

1           **Dense Point Cloud Quality Factor (DPQF) as A Proxy for Accuracy**  
2                           **Assessment of Image-based 3D Reconstruction**

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5 Parrish<sup>1</sup>  
6

7   **Abstract**

8           Small unmanned aircraft systems (UAS) carrying consumer-grade nonmetric cameras are  
9 increasingly being used to generate high-resolution 3D geospatial data. The photogrammetry and  
10 computer vision techniques Structure from Motion (SfM) and Multi-View-Stereopsis (MVS) can  
11 recover structure from a set of overlapping, un-oriented, and uncalibrated images. Those  
12 techniques have been widely adopted for UAS-based photogrammetry. It is possible to generate  
13 accurate reconstructions of sparse points using mathematically robust bundle adjustment (BA)  
14 procedures together with accurate surveying control data. However, MVS, which recovers the  
15 dense geometry by matching and expanding between sparse points through enforcing epipolar  
16 geometry constraints based on camera exterior orientation (EO) parameters from the initial BA is

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17 prone to additional error. It has been shown that factors such as image overlap, number of ground  
18 control points (GCPs), scene complexities, camera standoff distances, lighting condition, lack of  
19 texture in the scene, shadowing, etc. may introduce random noise or smoothing effects that can  
20 locally degrade the accuracy of the dense point cloud. This paper introduces dense point cloud  
21 quality factors (DPQF) as proxy indicators for assessing the accuracy of SfM-MVS dense point  
22 clouds. Simulated and empirical experiments are used to assess the accuracy of image-based 3D  
23 reconstructed models under various data collection and site condition scenarios. The spatial  
24 correlation between the DPQFs and the reconstruction error is investigated and interpreted for  
25 multiple experiments. The results of this study show that the DPQF can be a helpful additional  
26 field of information for 3D point clouds. The advantage of the DPQFs is that the factors can be  
27 defined solely based on the inputs and results of the SfM-MVS processing without prior  
28 knowledge about the error. Visualization of the factors may provide a proxy indicator for accuracy,  
29 while the error estimation for dense point clouds is more challenging than error propagation  
30 computations in BA. Further comprehensive experiments and studies are required to draw firm  
31 conclusions for a better application on DPQFs for accuracy assessment. However, the use of  
32 DPQFs that have physical definitions enables the development of more tangible intuitions  
33 regarding factors, which influence 3D reconstruction accuracy.

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## 35 **1. Introduction**

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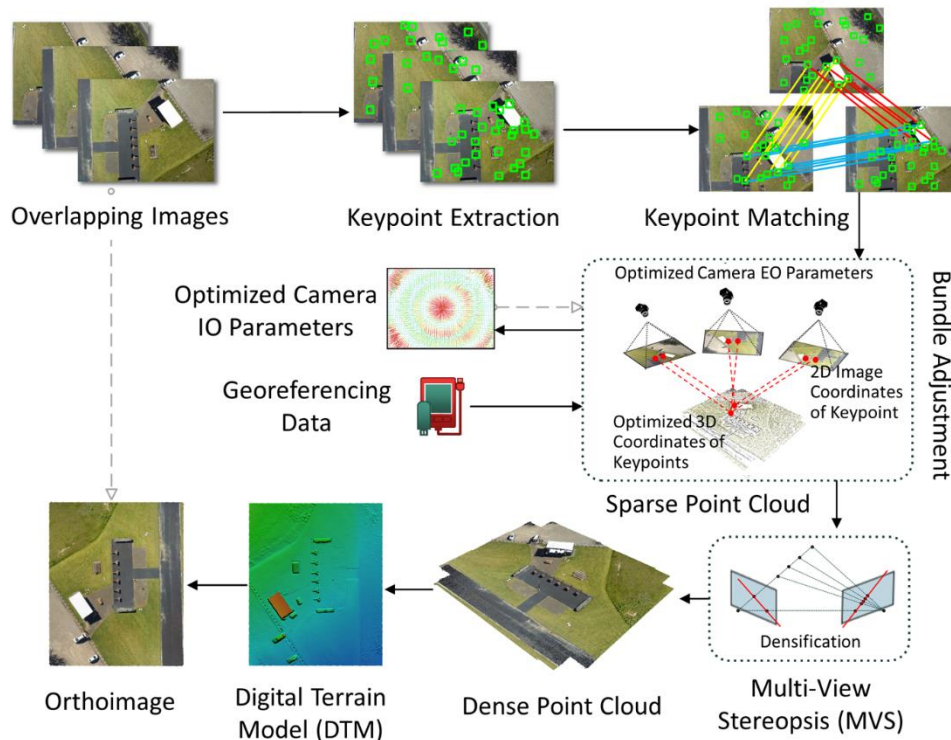
37 High-resolution three dimensional (3D) data is essential for detailed spatial interpretation  
38 and analysis in many geomatics applications (Abellan et al. 2016; Che and Olsen 2017; Javadnejad  
39 2013; Javadnejad et al. 2017b; Mahmoudabadi et al. 2016; McCaffrey et al. 2005; O'Banion 2017;

40 Olsen et al. 2015; Omasa et al. 2006; Sohn and Dowman 2007; Wood et al. 2017). Increasingly,  
41 modern photogrammetry utilizing consumer-grade nonmetric cameras mostly mounted on the  
42 small unmanned aircraft systems (UAS), is being used to generate the high-resolution 3D data.  
43 Because of the widespread availability, low maintenance cost, ease of operation, low altitude  
44 maneuvering capability, and flexible and frequent data collection, UAS-based photogrammetry is  
45 changing the surveying and mapping research and industry (Colomina and Molina 2014; Pajares  
46 2015). UAS-based photogrammetry has been tested in different environments, and its advantages  
47 and disadvantages have been explored (Faraji et al. 2016; Gao et al. 2017; Griffin 2014; Javadnejad  
48 2018; O'Banion et al. 2018; Shi et al. 2016; Wood et al. 2017). There are a number of open source  
49 programs such as *VisualSfM* (Wu 2011) and *Bundler* (Snavely et al. 2006), as well as commercial  
50 packages such as *PhotoScan*, now named *Metashape* (Agisoft 2018) and *Pix4DMapper* (Pix4D  
51 2018) that are commonly used for processing the imagery collected from UAS platform to generate  
52 high-resolution mapping products such as 3D point clouds, mesh surfaces, digital terrain models  
53 (DTMs), and orthoimages.

54         The photogrammetry from the nonmetric digital cameras includes using structure from  
55 motion (SfM) and multi-view stereopsis (MVS) techniques that recover structure from a set of  
56 overlapping un-oriented and uncalibrated images, and generate 3D dense point clouds (Eltner et  
57 al. 2016; Furukawa and Ponce 2010; Seitz et al. 2006). The general steps for SfM photogrammetry  
58 are shown in Figure 1. The processing starts with automatic extraction of key features from the  
59 raw imagery (Harris and Stephens 1988; Lowe 1999; Snavely et al. 2008; Szeliski 2010; Tomasi  
60 and Kanade 1992). Then the extracted features are described in multidimensional descriptors, e.g.,  
61 SIFT (Lowe 1999, 2004). The procedure is followed by matching the features (Snavely et al. 2008;  
62 Szeliski 2010) and outlier rejection (Crandall et al. 2013; Fischler and Bolles 1981). Later, the

63 bundle adjustment (BA) (Heung-Yeung Shum et al. 1999; Triggs et al. 1999) simultaneously  
 64 solves for the intrinsic orientation (IO) and extrinsic orientation (EO) parameters of the cameras  
 65 and to generate a sparse point cloud (Crandall et al. 2013; Snavely et al. 2008; Szeliski 2010). The  
 66 reconstructed model is transformed to a real-world coordinate system using either known ground  
 67 control points (GCPs) coordinates or via UAS-based GNSS camera pose estimations (Javadnejad  
 68 and Gillins 2016). In addition to georeferencing the SfM point clouds, use of GCPs can also  
 69 improve the accuracy of the model during the second step of the BA by performing a nonlinear  
 70 optimization on the estimation of sparse points coordinates, and the camera IO and EO parameters.  
 71 In this step, the camera parameters and the constructed geometry are reoptimized by minimizing  
 72 the reprojection error of feature matches in the form of a sparse point cloud after importing the  
 73 GCPs (Heung-Yeung Shum et al. 1999).

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**Figure 1: Steps of SfM-MVS processing (Javadnejad 2018)**

77           The sparse point cloud is ordinarily complemented with a densification step through MVS  
78 processing, which generates a depth map for pixels of the image based on photo-consistency in an  
79 oriented block obtained from bundle adjustment (Furukawa and Ponce 2010; Remondino et al.  
80 2014; Snavely et al. 2008). Together with accurate input data, it is possible to produce an accurate  
81 reconstruction of sparse points using the mathematically robust BA procedure. On the other hand,  
82 MVS, which recovers the densified point cloud by matching and expanding between sparse points  
83 may yield results with different and inconsistent accuracies. MVS works by enforcing epipolar  
84 geometry constraints obtained from BA solution, and later filtering the outliers (Furukawa and  
85 Ponce 2010). MVS algorithms are continuously being improved regarding accuracy and  
86 completeness, identifying and recognizing the sources reconstruction error, and estimating the  
87 accuracy and reliability are still ongoing research topics (Furukawa and Ponce 2010; Yao et al.  
88 2014; Zhu et al. 2015). A good number of research studies have investigated the applicability of  
89 the technique for a variety of mapping instances; also more researches are being carried out on the  
90 accuracy of SfM-MVS products (e.g., Javadnejad (2018) and references therein).

91           Uncertainty in image-based 3D reconstruction is function of various input data factors and  
92 processing parameters such as the quality of the input image, accuracy, number and distribution of  
93 ground control points (GCPs), the accuracy of IO and EO parameters, certainty in feature detection  
94 and matching in overlapping images or even choice of reconstruction technique (Hofsetz et al.  
95 2004; Seitz et al. 2006). The accuracy of SfM-MVS can be defined as the closeness of the empirical  
96 measurements to a reference ground truth, such as a comparison between SfM-MVS and a lidar  
97 point cloud or total station and/or GNSS measurements at checkpoints (CPs). Unlike GCPs, the  
98 CPs are not used for georeferencing or sparse point cloud optimizing; however, the known values  
99 of the CPs are compared to the coordinate measurements of the same points in the 3D geometry

100 and is reported for accuracy assessment purposes (Javadnejad and Gillins 2016). Besides empirical  
101 comparisons, Slocum and Parrish (2017) proposed a workflow, which uses a simulated graphics  
102 environment for generating virtual UAS surveys to overcome challenges of empirical surveys such  
103 as the time and cost of data collecting campaigns. This approach incorporates the systematization  
104 of a reliable ground truth reference, and also leverages the isolation of environmental factors that  
105 contribute to error budget in SfM-MVS models while it enables investigation of the impact of  
106 individual parameters (Slocum and Parrish 2017).

107         In mapping and surveying applications, it has been found that the accuracy of SfM-MVS  
108 sparse and dense point clouds is influenced by many factors such as image overlap, number of  
109 images, lens distortion model, number of ground control points (GCPs), geometry of GCP  
110 distribution, geometry of camera distribution, accuracy of GCP or camera positions, image  
111 resolution, blurriness of imagery, noise of imagery, lighting condition, shadow effect, scene  
112 complexities, standoff distances, image-matching performance, image texture, presence of dense  
113 vegetation, moving objects in the scene, and user errors in selecting the image coordinates of GCPs  
114 (Agüera-Vega et al. 2017; Carbonneau and Dietrich 2017; Clapuyt et al. 2016; Dandois et al. 2015;  
115 Eltner et al. 2016; Fonstad et al. 2013; Harwin et al. 2015; Harwin and Lucieer 2012; James et al.  
116 2017a; b; Javadnejad et al. 2017a; Smith and Vericat 2015; Tonkin and Midgley 2016; Westoby  
117 et al. 2012).

