, fisheries scientists since the 1950s have sought alternative approaches to informing natural mortality for a given species. One common approach is to infer a likely rate of natural mortality from the value of correlated life-history parameters. A recent review identified 29 different methods for estimating natural mortality from limited data, and concluded that many of these methods fail for at least some species because they were derived based on small samples with limited taxonomic breadth (Kenchington 2014). As an alternative, authors have simultaneously estimated relationships among multiple life- history parameters, but these studies have generally involved small groups of closely-related species rather than fishes in general (Nadon and Ault 2016).

 Fortunately, life-history variables (including estimates of life-history parameters from field-measurements as well as environmental conditions, e.g., temperature) have been compiled into public databases (e.g., Fishbase; Froese 1990, 2011), that include species within most major fish taxa. In principle, these databases allow us to use phylogenetic information and correlations among life history parameters to obtain better estimates of vital rates for both well-studied and poorly-studied species. Meta-analyses of life history parameters in fishes indicate that multiple parameters can be described using a reduced set of ordinates or groups (Winemiller and Rose 1992). Life history theory (e.g., Roff 1984, Charnov 1993, Jensen 1996) and more recently the metabolic theory of ecology (Brown et al. 2004) have provided a conceptual framework that explains observed correlations among traits. In particular, ratios of particular life-history parameters are often theorized to be nearly constant across widely different taxa (Charnov et al. 1991). A special case of life-history relationships occurs when the units are identical for parameters in the numerator and denominator, such that the ratio is dimensionless. Charnov (1993) and others have defined these as "life-history invariants" (LHI), and claimed that these dimensionless numbers in some cases will be conserved across taxa. In practice, life-history theory, LHI, and observed correlations suggest that if we know the adult mass and average environmental temperature of a given species (which are relatively easy to measure in the field), we can use this sampling coloner multysis, multispectics models of prediction rates, or data-integrated stock
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measurements. As it entiregrame and

 available regarding additional life-history parameters for that given species (e.g., local measurements of age at maturation), then these local results may also improve estimates of other parameters for that species.

 Many studies have shown that relationships among life-history parameters vary taxonomically, so that relationships are more similar for closely related species than otherwise unrelated species (e.g., Stearns 1983, Crespi and Teo 2002, Thorson et al. 2014b, Nadon and Ault 2016, Dick et al. 2017). Intuitively, this suggests that taxonomic information can also be useful when we predicting life history parameters. One approach to formalize this intuition is to apply a multivariate extension of the standard phylogenetic model for genetic evolution (Felsenstein 1985, Grafen 1989). Using this model, we predict that parameters follow a random evolution over time, where correlations among life-history parameters encode "neutral" (i.e., variable) or "conserved" (constant) relationships. Given information regarding the evolutionary lineage of a given taxonomic tree (i.e., time since divergence for every included species), this evolutionary model can inform correlations among two species by the time that has elapsed since their most recent common ancestor. In the absence of detailed evolutionary information, this tree can instead by approximated using taxonomic information (e.g., by assuming that two species from the same genera are more related than two species in the same family but not the same genera).

 In this study, we predict life history variables (temperature and life-history parameters) for all >32,000 fishes worldwide while accounting for similarity in the relationships among life-history parameters for fishes that are taxonomically related, explicitly representing residual error including correlations among parameters, and accounting for missing data. We then use results to address the following outstanding questions regarding life-history parameters in fishes: (1) What is the average relationship between the Brody growth 122 coefficient, K, and maximum size, L_{∞} (for which different theories have previously been 123 published)? (2) Is the ratio of natural mortality M and the growth coefficient K a LHI, or does it vary systematically (e.g., based on timing of maturity or other life-history 125 parameters)? and (3) How many variables must be known per species to precisely predict remaining life-history variables (i.e., how many parameters are "neutral" vs. "conserved")? We also distribute an R package *FishLife* [\(https://github.com/James-Thorson/FishLife\)](https://github.com/James-Thorson/FishLife) containing code to re-run this multivariate life-history model as well as results from fitting it to data from FishBase. This R package can also be used to update predictions for individual 102 has mominally even base relationships are more similar for closely related species than 1014h, Nadon and AME2015. Dietas de, 8, Stack-specific studies, and Tomon et al. 2014h, Nadon and AME2015. Dietas de, 8, Stack-sp

- research on fishes worldwide. It also includes an Rshiny graphical user interface, which can
- be used to visualize predicted life-history parameters for any fish species worldwide.

Methods

Life history theory and invariants

135 There are four main difficulties when predicting life-history parameters for all fishes

- worldwide:
- 1. *Measurement errors*: Field-measurements of life history parameters are imprecise and often correlated (e.g., a negative correlation in estimates of the Brody growth coefficient and asymptotic length). We therefore treat existing records of life-history parameters as imperfect measures of "true" parameters for a given species, where residual errors covary among parameters. Estimating residual covariance is feasible whenever multiple measurements are available for a single species (where we assume a single "true" parameters-value for each species; Thorson et al. (2014a)). This residual covariance is a combination of measurement errors (covariance in field measurements) and biological variation within a given species (e.g., variation in parameters among years or populations). In the following, we assume that field measurements of life-history parameters are unbiased, although future work could address this assumption by estimating average bias relative to a more-trusted data set (see e.g., Thorson et al. 2014a). 2. *Incomplete data*: No single field-study will provide estimates of all important life-history parameters – instead, a study will generally report a subset of parameters that are estimated using data from that study's design. To calculate the probability distribution of available data while accounting for covariation in field measurements (as necessary in a likelihood or Bayesian approach to parameter estimation), we must therefore account for "incomplete data", i.e., the values of life-history parameters that are not reported for each individual study. We therefore use standard missing-data methods under the assumption that data are "missing at random" (i.e., that missing values can be imputed from reported values). 135 There are repair running that the measurements of ite history parameters for all fishes wordwise.

