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Pseudo-Kinematic GPS Results using the Ambiguity Function Method

Benjamin W. Remondi

Rockville, MD
May 1990

DEPARTMENT OF COMMERCE
UNITED STATES OF AMERICA

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CONTENTS

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PSEUDO-KINEMATIC GPS RESULTS USING THE AMBIGUITY FUNCTION METHOD

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ABSTRACT. Pseudo-kinematic positioning with the Global Positioning System (GPS) is a survey technique employed by the National Geodetic Survey. This technique, which could also be called broken static (or intermittent static), must not be confused with kinematic surveying. The kinematic survey method requires continuous carrier phase tracking while moving between survey monuments and places strong restrictions on the number of satellites without cycle slips. The pseudo-kinematic method is a static technique whereby a geodetic monument is occupied for, typically, 1 to 5 minutes for two or more occupations separated substantially in time (e.g., 1 hour). Tracking during the transitions between monuments is categorically not required but is often done for practical reasons (e.g., to avoid having too many files and to achieve more rapid reacquisition of those satellites which were obscured en route). In fact, in pseudo-kinematic mode the rover receiver may be turned off while traveling between monuments.

Pseudo-kinematic surveying (PK) promises substantial productivity gains over classical static surveys and can be employed where the regular kinematic method is impractical. (However the static GPS survey method will remain as the method of choice in a wide variety of situations.) Furthermore, PK is safer than the regular static mode in that a constant antenna height is normally involved. It is more accurate than the regular static mode where the last millimeter is important. These advantages are discussed. The paper also defines the PK method, provides a very brief history, considers alternative possible names, presents the mathematical and physical basis for the method with emphasis on what to do about all the cycle slips between occupations separated by an hour or more, and provides a number of examples and results.

INTRODUCTION

Pseudo-kinematic surveying (or intermittent static, broken static or snapshot static surveying) was first conceived in the 1982-83 time frame (Remondi, 1988). The idea sprung from research related to the ambiguity function method (AFM) (Remondi, 1984). After months of processing first simulated data (1982) and later (1983) real data it became obvious that the effectiveness of the method depended primarily on geometric change; that is, the satellite-receiver geometry needed to change. It became clear that a few minutes of data at one time and a few minutes of data an hour later were nearly as good, geometrically, as the full hour of data. Thus the original conception of the pseudo-kinematic survey technique (hereafter PK) was associated with the AFM. Because the receivers at that time did not lend themselves to rapid survey techniques and because single-difference, double-difference and triple-difference processing methods proved superior to the AFM, PK remained no more than an interesting idea. PK and AFM were closely associated because the AFM was immune to cycle slips. If one were to revisit sites with a period of an hour (or more) between visits, fixing cycle slips could prove to be difficult with the standard least-squares postprocessing methods.

In 1983 PK was experienced for the first time in an unexpected way. A data set had two gaps of possibly 30 minutes or more. Fixing cycle slips over these gaps was possible, making it clear that the receiver could have been elsewhere during those outages. In 1984 NGS set up a test network in California for testing existing GPS receivers. Thus numerous monuments were established within tens of meters of each other using NGS Macrometers[TM][1]. In fact, each of these monuments were occupied for hours. I had considered demonstrating PK at that time and only the practical inconveniences kept NGS from doing so. In retrospect, the inconveniences were relatively minor.

In 1985 PK processing was accomplished successfully for the first time. The success, however, was based on achieving a first estimate using a triple-difference least-squares (TLSQ) processing program. Using this first estimate, cycle slips were fixed and definitive processing was performed with a double difference least-squares (DLSQ) processing program.

During this period it was obvious that cycle slip fixing based on the TLSQ solution was a limiting factor and AFM or similar search techniques would be required to expand the method for general use. It is remarkable how well the TLSQ solution has worked. This is particularly true where satellite visibility is superior (e.g., Arizona). Obtaining a TLSQ solution accurate enough to fix cycle slips is limited by baseline length, duration of gap, and the number of satellites. With these limitations in mind one can employ PK safely and productively based on TLSQ and fixed DLSQ. In spite of reasonable success, PK was never intended to be used in this way and especially to be under restrictions of the duration of the gap. PK was intended to be useful over the same baselines as "classical" static surveys (i.e., without gaps) where integers would be clearly determined using conventional DLSQ processing. PK can be used (potentially) up to 100 km or more with single frequency data under ideal conditions where the error sources are minimal or can be minimized. Obviously with dual frequency processing this could be extended considerably.

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[1] [TM] Macrometer is a trademark of Aero Service Division, Western Atlas International, Houston, TX.

On a practical matter, PK is normally performed with fixed-height survey poles. NGS is considering performing classical static surveying with fixed-height survey poles as well. Up to now most of the regular static surveys have been done with tripods.

THE PSEUDO-KINEMATIC METHOD

The PK method is a static survey method. This technique is often confused with the kinematic survey method that requires continuous carrier phase tracking while moving between survey monuments and has strong restrictions on the number of satellites without cycle slips. Pseudo-kinematic means false kinematic and hence not kinematic but, rather, static. It is a static technique whereby a geodetic monument is occupied for, typically, 1 to 5 minutes for two or more occupations separated substantially in time (e.g., 1 hour). Tracking during the transitions between marks is categorically not required but is often done for practical reasons. In fact, in PK mode the rover receiver (the receiver that travels) may be turned off while traveling between monuments.

Pseudo-kinematic surveying promises substantial productivity gains over regular classical static methods where the receiver stays at a mark for an hour or two. Field productivity improvements can be tenfold for certain surveying activities. Postprocessing in PK mode can be faster for those cases where searches are not needed and tend to be slower where searches are involved. Search techniques, while historically slower, are not as inherently slow as one might imagine. Even in sequential processing computers, search techniques can be made acceptably fast. In a parallel processing environment they can be made almost arbitrarily fast.

The PK method can be distinguished from the single occupation rapid static methods whereby a station is occupied for 5 minutes or less. These methods are equally valid and will yield even more productivity gains. These methods rely either on a high-quality pseudorange differential solution or search techniques based on dual frequency data. They will be extremely effective in near-real-time processing. These methods may be somewhat less accurate than PK however. These kinds of one-visit surveys are not the subject of this paper.

