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Appendix S2

Supplementary material for:

Title:
Stream and ocean hydrodynamics mediate partial migration strategies in an amphidromous Hawaiian goby.

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Contents:

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Modeling the proportion of resident adults

Detailed derivations of a single-watershed model under an odd scale formulation

For the case of a single isolated watershed, we ask what controls the proportion of adult *Awaous stamineus* that went to sea as larvae. The proportion of resident adults, *R*, and s_f and s_o are the survival rates of resident (freshwater) and seagoing (oceanic) larvae, respectively, and α is the larval amphidromy rate i.e. the proportion of larvae that go to sea.

$$R = \frac{\left(s_f / s_o\right)(1-\alpha)}{\left(s_f / s_o\right)(1-\alpha)+\alpha} \text{ ,} \qquad \text{equation S1}$$

which is equation (1.1) in the text. This can be further rearranged to

$$R = \frac{\left(s_f / s_o\right)}{\left(s_f / s_o\right)+\left(\alpha / (1-\alpha\right)} \text{ ,} \qquad \text{equation S2}$$

which has the advantage of separating the terms and in so doing, expresses *R* in terms of the odds of larval amphidromy.

Further mathematical simplification is possible by also putting adult residency on the odds scale, (*R*/(1-*R*)). Similar multiplicative relationships between probabilities are already present in the survival ratio, although this is not an odds because the values of s_f and s_o are independent. Transforming equation S2 to consider the odds of adult residency yields

$$\frac{R}{1-R} = \frac{\left(s_f / s_o\right)}{\left(\alpha / (1-\alpha)\right)} \text{ .} \qquad \text{equation S3}$$

Thus, when the odds rather than the probabilities of *R* and *α* are considered, the relationship between the three quantities is proportional, and the expression can be rearranged to solve for the other two quantities – the survival ratio and odds of larval amphidromy:

$$\frac{s_f}{s_o} = \left(\frac{R}{1-R}\right)\left(\frac{\alpha}{1-\alpha}\right) \text{ ,} \qquad \text{equation S4}$$

And

$$\left(\frac{\alpha}{1-\alpha}\right) = \left(\frac{s_f}{s_o}\right) \Big/ \left(\frac{R}{1-R}\right) \text{ .} \qquad \text{equation S5}$$

Plotting the same relationships as Figure 2 in the text on the log odds scale illustrates this proportionality (panel a, Figure S1).