139 where the correlated (e.g., a negative correlation in estimates of the Brody growth coefficient

143 of the correl
- 3. *Taxonomic similarity:* Many parameters are more similar for taxonomically related species than otherwise unrelated species. Importantly, this similarity is apparent at every taxonomic level, e.g., among taxonomic classes (Charnov et al. 1991), families (Beverton 1992), and species (Thorson et al. 2014b, Dick et al. 2017). We therefore specify that parameters vary "hierarchically", e.g., are different for Orders within a Class, Families

 by Hoenig et al. 2016). Measurements are only available for species, so average parameters for higher-level taxa are never directly observed.

- 4. *Unknown functional forms*: There is little agreement about the exact functional-form of the relationships among parameters. For example, recent studies have argued for (Hordyk
- et al. 2015) and against (Nadon and Ault 2016) a purported dependence of rate-
- parameters (M/k) and the ratio of length-at-maturity (L_{mat}) and asymptotic maximum
- 170 length (L_{∞}) . We therefore use a life-history model that can potentially include
- covariation among multiple parameters, rather than only modelling ratios among pairs of
- parameters. Published life-history relationships are often multiplicative (i.e., the product
- of age-at-maturity and natural mortality is approximately constant among species,
- Beverton and Holt (1959)), so taking logarithms often results in a linear (additive)
- relationship among parameters, or at least results in relationships that can be closely approximated as being linear.
- 177 We overcome these four difficulties by specifying a multivariate model for trait evolution
- along a taxonomic tree, while using replicated samples for each individual species to
- distinguish trait evolution from residual covariance (e.g., estimation errors when calculating
- parameters from field data).

Multivariate model for trait-evolution along taxonomic trees

 Taxonomies are defined such that, for any taxonomic classification, it belongs to exactly one classification for any higher taxonomic level (e.g., every family belongs to exactly one order, etc). Therefore, we can define a set of taxonomic levels (e.g., Class, Order, Family, Genus, and Species), and every taxon (e.g., genus *Sebastes*) is the "child" of exactly one "parent" taxon (e.g., family *Sebastidae*). It is therefore possible to factor the probability distribution for all life-history variables within a taxonomy by specifying a series of conditional probabilities for variables for a given taxon via the variables of "ancestors" (the "parent" taxon, its "parent", etc) (Felsenstein 1985). In the following, we specify a first-order Markov process for trait evolution (i.e., variables for a "child" taxon depend only upon values for its "parent" taxon). 1968 craft (Martim Christophera) and Ault 2016) a purported dependence of rationary (*L_{one}ry*) and the ration of length at maturity (*L_{oner}y*) and assumption maturity (*Lonery*) and assumption any Author Christopy (ma

 We approximate trait evolution via a multivariate random-walk process. However, in the absence of a high-quality chronology representing time-since-divergence for all fishes, we have approximated trait-evolution by estimating covariation for taxonomic Classes, Orders within Classes, families within Orders, genera within families, and species within genera (i.e., 197 Order, etc.). Specifically, we define the true (unobserved) value $x_{q,i}$ of variable j (of n_l 198 modelled variables) and taxon g (of n_G modelled taxa), where x_g is the vector of true 199 variables for that taxon. Variables then depend only upon the value of variables for the 200 parent-taxon:

$$
\mathbf{x}_g \sim MVN(\mathbf{x}_{p(g)}, \Sigma_{l(g)})\tag{1}
$$

201 where $p(g)$ is the index of the parent-taxon for child g, and $l(g)$ indicates the taxonomic 202 level of the child-taxon (e.g., $l(g) = 1$ when the child-taxon is a taxonomic Class, and $l(g) = 5$ when the child-taxon is a taxonomic species). The evolutionary-covariance, $\sum_{l(g)}$, may itself vary along the taxonomic tree. For simplicity, we assume that the correlation among variables is identical for all taxonomic levels, but that the variance changes among 206 levels. Specifically, we decompose evolutionary-covariance $\Sigma_{l(g)}$ into components caused by common factors (sensu "exploratory factor analysis," Lawley 1940) or independent evolution of each variables:

$$
\Sigma_{l(g)} = \lambda_l (\mathbf{L}_{\Sigma} \mathbf{L}_{\Sigma}^{\mathrm{T}} + \mathbf{D})
$$
 (2)

209 where L_{Σ} is a lower-diagonal n_l by n_{Σ} matrix $(0 \le n_{\Sigma} \le n_l)$ approximating covariance 210 among variables (where L_z is the Cholesky decomposition of $\lambda_l^{-1} \Sigma - D$ when $n_z = n_j$), **D** is 211 a diagonal matrix with diagonal elements σ_j^2 representing independent component of 212 variance in trait-evolution, and λ_l is the relative covariance explained by taxonomic level l 213 relative to the variance explained by taxonomic Order (i.e., we specify that $\lambda_{l=1} = 1$ to 214 ensure variable identifiability). The assumption that evolutionary-correlation is identical 215 among taxonomic levels is analogous to the assumption in conventional coalescent models 216 (Felsenstein 1973) that the evolutionary covariance does not change over time. Future 217 analyses could explore relaxing this assumption, although it would presumably require a 218 large increase in estimated coefficients (fixed and random effects). **226 Example 1226 Exam**