PSEUDO-KINEMATIC PROCESSING BASED ON TLSQ/DLSQ

In its most restricted form PK postprocessing is accomplished as follows. One first obtains a TLSQ solution. With only, say, 2 minutes of carrier phase data (for each of two visits) this solution can be expected to be poor (e.g., 50 cm). (The measurement recording interval is not too critical; 10-20 seconds is satisfactory.) Next one makes the cycle connection between visits based on the TLSQ solution. A correlated TLSQ solution will obviously be more accurate than a TLSQ solution which ignores the correlation of triple differences. Roughly speaking the TLSQ solution must be good to 10 to 20 cm if the site visitations are an hour apart. For example, suppose a site is initially occupied for 2 minutes and suppose 30 minutes later the site is occupied a second time for 2 minutes. The 4 minutes of data are processed in TLSQ mode. Assuming the solution is good to roughly 30 cm, the cycle slips (we are imagining, here, that the receiver has actually been turned off) normally are correctly fixed. Generally speaking, this is a short baseline (1 to 10 km) technique which requires good satellite visibility. Correct fixing of cycle slips is normally obvious under these assumptions. The TLSQ solution does not have to be as good if the time period between visits is shorter (e.g., 15 minutes). This however limits the full use of this method. Time gaps of 1 to 2 hours and longer, if they can be overcome, are obviously going to result in higher productivity and accuracy. It should be clear that this restricted mode (i.e., TLSQ/DLSQ) is not compatible with single frequency receivers over long baselines and with 1 to 2 hour time periods between visits. (This would require extremely good error models.)

A BRIEF DESCRIPTION OF THE AMBIGUITY FUNCTION METHOD (AFM)

Refer to Remondi (1984) for complete details on the AFM method. Only recent algorithmic advances to speed up the computations are missing from that document. The ambiguity function is simply:

$$f(\vec{r}) = \sum_{i=1}^{nepochs} \left| \sum_{k=1}^{nprns} e^{j\left(O^k - \frac{\rho^k(\vec{r})}{\lambda}\right)} \right|$$

where O^k is the single difference phase based on receivers 1 and 2 and satellite k, ρ^k is the difference between the receiver-satellite ranges, and λ is the nominal carrier wavelength. Note that integer ambiguities are absent from the model. This is because the phasor $\exp(j\Phi)$ is identical to $\exp[j(\Phi+N)]$, where N is an integer number of cycles. Of course this is a primitive model and for longer baselines refraction would be included.

The AFM works as follows. Select the next candidate position r and compute f(r). Realize that f(r) is bounded by the number of single difference measurements in that one single difference measurement can increment f(r) by at most unity. Thus, for 200 single difference measurements f(r) can achieve a value not to exceed 200. The concept of the AFM is really very simple. The difficulty is in achieving computational speed. This has been achieved. Further speed enhancements are being investigated and it seems possible that a goal of considering a million positions in 1 minute is possible on existing fast ($12,000) desk top computers. Generally speaking there is no current need to consider a million positions and, for present applications, 125,000 positions seem completely adequate. I intend to document these findings in the near future.

To get the full potential of this technique we first obtain the TLSQ solution (correlated or uncorrelated). After this we use a search technique. There are many possible search techniques capable of performing this search. To keep this discussion focused I shall limit it to the AFM (Remondi, 1984). Only recent improvements with respect to computational efficiencies are missing.

Let us assume sigmas resulting from TLSQ have been normalized so as to be realistic and believable. The AFM then searches all space around the TLSQ solution (correlated or uncorrelated) inside a box whose sides are $4*\sigma_x$, $4*\sigma_y$, and $4*\sigma_z$. Let us assume the rectangle is divided into a grid of points separated in all directions by 0.25 to 0.33 cycles. For L1 processing this could be, for example, 5 to 6 cm. A typical search volume might be a cube with 2 meter sides. Thus, for example, 41**3 or 68,921 positions would be considered. At each of these 68,921 points all the relevant single difference measurements (e.g., 200 of them) will be evaluated. Although this seems to be a Herculean task it can be performed on a fast ($12,000) desk top computer in less than 10 seconds (and on a regular 20 mHz 80386 PC in less than 1 minute). A maximal search volume might be a 4m*4m*2m box. This search technique will uncover those positions where the fractional phase measurements agree with the fractional phase based on the model. In such a search there is no such thing as a cycle slip in that only the fractional phase measurement is employed. A search will uncover numerous functional maxima. For short baselines and two visits there will be a predominant peak and possibly one or more significant but lesser peaks (this assumes reasonable satellite visibility; a representative example would be five satellites on both visits and perhaps an additional satellite on one of the two visits). If a peak is clearly greater than other peaks, this will almost certainly be the correct peak. Where necessary a candidate peak can be verified by double-difference processing (DLSQ). The candidate peak is first used to fix cycle slips. Next DLSQ processing follows. For short baselines, the float-DLSQ solution will achieve almost perfect integers if the cycle slips were fixed correctly (which will only be achieved at the correct peak).

RESULTS

In the following the results of four experiments will be discussed. In all cases the data were collected in preparation for PK postprocessing.

Experiment (April 24, 1989). The first experiment involves an airport survey performed in Apalachicola, FL. These data were collected consistent with kinematic processing (i.e., no cycle slips) and PK processing (i.e., two visits). Although there were no cycle slips in this data set, it is still possible to process the data while not benefitting from the fact that there were no cycle slips. In this survey the reference receiver was situated at mark APAL for the duration of the test with the exception of antenna swaps which took only a few minutes to perform. The rover receiver started at station APAZ and then performed two antenna swaps with the reference receiver (consequently returning the reference receiver to APAL and the rover to APAZ).

Then the rover occupied marks AP18, AP36, AP31, AP13, AP06, AP24 returning to APAZ. This circuit was then repeated. With the exception of APAZ all marks were occupied two times for 2 minutes each time. APAZ is occupied four times for 5 minutes each time. All marks are within 2 km of APAL.

These data were processed routinely in kinematic mode as part of routine NGS operations (Remondi, 1990). These kinematic solutions, though accurate to possibly 1 to 3 cm, will provide approximate truth for the PK processing even though PK processing would normally be more accurate than kinematic processing. The rationale for using a less accurate solution for truth is not accuracy but, rather, to verify that our solution is the correct one.

Table 1 comprises seven baselines with five entries for each baseline. The first entry is the triple difference (TLSQ) solution. This solution did not use a weight matrix. A weight matrix would be helpful in reducing the dimension of the search rectangle possibly by a factor of two (and the volume by eight). This would be an important consideration in an operational setting but was unimportant in isolating the correct position. Note that the TLSQ solution is relatively poor. The 1-sigma uncertainties associated with the TLSQ solution have been included in parentheses. The search volume was approximately four times these values.

The next entry is the DLSQ solution based on using the TLSQ solution for cycle slip repair. Notice that this failed for baselines APAL-AP06 and APAL-AP13. For the other baselines this step was successful and provided a definitive solution. For the two cases which failed the TLSQ/DLSQ processing, cycle slips of magnitude 1 or 2 were wrongly introduced; this naturally, was the cause of the failure. The AFM, of course, remedies this.

