Hybrid Survey Networks: Combining Real-Time and Static GNSS Observations for Optimizing Height Modernization

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Abstract: To derive ellipsoid heights on passive marks with cm-level accuracy, many current specifications require the collection and adjustment of long-duration, static post-processed Global Navigation Satellite System (GNSS) sessions. To increase efficiency, a campaign-style survey procedure that includes real-time kinematic (RTK) vectors from a real-time GNSS network was evaluated. Thirty different "hybrid" networks involving three to nine Network RTK (NRTK) vectors per mark and some static GNSS vectors were developed from surveys completed in Oregon and South Carolina. The variance-covariance matrices of the static and kinematic vectors were scaled by variance component estimation procedures to produce realistic error estimates for stochastic modeling. After least squares adjustment and formal random error propagation of the networks, the resulting ellipsoid heights on the passive marks had network accuracies ranging from 0.6 to 3.6 cm (95% confidence). These network accuracies reduced to < 2 cm when using six or more NRTK observations per mark. Further, the use of NRTK vectors obtained from observables of both the United States' Global Positioning System (GPS) and

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Russia's GNSS (GLONASS) were, on average, 19.2% more accurate vertically than vectors obtained solely from GPS observables.

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Introduction

Differential geodetic leveling provides precise height differences between marks; however, it is a tedious process that requires line-of-sight optical measurements every 90 meters or less, and significant errors may accumulate when leveling a long distance. When leveling, crews must take great care in setting up the rods, reading and operating the instrument, and recording the observations. For higher accuracy, other types of measurements or models are also required for correcting leveling observations, such as, but not limited to, a model or measurements of surface gravity and of the temperature gradient at the leveling equipment. Because differential leveling is tedious and prone to blunders, many researchers and practitioners have investigated methods for using advances in Global Navigation Satellite Systems (GNSS) technology to more efficiently derive orthometric heights on marks (e.g., Henning et al. 1998; Martin 1998; Ollikainen 1997; Baryla et al. 2013; Gillins and Eddy 2016). GNSS is an attractive and cost-effective alternative because it enables the simultaneous determination of both vertical and horizontal positions without the need for maintaining line-of-sight between marks on the ground. A report (NGS et al. 1998) to the United States Congress concluded that static GNSS surveys are more cost-effective than differential leveling surveys for projects with distances greater than 4 km in length. The report also noted 88% cost savings when using static GNSS surveys rather than differential leveling when measuring orthometric height differences between four marks spaced only 10 km apart.

Unfortunately, GNSS does not directly measure orthometric heights; rather, GNSS measures heights relative to the ellipsoid, a simple, geometric shape aimed to approximate the shape of the geoid (nominally global mean sea level). However, ellipsoid heights can be converted to orthometric heights using Eq. 1:

$$H = h - N \tag{1}$$

where H is the orthometric height measured along a plumb line from the geoid to the mark; h is the ellipsoid height measured along the ellipsoid normal (a line perpendicular from the ellipsoid to the mark), and N is the geoid height measured along the ellipsoid normal from the ellipsoid to the geoid. Technically, Eq. 1 is an approximation because the plumb line is not coincident with the ellipsoid normal and is slightly curved; however, the error is considered insignificant, likely less than 1 mm in even the most extreme settings on Earth (Jekeli 2000).

Minimizing the error in h will ultimately reduce error in estimating H per Eq. 1. This paper focuses exclusively on evaluating and comparing methods for measuring h and estimating its error with GNSS. Further discussion is not given in this paper on deriving H at a mark, but it can be computed per Eq. 1 by differencing h from direct GNSS measurements with a value of N at the latitude and longitude of the mark from a geoid model. Both the estimated error for h as well as the estimated error for N must be propagated when estimating the error for H.

In an effort to recommend procedures for reliably measuring h with GNSS, the United States National Geodetic Survey (NGS) developed detailed static GNSS surveying guidelines, referred to herein as NGS-58 (Zilkoski et al. 1997). These guidelines are followed for performing so-called GNSS "height modernization" surveys in the United States. The guidelines state that *h* measured on project marks are expected to have local and network accuracies less than 2 cm and 5 cm at 95% confidence, respectively. NGS-58 recommends observing multiple baselines at least twice between project marks assigned to four levels of hierarchy: control, primary, secondary, and local (Zilkoski et al. 1997). The guidelines also recommend long-duration static GNSS sessions depending on baseline length: baselines between 15 and 40 km in length should be observed for 5 hours on three consecutive days; baselines between 10 and 15 km in length should be observed twice for at least 1 hour, and baselines less than 10 km in length should be observed twice for at least 30 to 45 minutes (Zilkoski et al. 1997). Afterwards, the precision of the repetitive baseline observations should be verified to be less than 2 cm vertically, and NGS-58 then recommends performing minimally and fully constrained least squares adjustments of the survey network in order to derive most-probable ellipsoid heights at each mark.

More recently, in 2013, NGS released *OPUS-Projects*, which is a free, web-based application that allows users to upload, manage, post-process, and adjust multiple static GNSS observations on numerous marks in a survey project. *OPUS-Projects* is becoming popular in the U.S., and many practitioners desire to use it for height modernization surveys. Gillins and Eddy (2016) showed in a case study in Oregon that ellipsoid heights derived by post-processing in *OPUS-Projects* following NGS recommendations (i.e., Armstrong et al. 2015) generally matched to within ± 1 cm with ellipsoid heights derived following the NGS-58 guidelines.

Although *OPUS-Projects* is a valuable tool, its baseline processing engine, *PAGES*, requires at least a 2-h static GNSS session in order to produce reliable results (Soler et al. 2006). Such long-duration sessions are less attractive to surveyors accustomed to deriving geodetic coordinates on marks in less than a few minutes by obtaining real-time kinematic (RTK) GNSS observations utilizing a real-time network (RTN). An RTN consists of a set of continuously operating GNSS reference stations that are combined and used to generate network RTK (NRTK) solutions. RTN technology has matured over the last several years and is widely used by surveyors and other geospatial professionals for high-accuracy applications. As discussed later in this paper, a number of studies (Allahyari 2016, Smith et al. 2014, and Janssen and Haasdyk 2012) have been completed showing cm-level accurate geodetic coordinates can be obtained in minutes or less using an RTN.

There are considerable indications that NRTK baseline observations would greatly optimize the derivation of cm-level accurate ellipsoid heights on marks--possibly reducing the required duration of the GNSS observation per mark from hours (as recommended in NGS-58 or required in *OPUS-Projects*) to only minutes. However, RTNs do have some limitations worth consideration. First, any error between the position of the RTN reference stations as provided by the RTN network manager and the National Spatial Reference System (NSRS) is propagated to the user. NGS defines its network of active stations, referred to herein as continuously operating reference stations or “CORS,” as the backbone of the NSRS. Ideally, the network manager should ensure that the positions of the RTN reference stations are accurately referenced to the NSRS; however, this may not necessarily always occur. Second, to mitigate the first issue and check the alignment with the NSRS, it is best practice to tie the control survey project to multiple CORS. Because the spacing of the CORS often exceeds the spacing of the RTN references stations, tying a survey project network to multiple CORS will likely require some static GNSS observations and post-processing.

The objective of this paper is to develop an efficient, campaign-style surveying approach involving the inclusion of NRTK GNSS vectors with some static GNSS observations referenced to multiple CORS for deriving high-accuracy ellipsoid heights on marks. The intent is to evaluate the accuracy of combining numerous NRTK and static baseline observations on multiple marks into a survey network, referred to herein as a "hybrid network," that is subsequently adjusted by least squares. The approach allows holding the coordinates of the CORS as control in the adjustment. If sufficiently accurate, the hybrid network surveying approach could greatly optimize height modernization surveys because NRTK observations are much faster than static observations, and the method may lead to future revision or replacement of NGS-58.

To accomplish this objective, static and NRTK data were evaluated from two GNSS surveys in South Carolina and Oregon. The static data were post-processed in *OPUS-Projects* to derive geodetic coordinates on each mark following conventional practices. For comparison, several "hybrid networks" were developed following the methods proposed herein using only the static baseline observations at the active stations and repetitive NRTK GNSS vectors to the passive marks. Each hybrid network was adjusted by least squares, and the resulting coordinates were then compared with the coordinates of the static-only network from *OPUS-Projects*. In addition, error estimates for the coordinates were computed by formal random error propagation, and plots were drawn depicting horizontal and vertical errors as a function of the number of NRTK baseline observations to each passive mark in the hybrid networks.

Background on Real-Time Networks

Over the past 20 years, GNSS surveying with RTK technology has become extremely popular for numerous fields, such as in surveying engineering, mobile mapping, machine control, precision agriculture, mining, and construction. Conventional RTK utilizes a stationary, single reference station, or "base" station, which transmits its precise coordinates and GNSS observables to a moving, "rover" receiver, enabling real-time derivation of GNSS baselines. However, such a configuration usually limits baseline lengths to 10 - 20 km due to the fact that at greater baseline lengths, broadcast satellite orbits and atmospheric delay errors do not sufficiently cancel by differencing GNSS observables collected at the base and rover, resulting in greater difficulty resolving integer ambiguities.

To overcome this limitation, an RTN uses a network of continuously operating reference stations to interpolate atmospheric delay, and it broadcasts ultra-rapid ephemerides to reduce satellite orbit error (Zhang et al. 2006; Janssen 2009). Current practice is to space the reference stations every 70 km or less, enabling derivation of sufficiently accurate baselines up to approximately 40 km in length. In an RTN, the set of continuously operating GNSS reference stations send data to a centralized network processing server. The rover receiver also transmits and receives data from this server using wireless communication, such as via a cellular data plan. RTN software uses the data from the reference stations to generate NRTK corrections by fixing integer ambiguities of double-differenced GNSS phase observables. Using the correction messages and published geodetic coordinates of the RTN base stations, the rover can compute its precise coordinates quickly.

RTN implementation commonly involves one of two popular methods for providing solutions: the Virtual Reference Station (VRS) method or the Master-Auxiliary Concept (MAC) method. These methods have important differences worth noting.

For the VRS method, the rover first transmits an uncorrected point position to the service. To reduce distance-dependent errors associated with relative positioning, the service then assigns this position as the location of an imaginary base station, and interpolates network corrections at this virtual location. These interpolated corrections are used to generate corrected pseudo-observables that are transmitted from the virtual base to the rover to be processed using conventional single-base RTK algorithms to solve for precise Earth-centered, Earth-fixed (ECEF) coordinates at the rover (Wang et al. 2010). Thus, a very short GNSS vector (i.e., ~1-3 m) from the VRS to the rover is obtained. The position of the VRS may change often, such as each time the rover is powered on or moved a certain amount of distance requiring a new uncorrected rover point position to be transmitted to the service to maintain quality network solutions (Leica 2005, Janssen 2009). The network server broadcasts the ECEF coordinates of the VRS for each NRTK solution, and it also broadcasts a separate record indicating the nearest physical reference station (PRS) from the rover and its ECEF coordinates (computed and given by the network manager). Application software can be used to move the tail of the GNSS vector from the ECEF coordinates of the VRS to the ECEF coordinates of the PRS (Graham Briggs, personal communication, March 6, 2017). The final result is a delta ECEF vector and its variance and covariance values from the coordinates of the idealized isotropic absolute “null” antenna, named by the International GNSS Service (IGS) as “GPPNULLANTENNA” (IGS 2017), for the PRS to the antenna reference plane (ARP) of the rover antenna.

In the MAC method, the rover transmits an uncorrected point position to the central server, and then the server searches and selects a “cell” of RTN reference stations for generating corrections. Typically, the nearest reference station is assigned as the master station, and several additional auxiliary reference stations within the appropriate cell are chosen. The phase ranges from all selected stations are reduced to a common ambiguity level, and dispersive and nondispersive errors for each frequency and satellite-receiver pair are computed relative to the master station. Afterwards, the residual corrections between the auxiliary stations and the master station, as well as the full corrections and coordinates at the master station are transmitted to the rover. The rover then computes a double-differenced solution based on the transmitted observables from the master station corrections and network information to derive its precise position (Janssen 2009). The final result is a delta ECEF vector and its variance and covariance values from the coordinates of the ARP of the master station antenna to the ARP of the rover antenna.

Although the VRS and MAC method have important differences, Edwards et al. (2010), Wang et al. (2010), and Martin and McGovern (2012) found that both methods produce coordinates at the rover with similar accuracies. Table 1 summarizes several recent studies where investigators evaluated the accuracy of NRTK data by comparing with coordinates derived from previous static GNSS surveys. The results summarized in Table 1 include tests using an RTN with a recommended average reference station spacing (i.e., interstation spacing) approximately equal to 70 km or less. A small portion of the NRTK data in these studies were filtered prior to evaluation using various criteria, such as rejection of data with poor coordinate quality from GNSS software reports, high position dilution of precision (PDOP), or significant differences from published coordinates at a mark. Some studies (Wang et al. 2010; Janssen and Haasdyk

2012) also tested RTNs with greater than 70 km reference station spacing and found the resulting NRTK data were less accurate; these statistics are not shown in Table 1. Although each study was done with different RTNs using varying solution methods and observation durations, it is interesting that nearly every study found the horizontal root-mean-square error (HRMSE) equal to roughly 1 - 2 cm and the vertical root-mean-square error (VRMSE) equal to roughly 2 - 3 cm. It is also noteworthy that several of the studies found that the error hardly reduced when increasing the duration of the observation session from just 6 s to as long as 600 s (e.g., Janssen and Haasdyk 2012; Smith et al. 2014; Allahyari 2016).