118         This paper introduces seven different factors, named dense point cloud quality factors  
119 (DPQFs), as proxy indicators for the accuracy of image-based reconstruction. The factors include  
120 (1) distribution of the keypoint features resulting from the BA, (2) distribution of GCPs, (3) scene  
121 geometry, (4) camera stand-off distances, (5) number of images, (6) brightness index, and (7)  
122 darkness index. Although other studies presented the impact of some of these factors, this study

123 investigates the correlation between these factors to outline the importance of each factor for  
124 different scenarios holistically. This research study investigates the correlation of the DPQF with  
125 reconstruction error for sets of simulated and empirical data with multiple scenarios. The work is  
126 also meant to help surveyors with planning the best data collection strategies for UAS/SfM  
127 photogrammetry projects to minimize error by providing definition of indices that can be  
128 optimized during data collection for an improved accuracy. Also, it is within the scope of this  
129 paper to supplement the point cloud with additional quality indices that can be helpful for  
130 visualization and identifying points that are prone to inaccurate reconstruction, and weighting for  
131 mesh generation. An advantage of the DPQFs is that the factors can be defined solely based on the  
132 inputs and results of SfM-MVS processing. They can provide a proxy indicator for dense point  
133 clouds quality, while the estimation of error for dense point cloud is more challenging than the  
134 error propagation estimation in BA.

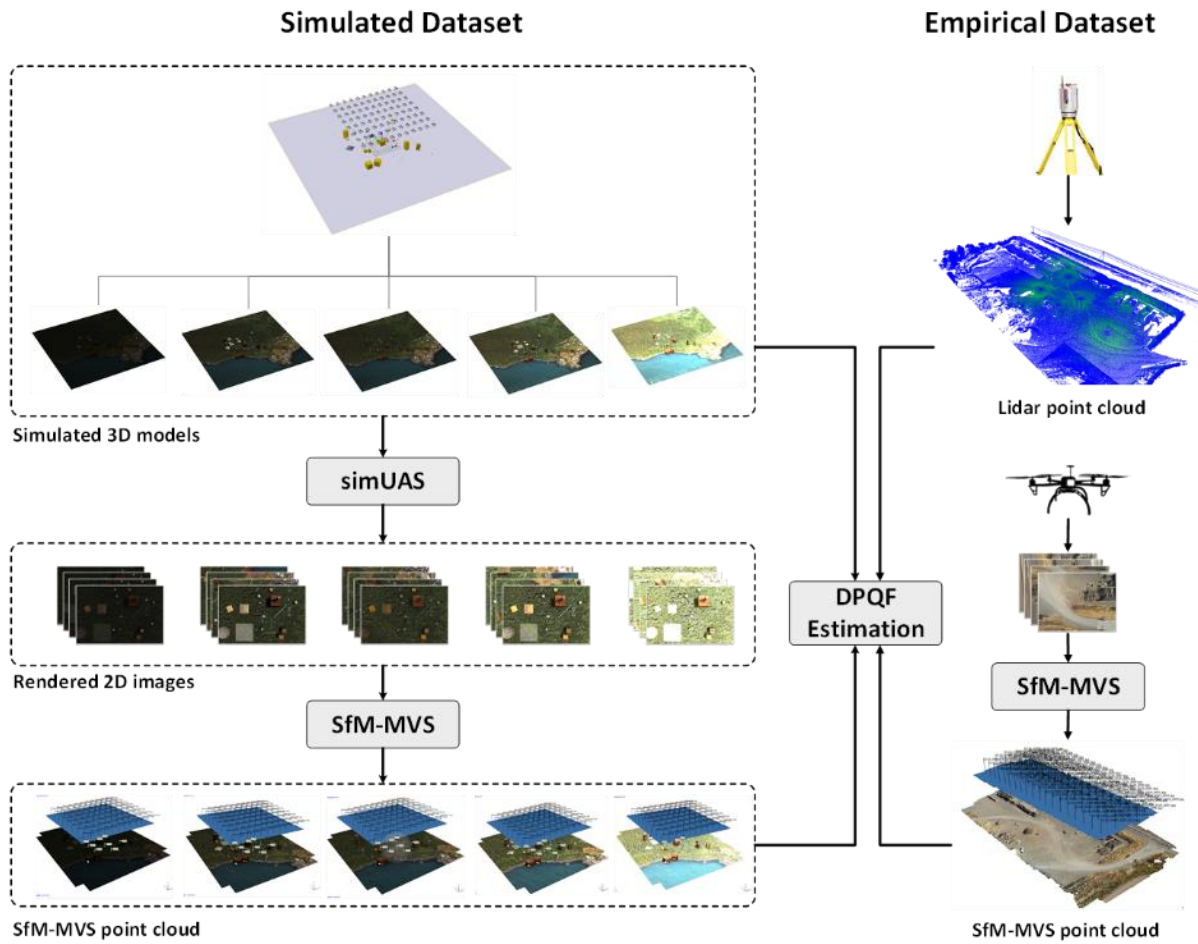
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## 136 **2. Materials and methods**

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138 This paper presents a simulated experiment based on the simUAS approach (Slocum and  
139 Parrish 2017), where the different scenarios for a scene are created, as well as an empirical case  
140 study exploring the use of both lidar and UAS for surveying a construction site. Digital  
141 photographs acquired from both datasets are post-processed using SfM techniques to produce a  
142 high-resolution point cloud. The error is defined as distance to the control data. The ground truth  
143 for the simulated data is perfectly known and can be exported as a mesh model from the simulation  
144 environment, and the ground truth for the empirical data is the lidar. The resultant dense point  
145 clouds and error indices are used to estimate the DPQF value, which is later investigated for

146 statistical correlation between all the factors and the error index to identify impact and importance  
 147 of each factor. Figure 2 shows the approach used for estimating DPQF for both the simulated and  
 148 empirical datasets.  
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150  
 151 **Figure 2: Workflow for data preparation of simulated and empirical datasets, and DPQF**  
 152 **estimation**

153  
 154 **2.1. Datasets**

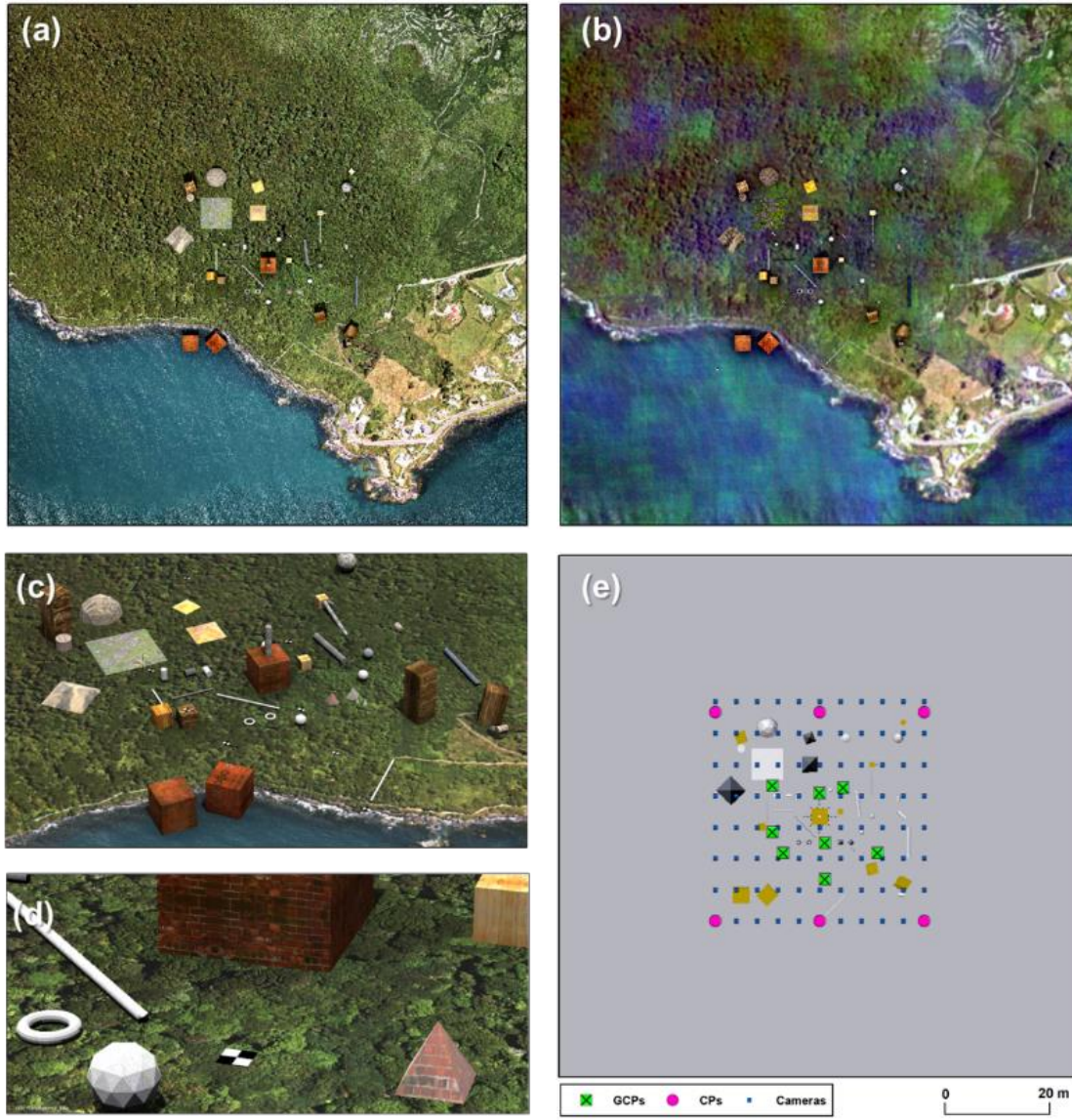
155 - *Computer graphics simulation*

156 The simulations were performed using the Blender® software (Blender 2017), a free open-  
 157 source 3D computer graphics software, following the simUAS approach (Slocum and Parrish



158 2017). The simulated scene is a site that consists of a flat plane surface, 11 boxes, 7 horizontal  
159 cylinders representing pipes, 6 vertical cylinders, 3 spheres, 5 pyramids, 2 cones and 3 icosphere  
160 of varying sizes (Figure 3). The objects were textured using a variety of texture images acquired  
161 from freely available online datasets such as brick, concrete, metal, wood, gravel, asphalt, and soil  
162 (Figure 3c). Two different scenes named “A” and “B” were generated by applying different texture  
163 to the ground plane. In scene A, the plane was textured using a 9620 by 9620 pixels image that  
164 was generated by tiling 6 high-resolution aerial images with 10cm pixel resolution (0.1m GSD)  
165 acquired over the Invercargill City in New Zealand that includes forest as well as industrial, and  
166 water textures (Figure 3a) (LINZ 2016). For the scene B, a 50 by 50-pixel Gaussian Smoothing  
167 filter with Sigma 0.5 was applied to the aerial image. Then it was overlaid with a transparent  
168 random noise image (Figure 3b). In both scenes, 14 black and white iron cross targets, similar to  
169 the one shown in Figure 3, were placed in the scene.

170



171  
 172 **Figure 3: (a) overview of simulated scene A, (b) overview of simulated scene B, (c) different objects**  
 173 **placed in the simulated scene with surface textures, (d) close-up view of textured objects as well as a**  
 174 **black and white GCP on the scene, and (e) the location of objects, GCPs, CPs, and the position of**  
 175 **cameras**






176

177 Both scenes were illuminated with 5 different sun and ambient light intensities to create  
 178 variation in lighting and shadows. Table 1 shows the different lighting conditions in the simulation.  
 179 Scenarios 1 and 5 are the darkest and brightest allowing the existence of underexposed and  
 180 overexposed object at the scene. Underexposure and overexposure, occur when camera sensor

181 does not record enough details in the darkest and rightest part of the images, respectively. Scenario  
 182 2 is brighter than the first scenario with stronger sunlight and lower ambient light creating strong  
 183 shadow areas. The higher ambient light intensity in scenario 3 minimizes the intensity of shadow,  
 184 and scenario 4 has high sun and ambient light intensity creating appropriate contrast on textures  
 185 (proper exposure). The impact of the different light on the accuracy of reconstruction is later  
 186 studied.

