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# NOAA Technical Memorandum ERL NHRL-91

**U.S. DEPARTMENT OF COMMERCE**

NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION  
Environmental Research Laboratories

## Comparison of Draft Scale Vertical Velocities Computed From Gust Probe and Conventional Data Collected by a DC-6 Aircraft

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June 1971





## NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION

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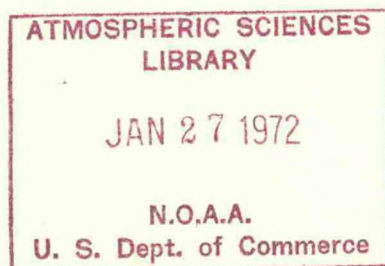
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COMPARISON OF DRAFT SCALE<sup>1</sup> VERTICAL VELOCITIES  
COMPUTED FROM GUST PROBE AND CONVENTIONAL DATA  
COLLECTED BY A DC-6 AIRCRAFT

Toby N. Carlson and Robert C. Sheets

In an experiment involving six passes through cumulus clouds on October 27, 1969, the draft scale vertical motions determined from the RFF inertial platform gust probe were compared with those computed from the conventional DC-6 output of pitch angle, true air speed, and radio altimeter. The results showed a high correlation between the two records of vertical motion particularly where no power setting changes were made during straight line flights through six cumulus clouds over the Florida Everglades. Indications are that the conventional method of measuring vertical motions provides useful accuracy in the study of individual cumulus clouds. An analysis of vertical draft velocity in Hurricane Debbie (1969) yielded reasonable profiles of vertical motions across the eye and wall cloud of the hurricane.

## 1. INTRODUCTION

Various investigators have spent considerable time and energy in attempts to measure selected scales of vertical motion in the atmosphere based on aircraft displacement. A properly instrumented aircraft is thought to be capable of providing information on the vertical motions associated with horizontal scales of motion ranging from about 10 meters to 10 meters<sup>?</sup> in length where the magnitude of the vertical motions range from 10 to 10 cm/sec. This part of the vertical motion spectrum encompasses cumulus motions as well as lee waves and clear air turbulence. A leading investigator in the field of aircraft measurement of turbulent motions in the lower atmosphere is Bunker (1955; 1960; 1968; 1969). Others such as Jones (1954), Malkus (1954), Carlson and Glass (1962), Telford and Warner (1962), Cunningham, Glass, and Carlson (1963)

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<sup>1</sup>The draft scale is that scale of motion in the atmosphere which lies roughly between 0.1 and 10 km in length.



Ross (1966), and Axford (1968) have attempted to measure the draft scale motions in cumulus clouds and thunderstorms while Vergeiner and Lilly (1969) have used aircraft measurements in a study of mountain waves. Gray (1965) computed vertical velocities along radial legs in hurricanes using data from the NHRL files which were gathered on board a B-50 aircraft. One of the earliest attempts to measure draft scale motions by aircraft was in the Thunderstorm Project (Byers, 1949).

The basic approach for computing vertical motion with aircraft data is to measure the vertical motion of the aircraft relative to the ground and to calculate the vertical motion of the air relative to the aircraft as determined from the pitch, roll, and angle of attack of the aircraft. The sum of these two components is equal to the vertical motion of the air relative to a fixed horizontal plane. In two dimensions (roll neglected) the angular component of the vertical velocity is represented simply in figure 1 which shows the pitch angle  $\theta$ , defined as the inclination of the longitudinal axis of the plane to the horizontal, and the angle of attack  $\alpha$ , the latter being the inclination of the longitudinal axis to the relative wind. In calm air, with no vertical component of air motion, the aircraft will assume an equilibrium position to maintain constant altitude. This equilibrium position will differ somewhat from the horizontal and may change with time as fuel is consumed. Following Gray (1965), the deviational quantities of pitch angle of attack  $\alpha$  and true air speed  $V_t$  are each defined with respect to an equilibrium value of that quantity or that

$$\theta_d \equiv \theta - \theta_e \quad (1)$$

$$\alpha_d \equiv \alpha - \alpha_e \quad (2)$$

$$V_{td} \equiv V_t - V_{te} \quad (3)$$

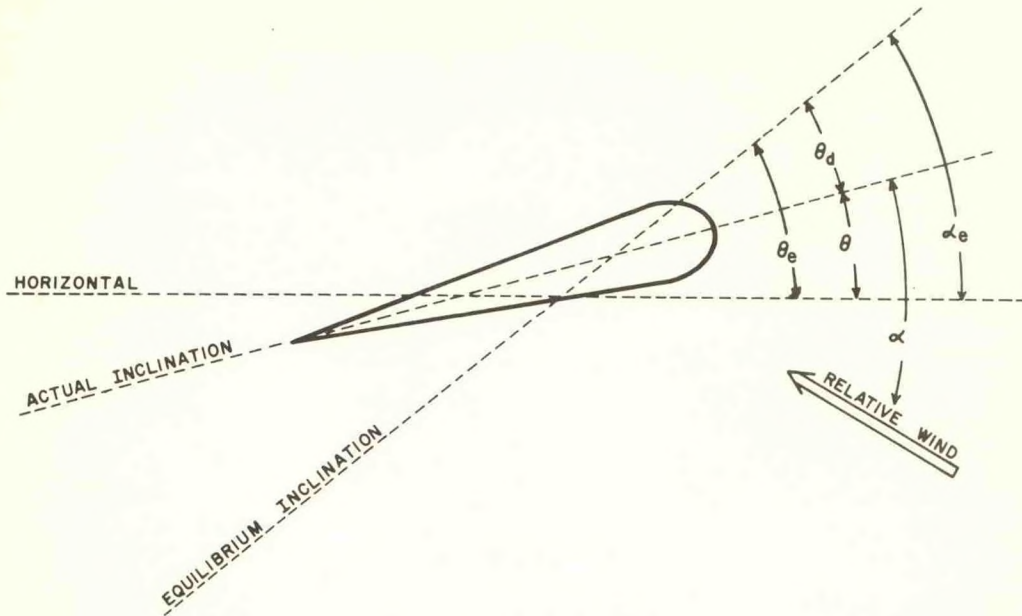


Figure 1. Airfoil in an updraft.

where the subscripts d and e refer to deviational and equilibrium values, respectively.

For small angles, the vertical gust velocity,  $W$ , can be expressed as

$$W = V_t (\alpha_d - \theta_d) + W_p \quad (4)$$

where  $W_p$  is the vertical velocity of the aircraft. (Note that since  $\theta_e \approx \alpha_d$ ,  $\alpha_e - \theta_d \approx \alpha - \theta$ .) On the RFF DC-6 aircraft the true air speed, pitch angle, and radio altitude are measured respectively by the aircraft sensors and recorded digitally on tape at one second intervals. The pitch angle measurement comes from an APN-81 pitch gyro located near the center of gravity of the plane and the radio altitude determined from a Stewart-Warner APN-159 instrument. The angle of attack is not directly measured on the aircraft but a rough estimate can be made from the aircraft equation of motion (see next section). The lag times of the pitch and radar altitude are probably less than one second but the



rapid pulses received by the radio altimeter are not electronically integrated and therefore the one second values of altitude are subject to some spurious fluctuations.

A lot of controversy has surrounded vertical velocity computations derived from the conventional aircraft instrumentation. As the result of numerous uncertainties in measuring the vertical wind there has been a widespread desire among meteorologists to see a more reliable system developed, one which would be as independent of the aircraft response as possible. Recently, various measurements in inertial Platform Gust Probe systems have been developed in England (Ross, 1966; Axford, 1968) and in the United States by the Air Force (Dutton, 1967) for its project HICAT. A modification of the latter system was adapted by the Research Flight Facility (RFF) of NOAA for use during BOMEX and is described by Lappe (McFadden et al., 1970). According to Axford (1968) the maximum error in vertical velocities determined by data from such a gust probe will be less than 1 m/sec. The average uncertainty inherent in the measurements should be much less than this figure, possibly no more than 10 or 20 cm/sec. In the investigation of turbulent conditions associated with cumulus clouds or clear air thermals, an uncertainty in  $W$  of at least 20 to 40 cm/sec can be tolerated without serious harm being done to the flux and energy calculations. Accepting the uncertainty, it becomes permissible in dealing with Axford's gust equations to permit the cosines of angles to be zero and the sine (or tangent) to be equal to the angle itself. Thus Axford's equation for  $W$  can be reduced to

$$W_g = V_{tg} (\alpha_g - \theta_g) + W_p + \lambda(d\theta/dt)_g \quad (5)$$

where  $\ell$  is the distance between the center of gravity of the aircraft to the tip of the boom and the subscript g refers to the measurements made with the RFF gust probe package.

On October 27, 1969, the ESSA DC-6 (39C) aircraft made a series of six passes through individual cumulus clouds at 11,000 ft over the Florida Everglades. Mounted on board the aircraft was the gust probe package which provided a separate but parallel set of measurements to those obtained from the conventional system. It is our objective in this paper to compare, both visually and statistically, gust probe and "conventionally-determined" draft scale vertical motion and to assess the usefulness of the conventional system in terms of its ability to duplicate the profiles of vertical motion which were obtained from the more reliable gust probe system.



## 2. COMPUTATIONS

Vertical motions derived from the gust probe system were processed by Turbulence Consultants, Inc. and Rinaldi Consultants, Inc. A low frequency band pass filter was designed to sharply remove all scales of motion in the gust probe data which were less than 1 second duration (about 100 meters), the rate at which data pieces are digitally recorded in the conventional RFF system. No filtering was done on scales of motion larger than one second other than the averaging which was taken over the total length of each pass. In the processing of gust probe data the individual components were treated as deviations from their respective means over the 4 or 5 minute duration of the pass.

In the gust probe system, the vertical velocity of the aircraft was measured from an accelerometer located on the boom structure and evaluated using the customary relationship  $W_{pg} = \int_{t_1}^{t_2} a_z dt$ , where  $a_z$  is the vertical acceleration of the aircraft. Scale analysis suggests that the torque term,  $(ld \theta/dt)_g$ , is negligibly small, a fact confirmed by the processed gust probe data. Neglecting the torque term, (5) is quite similar to (4). In the conventional system, however, the angle of attack is computed indirectly and the aircraft displacement is measured by a radio altimeter instead of being determined by integrating the acceleration. Because of spurious high frequency fluctuations contained in the data the basic one second values of  $\theta$  and  $V_t$  were smoothed using a 4-second running mean. Aircraft displacement was computed from the difference in radio altitude between the beginning and end of the 4-second period using a centered 3-second running mean to compute both the initial and

final altitude along the segment. (For hurricane work, a straight 6-second average, instead of the 4-second running mean, is used in making the vertical velocity computations.) Vertical velocity measurements were still generated at one-second intervals.