It is clear that ellipsoid heights with VRMSE less than 3 cm could be obtained in mere seconds using an RTN. However, typical height modernization survey guidelines require development and least squares adjustment of a campaign-style survey network consisting of numerous repeat baseline observations on several marks in a project. Repeat observations and adjustments are beneficial for identifying blunders, outliers, or poor session solutions. Prior work (Table 1) does not evaluate the accuracy of coordinates obtained from adjustment of such a survey network.

Source of Survey Data for Analysis

To develop and evaluate the accuracy of the hybrid networks, both static and NRTK GNSS data were compiled for analysis from separate surveys completed in South Carolina and Oregon. This section provides details on these GNSS surveys and the resulting data.

South Carolina

The authors began by compiling static GNSS data collected from December 11 to 17, 2013 (days of year 345 to 351) on 20 passive marks in South Carolina (Dennis 2014 and GCT

2014). For the 2013 survey, the surveyors attempted to collect GNSS baseline observations at several marks with unfavorable conditions, such as at marks set within roughly two meters of wooden power poles or under some moderate tree canopies. The idea was to collect data at these challenging locations so that resulting recommendations would be conservative in case surveyors needed to collect GNSS data at similar places. Six of the 20 marks (3201, LEX, PELI, SURV, AIKP, D138) had minimal obstructions 15 deg. above the horizontal of the antenna, 12 marks were located near power poles or under tree canopies that obstructed up to 25% of the view of the satellites, and two marks (L186, BUTL) were under canopies obstructing up to 50% of the view of the satellites.

A minimum of 30 h of static GNSS data were collected at each passive mark through several occupations. Two 5-h-duration sessions were conducted on each day of the survey. Ten marks were simultaneously observed for both 5-h sessions on days of year 345-347, and the other ten marks were simultaneously observed for both 5-h sessions on days of year 348, 350, and 351. The survey used three different types of Trimble antennas (official IGS antenna names are given in parentheses): R8-Model 2 GNSS/SPS88x Internal (TRMR8_GNSS NONE), Zephyr Geodetic 2 (TRM55971.00 NONE), and Zephyr Geodetic 2 RoHS (TRM57971.00 NONE). The antennas were attached to 2-m fixed-height tripods that had been calibrated prior to the campaign. During each session, both GPS and GLONASS observables were logged at a 1-Hz rate, later converted to receiver independent exchange (RINEX) format, version 2.11.

In addition to compiling the static GNSS data at the passive marks, 12-h-duration static GPS data files (overlapping in time with the sessions on the passive marks) at five reference stations in the South Carolina Real-Time Network (SCRTN) and 24-h static GPS data files collected at 14 CORS for each day of the survey were downloaded for future post-processing.

NRTK data utilizing the SCRTN collected at the same 20 passive marks from December 4 to 10, 2013 (Geoghegan 2014 and GCT 2014) were also used for developing the hybrid networks. It is worth noting that these same NRTK data were studied in Bae et al. (2015) and Allahyari (2016). The SCRTN is managed by the South Carolina Geodetic Survey, has base stations spaced less than 70 km, used *Trimble VRSNet3* software at the time of the NRTK survey, and provided network corrections using the VRS method. The aim of the 2013 survey was to investigate the accuracy of NRTK vectors versus the duration of the observational session, as well as to compare the NRTK vectors collected using differing RTN settings. For three days at each mark, the surveyors collected a “series” of six NRTK observations for the following durations at a 1-s epoch rate using a MAC method: 5, 30, 60, 180, 300, and 600 s. Afterwards, they would invert the antenna, rotate the tripod 120 degrees, re-set the antenna, and repeat the observation series. When using the full network of reference stations in the RTN, the surveyors repeatedly cycled through using only GPS and using both GPS and GLONASS observables. Five of the series of baseline observations were completed using GPS-only and five of the series of observations were completed using GPS+GLONASS per day, resulting in 12 distinct observational samples, and 15 independent observations (i.e., 5 per day, each one collected roughly every 2 hours) in each of these distinct samples per mark. All of the NRTK observations were stored as GNSS vectors from the PRS to the rover for later development of survey networks for least squares adjustment. Every NRTK observation was stored in the data collector with a unique “Point ID” so as to prevent the data collector from automatically averaging or overwriting the repetitive observations on the each mark.

Unfortunately, as explained in Allahyari (2016), it was discovered that four of the Trimble R8-Model 2 GNSS receivers utilized in South Carolina had out-of-date firmware (v.

4.12) which caused the rover to not correctly recognize the antenna model of the RTN base stations. After correspondence with Trimble engineers, the NRTK baseline observations produced from outdated firmware were likely biased in ellipsoid height by +8.546 cm. This positive bias is equal to the nominal vertical antenna phase center offset of the Trimble Zephyr Geodetic 2 (TRM55971.00 NONE) base antennas in the RTN as defined by Trimble. To correct this problem, the bias was subtracted from the ellipsoid heights of all NRTK vectors to seven stations G176, L186, W186, W53_, Q176, HUNT and PELI (Allahyari 2016). It is likely that subtracting the 8.546 cm resolved the vertical bias problem, but it is important to point out that this is a complex problem with many possible permutations. The observed height is affected not just by the rover firmware, but also by the network server software and its settings. Even if all software versions and settings at that time were known, it would still be necessary for Trimble engineers to analyze the exact version of the GNSS firmware code to determine exactly how each receiver handled the antenna models in real time (Graham Briggs, personal communication, Oct. 13, 2016). Unfortunately, this was not possible and because of such uncertainties, some small ellipsoid height bias may remain. Although this was an unfortunate occurrence for this research, it serves as a valuable example of the complexity of real-time solutions and the importance of using current software and firmware.

Oregon

This study also used static GNSS data collected from Oct. 7 to Nov. 8, 2014, on 18 passive marks (Fig. 1) in the Willamette Valley, Oregon, during a previous research study documented in Gillins and Eddy (2015, 2016). The 18 marks selected for the 2014 survey were located at sites that were considered suitable for obtaining satellite observations. Fifteen of the marks had only a few minor overhead obstacles (e.g., distant tree canopies) more than 15 degrees above the horizontal of the GNSS antenna. However, two marks (i.e., point names LBCC and GLAS) were located next to traffic signs and had nearby tree canopies as tall as 45 degrees above the horizontal, and one mark (B726) was next to a wooden telephone pole. The three marks with the less-ideal overhead obstacles and nearby features that could cause some multipathing were included in the survey study to simulate some typical types of field challenges surveyors encounter when attempting to make GNSS baseline observations on existing passive marks. However, these field challenges were less severe than at some of the marks in the aforementioned South Carolina survey.

During the 2014 static GNSS survey, 17 of the 18 marks were observed for a minimum of four sessions lasting 10 h in duration. One mark (D728) was observed for only three 10-h sessions. For each of the sessions, five to six Leica GS14 (IGS name “LEIGS14 NONE”) integrated receiver/antennas were attached to calibrated, 2-m fixed-height tripods set up over marks per the schedule in Gillins and Eddy (2016). During each session, both GPS and GLONASS observables were collected, stored both in a raw Leica proprietary format as well as in RINEX format, version 2.11.

In addition to the static data collected at the passive marks, 24-h static GPS data files collected at 11 active stations for every day of each of the survey sessions were gathered for future post-processing. Seven of the 11 active stations are part of the CORS Network, and four are base stations in the Oregon Real-Time GNSS Network (ORGN).

To develop the hybrid networks, NRTK observations utilizing the ORGN were made by the authors on the same 18 passive marks from July 6 to July 8, 2016. The ORGN is managed by the Oregon Department of Transportation (ODOT), has base stations spaced at 70 km or less, uses *Leica GNSS Spider* RTN software, and only used GPS observables at the time of the survey to provide NRTK corrections. Three field crews were deployed for three consecutive days, each using a Leica GS14 integrated receiver/antenna that was receiving NRTK corrections from the ORGN utilizing a MAC method. Each field crew was responsible for conducting nine 180-s-duration NRTK sessions per day at an epoch rate of 1 s, on six passive marks. In order to collect data using different overhead satellite geometries, each crew was assigned to drive a “loop” between six marks (Fig. 1a). At each mark, the crew would set up the receiver on a 2-m fixed-height tripod and then would wait for the receiver to initialize and fix integer ambiguities (Fig. 1b). A 180-s duration NRTK session was then performed, and then the antenna was inverted such that it lost initialization. Afterwards, the antenna was set back up and the procedure repeated until three 180-s-duration, fixed NRTK vectors were obtained. Similar to the South Carolina survey, every NRTK observation was stored in the data collector with a unique “Point ID”. The crew would then drive to the next mark in the loop and repeat the process. It took approximately 2-3 h for the crew to complete one revolution around the loop. Three revolutions were completed per day so that each mark was observed for nine 180-s NRTK sessions per day (i.e., three sessions in the morning, three in the afternoon, and three in the evening per day). Each day, the crews rotated equipment to see if any receiver noise could be detected in the data. After three days, twenty-seven 180-s-duration NRTK vectors with fixed integer ambiguities were acquired on each of the 18 passive marks.

Development of the Hybrid Networks

Because of the abundance of static and NRTK GNSS data collected in South Carolina and Oregon, numerous hybrid networks were created per the simple schematic in Fig. 2. Each of the hybrid survey networks were developed for testing by including: (1) a differing number of repeat NRTK vectors to each mark with fixed integer ambiguities; and (2) the same set of baseline observations post-processed in *OPUS-Projects* using the static GNSS observations collected at only the active stations in the survey network. The purpose for constructing numerous hybrid networks with a varying number of NRTK vectors to each mark was to evaluate how the number of NRTK observations per mark affects the accuracy of the final adjusted coordinates. It was intended to answer the question of what happens, in terms of accuracy, when as few as 3 repeat NRTK observations are collected on each mark as opposed to as many as 9 repeat NRTK observations on each mark.

The following describes the procedure for preparing and combining the data from these two sources into the hybrid test networks. Fig. 3 provides a summary flowchart for how to build a hybrid network.

Preparation of the NRTK Data

Thirty separate hybrid networks were developed using 3 to 9 NRTK vectors per mark. Again, all of the NRTK vectors in this experiment were collected utilizing network corrections from an RTN, and the settings were configured so that every baseline began at the physical location of a reference station (as illustrated in Fig. 2a).

To help organize the data and report the results, ten “network designations” were created: 3A, 3B, 4A, 4B, 5A, 5B, 6A, 6B, 9A, and 9B. The intent of this naming convention is to show how many NRTK vectors were taken to each passive mark in a given hybrid network, and the letter in the designation is simply meant to distinguish the first set of selected vectors from the second set of selected vectors. Starting with network designation “3A” for Oregon, three NRTK vectors to each of the passive marks in Oregon were strategically selected from the total number of available 180-s-duration, GPS-only, fixed-integer NRTK vectors. The vectors were selected from different times of the day (e.g., one in the morning, afternoon, and evening per mark). This selection strategy simulates typical surveying guidelines (e.g., NGS-58) which recommend that surveyors collect repeat GNSS baseline observations on a mark at different times of the day so that independent measurements are made with significantly different satellite geometries. Such a strategy is especially important since dilution of precision and multipathing varies with time, and short-duration baseline observations are more vulnerable to multipath. It also ensured that each vector in the set was acquired with an independent setup of the rover antenna (in order to simulate instrument setup errors) as well as with an independent NRTK initialization. For network designation "3B," a set of 3 different NRTK vectors than the ones chosen for "3A" were selected to each mark from the dataset in Oregon following the same sampling rules. Afterwards, the same strategy was followed for selecting two different sets of 4 vectors to each mark for network designations "4A" and "4B", and the pattern was then followed for selecting two sets of 5, 6, and 9 vectors to each mark for network designations 5A, 5B, 6A, 6B, 9A, and 9B.

Significantly more data are available from the South Carolina NRTK survey. However, for this study, only the 180-s-duration NRTK vectors were used in developing the hybrid networks. The reasons are: (1) previous research has found that the root-mean-square error (RMSE) of NRTK vectors hardly improves after averaging single-epoch (1-s) solutions into 60 to 300-s duration, multi-epoch solutions (Edwards et al. 2010; Janssen and Haasdyk 2012; Smith et al. 2014; Allahyari 2016); (2) this duration is consistent with the available NRTK vectors in Oregon; and (3) although a subjective reason, it seemed unnecessary during the survey campaign in Oregon to shorten the observation duration any further given the amount of time required to write some notes and/or collect photos at each mark.

