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REPLY

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This article is a reply to a comment by Anand et al. (2022), https://doi.org/10.1029/2022JF006669.

Key Points:

- The dimensionless number C_I can be understood as a dimensionless domain size
- For large enough domains $\left(C_I \gtrsim 10^3\right)$ the topographic structure is indeed insensitive to C_I
- The *NoHyd* results presented in our paper are for domains large enough that C_I is unimportant

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Reply to Comment by Anand et al. on "Groundwater Affects the Geomorphic and Hydrologic Properties of Coevolved Landscapes"

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Abstract Here we acknowledge an omission and offer a perspective on the importance of domain size for the simple landscape evolution model that appears in our original manuscript. For completeness, the domain size should have been included in the nondimensionalization. However, it can be shown to be of little consequence in the region of the parameter space that we examined. Furthermore, we show that our choice of nondimensionalization (with landscape evolution process rates rather than domain size) allows for a clearer view of controls on intrinsic features of the results, including valley spacing and channel head contributing areas, which were the focus of our original analysis.

Plain Language Summary Here we acknowledge an omission and offer our perspective on the importance of domain size for the simple landscape evolution model that appears in our original manuscript. Boundary conditions are an necessary component of the solutions to differential equations, however, important solution features are not necessarily sensitive to them. We show that the size of the modeled domain does not generally affect our results, and that interpreting the model with respect to length scales related to process rates rather than domain size is advantageous for understanding the features of model results that we are interested in, including the spacing of valleys and the size of areas upslope of channel heads.

1. Introduction

We appreciate the opportunity to further explore important details of our analysis where it connects with the work of Bonetti et al. (2020). Regarding the relationship between A and a, we indeed should have stated this more precisely. A better wording might have been: "The accumulated area A above a short contour width v_0 can be approximated by $A \approx av_0$ where a is the specific contributing area evaluated somewhere on the reference contour. This approximation becomes exact in the limit of small v_0."

The other point made by Bonetti et al. (2020) regarding our neglect of the importance of the domain size is worth spending some more time considering. Central to this conversation is the importance of the dimensionless parameter C_I. This number is defined in Bonetti et al. (2020) as a measure of channelization, but from our perspective C_I (or more precisely $C_I^{2/3}$) can be most straightforwardly understood as a dimensionless domain size.

The comment might appear to show that our paper omitted a major control on evolved landscape morphology. In fact this is not the case. We believe that the basic points made by Anand et al. (2022) are valid, but we disagree about their importance. We argue that

1. C_I appears to be of limited importance unless one is concerned about the impact of the domain boundary conditions in very small domains (i.e., those encompassing only a few ridges and valleys);
2. Using the domain size to non-dimensionalize model variables (as suggested by Bonetti et al. (2020) and Anand et al. (2022)) introduces dependencies on C_I that we believe obscure more physically meaningful relationships; finally,
3. If domain size is kept constant, increasing C_I amounts to shrinking the characteristic scale of the topographic features without changing their intrinsic properties.

Before expanding on these points, we should note that Anand et al. (2022) are entirely correct in pointing out that our dimensional analysis did not consider the domain size, and the effect of $\mathcal{C}_\mathcal{I}$ was not explored. Their comment motivated us to revisit that oversight. We can report that the *NoHyd* results presented in Litwin et al. (2022) were obtained from model simulations for which $\mathcal{C}_\mathcal{I} = 820$–1,800. As we demonstrate below, this situates them in the part of the parameter space where $\mathcal{C}_\mathcal{I}$ is of limited importance for the landscape properties of interest to us.

2. $\mathcal{C}_\mathcal{I}$ as a Dimensionless Domain Size

The point of more substantive disagreement relates to the use of the domain size in the non-dimensionalization of the governing equation and of metrics derived from the results. When expressing an equation or quantity in a dimensionless form one is at liberty to choose which variables will be used to set the fundamental scales of each dimension. That is, you are allowed to choose your preferred set of "units" when expressing dimensioned quantities. Bonetti et al. (2020) chose to set the domain width l as the fundamental length scale. Inspired by Theodoratos et al. (2018), we chose to use process rates to define the fundamental length scale, selecting $(D/K)^{2/3}$ for this purpose, which we refer to as ℓ_g for convenience. One consequence of this is that $\mathcal{C}_\mathcal{I}$ appears in Bonetti et al. (2020)'s dimensionless governing equation, but does not in that of Litwin et al. (2022). This difference is no difference at all in a strict sense, since they are in effect still the same equation. However the presence of $\mathcal{C}_\mathcal{I}$ in the governing equation of Anand et al. (2022) suggests it may be an important control on the solution. Here we demonstrate that this is the case only when the domain is "small" relative to the characteristic scale of the geomorphic features.