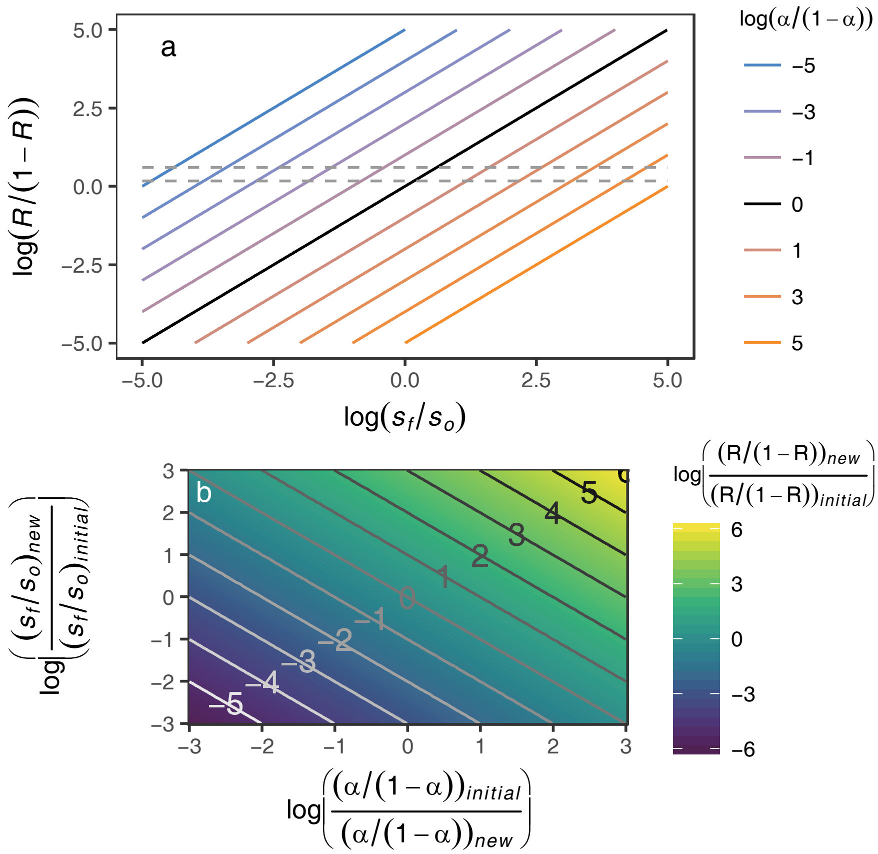

Figure S1. Analogous to Figure 2 in the text, but here plotted as the log odds of larval amphidromy rate and log odds of resident adults. Dashed lines are $R = 0.6$ and $R = 0.8$. On this scale, the relationships are symmetric and additive, as shown in equations S3) and S5). Moreover, the particular *initial* and *new* values of R do not matter for panel b; multiplicative changes in $R/(1-R)$ depend only on the multiplicative changes in the other variables, not their values.

Achieving a given change in R

In the text, we consider the changes in survival ratio, s_f/s_o, and larval amphidromy rate, α, that could yield the observed changes in R from 0.6 to 0.8. Here, we develop general formulae to understand any given change from from $R_{initial}$ to R_{new}, where these values are the proportion of resident adults at two different points in time. Although the text discusses the arithmetic differences in survival ratio and larval amphidromy, the mathematical development is clearest in terms of multiplicative changes on the odds scale. Equation S3 naturally leads to

$$\frac{R_{new}/(1-R_{new})}{R_{initial}/(1-R_{initial})} = \left(\frac{(s_f/s_o)_{new}}{(s_f/s_o)_{initial}}\right)\left(\frac{\alpha_{initial}/(1-\alpha_{initial})}{\alpha_{new}/(1-\alpha_{new})}\right), \quad \text{equation S6}$$

Which express how many times greater (or lower) the odds of adults being residents are in terms of multiplicative changes in the survival ratio and the odds of larvae going to sea. Just as equation S3 gives a log-linear relationship between residency odds , survival ratio, and amphidromy odds, equation S6 shows that multiplicative changes in residency odds are likewise log-linearly related to multiplicative changes to survival fraction and amphidromy odds, illustrated in panel b of Figure S1. For clarity, the subscripts on larval amphidromy are inverted compared to the subscripts on the survival ratio and adult residency, so that larger values yield increased adult residency. This arrangement is needed because increased amphidromy sends more larvae to sea. The subscripts would be the other way around if we were considering the stay-home larval probability rather than the amphidromy probability. Turning the question around, in the situation where we have some multiplicative change x in the odds of adults being residents, e.g.

$$R_{new} / (1 - R_{new}) = x\left(R_{initial} / (1 - R_{initial})\right) , \qquad \text{equation S7}$$

we can ask what changes in survival ratios and the odds of larval amphidromy are needed to achieve the change x. Substituting those terms S1.7 yields

$$R_{new} / (1 - R_{new}) = x\left(\frac{(s_f / s_o)_{initial}}{\alpha_{initial}(1 - \alpha_{initial})}\right) . \qquad \text{equation S8}$$

Thus, an x-fold change in the quotient of survival ratio to the odds of larval amphidromy yields an x-fold change in the odds of adult amphidromy. This shift could occur if the new survival ratio is x times greater than the initial, or if the new odds of amphidromy are $1/x$ times the initial, or any combination of changes yielding the required x-fold change in their ratio. Said another way, expressions S1.6 and S1.8 say that when the survival ratio doubles or larval amphidromy becomes half as likely, the odds that an adult was a resident larva will double. One important outcome of this expression is that it is not contingent on the starting values of any variable; instead, all that matters is their proportional change. Thus, we can plot the multiplicative change in residency odds against the multiplicative changes in the other two terms without needing to specify particular values for the *initial* or *new* proportion of resident adults (R) (panel b, Figure S1).