The next entry is the solution of the AFM search. Note that this step was always successful. The percentage represents how close f(r) came to its maximum theoretical peak (when multiple significant peaks were encountered, the second highest peak is also noted). For a specific value of r the percentage is computed by 100*f(r)/number of measurements.

The next to the last entry is the result achieved when the AFM solution is passed to fixed-DLSQ processing. In general, the fixed-DLSQ solution was neither better nor worse than the AFM solution.

The last entry is approximate truth. In this example the survey marks were occupied two times for 2 minutes each time. The occupations were spaced approximately 50 minutes apart. Either five satellites or six satellites were common to the two visits. In the case where five satellites were common a sixth satellite was tracked during one of the two occupations. Measurements were recorded every 15 seconds.

Experiment (February 21, 1990). In this experiment the PK method was an important part of the experiment design. The objective in this experiment was to measure the deflection of the vertical. The accuracy objective was 0.25 arc second. Geodetic leveling was also performed at these sites. This test was performed at the Naval Surface Warfare Center (NSWC), Dahlgren, VA.

The actual geodetic results will be reported by Dr. Alan G. Evans and Dr. Benjamin W. Remondi in the months ahead.

Five receivers were involved. The reference receiver was situated at site MBRE on a tripod and never moved; the remaining four receivers were rovers. Within 2 km of MBRE, in a loop roughly centered about MBRE, were 12 survey marks. Every other one was designated blue and red. Two of the rover receivers were to occupy the six red marks following a counter clockwise route which would complete the loop eight times. Two of the rover receivers did the same with respect to the blue marks. Based on the previous experiment only two visits were needed to determine the location of these marks to a couple of centimeters. Nevertheless, eight occupations were decided upon since we wanted to achieve the highest possible accuracy. There were numerous tall trees and buildings en route. In fact, there were so many obstructions that almost every receiver had cycle slips on almost every satellite during almost every transition between marks. In short, there were many dozens of cycle slips for each receiver (possibly more than 100). This presented no problem whatsoever; in fact, it was expected. The receiver could just as well have been turned off en route but was not so as to avoid a proliferation of data files. Each occupation took 4 minutes and data were collected every 10 seconds. Clearly this quantity of data was excessive and not required. It should also be pointed out that the number of satellites tracked during any specific occupation varied from three to six. In the most favorable situations the same six satellites were observed on consecutive occupations of a mark.

Finally, the red team completed its loop in approximately 48 minutes whereas the blue team took approximately 55 minutes. It is not surprising, therefore, that in the one instance the TLSQ solution was unable to correctly fix cycle slips was for one of the blue teams. For a given accuracy of the TLSQ solution the length of the time gap between revisits is bounded. In this case AFM was employed. This survey was performed in L1 and L2 and on two consecutive evenings. It should be clear that once the solution was obtained for L1 processing, for the first night, there was no need of TLSQ or AFM processing for the L2 solutions of the first night or for the L1 or L2 solutions of the second night.

Let us designate the reference mark by R, the blue marks by B1 through B6 and the red marks by R1 through R6. If, furthermore, we distinguish between the two blue rovers, we can designate the resulting solutions as R-B1I and R-B1II and similarly for red. With this in mind an important aspect of this experiment was to isolate solutions based on one rover (e.g., R-B1II, R-B2II, ..., R-B6II). To eliminate any possibility of instrument bias the differences such as [(R-B1II) - (R-B4II)] were formed. This, in effect, differences out the reference receiver and places the same instrument at both ends of a baseline. The rover antennae were placed on top of a <u>fixed height</u> survey pole so that the vertical height above the mark was constant. Of course we recognize the phase center still is a function of the direction to a satellite, but this procedure should eliminate some amount of bias.

Because two blue teams and two red teams were employed, two frequencies observed, and the survey performed twice, eight solutions were obtained for each baseline. These eight solutions are treated as being independent even though there may be some correlation involved. Table 2 lists means and standard deviations of solutions based on a given rover receiver after differencing out the reference receiver. The standard deviations in length (upper triangle) tend to be about 3 mm and those in height (lower triangle) tend to be about 6 mm. Although a final statement cannot be made at this time, preliminary results indicate that the (ensemble) means agree in height at the 2 to 3 mm level. This is based on a priori truth of the deflection of the vertical at the test sitee It is known to approximately 0.1 arc second (equivalent to 1 mm over 2 km). On the other hand baseline length truth was not available except for MBRE to HERO where there was 1 mm agreement. For these baseline lengths 2 to 3 mm agreement is anticipated. Prior tests were conducted to verify that the survey rods employed can be set up, horizontally, at the 1 mm level. The vertical set up error is essentially 0.0 mm.

Experiment No. 3 (February 23, 1990). This experiment was similar to experiment No. 2 from a GPS standpoint. Here a central site, CQCP, was encircled, more or less, by five other sites of interest. The entire experiment was restricted to a 200-meter network. CQCP was the site of the reference receiver. Four rover receivers were used. Their antennas were mounted using forced-centering devices. This guaranteed that, on reoccupations, the physical device would have no significant error (perhaps 0.1 mm). Thus errors were not due to mechanical set up but due to phase center variations, multipath, refraction, and so forth. The aim of this was accuracy. The truth in this case is known at the 0.5-millimeter level by careful terrestrial survey methods and modern instrumentation. The five monuments were ASTW, CQD1, CQD2, CQD3, and CQD4, located at the NOAA/NGS Instrumentation and Equipment Section facility in Corbin, VA.

The test was designed as a PK test. Thus cycle slips between marks were not a factor and the receivers could have been shut off between sites. Each receiver occupied a site for exactly 3 minutes and then moved clockwise to the next one. Exactly 3 minutes were allowed for travel time. In all cases set up was completed in less than the allotted 3 minutes. All four rover receivers completed the circuit exactly 10 times. Thus all baselines from CQCP to the remote sites were measured by the four receivers. Subsequently the reference receiver was differenced out. Before presenting the results, it should be stated that the satellite visibility was identical to that of experiment No. 2 in that the survey was performed on February 23, 1990, just one night after the NSWC deflection of the vertical test (February 21-22, 1990) and only 100 km away. Thus only three satellites were tracked during some occupations and a maximum of six satellites were tracked during some occupations. Because only 30 minutes separated occupations of a given mark by a specified receiver (i.e., once per loop), a maximum of six satellites was common to as many as three occupations.

Table 3 gives results from the reference mark to the remote sites. In the left-hand column are the means associated with four L1 solutions and four L2 solutions. The next column contains the associated standard deviations in millimeters. The final column contains the truth. Table 3a includes the distance results and table 3b the height results. The accuracy of the azimuth component can be assumed to be somewhere in-between.