187

188 **Table 1: Sunlight and ambient light settings used for scenarios 1 to 5 of A and B, and the resultant simulation**  
 189 **for scenarios A1 to A5. The intensity values are in the range between 0.0 to 10.0 with low values the sky has**  
 190 **no sun, and with high values the sky only has sun.**

Scenario	1	2	3	4	5
Sun Light	0.2	0.9	0.1	0.9	2.5
Ambient Light	0.2	0.1	0.9	0.9	2.0
Output					

191

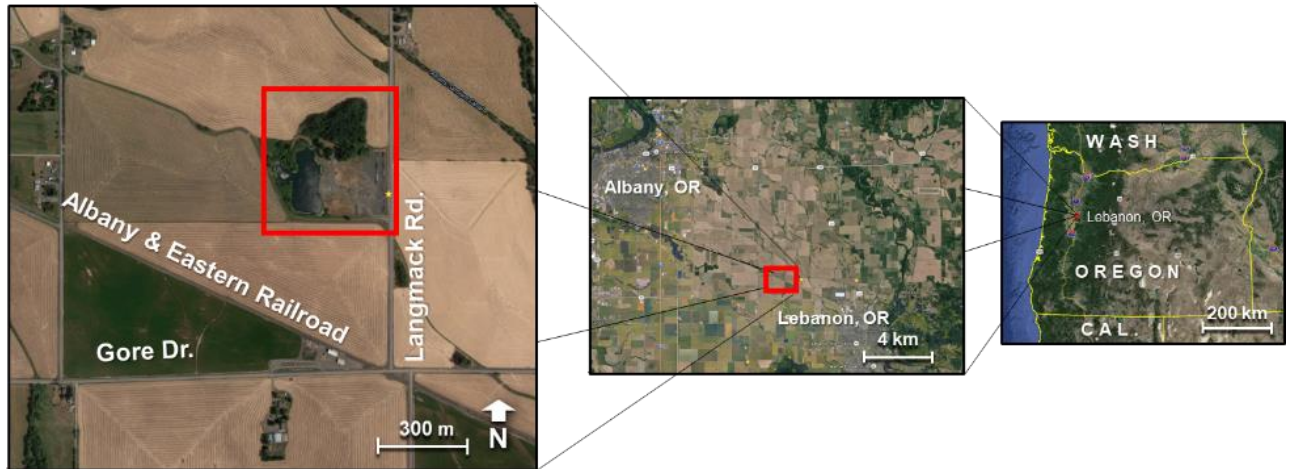
192 To simulate the UAS-based data collection, 88 cameras were positioned across the area  
 193 including the simulated objects (Figure 3). The camera was set to mimic the Sony A5000 with 16  
 194 mm focal length and output image sizes of 5456 by 3632 pixels. The simulated cameras were  
 195 placed at 20 m AGL altitude and the flight lines with respect to the height and width of the images  
 196 to keep 80 % of side and forward overlap. For the scenes A and B with 5 different lighting  
 197 scenarios, images were rendered at the given cameras locations using the Blender® Internal  
 198 Render Engine and the resulted images were post-processed using simUAS in MATLAB®  
 199 (MathWorks 2017) to add non-linear Brown’s lens distortion to the images. This approach has  
 200 been described in details by Slocum and Parrish (2017) for interested readers. Overall, it took about

201 4 hours to perform both the image rendering in Blender and the post-processing in MATLAB for  
202 each experiment. In total 10 experiments were simulated with A and B scenes, five different  
203 scenarios each.

204 - *Empirical case study*

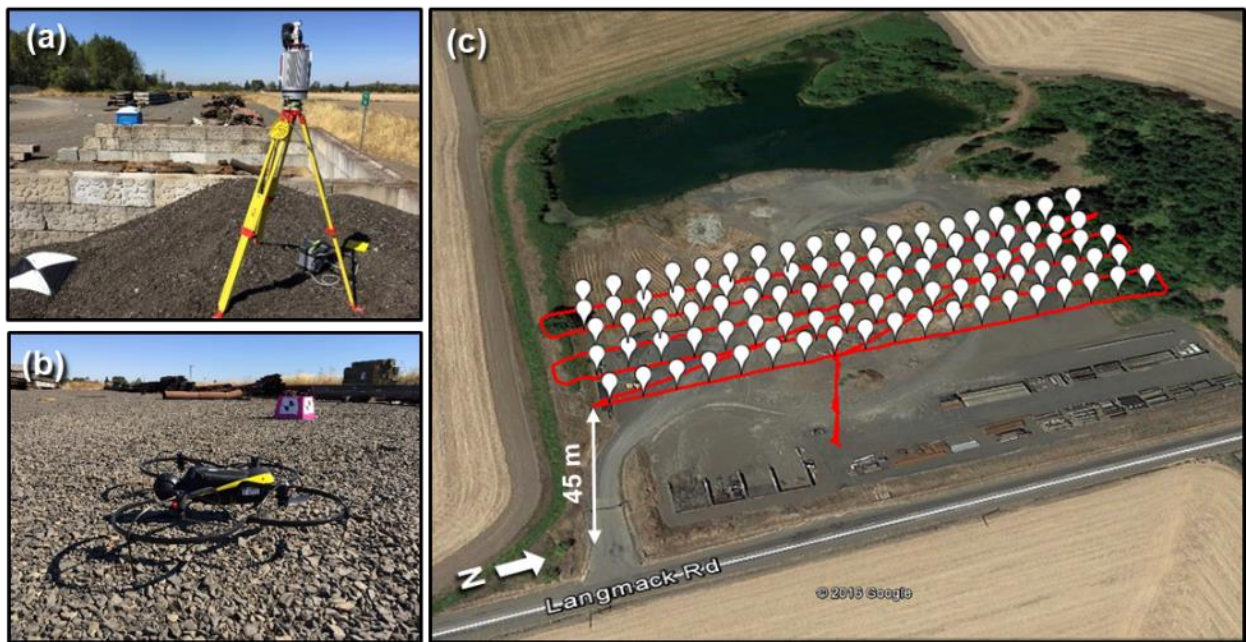
205 The study area for the empirical assessment is a storage yard utilized by Linn County in  
206 the State of Oregon to store gravel, asphalt grindings, debris, spare concrete bridge parts, and  
207 piping material. It is located approximately 5 km northwest of the City of Lebanon (Figure 4). The  
208 data for this site consists of terrestrial lidar data as well as the aerial imagery collected from a UAS.  
209 Prior to data collection, 18 aerial targets (Figure 5a), and 12 boxes with patterned targets (Figure  
210 5b) were distributed throughout the scene. The aerial targets, approximately 1 square meter in size,  
211 were nailed to the ground to ensure stability throughout the flight (Figure 5a). In addition, 38 color  
212 cross markings (such as the markings in Figure 10.f) were placed at two ends and the center of  
213 pipe sections to check the accuracy of SfM results. Figure 6 shows the layout of the control  
214 network. Repetitive, 3-minute observations were acquired on all aerial targets using a Leica GS14  
215 survey-grade GNSS receiver obtaining real-time kinematic corrections from the Oregon Real-  
216 Time GNSS Network (ORGN). The ORGN is a statewide real-time network managed by the  
217 Oregon Department of Transportation (ODOT 2017). Also, all the aerial targets, markings and box  
218 targets were surveyed using radial traversing methods with a Leica TS15 Total Station. It took 3.5  
219 hours in total to complete the ground surveying campaign to establish coordinates for the targets  
220 and markings. The approach for a similar ground control network survey has been described in  
221 more detail by Javadnejad and Gillins (2016) for interested readers.

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**Figure 4: Langmack storage facility (image data ©2017 Google Earth Landsat/Copernicus Data SIO, NOAA, U.S. Navy, NGA, GEBCO and ©2017 Google Maps)**



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**Figure 5: (a) The Riegl VZ400 and a nearby GCP, (b) the Albris and a target box, and (c) the flight pattern (image data Landsat/Copernicus ©2016 Google Earth)**



231  
 232 **Figure 6: Location of the established aerial targets, cross markings, box targets, and scan positions**

233

234 The box targets (Figure 5b) were placed strategically to be seen by the majority of the scans

235 throughout the site by distributing the targets across the study area in a 15 to 20 m grid pattern

236 (Figure 6). The lidar data of the site was collected via 6 separate scans using a Riegl VZ-400

237 scanner (Figure 5a). Each scan was 7 minutes in duration. In total, it took 1.5 hours for the entire

238 scan data collection, including setup and data capture times. The SenseFly Albris (formerly known

239 as the eXom) (Figure 5b) was used for collecting the aerial images. The flights were completed on

240 August 17, 2016, under the Oregon State University (OSU) Certificate of Authorization (COA)

241 for public UAS operations. The Albris has an integrated GNSS receiver and inertial navigation

242 system that allows the craft to fly to predefined mission waypoints. The visible sensor on this

243 platform is a Nokia camera with 10.01 by 7.51 mm sized sensor and 7.9 mm focal length,

244 producing 7152 by 5368 pixel images. The flights were performed such that nadir photographs

245 were systematically collected with 80% side and 80% forward overlap at 45 m above the ground

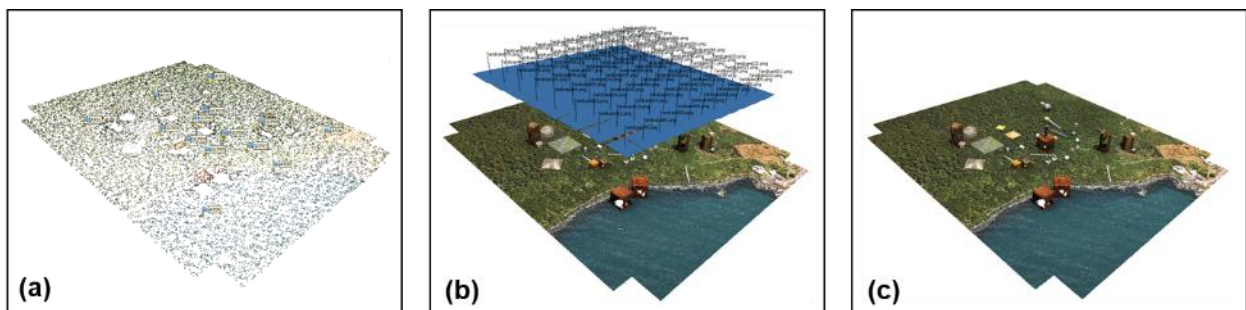
246 level, resulting in a ground sampling distance (GSD) of 8 mm per pixel. The camera was triggered  
247 with automatic focus and exposure modes by the onboard autopilot system when the aircraft  
248 reached the preplanned flight mission waypoints (Figure 5c). In total, 95 images were collected  
249 during an 11-minute flight.

250

## 251 2.2. Point cloud generation

### 252 - *Simulated data*

253 The resulted imagery from simulations and the actual UAS operation at the Langmack site  
254 were processed using the commercial software *PhotoScan Pro 1.4* (Agisoft 2018). *PhotoScan* can  
255 generate point clouds, textured polygonal models, georeferenced orthoimages and DEMs. In  
256 *PhotoScan*, the georeferencing is performed by identifying the GCPs in the photographs, inserting  
257 the coordinates and their standard deviation values, and/or by providing the location and  
258 orientation of the cameras. The exact position of the cameras, the camera calibration models, 3D  
259 coordinate and pixel coordinates of the GCPs and CPs of simulated scenes were imported to  
260 *PhotoScan*. The statistics of SfM processing are shown in Table 2. The final dense point clouds  
261 were processed using “medium quality” and “aggressive-depth filtering” settings. The total  
262 processing time was 15 minutes for each scenario.