The proportionality and independence from initial conditions exposed in equation S6 has important implications for the specific shift in R from 0.6 to 0.8 we explore in the text. In Figure S3, we show that the values of survival ratio and larval amphidromy depend on each other to yield a given R, as well as a given change in R. However, as stated in the text and demonstrated here, the multiplicative changes on the odds scale for R are invariant. This conclusion follows directly from equations S6 and S8, and is shown in general in panel B of Figure S1. For the specific case of shifting R from 0.6 to 0.8, the odds that an adult is a resident shift from 1.5 to 4, yielding a multiplicative change in the odds (the LHS of S6 and the x of S7) of 2.67. If only one term changes, it must be by 2.67x, as shown in Figure S2 and equation S8. If both change, their joint multiplicative change must be 2.67x, as shown in general in panel (b) of Figure S2 and equation S8.

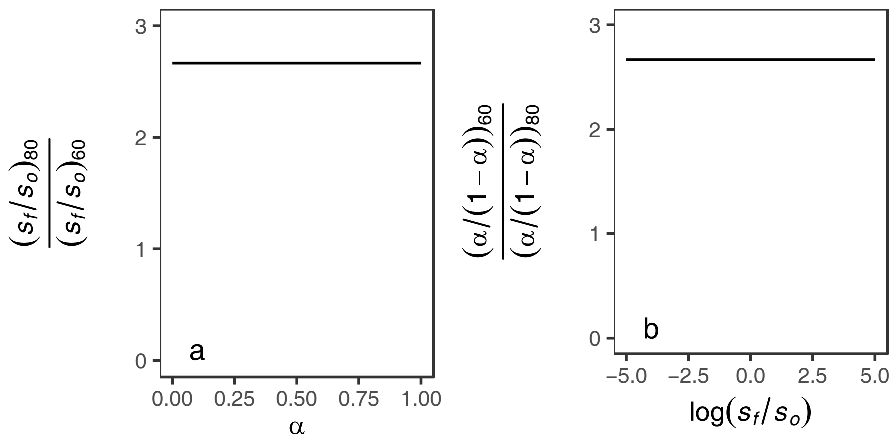

Figure S2. Multiplicative changes in adult survival ratio, panel a, and odds of larval amphidromy, panel b, necessary to shift the proportion of resident adults from $R = 0.6$ *to* R = 0.8*,* while holding the other variable constant at its value on the x-axis. Survival ratio $(s_f/s_o)_{80}$ is always 2.67x larger than $(s_f/s_o)_{60}$, and the odds of larval amphidromy at $R = 0.6$ are always 2.67x larger than at $R = 0.8$. Note that the larval amphidromy odds ratio is inverted for clarity, so that larger values indicate an increase in resident adults.

As we have seen here, equation S6 has useful mathematical properties and yields the mathematically simplest, most general results. However, it is often unintuitive to think in terms of multiplicative changes to odds, particularly in relating these modeling results to empirical measurements. What does it mean for the odds of larval amphidromy to change by 2.67x? How does that relate to empirical estimates of larval amphidromy rates? For this reason, we focus in the text on how arithmetic changes to the amphidromy rate itself and the survival ratio change the proportion of resident adults. We change one or the other variable while holding the other constant for clarity. We can rearrange equation S6 to yield

$$\left(s_f / s_o\right)_{new} - \left(s_f / s_o\right)_{initial} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{R_{new}}{1-R_{new}} - \frac{R_{initial}}{1-R_{initial}}\right) \qquad \text{equation S9}$$

