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The Development of Asymmetries in a Three-Dimensional Numerical Model of the Tropical Cyclone

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December 1971

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THE DEVELOPMENT OF ASYMMETRIES IN A THREE-DIMENSIONAL NUMERICAL MODEL OF THE TROPICAL CYCLONE

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Richard A. Anthes

Notable asymmetric features of an early experiment with a three-dimensional hurricane model (Anthes et al., 1971a) were spiral bands of convection and large scale asymmetries (eddies) in the outflow layer. Using an improved version of the model, the formation and maintenance of these features are described in greater detail in this paper. The spiral bands in the model are found to propagate cyclonically outward in agreement with bands in nature. The breakdown of symmetry into a chaotic pattern of eddies in the outflow region is shown to be the result of dynamic (inertial) instability, with the eddy kinetic energy derived from the kinetic energy of the azimuthal mean flow. This instability does not contribute to the overall intensification of the model storm, however.

A curious anticyclonic looping of the vortex center is observed in these experiments. This looping appears to be associated with asymmetries in the divergence pattern associated with the eddies in the outflow layer.

This paper also summarizes improvements made in the original version of the model. In contrast to the earlier model, the current version contains an explicit water vapor cycle. A staggered horizontal grid is utilized to provide for a higher resolution in evaluating the pressure gradient forces. Some of the pragmatic assumptions made in the earlier model, notably those involving horizontal diffusion of heat and momentum, have been eliminated in the current version.

1. INTRODUCTION

A preliminary version of an isolated, asymmetric hurricane model (Anthes et al., 1971a, hereafter referred to as paper A) reproduced at least two prominent asymmetric features associated with natural storms, i.e., spiral bands of upward motion and fairly large scale asymmetries in the outflow layer. However, detailed analysis and interpretation of these features were deferred until additional experiments with an improved version of the model could be carried out. Especially conspicuous

A curious feature of the later experiments, which was not present in the preliminary experiment (paper A), is an anticyclonic looping of the vortex center about the center of the grid. Figure 8 shows the path for Experiment 6, which approximates a circle of radius 75 km. The commencement of the looping is coincident with the formation of outflow asymmetries, suggesting that the eddies, which drift with the anticyclonic flow of the upper levels are controlling the looping of the vortex center. It is possible that the lateral boundary conditions are responsible for the tight, circular looping, and with a much larger domain the vortex center might meander in a much less organized pattern.

The time variation of the mass-integrated total kinetic energy budget for Experiment 6 is shown in figure 9. The kinetic energy equation for this model is presented in paper A. The difference between the observed kinetic energy rate of change and the tendency computed from the kinetic energy equation is small (see fig. 9, top). This close agreement indicates that truncation errors in the model, including those associated with the time integration, are small. The important components of the energy budget, are also shown in figure 9. The dissipation due to horizontal eddies is about half that due to vertical eddies, which includes dissipation at the surface. The values of the components are reasonable compared to observations and to symmetric model results (see Anthes, 1971 for a summary of empirical results). The following sections discuss the development of the asymmetric stage and investigate the energetics of the asymmetries.

among the deficiencies in the preliminary model were the low vertical (3 levels) and coarse horizontal (30 km) resolution, the absence of a water vapor cycle, and a very pragmatic treatment of the lateral mixing process for heat and momentum. At present, however, the model has been substantially improved with the exception of increasing the vertical resolution. In particular, improved horizontal resolution is attained through a staggering of the horizontal grid, an explicit water vapor cycle is added, and a formulation of the horizontal diffusion processes similar to that used by Smagorinsky et al. (1965) is adopted. In experiments with the improved version of the model, several relationships have emerged involving the development and interaction of the asymmetries. This paper examines, in detail, the development of the asymmetric features using the current version of the model. The asymmetries in the outflow arise from the dynamic instability of the mean flow, and the predominance of wave numbers one and two is unrelated to the form of the initial asymmetries. The changes in grid structure, treatment of water vapor, and horizontal mixing are also discussed.

A curious asymmetric feature not found in the preliminary experiment is an anticyclonic looping of the vortex center during the mature stage of the model storm. This looping seems to be associated with asymmetries in the upper level divergence pattern.

Well formed spiral bands are present in these experiments. These bands rotate cyclonically and propagate outward with a phase speed of about 24 knots.

Symmetric hurricane models (Ooyama, 1968) have shown a strong relationship between model storm intensity and the sea surface temperature. A symmetric model cannot, however, investigate the effect of sea surface temperature variations on the movement of the storm. A simple experiment is made to determine the effect of variations in sea temperature on the looping motion of the storm. As the looping storm passes over warmer water, an intensification occurs but there is no discernable effect on the motion of the storm.

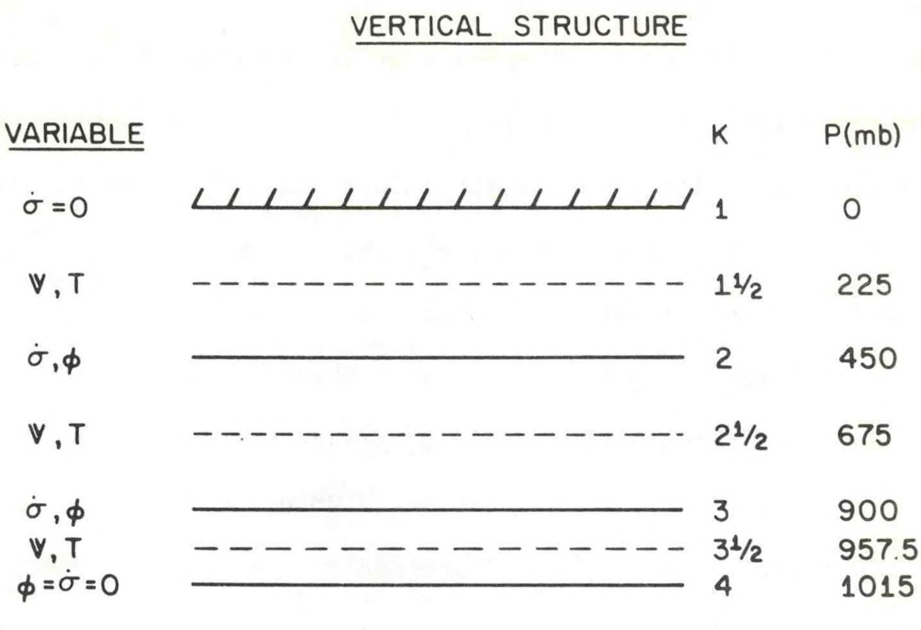
2. REVIEW OF MODEL

2.1 Basic Equations

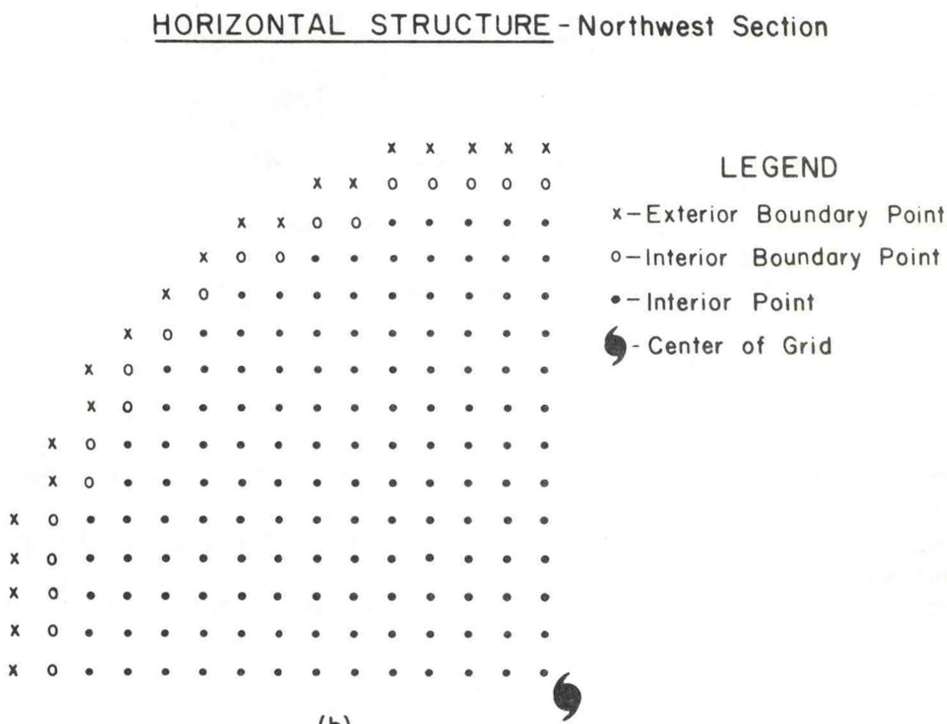
The equations of motion are written in σ-coordinates (Phillips, 1957) on an f-plane, where f, the Coriolis parameter, is appropriate to approximately 20°N (5×10^{-5} sec^{-1}). The equations of motion, the continuity equation, the thermodynamic equation, and the hydrostatic equation are identical to those employed by Smagorinsky et al. (1965) for general circulation studies. The basic equations are given in paper A and are not repeated here.