As Anand et al. (2022) point out, $\mathcal{C}_\mathcal{I}^{2/3} = l/\ell_g$, and so, from the perspective of our non-dimensionalization, $\mathcal{C}_\mathcal{I}^{2/3}$ is simply the domain width expressed in dimensionless form. The direct consequence of this choice is that the model results shown in Bonetti et al. (2020) and Anand et al. (2022) all have domains of width 1 when expressed in their preferred form of non-dimensionalization (since their length scales are expressed as multiples of l), but would be of different dimensionless widths when expressed in ours (since our length scales are expressed as multiples of ℓ_g).

While it may seem that this difference is trivial (something akin to the difference between expressing equivalent lengths in meters vs. miles) it actually has a significant effect on how one views the sensitivity of the model to certain variables. As we show in Litwin et al. (2022), ℓ_g scales with geomorphic elements like valley spacing and the contributing area at channel heads. Consequently, for Anand et al. (2022) increasing $\mathcal{C}_\mathcal{I}$ with constant l means shrinking the expected scale of the geomorphic features relative to the size of the domain. We would therefore expect to see a denser spacing of ridges and valleys when $\mathcal{C}_\mathcal{I}$ is larger, which is precisely what is shown in Anand et al. (2022) in their Figures 1a–1c.

From the perspective of our non-dimensionalization, however, increasing $\mathcal{C}_\mathcal{I}$ while keeping ℓ_g constant means increasing the size of the domain relative to the characteristic scale of geomorphic features. This is illustrated in Figure 1. Simulated landscapes on large and small domains (with different values of $\mathcal{C}_\mathcal{I}$) have large and small counts of geomorphic features, but the scale and properties of these geomorphic features do not appear to change appreciably.

3. Is (Dimensionless) Domain Size Important?

Detailed examination of two metrics derived from landscape form in Figure 1a suggest that there may be a subtle dependence on $\mathcal{C}_\mathcal{I}$ when $\mathcal{C}_\mathcal{I}$ is less than 10^3. Figure 2a shows how the characteristic drainage area per contour width at channel heads $\overline{a}_c$ and the mean drainage area per contour width on hillslopes $\overline{a}_h$ vary with $\mathcal{C}_\mathcal{I}$ when they are normalized by ℓ_g. Both quantities fluctuate a little for the two smallest $\mathcal{C}_\mathcal{I}$, but are fairly steady for larger values. This suggests that for sufficiently large domains the impact of the domain size on hillslope and headwater morphology is minimal.

Consider now what conclusion we might reach if we instead use the domain size l as the basis for normalizing $\overline{a}_c$ and $\overline{a}_h$ instead of ℓ_g. Figure 2b shows that $\overline{a}_c/l$ and $\overline{a}_h/l$ decrease precipitously with $\mathcal{C}_\mathcal{I}$. One might draw the conclusion that $\mathcal{C}_\mathcal{I}$ is an important control on the landscape morphology. However it is clear that this apparent control is largely the result of the chosen non-dimensionalization. For this reason we would push back on Anand et al. (2022)'s claim that we were mistaken in saying that "there is a single typology of the *NoHyd* model which