219 We then estimate variables by specifying a distribution for augmented data \tilde{y}_i for the *i*th 220 study:

$$
\tilde{\mathbf{y}}_i \sim MVN(\mathbf{x}_{g(i)}, \mathbf{V})\tag{3}
$$

221 where **V** is the residual covariance, which also includes factor-model (\mathbf{L}_V with rank n_V) and 222 independent variance components (v_j^2) . Many studies have missing values (i.e., instances 223 where $y_{i,j} = NA$), and we assume that these are missing-at-random (i.e., available data are 224 representative of missing data). Given this assumption, missing data can be treated as

$$
\tilde{y}_{ij} = \begin{cases} \varepsilon_{ij} & \text{if } y_{ij} = NA \\ y_{ij} & \text{if } y_{ij} \neq NA \end{cases} \tag{4a}
$$

226 where ε_{ij} is the missing value for variable *j* or observation *i*, and

$$
\varepsilon_{ij} \sim Uniform(lb, ub) \tag{4b}
$$

227 where ℓb and $\ell w b$ are set far below and above the range of data observed, such that further 228 decreases/increases in their values has no impact on model results.

 We then estimate fixed effects by identifying values that maximize the marginal 230 likelihood function. Fixed effects include evolutionary covariance (\mathbf{L}_{Σ} and σ_j^2), residual 231 covariance (\mathbf{L}_V and v_j^2), and the ratio of evolution covariance among taxonomic levels (λ_l). 232 We also treat as fixed the average value μ_i for each life-history variable *j* for the common ancestor of the taxonomic tree (Phyllum Chordata). The marginal likelihood is calculated while integrating across the joint probability of data and random effects with respect to 235 random effects. We treat as random the true (latent) value of variables for each taxon (\mathbf{x}_q) as well as all missing data (see Eq. 3 and 4b for their distribution). We approximate the marginal likelihood using the Laplace approximation (Skaug and Fournier 2006), implemented using Template Model Builder (Kristensen et al. 2016) within the R statistical environment (R Core Team 2015). TMB also provides the gradients of the marginal likelihood with respect to fixed effects, and we use these gradients in a conventional nonlinear optimizer to identify maximum likelihood estimates (MLE) for fixed effects. Given the MLE for fixed effects, we then calculate standard errors using a generalization of 243 the delta-method and the matrix of $2nd$ derivatives of the marginal likelihood function with respect to fixed effects. *lb* and *ub* are set far below and above the range of data observed, such that further
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245 **Database**

246 We compile all available records of eight life-history variables (seven life history parameters 247 and temperature, $n_1 = 8$): natural mortality rate (*M*), Brody growth coefficient (*K*),

248 asymptotic maximum length (L_{∞}) , asymptotic maximum mass (W_{∞}) , length at maturity

249 (L_{mat}) , age at maturity (a_{mat}) , maximum age (a_{max}) , and average temperature (T). We

250 apply the life-history model to data for all eight variables, although we also report results

- 251 from a sensitivity analysis where we analyse only the seven life-history variables (i.e.,
- 252 excluding temperature). Subsets of these variables are often obtained from the same study
- 253 (e.g., a study providing both k and L_{∞} for a single population of a given species), and we
- 254 record all measurements from a single study as a vector of data

$$
y_i = \{\ln(M), \ln(k), \ln(L_{\infty}), \ln(W_{\infty}), \ln(L_{mat}), \ln(a_{mat}), \ln(a_{max}), T\}
$$
 (5)

- 255 for the th study, while recording missing variables for each study as NA. We use a log-
- 256 transformation for all variables (except temperature, T) because (1) this transforms all $y_{i,j}$ to

257 have unbounded support (matching the support of the multivariate normal distribution from

- 258 Eq. 3), and (2) because this results in life-history invariants being expressed as linear
- 259 constraints that can be approximated using linear equations (e.g., $ln(M) ln(K)$ is
- 260 approximately constant if M/K is a life-history invariant). All variables were downloaded
- 261 from FishBase (Froese 1990) using the package *rfishbase* (Boettiger et al. 2015) on Aug. 25,
- 262 2016, and our database is distributed with R package *FishLife*. In total, we obtained 29,196
- 263 variables (life-history parameters and temperature) from 9,853 studies ($n_i = 9.853$) for 3,551
- 264 unique species of the 33,102 fish species that are listed within FishBase. For each species,
- 265 we also noted the taxonomic Class, Order, and Family for a given Latin binomial (Genus-
- 266 species) name.