2.2 Structure of the Model

The vertical structure of the model is shown by figure 1a. The atmosphere is divided into upper and lower layers of equal pressure depth and a thinner Ekman boundary layer. The information levels for the dynamic and thermodynamic variables are staggered according to the scheme used by Kurihara and Holloway (1967).



(a)



(b)

Figure 1(a). Vertical information levels.

(b). Northwest quadrant of horizontal grid (non-staggered).

In an effort to economically achieve an increase in the horizontal resolution, three horizontal grids are tested in the experiments discussed in this paper. The first is the grid utilized in paper A, in which all the variables are defined at all the grid points. Two staggered horizontal grids are tested and shown to provide for a better resolution of the pressure gradient forces. These grids, and their associated finite difference equations, are discussed in section 3.2.

2.3 Vertical Diffusion of Momentum

As in the preliminary version of the model, the vertical diffusive and "frictional" effects are due to the vertical transports of horizontal momentum by subgrid eddies smaller than the cumulus scale. The most important aspect of these effects is the surface drag which produces frictional convergence in the cyclone boundary layer and, therefore, a water vapor supply which controls the parameterized cumulus convection (Charney and Eliassen, 1964; Ooyama, 1969; Rosenthal, 1970b).

In vector notation, these terms are written for the σ-system as

$$\vec{F}_v = -g \frac{\partial \tau^z}{\partial \sigma} \tag{1}$$

where g is the acceleration of gravity and τ^z is the vector Reynolds stress. The quadratic stress law, with the surface wind speed approximated by the speed at level 3, is employed for the stress at $\sigma = 1$,

$$\tau^z_{k=4} = \rho^* C_D |\vec{V}_3| \vec{V}_3 \ , \tag{2}$$

where $\vec{V}$ is the horizontal wind vector. A value of 3×10^{-3} is adopted for the drag coefficient, C_D, and a standard value of 1.10×10^{-3} ton-m^{-3}

is used for the surface density, ρ^*, in (2). As an upper boundary condition,

$$\tau^z_{k=1} = 0 \ . \tag{3}$$

For the remaining σ-levels,

$$\tau^z_{k=2,3} = \rho(z)\mu(z)\frac{\partial\vec{V}}{\partial z} \ . \tag{4}$$

Here, $\rho(z)$ is density, z is height, and $\mu(z)$ is the kinematic coefficient of eddy viscosity.

Following Smagorinsky et al. (1965), $\mu(z) = \ell^2\left|\frac{\partial\vec{V}}{\partial z}\right|$ where ℓ is the mixing length. In this model, ℓ need only be assigned at levels 2 and 3. In the preliminary report, ℓ varied linearly from a maximum value of 35.5 at level 3 to 0 at level 1, yielding a value of 17.8 at level 2. In later experiments, the values of $\mu(z)$ at levels 2 and 3 were increased by an order of magnitude to reduce what subjectively appeared to be excessive vertical shear. The form currently in use for $\mu(z)$ is

$$\mu(z) = 25 + \alpha^2\left|\frac{\partial\vec{V}}{\partial z}\right| \quad m^2\ sec^{-1}, \tag{5}$$

with $\alpha^2 = 4 \times 10^4\ m^2$.

The value of $\mu(z)$ computed from (5) is about twenty times the value of $\mu(z)$ computed in the original experiment at level 3 and about forty times the original value at level 2 during the mature stage of development.

It is recognized that the form of $\mu(z)$ given by (5) is, at best, a temporary representation of the total effect of vertical mixing of momentum in the hurricane, since cumulus clouds having the same vertical scale as the hurricane itself play an important role in the vertical transfer of momentum. However, this formulation yields an order of magnitude of $\mu(z)$ (50-100 $m^2\ sec^{-1}$) found to give vertical shears representative of real hurricanes in symmetric models (Rosenthal, 1970a; Anthes, 1971). It is noteworthy that experiments with the symmetric analog (Anthes et al., 1971b) show that the exact form of $\mu(z)$ above the boundary layer is relatively unimportant. Finally, the finite difference expression for the vertical "frictional" force is

$$\vec{F}_v = -g \frac{\delta \tau^z}{\delta \sigma}, \qquad (6)$$

where the vertical differencing operator, δ, is defined in appendix A.

2.4 Horizontal Diffusion of Heat and Momentum

After Smagorinsky et al. (1965), the lateral exchange of horizontal momentum, F_H, by subgrid scale eddies is

$$F_H(\vec{V}) = \frac{\partial}{\partial x} K_H \frac{\partial p^* \vec{V}}{\partial x} + \frac{\partial}{\partial y} K_H \frac{\partial p^* \vec{V}}{\partial y}. \qquad (7)$$

The formulation of K_H in paper A, based on early tests and on results from symmetric model experiments (Rosenthal, 1970a), was

$$K_H = C_1 |\vec{V}| + C_2 \qquad (8)$$

where $C_1 = 10^3$ m and $C_2 = 5 \times 10^3\ m^2 sec^{-1}$. The current version of the model, however, utilizes a non-linear form similar to that used by Smagorinsky et al. (1965),

$$K_H = 5 \times 10^3 + \frac{k_o^2}{2} (\Delta S)^2 \ |D| \qquad (9)$$

where ΔS is the grid spacing (30 km),

$$D = \left\{ \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 \right\}^{\frac{1}{2}} \qquad (10)$$

and k_o, the von Karman constant, equals 0.4.

In (9), the constant part is important only near the outer boundary where the kinetic energy and horizontal shear are small.

For simplicity, the diffusion of heat in the original version of the model was modelled using a constant thermal diffusivity,

$$F_H(T) = p^* \left(\frac{\partial K_T \frac{\partial T}{\partial x}}{\partial x} + \frac{\partial K_T \frac{\partial T}{\partial y}}{\partial y} \right) \tag{11}$$

with $K_T = 5 \times 10^4$ $m^2 sec^{-1}$.

In the current version of the model, however, the diffusivities for heat, momentum, and water vapor are equal to K_H computed from (9).

2.5 Time Integration

As in paper A, the Matsuno (1966) simulated forward-backward scheme is utilized for the time integration. This scheme damps the very high frequency gravity waves, without significantly damping the low and medium frequencies.

2.6 Lateral Boundary Conditions

The small domain size and the irregular boundary make the choice of lateral boundary conditions very important. In paper A, the components of momentum were extrapolated outward from interior grid points regardless of the direction of the flow. A subsequent experiment, however, produced a more intense storm than the preliminary model storm, and the extrapolation outward of the momentum in areas of inflow led to an instability in which a rather intense jet formed near the boundary. The source of energy for this jet was apparently the unlimited supply of kinetic energy from the environment. In subsequent experiments, therefore, the momentum components on the boundary are extrapolated only where the normal flow is outward. For inflow, the momentum on the boundary is set to zero.