Except for Gray's (1965) work, the radio altimeter has not recently been used aboard aircraft to measure air motions. The radio altimeter is obviously useless for this purpose over hilly or irregular terrain. Both the Everglades and the ocean, however, are exceedingly flat. Gray has stated that the term  $\alpha_d$  is usually less than  $0.2^\circ$  when averaged over several seconds or more. Examination of the data for October 27 showed that  $\alpha_d$  was considerably smaller than  $\theta_d$  most of the time although the former occasionally attained values of a degree or more for a period of 2 or 3 seconds. An attempt was made to reduce spurious fluctuations in the altimeter data. Prior to determining each value of  $W_p$ , the basic unsmoothed height measurements were subjected to a least squares fit of the data taken over a 6-second running interval which overlapped the basic 4-second interval used to form the vertical displacements of the aircraft. Height values which differed from the linear trend by an amount which was considered to be excessive were rejected in favor of the least squares value, the latter being subsequently used in determining  $W_p$ . Less than one value in 20 was rejected in this way.

A serious difficulty arises in the use of (4) with respect ~~respect~~ to the evaluation of  $\theta_e$ . Because of fuel consumption and changes in power setting,  $\theta_e$  will vary gradually during the course of an extended flight though not appreciably so in short cumulus passes. On longer



flights, however,  $\theta_e$  may suffer a considerable variation with time. A direct measurement of  $\alpha_d$  would circumvent this problem in view of the equivalence of  $\theta_e$  and  $\alpha_e$  but this is not yet possible on the RFF aircraft. It should also be pointed out that  $\theta_e$  can be seriously affected by turns or changes in power setting.

For straight (radial, in the case of hurricanes) passes we make use of (4) to determine  $\theta_e$  by assuming that the calculated value of  $W$  is negligibly small when averaged over a sufficiently large interval or that

$$\tilde{W} \equiv \frac{1}{T} \int^T W dt \approx 0. \quad (6)$$

Here  $W$  refers to a running average vertical motion over the interval  $T$  and applies to the mid point of that interval.

Equation (4) becomes in view of (6)

$$0 + \overbrace{[V_t \theta - V_t \alpha_d - W_p]} = \tilde{V}_t \theta \equiv \tilde{V}_t \theta_e \quad (7)$$

The equilibrium value of  $\theta$ , solved for in (7) refers to the mid point of the sliding scale of length  $T$  which may be less than the total pass duration. At that point the calculated vertical motion is computed using an equilibrium value of  $\theta$  which requires that the mean vertical motion over the interval  $-T/2$  to  $+T/2$  be zero. Alternatively,  $\theta_e$  could be computed by simply forming an average of  $\theta$  as in (6). However, conditions which would result in  $\tilde{W}$  differing appreciably from zero would also result in  $\tilde{\theta}$  differing appreciably from  $\theta_e$ ; thus the predicament remains.

In hurricane flights of a half hour or more the value of  $T$  was chosen to be 15 or 20 minutes (50-70 miles). Obviously, the computations are unable to be determined by this method at times closer than  $T/2$  from

the end points of the passes. In practice the original equilibrium interval of  $T$  was allowed to compress progressively to a value of  $T/2$  between times  $T/2$  and  $T/4$  from the end points, allowing the equilibrium pitch angle to be continuously measured in that interval. Between  $T/4$  and 0 seconds from the end points  $\theta_e$  was fixed at its value at  $T/4$ . In short cumulus passes, such as the ones made on October 27, 1969, the value of  $T$  was set equal to the duration of the pass and the single value of  $\theta_e$  derived thereof was used as a constant for that pass. It is evident, therefore, that scales of motion of length greater than  $T$  can not be accounted for by these calculations. For passes which are large compared to the scale of the vertical motions the residual error between the true vertical velocity and the measured  $W$  will be rather small, i.e. it will be below the resolution of which the system is capable. In hurricanes, however, and in flights along the axis of a cloud band the residual may be rather large. Adjustment of the zero line may be necessary in such instances provided that a reasonable physical basis for determining this displacement can be established. One criterion for this is to assume that the vertical motion profile approaches zero at a great distance from the cloud or, in the case of hurricanes, near the center of the eye.

Gray (1965) was unable to measure  $\theta$  or  $\alpha$  directly but he calculated the former using the lifting equation for aircraft motion. Our procedure makes use of the normal equation for aircraft motion to calculate  $\alpha_d$ .



This equation is written

$$\Delta N = \frac{1}{2} \frac{\rho S}{m} (V^2 \frac{dC_L}{d\alpha} \alpha_d + 2V_t C_L V_{td}) - V_t d\theta_d/dt, \quad (8)$$

where  $\Delta N$  is the normal acceleration,  $\rho$  the air density,  $S$  the wind area,  $C_L$  the coefficient of lift and  $m$  the aircraft mass. The various constants were obtained from RFF and, to the best of our knowledge, are appropriate to the DC-6. These are  $m = 4.5 \times 10^7$  g ( $\pm 10$  percent),  $S = 1.36 \times 10^6$  cm<sup>2</sup>, and  $C_L = 0.092 + 0.368 (\alpha \text{ in degrees})$ . If one assumes that the normal acceleration is approximated by the second derivative of the RA,  $d\theta/dt \approx d\theta_d/dt$ , and  $\alpha \approx \theta_e$  in the expression for  $C_L$  then (8) can be solved for  $\alpha_d$  in terms of known quantities. The expression we obtain is

$$\alpha_d \text{ (degrees)} = \frac{720}{\rho V_t} \left( \frac{\Delta N}{V_t} - \rho V_{td} [0.00278 \theta_e + 0.0112] + d\theta/dt \right) \quad (9)$$

where the components are in cgs units and the angles in degrees. The deviational air speed was approximated by taking the departure of the true air speed from a 20-40 second least square fit of  $V_t$ . The other terms were determined from a least squares fit of the data taken over the small (4 seconds for cumulus passes) smoothing interval for  $\theta$  and altitude of flight.

Table I. Statistical summary of cloud run data showing a comparison between gust probe data (subscript g) and conventional data (subscript c) for the vertical motion (W) and for the components of the gust equation, pitch angle ( $Vt \theta_d$ ), vertical aircraft velocity ( $W_v$ ) and angle of attack ( $Vt \alpha_d$ ).\*

PASS NO.	VERTICAL GUST VELOCITY (W)							PITCH ANGLE ( $Vt \theta_d$ )				
	$\sigma_c$	$\sigma_g$	$\sigma_e^*$	$(\sigma_e^*/\sigma_g)$	R	$F_c$	$F_g$	$\sigma_c$	$\sigma_g$	$\sigma_e^*$	$(\sigma_e^*/\sigma_g)$	R
90 SEC. + PASS #1	90	59	58	1.06	0.70	-0.1	-0.2	44	44	18	0.45	0.87
131 SEC. + POWER SET CHANGE PASS #2	92	37	68	2.05	0.73	-0.6	0.0	48	42	25	0.61	0.83
78 SEC. PASS #3	82	45	72	1.60	0.48	-0.4	-0.2	63	55	33	0.60	0.85
80 SEC. POWER SET CHANGE PASS #4	75	64	114	1.78	-0.33	-3.6	3.8	29	31	19	0.61	0.80
55 SEC. PASS #5	44	56	51	0.91	0.50	-0.3	-0.4	32	26	9	0.35	0.97
84 SEC. + POWER SET CHANGE PASS #6	70	42	56	1.46	0.48	-0.4	0.0	27	23	16	0.66	0.84

PASS NO.	VERTICAL AIRCRAFT VELOCITY ( $W_p$ )						COMPUTED/MEASURED ATTACK ANGLE ( $Vt\alpha_d$ )				
	$\sigma_c$	$\sigma_g$	$\sigma_e^*$	$(\sigma_e^*/\sigma_g)$	R	—	$\sigma_c$	$\sigma_g$	$\sigma_e^*$	$(\sigma_e^*/\sigma_g)$	R
90 SEC. + PASS #1	57	49	44	0.90	0.64	—	** (40 SECONDS)				
							34	24	41	1.73	0.04
131 SEC. + POWER SET CHANGE PASS #2	74	44	53	1.32	0.71	—	** (81 SECONDS)				
							44	23	40	1.75	0.40
78 SEC. PASS #3	67	57	52	0.91	0.66	—	52	21	47	2.24	0.44
60 SEC. POWER SET CHANGE PASS #4	56	71	90	1.27	0.02	—	49	25	49	1.96	0.28
55 SEC. PASS #5	50	61	46	0.75	0.67	—	36	12	32	2.65	0.43
84 SEC. + POWER SET CHANGE PASS #6	64	38	46	1.29	0.77	—	** (44 SECONDS)				
							37	17	30	1.76	0.59

+

AVERAGE OF TWO SEGMENTS

++

TWO SEGMENTS MADE, COMPUTED OVER ONE SEGMENT

+ AVERAGE OF TWO SEGMENTS

\*\* TWO SEGMENTS MADE, COMPUTED OVER ONE SEGMENT

\* Parameters listed in the table are for the standard deviation ( $\sigma$ ) of the respective samples and the adjusted error deviation ( $\sigma_e^*$ ) which is defined in the text. R is the linear correlation coefficient between the two records and F the water vapor flux in m cal/cm/sec. Small figures in parenthesis listed under R are the linear correlation coefficients in the case where the derived angle of attack was omitted from the conventional calculations. The duration of the flight segment and other pertinent information is listed under the column labelled Pass No.

Table II. Same as Table I but for clear air segments.

