

1 **Lateral Spreading within a Limit Equilibrium Framework: Newmark Sliding Blocks**
2 **with Degrading Yield Accelerations**

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16 **ABSTRACT**

17 Lateral spreading is a prevalent geotechnical problem associated with earthquake-
18 induced liquefaction, often occurring at gentle slopes of loose, saturated sand near bodies
19 of water and causing significant damage to buried utilities. This study presents a
20 deterministic approach to analyze lateral spreading behavior using a modified Newmark
21 analysis applied to a column of sliding blocks with degrading yield accelerations. The
22 proposed sliding column approach exhibits reasonable agreement with a well-
23 instrumented, centrifuge test evaluating free-field lateral spreading. The analysis captures
24 lateral spreading displacement throughout a soil profile as well as shear strains and
25 simplified earth pressures. The effect of light cementation is investigated, demonstrating
26 notable arrest of lateral spreading displacements and pressures. Free face effects are
27 also evaluated for a liquefying layer of soil beneath a gentle, competent crustal slope,
28 demonstrating notable lateral spreading behavior with larger inclinations of liquefying soil.
29 However, lateral spreading still occurred when considering a horizontal liquefying layer,
30 realized due to inertial loading and differences between confining boundary forces. The

31 approach can be utilized to efficiently analyze lateral spreading across a large spatial
32 extent.

33

34 **Keywords:** Limit Equilibrium, Lateral Spreading, Liquefaction, Earthquakes, Natural
35 Hazards

36

37 **INTRODUCTION**

38 Lateral spreading is generally considered the most pervasive type of liquefaction-
39 induced ground failure generated by earthquakes (NRC 1985). When lateral spreading
40 occurs, mostly intact blocks of surficial, crustal soil situated atop weak, liquefied strata
41 displace down gentle slopes. Lateral spreading often occurs towards areas of large
42 topographic relief (e.g. river channels, marine structures). The laterally-spreading blocks
43 may encompass large areas, displacing several meters and generating large lateral loads
44 on buried structures, resulting in considerable damage to bridges, buildings, pipelines,
45 roadways, marine structures and more. Assessing lateral spreading potential and
46 displacements has primarily been performed using case history-based statistical
47 methods, experimental approaches and analytical approximations using Newmark sliding
48 block analyses.

49 A promising means of evaluating lateral spreading, while incorporating aspects of
50 topography, soil conditions and porewater pressures, is the use of a limit equilibrium
51 analysis that accounts for time-dependent excess porewater pressure (PWP) buildup and
52 inertial loading for a series of sliding blocks. These analyses frequently implement rigid
53 body mechanics (e.g. a “rigid” sliding block) considering limit state equilibrium with a sum

54 of forces, moments or both. Unfortunately, the typical rigid body assumption is a limitation
55 when considering liquefying sandy soil, which has a relatively low volumetric threshold
56 strain between elastic and plastic response (i.e., approximately 0.01 to 0.02%; Vucetic
57 1994). Limit equilibrium approaches are also hindered by an inability to determine
58 displacements; i.e., limit equilibrium analyses implicitly only capture the ultimate limit
59 state, which is representative of complete collapse and instability, and neglects
60 displacements at failure. Fortunately, the modified Newmark approach (Newmark 1965)
61 merges the simplicity of limit equilibrium analyses with displacement estimation via a
62 determined yield acceleration, k_y , for a given sliding block. Note that k_y is the horizontal
63 pseudostatic seismic coefficient, k_h , that causes limit equilibrium failure for a given sliding
64 block. To determine global displacement of a sliding block with the modified Newmark
65 method, the earthquake motion acceleration values that exceed the yield acceleration are
66 double-integrated as a function of time.

67 A potential means for capturing the time-dependent interaction of excess porewater
68 pressure with inertial loading in a limit equilibrium analysis is the consideration of time-
69 dependent yield accelerations. A buildup of excess porewater pressure reduces the
70 frictional shear strength of a given soil, ultimately lowering the yield acceleration required
71 for permanent displacement to occur. Matasovic and Kavazanjian (1997) evaluated
72 Newmark displacements considering degrading yield accelerations. Biondi et al. (2000)
73 extended this approach to an infinite slope analysis for coarse-grained, saturated soil,
74 further compounding the effects of excess porewater pressures on slope stability and
75 realized displacements. Kramer and Smith (1997) expanded upon the concept of a
76 singular sliding block for slope stability analyses by considering a series of coupled sliding

77 blocks with dashpots to evaluate displacements above a given sliding plane. The Kramer
78 and Smith (1997) method is particularly useful for large landfill slopes that are
79 characterized by low natural frequencies.

80 The present study extends the previous research to a column of sliding blocks, which
81 is used in a limit equilibrium/modified Newmark framework to evaluate displacement
82 profiles, shear strains, liquefaction depth and earth pressures. The primary goal of the
83 article is presenting the mechanics used to develop the modified Newmark framework for
84 predicting liquefaction-induced lateral spreading displacements (i.e., the methodology).
85 In addition, a simple verification exercise is performed using results from a well-
86 documented centrifuge test (Sharp and Dobry 2002) and some preliminary results are
87 discussed.

88

89 **BACKGROUND**

90 Due to the complexity associated with observed post-earthquake lateral spreading,
91 displacement models are mostly limited to statistical regression analyses of well-
92 documented case histories (Bartlett and Youd 1995, Rach and Martin 2000, Youd et al.
93 2002, Zhang et al. 2004, Gillins and Bartlett 2013). Regression models used to estimate
94 lateral spreading-based displacements are often based on topographic, geotechnical,
95 and seismic data. Important input parameters used with empirical and semi-empirical
96 models for estimating lateral spread displacements include: free-face ratio, ground slope,
97 standard penetration test (SPT) blow counts ($N_{1,60}$), mean grain size (D_{50}), fines content
98 (F), earthquake magnitude (M), and distance to the seismic source (R), within liquefiable
99 layers (Youd et al. 2002, Bartlett and Youd 1995, Gillins and Bartlett 2013). The most

100 widely used technique in practice is the Youd et al. (2002) empirical procedure (Olson
101 and Johnson, 2008), which was developed by multilinear regression (MLR) of a large
102 case history database. Using the same case history database, Gillins and Bartlett (2013)
103 revised the MLR empirical procedure to use more widely available geotechnical data,
104 thereby making the procedure more implementable for regional mapping of lateral spread
105 hazards. Following the work of Gillins and Bartlett (2013) and Franke and Kramer (2014),
106 Ekstrom and Franke (2016) used the empirical lateral spread displacement database to
107 develop a simplified performance-based prediction methodology, which is also applicable
108 to regional-scale hazard mapping. Such maps are useful for conveying hazards to public
109 utility companies, risk managers, and the earthquake engineering community.
110 Nonetheless, there is still limited susceptibility and hazard mapping of liquefaction-
111 induced lateral spread based on generalized, mechanistic analyses.

112 Within regions susceptible to liquefaction, it is critical to evaluate potential
113 displacements and structural loads induced by the temporary soil instability.
114 Displacements due to lateral spread have been evaluated using a variety of methods,
115 particularly using a modified Newmark (1965) analysis, which is an accepted practical
116 approach to estimating displacements of slopes (Wartman et al. 2003, Kramer and Smith
117 1997), walls (Rathje and Bray 2000, Matasovic and Kavazanjian 1997, Whitman and Liao
118 1985) and other structures under seismic loading. Olson and Johnson (2008) found that
119 classical Newmark-based back-calculations coincided well with lateral spreads from
120 various well-documented case studies and suggested that cone penetration tests (CPT)
121 were particularly useful for determining liquefaction potential. Brandenberg et al. (2007)
122 used a sliding block and wedge failure method to evaluate the moment and shear profiles

123 on a series of piles underneath a pile cap subject to lateral spreading. Notably, the cap
124 was treated as a sliding block subject to inertial loading, and Brandenberg et al. (2007)
125 corroborated analytical models with centrifuge testing. Taboada and Dobry (1998) used
126 centrifuge testing to evaluate lateral spreads for various slopes and configurations, and
127 found that dilation during lateral spreading possibly reduces excess porewater pressures
128 during shear; thus, potentially arresting the lateral spreads. Similar approaches for non-
129 linear resistance of sliding blocks have been implemented for other sliding block analyses
130 (e.g. Rathje and Bray 2000). However, these approaches have generally been related to
131 centrifuge tests or case studies, which are limited in application for evaluating regions of
132 lateral spread. Despite the prevalence of lateral spreading of gentle slopes and slopes
133 near free face relief, research evaluating lateral spreading as a generalized slope stability
134 problem is scarce. Development of a limit equilibrium analytical framework for evaluating
135 lateral spread-induced displacements and loading as a time-dependent problem enables
136 hazard assessment based on principles of soil mechanics, which broadens the
137 applicability of the methods and enhances existing geotechnical design approaches.