In a similar method to the Oregon data, two different sets of 3, 4, 5, 6, and 9 vectors to each passive mark were made using the fixed, 180-s-duration, GPS-only, NRTK solutions in South Carolina. Like Oregon, these sets were assigned to network designations 3A and 3B through 9A and 9B. In addition, in order to examine the influence of using GLONASS, two more sets of 3, 4, 5, 6, and 9 NRTK vectors to each mark were made using the fixed, 180-s, GPS plus GLONASS NRTK solutions. These GPS+GLONASS sets were assigned to network designations 3A and 3B through 9A and 9B.

Each of the resulting 30 sets of unadjusted NRTK vectors were next exported to 30 separate “G-files” (Step 1B, Fig. 3). A G-file is an NGS GNSS data transfer file format described in detail in the NGS Bluebook (NGS 2016a). The G-file contains the GNSS vector components in delta ECEF coordinates in a specified reference frame as well as the variance-covariance (VCV) matrix of each component of the vector. For this case, the reference frame for the vectors was the International GNSS Service of 2008 (IGS08) at the mean epoch of the measurement.

NGS (2015) recommends transforming baseline observations to a common epoch in the western United States that experiences significant crustal motion using an NGS utility, Horizontal Time-Dependent Positioning (*HTDP*). *HTDP* transforms the baseline observations made at various epochs in one geodetic datum into observations made at a common epoch in another geodetic datum by accounting for time-dependent horizontal velocities at the ends of the baseline and the transformation parameters between datums (Snay and Pearson 2012). The baseline components in each of the G-files were transformed in *HTDP* (version 3.2.5) to the current geometric reference frame and standard epoch of the NSRS, NAD 83(2011) Epoch 2010.00 (Step 1C, Fig. 3).

Next, each G-file was loaded in *ADJUST*, NGS software for performing least squares adjustments of GPS surveys (Milbert and Kass 1993). Using *ADJUST*, a minimally constrained least squares adjustment of the vectors was performed (Step 1D). This preliminary adjustment has two important purposes.

First, the preliminary adjustment assists with the determination of whether or not a poor NRTK vector (i.e., outlier or blunder) is present in the network. For this study, the 3-D residual for each adjusted vector was computed ($3D\text{-}res_i$) as well as the 3-D RMSE for all residuals of all adjusted vectors in the network ($RMSE_T$). If $3D\text{-}res_i > 3\text{x}RMSE_T$, then the vector was considered an outlier and was rejected, and a new vector was selected as replacement. This iterative process (Step 1E) was repeated until $3D\text{-}res_i \leq 3\text{x}RMSE_T$ for all vectors in the network. This criterion resulted in only a small number of rejections. Only 0.2% (2/972) of the NRTK vectors in Oregon were rejected and replaced, and only 0.9% (10/1080) and 0.4% (4/1080) of the NRTK GPS-only and GPS+GLONASS vectors in South Carolina were rejected and replaced, respectively.

The second reason for the preliminary minimally constrained least squares adjustment of the GNSS baselines is to mitigate inconsistencies and other problems with the VCV matrices generated by the baseline processor. Han and Rizos (1995) showed that the VCV matrices resulting from different GNSS surveying modes (e.g., static versus kinematic) are incompatible. Furthermore, it is also a well-known problem that the VCV matrices output by GNSS baseline processing software are usually overly-optimistic, and that differing software programs have varying levels of over-optimism (e.g., El-Rabbany and Kleusberg 2003; Craymer et al. 1990; Kashani et al. 2004). For example, Kashani et al. (2004) found that the VCV matrices output by static baseline processing in *Bernese GNSS Software* were over-optimistic by a factor of 23.0, whereas the VCV matrices output by processing the same static baseline observations in *GAMIT* were over-optimistic by a factor of 1.9. The reason for the over-optimism is believed to be because software typically underestimates and handles systematic errors differently, and *a priori* error estimates are sometimes arbitrary, thereby affecting the stochastic model used during baseline processing (El-Rabbany and Kleusberg 2003; Han and Rizos 1995).

One common remedy to this problem is to iterate a scale factor to apply to the VCV matrix until the standard deviation of unit weight of a minimally constrained least squares adjustment equals 1 (Kashani et al. 2004; Craymer et al. 1990). Such an approach is often referred to as variance component estimation. Although this approach was used, it is important to note that other techniques are available and may be worth future testing (e.g., Ananga et al. 1994; Snow and Schaffrin 2007).

The standard deviation of unit weight (σ_0) is computed as

$$\sigma_0 = \sqrt{\frac{V^T W V}{r}} \qquad (2)$$

where V is a vector of length $3n$ containing the adjusted residuals of the baseline observation components, n is the total number of baseline observations in the survey network for adjustment, W is the weight matrix (the inverse of the VCV matrix), and r is the degrees of freedom (redundancy) of the survey network. If $\sigma_0 \approx 1$ after an adjustment, then the overall estimated error of the baseline components is consistent with the residuals of the adjusted components.

The relative weights derived from the VCV matrices for an individual baseline processing software program are internally consistent (Han and Rizos 1995). Thus, only the

NRTK vectors were used in the preliminary minimally constrained least squares adjustment. Using a setting described in Pursell and Potterfield (2008), Milbert (2009), and NGS (2016b), *ADJUST* was set to iterate and solve for both a horizontal and vertical "standard deviation factor." Scaling the VCV matrix of the NRTK vectors by the square of these standard deviation factors results in $\sigma_0 = 1$. Table 2 presents the standard deviation factors found by *ADJUST* from the preliminary minimally constrained adjustment of each of the 30 sets of NRTK vectors. The horizontal and vertical factors were, on average, both equal to approximately 2. The square of these factors were applied to scale the VCV matrix in each of the 30 G-files.

Preparation of the Static Data

Next, the aforementioned and simultaneous static data collected at each of the active stations (i.e., RTN base stations and the CORS) were uploaded to *OPUS-Projects*, version 2.6 (Step 2A, Fig. 3). One CORS was identified as the "hub" station for each network in Oregon (CORV) and South Carolina (COLA), and each of the static sessions were post-processed in *OPUS-Projects* following the hub design depicted in Fig. 2b and recommended in the *OPUS-Projects User Manual* (Armstrong et al. 2015) (Step 2B). As recommended in this manual, data from multiple nearby and distant CORS were added for post-processing in each static session. The use of nearby CORS helps reference the network to the geometric frame of the NSRS, and the use of data from at least one long-distance (i.e., ~1,000 km) CORS helps the baseline processor in *OPUS-Projects*, *PAGES*, solve for the wet component of its tropospheric modeling corrections (Ugur 2013). During session baseline processing, only the published coordinates of the CORS were held with "normal" constraint weights in *OPUS-Projects*.

OPUS-Projects output two G-files containing the session solutions in the IGS08 reference system at the weighted mean epoch of the measurement; one from the static data of the active stations in Oregon and one from the static data of the active stations in South Carolina. Using *HTDP*, the baseline components in the G-files were transformed to NAD 83(2011) Epoch 2010.00 (Step 2C).

Similar to the NRTK data, each G-file was then input in *ADJUST* and a preliminary minimally constrained least squares adjustment was performed (Step 2D). The same outlier criterion ($3D\text{-}Res_i$ exceeding 3x$RMSE_T$) was applied for detecting possible blunders (Step 2E); fortunately, all of the vectors in both networks satisfied this criterion. The lack of rejections was not surprising, as all of the baseline observations were derived from long-duration, 12 to 48-h static observations at active stations with unobstructed view of the satellites.

For both sites, *ADJUST* was again set to iteratively solve for horizontal and vertical standard deviation factors that were necessary to scale up the VCV matrix of only baseline observations output from *OPUS-Projects* until σ_0 = 1. For Oregon, the horizontal and vertical standard deviation factors equaled 20.33 and 4.83, respectively; for South Carolina, the factors were 13.75 and 6.24.

Based on the standard deviation factors found in *ADJUST*, it can be concluded that the standard deviations of the vectors output by *OPUS-Projects* were 2.5 to 12 times more optimistic than the standard deviations of the NRTK vectors. Since the variances in the VCV matrix are derived from the square of the standard deviations output by these processors, this finding underscores the importance of correctly scaling VCV matrices prior to combining vectors from different sources in a survey network for adjustment. Otherwise, the vectors from *OPUS-Projects* would appear overly accurate relative to the NRTK vectors, and they would be weighted too high in an adjustment and would warp the results. In addition, the relative scaling within the different baseline processors are not the same. The horizontal and vertical standard deviation factors were nearly equal for the NRTK vectors, whereas the horizontal factors were about 2 to 4 times greater than vertical for *OPUS-Projects*.

Combination of the Data and Hybrid Network Adjustment

Satisfied that the poor baseline observations had been replaced and that the VCV matrix in the G-files were scaled appropriately, the next steps (Steps 3A - 3B, Fig. 3) were to simply merge each NRTK G-file with either the Oregon or South Carolina *OPUS-Projects* G-file and perform the final hybrid (static+NRTK) survey network adjustments in *ADJUST*. Figures 4 and 5 illustrate a final hybrid survey network in Oregon and South Carolina, respectively.

Holding the published coordinates of the hub station fixed, a minimally constrained least squares adjustment of all 30 hybrid networks was first completed. Afterwards, fully constrained adjustments were completed by holding the published coordinates of the CORS in each network as control and by using the published standard deviations of these coordinates for developing weights on the constraints. If the *a priori* VCV matrices for the NRTK vectors, static baseline observations, and coordinates for the control were realistic, then σ_0 for the least squares adjustments should remain approximately equal to 1. Table 3 presents σ_0 values for the minimally and fully constrained adjustments of each of the 30 hybrid networks. For all cases, $\sigma_0 \approx 1$ and passed the χ^2 statistical hypothesis test at the 5% level of significance.

Results and Discussion

There are two common methods to estimate the accuracy of the unknowns (i.e., coordinates at each mark) determined by a least squares adjustment: (1) using formal random error propagation theory; and (2) by comparison with coordinates of higher accuracy. The accuracy of the coordinates derived from the fully constrained hybrid network adjustments were evaluated according to both of these methods.

Accuracy of Results per Formal Error Propagation

After a least squares adjustment, the VCV matrix of the coordinates at each mark may be computed by multiplying the variance of unit weight (σ_0^2) by the inverse of the normal equation coefficient matrix. Using such a procedure to obtain realistic accuracy estimates requires critical assumptions that only random errors were present in the baseline observations and that the observations were properly weighted. *ADJUST* computes the VCV matrix of the unknowns after each adjustment and uses the resulting variances and covariances of the derived coordinates at each mark to compute standard deviations in north, east, and up components, as well as the horizontal correlation coefficient. *ADJUST* then uses these horizontal values to compute a horizontal error ellipse, and it then converts each ellipse into a horizontal error circle. For each mark in the network, *ADJUST* then reports the radius of the horizontal error circle at the 95% confidence level as the "horizontal network accuracy," and it reports the value of the standard deviation in up at the 95% confidence level as the "vertical network accuracy." These two values meet the definitions for so-called horizontal and vertical network accuracy given in U.S. federal standards (FGDC 1998) and are required for reporting the accuracy of a federal geodetic survey. Hence, the phrase in this paper "horizontal or vertical network accuracy" has a very specific meaning, and values of horizontal or vertical network accuracy are at the 95% confidence level unless explicitly stated otherwise.

Figs. 6 through 8 show the estimated errors by formal error propagation, expressed in terms of horizontal and vertical network accuracy, at each mark and for each of the 30 hybrid test networks. The error values generally become smaller as the number of NRTK vectors to each mark increases. As expected with GNSS data, the horizontal errors are 2 to 3 times smaller than the vertical errors. Per Figs. 6a through 8a, vertical network accuracies at every mark in each hybrid network test were less than 3.6 cm, indicating that the hybrid network survey approach can yield results that satisfy standards in the NGS-58 guidelines. For the test networks consisting of at least six NRTK vectors to each mark (i.e., networks designated as 6A, 6B, 9A, 9B), the vertical network accuracies were less than 2 cm and the horizontal network accuracies were less than 1 cm at every mark.

For comparison purposes, Fig. 9 presents the average of the network accuracies grouped by number of NRTK vectors to each mark well as error bars at one standard deviation. As shown, the network accuracies for the marks in the Oregon networks are, on average, smaller than the network accuracies for the South Carolina networks. This is likely because more marks in South Carolina were under moderate tree canopies or next to wooden power poles that obstructed more satellites or induced multipathing errors. Another interesting pattern observed in Fig. 9 is that for the hybrid networks in South Carolina, the average network accuracies were consistently smaller when using GPS+GLONASS NRTK vectors than when using GPS-only NRTK vectors. The inclusion of GLONASS was found to reduce the vertical network accuracy by an average value of 19% (4 mm) and the horizontal network accuracy by an average value of 7% (0.7 mm).