267 **Model exploration**

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268 Next, we fitted the evolutionary model for life-history variables to this entire database while 269 varying model complexity in two ways: varying the number of fixed effects used to 270 approximate covariance in trait-evolution (n_x) or residual covariation (n_y) . We specifically 271 perform a grid-search across all eight combinations for each configuration ($n_z = \{1,2,...,8\}$) 272 and $n_v = \{1, 2, ..., 8\}$, for a total of 64 models. For each model we confirm that the final 273 gradient of the marginal likelihood with respect to each fixed effects was low (<0.00001), and 274 that the hessian of the negative log-likelihood with respect to fixed effects was positive 275 definite. We then record Empirical Bayes predictions of values x_g for each taxon, and use 276 the generalized delta-method to calculate predictive covariance $Cov(\mathbf{x}_q)$ for these variables. 277 To calculate the variance in life-history variables explained on average by knowing 278 taxonomic information for a fish, we calculate the total covariance among variables as: nts that can be

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$$
Cov_{total} = \sum_{l=1}^{5} \Sigma_l + V
$$
 (6)

279 where the total variance for each individual life-history variable correspond to diagonal 280 elements of Cov_{total} . The variance explained by perfect knowledge of average life-history 281 variables for the highest l^* taxonomic levels then corresponds to:

$$
Cov_{explained}(l^*) = \sum_{l=1}^{l^*} \Sigma_l
$$
 (7)

such that the proportion of variance explained by perfect information about the highest l^* 282 283 taxonomic levels for variable *j* is $\sum_{l=1}^{l^*} \Sigma_l(j,j) / [\sum_{l=1}^5 \Sigma_l(j,j) + V(j,j)]$. Similarly, the 284 residual covariance in predictions given perfect information about the first l^* taxonomic 285 levels for a given species $Cov_{pred}(l^*)$ is expected to be:

$$
Cov_{pred}(l^*) = Cov_{total} - Cov_{explained}(l^*) - V = \sum_{l=l^*+1}^{5} \Sigma_l
$$
\n(8)

286 However, in practice we have imperfect information about average life-history variables for 287 higher taxonomic levels, such that $Cov_{pred}(l^*) > \sum_{l=l^*+1}^{5} \Sigma_l$, where $Cov_{pred}(l^*)$ – 288 $\sum_{l=l^*+1}^{5} \Sigma_l$ measures our imprecision when estimating average values for higher taxonomic 289 levels.

290 To interpret relationships among different life-history variables, we first predict variables 291 x_{pred} for a species with an Class, Order, family, genus, and species that is not within the 292 database (where $\mathbf{x}_0 = \boldsymbol{\mu}$), as well as the predicted covariance $Cov(\mathbf{x}_{pred})$ for these variables 293 (where $Cov(\mathbf{x}_{pred}) = Cov_{total} - \mathbf{V}$). This $Cov(\mathbf{x}_{pred})$ represents the predicted covariation 294 for life-history variables for a fish about which nothing is otherwise known, and it has rank 295 n_j because it is calculated from Σ_l (which itself has rank n_j because Σ_l includes a diagonal 296 component \mathbf{D}). We then calculate the eigen-decomposition of this covariance, and examine 297 the relative magnitude of eigenvalues to determine how many dimensions are necessary to 298 explain the vast majority (99%) of trait-covariance. For eigenvalues that explain a substantial 299 proportion of covariance, we then inspect their eigenvectors, where the ratio of two values in 300 a given eigenvector corresponds to the slope of a given axis of covariation. Inferring 301 relationships among variables from their predicted covariation, $Cov(\mathbf{x}_{pred})$, is similar in 302 some ways to major axis regression (MAR), and MAR is a useful for estimating relationships 303 among variables Y and Z whenever (1) an analyst seeks a relationship among Y and Z 304 without specifying either variable as "fixed" or "random", and (2) it is unknown whether 305 future users will be predicting Y from Z or Z from Y (Warton et al. 2006). Unlike 306 conventional MAR, however, we use multiple records of each variable for a given species 307 (several y_i for the same taxon g) and multiple taxa for the same parent-taxon (several x_g for 308 the same $\mathbf{x}_{p(g)}$ to separately estimate evolutionary and residual covariance (Σ_l and **V**), and 309 our ability to separately estimate these two sources of covariance eliminates one major 310 drawback when using conventional MAR (Seim and Sæther 1983, McArdle 1988). 311 **Results** Cov_{pred}
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Descriptive results

 We first demonstrate our predictions of all eight variables (seven life-history parameters and temperature) for six species, chosen for illustration purposes from the approximately 33,104 fish species listed in FishBase (Fig. 1-2). Model selection using the Akaike information criterion (AIC) supports using a model where (1) the evolutionary covariance is the sum of 317 five factors $(n_{\Sigma} = 5)$ and additional, independent variance for each variable, and (2) the 318 residual covariance is the sum of three factors ($n_V = 3$) and additional variance for each factor. We use this model in the following, but confirm that results are essentially unchanged 320 when using other models with similar ΔAIC (Table 1).

 Model predictions follow a multivariate normal distribution (as implied by the structure of our model), and we visualize this predictive covariance using ellipses to represent a region with a desired coverage (in this case, a 95% predictive ellipse). Predictions at high taxonomic levels (e.g., for all species in class Actinopterygii) have wide confidence ellipses (due to the accumulation of variance when predicting variables based only on high taxonomic levels, see Eq. 7). Similarly, lower taxonomic levels generally have more precise (smaller) predictive ellipses (as expected given Eq. 8). The exception is when predicting variables for a species with no available data. In this case, the prediction is identical to the next-nearest ancestor with available data. For example, any species of genus *Sebastes* without available data has predictions equivalent to the predictive distribution of *Sebastes* (results not shown). Similarly, in the unlikely event that an entire family has no available data, predictions for any species within that family are identical to the predictions for that taxonomic order. In this way, species without available information are guaranteed to have less-precise predictions of life-history variables than species with abundant data. 314 criterion **exic** surfacent using a model where (1) the evolutionary covariance is the sum of
314 fiven factors $\overline{m}_0 = S$) and additional, independent variate for each variate is ard (2) the
314 residual coordinates