Notice that the distance agrees at the 1-millimeter level except for CQD4 where the disagreement is 3 mm. Remarkably, all heights agree to about 1 mm except for CQD4 where the discrepancy is 9.7 mm. These results lead one to believe these multiple measurement (i.e., averaging) methods can deliver 1 mm distances and possibly 2-3 mm heights over short (100 m) distances.

Table 4 contains the results of this same experiment where the reference site has been differenced out. Here we are trying to eliminate small instrumentation biases.

A linear combination of noisy quantities yields a noisier quantity. Thus if no instrument-specific biases existed, the differenced quantities would be expected to have larger standard deviations than those quantities prior to differencing. The fact that the standard deviations have actually been reduced should point to the removal of biases. Clearly each monument has its biases and each instrument has its biases. Although there are monument height biases it is the removal of instrumentation biases has led to smaller standard deviations.

This experiment has one disadvantage over experiment No. 2. In experiment No. 2 we maintained exactly the same height above all remote marks since the same unadjustable rod was used by a given rover at all sites. At the Corbin facility a platform was set up over each survey mark to accommodate the forced-centering devices. These platforms are semi-permanent and extreme care was uaed in measuring these heights. I believe these errors are below 0.5 mm (in the previous experiment the vertical setup error was essentially 0.0 mm).

This test will be repeated. It is important to know what procedures are required to achieve 1 mm distances consistently and, possibly, 2 mm heights over short baselines (200 m). Experiments No.e2 and No.e3 should remove any question about the ability to measure distances and even ellipsoidal height differences to a few millimeters. The appropriate question, especially for short baselines (e.g., 3 km), is what procedures ahould be followed if one is to measure distances to, say, 2 mm and ellipsoid height differences to, say, 5 mm. Clearly these accuracies can be achieved over short distances, especially where the elevation changes are small and the meteorology is favorable.

Experiment No. 4 (February 9, 1990). This is the most important example with respect to supporting the theme of this paper.

A reference receiver was situated at S274, which is a National Geodetic Reference System first-order horizontal control station and bench mark. At a distance of approximately 40 km is Malden Municipal Airport, where the primary site is MALD. Another reference receiver was situated at MALD for the duration of this test. A third receiver wasea rover receiver. This rover began at site MAAZ and proceeded to sites MA04, MA13, MA18, MA22, MA31 and MA36, finally returning to MAAZ. This circuit was completed a total of four times. Each site was occupied for 5 minutes although thia length of atay was not required. Thus each loop took about 62 minutes and the entire experiment took about 4.25 hours. In this experiment there was no a priori truth. For approximate truth the entire data set for the baseline S274-MALD was used to determine this vector with sufficient accuracy. This experiment was conducted at night when the ionospheric refraction was minimal. As a consequence the integer ambiguities were obvious and determined to within 0.1 cycle in normal processing. This solution will provide truth in the sense that the same solution shall be attempted with only a few minutes of data. With MALD known approximately it was a simple matter to determine the other (local) seven airport marks. This was achieved using PK processing with the TLSQ and DLSQ solutions. This was easy to achieve since there were four occupations of 5 minutes each and the baselines were short (e.g., less than 2 km). Thus the approximate vector truth is obtained from S274 to MALD as well as the approximate vector truth from S274 to the other seven airport monuments. The satellites tracked during the four occupations varied from three or four to a maximum of six at any one time. Altogether seven satellites were observed.

In the easy part of the experiment the seven baseline vectors were determined from MALD to the other runway monuments using four occupations, three occupations, and finally two occupations of 5 minutes each using the AFM. Although not included in this report the results are roughly the same when only 2-minute occupations are assumed and where the remainder of the data is simply ignored.

For the case where two occupations are assumed, either six satellites are common to two occupations or there are at least five common satellites and an additional satellite which is not common. Thus the geometry is quite good but not significantly better than will be commonly available when the GPS constellation is complete.

Table 5 lists the results of the four-, three-, and two-occupation studies for the local MALD airport processing. For each baseline vector there are seven entries. The first line lists the TLSQ solution and realistic sigmas for the four-occupation case (denoted 4V for four visits). This is followed by the AFM solution for the four-occupation case. The next two lines reflect the three-occupation case (denoted 3V). This is followed by the two occupation case (denoted 2V), and finally the truth vector.

Each AFM solution includes the amplitude of the highest maximum in comparison with the theoretical maximum and also the second-highest peak. Notice that as the number of occupations is reduced the first and second maxima are less distinct. What may be surprising is that even when the peaks become essentially the same the correct solution can be isolated (normally) by DLSQ processing. The right peak will fix cycle slips correctly and go on to yield a fixed double difference solution with small residuals. The wrong peak will not behave this way.

The AFM search volume is based on the sigmas from TLSQ. These sigmas are realistic and even conservative. The search took place over approximately 4-sigma space. In fact the correct solution was normally well within the search volume. Clearly, if a satisfactory peak was not achieved a larger search volume would have been used. In none of the cases presented here was an expanded search volume needed because the search volume used was so conservative.

This processing was straightforward and, as stated earlier, only 2-minute occupations were generally required; possibly this will be documented in an upcoming report. It is important to note that the AFM solution is accurate and generally within the accuracy expectations of the fixed double difference solution.

S274 to Mald Airport

The final and most important aspect of this processing was in determining the baseline vectors from S274 to the eight airport marks with four, three, and finally two 5-minute occupations. These baselines are approximately 40 km in length. Table 6 lists the results.

In the four-occupation case the solution was extremely easy and obvious. The maxima were generally at the 90-percent level whereas other maxima were 80-percent or so, and below. Nevertheless the AFM solution was input into the DLSQ processing as a check. Correct cycle slips, ambiguities which are nearly integers, and small residuals are all required. In all cases these were achieved and secondary peaks failed. Notice, again, ambiguity function solutions compared favorably with the truth.

In the three-occupation case, notice that the maximum peak and the secondary peak are somewhat closer. Nevertheless the maximum peak in all cases was the correct one. In fact, in this experiment there was not much difference between the four-occupation case and the three-occupation case. As in the four-occupation case all strong secondary peaks were input to DLSQ processing as a test along with the maximum. In all cases the AFM maxima passed these tests whereas the other lesser maxima did not.

In the two-occupation case the primary and secondary maxima are nearly indistinguishable. Nevertheless in all cases except one the maximum was the correct solution. In that exceptional case the AFM maximum failed the DLSQ step in that large residuals were encountered. The second peak passed the DLSQ test in that small residuals were achieved. It should be emphasized that the two-occupation case was barely strong enough to succeed even though this was a favorable data set taken during the quiet portion of the day. In spite of this, these results are most encouraging:

They indicate that dual-frequency AFM processing over moderate baselines will be very effective.