138 The proposed limit equilibrium approach evaluates yield accelerations of multiple
139 thin, rigid slices within a soil column. The time-dependent yield accelerations are
140 determined throughout the soil column, and ultimately, the displacement for each
141 discretized slice for a given acceleration-time series is determined. The proposed limit
142 equilibrium approach assumes liquefying soil is a viscous material that does not fail as a
143 rigid element. By evaluating numerous thin slices placed in a column of sliding blocks and
144 solving for boundary forces on each slice, the liquefiable layer is “laminar,” which enables
145 the evaluation of equilibrium within each thin sliding element (i.e., effectively discretizing

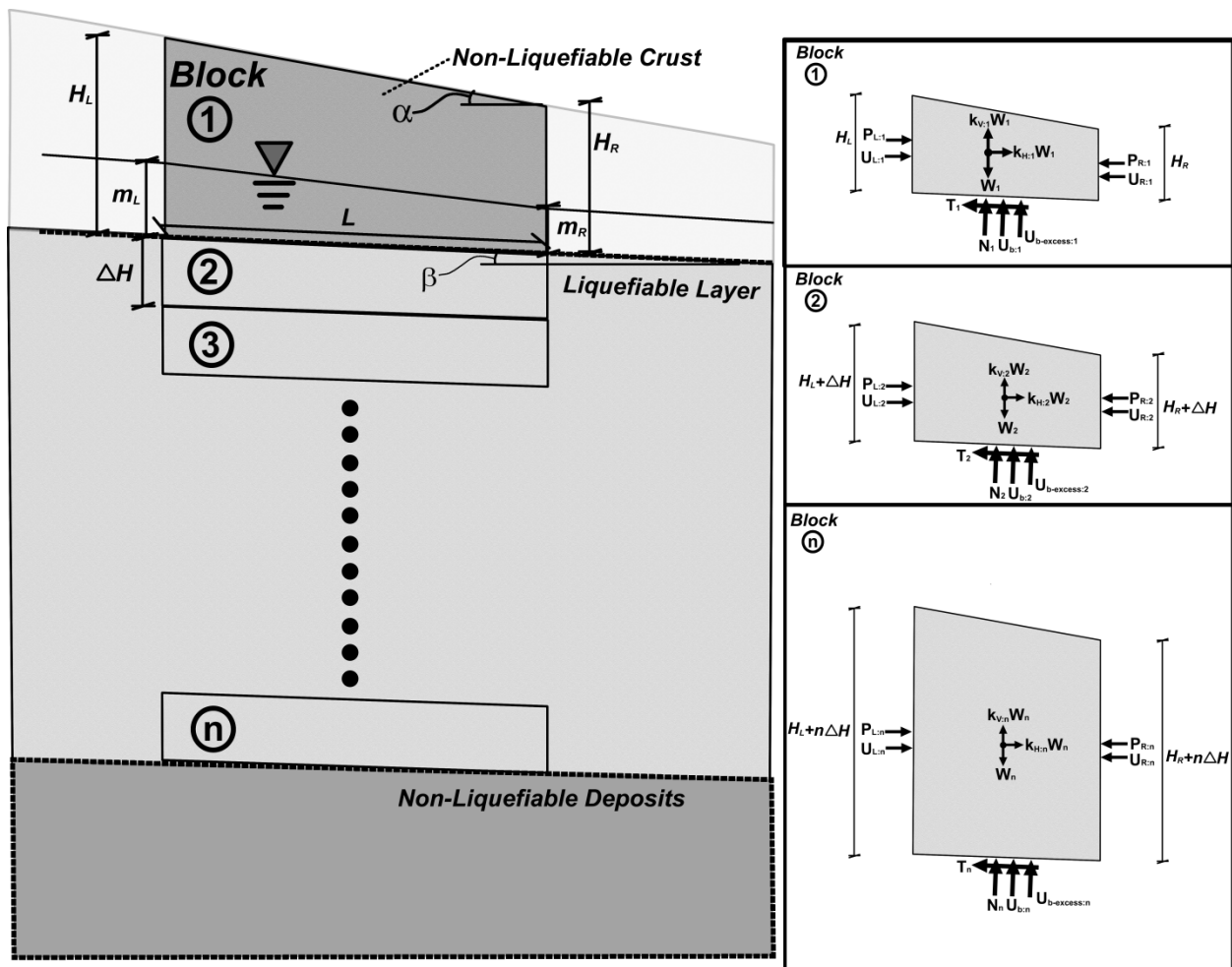
146 the liquefying soil). Furthermore, evaluation of boundary forces (e.g., slice weights, lateral
147 earth pressure resultants, shear resistance, pseudo-static seismic acceleration) and
148 boundary neutral forces (e.g., porewater pressure, excess porewater pressures)
149 surrounding each slice enables a time-dependent analysis of displacements under
150 seismic acceleration as well as the determination of a time-dependent yield acceleration
151 that accounts for the deleterious effects of excess porewater pressure buildup. That is,
152 the column of slices representative of liquefiable soil are evaluated in a limit equilibrium
153 framework for specified time increments using a modified Newmark approach for sliding
154 blocks with changing yield accelerations from excess porewater pressure buildup. Thus,
155 it captures not only the laminar, “flexible” displacements throughout the depth of a
156 liquefying, lateral spreading layer, but related earth pressures, depth of liquefaction front
157 (i.e. where the lateral spreading has initiated), and shear strain throughout a given soil
158 profile. The analyses performed herein often overestimates lateral spreading
159 displacements due to omission of complex behavior (e.g., dilative and viscous behavior
160 of liquefied soil); nevertheless, the analysis framework presents a useful deterministic
161 means of evaluating lateral spreading and related trends.

162

163 **METHODOLOGY**

164 The proposed framework evaluates lateral spreading using a limit equilibrium
165 analysis similar to that presented by Biondi et al. (2000), but applied to a column of
166 Newmark sliding blocks with time-dependent shear strength (dependent on input excess
167 pore water pressures) and time-dependent driving forces (seismic acceleration). The
168 column of sliding blocks (Figure 1) is analyzed with the modified Newmark approach with

169 time-dependent yield accelerations subject to excess pore water pressures and seismic
 170 motions. Cumulative displacements of both crustal layers and liquefying soil is
 171 subsequently used to generate (1) displacement profiles, (2) shear strain profiles, (3)
 172 liquefied earth pressures, and (4) liquefaction front depths. The results demonstrate good
 173 agreement with well-instrumented centrifuge tests investigating lateral spreading for soil
 174 having a small inclination ($\sim 5^\circ$) with buildup of excess porewater pressure and seismic
 175 excitation (Sharp and Dobry 2002).
 176



177

178

179

Figure 1. Free-body diagrams of sliding blocks under liquefaction conditions.

180 As previously mentioned, lateral spreading occurs when relatively intact blocks of
 181 soil slide as excess porewater pressures increase in subsurface layers, which reduces
 182 the shear strength and can lead to liquefaction. These preceding conditions can be
 183 modeled by a simplified sliding block analysis (Figure 1) defined by a trapezoidal block
 184 with boundary depths, H_L and H_R , a surface slope, α , above a layer of liquefiable soil with
 185 slope angle, β , and length, L . The weight of the block is W , the inertial forces are $k_v W$
 186 and $k_h W$, the normal resultant force is N , the shear resistance force is T , the boundary
 187 earth pressure resultants are P_L and P_R , the lateral boundary hydraulic resultant forces
 188 are U_1 and U_2 , the base boundary hydraulic resultant force representative of a static
 189 phreatic surface is U_b , and an excess pore water pressure resultant force that may induce
 190 lateral spread is $U_{b-excess}$. These porewater pressures are not explicitly determined in the
 191 presented framework, but can be determined from experimental results, numerical
 192 models, site response analyses, or simplified theoretical approaches such as Seed et al.
 193 (1977). A series of n sliding blocks can be represented similarly by adding perpendicular
 194 forces on top of a block, which represents the overburden of both the intact surficial block
 195 and the liquefied soil above.

196 Considering pseudostatic forces perpendicular and parallel to the basal, liquefied
 197 slip surface, static equilibrium can be determined for each slice as:

198

$$199 \quad \sum F_{\parallel} = 0 = (P_{L:n} + U_{L:n})\cos\beta - k_{v:n}W_n\sin\beta + k_{h:n}W_n\cos\beta + W_n\sin\beta - (P_{R:n} + U_{R:n})\cos\beta -$$

$$200 \quad T_n \quad (1)$$

201

202 $\sum F_{\perp} = 0 = N_n + U_{b:n} + U_{b:n-excess} + (P_{L:n} + U_{L:n})\sin\beta + k_{v:n}W_n\cos\beta + k_{h:n}W_n\sin\beta -$
 203 $W_n\cos\beta - (P_{R:n} + U_{R:n})\sin\beta \quad (2)$

204

205 where W_1 is the weight of the crustal block and W_n is the summed weight of laminar
 206 slices, which are both defined for a unit depth (i.e., for a 2D problem) as:

207

208
$$W_n = W_1 + \sum_2^n [0.5\gamma_n L(H_{L:n} + H_{R:n} - m_{L:n} - m_{R:n})(1) + 0.5\gamma_{sat:n} L(m_{L:n} + m_{R:n})(1)]$$

209 (3)

210

211 Herein, for simplicity, a linear phreatic surface is assumed where the basal hydrostatic
 212 pore pressure resultant, $U_{b:n}$, is defined as:

213

214
$$U_{b:n} = U_{b:1} + \sum_2^n 0.5\gamma_w L(m_{L:n} + m_{R:n})(1) \quad (4)$$

215

216 The shear force resultant is:

217

218
$$T_n = \tau_n L \quad (5)$$

219

220 Substituting (5) into (1), static equilibrium parallel to the basal liquefying surface can be
 221 determined as:

222

223 $\sum F_{\parallel} = 0 = (P_{L:n} + U_{L:n})\cos\beta - k_{v:n}W_n\sin\beta + k_{h:n}W_n\cos\beta + W_n\sin\beta - (P_{R:n} +$
 224 $U_{R:n})\cos\beta - \tau_n L$ (6)

225

226 where mobilized shear stress is defined as:

227

228 $\tau_{n-mob} = \frac{c_n' + \sigma_n' \tan\phi_n'}{FS}$ (7)

229

230 Assuming a length, L , the mobilized shear force resistance becomes:

231

232 $T_{n-mob} = \tau_{n-mob} L = \frac{c_n' L + \sigma_n' L \tan\phi_n'}{FS}$ (8)

233

234 Because the normal force, N_n , is equal to $\sigma' L$, equation 8 can be rewritten as:

235

236 $T_{n-mob} = \tau_{n-mob} L = \frac{c_n' L + N_n \tan\phi_n'}{FS}$ (9)

237

238 Substituting mobilized shear strength (Equation 9) into equation (6) yields:

239

240 $\sum F_{\parallel} = 0 = (P_{L:n} + U_{L:n})\cos\beta - k_{v:n}W_n\sin\beta + k_{h:n}W_n\cos\beta + W_n\sin\beta - (P_{R:n} + U_{R:n})\cos\beta -$
 241 $\frac{c_n' L + N_n \tan\phi_n'}{FS}$ (10)

242

243 By substitution, solving for N_n from equation (2) results in:

244

245 $N_n = W_n \cos\beta + (P_{R:n} + U_{R:n})\sin\beta - U_{b:n} - U_{b:n-excess} - (P_{L:n} + U_{L:n})\sin\beta - k_{v:n}W_n \cos\beta -$
 246 $k_{h:n}W_n \sin\beta$ (11)