263

264 **Figure 7: (a) Resultant sparse point cloud, (b) dense point clouds with the cameras locations and**  
265 **orientations, and (c) dense point clouds reconstructed for simulation A3**

266  
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**Table 2: General information of SfM-MVS processing and the resulted point cloud for scenarios of simulation scenes A and B**

Simulation	Attribute	1	2	3	4	5
A	No of points (Sparse)	$25.8 \times 10^3$	$25.5 \times 10^3$	$25.1 \times 10^3$	$24.8 \times 10^3$	$25.0 \times 10^3$
	RMSEGCP (mm)	0.05	0.03	0.02	0.05	0.06
	RMSECP (mm)	2.0	1.9	1.9	1.8	1.8
	No of points (Dense)	$9.5 \times 10^6$	$9.7 \times 10^6$	$9.4 \times 10^6$	$9.4 \times 10^6$	$9.2 \times 10^6$
B	No of points (Sparse)	$28.9 \times 10^3$	$26.7 \times 10^3$	$26.0 \times 10^3$	$25.6 \times 10^3$	$25.4 \times 10^3$
	RMSEGCP (mm)	0.09	0.03	0.04	0.06	0.03
	RMSECP (mm)	2.0	2.0	1.8	1.9	1.9
	No of points (Dense)	$9.9 \times 10^6$	$10.2 \times 10^6$	$9.6 \times 10^6$	$10.2 \times 10^6$	$9.2 \times 10^6$

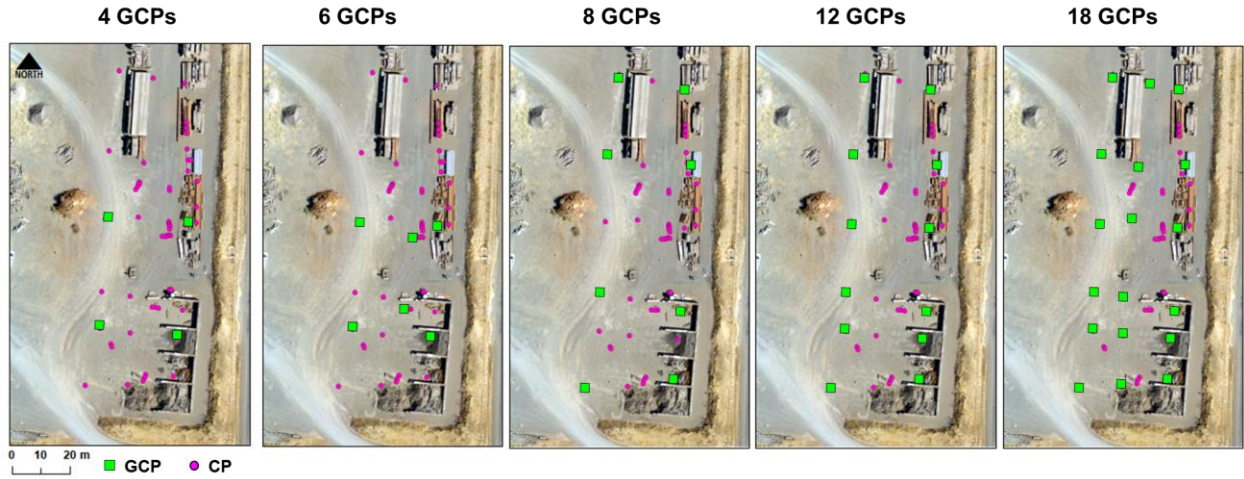
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269 - *Empirical data*

270 For the Langmack dataset, the coordinates of the targets and markings were determined by  
 271 performing a least square adjustment (LSA) on the network of Total Station and GNSS  
 272 observations within Star\*NET 8.0 using the approach described by Javadnejad and Gillins (2016).  
 273 The adjustment resulted in an estimated horizontal and vertical root mean square error (RMSE) of  
 274 1.97 mm and 2.8 mm, respectively, for the ground control network coordinates. The resultant  
 275 coordinates were used to post-process the 95 aerial images collected during the UAS flights in  
 276 *PhotoScan*. The data was reprocessed for five scenarios with selecting 4, 6, 8, 12 and 18 GCPs  
 277 from aerial targets and using the rest of aerial targets and/or markings and CPs. The total error on  
 278 GCPs and CPs is reported in Table 3. The final dense point cloud was processed using “high  
 279 quality” and “mild-depth filtering” settings. The total processing time for each scenario, was 1  
 280 hour to generate a dense point cloud.



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**Figure 8: The overview of GCPs selection for scenarios of empirical dataset**

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**Table 3: General information of SfM-MVS processing and the resulted point cloud for empirical data**

No GCPs	4	6	8	12	18
No of CPs	52	50	48	44	38
RMSEGCP (m)	0.004	0.009	0.011	0.012	0.013
RMSECP (m)	0.058	0.028	0.024	0.016	0.016
No of points (sparse)	$5.5 \times 10^3$	$5.5 \times 10^3$	$5.5 \times 10^3$	$5.5 \times 10^3$	$5.5 \times 10^3$
No of points (dense)	$84.6 \times 10^6$	$84.6 \times 10^6$	$84.6 \times 10^6$	$84.4 \times 10^6$	$84.4 \times 10^6$

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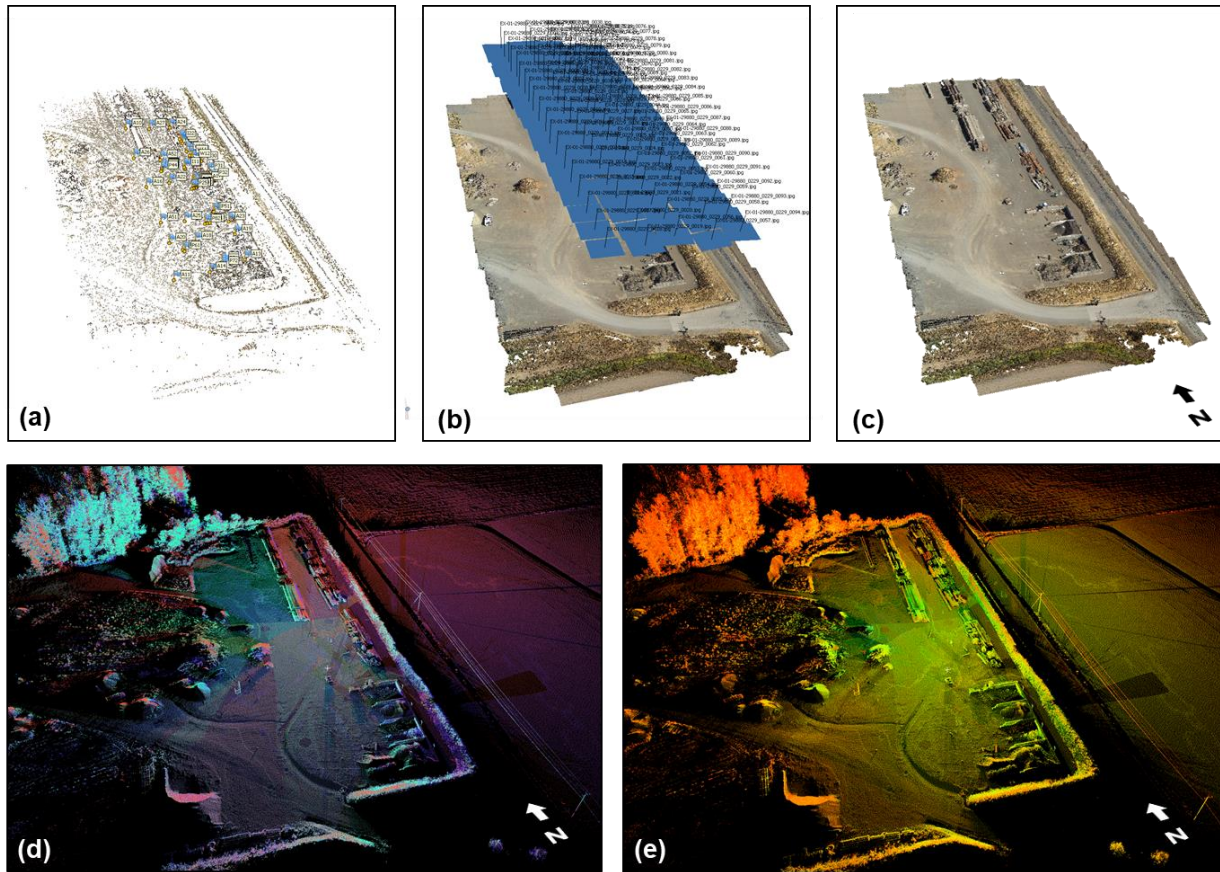
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The resultant coordinates from LSA were also used to register the lidar dataset to the same coordinates as the SfM data. All 6 scans were co-registered by constraining the clouds to the extracted targets and cloud-to-cloud registration in *Cyclone 9.1* (Leica Geosystems 2016). The registration resulted in a 3D RMSE of 1.4 cm and 1.5 cm for target only, and target and cloud-to-cloud (C2C) constraints, respectively. The composite point cloud of all scans contained 101 million points within the study area. The lidar point clouds were resampled at minimum space between points to be 2 cm to increase the processing speed; then the lidar point cloud was cropped to the area with existing SfM-MVS dense point clouds to avoid miscalculation of Lc2c for points

295 out of the SfM model. The resampled and cropped point clouds included about 14.2 million points  
 296 (Table 4).



297  
 298 **Figure 9: (a) Resultant sparse point clouds, (b) dense point clouds and cameras location and**  
 299 **orientation, and (c) dense point clouds reconstructed for simulation A3, (d) lidar point clouds**  
 300 **colored differently for each scanner, and (e) intensity-colored lidar point clouds**

301  
 302

**Table 4. General information of lidar for empirical dataset**

No of scans	No of targets	RMSE (targets)	RMSE (target and C2C)	No of points
6	21	1.4 cm	1.5 cm	$101.1 \times 10^6$

303  
 304

### 305 2.3. Dense point cloud quality factors (DPQF)

306 The overview of this approach is schematically shown in Figure 2. The following section  
 307 describes the DPQF and how they are calculated. The estimated quality indices are attributed as

308 additional fields together with the traditional  $X$ ,  $Y$ ,  $Z$ , and color values of the ground truth point  
309 cloud. For empirical data, the lidar was used as the ground truth model, and for simulation data,  
310 the simulated 3D geometry was exported as wavefront obj file, and then the obj model was  
311 resampled with 10 million points in the open source software CloudCompare (Girardeau-Montaut  
312 2017).

313 - *3D error ( $e$ )*

314 The error index is defined as a comparison between the ground truth model and the SfM-  
315 MVS dense point cloud. In this study, the error is calculated as the absolute 3D cloud-to-cloud  
316 distance (LC2C) of each point in ground truth data from the closest point in the SfM point cloud.  
317 Lague et al. (2013) recommended to for interested readers for more information about approaches  
318 on 3D cloud-to-cloud comparison.

319 - *Distance to keypoint features ( $d_{kp}$ )*

320 The LC2C for each ground truth point was calculated from the closest keypoint feature in  
321 CloudCompare.

322 - *Distance to GCP ( $d_{gcp}$ )*

323 The locations of the GCPs used for georeferencing the SfM models were imported to  
324 CloudCompare, and the LC2C between the ground truth point cloud and GCPs were calculated.

325

326 - *Scene geometry ( $\alpha$ )*

327 The scene geometry and the data collection pattern can be described by the angle of  
328 incidence between the surface normal at the point  $P$  and the line from the camera center  $C$  to the  
329 point using Eq. 1:

$$\alpha_{\text{inc}} = \tan^{-1} \left( \frac{\|\vec{N}_P \times \vec{u}_{CP}\|}{\vec{N}_P \cdot \vec{u}_{CP}} \right) \quad (1)$$

330 where  $\vec{N}_P$  is the surface normal vector at the point and  $\vec{u}_{PC}$  is the unit vector between the  
 331 point and the camera that is calculated using their position vector in Eq. 2:

$$\vec{u}_{CP} = \frac{1}{\|\vec{r}_P - \vec{r}_C\|} (\vec{r}_P - \vec{r}_C) \quad (2)$$

332 The point normal vectors were estimated by fitting quadratic surfaces to the neighboring  
 333 points (OuYang and Feng 2005) in +z orientation, and the camera positions were obtained from  
 334 the estimated camera EO from the SfM solution. The average of all angle of incidence between  
 335 the point and all the cameras that point is attributed to the point. If all the cameras are pointing to  
 336 a particular direction for example, in vertical photogrammetry, it is possible to use the  $-z$  unit  
 337 vector  $(0,0,-1)$  and just calculate the angle of the surface ( $\alpha_{\text{sur}}$ ) instead of the angle of incidence.

338 - *Camera stand-off distances ( $d_c$ )*

339 The stand-off distances between the point  $P$  in ground truth point cloud and the camera are  
 340 calculated using their 3D coordinates (Eq. 3), then the average distance between the point and all  
 341 cameras which are attributed to the point.

$$d_c = \|\vec{r}_C - \vec{r}_P\| \quad (3)$$

342 - *Number of images ( $n_{\text{img}}$ )*

343 Using the IO and EO parameters obtained from SfM solution as well as the real-world  
 344 coordinate transformation parameters, it is possible to back calculate the 2D pixel coordinates of  
 345 every 3D point in the scene. For each point, if the 2D coordinates are located in the image, the  
 346 counter is augmented by 1, and the final counter value of the number of images is attributed to the  
 347 point in the cloud. This approach lacks the ability to deal with occlusions; however, on approach  
 348 to deal with the problem is by considering the angle of incidences. The point is considered to be

349 seen in the image if the angle of incidence is less than 90 degrees. This criterion filter the points  
 350 that are on the surfaces that do not face the camera, while their pixel coordinates are within the  
 351 image plane. Future developments can incorporate more advanced ray tracking analysis in the  
 352 number of image calculations. For this study, there was minimal occlusion in the scene. Hence the  
 353 effect of those occlusions on the DPQFs is determined to be negligible.