PASS NO.	VERTICAL GUST VELOCITY (W)							PITCH ANGLE ( $Vt \theta_d$ )				
	$\sigma_c$	$\sigma_g$	$\sigma_e^*$	$(\sigma_e^*/\sigma_g)$	R	$F_c$	$F_g$	$\sigma_c$	$\sigma_g$	$\sigma_e^*$	$(\sigma_e^*/\sigma_g)$	R
70 SEC. PASS #1	278	251	142	0.57 (0.73)	0.86 (0.80)	149	156	174	173	39	0.23	0.98
92 SEC. POWER SET CHANGE PASS #2	124	104	86	0.83 (0.72)	0.73 (0.74)	6	-13	71	77	50	0.65	0.78
112 SEC. PASS #3	156	154	98	0.64 (0.55)	0.80 (0.86)	51	76	109	105	47	0.45	0.90
85 SEC. POWER SET CHANGE PASS #4	124	195	213	1.09 (1.21)	0.16 (0.02)	-33	-12	126	112	59	0.53	0.88
128 SEC. PASS #5	137	296	238	0.80 (0.81)	0.61 (0.61)	34	79	117	103	75	0.73	0.78
125 SEC. POWER SET CHANGE PASS #6	89	62	77	1.24 (1.17)	0.53 (0.37)	3	-4	55	50	33	0.46	0.81

PASS NO.	VERTICAL AIRCRAFT VELOCITY ( $W_p$ )							COMPUTED/MEASURED ATTACK ANGLE ( $Vt \alpha_d$ )				
	$\sigma_c$	$\sigma_g$	$\sigma_e^*$	$(\sigma_e^*/\sigma_g)$	R	—	—	$\sigma_c$	$\sigma_g$	$\sigma_e^*$	$(\sigma_e^*/\sigma_g)$	R
70 SEC. PASS #1	208	182	155	0.85	0.69	—	—	82	89	90	1.01	0.44
92 SEC. POWER SET CHANGE PASS #2	98	102	53	0.52	0.86	—	—	75	59	66	1.12	0.54
112 SEC. PASS #3	137	141	57	0.40	0.92	—	—	81	61	86	1.41	0.30
85 SEC. POWER SET CHANGE PASS #4	150	196	165	0.84	0.58	—	—	92	87	90	1.03	0.50
128 SEC. PASS #5	130	238	235	0.99	0.29	—	—	67	37	82	2.22	-0.17
125 SEC. POWER SET CHANGE PASS #6	81	62	51	0.82	0.78	—	—	58	44	59	1.34	0.35



### 3. STATISTICAL EVALUATION OF CONVENTIONAL DRAFT SCALE MEASUREMENTS

#### 3.1 A Total Comparison

Superimposed records of the gust probe and conventional vertical motion data are shown in figures 2 (a-f). As an aid in outlining the cloud dimensions, a graph of the absolute humidity is presented above each pair of vertical motion traces. Significant departures of the vapor content of the air from the ambient value of absolute humidity ( $3\sim\text{g}/\text{m}^3$ ) denotes the presence of cloud or evaporated cloud material; total saturation occurs at about  $7\text{ g}/\text{m}^3$ . Passes 1, 3, and 5 were made through cumulus clouds with the pilot attempting to maintain constant power setting and altitude; in passes 2, 4, and 6 the pilot was instructed to change the power setting at will. The purpose in doing this was to assess the effects of such power set changes on the calculations. Inadvertent or procedural augmentation and reductions in power are made in hurricanes when the aircraft penetrates the eye wall or enters a particularly intense band of cumulonimbus. It was unfortunate that the three passes chosen to make power setting changes were associated with the three weakest clouds, thus making it more difficult to assess the effect of changing power on the updraft profile.

Tables 1 and 2 statistically summarize the degree of match between the conventional and gust probe calculations for cloud and clear air regimes, respectively. The 'cloud runs' correspond approximately to the turbulent conditions in and around the cloud; the end points of the cloud runs are delineated in figures 2 (a-f) by the (dotted) vertical lines which are labelled accordingly. The clear air statistics are composed of an average of the two (approximately equal) data segments which lie on either side

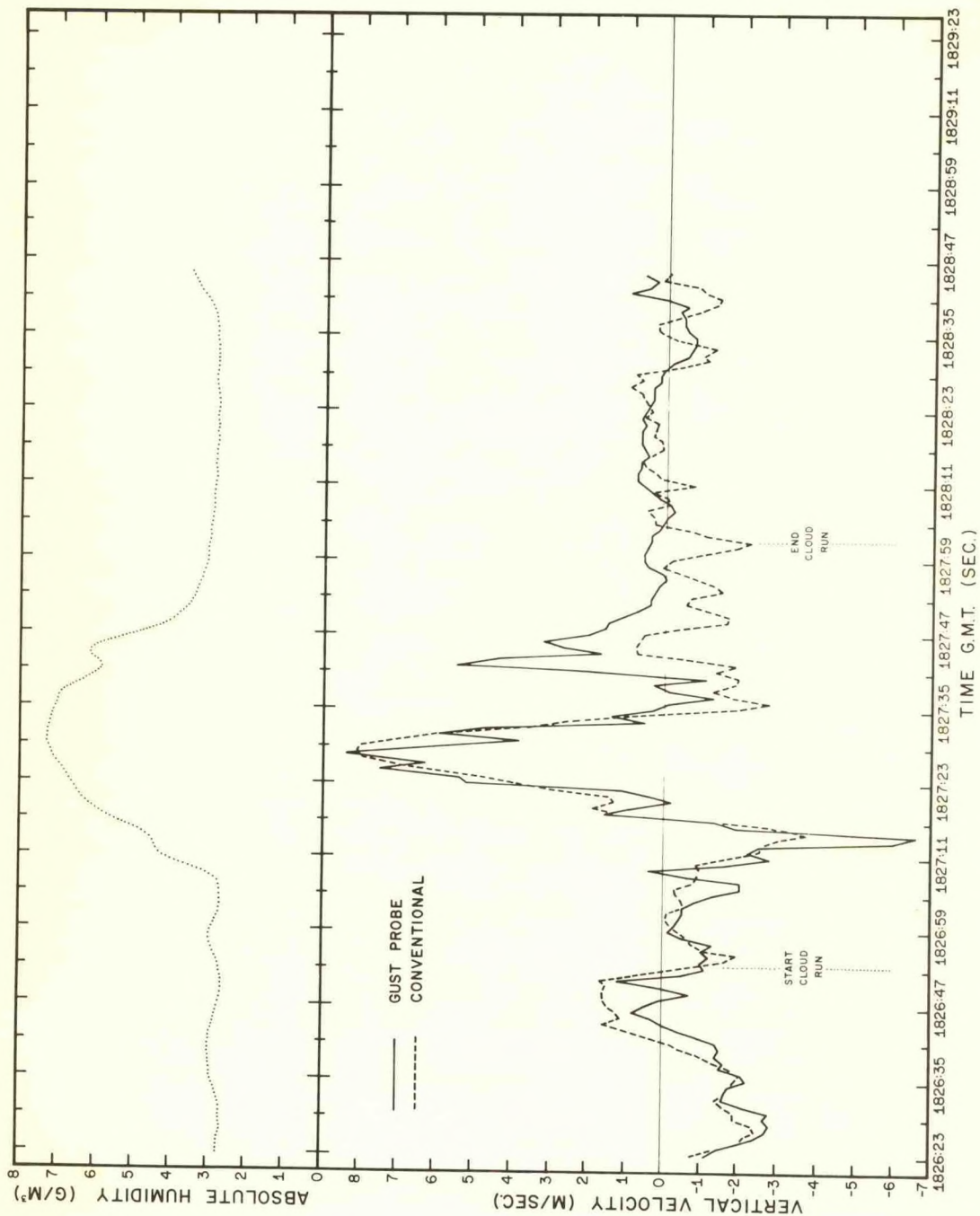


Figure 2(a). Profiles of draft-scale vertical motion and absolute humidity in pass 1. Statistics compiled in table 1 refer to segment of flight which lies between vertical pecked lines labelled start and end cloud run.



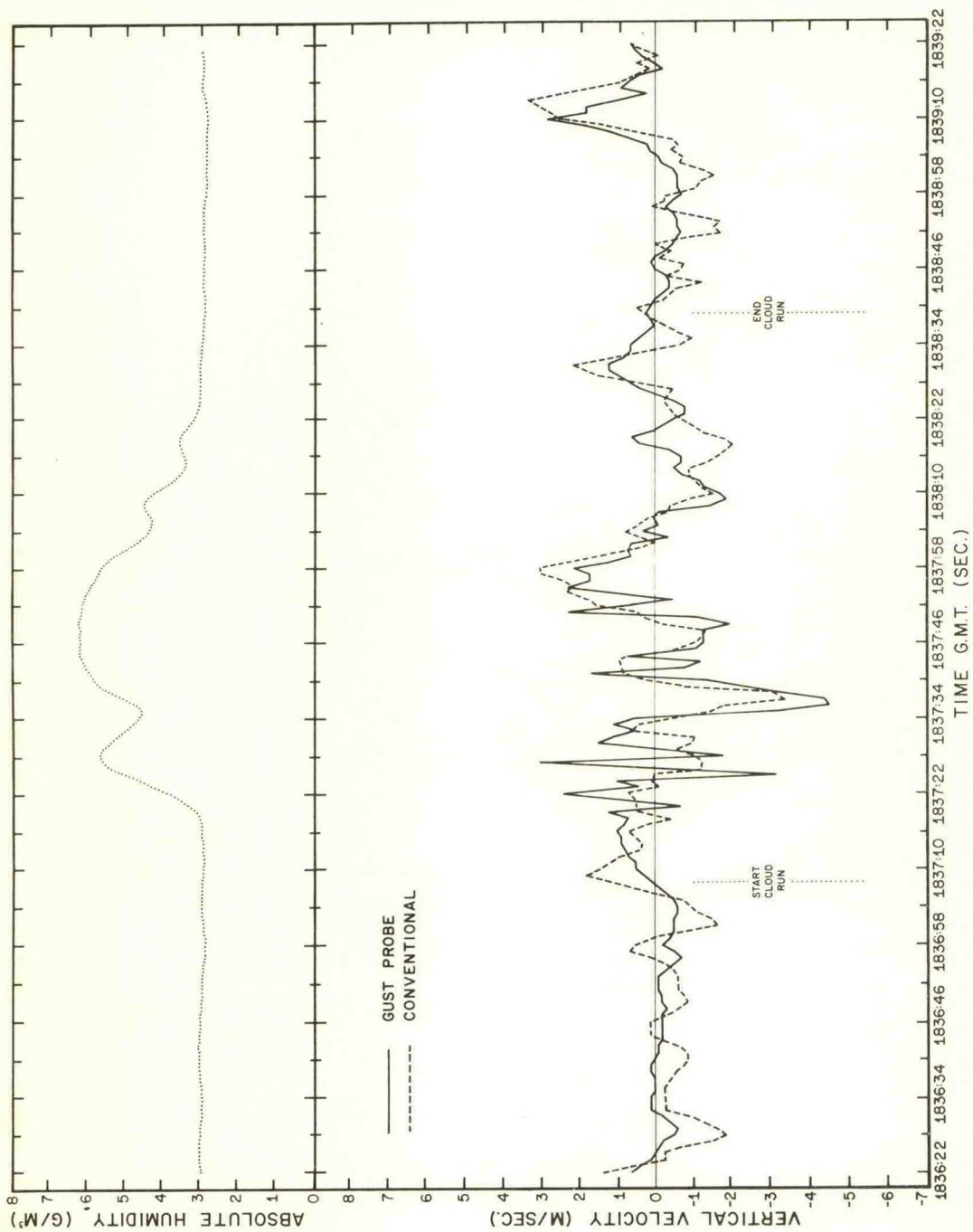


Figure 2(b). Same as figure 2a but for pass 2.