In another effort to highlight the benefit of using GPS+GLONASS instead of only GPS, Fig. 10 depicts the number of NRTK vectors required to a mark to yield an ellipsoid height with an average vertical network accuracy less than 2.0 cm at 95% confidence. The vertical network accuracy was less than 2.0 cm at 15 of 20 marks versus 7 of 20 marks when using three NRTK GPS+GLONASS baseline observations versus three GPS-only observations per mark. Further, this level of accuracy was achieved at every mark rather than only at 10 of the 20 marks when using five NRTK GPS+GLONASS observations per mark versus five GPS-only observations per mark.

Per Fig. 10, average vertical network accuracies were generally less than 2.0 cm after adjustment of three 180-s NRTK observations at each mark; however, at some marks, additional NRTK observations were required. Multiple factors affect the accuracy of GNSS baseline observations, but a portion of the error is modestly correlated with baseline length (Fig. 11). To create Fig. 11, the average vertical network accuracy at each mark from the constrained adjustments of the 10 hybrid networks involving GPS-only NRTK vectors, and the 10 hybrid networks involving GPS+GLONASS NRTK vectors were computed. Afterwards, linear regression lines were fitted to these accuracy values at each mark versus baseline length. As the baseline length or distance from the RTN base station increases, the network accuracy tends to worsen. Of course, there is considerable scatter in Fig. 11, likely due to local conditions such as overhead obstructions or nearby features that may have induced additional multipathing errors. Nonetheless, the modest correlation shows that minimizing the distance from the rover to a reference station in the RTN will improve the accuracy of an NRTK solution.

Comparison with Coordinates of Higher Accuracy

Ideally, exact or known coordinates at a mark could be used as a basis for evaluating the accuracy of the coordinates derived at a mark from a survey adjustment. Unfortunately, there is no method for deriving exact coordinates at a mark because all measurements contain some amount of error. This problem is often mitigated in practice by using coordinates of a higher-order of accuracy (rather than exact coordinates) as a basis for evaluating the accuracy of coordinates of a lower-order of accuracy. However, according to formal error propagation, the coordinates of the marks derived by adjustment of the hybrid survey networks had network accuracies ranging from only 0.6 to 3.6 cm in ellipsoid height and 0.3 to 1.7 cm horizontally at 95% confidence. Deriving coordinates at each mark of a higher-order of accuracy (i.e., 0.03 to 0.36 cm at 95% confidence) is practically impossible.

The best available option was to add all of the aforementioned static GNSS data to *OPUS-Projects*, including the 30 to 100 h of static observations collected at each passive mark and the six to 15 days' worth of static observations at the active stations. Then, all of the static sessions were post-processed in the same manner as was done previously when preparing the session solutions for only the static GNSS data collected at the active stations for the hybrid survey networks. The same hub station was used in Oregon (CORV) and South Carolina (COLA), and only the published coordinates of the same CORS (but not the hub) were held with "normal" constraint weights in *OPUS-Projects*. The baseline components in the resulting G-files were transformed to NAD 83(2011) Epoch 2010.00 using *HTDP*, and the transformed G-files were input in *ADJUST*. Then, using *ADJUST*, a minimally constrained adjustment of each network was performed to solve for horizontal and vertical standard deviation factors for scaling the VCV matrix output from *OPUS-Projects*. After multiplying the VCV matrix by the square of the standard deviation factors, a fully constrained adjustment of each network was performed using identical control as was used for the fully constrained adjustments of the hybrid networks.

ADJUST output coordinates at each passive mark with network accuracy estimates ranging from 0.5 to 1.7 cm in ellipsoid height and 0.2 to 0.6 cm horizontally at 95% confidence. These error estimates are only 1.2 to 3.1 times smaller than the error estimates for the coordinates derived by the hybrid survey network approach--not an order of magnitude smaller.

For each hybrid network, $HRMSE_j$ and $VRMSE_j$ of the coordinates of the passive marks were computed according to Eqs. 3 - 4.

$$VRMS_j = \sqrt{\frac{\sum_{i=1}^{n}\left(h_{hybrid,i} - h_{static,i}\right)^2}{n}} \tag{3}$$

$$HRMS_j = \sqrt{\frac{\sum_{i=1}^{n}\left(n_{hybrid,i} - n_{static,i}\right)^2 + \sum_{i=1}^{n}\left(e_{hybrid,i} - e_{static,i}\right)^2}{n}} \tag{4}$$

where $h_{hybrid,i}$, $n_{hybrid,i}$, $e_{hybrid,i}$ is the ellipsoid height, northing, and easting, respectively, at passive mark *i* from the constrained adjustment of hybrid network *j*; $h_{static,i}$, $n_{static,i}$, $e_{static,i}$ is the ellipsoid height, northing, and easting at passive mark *i* from the constrained adjustment of the static-only network, and *n* is the number of passive marks in hybrid network *j*.

Figure 12 presents $HRMSE_j$ and $VRMSE_j$ for each hybrid network versus the number of 180-s NRTK observations per passive mark. The VRMSE values, which ranged from 1.3--2.2 cm for all hybrid networks, are nearly 1 cm smaller than the values reported in Table 1, which are based solely on evaluations of the accuracy of baseline observations obtained in real-time without adjustment. This finding shows how much vertical error was reduced by developing and adjusting a network of repeat baseline observations. HRMSE values ranged from only 0.6 to 1.0 cm with sub-millimeter reduction as the number of NRTK observation per mark increased.

Comparison of Methods for Estimating the Accuracy of the Results

In order to compare the two methods presented in this paper for estimating the accuracy of the adjusted coordinates, the network accuracies obtained from formal error propagation were reduced from the 95% confidence level to the 68% confidence level. This is because if systematic errors were removed (or mostly removed), then the values of RMSE should approximate accuracies at the 68% confidence level.

Dividing the formal vertical and horizontal network accuracies by 1.96 and 2.45, respectively, results in formal vertical network accuracies ranging from 0.3 to 1.8 cm and horizontal network accuracies ranging from 0.1 to 0.7 cm (68% confidence). Thus, the formal accuracy estimates are somewhat smaller than the empirical accuracy estimates found by computing $VRMSE_j$ and $HRMSE_j$ (Table 4). On one hand, it is reasonable based on these results to argue that the formal accuracy estimates are somewhat optimistic. On the other hand, it is also possible that some systematic error remained, thus causing the values of $VRMSE_j$ and $HRMSE_j$ to be slightly greater than the formal accuracies at 68% confidence. The coordinates derived from least squares adjustment of only the static GNSS baseline observations were treated as if they were errorless when used as a basis for computing $VRMSE_j$ and $HRMSE_j$. However, these coordinates have formal errors near the same order of magnitude as the formal errors of the coordinates derived from the hybrid survey networks and likely introduced some bias in RMSE.

Conclusions and Recommendations

Existing NGS height modernization surveying guidelines require long-duration static GNSS baseline observations (e.g., NGS-58) in order to derive ellipsoid heights with network accuracies less than 5 cm at 95% confidence. In an effort to develop a more efficient approach, this paper explored the idea of creating a "hybrid network" by combining NRTK baseline observations from an RTN with static post-processed baseline observations and performing a simultaneous least squares adjustment. In order to test the idea, a total of 30 hybrid networks were developed using static GNSS data from active stations along with repeat 180-s-duration NRTK baseline observations on 18 passive marks in Oregon and 20 passive marks in South Carolina. The results of the constrained least squares adjustments of the hybrid survey networks show great promise for significantly reducing the observational session per mark from (i.e., from hours to minutes) in order to derive ellipsoid heights with accuracies similar to values obtained from following NGS-58.

Fig. 3 shows the recommended workflow for developing a hybrid network; critical steps in this workflow are to use variance component estimation procedures to scale the VCV matrix output from the static baseline processing software (Step 2D) as well as the VCV matrix of the NRTK vectors (Step 1D). Such scaling is crucial in order to estimate realistic and compatible variances and covariances for development of an appropriate stochastic model for the hybrid network adjustment. Another important step in the hybrid network development is to ensure that the RTN is configured so as to provide NRTK vectors that are referenced to a physical base station in the RTN.

According to formal error propagation, network accuracies (at 95% confidence) on the passive marks in the hybrid networks ranged from 0.6 to 3.6 cm in ellipsoid height and 0.3 to 1.7

cm horizontally. These network accuracies are only 1.2 to 3.1 times larger than network accuracies achieved when post-processing and adjusting at least 30 h of static GNSS observations on the same passive marks in the test networks. Further, when comparing coordinates from the constrained adjustments of the hybrid networks with the coordinates from the constrained adjustment of a network of > 30 h of static GNSS observations, VRMSE ranged from 1.3 to 2.2 cm, and HRMSE ranged from 0.6 to 1.0 cm as the number of NRTK baseline observations per mark decreased from 9 to 3.

Generally, three to six 180-s-duration NRTK vectors to each passive mark were required to reduce the estimated vertical network accuracy to less than 2 cm (95% confidence). If survey specifications or guidelines were to be developed using the hybrid network approach, then for conservatism, the authors recommend collecting six 180-s-duration network NRTK vectors to each mark to achieve a vertical network accuracy of less than 2 cm (95% confidence). This conservatism is meant to account for the fact that site conditions at passive marks can be highly variable due to trees or other overhead obstacles, presence of power lines, etc., and that short occupations are more vulnerable to multipath. Since multipathing varies throughout the day and in an effort to make independent observations, the authors also recommend that the NRTK baseline observations on a mark are taken at different times of the day with independent antenna setups and NRTK initializations. Lastly, as a quality-control measure, the authors recommend checking the precision of the repeat NRTK observations at each mark while in the field in order to identify possible blunders or poor real-time solutions. Such a quality-control measure is one of the major benefits of conducting a real-time survey campaign and cannot be done in a static survey campaign which requires post-processing.

In addition, when available, it is recommended to use both GPS and GLONASS observables. The use of GPS and GLONASS NRTK vectors improved the network accuracy of the coordinates, on average, by 19% (4 mm) in ellipsoid height and by 7% (0.7 mm) horizontally versus the use of GPS-only observables. The use of GLONASS increases the number of available satellites on different orbital planes for deriving NRTK solutions which is especially beneficial at marks with overhead obstructions.

Although RTNs were shown to greatly optimize the derivation of cm-level geodetic coordinates on marks, improper settings or outdated firmware can lead to significant biases in the coordinates. In the real-time GNSS survey in South Carolina, four rovers with outdated firmware led to biases in the measured ellipsoid heights of the observed marks of up to +9 cm. This positive bias occurred because the firmware could not properly apply the nominal vertical antenna phase antenna offset for the base antennas in the SCRTN. Unfortunately, the biased observations made in real-time were highly precise; thus, these biases could not be identified in the field by simply taking several repeat observations and checking the precision of the results. Thus, an important recommendation is that users check each rover prior to a survey campaign by collecting NRTK observations on a passive mark with reliable geodetic coordinates. Checking that the coordinates derived from the RTK observations compare well with the "known" coordinates at control mark(s) minimizes the risk of collecting biased data due to faulty firmware or network configuration settings.

The hybrid networks used in this study only involved data collected from RTNs in Oregon and South Carolina. Future research should include testing the accuracy of hybrid networks constructed using NRTK vectors from other RTNs in differing geographic, topographic, and climatic settings. In addition, more testing is required to find the recommended separation of time between repeat NRTK observations on a passive mark. All of the hybrid networks described in this paper had a minimum time separation between repeat NRTK observations at a mark of approximately 2 hours. Further research could also analyze NRTK solutions using additional types and combinations of other viable GNSS (e.g., Galileo, BeiDou). Methods other than variance component estimation could also be explored for adjusting the GNSS vectors from the differing GNSS surveying modes (i.e., kinematic versus static) or baseline processing software which output incompatible VCV matrices.

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Table 1. Summary of empirical studies on the accuracy of NRTK baseline observations.

Source	**Location**	**HRMSE (cm)**	**VRMSE (cm)**	**Session Dur. (sec)**	**Comments**
Edwards et al. (2010)	Great Britain	1.0 - 2.0	1.5 - 3.0	1	Tested MAC and VRS; found 1 - 4 mm reduction in root-mean-square error (RMSE) when averaging 300 s of data
Wang et al. (2010)	Brisbane, Australia	2.0 - 3.0	4.0 - 5.0	1	Tested both MAC and VRS. Also tested longer than recommended interstation distances but results not shown here
Janssen and Haasdyk (2012)	New South Wales, Australia	0.5 - 1.2	0.9 - 2.1	1 - 600	VRS; collected NRTK data for 3 full days at each test mark
Martin and McGovern (2012)	Ireland	2.2	2.9	5	Tested both VRS and MAC using three independent RTNs; investigated effects of including GLONASS observables
Smith et al. (2014)	Texas, U.S.A.	1.5	2.7 - 2.8	6 or 180	VRS; data compared with 48-h static GNSS baseline observations processed in *OPUS-Projects*
Allahyari (2016)	Oregon, U.S.A.	1.1 - 1.6	2.0 - 2.7	5 - 900	MAC, GPS-only; compared with >40-h static GNSS baseline observations processed in *OPUS-Projects*
Allahyari* (2016)	South Carolina, U.S.A.	1.1 - 1.6	2.1 - 2.8	5 - 600	VRS; collected GPS-only and GPS+GLONASS data; compared with >30-h static GNSS baseline observations processed in *OPUS-Projects*

* = The South Carolina NRTK data evaluated in Allahyari (2016) were also used in this study

Table 2. Standard deviation factors for the VCV matrix of the NRTK vectors in each hybrid network; scaling the VCV matrix by the square of these factors and performing a minimally constrained least squares adjustment of the NRTK vectors resulted in σ_0 equal to 1.