Gaps of 2 to 3 hours in metropolitan surveys (20 km) will be productive, efficient, and accurate especially with a full GPS constellation.

As the author has previously stated, they prove (Remondi, 1988) that PK works and that two or three occupations of 2 to 3 minutes are adequate to determine baselines of moderate length and, in fact, the GPS receiver may be turned off between visitations.

OTHER USEFUL TECHNIQUES

AN ALTERNATIVE SEARCH METHOD

Previously it was stated that many alternative search techniques can also isolate the correct solution. Here I shall discuss one that I have occasionally used in my PK research. This method assumes two or more occupations, as its objective is to make the cycle connection between occupations.

The method is conceptually simple. After the TLSQ solution is used to fix cycle slips the DLSQ solution follows. If double-difference integers are very poor, the user should assume the cycle slips were not fixed correctly and add a vector increment to the TLSQ solution and attempt to fix cycle slips again. The DLSQ solution would be attempted again.

In PK processing this method can be effective in that only a few observations are processed and success is known after just one iteration. In fact success or failure is known within seconds. In effect one searches a box about the TLSQ solution but uses these candidates in a different way than in ambiguity function processing. The search grid can be very crude if the time period between occupations is not too large. The longer the gap the more fine the search grid. A typical example would be for 30 minute gaps. In this case if the TLSQ solution does not repair cycle slips correctly, possibly a 20 cm search grid would be appropriate. The granularity of the search grid can easily be computed based on the amount of integrated Doppler error one can tolerate over the period between occupations. Without providing specific examples, I shall simply state that the method works rather well in a variety of situations but the AFM and other search methods, not given here, are more general.

THE TANDEM PSEUDO-KINEMATIC METHOD (TPK)

The PK method as described so far in this paper assumes the rover receiver will occupy a given mark two or more times.

An extension to this, which I have used originally out of necessity and later by design, is as follows. A hypothetical example will be used to describe the method.

The U.S. Capital Beltway around Washington, DC, is approximately 100 km in circumference. Suppose there were geodetic marks located approximately 1 km apart along the circumference. Suppose the objective was to determine the locations of these 100 marks to within a couple of centimeters with respect to a central geodetic monument. One could use static or a combination of static and kinematic methods (since there are many overpasses). Clearly PK would be more productive. Still more productive would be TPK which is executed as follows.

In the above example, the reference receiver is situated in downtown Washington, DC, near the center of the beltway. Two or more rovers will be assumed. The first rover begins the loop at a certain time. One or two hours later a second rover occupies the same marks as the first rover and in the same order. For increased accuracy and reliability an optional third tandem rover could also follow the second. During postprocessing of a given geodetic mark the carrier phase data are used from any receiver which occupies that mark. Naturally it would not matter very much that these data were collected by many receivers as opposed to the traditional single receiver. One savings over PK is that the rover does not have to waste time in returning to the original mark. Another factor is that the survey would be completed in less than half the time of a one-rover PK survey. (Of course the total amount of person-hours would be nearly the same.) Planning and executing a TPK survey would be simple.

THE SIMULTANEOUS TANDEM PSEUDO-KINEMATIC METHOD (STPK)

This method is similar to the TPK method described above except the tandem rovers occupy marks simultaneously. For example, this would be accomplished either by a rigorous preschedule plan, or with the aid of cellular phones. This method differs in another subtle way. Where TPK above can be executed with either two or three rovers this method is more effective with three (or more) rovers. Three simultaneous rovers provide two occupations of all circumferential neighboring vectors allowing subcentimeter accuracies. Note that even two rovers, operating in STPK mode, can be effective (assuming enough satellites) since the a priori vectors will be determined with respect to the (fixed) reference receiver (situated at some central location). The combination of the radial vectors and the high-precision circumferential vectors should yield a powerful network combination.

It should be obvious that the planning and executing of an STPK survey could be tricky. An STPK survey must have simultaneity, uniform travel times between marks, and a reasonable time gap between when the tandem rovers occupy a given mark. Thus, while this technique could be useful, it may have limited applications.

The circumferential survey example of course is only illustrative. One does not actually have to circumnavigate the reference receiver. Best results would normally be achieved if the rovers ultimately returned to their starting marks. This is not strictly required.

SUMMARY

Four examples have been presented to substantiate earlier claims that PK is a potentially powerful static survey method where only two or three occupations for only 2-3 minutes are required, allowing the rover, at least in theory, to turn off the GPS receiver between visits.

The first example demonstrated that the PK method worked with two visits of 2 minutes each. A search method was needed for two cases since, for those cases, the TLSQ solution was not sufficiently accurate to fix cycle slips between occupations. Either the alternative search method, as described above, or the ambiguity function method easily solves this problem. The ambiguity function method is preferred because it is more general. The latter method (i.e., the alternative search method) will encounter problems when attempting to fix cycle slips over baselines of moderate lengths where gaps of 2 to 3 hours are involved; the AFM would not encounter such limitations.

The second and third examples demonstrated that the PK method can yield accurate survey results and that loss of lock between occupations does not present a problem.

The fourth example was potentially the most informative. It demonstrated that the AFM allowed PK processing from short to moderate baseline lengths (i.e., 40 km). To some extent this example proved that PK is not subject to working only some of the time as has often been conjectured. It can be expected to work all the time if certain limitations are respected.

The PK method has evolved over recent years into a technique that is useful today and has a bright future. The AFM has a long history and the technique was studied a great deal in the early 1980s (Remondi 1984, Counselman and Gourevitch 1981). In the author's opinion PK was always intended to be combined with a search method to achieve its full effectiveness. The ambiguity function method is a good candidate for this now that the method is fast enough to do complex searches in as little as 1 minute.

A less general but alternative search method was presented. Abbreviated as TPK and STPK, two variations of the pseudo-kinematic method were introduced.

CONCLUSIONS

The PK method can achieve its full effectiveness when combined with an appropriate search method. Although the AFM is not the only search method it is fast and effective; in a parallel processing future it could possibly be performed in less than a second. These search techniques easily extend to the determination of integer ambiguities on the fly in that one can search for the correct starting location in the same way one searches for the static vector solution. The reason for this is simple: The starting position is not known but one does know, precisely, the change in position from the starting position to the position a few minutes later.

The PK method based on an effective search technique will be an effective method for performing rapid and accurate surveys on metropolitan scales (e.g., Washington, DC, and vicinity to, about, 25 km), based on a central reference site. This will become clearer as the full GPS constellation is deployed and advances in postprocessing software are developed.

