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Airborne Kinematic GPS Positioning for Photogrammetry The Determination of the Camera Exposure Station

Lewis A. Lapine, PhD

AIRBORNE KINEMATIC GPS POSITIONING FOR PHOTOGRAMMETRY THE DETERMINATION OF THE CAMERA EXPOSURE STATION

Lewis A. Lapine, Ph.D.
Chief, National Geodetic Survey
Silver Spring, MD 20910

ABSTRACT

Kinematic GPS positioning of airborne platforms coupled with aerial photogrammetry has become an operational reality within NOAA. Real time GPS positioning is utilized for navigation, while post processed carrier phase differencing is employed for exposure station locations. Since it is not presently practical for the exposure station event to control the GPS collection, the antenna phase center position at the time of exposure must be computed by interpolating the aircraft trajectory after the appropriate timing biases have been added to the observed exposure time. This paper will discuss present and future methods for determining the position of the exposure station event.

INTRODUCTION

Kinematic GPS positioning of airborne platforms coupled with aerial photogrammetry has become an operational reality within NOAA (Lapine 1990). Real time GPS positioning is utilized for navigation, while post processed carrier phase differencing is employed for exposure station locations. Nearly every NOAA aerial mapping project employs some combination of airborne GPS and ground control for the aerotriangulation process. In every case, the requirement for ground control has been reduced. Operational efficiency has increased as the need for expensive and labor intensive ground control has diminished. This technology is rapidly maturing such that it will become the most commercially viable approach for aerial mapping. Commercialization will be heavily dependent on instrumentation, capitalization costs and processing efforts, all of which are now in the favor of the technology. Instrumentation is available as a result of other GPS and photogrammetric applications. Capitalization costs have decreased as a result of the increased market share of GPS in the surveying and mapping fields. Post processing software has been improved by the manufacturers, academia and federal sector (Mader 1992) to the point where user friendly software is efficient and readily available. Since it is not presently possible to sample the GPS signal at the instant of exposure, some form of interpolation of the GPS positional information is required. The ultimate goal is to select a sampling rate and interpolation model which yield exposure station accuracy commensurate with the final mapping product while at the same time minimizing post processing effort.

RELATIONSHIP OF THE GPS ANTENNA PHASE CENTER TO EXPOSURE STATION

The GPS receiver collects the carrier phase and pseudo-range information pertaining to the trajectory of the aircraft throughout the photo mission. The raw data is post processed into trajectory information consisting of GPS time-tagged geocentric positions for the GPS antenna phase center. This position is correlated to the camera exposure station through time and orientation of the spatial offsets between two origins, earth center and photo center. Exposures rarely coincide with the times at which the antenna positions are recorded. Therefore, the antenna position at the time of exposure must be computed by interpolation. The camera exposure station position can then be determined using an orthogonal three-dimensional transformation incorporating the spatial offsets between the entrance node of the lens system and GPS antenna phase center and *a priori* estimates for the elements of exterior orientation. These *a priori* estimates may be refined during aerotriangulation and more accurate exposure stations computed after each iteration of the aerotriangulation solution (Lucas 1989). The error of the individual component observations can be propagated during this process to yield a variance-covariance matrix for the camera exposure station.

INTERPOLATION ACCURACY AS A FUNCTION OF GPS SAMPLING RATE

Since it is not presently practical to control the GPS collection by the exposure event, the antenna phase center position at the time of exposure must be computed by interpolating the aircraft trajectory after the appropriate timing biases have been removed from the observed exposure time. The exposure station position accuracy is a function of the kinematic positioning accuracy (1 or 2 cm relative accuracy for most geodetic quality GPS receivers), sampling rate and interpolation model. GPS receiver manufactures have increased sampling rates in part to enhance the accuracy of interpolation. The inclination would be to sample the GPS receiver at as high a frequency as possible to minimize the time difference between GPS epochs and exposure times. This practice also increases the processing burden. A more practical solution would be to select a sampling rate and interpolation model which yield exposure station accuracy commensurate with the final mapping product while at the same time minimizing post processing effort. Operational experience gained by NOAA has demonstrated that a 1 hertz GPS sample is adequate for photo scales as large as 1:10,000 when using the interpolation model discussed in this paper. Higher sampling rates or more different interpolation models may be necessary for low altitude (larger scale) photography to accommodate aircraft trajectories influenced by short period turbulence. To this end, the following analysis of the interpolation process is presented.