354 - *Brightness ( $I_b$ ) and darkness ( $I_d$ ) indices*

355 RGB color values were used to define the brightness and darkness factors. The RGB values  
 356 were converted to a normalized grayscale scale intensity values between -1 and 1 (Eq. 4)  
 357 (Anderson et al. 1996), then the brightness and darkness indices were calculated for each point  
 358 using Eq. 5.

$$\tilde{I}_{\text{gray}} = \frac{2 \times (0.2126 \times R + 0.7152 \times G + 0.0722 \times B)}{255} - 1 \quad (4)$$

$$\begin{cases} I_b = \tilde{I}_{\text{gray}}, I_d = 0 & \tilde{I}_{\text{gray}} > 0 \\ I_d = \tilde{I}_{\text{gray}}, I_b = 0 & \tilde{I}_{\text{gray}} < 0 \end{cases} \quad (5)$$

359

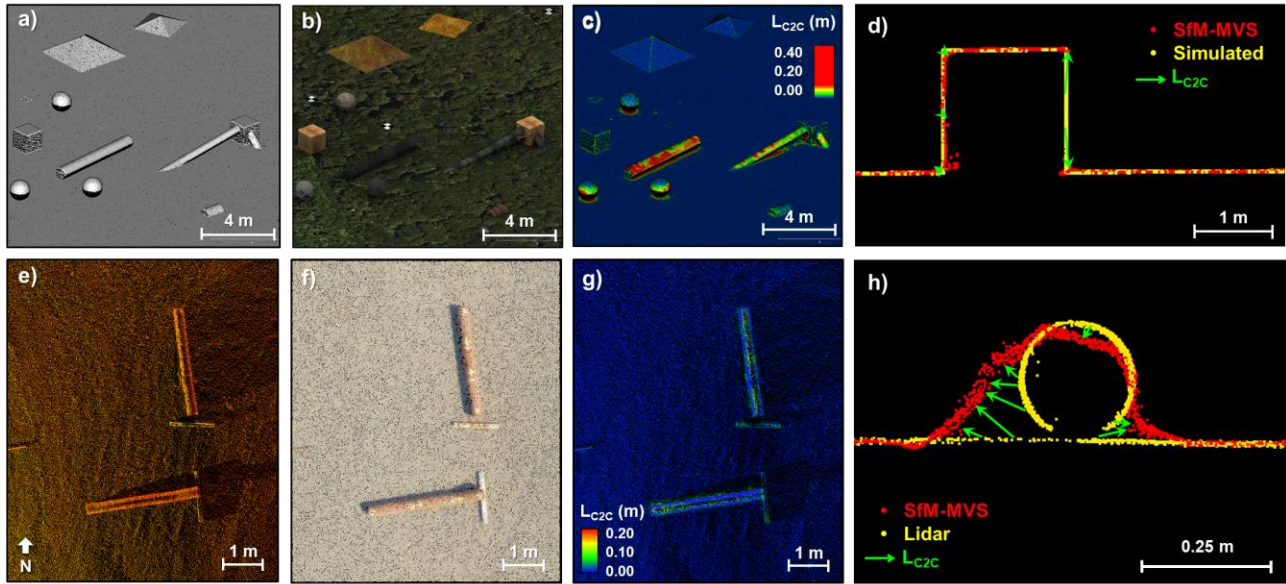
### 360 **3. Results and discussion**

361

362 Figure 10 shows the ground truth models (a, e), SfM-MVS dense point clouds (b and f),  
 363 and the calculated 3D error for sections of the scene A3 of simulated (c) and the empirical datasets  
 364 (g). Also, a profile view of a box in the simulated data (d) and a pipe in the empirical data (h) are  
 365 shown, and the LC2C calculations are schematically illustrated. Figure 11 and Figure 12 shows  
 366 the maps of error and the DPQF indices including, keypoint features distribution  $d_{\text{kp}}$ , GCP  
 367 distribution  $d_{\text{gcp}}$ , scene geometry  $\alpha_{\text{sur}}$ , camera stand-off distances  $d_c$ , image coverage  $n_{\text{img}}$ ,

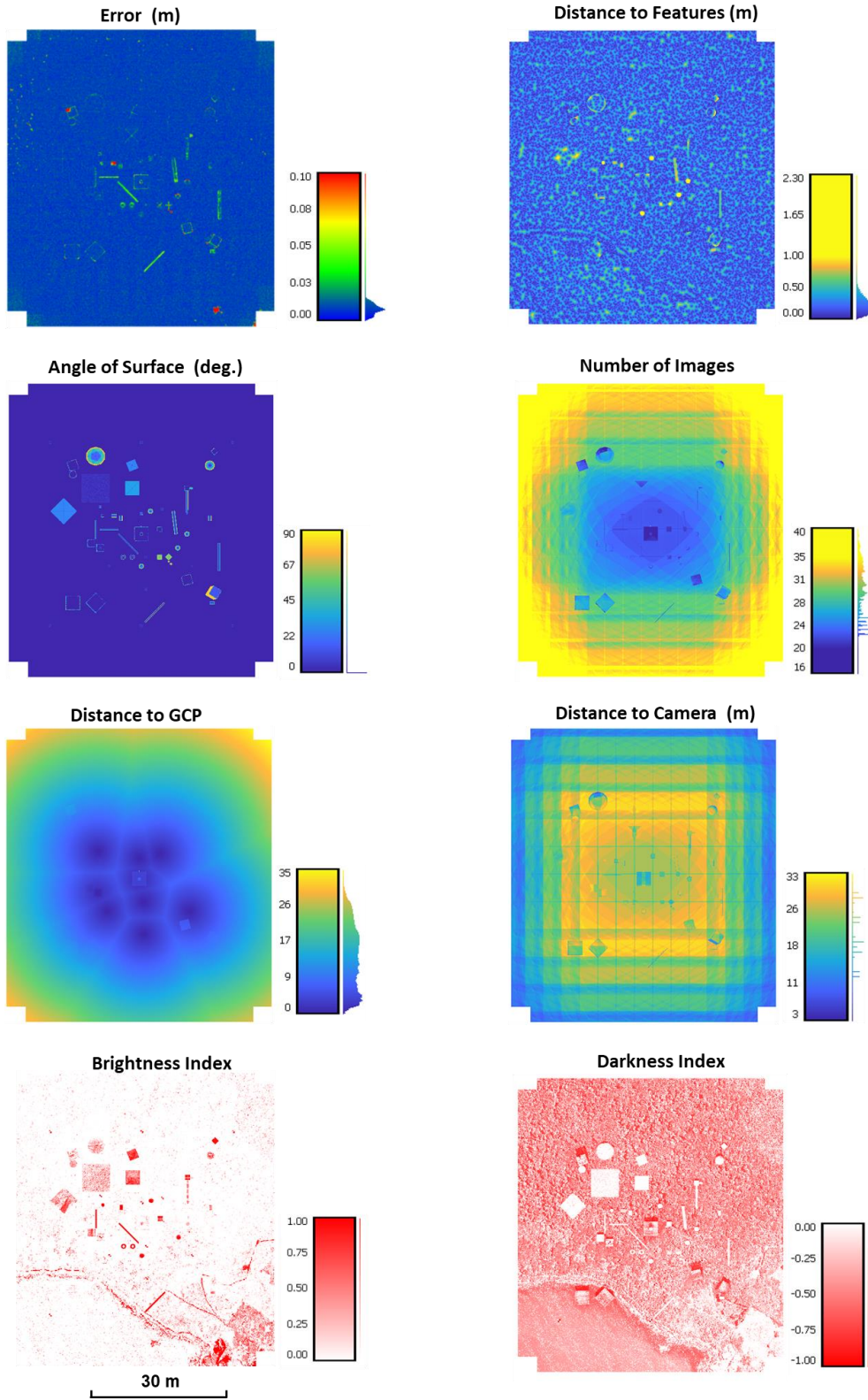
368 brightness index  $I_b$  and darkness index  $I_d$  for a simulated scene (A3 scenario) and an empirical  
 369 dataset (6 GCP scenario).

370



371  
 372 **Figure 10: (a) ground truth model for a section of scenario A3, (b) SfM-MVS dense point cloud for**  
 373 **scenario A3, (c) 3D error visualization for scenario A3, (d) a crosssection of reconstructed and**  
 374 **ground truth point clouds and schematic error calculation for scenario A3, (e) lidar point cloud for**  
 375 **a section of Langmack dataset with 18 GCPs, (f) SfM-MVS dense point cloud for a section of**  
 376 **Langmack dataset with 18 GCPs, (g) 3D error visualization for a section of Langmack dataset with**  
 377 **18 GCPs, (h) a crosssection of reconstructed and ground truth point clouds and schematic error**  
 378 **calculation for a section of Langmack dataset with 18 GCPs**

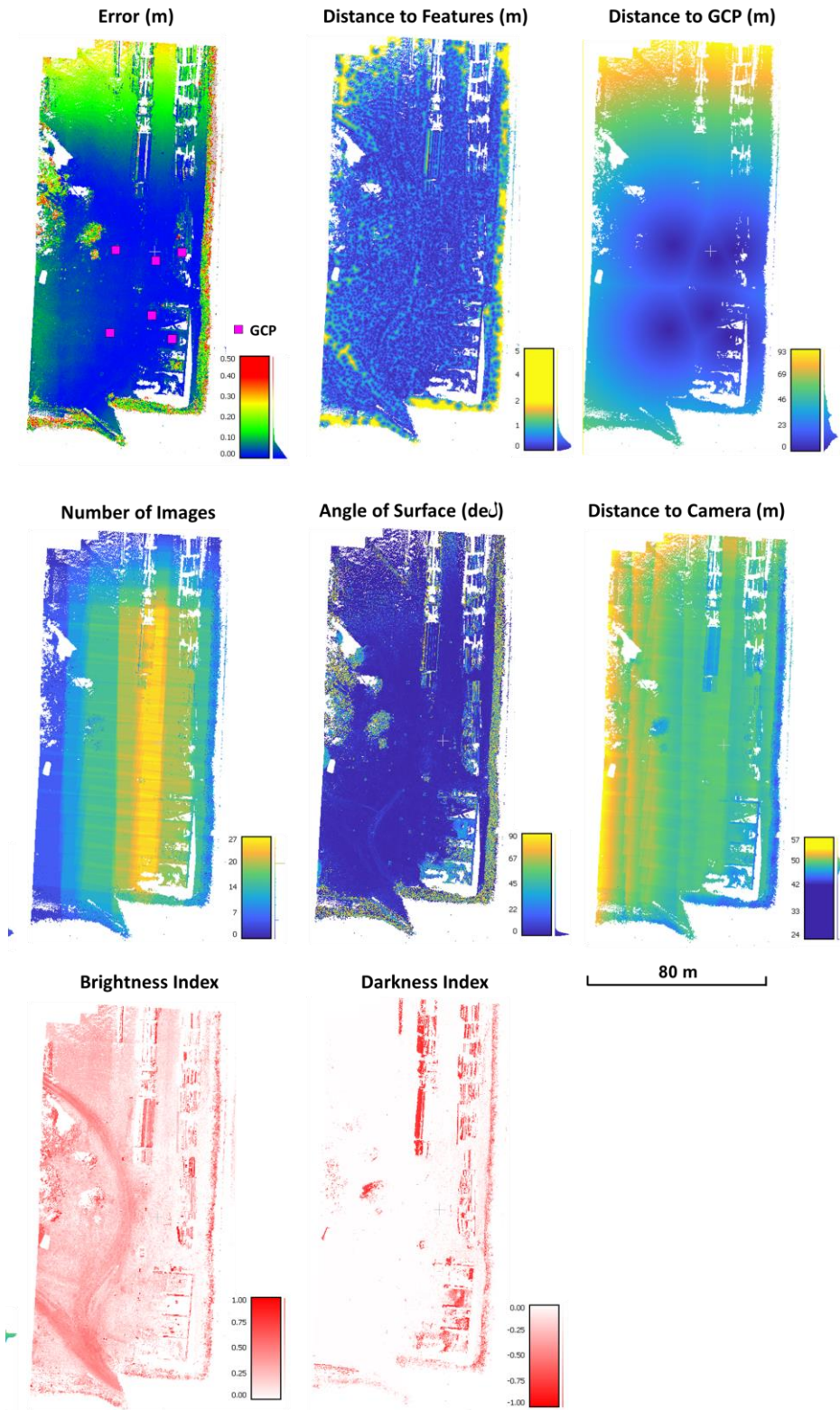
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380