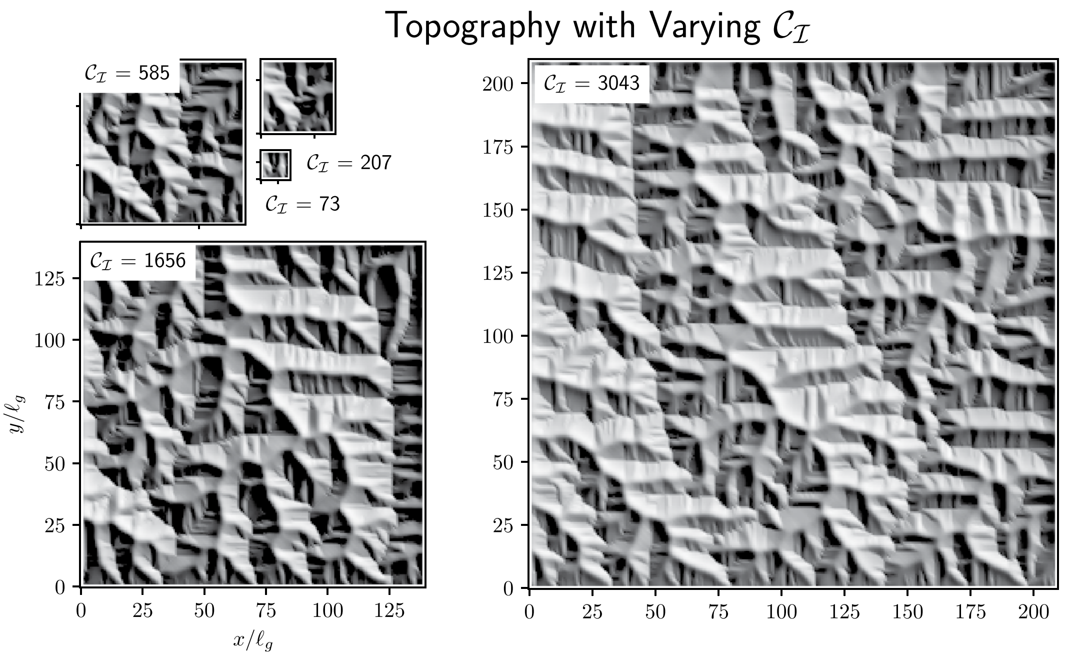

Figure 1. Five simulations from the NoHyd model of Litwin et al. (2022) using the same boundary conditions and a fixed value of ℓ_g, but varying $\mathcal{C}_\mathcal{I}$. Left, right, and top are zero flux boundaries and the bottom is held at constant elevation. Results show an increase in the number of features captured with increasing $\mathcal{C}_\mathcal{I}$, though not a fundamental change in character of the solution.

can be rescaled to obtain all results the model may produce". Our statement lacks only the qualification that this is true once the domain size is large enough to contain a representative sample of geomorphic features.

In fact, results presented in Bonetti et al. (2020) support our contention that the main effects of $\mathcal{C}_\mathcal{I}$ highlighted by Anand et al. (2022) can be attributed to scaling with the domain size (at least for large enough $\mathcal{C}_\mathcal{I}$). For example, Figure 2j from Bonetti et al. (2020) (reproduced here in Figure 3) suggests that the number of channels along each side of the domain increases with $\mathcal{C}_\mathcal{I}$. Now, if we assume the spacing between channels scales with ℓ_g, then we would expect the number of channels N_{channel} along a boundary of length l to vary as $N_{\text{channel}} \sim l/\ell_g$. Since $\ell_g = l/\mathcal{C}_\mathcal{I}^{2/3}$ we would therefore expect $N_{\text{channel}} \sim \mathcal{C}_\mathcal{I}^{2/3}$. Figure 3 shows the curve defined by $N_{\text{channel}} = \frac{1}{6}\mathcal{C}_\mathcal{I}^{2/3}$ superimposed on Bonetti et al. (2020)'s original figure (the $\frac{1}{6}$ coefficient was chosen by eye).