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Table 1.--Ambiguity function method results from the April 24, 1989, Apalachicola, FL, survey

APAL-APAZ	dx (m)	dy (m)	dz (m)	Remarks
TLSQ	18.964 (10 cm)	-498.574 (14 cm)	-870.138 (7 cm)	resid=0.011
DDX(TLSQ)	18.918	-498.504	-870.136	resid=02041
AF(TLSQ)	18.926	-498.496	-870.144	94%, 88%
DDX(AF)	18.918	-498.504	-870.136	resid=02041
TRUTH	182922	-498.506	-8702132	Approximate

APAL-AP06	dx (m)	dy (m)	dz (m)	Remarks
TLSQ	-1259.255 (25 cm)	-866.540 (31 cm)	-1316.832 (22 cm)	resid=02015
DDX(TLSQ)	-1259.337	-866.857	-1316.802	resid=0.280 bad cycle fix
AF(TLSQ)	-1259.691	-866.184	-1316.800	93%
DDX(AF)	-1259.690	-866.190	-1316.793	resid=02020
TRUTH	-1259.690	-866.182	-1316.798	approximate

APAL-AP13	dx (m)	dy (m)	dz (m)	Remarks
TLSQ	-1051.322 (20 cm)	-271.354 (24 cm)	-312.845 (16 cm)	resid=02012
DDX(TLSQ)	-1051.308	-271.676	-312.832	resid=02146 bad cycle fix
AF(TLSQ)	-1051.288	-271.126	-312.921	93%
DDX(AF)	-1051.287	-271.122	-312.916	resid=02030
TRUTH	-1051.289	-271.130	-312.914	Approximate

APAL-AP18	dx (m)	dy (m)	dz (m)	Remarks
TLSQ	55.464 (23 cm)	27.096 (26 cm)	38.879 (16 cm)	resid=0.012
DDX(TLSQ)	55.670	26.837	38.907	resid=02016
AF(TLSQ)	55.668	26.841	38.907	94%, 85%
DDX(AF)	55.670	26.837	38.907	resid=0.016
TRUTH	55.671	26.833	38.907	Approximate

Table 1.--Ambiguity function method results from the April 24, 19892
Apalachicola, FL, survey (continued)

APAL-AP24	dx (m)	dy (m)	dz (m)	Remarks
TLSQ	94.780 (31 cm)	-352.720 (45 cm)	-626.084 (24 cm)	resid=02016
DDX(TLSQ)	94.677	-352.466	-626.003	resid=0.020
AF(TLSQ)	94.670	-352.447	-626.018	94%, 88%
DDX(AF)	942677	-352.466	-626.003	resid=0.020
TRUTH	94.675	-352.460	-626.007	Approximate

APAL-AP31	dx (m)	dy (m)	dz (m)	Remarks
TLSQ	121.658 (17 cm)	-738.019 (20 cm)	-1302.870 (13 cm)	resid=0.010
DDX2TLSQ)	121.730	-738.090	-1302.889	resid=02023
AF2TLSQ)	121.736	-738.080	-1302.898	93%, 89%
DDX(AF)	121.730	-738.090	-1302.889	resid=0.023
TRUTH	121.735	-738.090	-13022890	Approximate

APAL-AP36	dx (m)	dy (m)	dz (m)	Remarks
TLSQ	115.874 (28 cm)	-765.752 (31 cm)	-1350.151 (20 cm)	resid=0.015
DDX(TLSQ)	115.937	-765.755	-1350.216	resid=0.025
AF(TLSQ)	115.936	-765.738	-1350.230	93%, 87%
DDX(AF)	115.937	-765.755	-1350.216	resid=0.025
TRUTH	115.938	-765.755	-1350.218	Approximate

Table 2.--Results from the February 21-22, 1990, deflection of the vertical test

RED	AAFR	DCKR	PBRR	PNDR	RUNR	WHYR
AAFR		1044.251 (1.2 mm)	1494.963 (2.5 mm)	275.401 (2.4 mm)	1147.797 (1.9 mm)	1852.769 (1.6 mm)
DCKR	-0.734 (4.7 mm)		2261.520 (2.8 mm)	1260.200 (3.2 mm)	837.213 (2.8 mm)	2075.090 (2.5 mm)
PBRR	0.509 (5.1 mm)	1.243 (4.9 mm)		1227.171 (2.5 mm)	1725.680 (2.7 mm)	1145.879 (2.0 mm)
PNDR	-0.488 (6.0 mm)	0.246 (7.5 mm)	-0.997 (6.2 mm)		1192.328 (1.0 mm)	1690.531 (2.0 mm)
RUNR	0.741 (3.9 mm)	1.475 (5.8 mm)	0.232 (7.4 mm)	1.229 (7.1 mm)		1264.401 (2.5 mm)
WHYR	0.164 (4.5 mm)	0.898 (4.8 mm)	-0.244 (5.3 mm)	0.653 (3.7 mm)	-0.576 (4.9 mm)	

BLUE	AAFB	DOCK	HERO	PNDB	RUNB	SHL3
AAFB		1152.020 (1.72mm)	1621.348 (3.02mm)	210.650 (2.12mm)	1828.959 (2.72mm)	1692.221 (1.92mm)
DOCK	2.464 (6.4 mm)		1277.366 (4.2 mm)	1321.689 (2.8 mm)	2020.425 (2.8 mm)	2628.451 (2.9 mm)
HERO	0.853 (4.9 mm)	3.217 (6.8 mm)		1631.203 (2.9 mm)	927.280 (2.5 mm)	2189.166 (2.8 mm)
PNDB	-0.036 (4.0 mm)	2.429 (5.2 mm)	-0.289 (4.7 mm)		17282934 (3.2 mm)	1483.097 (3.0 mm)
RUNB	0.341 (4.5 mm)	2.805 (5.9 mm)	-0.212 (3.7 mm)	0.377 (4.5 mm)		1555.228 (3.0 mm)
SHL3	-0.875 (2.3 mm)	1.589 (5.9 mm)	-1.729 (5.5 mm)	-0.240 (3.5 mm)	-1.217 (5.7 mm)	

Table 3a.2-Distance results from the February 23, 1990, Corbin, VA test

Baseline	Mean (m)	Standard deviation (mm)	Truth
CQCP-ASTW	34.9458	1.1	34.945
CQCP-CQD1	95.4716	1.2	95.472
CQCP-CQD2	86.9092	1.1	86.910
CQCP-CQD3	79.7833	1.5	79.783
CQCP-CQD4	106.9900	2.1	106.987

Table 3b.--Differential height results from the February 23, 1990, Corbin, VA test