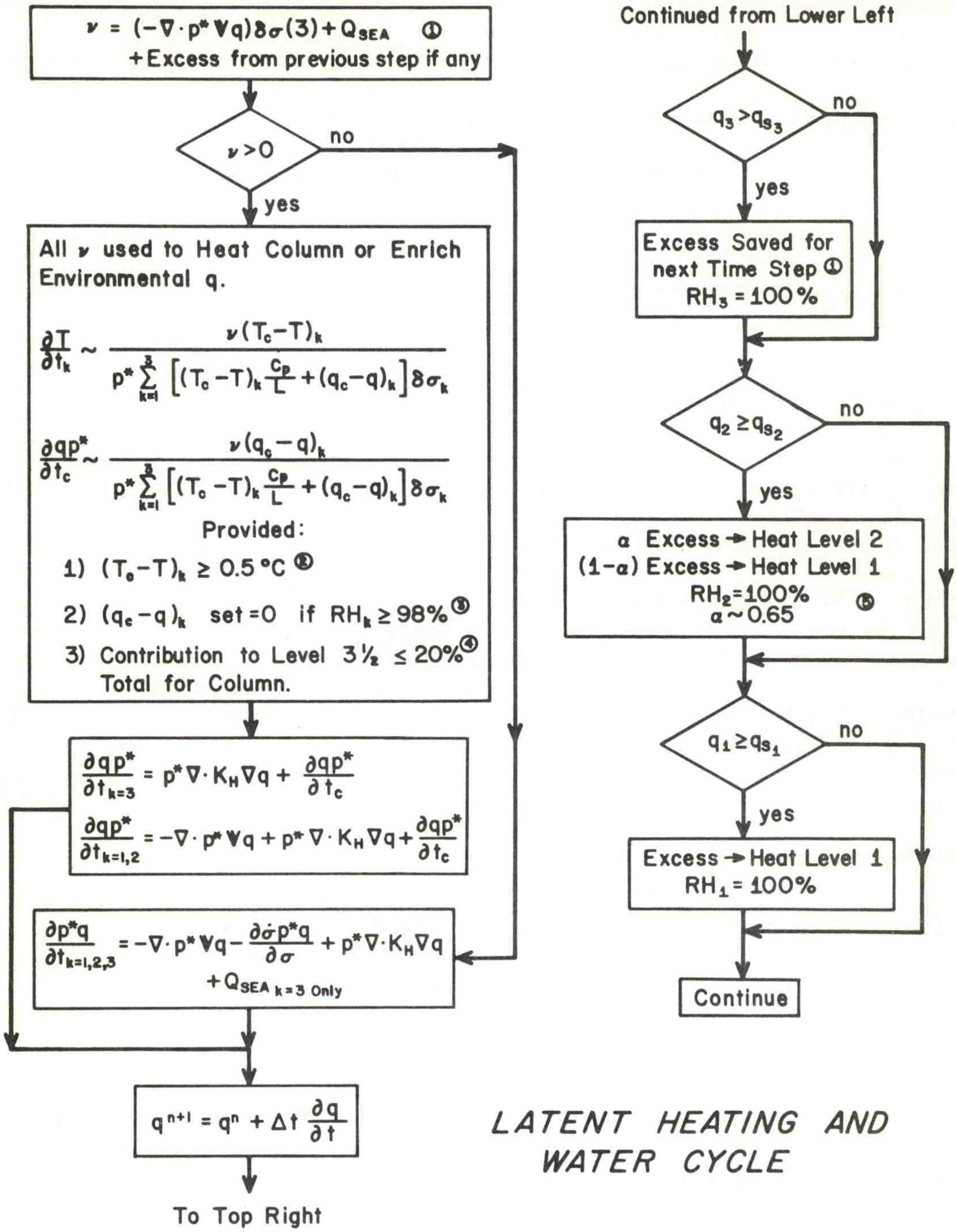
As in paper A, the boundary values for pressure and temperature are in uniform steady state. In experiments which include an explicit water vapor cycle, the relative humidity on the boundary is fixed at 90%.

2.7 Parameterization of Cumulus Convection

A major improvement in the model experiments discussed in this paper is the addition of an explicit water vapor cycle. This change eliminates the assumptions concerning boundary layer water vapor content that were necessary in paper A, and allows for simulation of non-convective release of latent heat. This section describes the parameterization of the feed-back between convection and the large-scale temperature and moisture fields. The following section describes the water vapor cycle and the non-convective release of latent heat. The flow chart for both schemes is summarized in figure 2.

The parameterization of the cumulus convection closely follows the scheme used successfully by Rosenthal (1970b), although some slight modifications are necessary because of the reduced vertical resolution. The basic characteristics of the scheme have been thoroughly discussed elsewhere (Rosenthal, 1969, 1970a) and will not be elaborated upon here. The two most important aspects of the scheme may be summarized, however:

1) In the convective parameterization, the vertical integral of latent energy is conserved.



LATENT HEATING AND WATER CYCLE

Figure 2. Flow chart illustrating water vapor cycle and parametrization of convective and non-convective latent heat release. See sections 2.7 and 2.8 for details.

2) The heat and moisture made available to the environment are distributed vertically in such a way that the environment is driven toward an ultimate state in which the humidity and temperature are those defined by the equivalent potential temperature of the surface air.

The convective adjustment for the σ-system is

$$I = \frac{\nu}{\sum_{k=1}^{3} (T_c - T)_k \frac{C_p}{L} + (q_c - q)\,\delta\sigma_k} \tag{12}$$

where T_c and q_c are the temperature and specific humidity, respectively, of the pseudo-adiabat with the equivalent potential temperature of the surface, q is the environmental specific humidity, L is the latent heat of vaporization, and C_p is the specific heat for dry air at constant pressure. The boundary layer convergence of water vapor and the latent heat addition from the sea (Q_{sea}), is

$$\nu = -\nabla \cdot p^* \vec{V} q_3 \delta\sigma_3 + Q_{sea} \ . \tag{13}$$

Then, if $\dot{Q}_c$ is that part of the diabatic heating due to convection,

$$p^*\dot{Q}_c \begin{cases} = C_p I (T_c - T) & \text{if } \begin{array}{l}(T_c - T) \geq 0.5°C \\ \text{and } I > 0\end{array} \\ = 0 & \text{otherwise.} \end{cases} \tag{14}$$

Letting $(\frac{\partial p^* q}{\partial t})_c$ represent the addition of moisture to the environment by cumulus convection,

$$\left(\frac{\partial p^* q}{\partial t}\right)_c \begin{cases} = I(q_c - q) & \text{if } \begin{array}{l} I(q_c - q) > 0 \\ \text{and RH} \leq 98\% \end{array} \\ = 0 & \text{otherwise.} \end{cases} \tag{15}$$

Equations (12) through (15) are utilized only when the atmosphere is conditionally unstable, i.e., $(T_c - T)$ and $(q_c - q)$ are positive for some value of k. In practice, convection occurs only if $(T_c - T)$ equals or exceeds 0.5°C at either level $1\frac{1}{2}$ or $2\frac{1}{2}$, in order to avoid numerical difficulties with small values of the denominator of (12). As an additional constraint on the vertical partitioning of latent heat, under nearly moist neutral conditions ($T_c \approx T$ and $q_c \approx q$), no more than 20% of the total latent heat in the column is assigned to the boundary layer (level $3\frac{1}{2}$), and the remaining 80% is distributed equally between the middle and upper tropospheric layers.

An additional modification to the above scheme occurs under a nearly saturated environment, defined by a relative humidity (RH) equal to or greater than 98%. Under these conditions, $\left(\frac{\partial p^* q}{\partial t}\right)_c$ (which may be thought of as resulting from evaporating clouds) is set equal to zero, and all of the condensation heating is made available to the large scale flow.

The surface temperature, T*, needed to establish the surface equivalent potential temperature, is computed by a downward extrapolation from level $3\frac{1}{2}$,

$$T^* = T_{3\frac{1}{2}} + 3.636°K \qquad (16)$$

and the surface specific humidity is obtained by the assumption that the relative humidity at the surface equals the humidity at $k = 3\frac{1}{2}$.

2.8 Water Vapor Budget and Non-convective Latent Heat Release

The treatment of the water vapor cycle closely follows the scheme developed by Rosenthal (1970b) for the symmetric, 7-level model, again with some modifications due to the limited vertical resolution. The scheme is outlined in figure 2.

Under unsaturated conditions in the presence of convection, all of the boundary layer water convergence and the evaporation from the sea is utilized in convection, and the forecast equation for specific humidity at level $3\frac{1}{2}$ is simply,

$$\left(\frac{\partial p^*q}{\partial t}\right) \approx p^*\nabla\cdot K_H\nabla q + \left(\frac{\partial p^*q}{\partial t}\right)_c \quad . \tag{17}$$

In the middle and upper layers, the forecast equation is

$$\frac{\partial p^*q}{\partial t} = -\nabla\cdot p^*\vec{V}q - p^*\frac{\partial\dot{\sigma}q}{\partial\sigma} + p^*\nabla\cdot K_H\nabla q + \left(\frac{\partial p^*q}{\partial t}\right)_c \quad . \tag{18}$$

In columns which contain no convection, the forecast equation for level $3\frac{1}{2}$ is identical to (18) with the additional term, Q_{sea}, representing evaporation from the sea surface (see section 2.9). The interpolation of q to the $\dot{\sigma}$ levels, necessary for computing the vertical flux in (18), is obtained by assuming an exponential variation of q between the σ-levels,

$$q = q_o e^{\gamma(\sigma-\sigma_o)} ; \tag{19}$$

where the reference level is designated by the subscript o. Evaluating the constant, γ, we obtain for $\overline{q}_k$,

$$\overline{q}_k = q_{k+\frac{1}{2}}\left(\frac{q_{k-\frac{1}{2}}}{q_{k+\frac{1}{2}}}\right)^{\left(\frac{\sigma_k-\sigma_{k+\frac{1}{2}}}{\sigma_{k-\frac{1}{2}}-\sigma_{k+\frac{1}{2}}}\right)} , \tag{20}$$

and,

$$\frac{\partial\dot{\sigma}q}{\partial\sigma} \sim \frac{\delta\dot{\sigma}q}{\delta\sigma} \quad . \tag{21}$$

The release of non-convective latent heat is modeled in the following manner. After every forecast step, each grid point for specific humidity is checked for supersaturation. If supersaturation occurs in the boundary layer, and conditional instability is still present, the excess water vapor over saturation is used to fuel additional convection according to the convective scheme given by (14) and (15). In the event (rare) that supersaturation occurs in the boundary layer and the atmosphere is conditionally stable, the excess moisture is condensed in situ.