381

Figure 11: DPQFs for scenario A4 of simulated dataset



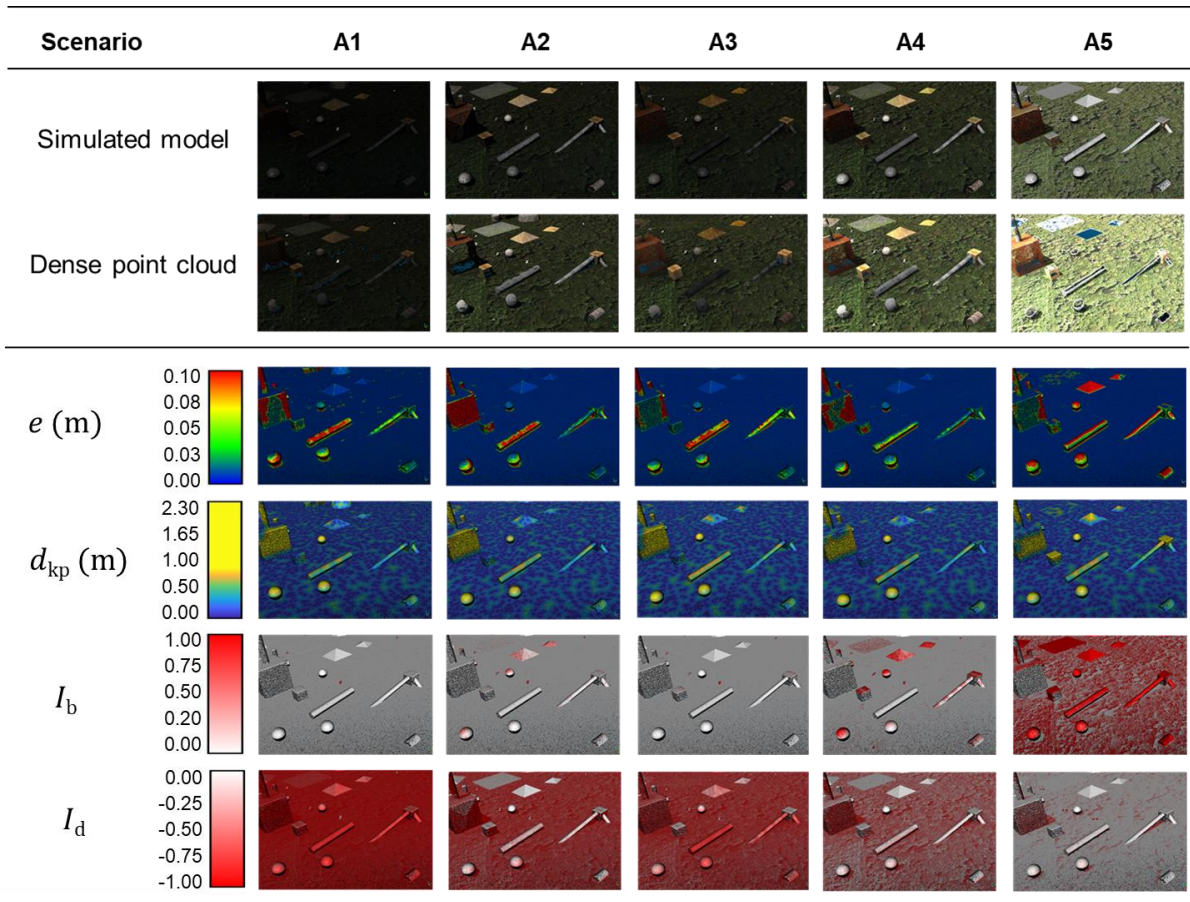
382

383

**Figure 12: DPQFs for the scenario with 6 GCPs for Langmack empirical dataset**



384 To better visualize the variations of DPQF with respect to change of lighting conditions,  
 385 the simulated model and the SfM-MVS point clouds for all five scenarios of scene A are shown in  
 386 Figure 13. Changing the lighting conditions from Scenario A1 to A5 impacted a number of DPQFs  
 387 including,  $d_{kp}$ ,  $I_b$  and  $I_d$ , as shown in Figure 13. If the ground truth point clouds are used for  
 388 estimating the DPQF, there will be no variations in  $d_{gcp}$  and  $\alpha_{sur}$  indices as they are independent  
 389 from change in lighting conditions. In addition, the factors depending on the locations of cameras,  
 390 i.e.  $d_c$  and  $n_{img}$  are minimally impacted within the simulated scenarios in this study.



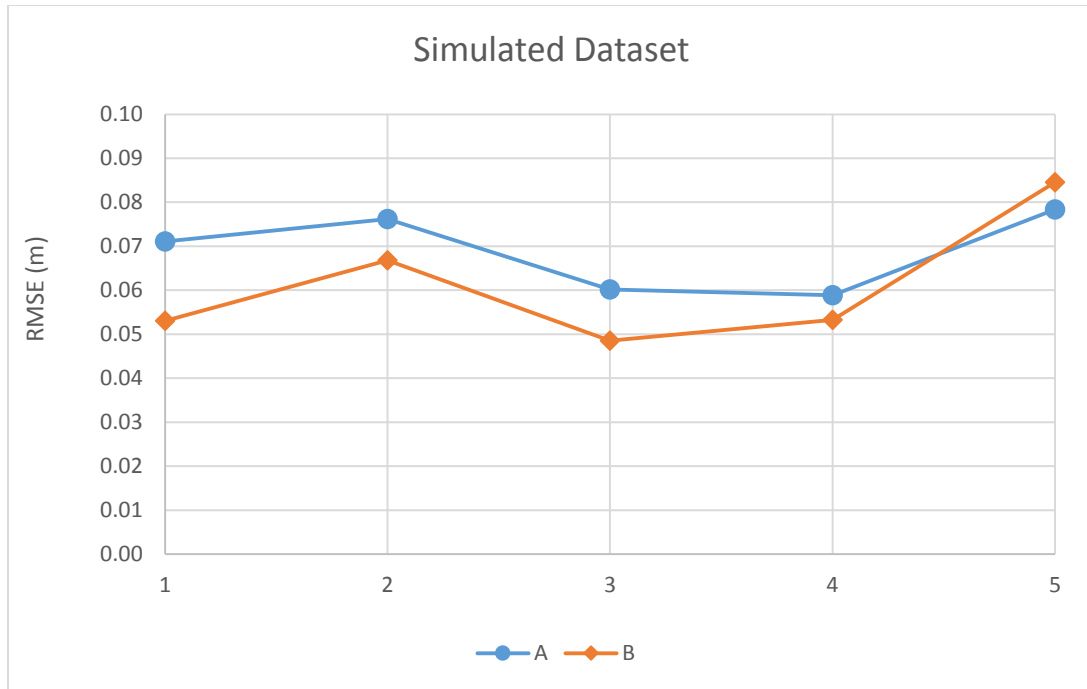
391  
 392 **Figure 13: Change of DPQFs for scenario of simulated scene A**

393  
 394 As shown in the Figure 13 the lighting condition had a distinguishable impact on the error  
 395 in scenes. For example, as shown in 3D error visualizations, it is evident that changing the lighting

396 conditions from scenario A1 to A2 that includes stronger shadows (lower ambient light), creates  
397 data gaps on the vertical faces. The shadow caused an error is a minimum in scenario A3, where  
398 the settings of simulation minimized the deepness of shadow by having same sun and ambient  
399 light intensity values. Increasing the sunlight intensity in simulation A5 improves the quality of  
400 reconstruction on the shadowed faces of boxes; however, at the same time, it adversely impacts  
401 the reconstruction quality on the brighter or shiny surfaces, such as the top of the pipes and the  
402 bright textured pyramids in 3D error visualizations for A5. The reason for poor reconstruction on  
403 light surfaces is that the excess of light masks features and minimizes the sparse point cloud  
404 density. Moreover, there is not enough texture to further perform matching and expanding between  
405 sparse points in densification step. As a result, data gaps and poor reconstruction occur in these  
406 overexposed regions. However, it appeared that this was not the case for areas on the flat ground  
407 surface where areas with higher brightness index still have proper 3D reconstruction from the  
408 images. This might be because of the better performance of SfM-MVS on the flat ground surface  
409 as it can be seen in both experimental and simulated datasets.

410         The total 3D RMSE for each simulated scenario is shown in Figure 14. It is found that both  
411 underexposed scenes (A1, A2, B1, and B2) and overexposed scenes (A5 and B5) include higher  
412 reconstruction error. This error appeared to be even higher for the overexposed situation. The  
413 scene with minimal shadow effect (A3 and B3) and the balanced sun and ambient light conditions  
414 (A4 and B4) include less error. One objective of this study is to investigate the correlations between  
415 aforementioned quality factor with the error. In addition, in order to study the relation of each  
416 factor with reconstruction error, the statistical relationships between the DPQF was calculated by  
417 developing the Pearson correlation matrices for each scenario. The correlation coefficients are  
418 between -1 and 1. The closer the coefficient is to either  $-1$  or  $1$ , there is a stronger dependency

419 between the variables, and the value 0 implies that the variables are independent. The correlation  
420 coefficient matrices of the simulated scenarios are shown in Table 5. The red and blue colors  
421 indicate the positive and negative correlation, respectively, where the darker colors stand for  
422 greater correlations of each, and white color indicates no correlation between variables.  
423



424  
425  
426

**Figure 14: RMSE of 3D error for all scenarios of A and B simulations**

**Table 5: Pearson correlation coefficients between DPQFs for simulations A and B (matrices are symmetric).**

<b>A1</b>	$d_{kp}$	$d_{rcp}$	$d_{img}$	$\alpha_{sur}$	$d_c$	$I_b$	$I_d$
$e$	0.4983	-0.1219	-0.1342	0.5103	-0.1598	-0.0021	-0.1018
$d_{kp}$		-0.1696	-0.1882	0.6222	-0.2270	-0.0101	-0.1730
$d_{rcp}$			-0.6074	-0.2668	0.8898	-0.0251	0.0222
$d_{img}$				-0.2693	-0.5050	0.0144	0.0264
$\alpha_{sur}$					-0.3490	-0.0027	-0.1516
$d_c$						-0.0177	0.0819
$I_b$							0.2958

<b>A2</b>	$d_{kp}$	$d_{rcp}$	$d_{img}$	$\alpha_{sur}$	$d_c$	$I_b$	$I_d$
$e$	0.4924	-0.1351	-0.1481	0.5029	-0.1688	-0.0102	-0.1561
$d_{kp}$		-0.1717	-0.1881	0.6434	-0.2203	0.0065	-0.2182
$d_{rcp}$			-0.6074	-0.2668	0.8898	-0.0598	-0.0141
$d_{img}$				-0.2693	-0.5050	0.0429	0.1065
$\alpha_{sur}$					-0.3490	0.0358	-0.2117
$d_c$						-0.0514	0.0473
$I_b$							0.3386

<b>A3</b>	$d_{kp}$	$d_{rcp}$	$d_{img}$	$\alpha_{sur}$	$d_c$	$I_b$	$I_d$
$e$	0.4498	-0.1080	-0.1006	0.4647	-0.1328	-0.0084	-0.0850
$d_{kp}$		-0.1957	-0.1655	0.6246	-0.2484	-0.0304	-0.1543
$d_{rcp}$			-0.6075	-0.2668	0.8898	0.0094	0.0989
$d_{img}$				-0.2691	-0.5053	-0.0167	-0.0760
$\alpha_{sur}$					-0.3488	-0.0156	-0.1385
$d_c$						0.0258	0.1604
$I_b$							0.2687

<b>A4</b>	$d_{kp}$	$d_{rcp}$	$d_{img}$	$\alpha_{sur}$	$d_c$	$I_b$	$I_d$
$e$	0.4421	-0.1100	-0.1053	0.4554	-0.1277	-0.0232	-0.1810
$d_{kp}$		-0.1786	-0.1714	0.6210	-0.2326	-0.0407	-0.2340
$d_{rcp}$			-0.6075	-0.2670	0.8898	-0.0216	0.0671
$d_{img}$				-0.2691	-0.5053	0.0003	0.0465
$\alpha_{sur}$					-0.3490	-0.0064	-0.2681
$d_c$						0.0202	0.1320
$I_b$							0.4348

<b>A5</b>	$d_{kp}$	$d_{rcp}$	$d_{img}$	$\alpha_{sur}$	$d_c$	$I_b$	$I_d$
$e$	0.3849	-0.1034	-0.0074	0.3305	-0.1335	0.0031	-0.1341
$d_{kp}$		-0.1857	-0.1598	0.6148	-0.2339	-0.1083	-0.2977
$d_{rcp}$			-0.6074	-0.2668	0.8898	0.0697	0.1487
$d_{img}$				-0.2693	-0.5050	0.0358	0.0876
$\alpha_{sur}$					-0.3490	-0.2444	-0.4619
$d_c$						0.1336	0.1926
$I_b$							0.5737