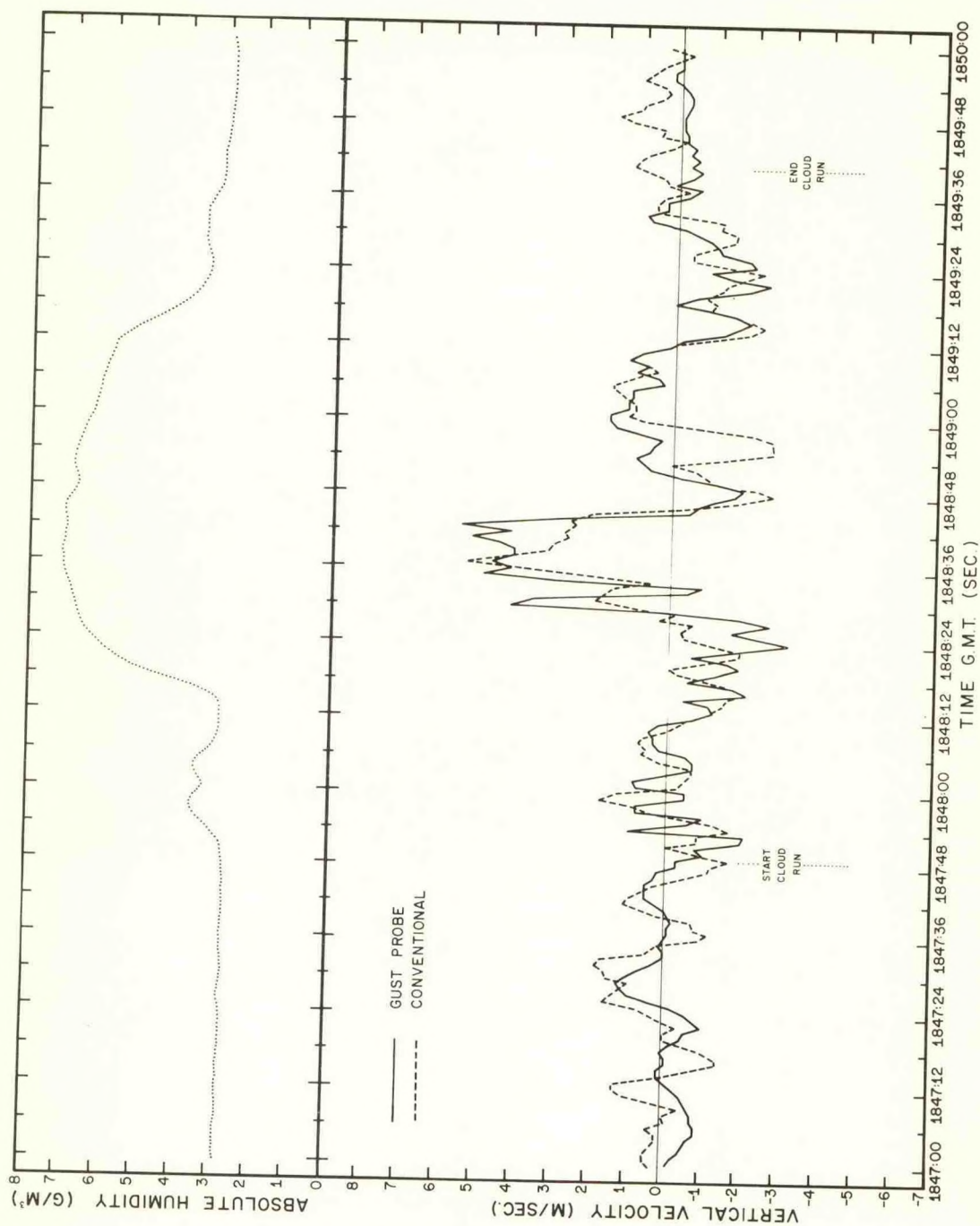


Figure 2(c). Same as figure 2a but for pass 3.



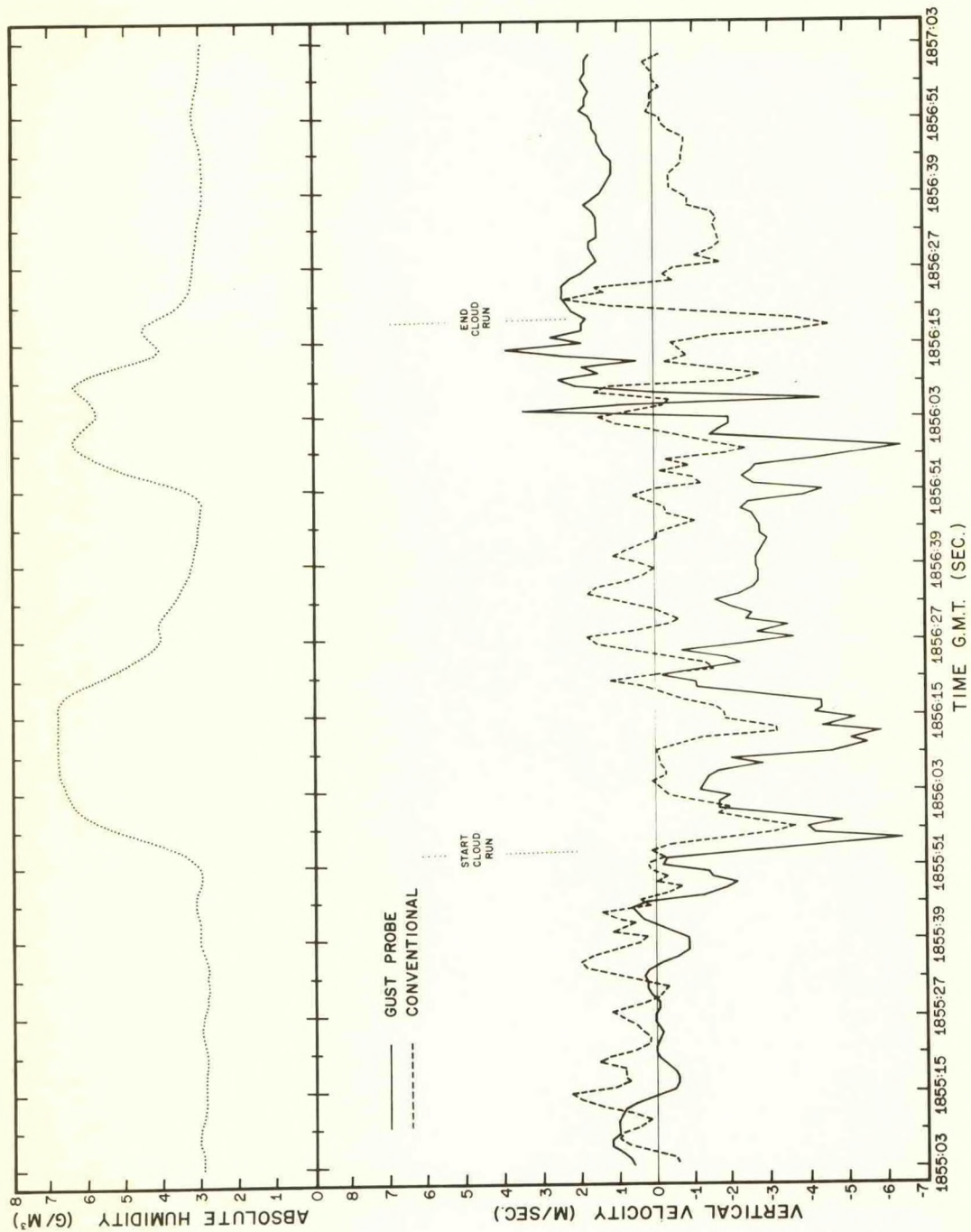


Figure 2(d). Same as figure 2a but for pass 4.