For supersaturation at levels $1\frac{1}{2}$ and $2\frac{1}{2}$, the excess vapor is assumed to condense as large scale precipitation rather than convection, since the atmosphere is conditionally stable above these levels. At level $1\frac{1}{2}$, all the excess is condensed and the latent heat is made available to the circulation at this level. For supersaturation at level $2\frac{1}{2}$, however, only part (65%) of the excess water vapor is condensed at this level, the remainder is assumed to condense at level $1\frac{1}{2}$. This partitioning follows from the assumption that the mechanism for the latent heat release is large scale ascent of saturated air. For typical hurricane soundings, a saturated parcel starting at level $2\frac{1}{2}$ (about 675 mb) and rising to the tropopause, condenses about 65% of its total water vapor below 450 mb (level 2) and the rest above this level.

2.9 Air-sea Exchange of Sensible and Latent Heat

The sensible and latent heat fluxes at the air-sea interface obey the bulk aerodynamic relationships, and decrease linearly with σ until they reach zero at the k = 3 level. This gives

$$p^*\dot{Q}_s = \begin{cases} \dfrac{gC_pC_E|\vec{V}|\rho^*(T_{sea}-T^*)}{\sigma_4-\sigma_3}, & T_{sea} > T^* \\ 0 & T_{sea} \leq T^* \end{cases} \tag{22}$$

and

$$Q_{sea} = \begin{cases} \dfrac{gC_E|\vec{V}|\rho^*(q_{sea}-q^*)}{\sigma_4-\sigma_3} & q_{sea} > q^* \\ 0 & q_{sea} \le q^*, \end{cases} \tag{23}$$

where $\dot{Q}_s$ and Q_{sea} are the sensible and latent heat added per unit mass and time at level $3\frac{1}{2}$. The exchange coefficient C_E is taken equal to C_D (.003) and the value of T_{sea} is 302°K for most of the experiments.

2.10 Initial Conditions

The initial conditions consist of an axisymmetric vortex in gradient balance, with a maximum wind speed of 18 m sec^{-1} at a radius of 240 km. The details are presented in paper A. For initially symmetric conditions, the solutions to the differential equations remain symmetric for all time. However, asymmetries in the truncation and roundoff errors as well as in the lateral boundaries produce weak asymmetries (on the order of 10^{-10}%) in the finite difference equations after the first time step. These perturbations may then grow with time and become a significant part of the total circulation. In a later section, the mechanism for this observed growth is investigated. It is also shown that the initial form of the perturbation is unimportant in determining the final form of the asymmetries.

3. EXPERIMENTAL RESULTS

3.1 Addition of Explicit Water Vapor Cycle

The next two sections briefly describe the effects of the major modifications to the original version of the model, specifically, the addition of the water vapor cycle and the horizontal staggering of the grid. The details of the model storm structures associated with these intermediate stages of the model are not presented. Sections 3.3-3.7 present detailed analyses of the model storm's life history as computed from the current version of the model. These sections are primarily concerned with the development and structure of the asymmetric features including rainbands and outflow eddies.

Table 1 lists some of the properties of each experiment discussed in this paper. These properties, appropriate to the mature stage of each model storm, serve as an overall comparison of the experiments, and are discussed individually in later sections.

Figure 3 shows the time variation of the minimum surface pressure and the maximum surface wind speed for each experiment. The effect of adding the explicit water vapor cycle alone is illustrated by the curves labelled "non-staggered grid", and, as suggested by the close similarity to the corresponding profiles for Experiment I, the water vapor cycle has a fairly small effect on the overall behavior of the model storm. The initial development is somewhat slower, since part of the water vapor convergence in the boundary layer is utilized to enrich the environmental water vapor content in the middle and upper troposphere (the initial relative humidity at all levels is 90%). In Experiment I, which does not contain a water vapor cycle, <u>all</u> the water vapor convergence is condensed and made available as latent heat to the large scale flow.

Although the initial development in the experiment with the water cycle is slower, the ultimate state is equal to, or slightly greater than in Experiment I. This difference is related to somewhat higher boundary

Table 1. Summary of Experiments

EXPERIMENT	GRID	MIN P (mb)	MAX S m sec^{-1}	10^{14} Watts				10^{12} Watts		
				Total heating rate	Convective heating rate	Non-Conv heating rate	Sensible heating rate	C(K)	D_H	D_V
1. (Anthes et al., 1971a) (228 h)	Non-stag	965	66	13.9	13.9	----	0.46	45	-11	-32
2. (H_2O cycle added) (282 h)	Non-stag	962	66	11.8	11.4	0.4	0.24	43	-16	-28
3. (Initial asymmetries 10^{-1}%) (174 h)	S-1	976	60	10.0	8.6	1.4	0.19	31	-10	-19
4. (Initial asymmetries 10^{-1}%) (120 h)	S-1	976	60	11.2	9.9	1.3	0.21	36	-11	-23
5. (Aborted due to lateral boundary problems) (87 h)	S-2	980	57	7.2	6.8	0.4	0.11	17	- 7	- 9
6. Non-linear-type Hor. Mixing (156 h)	S-1	971	63	12.6	10.6	2.0	0.22	36	-11	-24
7. Symmetric analog (96 h)	1 Horizontal Dimension Staggered Grid	969	53	15.1	12.4	2.7	0.30	39	-11	-28

Time in ( ) is time at which components of energy budget are given.

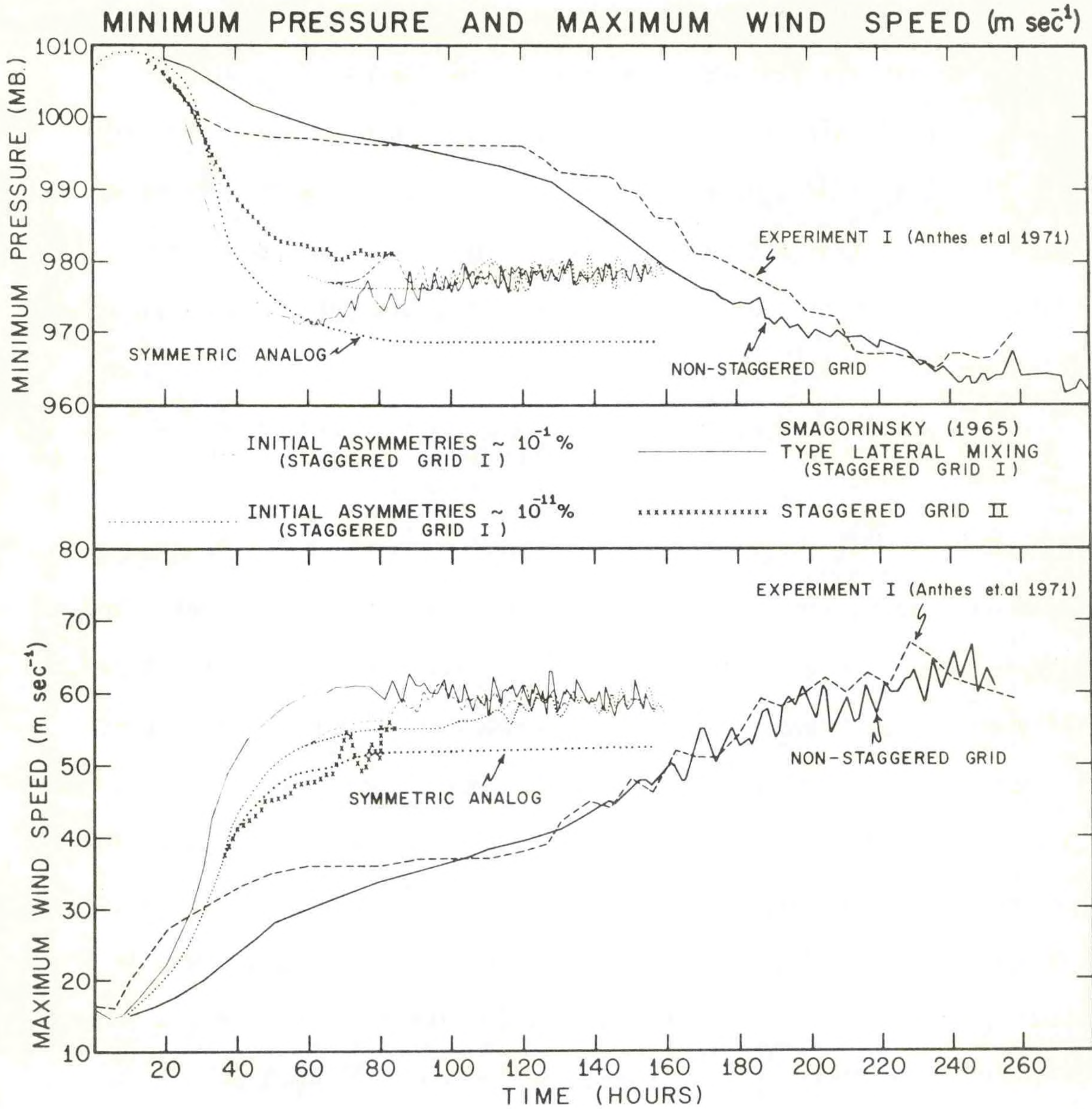


Figure 3. Time variation of (a) minimum pressure and (b) maximum surface wind speed for experiments discussed in this paper.

layer specific humidities in the later model. The structures of the two model experiments are quite similar and are not compared in detail. Therefore, the primary advantages of adding the water vapor cycle are: (1) the elimination of some arbitrary assumptions made in the first experiment, (2) the provision for a wider range of initial conditions (variations in initial humidity distributions), and (3) the provision for studying the hurricane water vapor budget.