The GPS signals are generally sampled on a nearly uniform time interval affected only by a very small (one usec/sec) (King and Durboraw 1988) drift in the receiver clock. The signals, subsequent to being post-processed generally have receiver clock drift removed and are time shifted so that all antenna positions are equally spaced in time. The interpolation becomes simplified since the data points are evenly spaced in time. As mentioned above, the length of the uniform interval (time between successive epochs of data) is governed by the GPS receiver hardware and may be the limiting factor for the accurate interpolation of the exposure station position from the aircraft trajectory. The interpolation precision is well correlated to the limit of positional resolution which can be expected from kinematic GPS (Lapine, 1991). Longer time intervals degrade the interpolation. The following example used GPS data collected at a 1-second rate and then thinned to 2-second and 5-second rates by removing the appropriate sample points. The thinned data sets were interpolated for the missing midpoint samples. The GPS data used for this test came from two data sets collected aboard the NOAA Citation II jet and one data set from the Texas Highway Department King Air Turbo-prop. Comparison of the interpolated positions with the observed positions indicate a standard deviation about the mean difference as great as 41 cm. The number of samples whose position difference was greater than 20 centimeters was recorded for each data set and reported in Table 1.

The Citation and King Air results are still commensurate with kinematic positioning expectations when thinned to 2 seconds. The sample standard deviations for the data thinned to 5 seconds for the King Air suggest a serious degradation in position. It is interesting to note that the magnitude of the standard deviations associated with the NOAA Citation are different from the Texas King Air. The population variances were tested and failed the equality test based on the F statistic (Hamilton 1964). The differences between the 2- and 5-second populations are most likely caused by the inability to model the aircraft trajectories over a time span greater than 2 seconds. The difference between the NOAA and Texas populations may result from the same inability to model the trajectories or may be caused by a larger signal-to-noise ratio in the receivers used for the tests (different manufactures). The trend indicates that sampling intervals greater than 1 second should be avoided if the full accuracy of kinematic GPS positioning is required. However, the good comparison between the data thinned to 2 seconds and observed positions does validate the ability of the interpolation process when sampling at 1 hertz.

An interesting alternative to the above timing situation would be to activate the camera shutter with the 1-second timing pulse generated by the GPS receiver. In this procedure one may be able to entirely eliminate the time difference between GPS fix information and exposure. Interpolation of the GPS navigation file would be eliminated except possibly for a constant time offset between the timing signal and the camera response to the signal. A short test conducted aboard the NOAA aircraft indicated that the time delay would be on the order of 0.1 second for the particular Wild RC-10 camera used in the test. The camera service manual states that the maximum time delay between rotating shutter blade opening and capping shutter delay is on the order of 0.070 second. The time delay depends on the shutter speed, position of the rotating shutter blades at the time the pulse was initiated, and the vacuum status. One major problem with this procedure would be the inability to accurately control overlap.

Table 1
Evaluation of 2- and 5-Second Sample Rates for GPS Phase Information During a Photo Mission

Aircraft Type	Sample Size	Sample Rate	% of sample > 20 cm	Mean/Std Dev of Sample (m) X,Y, and Z
King Air	2086	2 Sec	15	0.000/0.055 -0.001/0.122 0.001/0.114
King Air	1039	5 sec	58	-0.002/0.270 0.004/0.412 -0.007/0.409
Citation	1715	2 sec	4	0.000/0.031 0.000/0.063 -0.001/0.055
Citation	1503	2 sec	5	0.000/0.032 0.001/0.084 0.000/0.063
Citation	873	5 sec	33	0.000/0.292 -0.002/0.261 0.002/0.281
Citation	748	5 sec	34	0.001/0.286 0.000/0.274 -0.002/0.261

INTERPOLATION ALGORITHM

The objective of the interpolation is to compute, using a limited portion of the navigation file, a position for the antenna phase center at the time of exposure which is within a half epoch of the central time of the limited data set i.e., less than 0.5 second for a 1-Hertz sample.

The interpolation algorithm uses a second-order polynomial whose three coefficients are solved for by least squares method. The polynomial represents a curve which fits the aircraft trajectory over a 5-epoch period. A different curve is fit to each coordinate axis. The coefficients of this polynomial can be used to compute the aircraft position offset, velocity and acceleration in each coordinate direction.