247

248 Substituting N_n into equation (10) and solving for the factor of safety (FS) yields:

249

250 $FS = \frac{c_n' L + [W_n \cos\beta + (P_{R:n} + U_{R:n})\sin\beta - U_{b:n} - U_{b:n-excess} - (P_{L:n} + U_{L:n})\sin\beta - k_{v:n}W_n \cos\beta - k_{h:n}W_n \sin\beta] \tan \phi_n'}{(P_{L:n} + U_{L:n})\cos\beta - k_{v:n}W_n \sin\beta + k_{h:n}W_n \cos\beta + W_n \sin\beta - (P_{R:n} + U_{R:n})\cos\beta}$ (12)

251

252 The FS represents the sliding stability of a given block under given seismic and hydraulic
 253 conditions. The FS can be evaluated with a time series of excess pore water pressures
 254 (u_{excess}) at the base, which may increase due to the seismic accelerations (i.e.,
 255 liquefaction). A given excess pore water pressure profile is defined as an increase above
 256 hydrostatic water pressures, because pressurized water cannot dissipate rapidly during
 257 strong shaking. Ultimately, the excess PWP buildup affects the sliding resistance of the
 258 block by increasing its “buoyancy.” The excess PWP resultant force for a unit width is:

259

260 $U_{b:n-excess}(t) = (u_{n:excess}(t))(L)(1)$ (13)

261

262 The effective stress conditions can be defined by an excess porewater pressure
 263 coefficient, r_u , which is commonly used to quantify the onset of liquefaction. For this
 264 analysis, r_u is defined as:

265

266 $r_u = \frac{u_{excess}}{\sigma_{eff}} = \frac{U_{b:n-excess}}{W_n \cos(\beta) - U_{b:n}}$ (14)

267

268 The excess PWP for a given time increment can be computed and used in the FS
 269 equation (Figure 2b). Note that the excess PWP reduces the stability of the sliding block,
 270 possibly to the point of sliding (i.e., FS = 1). When FS = 1, the yield acceleration, $k_{H:n-yield}$,
 271 can be defined as:

272

$$273 \quad k_{H:n-yield}(t) = \frac{c_n' L + [W_n \cos\beta + (P_{R:n} + U_{R:n}) \sin\beta - U_{b:n} - U_{b:n-excess}(t) - (P_{L:n} + U_{L:n}) \sin\beta] \tan\phi_n' - (P_{L:n} + U_{L:n}) \cos\beta + (P_{R:n} + U_{R:n}) \cos\beta - W_n \sin\beta}{(W_n \sin\beta \tan\phi_n' + W_n \cos\beta)}$$

274

$$275 \quad (15)$$

276

277 which is a function of time and depth. For each thin slice, a Newmark sliding block analysis
 278 can then be performed for given time-dependent yield accelerations. This is performed
 279 by quantifying the relative acceleration at a given depth at a specific time, which is
 280 evaluated as the difference between a given time-dependent yield acceleration ($k_{h:n-yield}$)
 281 and a recorded acceleration ($k_{h:n-input}$); that is:

282

$$283 \quad k_{H:rel}(t) = k_{H:n-input}(t) - k_{H:n-yield}(t)$$

284 The relative acceleration, k_{rel} , can then be integrated once to determine slice relative
 285 velocity, v_{rel} , defined as:

286

$$287 \quad v_{rel}(t) = v_{rel}(t_o + \Delta t) + \int_{t_o + \Delta t}^t k_{H:rel}(t) dt$$

288

289 and then integrated a second time to determine the slice relative displacement, d_{rel} ,
 290 defined as:

291

292

$$d_{rel}(t) = \int_{t_0+\Delta t}^t v_{rel}(t) dt$$

293

294 The displacements of each thin slice can then be integrated from the base of the liquefying
295 layer to a depth of concern to evaluate a cumulative displacement profile along a given
296 soil column (Figure 2), defined as:

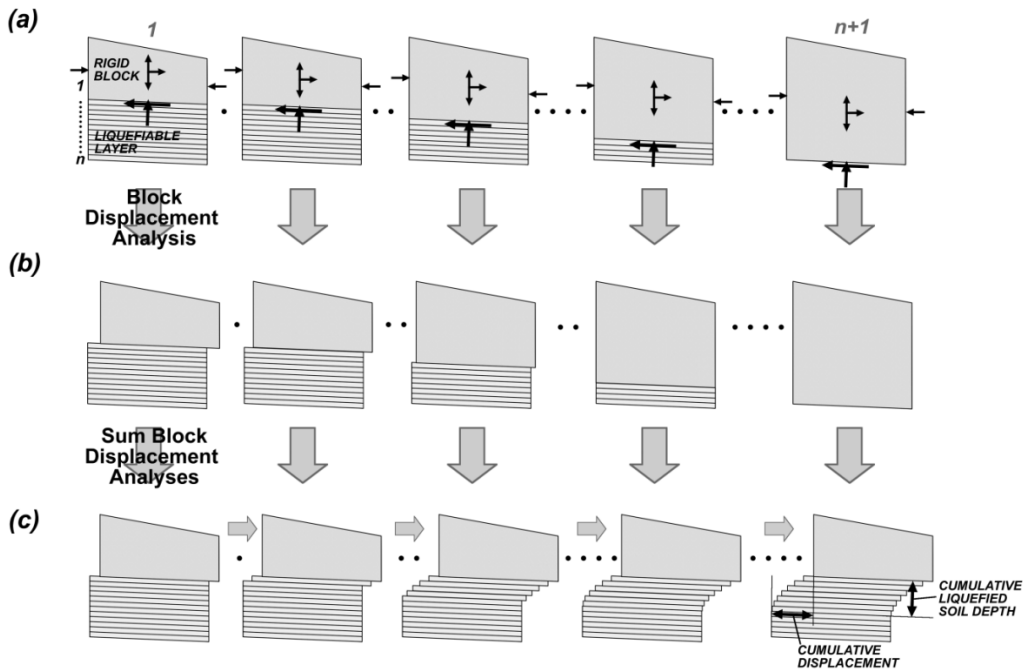
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$$d(t) = \int_{H+n\Delta H}^H d_{rel}(t) dH$$

298 In addition, the resultant of downslope liquefied earth pressures for each slice (P_R) can
299 be determined, which gives an estimate of the potential lateral loads on structural
300 elements subjected to lateral spreading (e.g., bridge piers, retaining walls, marine
301 structures). Furthermore, the acceleration-time series of aftershocks can be considered,
302 which enables the estimation of lateral spreading caused after the mainshock.

303

Equilibrium Analysis of Series of Rigid Blocks



304

305

Figure 2. Sliding block series used to estimate earth pressures or cumulative displacements.

306

307

308 **MODEL VERIFICATION: COMPARISON TO EXPERIMENTAL DATA**

309 To evaluate the sliding block model, a comparison with results from experimental testing

310 was performed. Contrary to the complexity and uncertainties associated with lateral

311 spreading in post-earthquake assessments, significant experimentation has been

312 performed under controlled conditions to isolate and study the mechanics of lateral

313 spreading. Notably, various researchers have used geotechnical centrifuge testing to

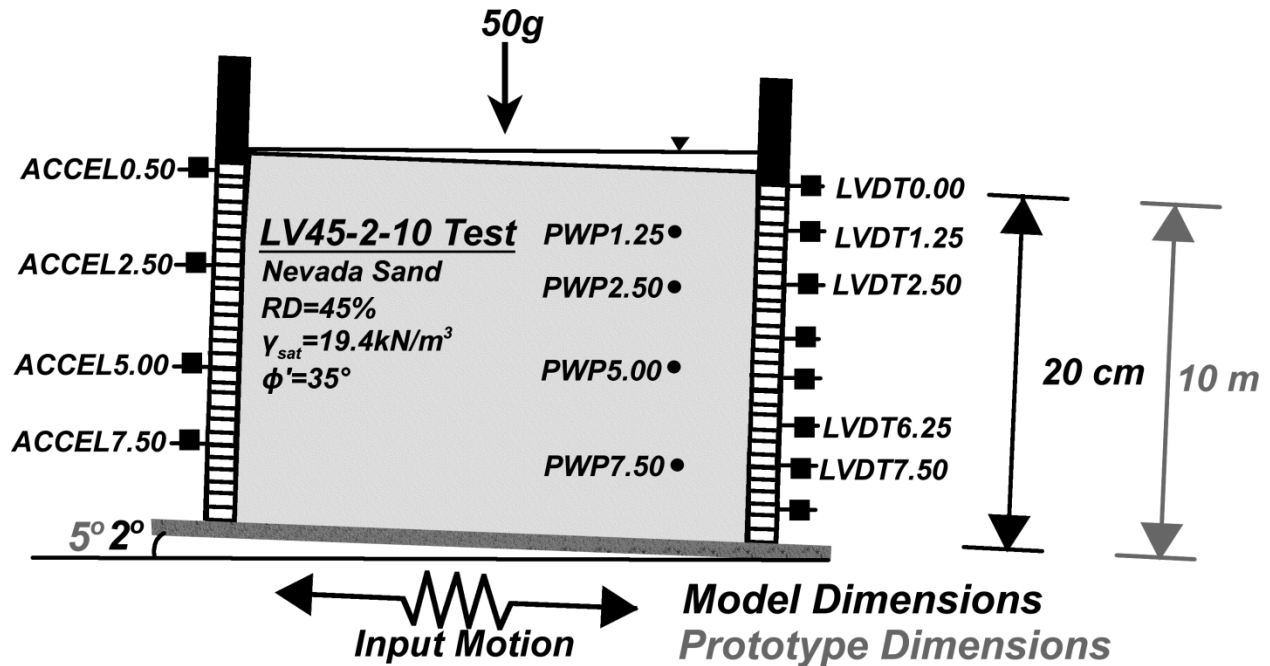
314 investigate lateral spreading, which is often evaluated for structural systems such as

315 bridge foundations (e.g., Abdoun and Dobry 2002, Abdoun et al. 2003, Brandenburg et

316 al. 2005, Brandenburg et al. 2007). To isolate free-field conditions, a well-instrumented

317 experimental series evaluating lateral spreading within a centrifuge was also performed