 We interpret the relationship among variables using the eigen-decomposition of the predictive covariance for all fishes (Table 2). This covariance among life-history variables is estimated while controlling for residual covariance, e.g., arising from estimation error when calculating life-history parameters from available field data (Table 3). The dominant eigenvector for trait-covariance is associated almost entirely with temperature (loading: -0.995). The value of this eigenvector for natural mortality indicates that a 1 °C increase in temperature is associated with a 5% increase in natural mortality rate on average. This increase in natural mortality is also positively associated with the growth coefficient, and negatively associated with asymptotic weight and maximum age (among other associations).

 temperature (or environmental characteristics associated with temperature, e.g., latitude) on metabolic rates.

 Question #1: What is the average association between growth rate and maximum size The second axis when analyzing all eight variables explains approximately 80% of remaining variance (after controlling for the impact of temperature), and has strongest association with asymptotic mass (Table 2). Loadings indicate proportional variation in length-at-maturity and asymptotic length, and a greater-than-cubic increase in asymptotic mass with increasing asymptotic length (i.e., 1% increase in asymptotic length is associated with a 3.25% increase in asymptotic mass, or a 3.21% increase for the sensitivity analysis excluding temperature, Table S1). A 10% increase in asymptotic mass is in turn associated with a 2.4% decrease in mortality rate and growth coefficient. Collectively, this axis suggests that body size is the dominant axis of variation for life-history parameters in fishes that are subject to the same temperature, and that increased size is negatively associated with mortality and growth rates. 358 Notably, the allometric-scaling of growth coefficient and asymptotic length $\left(\frac{-0.258}{0.204} = -1.24\right)$ is between the value of -1.0 predicted by Charnov et al. (2013) and -2.0 predicted by Pauly and Binohlan (1996), and this result is robust to excluding temperature (Table S1, showing -0.258 $\frac{-0.256}{0.217} = -1.19$. 349 variance informed that points in the impact of temperature), and has strongest association
379 asymptotic fungil). All adaptaticalle proportional variation in length-a-maturation
371 and asymptotic length (i.e., 1% in

Question #2: Is the ratio of natural mortality rate and growth coefficient a LHI?

 The first and second axes of variation explain 97% of total variance, and both have had 364 nearly proportional scaling of mortality (M) and growth coefficient (K) . In isolation, these 365 two axes therefore suggest that the ratio M/k is constant among fishes. However, species show large variation in the ratio of natural mortality and growth coefficient (Fig. 3), arising predictably along phylogenetic lineages. In particular, family Sebastidae has relatively low 368 M/k , Salmonidae has relatively high M/k , and Scombridae and Lutjanidae fall somewhere in 369 the middle $(3^{rd}$ panel of Fig. 3).

370 This variation in M/k is generated largely via the 3rd axis of variation (Table 2), which explains 1.6% of total variation. While small, this is nevertheless 60% of the variance remaining after accounting for mass and temperature (or after accounting for mass when excluding temperature, Table. S1). Importantly, this axis is the only component of evolutionary covariance to have substantially different scaling between asymptotic length and length-at-maturity, or between natural mortality rates and growth coefficients. Positive 376 movement along this axis represents a decrease in L_{mat}/L_{∞} , and we interpret this axis as

- 378 maturity (following Charnov and Berrigan 1991). A decreased L_{mat}/L_{∞} is then associated 379 with an increased value of M / k (Fig. 4). Importantly, related taxa (e.g., species within the 380 same taxonomic family) cluster closely together in values of L_{mat}/L_{∞} and M/k , suggesting
- that these parameters evolve gradually within lineages. This implies that species within the
- same taxon (e.g., species within the same genus) will be highly informative about the value of
- 383 L_{mat}/L_{∞} and M/k . For example, Sebastidae species have relatively high values for L_{mat}/L_{∞}
- 384 L_{∞} (where a value of 1 indicates that growth ceases at maturation), and also have relatively
- 385 low values for M/k compared to other fishes.
- **Question #3: How many parameters must be known to predict remaining parameters?**

 Inspecting the eigen-decomposition of predictive variance for fishes shows that three axes explain 99% of total variance in all eight variables (Table 2). Given results from this eigen- decomposition, we infer that we could eliminate 99% of total variance when predicting the remaining life-history variables if abundant sampling data allowed us to have near-perfect measurements of temperature, maximum size, and either natural mortality or individual growth rates for a given species. As sensitivity analysis, we also re-run the model while excluding temperature (Appendix S1 Table S1). In this case, the first two axes explain 95% of total variation, and the eigenvalues and eigenvectors for these axes are strongly similar to 395 the $2nd$ and $3rd$ axes of the model involving temperature.