<b>B1</b>	$d_{kp}$	$d_{rcp}$	$d_{img}$	$\alpha_{sur}$	$d_c$	$I_b$	$I_d$
$e$	0.4558	-0.1186	-0.0924	0.4241	-0.1393	-0.0013	-0.1192
$d_{kp}$		-0.1697	-0.1314	0.5216	-0.2045	-0.0132	-0.1700
$d_{rcp}$			-0.6075	-0.2669	0.8898	-0.0252	0.0238
$d_{img}$				-0.2691	-0.5053	0.0144	0.0328
$\alpha_{sur}$					-0.3489	-0.0024	-0.1754
$d_c$						-0.0178	0.0953
$I_b$							0.4037

<b>B2</b>	$d_{kp}$	$d_{rcp}$	$d_{img}$	$\alpha_{sur}$	$d_c$	$I_b$	$I_d$
$e$	0.4788	-0.1299	-0.1283	0.4696	-0.1590	-0.0056	-0.1751
$d_{kp}$		-0.1591	-0.1639	0.5689	-0.1941	0.0071	-0.2165
$d_{rcp}$			-0.6075	-0.2669	0.8898	-0.0861	-0.0266
$d_{img}$				-0.2692	-0.5053	0.0678	0.1302
$\alpha_{sur}$					-0.3489	0.0506	-0.2252
$d_c$						-0.0862	0.0428
$I_b$							0.3762

<b>B3</b>	$d_{kp}$	$d_{rcp}$	$d_{img}$	$\alpha_{sur}$	$d_c$	$I_b$	$I_d$
$e$	0.3814	-0.1178	-0.0605	0.3848	-0.1381	-0.0017	-0.1276
$d_{kp}$		-0.2194	-0.0786	0.4873	-0.2379	-0.0173	-0.0992
$d_{rcp}$			-0.6075	-0.2669	0.8898	-0.0205	0.1292
$d_{img}$				-0.2692	-0.5053	0.0122	-0.1210
$\alpha_{sur}$					-0.3489	-0.0014	-0.1459
$d_c$						-0.0159	0.2110
$I_b$							0.2277

<b>B4</b>	$d_{kp}$	$d_{rcp}$	$d_{img}$	$\alpha_{sur}$	$d_c$	$I_b$	$I_d$
$e$	0.4298	-0.1156	-0.0770	0.4123	-0.1300	-0.0006	-0.2172
$d_{kp}$		-0.2080	-0.1038	0.5203	-0.2362	0.0084	-0.2261
$d_{rcp}$			-0.6075	-0.2669	0.8898	-0.0807	0.0875
$d_{img}$				-0.2692	-0.5053	0.0481	0.0324
$\alpha_{sur}$					-0.3489	0.0608	-0.3153
$d_c$						-0.0759	0.1915
$I_b$							0.4323

<b>B5</b>	$d_{kp}$	$d_{rcp}$	$d_{img}$	$\alpha_{sur}$	$d_c$	$I_b$	$I_d$
$e$	0.3687	-0.0711	-0.0262	0.2565	-0.0782	0.0835	-0.1598
$d_{kp}$		-0.2401	-0.0783	0.5211	-0.2734	-0.0762	-0.3809
$d_{rcp}$			-0.6075	-0.2669	0.8898	0.0796	0.1972
$d_{img}$				-0.2691	-0.5053	0.0115	0.1415
$\alpha_{sur}$					-0.3489	-0.2747	-0.6152
$d_c$						0.2027	0.2244
$I_b$							0.4154

**Legend**



428

429

430

431

432

433 It is found that in almost all A and B simulations, a higher correlation exists between error  
434 and  $d_{kp}$  and  $\alpha_{sur}$  factors with a coefficient of 0.4 - 0.5. This correlation pattern appears to be similar  
435 in all simulations, as it can also be seen with coloring pattern of matrices in Table 5, indicating  
436 that most of the error happens near vertical surfaces or locations at a far distance from the keypoint  
437 features. The  $d_{kp}$  and  $\alpha_{sur}$  also have higher inter-correlation, indicating that the density of sparse  
438 point cloud is lower on vertical surfaces with a high angle of incidence value. The collection  
439 between  $I_d$  and  $\alpha_{sur}$  is at the highest for scenarios A5 and B5, where because of the lighting  
440 condition darker points mostly exist at the shadowed regions. Interestingly, there is a positive  
441 correlation between  $I_b$  and  $I_d$ , while these factors are supposed to negatively correlated. The reason  
442 for the correlation is because of the definition of these factors, as for most of the points the  $I_b$  and  
443  $I_d$  are small with a normalized grayscale value close to zero. This normalized value is assigned to  
444  $I_b$  and  $I_d$  whether the value is slightly positive or negative, and the other factor will be assigned  
445 zero value. The low values close to zero on one index and zero for the other index results in a high  
446 correlation between them. The high 0.90 correlation coefficient between  $d_c$  and  $d_{gcp}$  or the -0.60  
447 between  $d_{gcp}$  and  $n_{img}$  is resulted merely because of the geometry of the scene, where all the GCPs  
448 are located at the center of the scene with more image coverage and closer distance from the  
449 cameras.

450 Table 6 shows the variation of the correlation coefficients of  $e$  with other DPQFs for  
451 simulations of both A and B scenes. The coefficients are comparable in most of the scenes and  
452 almost without considerable, having the high correlations between the error with  $d_{kp}$  and  
453  $\alpha_{sur}$  variables. This correlation coefficient will change if the simulation is adjusted by adding  
454 oblique images that capture the vertical surfaces, which are not captured in nadir images to improve  
455 the construction quality. In that case, the correlation between error and  $\alpha_{sur}$  (or  $\alpha_{inc}$  for the case of

456 oblique and nadir imagery) can decrease. Then it is possible to analyze the correlation of the other  
 457 factors that are overlaid by the more significant error at a greater distance from the keypoint  
 458 features at vertical surfaces.

459

460 **Table 6: Changes in Pearson Correlation Coefficients of 3D error and DPQFs for all simulations of A and B.**

	<b>A1</b>	<b>A2</b>	<b>A3</b>	<b>A4</b>	<b>A5</b>
$d_{kp}$	0.4983	0.4924	0.4498	0.4421	0.3849
$d_{gcp}$	-0.1219	-0.1351	-0.1080	-0.1100	-0.1034
$d_{img}$	-0.1342	-0.1481	-0.1006	-0.1053	-0.0074
$\alpha_{sur}$	0.5103	0.5029	0.4647	0.4554	0.3305
$d_c$	-0.1598	-0.1688	-0.1328	-0.1277	-0.1335
$I_b$	-0.0021	-0.0102	-0.0084	-0.0232	0.0031
$I_d$	-0.1018	-0.1561	-0.0850	-0.1810	-0.1341

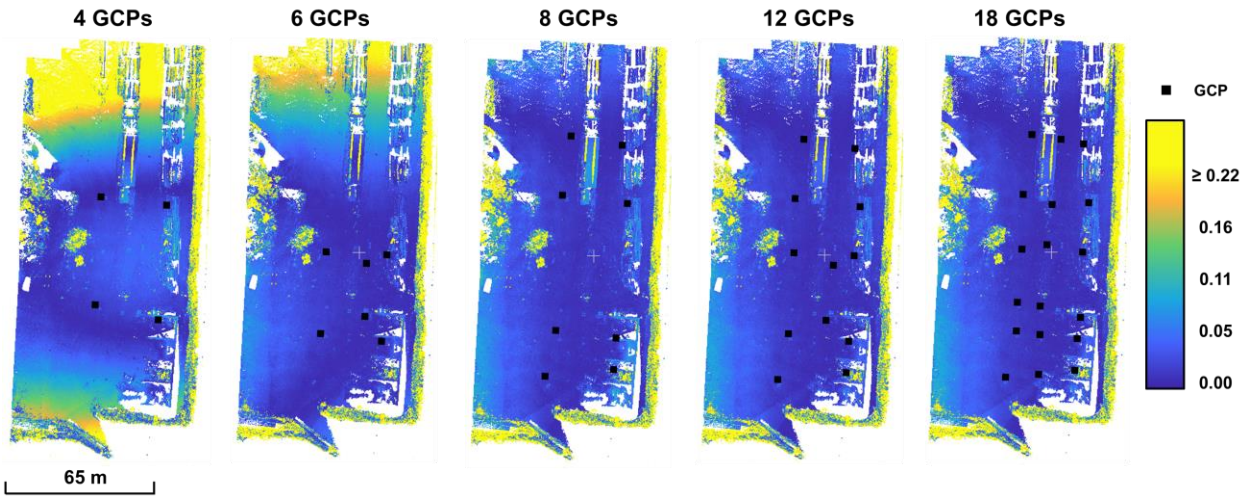
	<b>B1</b>	<b>B2</b>	<b>B3</b>	<b>B4</b>	<b>B5</b>
$d_{kp}$	0.4558	0.4788	0.3814	0.4298	0.3687
$d_{gcp}$	-0.1186	-0.1299	-0.1178	-0.1156	-0.0711
$d_{img}$	-0.0924	-0.1283	-0.0605	-0.0770	-0.0262
$\alpha_{sur}$	0.4241	0.4696	0.3848	0.4123	0.2565
$d_c$	-0.1393	-0.1590	-0.1381	-0.1300	-0.0782
$I_b$	-0.0013	-0.0056	-0.0017	-0.0006	0.0835
$I_d$	-0.1192	-0.1751	-0.1276	-0.2172	-0.1598

461

462

463 Figure 15 shows the geospatial distribution of the error in SfM-MVS point clouds of the  
 464 Langmack area for scenarios with different numbers of GCPs. It seems, as expected, having the  
 465 appropriate number of GCPs is essential for processing the empirical dataset, and the amount of  
 466 error and its distribution significantly controlled by the GCPs. For this dataset, the accuracy of 3D  
 467 reconstruction considerably improves for scenarios with 8 or more GCPs. For the simulation  
 468 dataset, the number of GCP did not appear to be a significant factor. This result is expected for the  
 469 simulated data because the locations of cameras are known and are provided to the software, and

470 the camera calibration model is accurately defined. However, for the empirical dataset, the impact  
471 is significant mainly because of using of an uncalibrated camera with unknown or less accurately  
472 known lens distortion parameters and low accuracy EO positions. In addition to error for scenarios  
473 with a limited number of GCPs, some points are mapped continuously as erroneous points. The  
474 point mostly belongs to the areas with high vegetation at the borders of the study area, as well as  
475 the piles of biomass located in the center. The problem with these points is that there is no well-  
476 defined surface to capture with lidar, and the lidar pulse can be from either outer or inner objects.  
477 So the ground truth lidar data for this regions is not reliable, and it is not expected to build SfM  
478 dense point cloud that reproduces the same results. In addition, there are significant errors for  
479 spaces between the concrete bridge objects, where there is no data in SfM dataset, but TLS was  
480 able to capture the geometry. The error resulted from covered objects and the error on vegetation  
481 is not meant to be included in the future analysis, so the regions with vegetation and biomass were  
482 cropped out for the final data, and the points with error larger than 21.7 cm were omitted from  
483 furthered analysis. The cut-off of 21.7 cm was decided based on the RMSE of total 3D error for  
484 cropped data of scenario with the smallest error ( $1.6166 \times \text{RMSE}_{3D}$ ), also based on the  
485 performance of selected cut-off in filtering the points with an error.  
486



487  
 488 **Figure 15: Changes in spatial distribution of 3D error with respect to the changes in the number of**  
 489 **GCPs**