318 (Sharp and Dobry 2002) within a laminar box of saturated Nevada sand excited with an
319 input seismic motion. Although the experimental results from several tests are presented,
320 the most comprehensive measurements were presented for model L45V-2-10, consisting
321 of cohesionless Nevada Sand at a 45% relative density ($e_{min}=0.516$, $e_{max}=0.894$
322 corresponding to $\gamma_{dry-max}=17.33$ kN/m³ and $\gamma_{dry-min}=13.87$ kN/m³, respectively) with a
323 saturated unit weight of $\gamma_{sat}=19.4$ kN/m³ and effective friction angle of $\phi'=35^\circ$ (Arulmoli et
324 al. 1992, Mikola and Sitar 2013). The L45V-2-10 model was 0.2 meters (model scale) in
325 height and experienced a centrifugal acceleration of 50 g (prototype height =10 meters),
326 as shown in Figure 3. The L45V-2-10 model, inclined at $\beta=2^\circ$, is scaled to a prototype
327 inclination of $\beta=5^\circ$ under centrifugal conditions when correcting for the weight of the
328 laminar rings and hydrostatic conditions (Taboada 1995, Sharp and Dobry 2002, Sharp
329 et al. 2003). During the test, excess porewater pressures were recorded at prototype
330 depths of 1.25, 2.5, 5.0 and 7.5 meters (Figure 4). Similarly, accelerations from a given
331 input motion ($a_{max}=0.2g$, $f=2$ Hz, 22 sinusoidal loading cycles) were recorded at depths of
332 0.5, 2.5, 5.0 and 7.5 meters (Figure 4). Lateral displacements were recorded at the
333 surface and prototype profile depths of 1.25, 2.5, 6.25 and 7.5 meters. This test, although
334 relatively simple, consisting of homogenous material, was selected for its well-monitored
335 response and well-characterized geotechnical conditions.

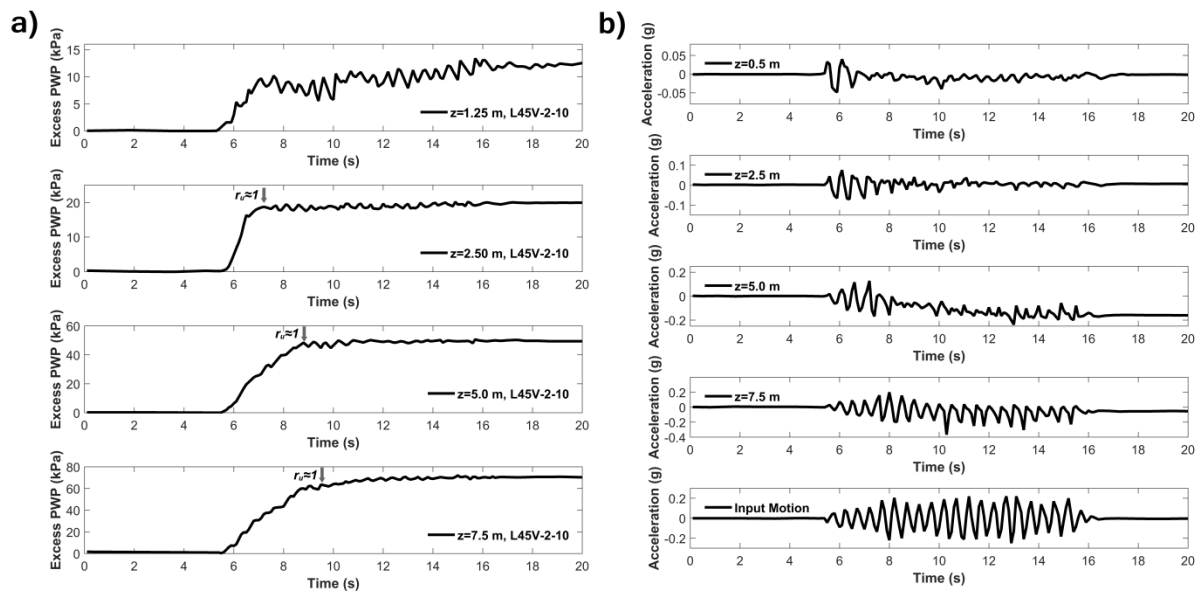


336

337 **Figure 3.** Schematic of LV45-2-10 centrifuge test and instrumentation (after Sharp and
 338 Dobry, 2002).

339 The Newmark sliding column model was evaluated using the soil conditions and
 340 prototype geometry of test L45V-2-10 with the measured excess porewater pressure and
 341 acceleration-time series as inputs. The analysis was coded within a MATLAB script that
 342 implemented the equations presented within the methodology section with the inputs from
 343 the experiment presented by Smith and Dobry (2002). The presented experimental data
 344 was recorded at discrete depths in the soil profile; accordingly, linear interpolation was
 345 performed between measured excess pore water pressures for the profile at given time
 346 increments ($\Delta t=0.1$ second), constrained to zero at the surface and kept constant from
 347 depths of 7.5 meters to 10 meters. The latter assumption is reasonable, because Sharp
 348 and Dobry (2002) report r_u values of less than 70% at depths greater than 7.5 meters in
 349 later stages of testing, which are agreeable with values computed by the sliding column
 350 model. The acceleration-time series were treated similarly; that is, they were linearly

351 interpolated between recorded depths at a specific time increment with the exception that
 352 the input motion was used at the full depth (i.e., 10 meters). The soil column was
 353 discretized into 100 increments; i.e., each increment is 0.1 meters in depth. The analysis
 354 did not demonstrate significant sensitivity after approximately 100 increments in depth.
 355 The surficial soil block was neglected as the experiment was performed on saturated sand
 356 with no coherent, crustal block.

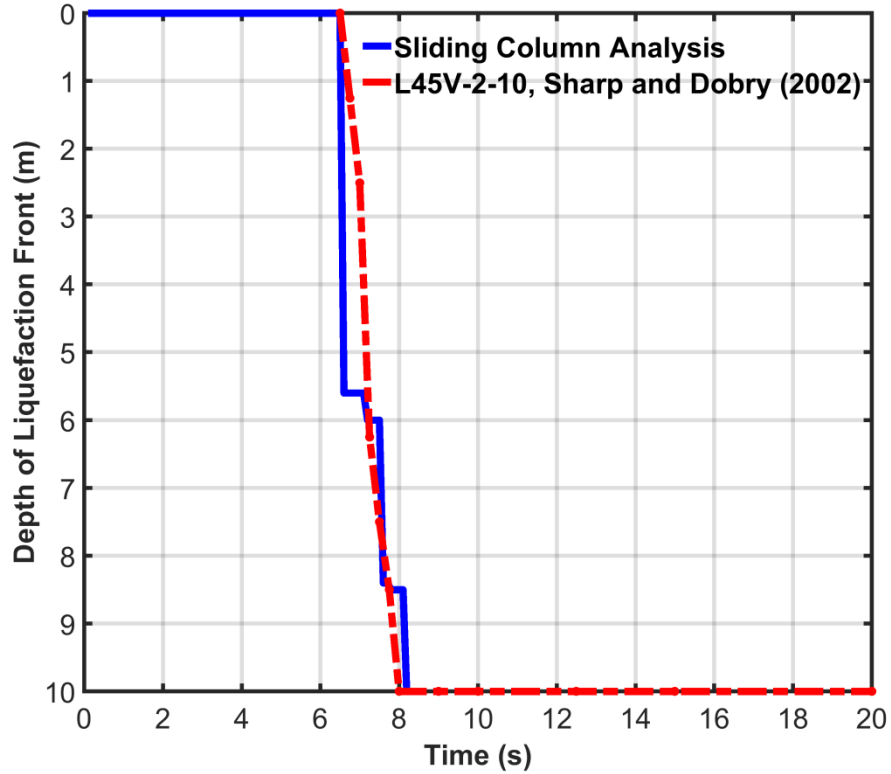


357
 358 **Figure 4.** Recorded (a) excess porewater pressure and (b) acceleration data at scaled
 359 prototype depths for centrifuge test L45V-2-10 (Taboada and Dobry 1998, Sharp and
 360 Dobry 2002).

361
 362 Comparison of the Newmark sliding column analysis with the experimental results
 363 for model L45V-2-10 from Sharp and Dobry (2002) demonstrate reasonable agreement,
 364 particularly for determining the initiation time and depth of lateral spreading. For example,
 365 Figure 4 shows the depth of the liquefaction front versus the depth where displacement

366 is occurring, and reasonable agreement between the model and the experiment can be
367 observed. More specifically, Figure 5 exhibits a relatively rapid transition towards a
368 liquefied state from the surface to the basal layer between times of 6.5 and 8 seconds.
369 Figure 5 shows that the aforementioned transition is observed in the displacement profiles
370 of the experimental layer as well, where lateral spreading initiates in the upper portion of
371 the soil, transitioning to increasing depths with additional loading cycles. The agreement
372 is notable for approximately the first eight seconds of shaking, but diverges for the final
373 three seconds as the Newmark analysis becomes unstable. For the upper regions of the
374 soil profile, small confining pressures and high porewater pressures result in excessive
375 model displacements after approximately 13 seconds, while the experimental prototype
376 demonstrated arrested movements after shaking. The model overprediction is more
377 muted at greater depths, exhibiting a similar arrest of displacement after shaking
378 cessation. Some of the disagreement between the modeled and experimental results is
379 likely caused by uncertainties in excess porewater pressures and accelerations between
380 transducers as well as the omission of complex parameters that may be associated with
381 liquefied soil, such as viscous and dilative behavior of the saturated sand. Furthermore,
382 the presented analysis does demonstrate a well-known limitation of Newmark
383 displacement analyses – a propensity to exhibit excessively large (and conservative)
384 displacements when notable difference exists between the yield and input accelerations
385 (Jibson 2011), as shown in Figure 6. Despite these drawbacks, the presented sliding
386 column analysis demonstrates a reasonable and simple deterministic means of
387 evaluating lateral spreading displacements in consideration of excess porewater