 Finally, we visualize the proportion of variance explained for each variable using this model (Fig. 5) when decomposing total covariance into covariance explained by Class, Order, family, genus and species, as well as residual covariance (where residual covariance is shown in Table 3). This decomposition shows that residual covariance explains approximately 10% of total variation on average across variables. Residual variation is 401 greatest (approximately 20%) for growth coefficients (K) and mortality (M) rates, where estimates of both are expected to be most variable given typical field-measurements, and is lowest (approximately 5%) for maximum length and length-at-maturity. Evolutionary- covariance is greatest for different family within a given order, and is relatively similar for other taxonomic levels. 382 same taxon (e.g., species within the same genus) will be highly informative about the value of $L_{\text{tot}}/L_{\text{tot}}$ and L_{tot} (where a value of 1 indicates that growth ceases at maturation), and also have relative L

Discussion

In this study, we have applied a multivariate model for the evolution of life-history

- parameters to a global database of fish variables including individual growth, size, mortality,
- maturity, and average temperature. We have then used this model to resolve three ongoing
-

 results as an R package (with a graphical user interface) that can be used to predict life- history parameters for any of the nearly 33,000 described fish species. These predictions are informed using data in FishBase as well as taxonomic similarity to other fish species with many or few data.

 Predictions from our model indicate that variation among all eight variables can be 416 described using three main dimensions, representing temperature $({}^{\circ}C)$, size (maximum body mass, W_{∞}), and the timing of maturation (covariation among $\frac{M}{k}$ and $\frac{L_{mat}}{L_{\infty}}$ ι_{∞} 417 mass, W_{∞}), and the timing of maturation (covariation among $\frac{m}{L}$ and $\frac{L_{mat}}{L}$). The first dimension conforms to the metabolic theory of ecology (Brown et al. 2004), which predicts that temperature controls metabolic rates underlying individual growth and activity levels among taxa worldwide. Similarly, the second dimension conforms to the size-spectrum theory of marine communities (Andersen et al. 2009), which predicts that growth and mortality are defined by asymptotic mass. However, we also find that a third dimension (the biological trade-offs between growth, maturity, and longevity) explains an important component of variation (60% of variance after controlling for temperature and maximum size). By identifying a third dimension of covariation, our results contrast with previous empirical studies of fish life history, which predict growth, mortality, and maturity from a species' asymptotic mass (e.g., Eq. 3 and 5c of Charnov et al. 2013). Interestingly, our three dimensions correspond to the three predictors for natural mortality estimated by Pauly (1980), although this latter study did not include maturity as either predictor or response variable. 445 described was from our model indicate that variation among all eight variables can be described with α ¹⁴³ derived from others) (reported from α ¹⁴³ derivational in the other main dimension content (α ¹⁴³

 We conclude that three dimensions are sufficient to capture 99% of covariation among our eight variables, and a natural question is whether this number of dimensions is consistent with existing life-history theory. We therefore compile a list of previously published relationships among these eight life-history variables for fishes that are based entirely upon theoretical optimization of fitness rather than empirical observations (Appendix S2). This table shows that reported relationships can often be derived from one-another. For example, Jensen (1996) attributes three LHI to Beverton and Holt (Eq. TB1-4, TB1-5, and TB1-6 in 438 Appendix $\overline{S2}$. However, the third LHI (TB1-6) can be derived from the others given that growth follows a von Bertalanffy growth curve (TB1-1). We therefore identify a minimal set of life history relationships that can be used to derive or approximate all other reported relationships (Table 4). We refer to this as a "minimal and sufficient" set of relationships, because it involves the minimal number of equations (i.e., eliminates equations that can be

444 relationships. These six relationships represent von Bertalanffy growth (T4-1); allometric 445 scaling of weight-at-length (T4-2), maximizing fecundity with respect to age at maturity (T4- 446 3) and the Brody growth coefficient (T4-4), the definition of maximum age (T4-5), and the 447 metabolic scaling with temperature (T4-6). We also hypothesize which variables in these six 448 equations are likely to be conserved or varying among fish species. By examining the 449 number of free parameters that can be used to calculate all eight variables in this study (see 450 Table 4 caption), we predict that all eight life-history variables can be well-described via 3-4 451 dimensions. This prediction is generally consistent with our empirical conclusion that all 452 variables can be well predicted given perfect knowledge of three variables for each species. 453 Across all fishes, we also conclude that asymptotic length $\ln(L_{\infty})$ increases 24% faster 454 than any decrease in growth coefficient $ln(K)$ among species, i.e., $ln(K) \propto -1.24 ln(L_{\infty})$ as 455 indicated by the 2^{nd} axis of variation. This estimate arises after using replicated sampling for 456 each individual species to estimate the residual covariance for these two parameters arising 457 from estimation errors: as expected, this residual covariance is negative between $ln(L_{\infty})$ and $\ln(K)$ (see Table 3). Following Charnov and Berrigan (1991), we interpret this covariance 459 between $ln(L_{\infty})$ and $ln(K)$ as species optimizing their fecundity with respect to growth rates, 460 with resulting impacts on asymptotic length and mass. Our predicted coefficient (using a 461 variant of major axis regression) is closer to the relationship $ln(k) \propto -ln(L_{\infty})$ predicted by 462 Charnov et al. (2013) than the relationship $ln(k) \propto -2 ln(L_{\infty})$ predicted by Pauly and 463 Binohlan (1996). The product of growth coefficient and asymptotic mass is the anabolic rate 464 for juveniles (Mangel 2006, Charnov et al. 2013), so our results suggests that large-bodied 465 species on average have a higher juvenile anabolic rate (i.e., are able to access greater 448 equations are likely to be conserved or v.