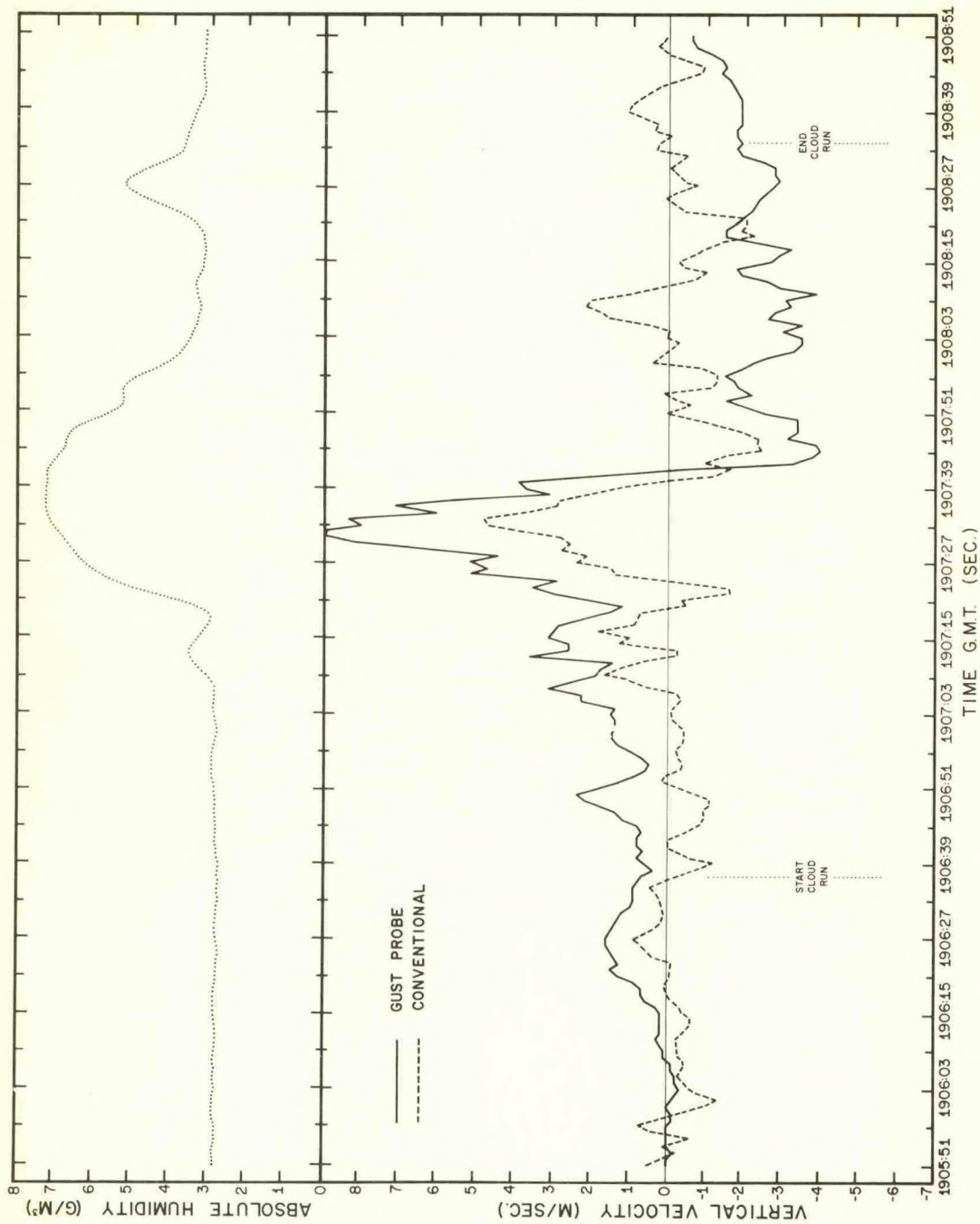


Figure 2(e). Same as figure 2a but for pass 5.



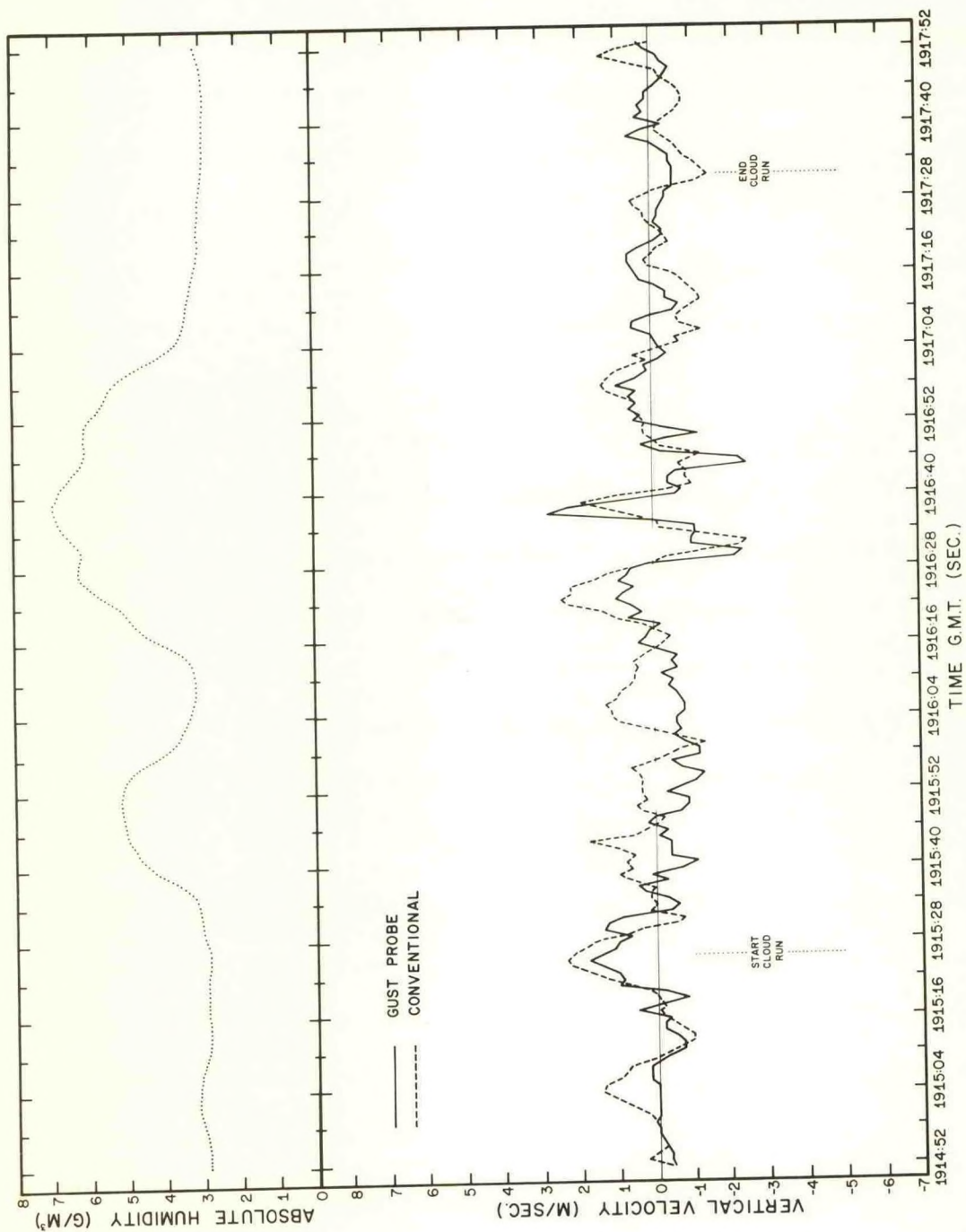


Figure 2(f). Same as figure 2a but for pass 6.

of the cloud run segment. In tables 1 and 2, statistics for each of the four components in the gust velocity (4) are presented separately, showing the standard deviation of the record  $\sigma$  (the square root of the variance), the adjusted standard deviation of the error  $\sigma_e^*$ , and the linear correlation coefficient R between the gust probe (subscript g) and the conventional system (subscript c). The error sigma,  $\sigma_e^*$ , is defined as the standard deviation of the point by point differences between the two records after the means have been subtracted out of both the records or that

$$\sigma_e^{*2} = \sigma_e^2 - (\bar{X}_c - \bar{X}_g)^2 \quad (10)$$

where the bar operator represents the record average over the particular segment. It should be noted again that the records are composed of data taken at one-second intervals. In determining tables 1 and 2 the original gust probe records, shown in figures 2 (a-f), have been smoothed by taking a 4-second running average in order to conform to the smoothing used in making the conventional calculations. In addition to the linear correlation coefficient R, a measure of record similarity is the error to signal ratio ( $\sigma_e^*/\sigma_g$ ), a concept which contains the implication that the gust probe data are essentially perfect. In actuality the gust probe will not yield perfect information and therefore the statistics will generally underestimate the reliability of the conventional system.

A comparison of the water vapor fluxes (F) for both records is also shown in tables 1 and 2. The fluxes (m-cal/cm<sup>2</sup>/sec) were computed using the formula  $F = \overline{Lw^1q^1}$ , where L is the latent heat of condensation for water, q is the mixing ratio in g/m<sup>3</sup>, and the bar operator refers to an average over the segment of the pass (clear or cloudy) in question.



Additional cloud physics information concerning maximum droplet size and liquid water content of the six cumuli are presented in figure 3. The liquid water content was computed from cloud particle samples collected by a continuous hydrometeor sampler which consists of a moving strip of soft aluminum foil exposed to the ambient airflow through a small slot. Cloud particles larger than 200 microns in diameter leave impressions when they impact on the foil, the liquid water content of the large (raindrop) sized hydrometeors is computed using a method similar to that used by Tacheuchi (1969). Maximum drop diameters were obtained at approximately 1.35-second intervals and liquid water contents were computed over each 5.4-second interval. Terminal velocities corresponding to the largest drop sizes were obtained from a relationship experimentally obtained by Gunn and Kinzer (Byers, 1965). Figure 3 therefore contains a qualitative but independent assessment of cloud activity and updraft strength based on the maximum fall velocity of the drops.

On the whole, passes 1, 3, and 5, show the better agreement between the gust probe and conventional data than passes 2, 4, and 6 both visibly, in terms of the location and strength of the maximum up- and downdrafts, and with respect to the statistical parameters of tables 1 and 2. In pass 1, the records are remarkably similar except that the secondary updraft at 1827:45 failed to be captured in the conventional record. Cloud pass 5 exhibits the strongest updrafts (9.2 m/sec) according to the gust probe data but is clearly less substantial than cloud 1 which contained a considerably higher liquid water content and much larger droplets. Although the gust probe is undoubtedly superior to the conventional system