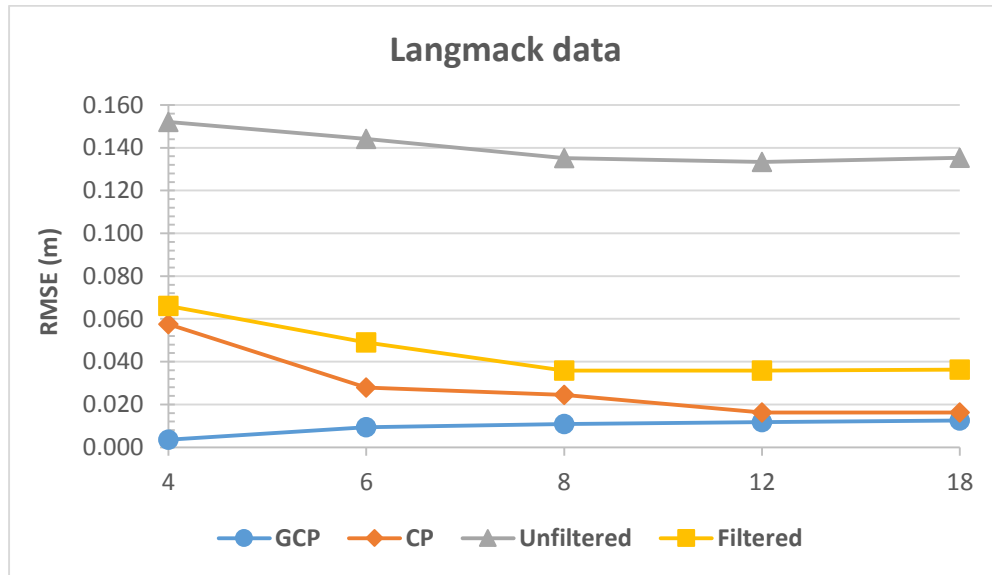
490

491 The RMSE for cropped and filtered empirical data, the georeferencing error and the RMSE  
 492 of CP error are shown in Figure 16. The error is at the highest for 4 GCPs, and it decreases as more  
 493 GCPs are used for georeferencing. The trend of the change for RMSE CPs follows the same trend  
 494 but with smaller values. This difference is expected because the RMSE only represents selected  
 495 CP in the model that usually exist in the sparse point cloud. In most cases, the CPs are distinct  
 496 objects at the scene creating key point features that can be extracted and use in sparse point clouds,  
 497 which have more confident 3D construction. The regions near the high contrast markings can help  
 498 to generate results that are more accurate at those locales; however, the smooth and featureless  
 499 regions might have a higher error. This study shows that comparing the SfM-MVS modeled  
 500 coordinates on CPs and comparing the accurately measured CPs can be a relatively good  
 501 representation of overall accuracy of the sparse pointcloud; however, the higher error should be  
 502 expected in dense point clouds. Interestingly, the RMSE of georeferencing increases as more GCPs  
 503 are included in the processing. The reported RMSE for georeferencing error must be handled with  
 504 care. This error might be because of having an error in GCP coordinates. Also, the error might



505 increase by adding more constraints to a least square solution for coordinate transformation by  
 506 providing a more realistic estimation of georeferencing error.

507



508

509

510

**Figure 16: RMSE georeferencing error, CP error, 3D error for unfiltered and filtered data of Langmack dataset**

511

512 The correlation matrices of Langmack data for scenarios with a different number of GCPs  
 513 are presented in Table 7. Supporting the trend and pattern of error shown in Figure 15 and Figure  
 514 16, there is high correlation coefficient of 0.65 between  $e$  with  $d_{gcp}$  for scenarios with 4 and 6  
 515 GCPs showing that point with higher error is concurrent with areas are that located in the distance  
 516 from GCPs. The results suggest that adding more GCPs significantly improves the quality of SfM  
 517 result; however, the number of improvement plateaus after 8 GCPs at this site (number of GCPs  
 518 is site specific and varies based on how big the site is). Also improving the accuracy by adding  
 519 GCPs decreases the correlation coefficients of error and  $d_{gcp}$ . The change of correlation  
 520 coefficients for  $e$  with other DPQFs are shown in Table 8. For scenarios with more GCPs, the error  
 521 becomes more independent from a distance to GCP. Meanwhile, the remainder of the error shows

522 higher correlation coefficients with  $n_{img}$ ,  $\alpha_{sur}$ , and  $I_d$ . The  $n_{img}$  factor is also correlated with  
523  $d_{gcp}$  for scenarios with 12 and 18 GCPs because more GCPs are located in the center of the area  
524 with a higher number of images. Also, the correlation between  $\alpha_{sur}$  and  $d_c$  is resulted because of  
525 the site setup as the center of the study area that is closer to the cameras is flat and different objects  
526 that create  $\alpha_{sur}$  are located around the edges of the area. However,  $I_d$  and  $\alpha_{sur}$  has a meaningful  
527 correlation, while points with  $I_d$  values closer to -1 are located at vertical faces or overhanging  
528 surfaces that are mostly in shaded areas.

529

530 **Table 7: Pearson correlation coefficients between DPQFs all scenarios of Langmack data with 4, 6, 8, 12 and**  
531 **18 GCPs (matrices are symmetric).**

4 GCP	$d_{kp}$	$d_{gcp}$	$d_{img}$	$\alpha_{sur}$	$d_c$	$I_b$	$I_d$
$e$	0.0917	0.6525	-0.0948	0.0196	0.0849	0.0470	-0.0698
$d_{kp}$		0.1145	-0.2521	0.0481	0.1269	0.0422	-0.0791
$d_{gcp}$			-0.2995	0.0178	0.3415	0.1421	0.0499
$d_{img}$				-0.2772	-0.1927	-0.2076	0.0500
$\alpha_{sur}$		Sym			-0.4291	0.0384	-0.4612
$d_c$						0.1702	0.2957
$I_b$							0.4111

6 GCP	$d_{kp}$	$d_{gcp}$	$d_{img}$	$\alpha_{sur}$	$d_c$	$I_b$	$I_d$
$e$	0.1167	0.6458	-0.1603	0.1643	0.0430	-0.0693	-0.1845
$d_{kp}$		0.1113	-0.2530	0.0499	0.1178	0.0398	-0.0806
$d_{gcp}$			-0.2447	0.1549	0.0816	0.0884	-0.0150
$d_{img}$				-0.2819	-0.1741	-0.2108	0.0526
$\alpha_{sur}$					-0.4284	0.0362	-0.4673
$d_c$						0.1647	0.3005
$I_b$							0.4113

8 GCP	$d_{kp}$	$d_{gcp}$	$d_{img}$	$\alpha_{sur}$	$d_c$	$I_b$	$I_d$
$e$	0.1545	0.1622	-0.3302	0.3027	-0.0047	0.0465	-0.2893
$d_{kp}$		0.1162	-0.2578	0.0394	0.1266	0.0399	-0.0760
$d_{gcp}$			-0.3114	-0.1069	0.4938	0.0901	0.1434
$d_{img}$				-0.2788	-0.1762	-0.2100	0.0433
$\alpha_{sur}$					-0.4333	0.0497	-0.4453
$d_c$						0.1590	0.2881
$I_b$							0.4040

12 GCP	$d_{kp}$	$d_{gcp}$	$d_{img}$	$\alpha_{sur}$	$d_c$	$I_b$	$I_d$
$e$	0.1733	0.2879	-0.3782	0.3057	0.0881	0.0586	-0.2829
$d_{kp}$		0.2029	-0.2618	0.0502	0.1203	0.0377	-0.0817
$d_{gcp}$			-0.5186	-0.0526	0.5436	0.1659	0.1405
$d_{img}$				-0.2729	-0.1779	-0.2085	0.0450
$\alpha_{sur}$					-0.4373	0.0460	-0.4557
$d_c$						0.1633	0.2947
$I_b$							0.4074

18 GCP	$d_{kp}$	$d_{gcp}$	$d_{img}$	$\alpha_{sur}$	$d_c$	$I_b$	$I_d$
$e$	0.1762	0.3346	-0.3958	0.2863	0.1124	0.0669	-0.2747
$d_{kp}$		0.2180	-0.2600	0.0438	0.1242	0.0399	-0.0770
$d_{gcp}$			-0.6389	-0.0658	0.5205	0.1370	0.1177
$d_{img}$				-0.2815	-0.1742	-0.2112	0.0458
$\alpha_{sur}$					-0.4333	0.0480	-0.4508
$d_c$						0.1623	0.2957
$I_b$							0.4057

**Legend**

-1.00 -0.75 -0.50 -0.25 0.00 0.25 0.50 0.75 1.00

532

533

534 **Table 8: Changes in Pearson Correlation Coefficients of 3D error and DPQFs for all scenarios of Langmack**  
535 **data**

	4 GCP	6 GCP	8 GCP	12 GCP	18 GCP
$d_{kp}$	0.0917	0.1167	0.1545	0.1733	0.1762
$d_{gcp}$	0.6525	0.6458	0.1622	0.2879	0.3346
$d_{img}$	-0.0948	-0.1603	-0.3302	-0.3782	-0.3958
$\alpha_{sur}$	0.0196	0.1643	0.3027	0.3057	0.2863
$d_c$	0.0849	0.0430	-0.0047	0.0881	0.1124
$I_b$	0.0470	-0.0693	0.0465	0.0586	0.0669
$I_d$	-0.0698	-0.1845	-0.2893	-0.2829	-0.2747

536  
537

#### 538 4. Conclusions and future works

539

540 This paper defines quality factor indices to be used as proxy indicators for assessing the  
541 accuracy of SfM-MVS dense point clouds. The dense point cloud quality factors (DPQF) include  
542 the geometry of GCPs, the geometry of keypoint features, number of images, distance to the  
543 camera, the angle of incidence, brightness index, and darkness index. In this study, simulated and  
544 empirical experiments were used to assess the accuracy of image-based 3D reconstructed models  
545 with respect to different data collection and site conditions. The data are used to estimate the DPQF  
546 that reflect the scenarios settings, then the spatial correlation between the DPQFs and the  
547 reconstruction error to investigate for multiple datasets.

548 A 3D computer graphics environment was used to generate a set of simulated scenarios  
549 with different lighting conditions. Then, the virtual cameras were placed at the scene to emulate a  
550 UAS-based imagery collection and the images rendered at the defined camera locations. In  
551 addition to simulated data, real-world UAS flights were performed at a construction site to collect  
552 aerial imagery in an empirical experiment. For the empirical dataset, accurate lidar data were also  
553 collected using a terrestrial lidar scanner as a ground truth dataset. Digital images generated for  
554 simulated scenarios were post-processed using SfM-MVS techniques to produce a high-resolution

555 3D point cloud for each scenario. Similarly, multiple SfM datasets were processed with empirical  
556 data by adjusting the number of GCPs for georeferencing and study the results were studied.

557         The results were used to estimate the error and DPQF indices. The error was defined as the  
558 closeness of SfM-MVS data to the ground truth model. The ground truth geometry was precisely  
559 known for the simulated scenarios, and lidar was used as the ground truth for the empirical dataset.  
560 The result of simulated dataset demonstrated that the lighting condition had a distinguishable  
561 impact on the error  $e$  in scenes. In general, scenarios with stronger shadows or overexposed objects  
562 create more error. The reason is that both cases have featureless regions without visible texture,  
563 which can locally degrade the accuracy of the point clouds. The results of the experiments  
564 demonstrated that having the appropriate number of GCPs is essential for the accuracy of 3D  
565 reconstruction; however, overuse of GCPs may reach a point of diminishing return. It seems the  
566 importance of the number of GCPs for dealing with an uncalibrated camera with unknown or not  
567 accurately known EO and IO is more essential.

568         To compare the total RMSE, in order to study the relation of each factor with reconstruction  
569 error was compared to assess the statistical relationships between the DPQF was calculated by  
570 developing the Pearson correlation matrices for each scenario. It is found that in almost all A and  
571 B simulations, higher correlation exists between error, and the distance to the keypoint features  
572 and the angle of incidence factors with correlation pattern that appears to be similar in all  
573 simulations. The results show that the error is more significant in the areas with a lower density of  
574 sparse point cloud as well as on vertical surfaces with a high angle of incidence value. The  
575 correlation coefficients of the empirical data showed a high correlation coefficient of 0.65 between  
576 error and distance to GCP for scenarios with a smaller number of GCPS, meaning that that point  
577 with higher error is concurrent with areas are that located in a greater distance from GCPs.

578 However, the error becomes independent from the number of GCP. Meanwhile, the remainder of  
579 error shows higher correlation coefficients with the angle of incidence, darkness factor, and the  
580 number of images.

581 The paper introduced new quality factor indices for assessing the accuracy of a dense point  
582 cloud by and visualizing the error proxy indices. Definition of quality factors has tangible physical  
583 meaning that can help surveyors with planning the best data collection strategies for UAS/SfM  
584 photogrammetry. Identifying the factor during the field work can help to optimize the negative  
585 factors and minimize the SfM-MVS error that may coexist with these factors.

586 The result of this study shows the initial development of DPQFs with the scope of indirect  
587 accuracy assessment. More studies with more comprehensive experiments are needed to draw firm  
588 conclusions of which factor are the best for accuracy assessment in various scenarios. The  
589 advantage of the DPQFs is that the factors can be defined solely based on the inputs as results of  
590 SfM-MVS processing. The factors may provide a proxy indicator for accuracy to estimate the error  
591 estimation for dense point clouds, which is more challenging than error propagation estimation for  
592 BA procedures due to the MVS processing.

593

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605

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