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467 Lastly, we conclude that the ratio of natural mortality rate and the Brody growth coefficient $\left(\frac{M}{U}\right)$ 468 coefficient $\left(\frac{m}{K}\right)$ is not itself conserved across species (i.e., not a "life history invariant" as 469 claimed by Jensen (1996 Eq. 8)). Instead species with slow growth relative to mortality 470 (relatively high $\frac{M}{K}$) also have lower maturation length relative to maximum length (relatively $\frac{L_{mat}}{$ 471 low $\frac{L_{mat}}{L_{\infty}}$, such that the ratio of age-at-maturity (a_{mat}) and maximum age (a_{max}) is 472 approximately conserved among species. This result explains large differences in $\frac{M}{K}$ among taxonomic groups, e.g., where *Sebastes* species have very low $\frac{M}{K}$ and high $\frac{L_{mat}}{L_{\infty}}$ ι_{∞} 473 taxonomic groups, e.g., where *Sebastes* species have very low $\frac{m}{r}$ and high $\frac{m}{r}$ relative to other taxa (Thorson et al. 2014b). This association between $\frac{M}{K}$ and $\frac{L_{mat}}{L_{\infty}}$ ι_{∞} 474 other taxa (Thorson et al. 2014b). This association between $\frac{m}{k}$ and $\frac{m}{l}$ is not a new

475 discovery (Holt 1958, Beverton 1992, Hordyk et al. 2015). In fact, it is easy to justify

476 theoretically given the assumption that lifetime reproductive output per recruit is proportional

477 to survival-to-maturity and mass at maturity (we follow the presentation from Mangel (2006

478 Chap. 2)):

$$
F(a_{mat}) = \exp(-Ma_{mat}) \times \alpha L_{mat}^{\beta}
$$

479 where α is the average mass per standardized volume and β represents ontogenic changes in 480 body shape. Optimizing fitness $F(a_{mat})$ with respect to a_{mat} yields:

$$
a_{mat} = \frac{1}{k} \log \left(\frac{M + \beta K}{M} \right)
$$

481 where substituting L_{mat} for a_{mat} and re-arranging then yields:

$$
\frac{L_{mat}}{L_{\infty}} = \left(\frac{\beta K}{M + \beta K}\right)
$$

This derivation therefore suggests that $\frac{L_{mat}}{$ ι_{∞} 482 This derivation therefore suggests that $\frac{L_{\text{mat}}}{L_{\infty}}$ and $\frac{M}{k}$ should have a nonlinear association. 483 Despite this theoretical evidence, however, authors have periodically suggested that $\frac{M}{k}$ is 484 constant among species (e.g., Jensen 1996). Our study also estimates taxonomic variation around this relationship, e.g., where salmonids have higher $\frac{L_{mat}}{I}$ ι_{∞} than predicted from their $\frac{M}{k}$ 485 486 (i.e., are closer to determinate growth than otherwise predicted), whereas Scombrids are the 487 opposite (less determinate growth than otherwise predicted). We hope that this combination 488 of theoretical and empirical evidence will put to rest the notion that $\frac{M}{k}$ is a life-history 489 invariant, and will encourage future research to explain residual patterns around life-history 490 relationships among taxa.

 Finally, we note several shortcomings of our current study, which could potentially be explored in future research. Most importantly, previous evolutionary studies have often used rates of molecular evolution rather than taxonomic information to account for correlations among species (Felsenstein 1973, 1985). Using the "coalescent model" for molecular evolution, an analyst can model an evolutionary tree as being shaped by standard population-496 genetic forces. For this reason, authors have argued that the coalescent model provides a coherent statistical basis for analyzing species lineages. However, the coalescent model may be difficult to apply in practice because it involves several unknown parameters including effective population size, recombination rate, and selection history (i.e., selective and random mutation rates). In particular, a large evolutionary tree (like we analyses here) may include F(a_{rmax}) = exp (--Mα_{rmax}) × α*H*_{caxa}r

479 where a is the average mass per standardized volume and β represents onogenic change

480 body shape. Optimizing fitness F (a_{rmat}) with respect to M_{max}

481 where

 difficulties in reconstructing the species tree, or in accurately approximating correlations among species based only on time since divergence (Takahashi et al. 2001, Rosenberg and Nordborg 2002). Therefore, we have instead used a taxonomic tree to approximate the evolutionary time since divergence for all pairs of species. Our current approach is similar to previous fisheries meta-analyses that have used nested random effects to explain variation in species parameters at different taxonomic levels (Foss-Grant et al. 2016), and we recommend future research conducting a detailed comparison of the two methods. We also note that future research could explore different evolutionary-covariance occurring at different taxonomic levels, where different covariance could be interpreted as representing different evolutionary constraints operating at short (species-level) or long (family- or class-level) evolutionary scales. This exploration would not be possible using the conventional coalescent model, so we see this potential for future research as one advantage to our current approach.