The following models are used for interpolating the antenna phase center coordinates:

$$X_1 = a_x + b_x t_1 + c_x t_1^2 \quad (1)$$

$$X_2 = a_x + b_x t_2 + c_x t_2^2 \quad (2)$$

$$\vdots$$

$$X_i = a_x + b_x t_i + c_x t_i^2 \quad (3)$$

where $t_i = time_i - time_3$; when $i = 1,2,...5$;

$time_3$ is the central time and

$t_1 \ldots t_5$ are the five consecutive time tags of GPS antenna phase center positional data

Similar equations could be written for the other two models:

$$Y = a_y + b_y t + c_y t^2 \quad (4)$$
$$Z = a_z + b_z t + c_z t^2 \quad (5)$$

The unknown parameters for each model can be related to distance, velocity and acceleration by differentiating the above equations as follows:

$$\text{distance from origin} = a, \quad (6)$$
$$\text{velocity} = dX/dt = b + 2ct, \text{ and} \quad (7)$$
$$\text{acceleration} = dX^2/d^2t = 2c \text{ at } t_3\ . \quad (8)$$

The observation equations are:

$$v_x = a_x + b_x t + c_x t^2 - X = 0 \quad (9)$$
$$v_y = a_y + b_y t + c_y t^2 - Y = 0 \quad (10)$$
$$v_z = a_z + b_z t + c_z t^2 - Z = 0 \quad (11)$$

The coefficient matrix elements for all three models are the partial derivatives of the model with respect to the unknowns. The coefficient matrix is the same for all three models, as follows:

$$A = \begin{matrix} -1 & -(t_1-t_3) & -(t_1-t_3)^2 \\ -1 & -(t_2-t_3) & -(t_2-t_3)^2 \\ -1 & -(t_3-t_3) & -(t_3-t_3)^2 \\ -1 & -(t_4-t_3) & -(t_4-t_3)^2 \\ -1 & -(t_5-t_3) & -(t_5-t_3)^2 \end{matrix} \quad (12)$$

From this point forward in the discussion, time differences will be denoted as simply "t" to simplify the expressions.

The observation vectors composed of the observed coordinate values for each GPS epoch are:

$$\begin{matrix} \text{for X;} & \text{for Y;} & \text{for Z;} \\ -x_1 & -y_1 & -z_1 \\ -x_2 & -y_2 & -z_2 \\ -x_3 & -y_3 & -z_3 \\ -x_4 & -y_4 & -z_4 \\ -x_5 & -y_5 & -z_5 \end{matrix} \quad (13)$$

A least squares solution minimizing the function

$$PHI = V'\ P\ \ V \quad (14)$$

is used to solve for the unknown parameters (Uotila 1986). The P matrix is the scaled inverse of the variance-covariance matrix (Sigma L_b) for the observed quantities. The scaling is represented by $Sigma_0^2$, the variance of unit weight, which in this case has the value of 1. The solution for the unknown parameters begins with the normal equations noted as follows:

$$V_x = AK_x + X \quad (15)$$
$$V_y = AK_y + Y \quad (16)$$
$$V_z = AK_z + Z \quad (17)$$

where;

$$K_x = -(A'PA)^{-1}(A'PX) \quad (18)$$
$$K_y = -(A'PA)^{-1}(A'PY) \quad (19)$$
$$K_z = -(A'PA)^{-1}(A'PZ) \quad (20)$$

and the ' symbol represents the transpose matrix. For the moment consider the weight matrix P to be the Identity matrix I,

$$A'IA = \begin{matrix} 5 & 5t & 5t^2 \\ 5t & 5t^2 & 5t^3 \\ 5t^2 & 5t^3 & 5t^4 \end{matrix} \quad (21)$$

$$A'IX = \begin{matrix} x_1 + x_2 + x_3 + x_4 + x_5 \\ x_1t + x_2t + x_3t + x_4t + x_5t \\ x_1t^2 + x_2t^2 + x_3t^2 + x_4t^2 + x_5t^2 \end{matrix} \quad (22)$$

similar equations can be written for A'PY and A'PZ.