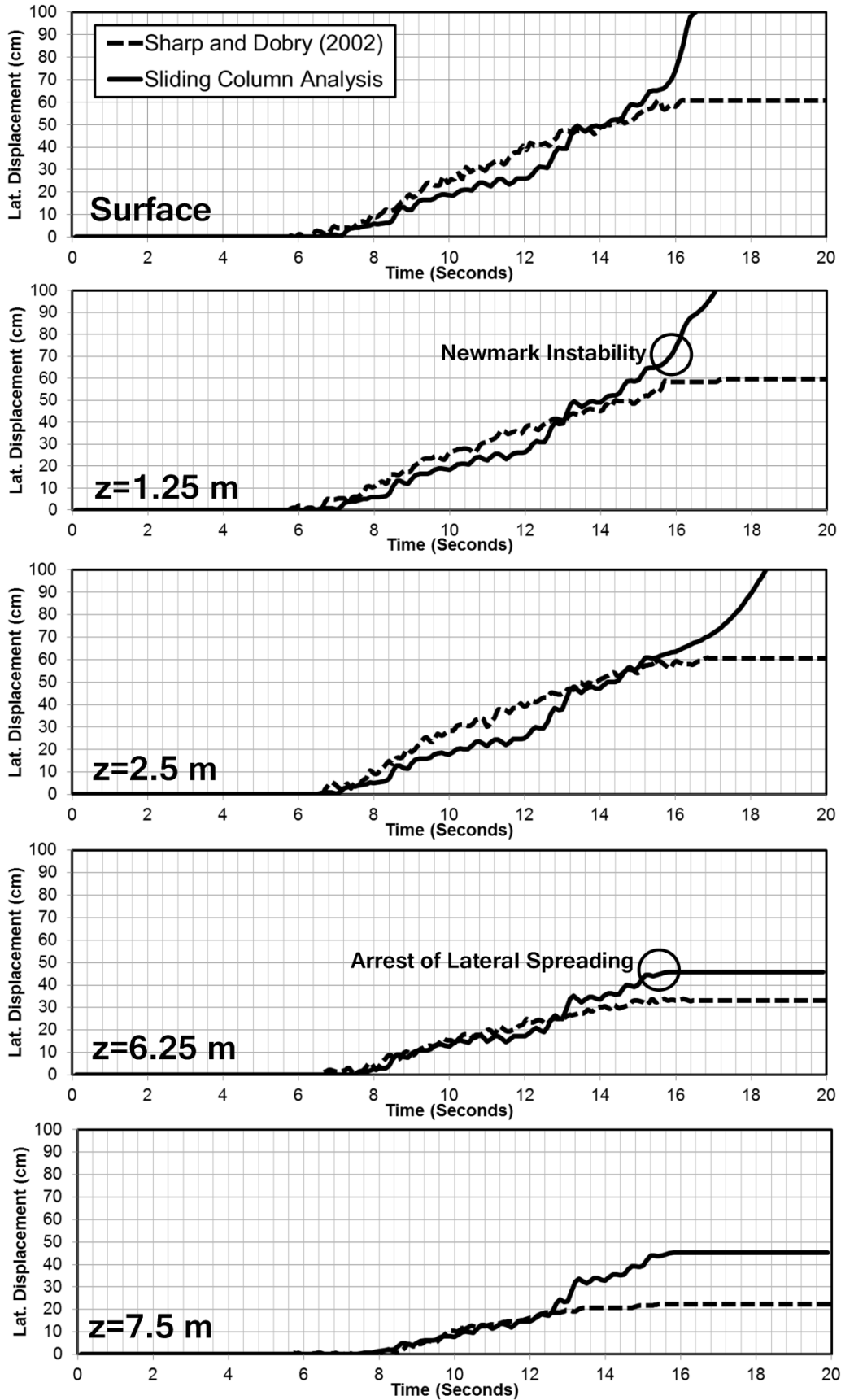
388 pressures and seismic accelerations, especially during the initial onset of lateral
389 spreading.



390

391 **Figure 5.** Depth of liquefaction front from centrifuge experiment L45V-2-10 (Sharp and
392 Dobry, 2002) and sliding column analysis.

393



395

396 **Figure 6.** Recorded movement from both experiment (Sharp and Dobry, 2002) and
397 proposed sliding block analysis.

398

399 **DISCUSSION**

400 The presented sliding column analysis demonstrates a means of evaluating lateral
401 spreading displacements under a variety of scenarios. Furthermore, it enables
402 assessment of various behaviors associated with lateral spreading other than lateral
403 displacement, including assessment of earth pressures and shear strain – parameters
404 that have notable implications on buried structures, such as sewers, pipelines,
405 foundations and marine shoring. The effects of soil unit weight, cementation, liquefying
406 layer slope and free face topography are investigated using the baseline prototype
407 geometry used for test LV45-2-10 (Sharp and Dobry 2002).

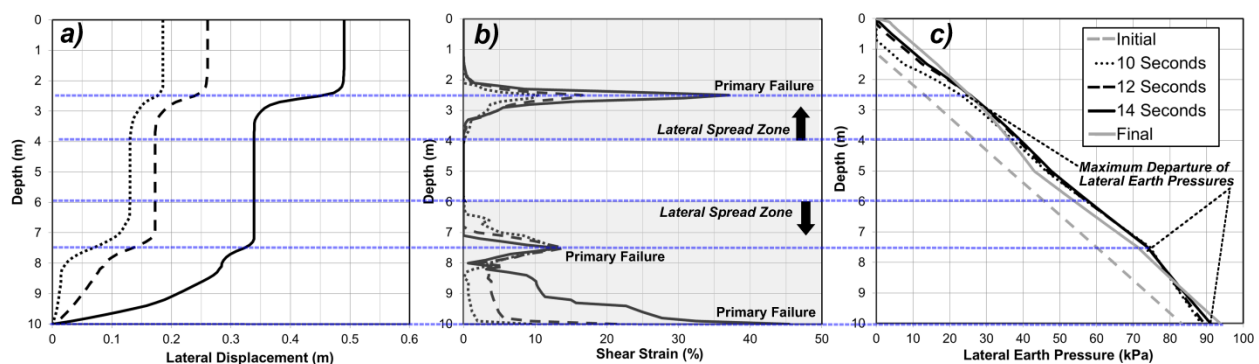
408

409 **Earth Pressures and Shear Strains**

410 A benefit of the proposed sliding column analysis is the direct assessment of earth
411 pressures and shear strains within a 10 meter-deep soil profile for test LV45-2-10. Shear
412 strains can be calculated by dividing the displacement increment for a given slice by the
413 slice depth. The calculated shear strain profile (Figure 7), highlights the notable planes of
414 liquefaction-induced failure. In the case of the LV45-2-10 model, the primary plane of
415 failure occurred at approximately 2.5 meters in depth with other notable planes occurring
416 7.5 and 9.5 meters in prototype depth. The peaks in shear strain correspond to inflections
417 in the displacement profile (Figure 7). Practically, shear strain and displacement profiles

418 are important for assessing the effects of lateral spreading on buried, linear utilities (e.g.
 419 sewers, cables) that cannot withstand significant deformation, particularly differential
 420 displacements. Earth pressures can be determined by setting the factor of safety, FS, in
 421 equation (12) to unity and solving for P_2 , which is in turn discretized into pressure by
 422 dividing the resisting force by a given depth increment. The simplification required to
 423 calculate earth pressures does not account for seismic boundary forces, but may be
 424 considered reasonable for a one-dimensional sliding column with equivalent left and right
 425 boundaries. The earth pressures calculated with the sliding column model tend to peak
 426 within regions of significant shear strains and displacements, because each sliding block
 427 is inherently unstable and exhibits larger earth pressures on its downslope end. This
 428 peaking effect is demonstrated at the base by a transition from initial earth pressures to
 429 increased pressures - 80 kPa to 95 kPa - between 10 and 14 seconds of testing,
 430 respectively. The demonstrated pressure distribution is representative of a mass of soil
 431 moving along the liquefying plane, not accounting for localized, wedge-type earth
 432 pressures (e.g. Brandenberg et al. 2007) that may impart different loading phenomena.

433



434

435

436 **Figure 7.** (a) Displacement profiles. (b) Associated shear strains for modeled lateral
437 spread. (c) Associated lateral earth pressures for modeled lateral spread.