3.2 Increased Horizontal Resolution Utilizing Staggered Horizontal Grids

The behaviors of the two experiments utilizing the non-staggered grid are more similar to each other than to any of the other experiments shown in figure 3. More significantly, during the early stages of the storms, in which asymmetries are negligible, the behavior of the non-staggered grid storms varies markedly from the behavior of the symmetric analog (Anthes et al., 1971b). The differences during the early stages of the storm are related to greater truncation errors in evaluating the pressure gradient force in the asymmetric model. The symmetric analog, which utilized a staggered grid in the radial direction to avoid computational difficulties at the origin, evaluated the pressure gradient over 30 km rather than the 60 km interval used by the asymmetric model. The comparison suggested a staggering of the pressure and velocity variables in the asymmetric model as well.

The staggering of variables in two horizontal dimensions is somewhat more complicated than in one dimension. Two types of staggered grids, shown schematically in figure 4, were tested. The first

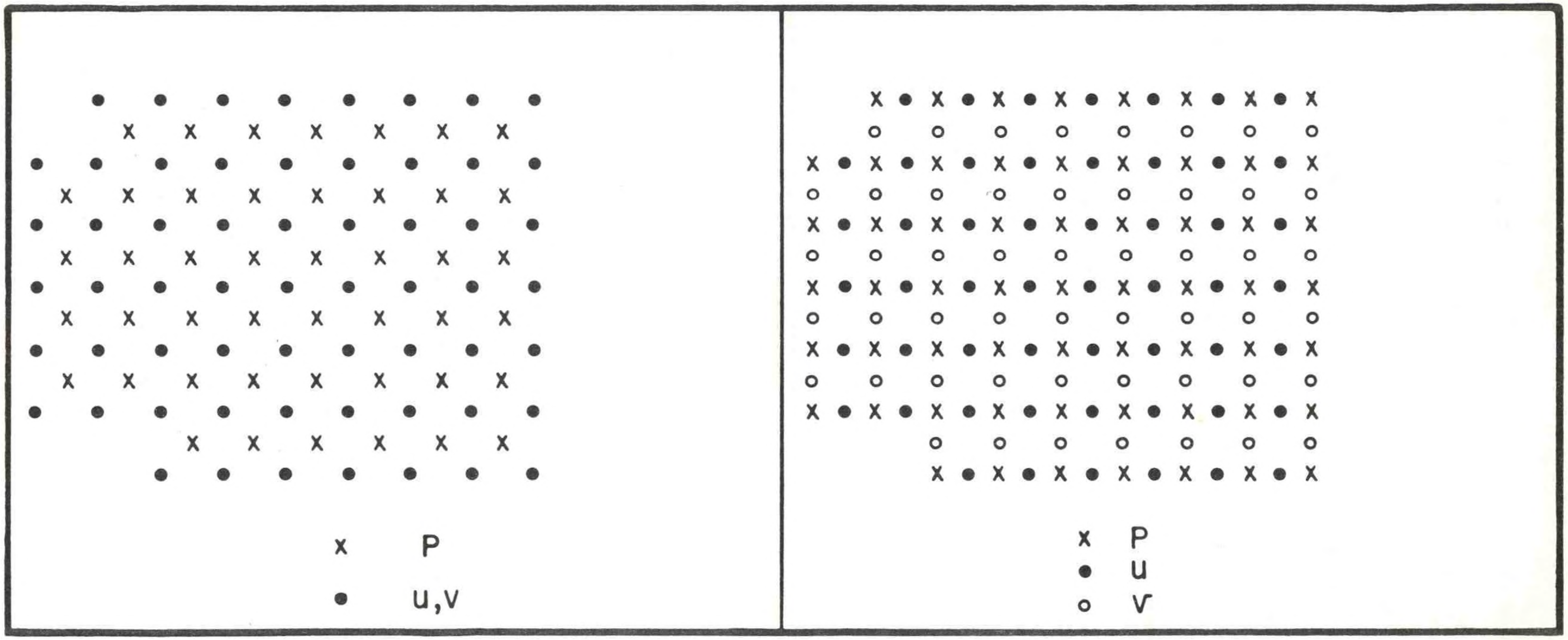


Figure 4. Schematic diagram for two staggered horizontal grids tested with hurricane model. S1 grid is shown on left, S2 grid is shown on right.

(designated by S1), consists of two sets of prediction points, one for the horizontal velocity components and one (offset by 45°) for all the other variables. The second (designated by S2) consists of three sets of prediction points, one for each horizontal velocity component and a third for the remaining variables. The S2 grid has been used by Lilly (1965) and tested by Grammeltvedt (1969). In both staggered grids, the evaluation of the pressure gradient and the horizontal divergence over smaller grid increments necessitates a reduction in the time step to maintain computational stability. Thus the time step of 60 s, which was adequate for the non-staggered grid, must be reduced to 45 s for S1 and to 30 s for S2. The alternate finite difference equations associated with each staggered grid are given in appendix A.

As seen in figure 3, both staggered grids yield very similar results during the first 48 hours of model time. Furthermore, both experiments are more similar to the symmetric analog than are the two experiments with the unstaggered grid. The solutions associated with the S1- and S2-grids diverge considerably after 48 hours, however, with the S2-storm reaching an asymmetric stage much earlier than the S1-storm. Furthermore, the solution associated with the S2-grid deteriorates after 72 hours and finally becomes unstable. The primary cause of this instability seems to be associated with the lateral boundary conditions, since the u and v components are not defined at the same points. This instability and the requirement for the small time step were the prime reasons for the choice of the S1-grid for the current version of the model. The S1-grid, then, provides for an economical increase in horizontal resolution, and the behavior of the symmetric stage of the model storm utilizing this grid compares favorably with the behavior of the symmetric analog.

3.3 Structure of Asymmetric Hurricane

The structures of the storms generated in the later experiments are similar to the storm structure discussed in paper A. An overall view of the three-dimensional, time-dependent structure of a typical experiment (Experiment 2, Table 1) is shown in figure 5, which shows the tracks of particles released in the hurricane circulation over an eight day period. The computed velocities are interpolated in space and time for the computation of the trajectories. (See Anthes et al., 1971c for more details.) Figure 5 reveals a nearly steady state, axisymmetric boundary layer in which air accelerates as it flows inward to the center. Reaching the center, the particles are carried rapidly upward, reaching the outflow layer in about two hours. (Note, the large scale, mean vertical velocities are used to compute the vertical displacements; in reality, a particle would probably be carried upward in a cumulonimbus updraft in considerably less time.) After the particles reach the outflow layer, they decelerate and move outward in a highly asymmetric, unsteady flow.

Figure 5 also shows the path of one particle that is released in the middle troposphere (about 500 mb) rather than in the inflow layer. This particle experiences very little radial motion, and is carried slowly upward as it spirals around the storm center.

Figure 6 shows a typical streamline and isotach pattern in the upper level during the mature stage (156 hours) of Experiment 6. Noteworthy is