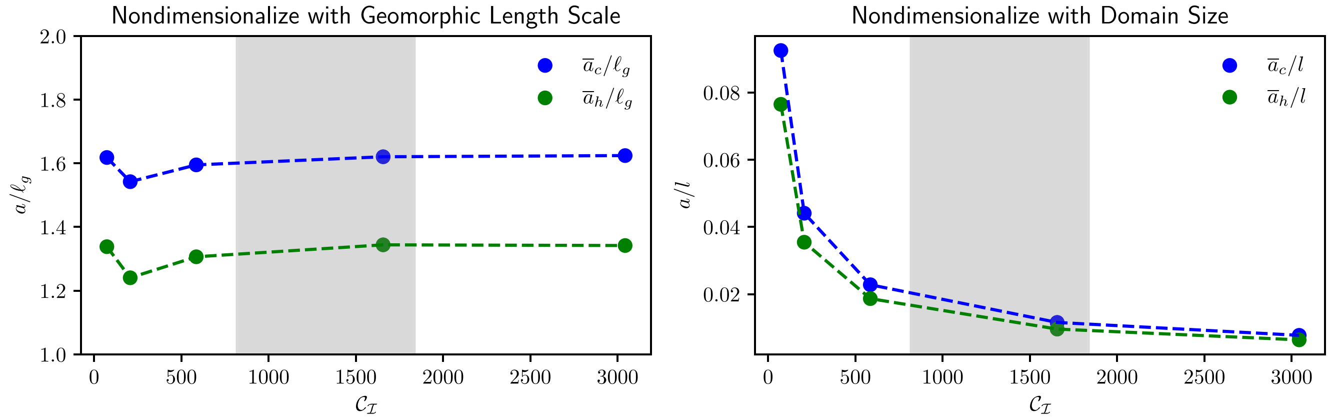

Figure 2. Left panel shows how the characteristic drainage area per contour width at channel heads $\overline{a}_c$ and the mean drainage area per contour width on hillslopes $\overline{a}_h$ vary with $\mathcal{C}_\mathcal{I}$ when they are normalized by the characteristic length scale ℓ_g, while the right shows how the quantities vary when they are normalized by the domain width l. Shaded regions show the range of $\mathcal{C}_\mathcal{I}$ values in Litwin et al. (2022).

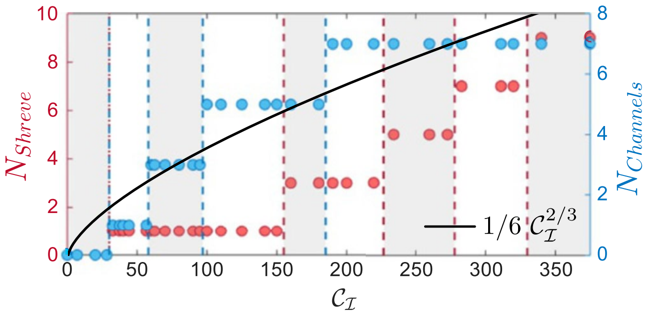

Figure 3. Reproduction of Figure 2J from Bonetti et al. (2020), with the curve $N_{\text{channel}} = \frac{1}{6}\mathcal{C}_{\mathcal{I}}^{2/3}$ superimposed. The good fit suggests that the characteristic scale ℓ_g explains controls the variation in the number of channels (since $\ell_g = l/\mathcal{C}_{\mathcal{I}}^{2/3}$ and so $N_{\text{channels}} \sim \mathcal{C}_{\mathcal{I}}^{2/3}$).

The good agreement of this prediction with Bonetti et al. (2020)'s results suggests that much of the apparent dependence of N_{channel} on $\mathcal{C}_{\mathcal{I}}$ is actually the result of the dependence of channel spacing on ℓ_g, and is not strongly influenced by the size of the domain (except in the trivial sense that you can fit more valleys along a longer boundary).

4. Asymptotic Behavior With Increasing $\mathcal{C}_{\mathcal{I}}$?

Anand et al. (2022) suggest that solutions exhibit self-similar behavior as $\mathcal{C}_{\mathcal{I}}$ becomes very large. Based upon this observation they draw further comparison between this landscape evolution model and other systems that have asymptotic behavior as a characteristic scale goes to infinity. It might be argued that it is only in this limiting regime that the effects of the domain size can truly be neglected. However there is a very important difference between solutions with large values of $\mathcal{C}_{\mathcal{I}}$ and those in the limit of $\mathcal{C}_{\mathcal{I}} \to \infty$ on finite domains. In the limit $\mathcal{C}_{\mathcal{I}} \to \infty$ it must be that $\ell_g \to 0$, which would seem to imply a solution topography infinitely dissected by channels, and drainage divides with infinite curvature. It is not clear to us whether such solutions to the governing equations can even be said to exist. Certainly the features of interest to us (particularly the morphology near the hillslope-channel transition) shrink and disappear in this limit. Therefore we question this comparison, and affirm that we are interested in the regime in which $\mathcal{C}_{\mathcal{I}}$ is large but finite.

5. Conclusion

So is $\mathcal{C}_{\mathcal{I}}$ important or not? Anand et al. (2022) are certainly correct to point out that in general we cannot discuss solutions to a set of governing equations without considering the boundary conditions, and in Litwin et al. (2022) we were remiss in not including $\mathcal{C}_{\mathcal{I}}$ in the set of governing dimensionless variables. However, it seems to us that $\mathcal{C}_{\mathcal{I}}$ has little effect on the landscape morphology unless the domain size is particularly small.

Data Availability Statement

No original data is presented here.

Acknowledgments
We would like to thank the authors of the comment for the opportunity to delve into these issues. We believe this comment/reply will be of use to the wider landscape evolution community. We would also like to thank the editors and anonymous reviewers for facilitating this exchange.

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