# FLIGHT 691027-A

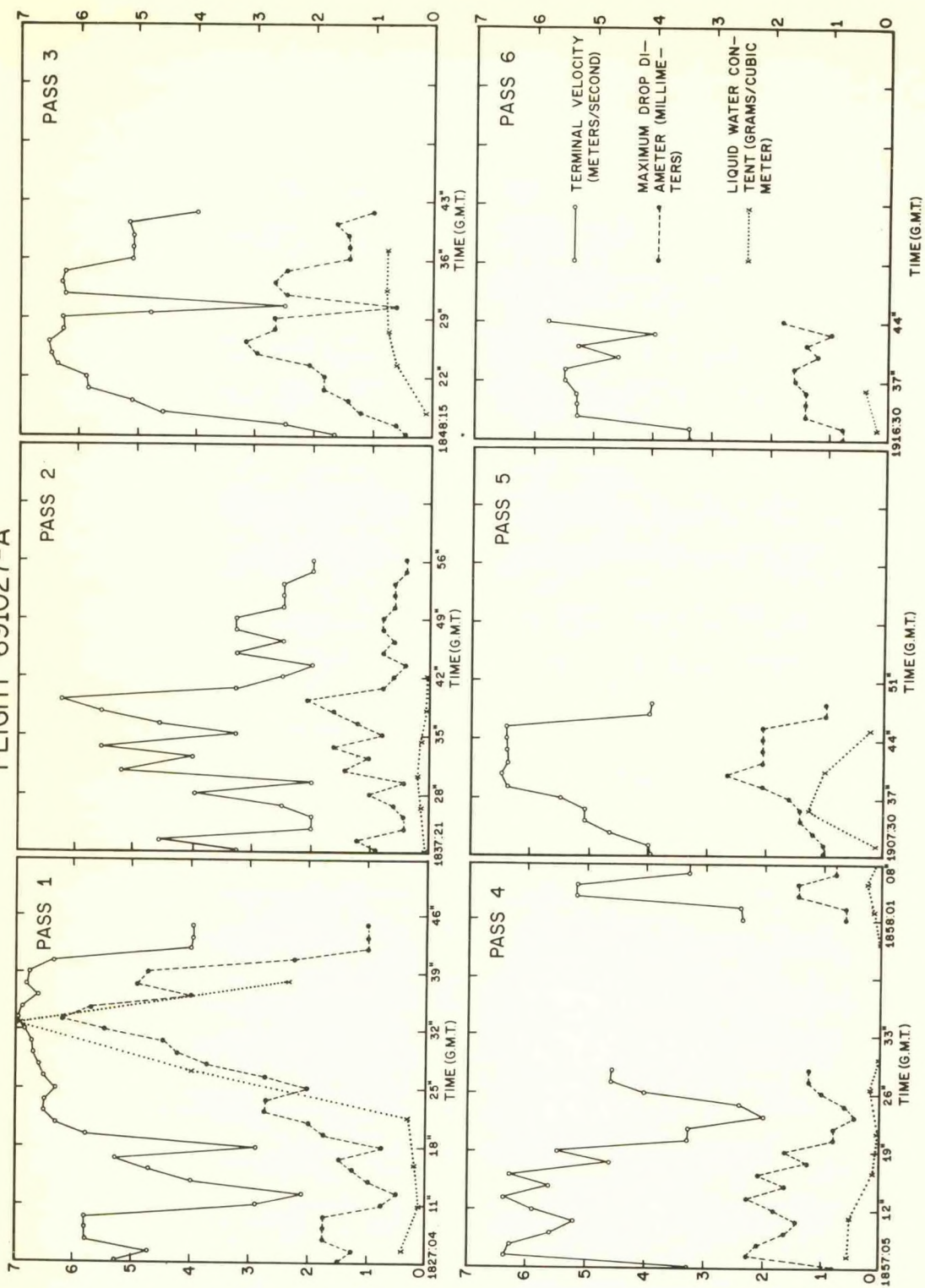


Figure 3. Selected hydrometeor data for passes one through six.



### 3.2 Agreement As A Function Of Wavelength

In an attempt to assess the dependency of  $\sigma_e$  and  $\sigma_e^*/\sigma_g$  on the wavelengths, the harmonic analysis (power spectra distribution) was computed for each of the six passes over the entire length of the pass using the generalized E-M method described by Evans et al. (1963). This method determines the autocorrelation coefficients in one step and then transforms them into power spectral density using Fourier techniques. Numerical uncertainties, probably resulting from the shortness of the records and the singularity of having a discrete cumulus cloud in the middle, made it necessary to average the power over discrete wave bands for each pass and then combine the averages for all six passes in order to arrive at a representative value for that wave band. In figure 4 the power density (variance) spectrum versus wavelength is presented for the gust probe measurements on a log-log scale. The mean variance in each wave band is shown by a series of horizontal line segments through which a smooth curve has been drawn. The figure shows the decrease in power with decreasing wavelength that customarily occurs in cumulus spectra (see Rhyne and Steiner, 1964).

The semi-log plot at the top of figure 4 shows the ratio  $(\sigma_e^*/\sigma_g)$  as a function of wavelength. Here  $\sigma_e^*$  was obtained from a power spectral analysis of the error variance  $\sigma_e^*$  comparable to the one done for  $\sigma_g$ . The ratio of the two was formed for each wave band and plotted as a series of single points. The pair of smooth curves drawn through the points represent values for all six passes combined and for just the three passes in which no power setting changes were made.

in measuring draft scale vertical motions the former may have suffered from a pronounced zero drift in passes 4 and 5. In that case, the major ascending plume at 1907:33 may have been overestimated by at least a couple meters per second in  $W_g$ ; after 1907:39 the zero shift seems to have been in the opposite direction. The same problem may also be afflicting pass 4 which yielded the worst comparison with conventional data in both clear and cloud runs. Pass 2, despite power set changes, is in better mutual agreement than pass 5 with respect to the correlations although  $(\sigma_e^*/\sigma_g)$  and the fluxes are slightly poorer for this run.

In clear air passes 1, 3, and 5 are generally superior to the other three runs even though the variance of the records are about the same. However, statistical agreement is notably poorer in the clear air runs than it is in cloud, signifying the improvement gained in the statistics with increasingly turbulent conditions. In the non-constant-power set runs the fluxes agree in sign and in magnitude to within a factor of about 2 in either regime. Even with power setting changes a certain amount of useful information is contained in the conventional data; for example, the agreement between the primary up- and downdraft profiles in passes 2 and 6 (figures 2b and 2e). Under constant power setting and in turbulent conditions ( $\sigma_c > 100$  cm/sec) the draft scale vertical motions computed conventionally and those which represent the true values are felt to be in reasonable agreement.



Agreement between the individual components in the gust probe (4) is by far the best with respect to the pitch angle term,  $(V_t \theta_d)$  as compared to the other terms. Unfortunately, most of the variance in  $W$  comes from the aircraft vertical velocity ( $W_g$ ), a component which has a poorer correspondence between conventional and gust probe values than does the pitch angle. It appears that the inability of the conventional system to attain more than about 5 m/sec in pass 5, as compared with 9 m/sec for  $W_g$  at the point of maximum updraft, is entirely attributable to the much smaller aircraft vertical motions determined from the radio-altimeter. Since the pitch angle variance was not particularly large in pass 5 there is additional reason to suspect the accuracy of the gust probe accelerometer in this run. By far the worst correspondence in the gust equation lies in the angle of attack values  $(V_t \alpha_d)$ , a fact not surprising in view of the vastly differing ways in which they are determined for each system. Nevertheless, there appears to be some information contained in the computed angle of attack. Draft velocities computed from (4), using the computed value of  $\alpha_d$  determined from (7), are found to be slightly superior to those generated with  $\alpha_d$  set to zero (see parenthesized figures located above the bold-faced numbers in table 1). Although the variance of  $\alpha_d$  is small compared to that of  $W$ , there is obviously no substitute for measuring this term directly.

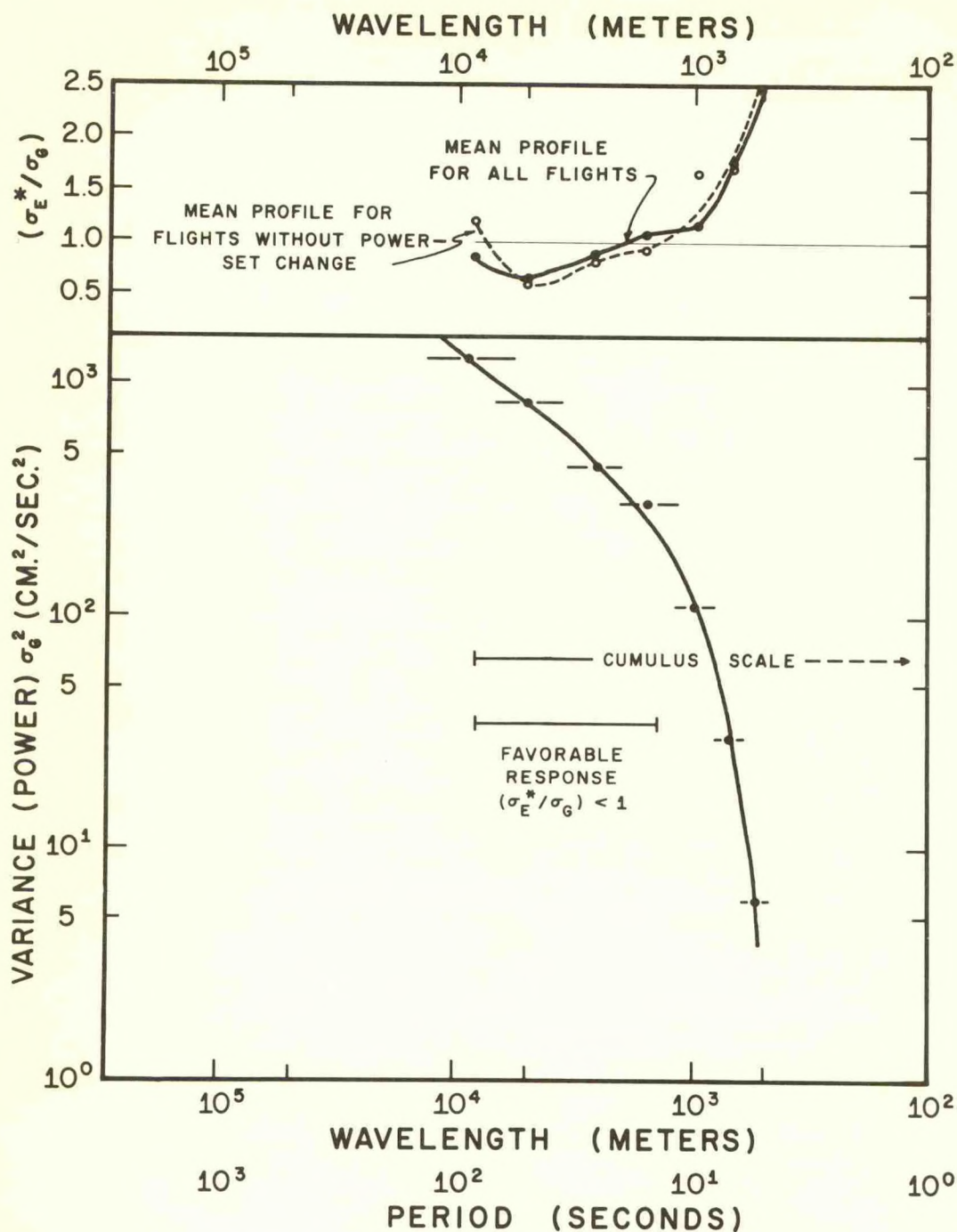


Figure 4. Variance (Power spectrum density) of the gust probe vertical motions versus wavelength (period) for the six cumulus passes (lower part of figure). Horizontal line segments indicate width of spectrum over which the average power (signified by the solid dot) was computed. The top pair of curves show the ratio  $(\sigma_E^*/\sigma_G)$  as a function of wavelength for all six passes (solid curve and filled dots) and for the three passes made without power setting changes (dashed curve and open dots).