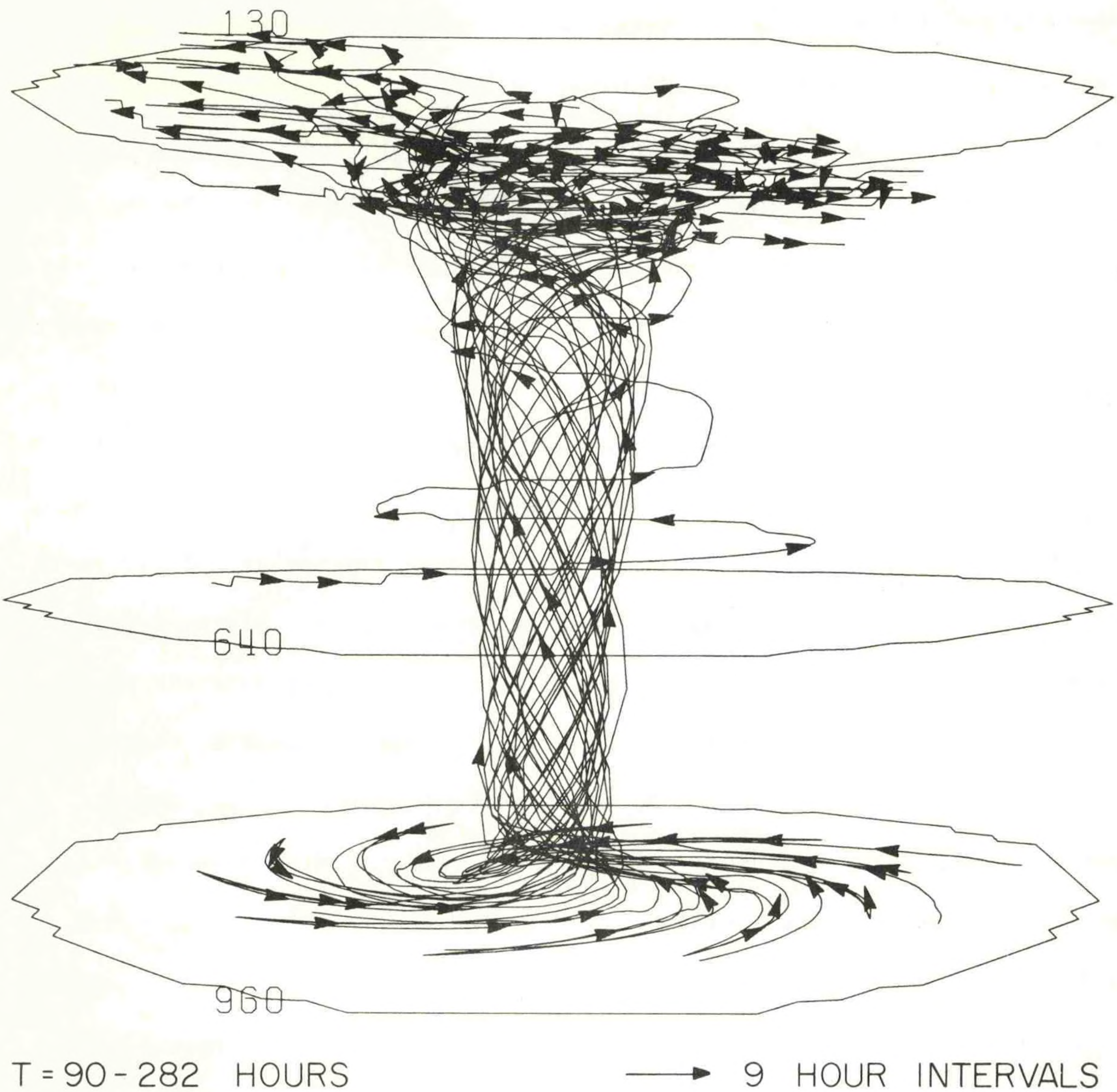


Figure 5. Particle trajectories calculated over a 9-day period in Experiment 2 (see Table 1). The three levels are labeled in mb (approximate). All particles start in boundary layer except one, which is started in the middle troposphere.

the anticyclonic eddy located to the "north" of the storm center. The outflow occurs mainly in two jets, in agreement with storms in nature (Black and Anthes, 1971).

Figure 7 shows the vertically integrated convective heat release, expressed as cm of rain per day. The semi-circle of rainfall rates over 200 cm/day corresponds well to the non-uniform eyewall convective region in real storms (see, for example, Hawkins and Rubsam, 1968). However, figure 7 shows that a rain-free "eye" is not present at this time, and only a region of relatively light rainfall occurs at the center of the storm. The absence of an eye is probably due to the coarse horizontal resolution. The notable spiral bands, with rainfall rates averaging about 2 cm/day, are approximately 90 km wide at large distances from the center, and somewhat wider closer to the center. These bands rotate cyclonically about the storm center and propagate outward at a speed of about 24 knots*. Although the outer rain bands in nature apparently propagate outward (Gentry, 1964; Senn and Stevens, 1964), we feel that the model rate of 24 knots is somewhat too high. Since the bands are undoubtedly internal gravity waves, improved vertical resolution may give a more realistic phase speed. The band thickness of 90 km is considered fairly acceptable compared to observations, when the coarse resolution of the model is considered. The rainfall rate of 2 cm/day is also considered acceptable, although possibly on the low side, for an average over 90 km.

*This speed is computed by measuring the normal displacement of the outer edge of the bands over a 6-hour period and hence includes the effect of rotation as well as outward propagation.

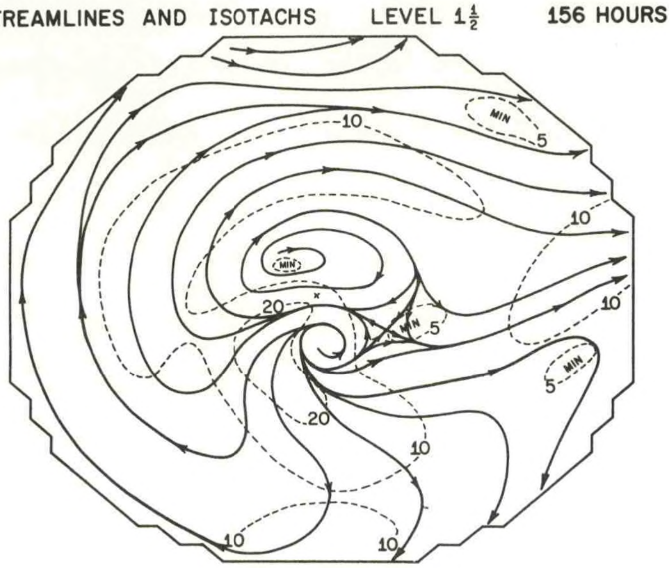


Figure 6. Streamline and isotach (m sec^{-1}) analysis for upper level in Experiment 6 (Table 1) during the mature asymmetric stage (156 hours).

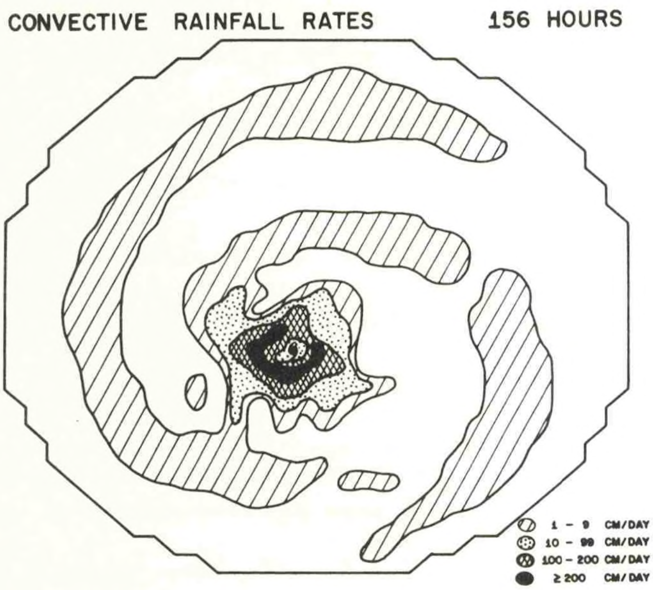


Figure 7. Vertically integrated convective heat release expressed as rainfall rates in cm/day at 156 hours of Experiment 6.

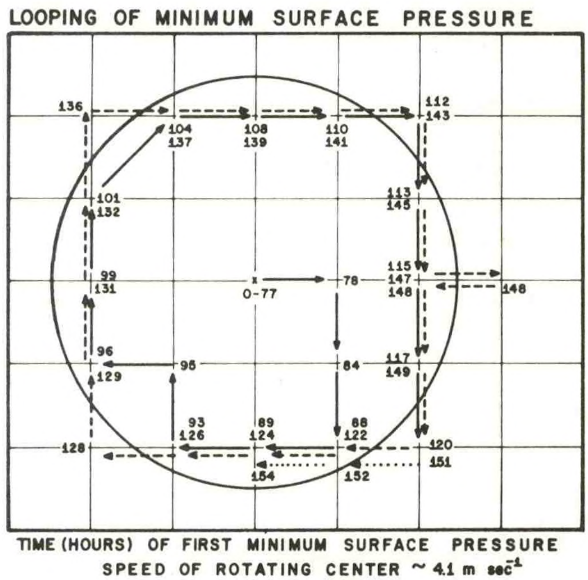


Figure 8. Positions of minimum pressure at selected times for Experiment 6 (Table 1), showing anticyclonic looping during asymmetric stage of storm.

The temperatures inside the bands are not appreciably different from the surrounding environment, in agreement with the mean thermal structure of outer bands in nature (Gentry, 1964). This uniformity in temperature may be related to the near compensation in the thermodynamic equation between the latent heat release and the adiabatic cooling in the region of upward motion, and the relatively short time (about 3 hours) required for the band to move past a particular point. It is noteworthy that, compared to earlier versions of the model, the current version yields spiral bands with the structure most like rain bands in nature.

Although the spiral bands in the model are undoubtedly internal gravity waves modified by latent heat release (Ogura and Charney, 1962), the mechanism for their generation is unknown. There does seem to be an interesting, although obscure, relationship between the bands, which are most pronounced at the top of the boundary layer, and the asymmetries in the outflow layer. In all experiments, the bands are conspicuously absent until the symmetric flow in the upper levels breaks down. Also, the number of bands (two) seems to be associated with the predominance of wave number two in the outflow layer.