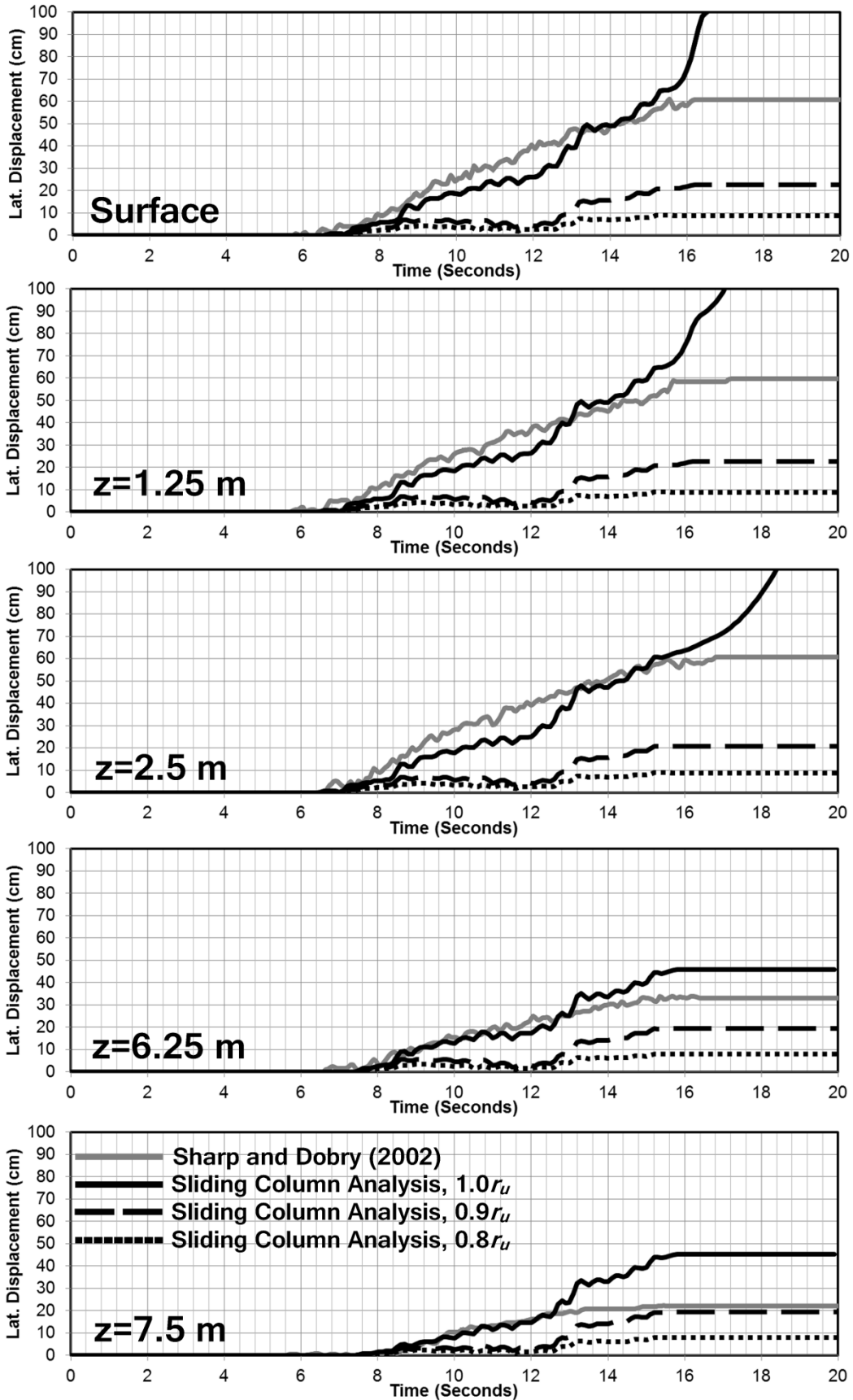
438

439 **Impact and Sensitivity of Pore Water Pressures**

440 The phenomenon of liquefaction and associated lateral spreading is dependent on
441 the generation of excess porewater pressures and effective stress, the latter of which
442 incorporates the unit weight of a soil. When the effective stress of a given soil approaches
443 zero (e.g. $r_u=1$), the mobilized frictional strength is neutralized, which can result in
444 weakened foundations, excessive displacement of buried utilities, and lateral spreading,
445 among other phenomena. To demonstrate the impacts of varying excess pore water
446 pressure ratios and the associated sensitivity of the sliding column model, an analysis
447 was performed using the same inputs for LV45-2-10, but reducing the maximum r_u values
448 recorded in the testing to 80% and 90% of observed levels. The results (Figure 8) exhibit
449 rapid and excessive lateral spreading when only 100% of observed r_u are used,
450 demonstrated by a steep displacement curve occurring between 16 and 18 seconds. It is
451 noted that the frictional and resisting effects of the laminar rings in the centrifuge testing
452 apparatus may restrict some of the large displacements; however, this observed behavior
453 is still likely an artifact of Newmark-type analyses. Use of 90% and 80% of observed r_u
454 values present a generally inverse trend; that is, lateral spreading displacements are
455 muted at all depths, limiting most of the observed displacements to beyond 7.5 meters in
456 depth. When excess porewater pressure ratios are smaller, the liquefaction front
457 progresses slightly less rapidly – taking two to three more loading cycles to produce lateral
458 spreading displacement at all depths (Figure 9). The preceding analysis demonstrates

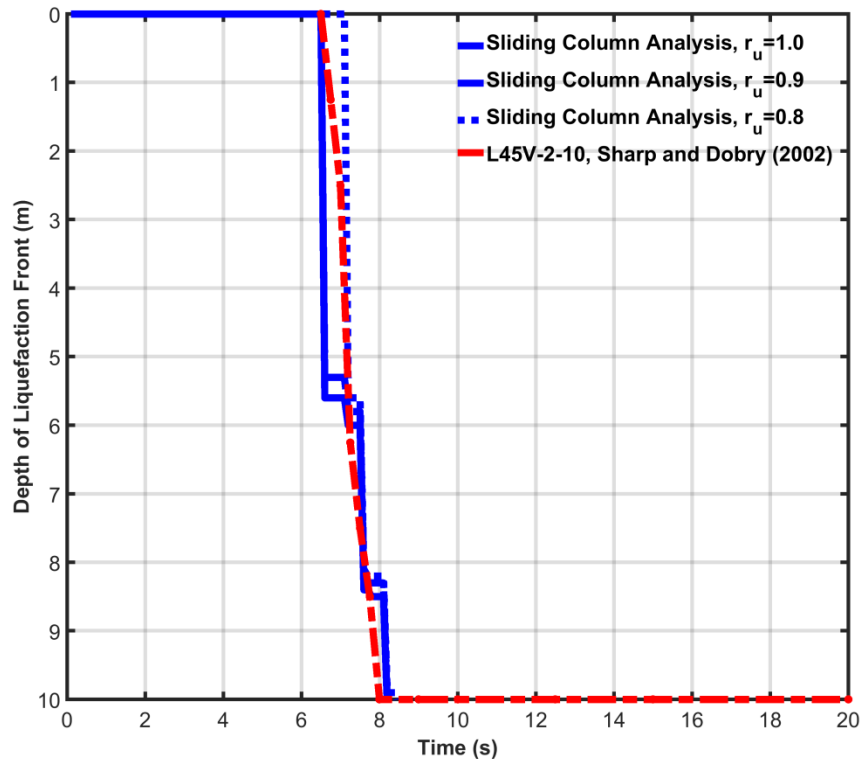
459 that the sliding column model is inherently sensitive to excess porewater pressures - a
460 property that is inherently variable in real geotechnical problems. Underestimation of r_u
461 can lead to underprediction of observed lateral spreading displacements, while higher
462 values of r_u may cause large displacements, particularly near the soil surface. The
463 sensitivity analysis does also exhibit a behavior that is intuitive – lower r_u values arrest
464 lateral spreading displacements. However, permanent displacements from r_u values less
465 than unity may still occur, demonstrating a need to evaluate excess porewater pressure
466 generation even when “full liquefaction” (i.e., $r_u = 1$) is not realized.

467



469 **Figure 8.** Effect of excess porewater pressure ratio, r_u , on lateral spreading
470 displacements.

471



472

473 **Figure 9.** Effect of excess porewater pressure ratio, r_u , on depth of liquefaction front.

474

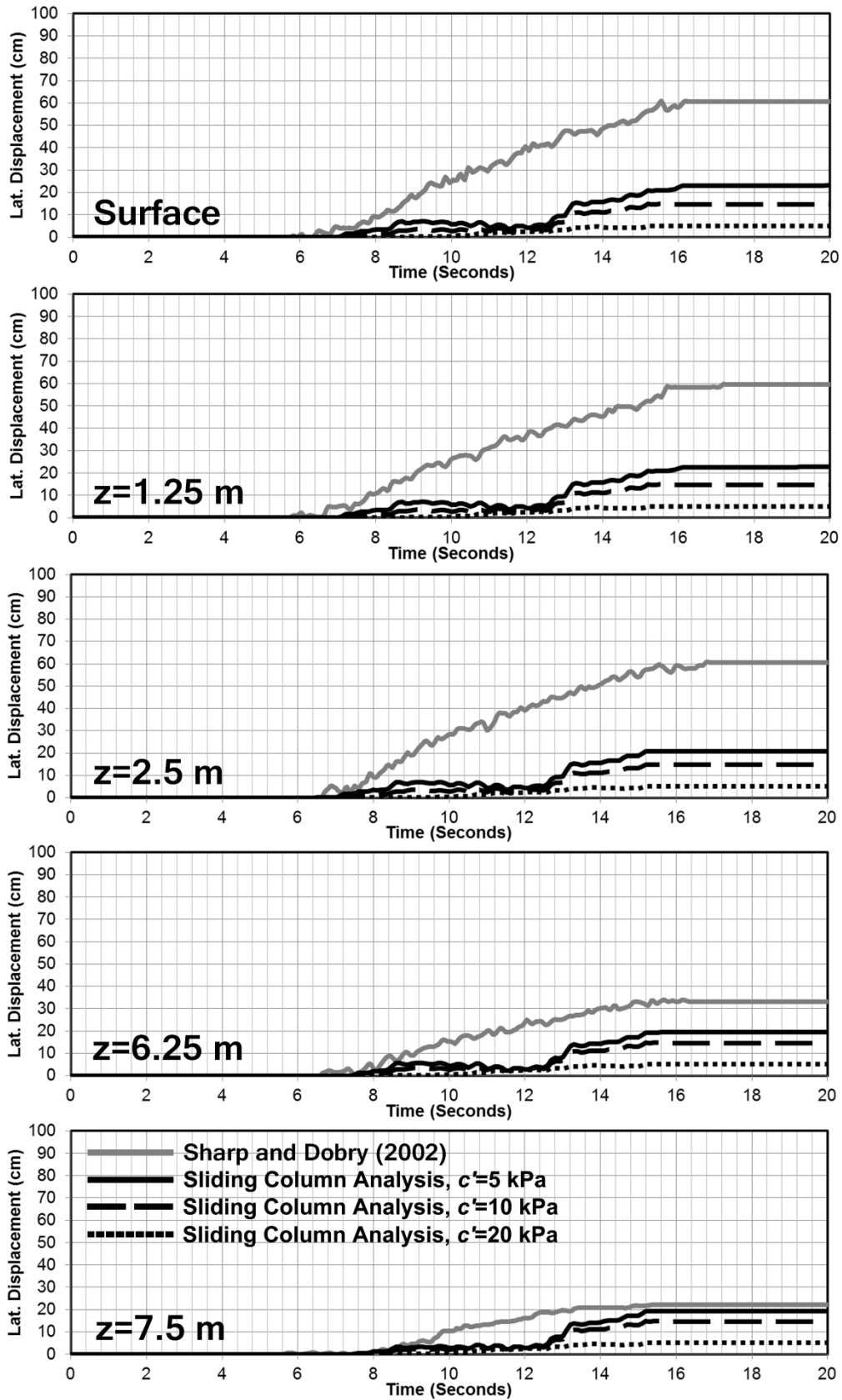
475 Implications of Cohesion and Cementation

476 The effects of true cohesion, primarily stemming from cementation, can
477 significantly influence the arrest of liquefaction (Clough et al. 1989, Mitchell et al. 1995).