Hybrid Network Designation	**No. of 180-s NRTK vectors per mark**	**Oregon (GPS-only)** Scale Factor		**South Carolina (GPS-only)** Scale Factor		**South Carolina (GPS+GLONASS)** Scale Factor	
		Horiz	*Vert*	*Horiz*	*Vert*	*Horiz*	*Vert*
3A	3	2.16	2.13	1.51	1.87	2.10	1.57
3B	3	1.74	2.41	1.53	1.75	1.61	1.76
4A	4	2.08	1.83	1.64	1.72	2.03	1.85
4B	4	1.97	2.40	1.52	2.29	1.98	1.65
5A	5	1.99	1.95	1.62	1.68	1.89	1.79
5B	5	2.07	2.40	1.52	2.11	1.92	1.65
6A	6	2.05	1.94	1.52	1.71	2.00	1.66
6B	6	1.98	2.31	1.54	1.58	1.97	1.87
9A	9	2.03	2.16	1.68	1.86	1.99	1.75
9B	9	2.09	2.29	1.50	1.78	1.88	1.84
	Average =	2.01	2.18	1.56	1.84	1.94	1.74

Table 3. Standard deviations of unit weight (σ_0) for the test hybrid networks from either a minimally constrained (MC) or fully constrained (FC) least squares adjustment

Hybrid Network Designation	No. of 180-s NRTK vectors per mark	Oregon (GPS-only) σ_0		South Carolina (GPS-only) σ_0		South Carolina (GPS+GLONASS) σ_0	
		MC	FC	MC	FC	MC	FC
3A	3	1.00	1.04	1.00	1.06	1.00	1.06
3B	3	1.00	1.01	1.00	1.06	1.00	1.06
4A	4	1.01	1.02	1.00	1.05	1.00	1.05
4B	4	1.01	1.02	1.00	1.06	1.00	1.05
5A	5	1.01	1.03	1.00	1.05	1.00	1.04
5B	5	1.00	1.02	1.00	1.05	1.00	1.05
6A	6	1.00	1.03	1.00	1.05	1.00	1.05
6B	6	1.00	1.01	1.00	1.05	1.00	1.04
9A	9	1.00	1.01	1.00	1.04	1.00	1.03
9B	9	1.00	1.02	1.00	1.04	1.00	1.03
	Average =	1.00	1.02	1.00	1.05	1.00	1.05

Table 4. Summary of the estimates of the accuracy of the adjusted coordinates from the 30 hybrid GNSS survey networks

	Range of Formal Network Accuracies (68% confidence)	Range of RMSE values
Vertical	0.3 - 1.8	1.3 – 2.2
Horizontal	0.1 - 0.7	0.6 – 1.0

Figure 1

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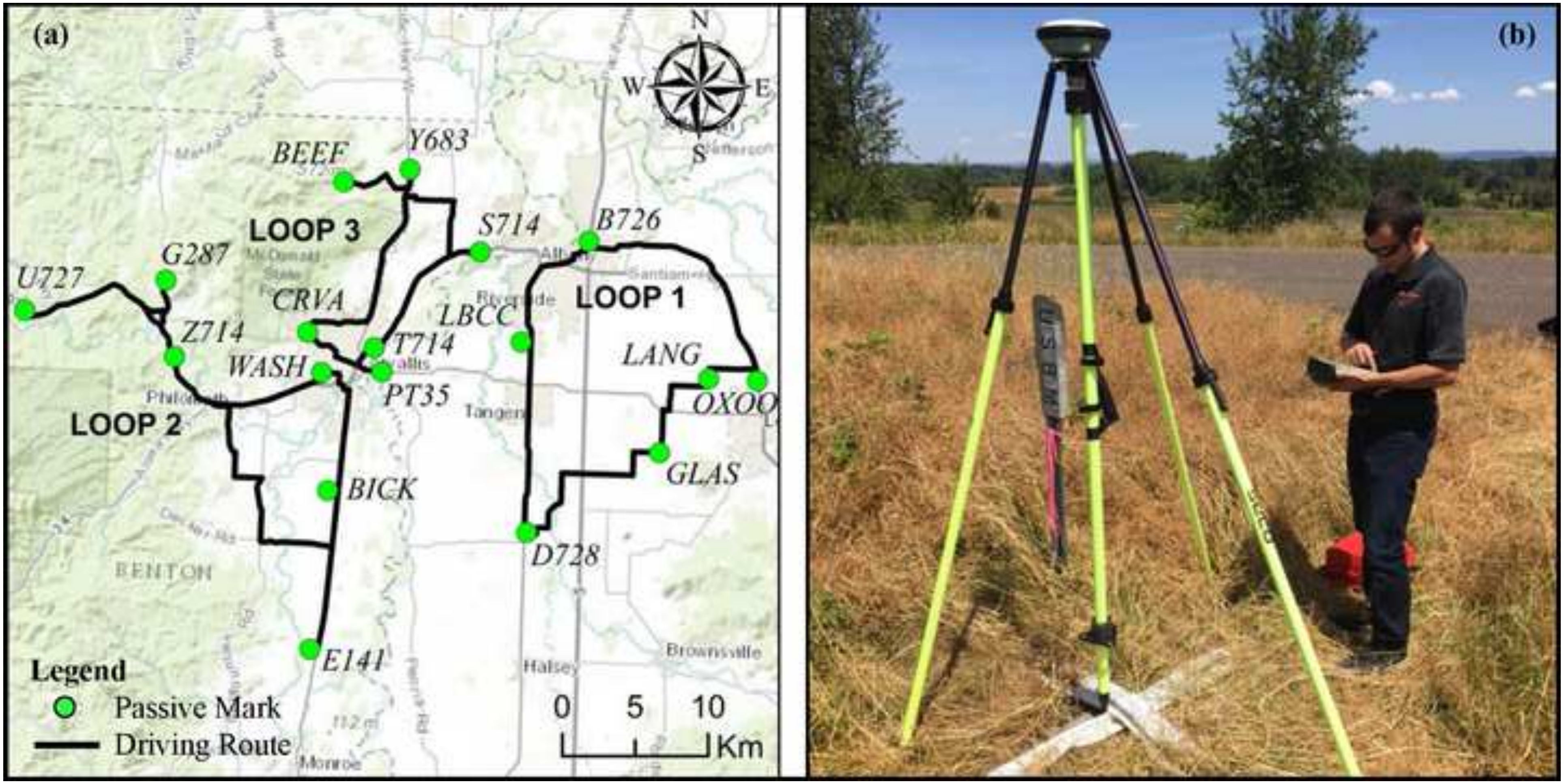

Figure 2

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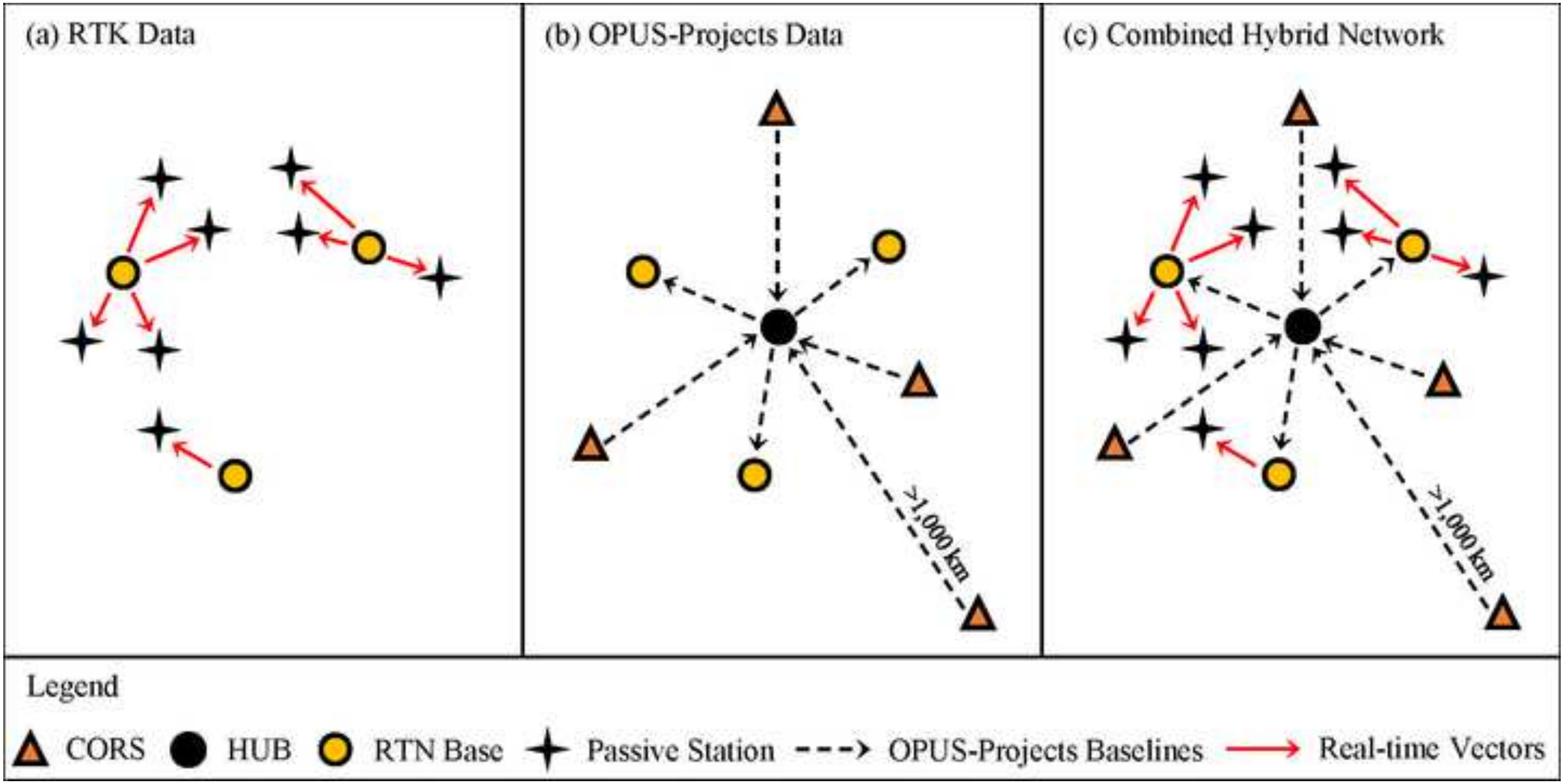