VARIANCE-COVARIANCE WEIGHT MATRIX FOR GPS OBSERVATIONS

The variance-covariance matrix, P, is used to weight the contribution of each observation considering the span of time between the central observation point and the camera exposure station. The assumption is made that the five observations are independent and, therefore, the co-variances between observations are zero. Several different choices for the variances were considered. The first choice was equal weights. This choice was not considered appropriate considering possible non-uniformity of the trajectory. A second choice was to compute the variances by giving more weight to the center value of the interpolation, a central weight scheme. The justification for this decision is based on the increasing difficulty to accurately model a trajectory as the distance between the central data point and its neighbors increases. This fact was confirmed in Table 1. The weight for the central value is, therefore, greatest with decreasing weights for the other data points as the time span from the data point to the central value increases. Several central weight systems were tried including the use of the Geometric Dilution of Precision (Spilker 1980), an estimate of GPS relative accuracy, obtained from the satellite geometry at the time of exposure. The final scheme weights the data points as a binomial expansion technique. The central variance was chosen to be 1.0 cm^2. The following formula for the variances of the weight matrix ($Sigma_{Lb}$) follows:

$$\begin{matrix} 2^2*0.01\ m^2 & 0 & 0 & 0 & 0 \\ 0 & 2^1*0.01\ m^2 & 0 & 0 & 0 \\ 0 & 0 & 2^0*0.01\ m^2 & 0 & 0 \\ 0 & 0 & 0 & 2^1*0.01\ m^2 & 0 \end{matrix} \quad (23)$$

$$0 \qquad 0 \qquad 0 \qquad 0 \qquad 2^2 * 0.01\ m^2$$

or

$$\begin{matrix} 4\ cm^2 & 0 & 0 & 0 & 0 \\ 0 & 2\ cm^2 & 0 & 0 & 0 \\ 0 & 0 & 1\ cm^2 & 0 & 0 \\ 0 & 0 & 0 & 2\ cm^2 & 0 \\ 0 & 0 & 0 & 0 & 4\ cm^2 \end{matrix} \qquad (24)$$

The time separation from the central observation is inversely proportional to the weight. No correlation was considered between observations. The P (weight) matrix is a diagonal matrix with the following elements when a 1 Hertz sample rate is used:

$$P_{11} = 1.0 / Sigma^2_1 = 1/(2^2 * 0.01\ m^2) \qquad (25)$$
$$P_{22} = 1.0 / Sigma^2_2 = 1/(2^1 * 0.01\ m^2) \qquad (26)$$
$$P_{33} = 1.0 / Sigma^2_3 = 1/(2^0 * 0.01\ m^2) \qquad (27)$$
$$P_{44} = 1.0 / Sigma^2_4 = 1/(2^1 * 0.01\ m^2) \qquad (28)$$
$$P_{55} = 1.0 / Sigma^2_5 = 1/(2^2 * 0.01\ m^2) \qquad (29)$$

The *a priori* variance of unit weight is 1.0. The validity of this weight system can be proven using a Chi Square test of the *a posteriori* variance of unit weight against the *a priori* variance of unit weight. The test indicated equal variances in 43 of 45 selected interpolation tests. The validity must be tempered by the knowledge that the degrees of freedom for the test is only 2 (number of observations - number of unknowns, 5 - 3 = 2).

The interpolated values for the antenna phase center at the time of the exposure are expressed as follows:

$$X_{exp} = K_x(1) + K_x(2)*(time_{exp} - time_3) + K_x(3)*(time_{exp} - time_3)^2 \qquad (30)$$

$$Y_{exp} = K_y(1) + K_y(2)*(time_{exp} - time_3) + K_y(3)*(time_{exp} - time_3)^2 \qquad (31)$$

$$Z_{exp} = K_z(1) + K_z(2)*(time_{exp} - time_3) + K_z(3)*(time_{exp} - time_3)^2 \qquad (32)$$

As a final note, the model used in this paper only approximates the actual trajectory of the aircraft. The mathematical modeling process acts as a filter, smoothing the aircraft trajectory using a second order polynomial. It would be interesting to examine the extent of smoothing which actually takes place. The model is validated by the results illustrated in Table 1. This same model was independently developed by James Lucas (Lucas 1989) and is currently used within the National Geodetic Survey.

ANTENNA PHASE CENTER TRANSFORMATION TO EXPOSURE STATION

The interpolated geocentric position of the antenna phase center at the exposure time can now be transformed through the previously determined system of spatial offsets to the position of the exposure station.

Errors introduced by the observed quantities and interpolation model can be propagated through the above system of interpolation equations if the variances for the unknown parameters and time can be estimated. The variance-covariance for the parameters is obtained from the inverse of the normal equation matrix scaled by the *a posteriori* variance of unit weight. The estimate for timing error is obtained empirically as follows:

1. Knowledge of the precision for the GPS time tags which are receiver dependent.

2. Knowledge of the uncertainty in timing delay between the camera and GPS which is GPS receiver and camera system dependent.