478 Light cementation is common in naturally deposited sands or sands improved with
479 admixtures (Huang and Airey 1998). The effects of cementation are evaluated from the

480 results of LV45-2-10, applied as 5, 10 and 20 kPa of cohesion. The results, shown in
481 Figure 10, demonstrate a significant arrest of displacements, particularly at the surface,

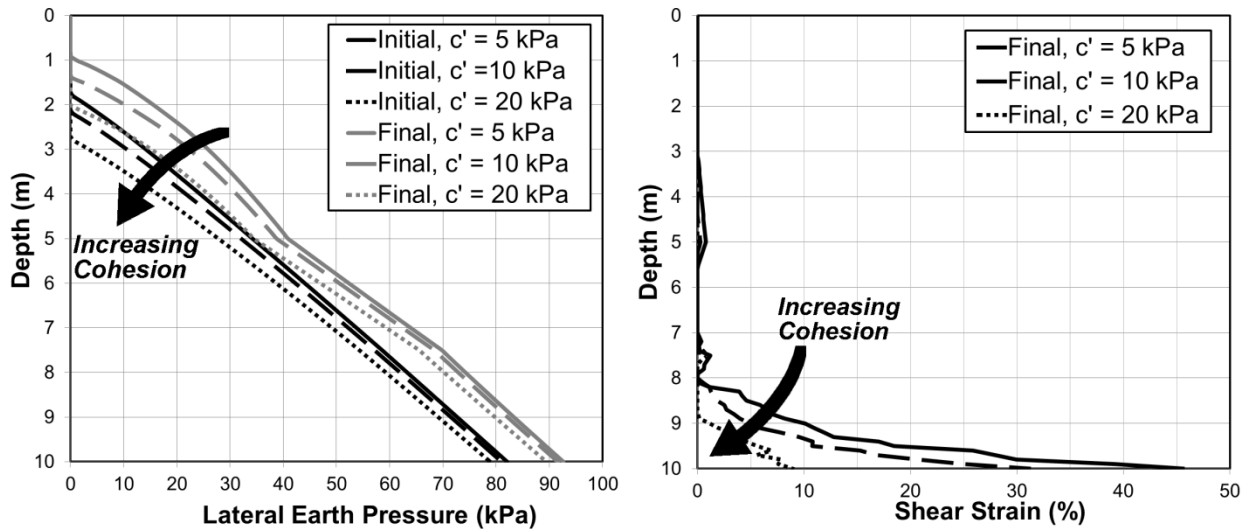
482 which exhibited large modeled displacements when cohesionless. The arrest of
483 displacements are amplified when cohesion is increased further, demonstrated by a
484 reduction of surface displacement from 23 cm to 5 cm for cohesion values of 5 and 20
485 kPa, respectively. Cohesion effects are also observed in the lateral earth pressure and
486 shear strain profiles (Figure 11), where cohesion stabilizes the failing surface layer and
487 mutes the shear strains at greater depths. Increasing cohesion, as expected, decreases
488 the earth pressures, particularly near the surface, both before and after shaking. The
489 effects of true cohesion stemming from natural or artificial cementation intuitively
490 demonstrate improved performance during seismic excitation and porewater pressure
491 buildup owing to increased internal shear strength and resistance to failure, particularly
492 at shallow depths. From a practical perspective, artificial and especially natural
493 cementation is an inherently spatially variable, suggesting that judgment must be used if
494 cementation is considered as a means of lateral spreading mitigation. However, the
495 benefits of even light cementation are significant.



497

Figure 10. Effect of cohesion on lateral spreading displacements.

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499

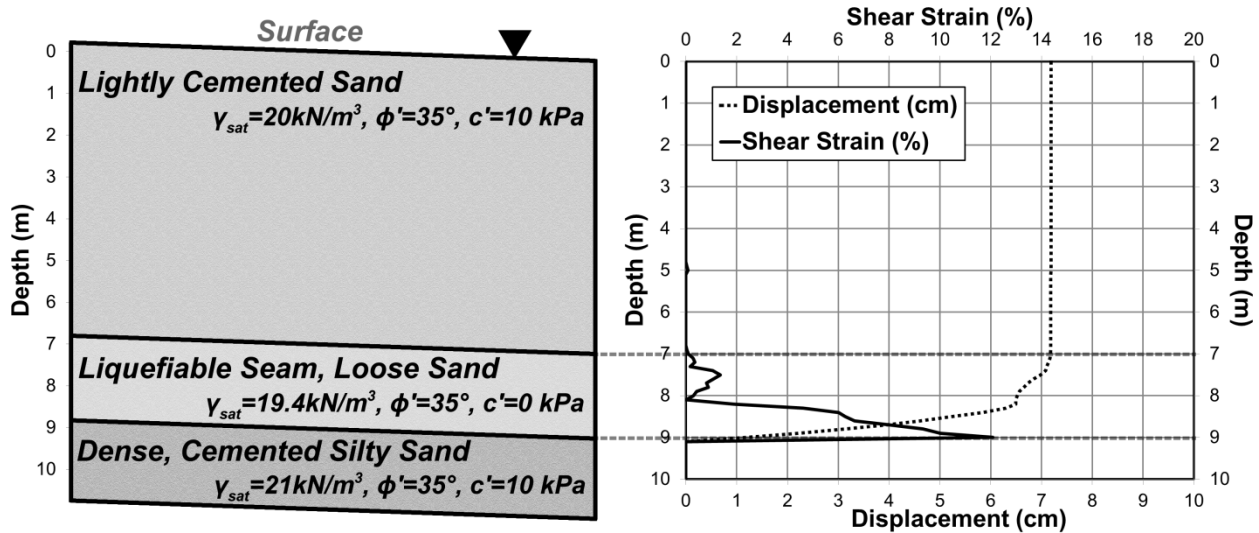
500

Figure 11. Effect of cohesion on lateral earth pressures and shear strains.

502 Application to Layered Soils

503 To illustrate application towards layered soils, a soil profile consisting of a two
 504 meter thick liquefiable seam is considered. The top seven meters consist of lightly
 505 cemented sand ($\gamma_{sat}=20\text{kN/m}^3$, $\phi'=35^\circ$, $c'=10\text{kPa}$), underlain by loose, liquefiable sand
 506 ($\gamma_{sat}=19.4\text{kN/m}^3$, $\phi'=35^\circ$, $c'=0\text{kPa}$) and a layer of dense, heavily cemented silty sand
 507 ($\gamma_{sat}=21\text{kN/m}^3$, $\phi'=35^\circ$, $c'=50\text{kPa}$), all inclined at a 5° angle – see Figure 12. For the given
 508 profile, the same excess porewater pressure regime and input motion used in previous
 509 sections was considered. At the end of the twenty seconds of shaking, there is a
 510 demonstrated concentration of shear strains (up to 12%) within the cohesionless,
 511 liquefiable layer (Figure 12). This results in permanent lateral displacements of
 512 approximately 7 cm constrained to the seam of liquefying soil. This example

513 demonstrates the importance of characterizing problematic seams of liquefiable soil
 514 within profiles of relatively competent material. When the stratigraphy and the properties
 515 of underlying soils are well-characterized, the proposed model may capture the localized
 516 failure and subsequent displacement of weak, liquefiable seams.



517
 518 **Figure 12.** Left: Example layered soil profile with a subsurface weak seam of sand.
 519 Right: Modeled shear strains and displacements for given profile.

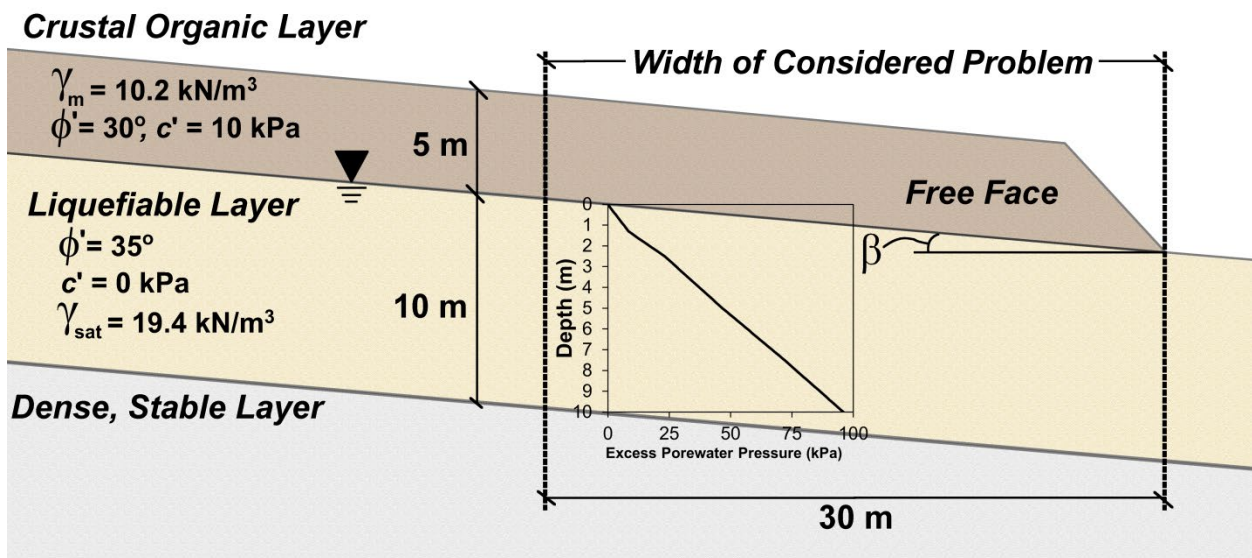
520 **Free Face Effects**

521 To exhibit the effects of a free face, an example consisting of a partially saturated,
 522 organic crustal slope that has a height of 5 meters, a width of 30 meters, and a steep 45°
 523 face is placed on a gently sloping layer of saturated sand (subsurface slope of β) overlying
 524 a competent, non-liquefiable basal layer (Figure 13). The crustal layer and saturated sand
 525 layer have total unit weights of 10.2 kN/m^3 and 19.4 kN/m^3 , respectively. The crustal layer
 526 has a cohesion of 10 kPa and an internal angle of friction of 30°. The saturated sand layer
 527 beneath the crustal layer is cohesionless and has an internal angle of friction of 35°. A
 528 sensitivity study was performed on several subsurface slopes ($\beta = 0^\circ, 1^\circ, 3^\circ$ and 5°) for the

529 same seismic inputs from LV45-2-10 (Sharp and Dobry 2002). The cyclic behavior of the
 530 excess porewater pressures within the soil profile is similar to that from that of Sharp and
 531 Dobry (2002), but the maximum magnitudes were changed to vary from zero to 90 kPa
 532 at the basal layer (Figure 13).