And

$$\alpha_{initial} - \alpha_{new} = \frac{\left(s_f/s_o\right)}{\left(s_f/s_o\right) + \left(R_{initial}/(1-R_{initial})\right)} - \frac{\left(s_f/s_o\right)}{\left(s_f/s_o\right) + \left(R_{new}/(1-R_{new})\right)} \qquad \text{equation S10}$$

These expressions are clearly more complex, and most importantly, the size of the change in either variable needed to shift R a given amount depends on the value of the other variable, as shown in Figure S3. The key issue here is that an initial value of, for example, R_{60}, requires a different survival ratio for each amphidromy rate. Because equation S6 says that the survival ratio needs to be 2.67x greater to yield R_{80}, the arithmetic difference between survival ratios will change depending on the amphidromy rate. This is seen in Figure S3.

The biological meaning of the terms are perhaps most intuitive to think of as multiplicative for survival ratios (*x* times more likely to survive in freshwater than ocean), while remaining on the scale of amphidromy rates (what proportion of larvae go to sea) and proportions of resident adults. We can plot the changes to *R* in this way, but as shown in equations S9 and S10, we need to specify starting values. This relationship is plotted in Figure S3.

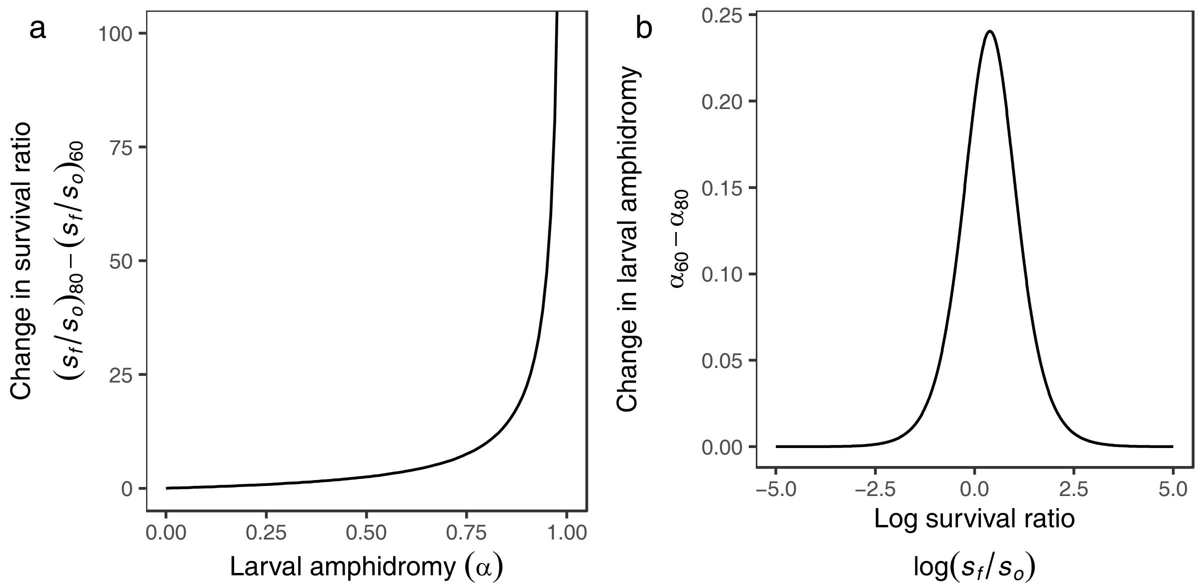

Figure S3. Arithmetic changes in survival ratio (panel a), and larval amphidromy (panel b), necessary to shift the proportion of resident adults from R = 0.6 to R = 0.8 (observed in 2009 and 2011 respectively), while holding the other variable constant at its value on the x-axis. Note that although these differences change along each x- axis, the survival ratio $(s_f/s_o)_{80}$ is always 2.67x larger than $(s_f/s_o)_{60}$, and the odds of larval amphidromy at $R = 0.6$ are always 2.67x larger than at $R = 0.8$, as shown in Figure S2.