3. Knowledge about the uncertainty of shutter timing offset measurement at the midpoint of shutter opening which is camera dependent (Taylor 1964).

The first two error sources are insignificant considering the velocity of the aircraft and the stability of the receiver and timing clocks. The largest contributing factor is, therefore, the shutter timing offset. This error is currently estimated to be 0.0005 sec (Lucas 1989). This estimate will decrease in magnitude as camera manufacturers refine internal timing techniques associated with forward motion compensation requirements (Coker 1989). The error estimate for the GPS position is given as 2 cm in planimetry and 4 cm in elevation (Mader 1992) . These values have been accepted as true. Assuming that there is no correlation between the position, timing and unknown parameters, a combined variance-covariance matrix can be derived. The derivation can be found in the author's dissertation (Lapine 1991) and will not be developed at this time.

Several assumptions are made about the camera and aircraft attitudes and the resultant contribution to the error in the exposure station position:

a. The exposure station will be defined as the entrance node to the lens system of the camera.

b. Vertical photography is assumed *a priori* and refined during the aerotriangulation. The initial swing angle is estimated by the azimuth between two consecutive antenna positions and can be refined as improved knowledge is obtained from aerotriangulation.

c. The pitch and drift angles are measured between the aircraft and the camera. The pitch and drift angles were zero when the spatial offsets were measured.

d. The variance-covariance matrix for the exposure station can be propagated through the various models using *a priori* estimates of the precision for the observations.

The antenna and camera coordinate systems are right-handed as are the three rotations about the camera axes. Three rotations and three translations are used in the transformation:

a. The antenna coordinates at the time of exposure are converted from WGS84 rectilinear to ellipsoidal latitude, longitude and elevation. The latitude (lat) and longitude (lon) are then used to rotate the WGS84 coordinates into an East, North, local vertical system.

b. The covariance matrix from the interpolation model error propagation is similarly transformed to obtain a covariance matrix in the local vertical system.

c. The spatial offset components between the camera and antenna are then rotated into the local coordinate system using the standard gimbal form. The rotation elements of Kappa and drift are combined into a single rotation. Rotational elements of Phi and pitch are also combined. No determination of roll was made during the flight, so Omega is treated independently.

It may be argued that the camera rotates about a set of mechanical gimbals whose rotational center may or may not be coincident with the exposure station. If there is any eccentricity between the gimbal rotational center and the exposure station, then the angles of exterior orientation can not be algebraically summed with the observed angles of pitch and swing. The eccentricity discussion will begin with the particular situation encountered in NOAA's application where pitch and swing angles are small (less than 2^{o}).

By design of the RC-10 camera mount, the optical axis, about which swing is measured also coincides with the vertical axis of the mount, therefore swing and kappa angles can be combined without an eccentricity correction. Such is not the case for the pitch angle. The exposure station has a vertical separation of approximately 27 mm from the gimbal origin (the gimbal center is probably at the camera's center of gravity). The aircraft pitch <u>relative to the camera mount</u> was measured during actual flight conditions and is referenced to the gimbals. A maximum and generally constant pitch angle of 1.5^{0} was measured and occurred at the lower limit of the Citation's operating speed. The consequences of not taking the eccentricity into account are as follows:

Maximum pitch error = 0.027m * sin(1.5^{0}) = 0.0007 m

A displacement error of the exposure station of less than 1 mm in the direction of flight is introduced if the eccentricity correction is neglected. In the most general case, a camera mount consists of a double concentric two orthogonal axis gimbal. The mechanical design of the gimbals and the magnitude of the rotation angles may dictate a more thorough investigation of the effect of the eccentricity.

The gimbal form rotation matrix (Merchant 1988) is formed by the product of the three independent rotations of pitch roll and swing. The order of multiplication is important only in the general case mentioned above. The order of rotation for the specific case is not critical, considering, the design of the camera mount, magnitude of the angles (generally less than 2^{o}), and the fact that the rotations are being treated in a purely analytical application.

The combined rotation matrix would transform survey coordinates to photo coordinates. Since the spatial offsets are measured in the photo system, the transpose of the combined matrix is required. The product of the transposed matrix times the spatial offset vector yields values for the spatial offsets in the survey system. The spatial offsets may now be algebraically added to the interpolated position of the antenna position at the time of exposure. The result is a set of GPS coordinates for the exposure station and the associated estimates for the position precision.

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