Baseline	Mean (m)	Standard deviation (mm)	Truth
CQCP-ASTW	-0.2297	5.4	-0.2292
CQCP-CQD1	1.6918	6.3	1.6924
CQCP-CQD2	0.7414	4.5	0.7427
CQCP-CQD3	-0.2945	4.9	-0.2964
CQCP-CQD4	0.6385	7.3	0.6482

Table 4.--Distance and differential height results from the February 23, 1990, Corbin, VA test where the reference receiver is differenced out

	ASTW	CQD1	CQD2	CQD3	CQD4
ASTW		106.1009	121.8316	82.5642	72.0967
		(0.8 mm)	(0.9 mm)	(0.92mm)	(1.7 mm)
TRUTH		106.102	121.832	82.563	72.095
CQD1	-1.9214		122.8378	17522505	150.0037
	(2.6 mm)		(0.8 mm)	(0.6 mm)	(0.9 mm)
TRUTH	-1.9216		122.838	175.250	150.002
CQD2	-029711	0.9503		123.4383	193.8942
	(3.6 mm)	(3.1 mm)		(1.2 mm)	(1.5 mm)
TRUTH	-0.9719	0.9497		123.439	193.892
CQD3	0.0649	1.9863	1.0360		127.2370
	(1.7 mm)	(2.2 mm)	(2.5 mm)		(1.8 mm)
TRUTH	0.0672	1.9888	1.0391		127.234
CQD4	-0.8682	1.0533	0.1030	-0.9330	
	(5.3 mm)	(4.8 mm)	(4.9 mm)	(4.2 mm)	
TRUTH	-0.8774	1.0442	0.0945	-0.9446	

Table 5.2--Ambiguity function method results from the February 9, 1990, Malden Municipal Airport test

MALD-MA04		dx(m)	dy(m)	dz(m)	Remarks
TLSQ	4V:	416.974	196.616	264.707	
		(14 cm)	(17 cm)	(7 cm)	
AF(TLSQ)	4V:	417.166	196.608	264.749	97% 79%
TLSQ	3V:	417.028	196.644	264.646	
		(16 cm)	(21 cm)	(8 cm)	
AF(TLSQ)	3V:	417.170	196.602	264.746	95% 85%
TLSQ	2V:	416.936	196.656	264.701	
		(29 cm)	(38 cm)	(15 cm)	
AF(TLSQ)	2V:	417.170	196.606	264.743	94% 89%
TRUTH		417.162	196.608	264.734	

MALD-MA13		dx(m)	dy(m)	dz(m)	Remarks
TLSQ	4V:	416.552	177.393	239.169	
		(17 cm)	(19 cm)	(9 cm)	
AF(TLSQ)	4V:	416.410	177.501	239.143	95% 80%
TLSQ	3V:	416.584	177.425	239.206	
		(18 cm)	(25 cm)	(10 cm)	
AF(TLSQ)	3V:	416.410	177.491	239.141	95% 85%
TLSQ	2V:	416.540	176.821	239.178	
		(30 cm)	(45 cm)	(19 cm)	
AF(TLSQ)	2V:	416.414	177.521	239.128	94% 89%
TRUTH		416.412	177.509	239.226	

MALD-MA18		dx(m)	dy(m)	dz(m)	Remarks
TLSQ	4V:	41.990	-264.539	-356.865	
		(13 cm)	(13 cm)	(6 cm)	
AF(TLSQ)	4V:	41.956	-264.575	-356.865	96% 78%
TLSQ	3V:	42.009	-264.568	-356.869	
		(13 cm)	(16 cm)	(7 cm)	
AF(TLSQ)	3V:	41.951	-264.558	-356.859	95% 85%
TLSQ	2V:	42.008	-264.560	-356.864	
		(18 cm)	(22 cm)	(10 cm)	
AF(TLSQ)	2V:	41.957	-264.560	-356.864	94% 90%
TRUTH		41.950	-264.556	-356.869	

MALD-MA22		dx(m)	dy(m)	dz(m)	Remarks
TLSQ	4V:	-672.718	-438.812	-591.041	
		(20 cm)	(20 cm)	(9 cm)	
AF(TLSQ)	4V:	-672.610	-438.602	-591.075	95%
TLSQ	3V:	-672.708	-438.859	-591.043	
		(19 cm)	(23 cm)	(11 cm)	
AF(TLSQ)	3V:	-672.611	-438.603	-591.075	95% 83%
TLSQ	2V:	-672.700	-438.857	-591.104	
		(25 cm)	(31 cm)	(15 cm)	
AF(TLSQ)	2V:	-672.614	-438.584	-591.090	93% 87%
TRUTH		-672.616	-438.593	-591.085	

Table 5.--Ambiguity function method results from the February 9, 1990, Malden Municipal Airport test (continued)

MALD-MA31		dx(m)	dy(m)	dz2m)	Remarks
TLSQ	4V:	-649.181	826.538	1113.925	
		(15 cm)	(19 cm)	(9 cm)	
AF(TLSQ)	4V:	-649.207	826.429	11132975	95% 79%
TLSQ	3V:	-649.191	826.611	1113.882	
		(17 cm)	(26 cm)	(11 cm)	
AF(TLSQ)	3V:	-649.207	826.419	1113.982	95% 79%
TLSQ	2V:	-649.224	826.716	1113.772	
		(14 cm)	(29 cm)	(12 cm)	
AF(TLSQ)	2V:	-649.210	826.429	11132970	93% 85%
TRUTH		-649.209	826.434	1113.968	

MALD-MA36		dx(m)	dy(m)	dz2m)	Remarks
TLSQ	4V:	58.284	645.367	869.596	
		(15 cm)	(19 cm)	(9 cm)	
AF(TLSQ)	4V:	58.426	6452457	8692630	95% 79%
TLSQ	3V:	58.355	645.395	869.601	
		(18 cm)	(26 cm)	(11 cm)	
AF(TLSQ)	3V:	58.421	6452453	869.627	95% 78%
TLSQ	2V:	58.324	645.288	869.699	
		(32 cm)	(45 cm)	(19 cm)	
AF(TLSQ)	2V:	58.416	645.446	869.631	92% 85%
TRUTH		58.419	645.465	869.614	

MALD-MAAZ		dx(m)	dy(m)	dz2m)	Remarks
TLSQ	4V:	8.175	378.968	510.962	
		(15 cm)	(16 cm)	(7 cm)	
AF(TLSQ)	4V:	8.209	379.160	510.970	95% 83%
TLSQ	3V:	8.239	379.045	510.994	
		(14 cm)	(16 cm)	(8 cm)	
AF2TLSQ)	3V:	8.214	379.151	510.978	94% 86%
TLSQ	2V:	8.309	378.901	511.078	
		(16 cm)	(21 cm)	(10 cm)	
AF(TLSQ)	2V:	8.215	379.165	510.963	92% 88%
TRUTH		8.214	379.254	510.963	