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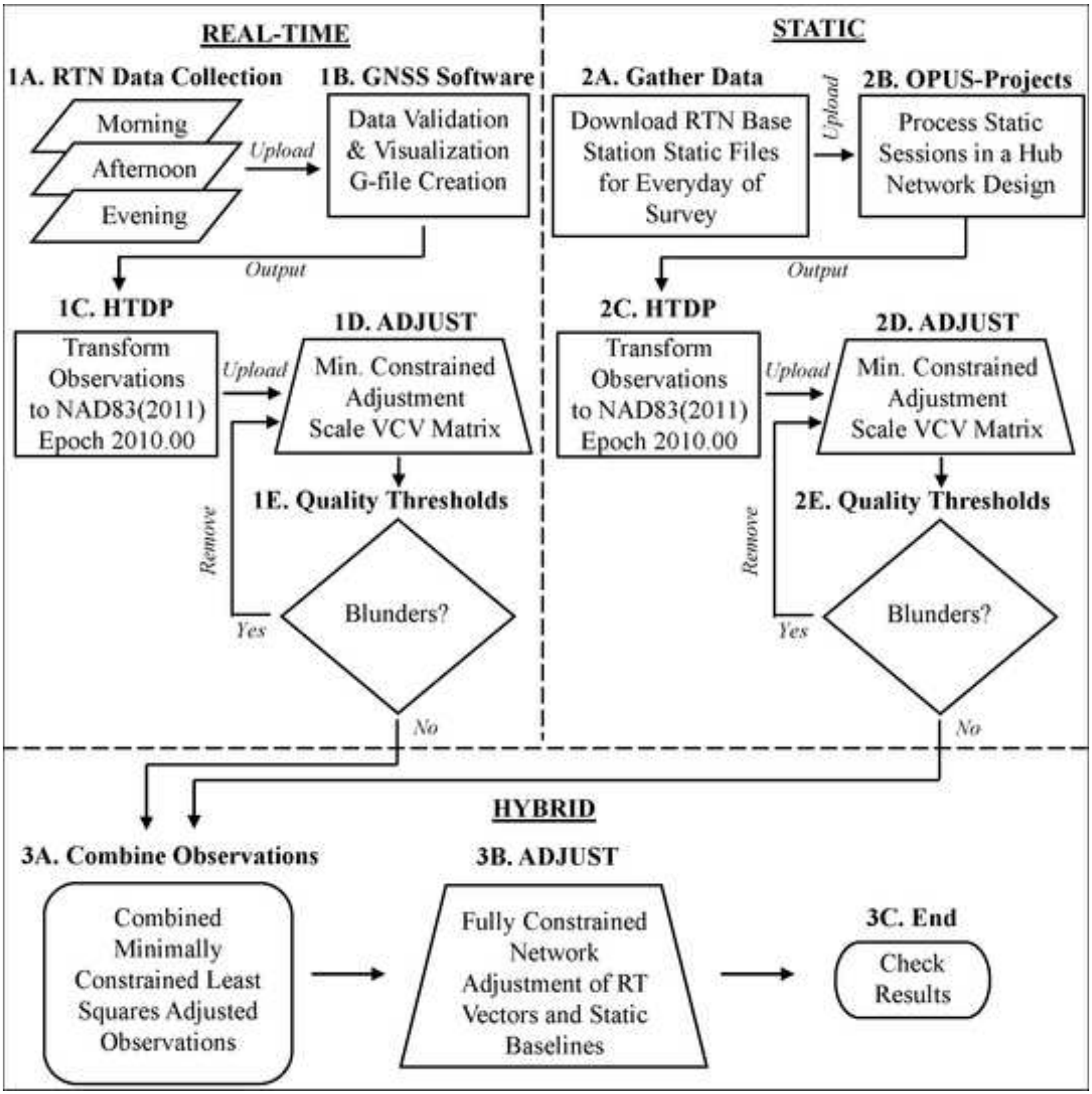
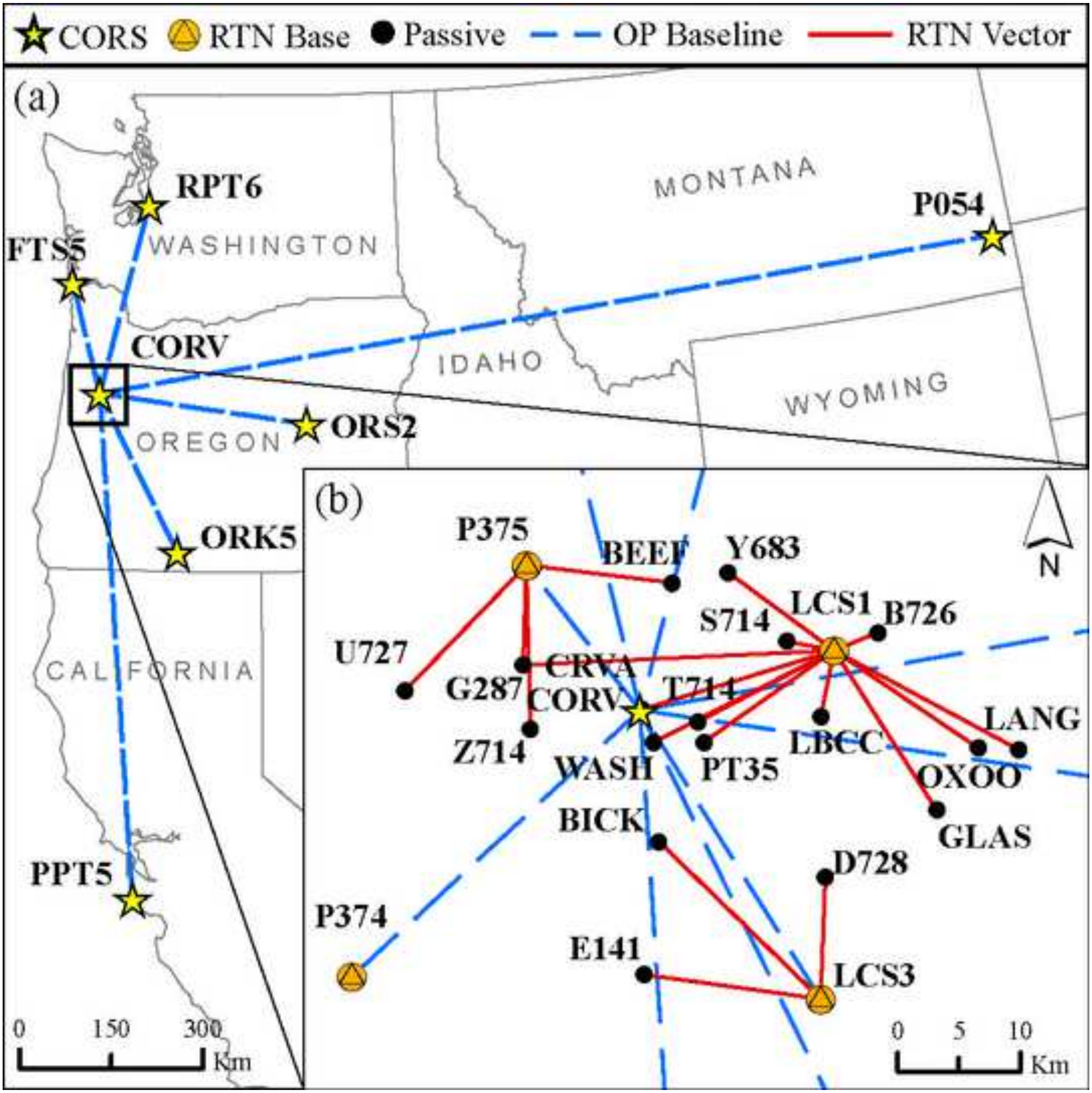


Figure 4 Click here to download Figure Fig4.tiff



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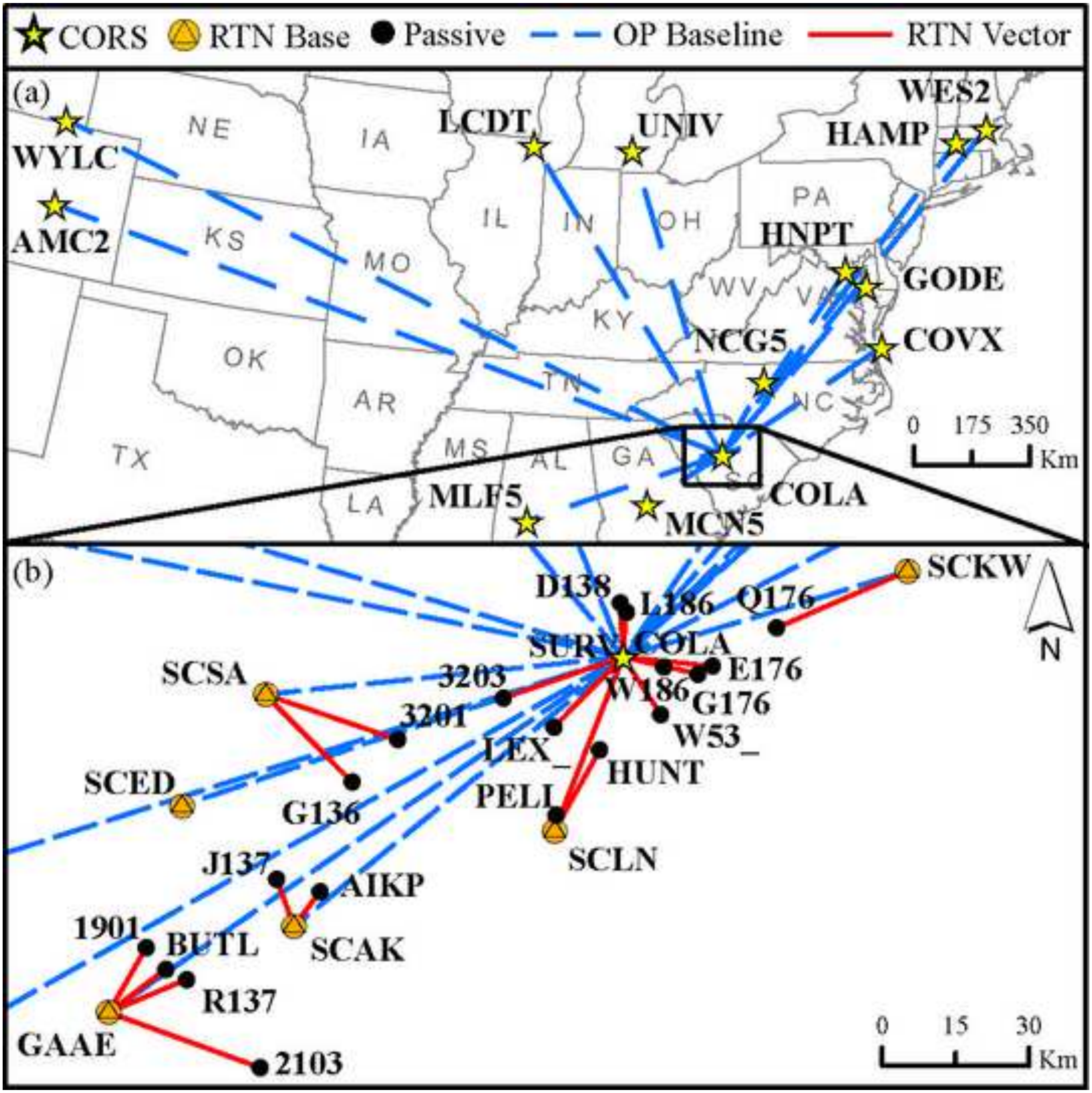

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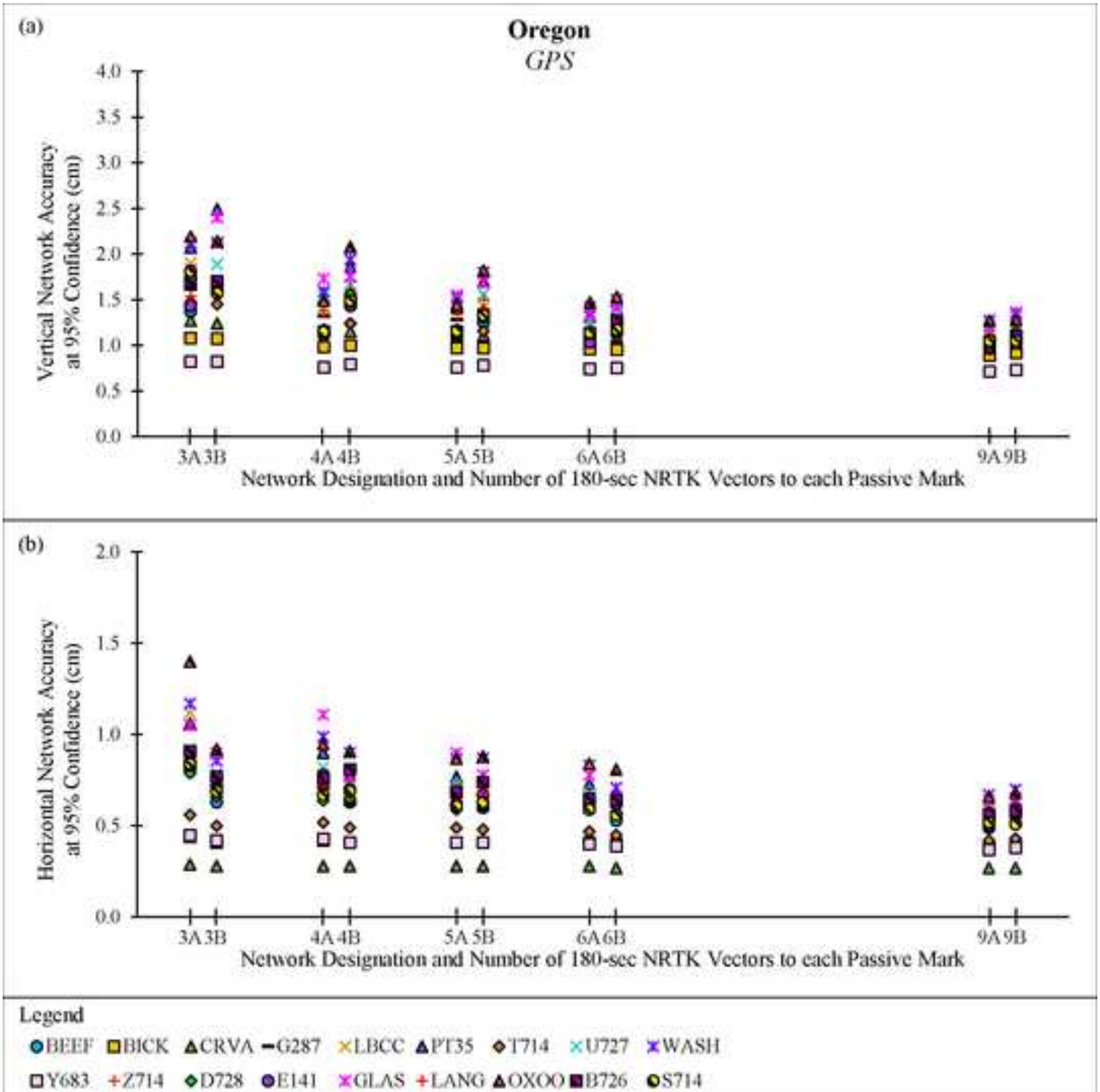