Lowest values ( $\sim 0.6$ ) are found near 4000 meters, the approximate width of the cumulus cloud and its surrounding perturbation in the clear air. Between 1000 and 4000 meters wavelength this ratio remains below 1.0, a value considered by this author to be a reasonable criteria for a favorable comparison between gust probe and conventional motions. Below 1000 meters the ratio rises sharply with decreasing wavelength. Thus, on the scale comparable to the width of a cumulus cloud the conventional system provides a favorable response to the pattern of updrafts and downdrafts particularly when power setting changes on the aircraft are reduced to a minimum. Although it is not possible to assess with any reliability the dependency of the forementioned ratio on the intensity of the turbulent motions it seems quite likely that the system responds more favorably when there is a greater amount of energy in the longer wave lengths.

Part of the difficulty in measuring the smaller scale of vertical motions lies in the inability of the conventional system to properly determine  $\alpha_d$ . Tables 1 and 2 show that the magnitude of  $\alpha_d$  is grossly overestimated in the conventional system during passes in clear air but is comparable to that obtained from the gust probe in cloudy regimes. Another factor in the poor agreement at short wavelengths may be the use of a running mean which acts as an imperfect filter and therefore distorts wavelengths near or below 400 meters (in the case of a 4 second running mean).<sup>2</sup>

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<sup>2</sup>At present a running binomial smoother is being used which is thought to be an improvement on the straight running mean.

#### 4. COMPUTATION OF DRAFT-SCALE VERTICAL MOTIONS IN A HURRICANE

Gray (1965) has attempted to measure draft scale vertical motions in hurricanes, arriving at some radial profiles of vertical motion from which he compiled a set of statistics on draft strength and width. With an improved system on board the DC-6, capable of making more accurate measurements than were available to Gray, and with some confidence gained in the validity of the data as the result of the gust probe comparison, we find it desirable to reopen the question of measuring hurricane vertical motions with an aircraft. In this section vertical motions associated with Hurricane Debbie (1969) will be discussed.

As stated in section 2, the vertical velocity is determined at the center point of a sliding time scale of length,  $T$ , over which the mean vertical motion is assumed to be zero. In cumulus work the basic data sub-unit was a 4 point running average of the one-second interval observations; the interval  $T$  is regarded as the length of the straight line pass--usually a few minutes duration. In hurricanes, the data are manifestly less steady. The smaller scales of motion investigated in cumulus passes are felt to be of even less general importance and have less reliability in the hurricane flights. Radial legs are typically 15 to 45 minutes in duration and a single pass may intersect rainbands, both eye walls, and the eye itself.

It was decided in hurricane work to use a 6-second (~600 meter) block average (not a running mean) for the basic data sub-unit and a 12-minute (~72 km) interval for  $T$ . As stated in a previous section, the value of  $\theta_e$  is determined by assuming that the mean vertical motion is



zero when averaged over the 72 km interval except for the interval centered between  $T/2$  and  $T/4$  from the end points. Here the original value of  $T$  is gradually reduced from  $T$  to  $T/2$  at  $\pm T/4$  from the end points; closer than  $\pm T/4$  from the end points  $\theta_e$  is held constant. This procedure permits  $\theta_e$  to vary slowly in response to a drift or to slow oscillations in the actual equilibrium pitch angle. The results of section III suggest that the effects of infrequent power setting changes by the pilot may not seriously compromise the results although it should be noted that power setting changes simulated in the October 27 passes were not likely representative of the actual power setting changes that take place in an eye wall penetration.

On August 20, 1969, ten passes were made through Hurricane Debbie by the RFF 39C aircraft during Project STORMFURY.<sup>3</sup> Figure 5 shows an example of the vertical motions, along with wind speed, temperature, and a composite of the vertical profile of the radar presentation, on a northeast to southwest traverse at 12,000 ft. In this figure the 6-second block values of vertical motion have been additionally smoothed using a centered 36-second running mean. Southwest of the eye center the tallest and widest isoecho contour coincides closely with the largest updraft ( $\sim 5$  m/sec). Elsewhere the principal updrafts tend to coincide with isoechoes and the stronger downdrafts with gaps between the cells. In the eye itself the computed vertical motions behave erratically, in contrast to one's expectations. Strongest winds recorded were located

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<sup>3</sup> See Project STORMFURY, Annual Report 1969.



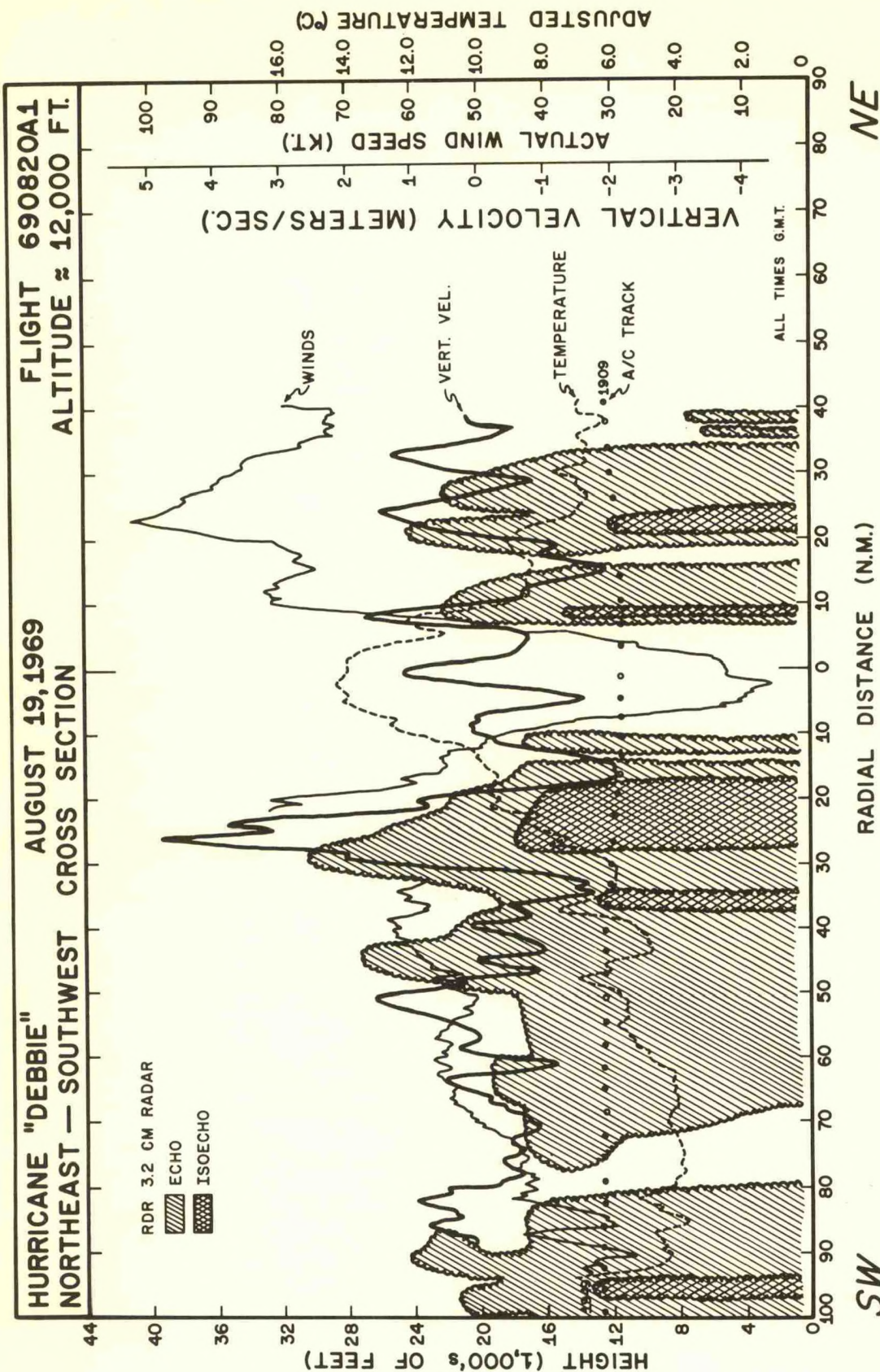


Figure 5. Profiles of vertical velocity (heavy solid line), wind speed (thin solid line), temperature (dashed line) along a northeast to southwest pass through Hurricane Debbie at approximately 12,500 ft. Range-height contours of low and high gain (iosecho) radar echoes are indicated by hatched and cross hatched lines, respectively. The aircraft track, starting at 1909 GMT and ending at 1940 GMT, is indicated by the row of closed circles spaced at one minute intervals.



on the northeast side of the storm; however, the wind speeds computed relative to the moving storm were about equal on both sides of the eye wall.

Altogether eight passes were made along the same azimuth, four approaching the eye from the northeast and four from the southwest. An arithmetic composite of the eight passes was made by first aligning the data for each radial segment such that the point of entry or exit through the physical eye wall was mutually in common and then averaging the vertical motions for all passes. Examination of the initial data, however, revealed that the eye of Hurricane Debbie consisted of an inner 'undisturbed region,' over which temperature, humidity and tangential wind varied slowly in the horizontal, and a transition region, across which the wind speed and dewpoint decreased rapidly (inward) and the temperature increased at a rate comparable to that found near the edge of the eye wall cloud. Since the eye itself varied in size from one pass to another the inner eye was composited separately and then linked together with the vertical velocity profile averaged relative to the eye wall in order to produce a mean profile of vertical motion across Hurricane Debbie. In figure 6 the solid curve is the composite vertical velocity profile with respect to the azimuth from the eye center. In order to examine the directional bias in the computations the vertical velocity profile was also formed without regard to azimuth, the starting and finishing points of the flight being on the left and right, respectively, of figure 6 (dotted curve). Ideally, the latter profile should be symmetric, unlike the former which would show real aximuthal variations.