Table 6.--Ambiguity function method results from the February 9, 1990, Malden Municipal Airport test

S274-MALD		dx(m)	dy(m)	dz(m)	Remarks
TLSQ	4V:	38350.787	-1640.139	-2400.060	
		(18 cm)	(22 cm)	(10 cm)	
AF(TLSQ)	4V:	38350.737	-1640.213	-2400.144	87%
TLSQ	3V:	38350.814	-1640.294	-2400.068	
		(19 cm)	(29 cm)	(12 cm)	
AF(TLSQ)	3V:	38350.738	-1640.236	-2400.133	85% 77%
TLSQ	2V:	38350.794	-1640.662	-2400.042	
		(18 cm)	(38 cm)	(15 cm)	
AF(TLSQ)	2V:	38350.734	-1640.238	-2400.138	88% 87%
TRUTH		38350.740	-1640.212	-2400.137	

S274-MA04		dx(m)	dy(m)	dz(m)	Remarks
TLSQ	4V:	38768.140	-1444.383	-2134.994	
		(24 cm)	(28 cm)	(12 cm)	
AF(TLSQ)	4V:	38767.890	-1443.626	-2135.378	91% 80%
TLSQ	3V:	38768.170	-1444.558	-2135.003	
		(26 cm)	(35 cm)	(14 cm)	
AF(TLSQ)	3V:	38767.894	-1443.635	-2135.371	90% 83%
TLSQ	2V:	38768.535	-1444.780	-2135.099	
		(39 cm)	(52 cm)	(22 cm)	
AF(TLSQ)	2V:	38767.899	-1443.656	-2135.367	91% 88%
TRUTH		38767.902	-1443.604	-2135.403	

S274-MA13		dx(m)	dy(m)	dz(m)	Remarks
TLSQ	4V:	38767.208	-1462.538	-2161.162	
		(24 cm)	(28 cm)	(12 cm)	
AF(TLSQ)	4V:	38767.150	-1462.737	-2160.980	91% 77%
TLSQ	3V:	38767.184	-1462.636	-2161.140	
		(27 cm)	(35 cm)	(14 cm)	
AF(TLSQ)	3V:	38767.150	-1462.736	-2160.982	91% 83%
TLSQ	2V:	38767.063	-1462.245	-2161.201	
		(36 cm)	(49 cm)	(22 cm)	
AF(TLSQ)	2V:	38767.151	-1462.761	-2160.969	91% 88%
TRUTH		38767.152	-1462.703	-2161.011	

S274-MA18		dx(m)	dy(m)	dz(m)	Remarks
TLSQ	4V:	38392.730	-1904.452	-2756.826	
		(18 cm)	(19 cm)	(8 cm)	
AF(TLSQ)	4V:	38392.696	-1904.768	-2757.002	90%
TLSQ	3V:	38392.764	-1904.519	-2756.947	
		(19 cm)	(22 cm)	(10 cm)	
AF(TLSQ)	3V:	38392.698	-1904.767	-2757.007	90% 81%
TLSQ	2V:	38392.603	-1904.252	-2756.919	
		(22 cm)	(26 cm)	(12 cm)	
AF(TLSQ)	2V:	38392.695	-1904.780	-2757.007	92% 91%
TRUTH		38392.690	-1904.768	-2757.006	

Table 6.--Ambiguity function method results from the February 9, 1990, Malden Municipal Airport test (continued)

S274-MA22		dx(m)	dy(m)	dz(m)	Remarks
TLSQ	4V:	37678.083	-2079.005	-2991.192	
		(22 cm)	(23 cm)	(10 cm)	
AF(TLSQ)	4V:	37678.125	-2078.780	-2991.234	92% 80%
TLSQ	3V:	37678.112	-2078.860	-2991.035	
		(21 cm)	(25 cm)	(12 cm)	
AF(TLSQ)	3V:	37678.120	-2078.768	-2991.235	91% 81%
TLSQ	2V:	37678.113	-2078.828	-2991.069	
		(24 cm)	(31 cm)	(15 cm)	
AF(TLSQ)	2V:	37678.121	-2078.800	-2991.225	92% 90%
TRUTH		37678.124	-2078.805	-2991.222	

S274-MA31		dx(m)	dy(m)	dz(m)	Remarks
TLSQ	4V:	37701.584	-813.518	-1286.309	
		(19 cm)	(23 cm)	(11 cm)	
AF(TLSQ)	4V:	37701.526	-813.778	-1286.175	88% 80%
TLSQ	3V:	37701.565	-813.570	-1286.305	
		(21 cm)	(32 cm)	(13 cm)	
AF2TLSQ)	3V:	37701.523	-813.804	-1286.171	87% 80%
TLSQ	2V:	37701.586	-814.131	-1286.210	
		(18 cm)	(38 cm)	(16 cm)	
AF(TLSQ)	2V:	37701.526	-813.791	-1286.182	90% 88%
TRUTH		37701.531	-813.778	-1286.169	

S274-MA36		dx2m)	dy(m)	dz2m)	Remarks
TLSQ	4V:	38409.424	-994.511	-1530.747	
		(17 cm)	(22 cm)	(10 cm)	
AF(TLSQ)	4V:	38409.158	-994.761	-1530.513	91% 81%
TLSQ	3V:	38409.450	-994.563	-1530.769	
		(20 cm)	(30 cm)	(12 cm)	
AF(TLSQ)	3V:	38409.158	-994.789	-1530.495	90% 83%
TLSQ	2V:	38409.558	-994.716	-1530.712	
		(39 cm)	(46 cm)	(20 cm)	
AF(TLSQ)	2V:	38409.155	-994.792	-1530.500	91% 88%
TRUTH		38409.159	-994.747	-1530.523	

S274-MAAZ		dx(m)	dy(m)	dz(m)	Remarks
TLSQ	4V:	38358.876	-1261.565	-1889.100	
		(18 cm)	(20 cm)	(9 cm)	
AF(TLSQ)	4V:	38358.937	-1261.064	-1889.183	87%
TLSQ	3V:	38358.892	-1261.531	-1889.061	
		(19 cm)	(22 cm)	(11 cm)	
AF(TLSQ)	3V:	38358.943	-1261.021	-1889.187	89% 81%
TLSQ	2V:	38358.997	-1261.760	-1889.079	
		(22 cm)	(29 cm)	(14 cm)	
AF2TLSQ)	2V:	38358.945	-1261.032	-1889.183	90% 88%
TRUTH		38358.954	-1261.058	-1889.174	