The release of latent heat in the spiral bands is entirely convective. The non-convective latent heat release (not shown) occurs entirely within 180 km of the center in a roughly circular pattern. The total non-convective heat release of 2.0×10^{14} watts (Table 1), represents a significant contribution to the total latent heat release (12.6×10^{14} watts), in agreement with observations (Hawkins and Rubsam, 1968) and symmetric model results (Rosenthal, 1970b).

3.4 The Development of the Asymmetric Stage

This section discusses the development and maintenance of the asymmetric features of the circulation using the current (Experiment 6) version of the model. A measure of the asymmetry is the standard deviation (from the circular mean) of any variable. Figure 10 shows the time variation of the standard deviations (σ) of the tangential and radial wind components and of the temperature at 105 km in the upper level. The early part of the storm's history is quite symmetric, with maximum σ of the wind components about 0.3 m sec^{-1}, and for the temperature, about 0.05°C. The storm becomes quite asymmetric after 60 hours. Thereafter, the σ for the wind components are about equal in magnitude to the azimuthal means at this level. However, the temperature field remains relatively symmetric even during the later stages, with σ rarely exceeding 1°C. This relative symmetry may be, in part, due to the symmetric boundary conditions on temperature.

Detailed analysis of the development of the asymmetries (Trout, 1972) shows that during the symmetric stage, when the variance of any quantity is small, wave number four accounts for nearly all of the variance. Because of the orientation of wave number four with respect to the four irregular corners of the grid, this early symmetry is probably due solely to the artificial aspects of the irregular boundary. Subsequent to the rapid growth of the variance, however, wave numbers one and two become dominant and account for most of the variance, in agreement with observations (Black and Anthes, 1971). See Trout (1972) for further details.

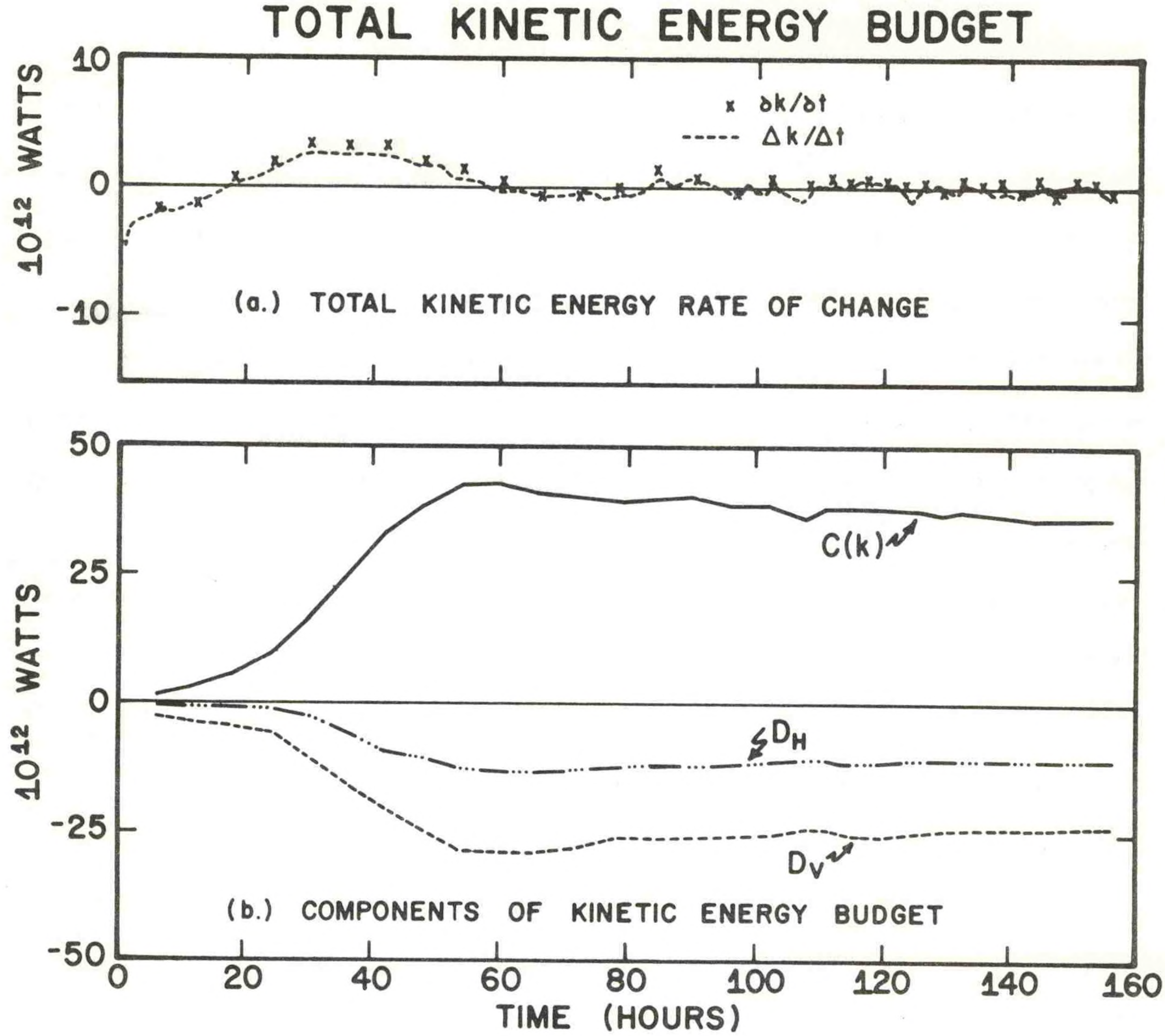


Figure 9(a). Time variation of the observed total kinetic energy change ($\Delta K/\Delta t$) and the change computed from the kinetic energy equation ($\partial K/\partial t$) for Experiment 6.

(b). Individual components of the kinetic energy tendency: C(K) is the conversion of potential to kinetic energy, D_H is the dissipation of kinetic energy through lateral eddy viscosity, D_V is the dissipation of kinetic energy through vertical eddy viscosity and includes the effect of surface drag friction. The flow of kinetic energy through the lateral boundary is negligible in this experiment.

The appearance of large scale asymmetries in the model storms which develop from initially axisymmetric conditions raises at least three questions: 1) What is the source of the initial perturbations? 2) Why do certain wavelengths become predominant? 3) What is the mechanism for growth of these disturbances? The next sections present results from two types of initial perturbations, and show that the initial form of the perturbations is unimportant in determining the dominant scale of the asymmetries. The growth of the initially small disturbances is shown to be a type of dynamic (or inertial) instability in which longer wavelengths are more unstable than the shorter wavelengths. The important mechanism for the growth of the eddies is the barotropic conversion of mean azimuthal kinetic energy to eddy kinetic energy.

3.5 The Initial Perturbations

The initial asymmetries in the preliminary experiment (I), and in the experiments discussed so far in this paper arise from non-symmetric truncation and roundoff errors in the finite difference schemes. These initial asymmetries (see fig. 11) after one time step are (in Experiment 1) on the order of $10^{-10}\%$ on the interior of the grid and $10^{-25}\%$ on the boundaries. Inspection of each term in the forecast equations showed these asymmetries to arise from the divergence terms in the continuity equation.

The magnitude and distribution of the initial perturbations, and consequently the time required for the disturbances to manifest themselves in the large scale flow, varies with the grid and finite difference scheme. The asymmetric stage begins at 120 hours for the non-staggered, grid, at 100 hours for the S1-grid and at 60 hours for the S2-grid.

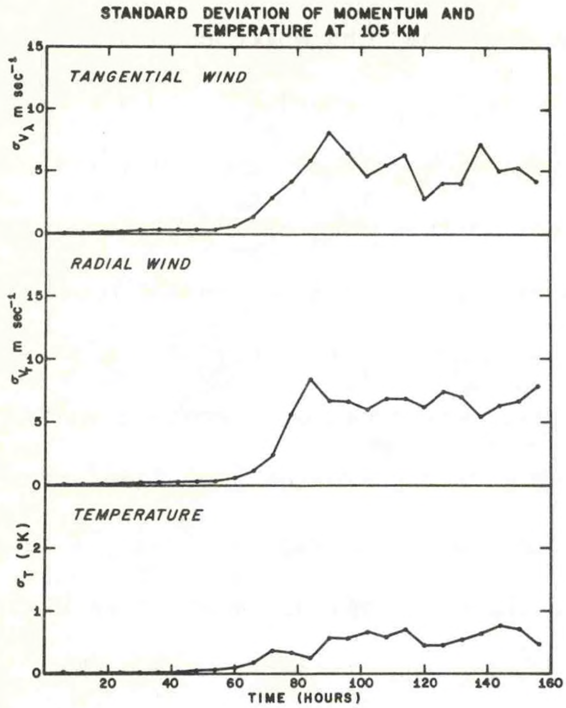


Figure 10. Time variation of standard deviations of tangential and radial wind components, and temperature for the upper level at a radius of 105 km in Experiment 6.