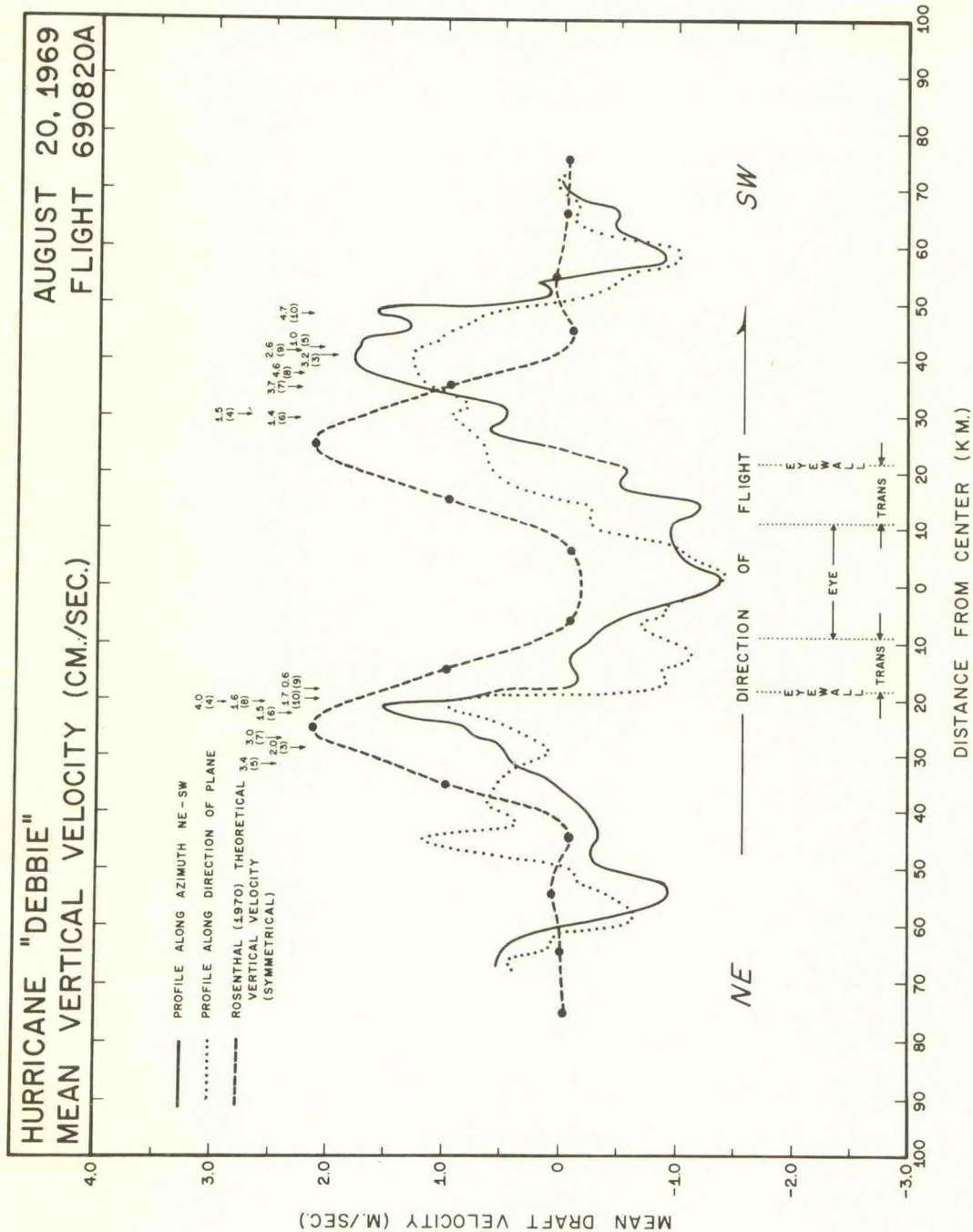


Figure 6. Composite profiles of vertical motion through Hurricane Debbie averaged from eight separate passes through the storm at approximately 12,000 ft. (For meaning of profile legends, see test.) Figures shown above profiles (with arrow showing location of point along azimuthal profile) give maximum strength of updraft (meters per second) within eye wall band for passes three through ten (pass number in parentheses). Dashed curve is for Rosenthal's (1970) symmetric hurricane model.



According to figure 6 the southwest portion of the hurricane appears to have been stronger than the northeastern sector in terms of the maximum value of  $W$  ( $\sim 2$  m/sec). The overall draft profile, however, resembles Rosenthal's (1970) symmetric model. Individual updrafts are shown to vary considerably in strength and location from pass to pass, figure 5 being representative of one of the strongest updrafts. Sinking motion of about 1 m/sec is found in the eye and on the outside edges of the eye wall cloud. This amount of subsidence in the eye seems excessive and may be attributed to the effects of power setting changes. Alternatively, it could be explained by the fact that the residual vertical motion on scales larger than  $T$  are not captured by the method of calculating  $W$  due to an incorrect value of  $\theta_e$ . The existence of scales of vertical motion having dimensions greater than  $T$  (as well as the presence of systematic fluctuations in power setting near the eye wall) can result in an underestimate of  $W$ , not only in the eye wall cloud but across the eye itself. In this event, a correction in the form of an undetermined amount of ascending motion may have to be added to the updraft profiles in figure 6 to make them more realistic. According to Miller (1963) the mean upward vertical motion at 700 mb in Hurricane Donna, 1960, averaged between 20 and 80 km, was slightly greater than 0.6 m/sec. An upward shift of the vertical motion profiles in figure 6 by this magnitude would evidently result in better degree of fit with the Rosenthal (1970) curve, thereby encouraging one to believe that such a correction to the computed values should be made.

Following Bunker (1968) the vertical transport of sensible heat  $H$  and water vapor  $E$  can be written

$$\begin{aligned} H &= c_p \rho (\overline{W'T'} + \overline{W^*T^*}) \\ E &= L (\overline{W'q'} + \overline{W^*q^*}) \end{aligned} \quad (11)$$

where  $c_p$  is the specific heat of air at constant pressure,  $\rho$  the air density,  $L$  the latent heat of condensation for water,  $q$  the water vapor content of the air,  $W$  the vertical velocity, and  $T$  the temperature. The fluxes are represented as energy per unit area per unit time and consist of two components, a small (cumulus) scale eddy term (signified by the primed quantities) and a hurricane-scale eddy term (signified by the product of the two starred quantities). The latter term represents the deviations in the mean of  $T$  and  $q$  in the eye wall cloud region from that in the mean tropical environment. In this case the bar signifies a linear mean taken between the eye wall and the outer edge of the wall cloud region, a distance of about 50 km. The above equations were evaluated for the individual passes and the quantities of  $E$  and  $H$  multiplied by the appropriate area, and averaged over all passes to arrive at a rough estimate for the total energy and moisture flux across the level of flight (640 mb). The values for the southwest and north-eastern portions of the storm are shown in table 3.<sup>4</sup>

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<sup>4</sup>Since the figures refer to a linear rather than a cylindrical average in the table the values are likely to be overestimated slightly. Recalculation of the fluxes in cylindrical coordinates showed little change in the figures, however.



Table 3. Small and Large (hurricane) scale sensible heat and moisture fluxes across the 640 mb surface for the portion of Hurricane Debbie lying between the edge of the eye wall (radius 20 km) and the outer edge of the principal hurricane clouds (radius 70 km).\*

	H (small scale)	H (hurricane scale)	H (total)	E (small scale)	E (hurricane scale)	E (total)
( $\bar{\sigma} = 161$ ) Southwest	0.3	1.2	1.5	3.7	31.5	35.2
( $\bar{\sigma} = 152$ ) Northeast	0.7	0.1	0.8	5.6	1.6	7.2
Average eye wall band	1.0	1.3	2.3	4.6	16.6	21.2

\* Units are in joules/sec  $\times 10^{13}$  for H and cm/day of rainfall for E (see (11) and (12) in text).

Surprisingly, the small-scale eddy fluxes are smaller than the band-scale fluxes. This could indicate that processes associated with the mean hurricane circulation and with the very large clusters of cumulonimbus are the dominant ones in the eye wall band. It is probably also true that the smaller scales of motion are less adequately represented in the calculations as has been suggested in a previous section. Nevertheless, the total values in table 3 are quite reasonable when compared to the results of Hurricane Hilda (Hawkins and Rubsam, 1969). In that storm the production of kinetic energy within the inner 80 km radius was about  $0.2 \times 10^{13}$  j/sec and the rainfall in the same annular ring about 25 cm/day.

## 5. CONCLUDING REMARKS

Based on a small sample of six passes through cumulus, it seems reasonable to conclude that the conventional method of computing draft scale vertical motion can yield values which are not only meaningful but are relatively quick and inexpensive to obtain. According to the results shown in this paper one can tentatively conclude that on a straight line flight through an individual cumulus cloud, the linear correlation between the vertical velocities computed by the conventional method and those which actually exist may be 0.7 to 0.9. Alternatively, the degree of fit between computed and actual vertical motion profiles as expressed by the ratio of error variance to the actual variance of the motions will be substantially less than 1.0 over the whole spectrum of wavelengths and lowest (less than 0.5) in the longer wavelengths that are comparable in scale to the width of a cumulus cloud. There is also some indication that the relative accuracy of the conventional measurements improves with increasing intensity of the vertical motions.

Power setting changes on the aircraft are shown to be detrimental to the conventional results, though less so in active cumulus clouds than in clear air where agreement between gust probe and conventional measurements was poor. It is anticipated that a direct measurement of angle of attack, using an angle of attack vane mounted on the aircraft, will be incorporated into the conventional system in the near future. This improvement would reduce the uncertainties caused by power setting changes on the aircraft and enable a better estimate of vertical velocity to be made at shorter wavelengths. Despite power setting changes and their adverse affects on the results, it was shown that reasonable profiles of vertical motions can be obtained across the principal cloud band outside the eye wall of a hurricane.



## 6. ACKNOWLEDGEMENTS

We are especially grateful to Dr. Joanne Simpson of NOAA's Experimental Meteorology Laboratory whose support made possible the comparison flight with the gust probe package and the later reduction of the gust probe data. The cooperation and support of the Research Flight Facility was also a critical factor in carrying out the project. A number of helpful suggestions and comments have been contributed by O. Lappe of Turbulence Consultants, Inc. and by Harry Hawkins of this laboratory. Mr. Robert Carrodus did the drafting and Mrs. Mary Jane Clarke typed the manuscript.

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