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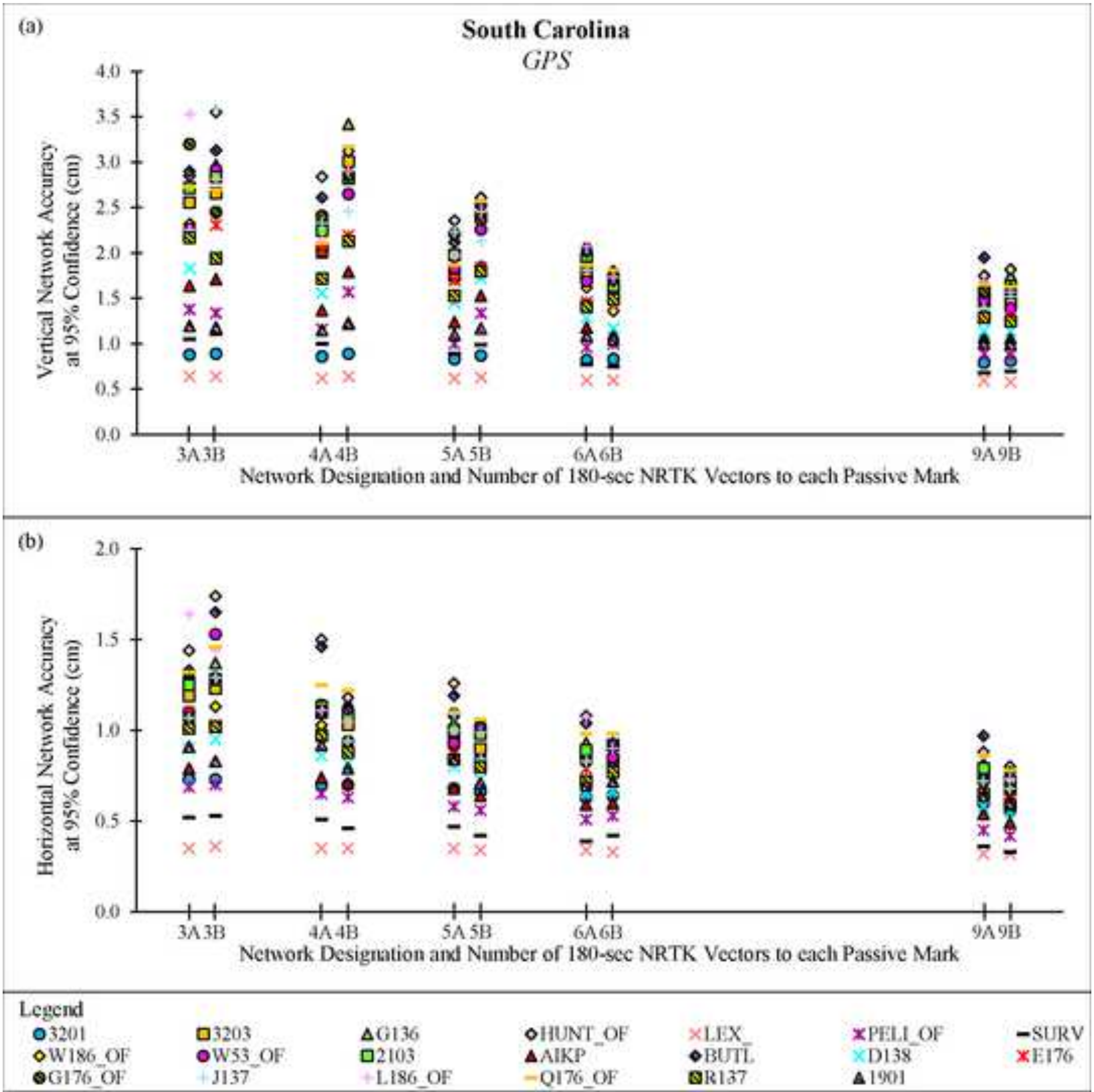

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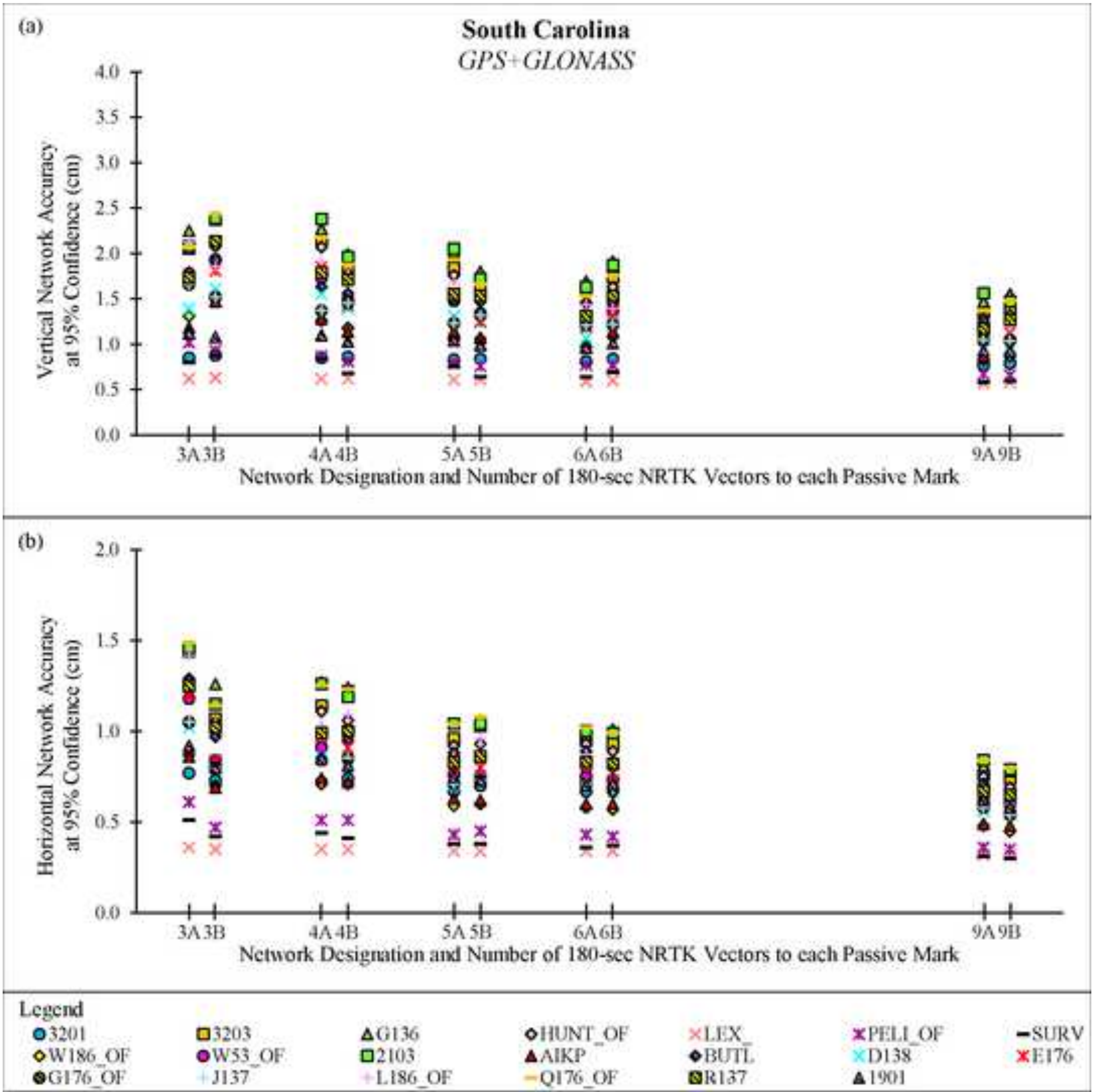

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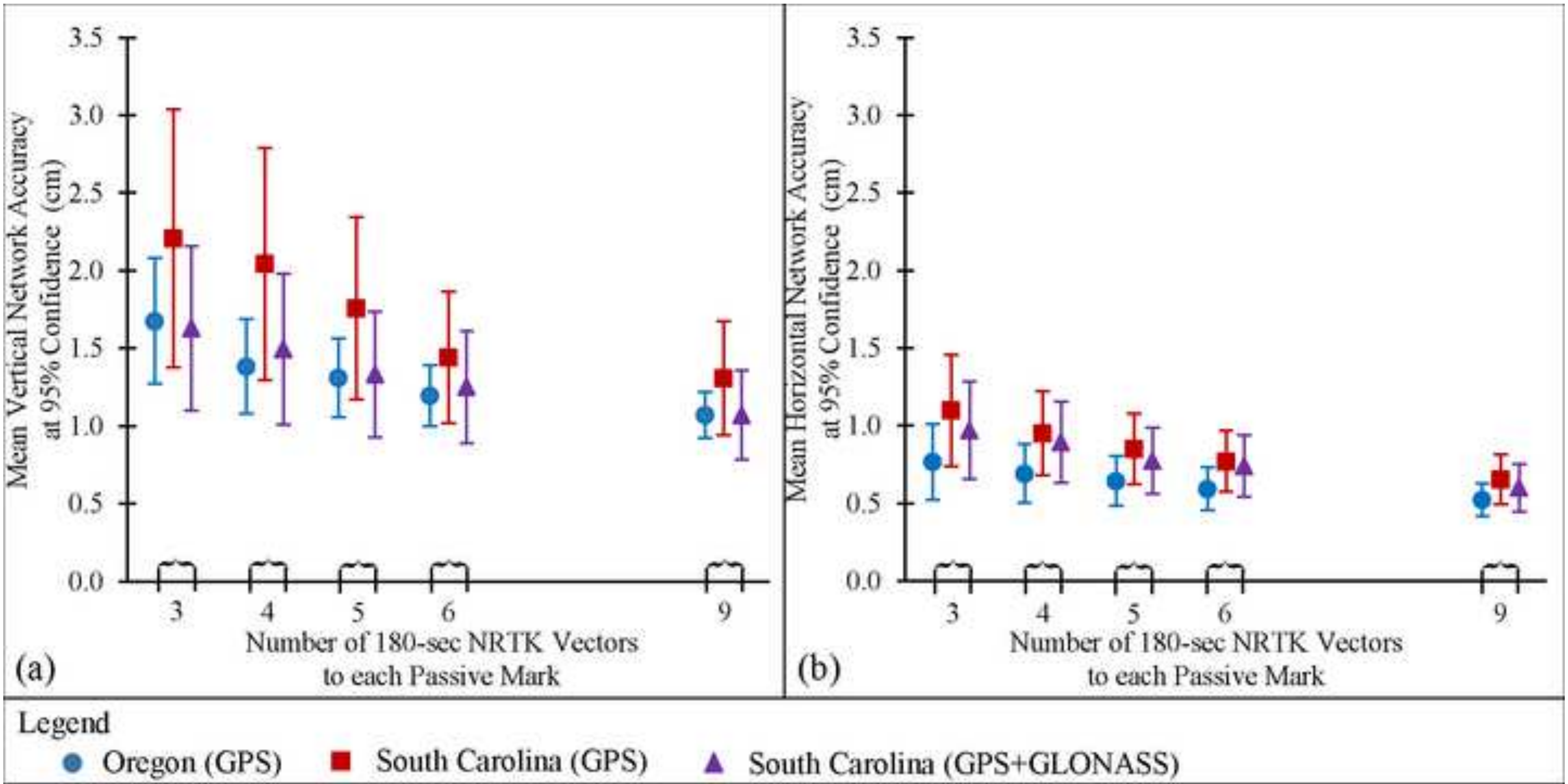