 Secondly, we have not explicitly included many variables that might be expected to control opportunities for growth or reproduction in fishes. For example, species parameters like maximum size are associated with depth and latitude (Cailliet et al. 2001, Gertseva et al. 2010). These patterns may be driven to some extent by temperature (Pauly 2010), and our model therefore indirectly accounts for depth and latitude via our inclusion of temperature measurements. However, other factors controlling opportunities for growth include predation and food availability (Winemiller 1989). We have not included any variable to directly measure differences in food availability or predation, and identifying such a variable (e.g., using regional variation in primary production) could perhaps improve life-history predictions in fishes. Likewise, human-induced changes in life history parameters could alter the relationship among traits (Conover and Munch 2002), so including a variable representing exploitation history may similarly improve predictions in some fishes. While a large portion of variability is captured in the three main dimensions presented here, further precision and ecological insight could be gaining by including additional, ecologically relevant variables in the life-history analysis. 566 previous fisheries metu-analyses that have used nested madam effects to explain variation in species parameters at different taxonomic levels (Tos-Cenar text cl10.5) and we recommen-
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 Thirdly, we have used data from FishBase without attempting to discriminate between high and low-quality studies (e.g., between estimates of natural mortality arising from high- quality tag-recapture vs. lower-quality catch curves for lightly impacted populations). Previous work suggests that FishBase records are on average an unbiased measure of high

 include a large proportion of missing values (i.e., 63% of all values are missing) and this high proportion of missing records may impact our ability to precisely estimate the number of dimensions for representing trait-covariation. To account for this high proportion of missing data, we used a "rank-reduction" technique to estimate covariation among variables (Zuur et al. 2003, Warton et al. 2015), and this allowed us to select a parsimonious number of parameters for both evolutionary and residual covariation. However, it is difficult to assess the potential impact of missing data on estimating the dimensionality of covariation, and we recommend future statistical exploration regarding this topic.

 We distribute an R package *FishLife* containing code to replicate or update our analysis, and packaging results when fitted to FishBase data. This R package includes predictions for all marine fishes, where species without any available data are predicted using information from all related taxa. The package also includes a function to update predictions of life- history parameters for a given species based on user-supplied data. In this way, it can be 548 used to generate species-specific predictions of life-history parameter that combine both global and local data. Many previous studies have developed relationships for predicting these same variables, e.g., where life-history data are combined with assumed functional relationships among variables to estimate a small set of model parameters, and these relationships and model parameters are then used to predict life-history variables for unobserved species (Pauly and Binohlan 1996, Froese and Binohlan 2000, Hordyk et al. 2015, Nadon and Ault 2016). By contrast, our approach uses both model parameters (fixed effects) as well as patterns in residuals among taxa (random effects) when predicting life- history variables for any species, and therefore bridges between analyses of all fish species (Pauly and Binohlan 1996, Charnov et al. 2013) and analyses restricted to specific taxa (Thorson et al. 2014b, Nadon and Ault 2016). We believe that our mixed-effect approach (and R package *FishLife*) has several benefits relative to previous regression-approaches including: (1) it predicts life-history variables probabilistically for species while using information about related taxa to inform predictions; (2) it predicts uncertainty for species based on the quantity of data that are available, so that species with many field-measurements of life-history variables have predictions that are more precise than species with only a few field measurements; (3) it uses the full set of life-history variables available for a given species to predict all unknown variables, e.g., rather than calculating natural mortality from a single variable (i.e., maximum age) and ignoring all others (e.g., individual growth rates). al. 2003, **Warton et al

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- We thank R. Froese for the tremendous effort in compiling records for all fishes globally into
- FishBase, and D. Pauly for decades of research that has highlighted the utility of FishBase of
- fish life-history analysis. We also thank P. Neubauer for preliminary discussions regarding
- phylogenetic regression, and O. Hamel, J. Hastie, M. Mangel, and one anonymous reviewer
- for comments on an earlier draft.

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711 Table 1 – Model selection results for the multivariate model of fish life-history variables.

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714 Table 2 – Summary of predictive covariance among all eight life history variables. Predictive 715 covariance is decomposed into eight orthogonal components, where the first component 716 explains the maximum variance, the second explains the maximum variance after accounting 717 for the $1st$ component, etc. The first three components explain 99% of total variance, and we 718 list the predicted variance for each (1st row), proportion of total variance (2nd row), 719 cumulative proportion of variance $(3rd row)$, and association of that component with different

721 average relationship between two variables along the axis of variation represented by that

722 eigenvector.

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724 Table 3 – Measurement covariance (lower-triangle, including diagonal) and resulting correlation (upper triangle, indicated by bold-font) for 725 replicated measurements of each life-history variable for a given species (e.g., the $2nd$ row and $1st$ column shows the negative measurement 726 covariance between $ln(K)$ and $ln(L_{\infty})$).

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730 Table 4 – Minimal (i.e., non-redundant) and sufficient (i.e., not missing any relations) set of 731 theoretical life-history relationships among the eight life history variables

732 $\{M, K, a_{mat}, L_{mat}, L_{\infty}, W_{\infty}, a_{max}, T\}$ analyzed in this study. These relationships involve six

733 exogenous parameters, $\{\delta, \alpha, \beta, E, A, C\}$, and if these parameters are constant for all fishes,

734 we expect that all eight variables are constrained within a subspace with 2 degrees of freedom

735 (= 8 variables – 6 relations). We hypothesize that: α (average mass per standardized

736 volume) does not vary systematically among fishes; β (ontogenic changes in body shape)

737 will vary systematically among fishes; δ (allometric scaling of pre-mature survival to length-

738 at-maturity) does not vary systematically; A (anabolic rate) does not vary systematically; E

739 (activation energy from metabolic theory) does not vary systematically; and C (average

740 mortality rate when $T \gg E$) may or may not vary systematically. We therefore hypothesize

741 that the value for all eight variables for each fish can be described via 3-4 coordinates,

742 { R, T, β } or $\{R, T, \beta, C\}$, depending upon whether C is constant or varies among fishes.

- 768 otherwise proportional among variables. "Comb." refers to the sum of variance for all
- 769 variables, and thus represents variance-explained on average among variables.

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