533 The results demonstrate that lateral spreading displacements decrease with
 534 diminishing subsurface slopes, but still demonstrate displacements on horizontal
 535 liquefying sand layers (Figure 14). When transitioning from $\beta=0^\circ$ to $\beta=5^\circ$, the maximum
 536 lateral displacements increase from approximately 4 cm to approximately 46 cm. Notably,
 537 the difference between driving and resisting at-rest forces due to uneven overburden
 538 along the width of a sliding block results in increased movement during liquefaction,
 539 particularly with seismic motions. Although the displacement for horizontal ground is
 540 limited in this example, very gentle, sloping liquefiable layers of 1° and 3° result in 9 and
 541 22 cm of displacement (Figure 14), respectively, which may compromise buried utilities
 542 and sewers.

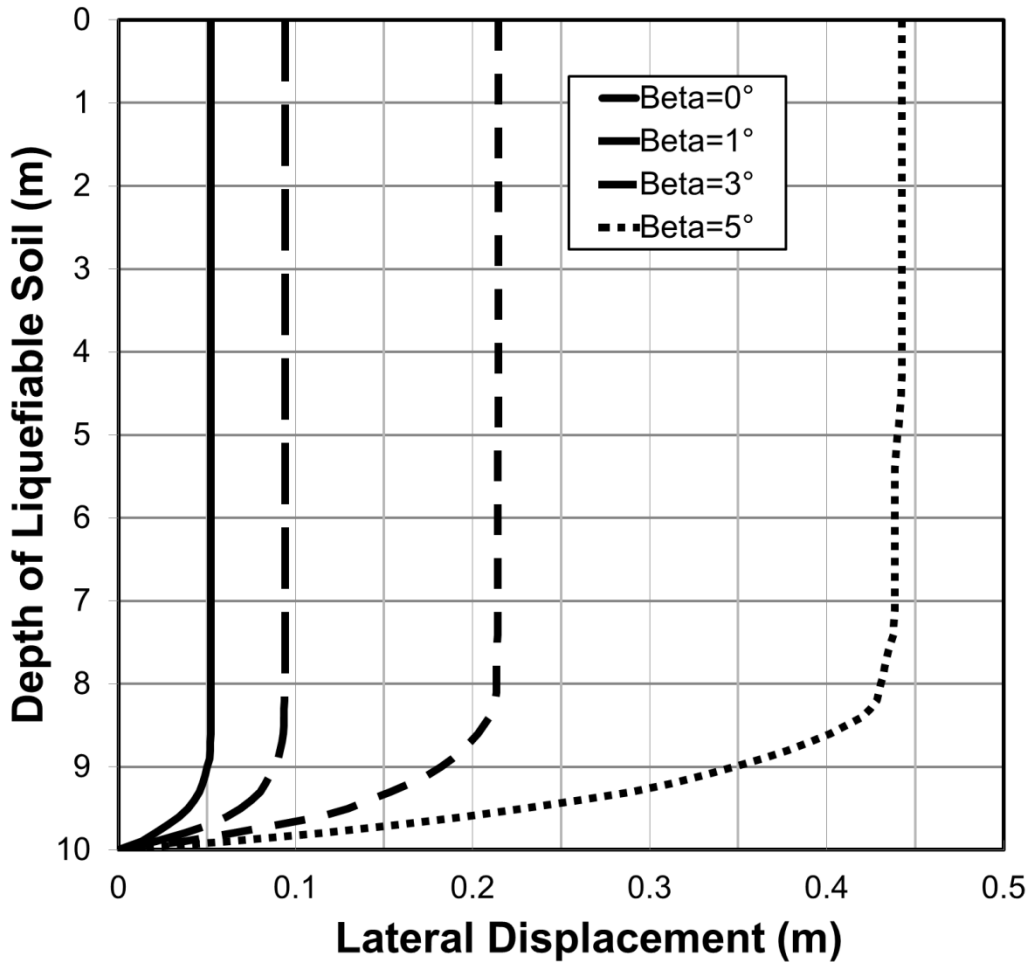
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546 **Figure 13.** Example profile analyzed in the free face lateral spread scenario.



547

548

549 **Figure 14.** Effect of liquefiable layer inclination on free face movements for discrete,
550 coherent sliding mass.

551

552 CONCLUSIONS

553 Liquefaction-induced lateral spreading behavior was investigated using a Newmark-
554 type limit equilibrium framework comprised of a column of sliding blocks subject to
555 changing yield accelerations and seismic accelerations. A contribution of this work is that

556 it analyzes lateral spreading within a deterministic, limit equilibrium-based slope stability
557 framework instead of traditional statistical or experimental approaches. Notably, the
558 primary focus of the article was presenting the application of the modified Newmark
559 framework for evaluating lateral spread displacements. In its current form, this tool
560 presents itself as a useful supplement, or check, for determining lateral spreading
561 displacements in comparison to more complex site-specific analyses that necessitate use
562 of numerical methods. In the future, this framework can be expanded to account for
563 different failure kinematics, incorporate more complex soil constitutive or hydraulic
564 properties, account for site response analyses, or even incorporate data from numerical
565 models. Because the methodology considers physical data, it may provide more reliable
566 results than empirical-based lateral spreading displacement procedures when a
567 subsurface is well-defined. The simplicity of the framework in its current form makes it
568 less suited for site-specific analyses, but transferrable towards regional assessments of
569 lateral spreading displacements. There are limited deterministic methods for regional
570 evaluation of lateral spreading displacements that can incorporate subsurface data – this
571 framework will enable such assessments. Such an analysis is useful for planning
572 purposes as well as for rapid analysis to support post-earthquake reconnaissance.

573 The presented sliding column analysis captures the general behavior of lateral
574 spreading in the free field when compared to well-instrumented centrifuge testing. Some
575 disagreement occurs due to instability of Newmark sliding blocks at the surface,
576 particularly for very low yield accelerations. Nonetheless, the sliding column analysis can
577 capture shear strains and displacements within a given soil profile during lateral spreading
578 and predicts larger earth pressures at notable liquefaction fronts. The presented analysis

579 is sensitive to the effective stress of the soil column, predicting significant instability for
580 excess porewater pressure coefficients near unity, with arrested, but still significant
581 displacements, for lower porewater pressures. The effects of soil cementation are
582 pronounced for arresting lateral spread displacements, and soil cementation effects are
583 more pronounced at the ground surface. The proposed model can incorporate layers of
584 soil with a well-characterized subsurface, isolating localized seams of unstable material.
585 The effects of free face on lateral spreading displacements are more pronounced when
586 the liquefying later inclination increases. However, lateral spreading still occurs for a
587 horizontal liquefying layer as the inertial loading and difference in surrounding at-rest
588 earth forces destabilize the block.

589 The sliding column analysis, although applicable with basic geotechnical
590 properties, has several constraints and uncertainties that deserve further refinement. In
591 particular, Newmark-type analyses for evaluating displacements of rigid block using a
592 limit equilibrium framework may diverge and overpredict displacements when the yield
593 acceleration is exceeded significantly (Kramer and Lindwall 2004). Incorporation of
594 material properties, such as dilation and the viscosity of liquefied soil, may mitigate
595 aforementioned weaknesses of Newmark-type analyses – leading to better lateral
596 spreading displacement and earth pressure predictions. Furthermore, seismic earth
597 pressures in saturated soils are dependent on more complex soil behavior, which is
598 difficult to capture within a limit equilibrium-based model; namely, the omission of viscous
599 soil behavior and incorporation of seismic boundary effects. Another potential weakness
600 includes the need to input an excess porewater pressure-time series. The excess
601 porewater pressure-time series can be developed from one-dimensional or two-

602 dimensional site response analyses for a given earthquake motion and soil profile with
603 appropriate dynamic soil properties (and an assumption for how strain is related to pore
604 water pressure generation). In addition, the excess porewater pressure-time series can
605 be developed using more complete numerical modeling techniques with soil constitutive
606 models. However, using the aforementioned methods may negate some of the benefits
607 of the modified Newmark method; mainly, the simplicity of the method. Accordingly, for
608 future application, we recommend using a simplified porewater pressure estimation
609 method (e.g., Seed et al. 1977) to develop reasonable excess porewater pressure-time
610 series.

611 Notwithstanding potential weaknesses of the Newmark-type analysis, the
612 presented analysis is appropriate for simple modeling of lateral spreading displacements.
613 The model can be used to investigate the effect of varying soil density, frictional strength,
614 and excess porewater pressures with depth, which is important for integrating
615 geotechnical site investigation data (e.g. CPT, SPT). Furthermore, the effects of a crustal
616 surface layer with two-dimensional topography can be implemented, providing a means
617 of describing the notable lateral spread occurrence at the confluence of gentle slopes and
618 bodies of water. The simplified analysis also demonstrates promise for applicability to a
619 larger scale, particularly implementation through use of combined digital elevation models
620 and site investigation databases, providing an opportunity to create a deterministic hazard
621 mapping procedure as had been done using statistical methods (e.g., Youd and Perkins
622 1987, Bardet et al. 2002, Rosinski et al. 2004, Olsen et al. 2007, Gillins 2012). Finally, the
623 model can be further enhanced to handle more complex problems by (1) evaluating
624 alternative failure mechanisms, (2) incorporating seismic boundary forces more robustly,

625 (3) incorporating models relating coupled or decoupled seismic accelerations throughout
626 the soil column, and (4) implementing models relating the buildup of excess porewater
627 pressures throughout the soil profile.

628

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