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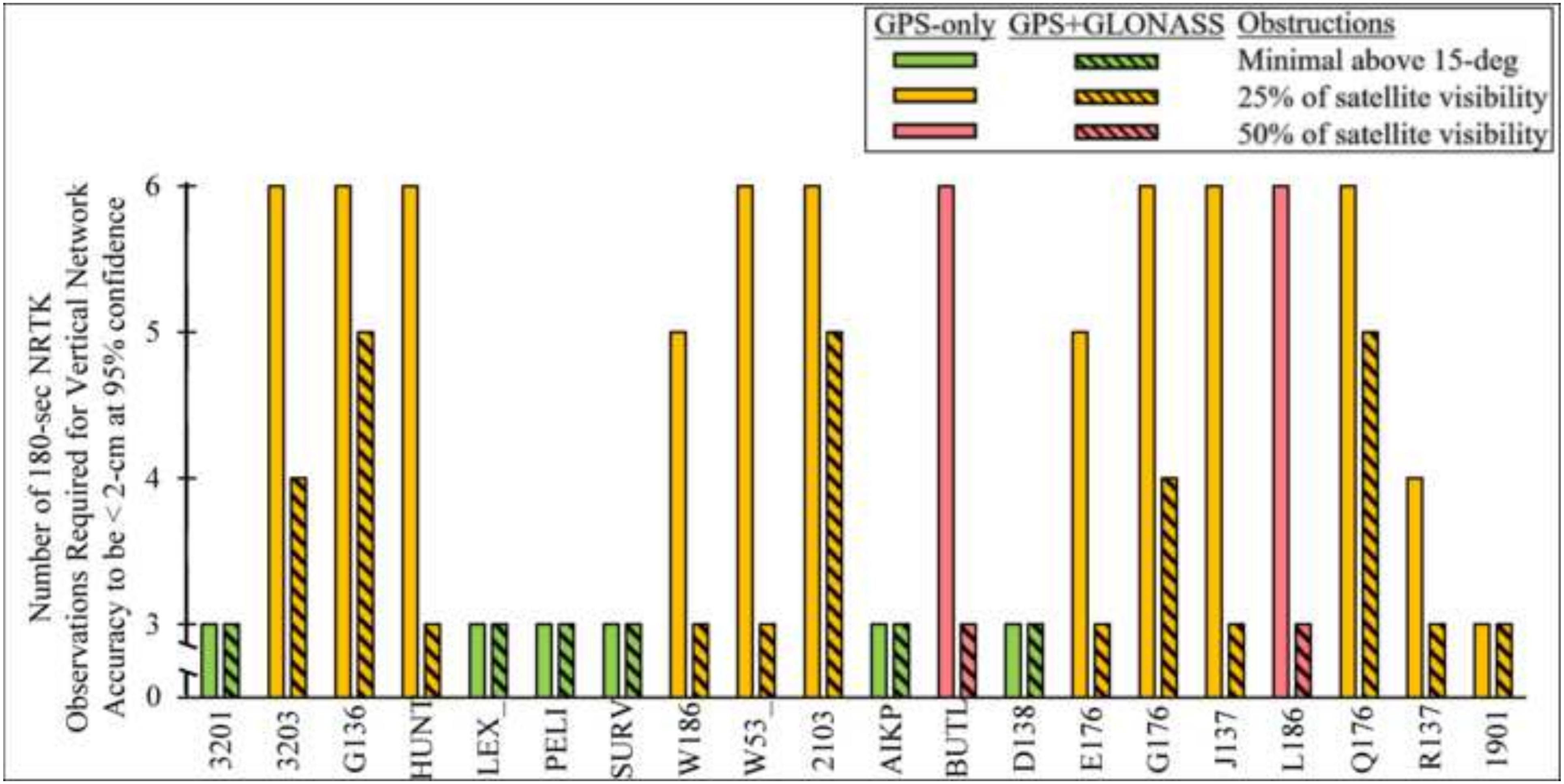

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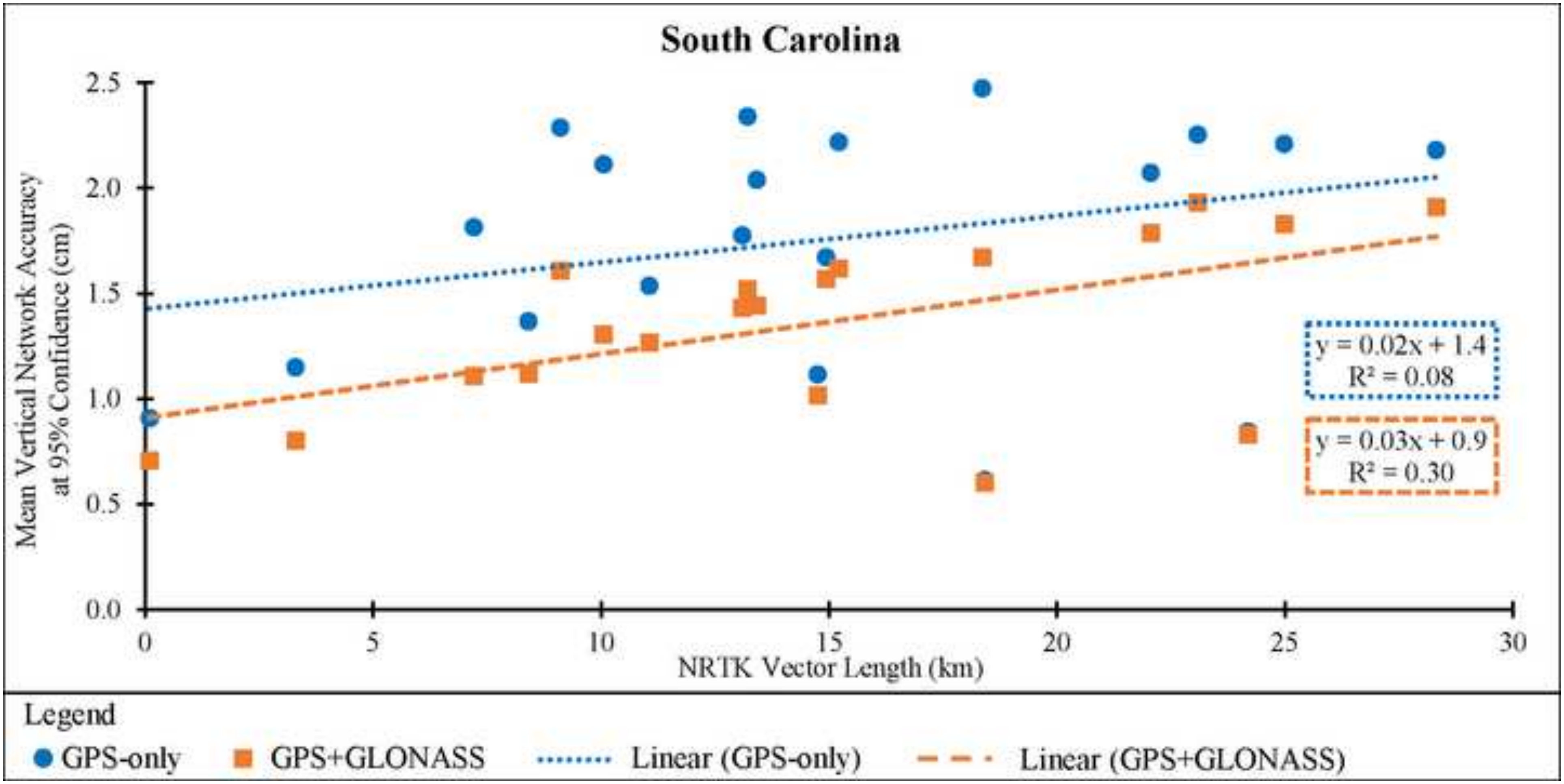

Figure 12

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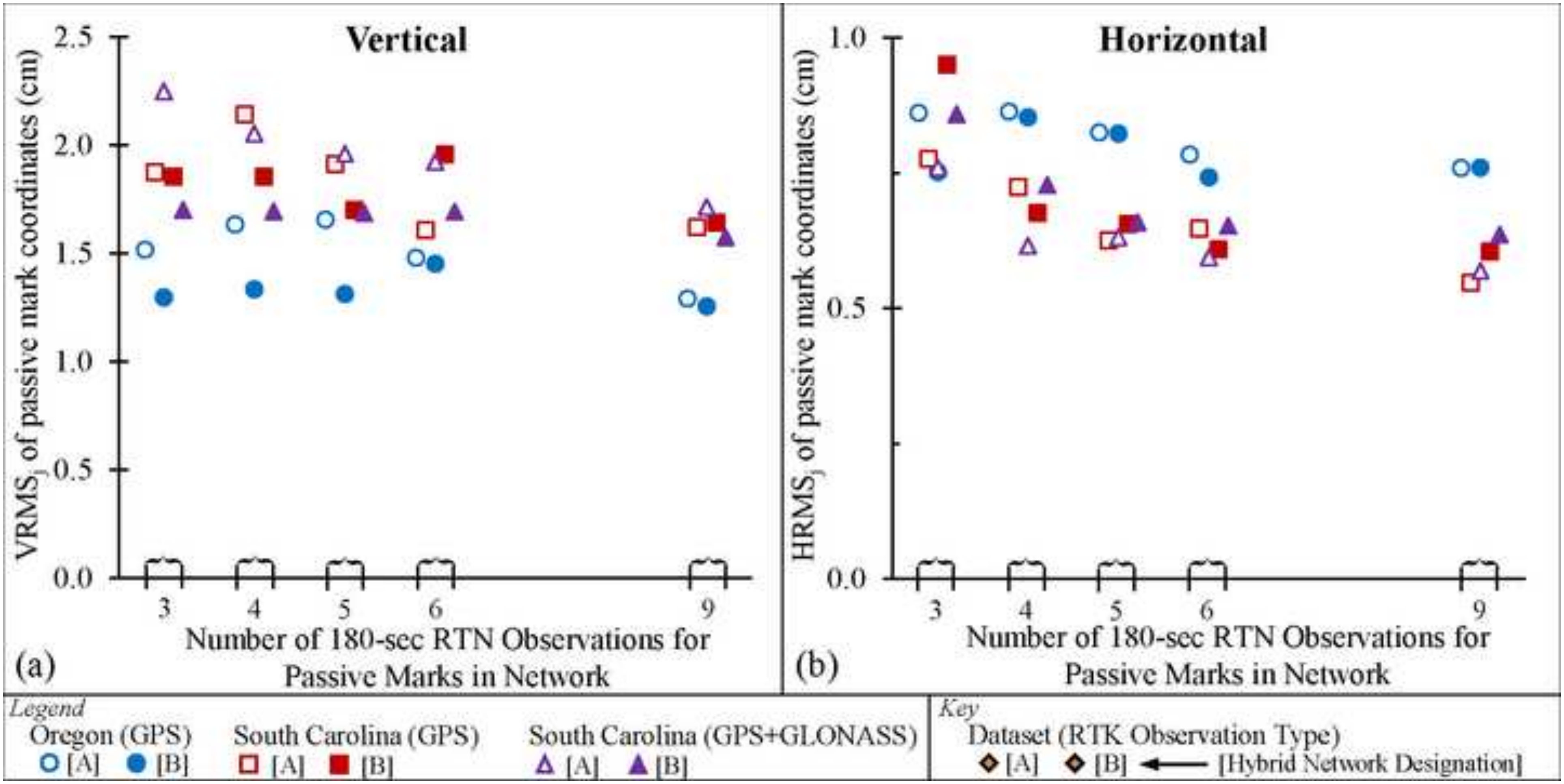

List of Captions

Fig. 1. (a) Driving loop assignments for each party during the 2016 NRTK survey in Oregon; (b) typical setup on a passive mark, involving a 2-m fixed-height tripod, Leica GS14 receiver/antenna, and Leica CS15 data collector (image by Daniel T. Gillins).

Fig. 2. Hybrid network conceptual design where (a) NRTK vectors are added with (b) vectors derived from post-processing static GNSS observations at the active stations to produce (c) a final combined or "hybrid" survey network for adjustment.

Fig. 3. Flowchart for developing and adjusting a hybrid survey network that consists of NRTK GNSS vectors and static GNSS vectors processed in *OPUS-Projects*

Fig. 4. Diagram of hybrid network 6A in Oregon: (a) six distant CORS and the hub (CORV); (b) detail of project area, passive marks, and active stations in the ORGN

Fig. 5. Diagram of hybrid network 6A in South Carolina: (a) CORS and the hub (COLA); (b) detail of project area, passive marks, and active stations in the SCRTN

Fig. 6. Resulting (a) vertical (in terms of ellipsoid height) and (b) horizontal network accuracy at each mark versus number of 180-s, GPS-only NRTK vectors to each passive mark; Oregon hybrid networks

Fig. 7. Resulting (a) vertical (in terms of ellipsoid height) and (b) horizontal network accuracy at each mark versus number of 180-s, GPS-only NRTK vectors to each passive mark; South Carolina hybrid networks

Fig. 8. Resulting (a) vertical (in terms of ellipsoid height) and (b) horizontal network accuracy at each mark versus number of 180-s, GPS+GLONASS NRTK vectors to each passive mark; South Carolina hybrid networks

Fig. 9. Mean (a) vertical (in terms of ellipsoid height) and (b) horizontal network accuracy (at 95% confidence) for the coordinates derived at the passive marks versus number of 180-s NRTK vectors to each passive mark; Oregon and South Carolina hybrid networks (error bars are 1 standard deviation)

Fig. 10. Number of 180-s NRTK observations required to produce an adjusted ellipsoid height with an average vertical network accuracy less than 2.0 cm; South Carolina hybrid networks

Fig. 11. Average vertical network accuracy at each mark from the constrained adjustments of the 10 hybrid networks involving GPS-only NRTK vectors or GPS+GLONASS NRTK vectors versus baseline length; South Carolina

Fig. 12. (a) $VRMSE_j$ and (b) $HRMSE_j$ of the coordinates of the passive marks derived from constrained adjustment of the hybrid networks