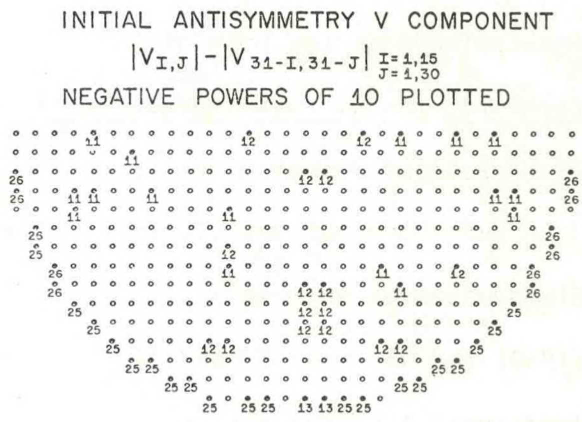


Figure 11. Asymmetries in the v-component after one time step in the preliminary experiment. These asymmetries are due to truncation and roundoff errors alone and are functions of the grid system and the finite difference scheme.

Although truncation errors produce perturbations which eventually grow to the interesting asymmetric features of the model, it is more appealing to deliberately introduce perturbations of known amplitude and variance. Therefore, initial asymmetries on the order of 10^{-1}% (ten orders of magnitude greater than the perturbations due to truncation errors) were introduced by adding random numbers to the initial u and v components. As shown in figure 3, the asymmetric stage is reached much earlier (60 hours rather than 100 hours) with the greater amplitude perturbations. (The onset of the asymmetric stage is marked in figure 3 by the rise in minimum pressure preceeding the oscillations in the pressure.) It is important to note, however, that during the asymmetric stage the structures of the model experiments are very similar (see also Table 1), indicating that <u>the form of the initial asymmetries is unimportant in determining the ultimate structure of the asymmetric circulation</u>.

3.6 Role of Dynamic Instability in the Development of the Asymmetries

The theory of dynamic instability has been investigated by many individuals over the years. The reader is referred to Kuo (1949), Godson (1950) and Van Mieghem (1951) for a review and discussion of the generalized concept of dynamic instability.

In the application of dynamic instability to the hurricane problem, meteorologists have generally referred to the growth of symmetric radial displacements in an axisymmetric vortex (Sawyer, 1947; Alaka, 1963; Yanai, 1964; Yanai and Tokioka, 1969). However, this instability, defined by the criterion

$$\left(\frac{\partial r\overline{v_\lambda}}{r\partial r} + f\right) \overline{Z} < 0, \quad Z = \left(\frac{2v_\lambda}{r} + f\right) , \tag{24}$$

where v_λ is the tangential wind, and the $\overline{(\ )}$ operator refers to an azimuthal average, does not appear to play an important role in the intensification of symmetric model storms (Yamasaki, 1968; Ooyama, 1969; Rosenthal, 1969a).

A second type of dynamic instability which seems relevant in the asymmetric hurricane model is the instability represented by the growth of azimuthal perturbations at the expense of the axisymmetric flow. For purely horizontal, non-divergent flow, a necessary and sufficient criterion is

$$\frac{\partial \left(\overline{\frac{\partial r v_\lambda}{r\partial r} + f}\right)}{\partial r} = 0 \tag{25}$$

somewhere in the domain (Kuo, 1949). However, the derivation of (25) depends on the horizontal perturbations vanishing at the outer boundary, which is not strictly the case in the model in which the winds on the boundaries vary through the extrapolation procedure discussed earlier (see Section (2.6)).

It may also be shown that, for non-divergent, horizontal flow (24) is a sufficient criterion for the growth of azimuthal perturbations, regardless of the form of the perturbations on the boundary.

In Experiment 6 (the current version of the model), we have considered the criterion defined by (24) and (25). Figure 12 shows the time variation of the minimum azimuthal averages of ζ_a, Z, and the minimum value of the product, $\overline{\zeta_a}\,\overline{Z}$ for the upper (level $1\frac{1}{2}$) and middle (level $2\frac{1}{2}$)

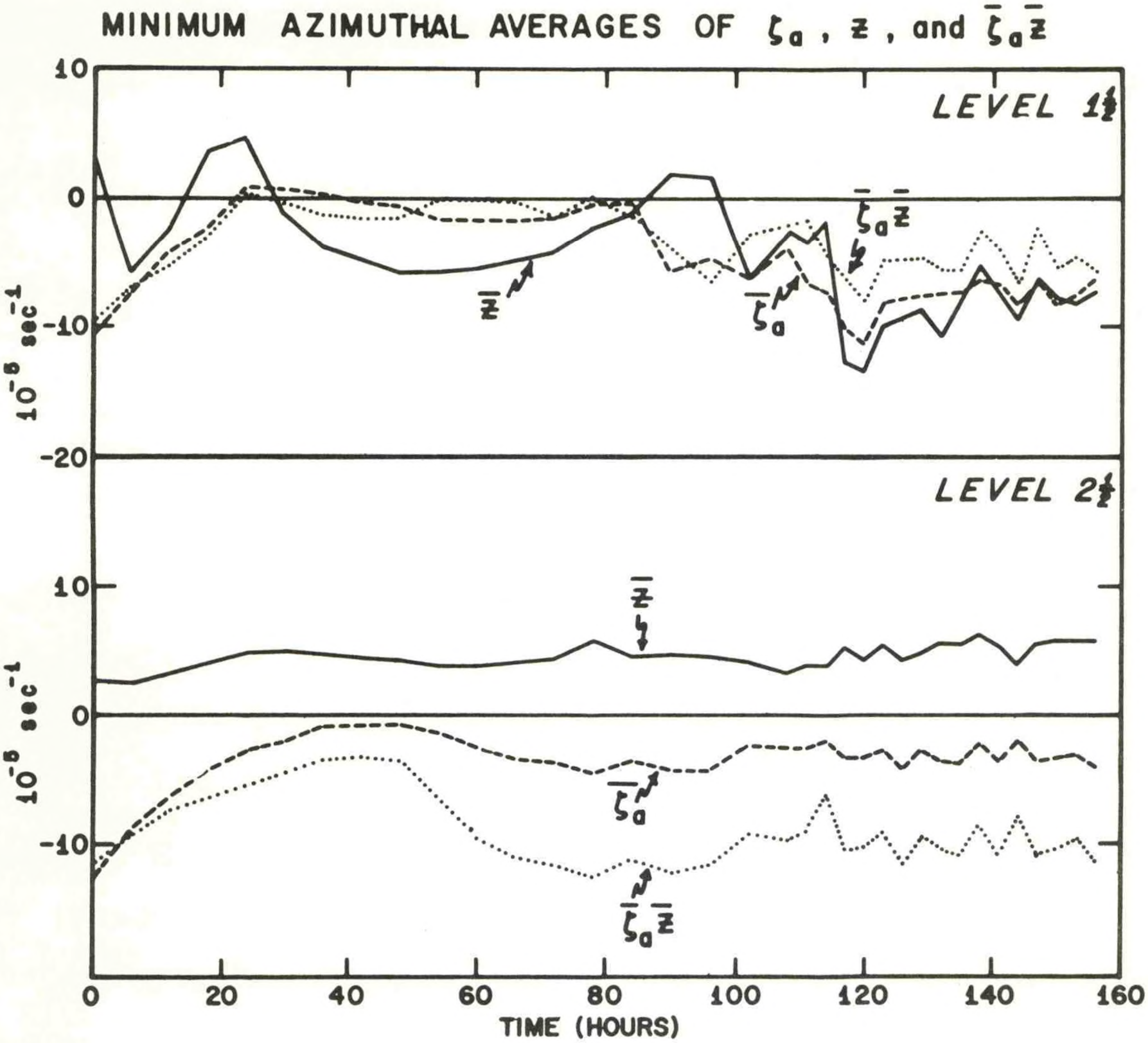


Figure 12. Time variation of minimum azimuthal averages of $\overline{\zeta_a}$, $\overline{Z}$, and $\overline{\zeta_a}\overline{Z}$ for levels $1\frac{1}{2}$ and $2\frac{1}{2}$ in Experiment 6.

tropospheric layers. Negative absolute vorticity is present in the initial conditions at both levels beyond 300 km radius. During the first 40 hours this outer area of negative ζ_a decreases in intensity in both layers. After about 48 hours, a new region of negative ζ_a appears close to the storm center, at a radius of about 150 km. This inner region persists for the remainder of the forecast.

Figure 12 also shows that negative azimuthal averages of Z exist in level $1\frac{1}{2}$, but never at level $2\frac{1}{2}$ where the flow is always cyclonic. There are regions in both levels where the product is negative, thus criterion (24) is satisfied in both levels over most of the forecast period.

Although the asymmetries at level $1\frac{1}{2}$ have been emphasized, there is significant eddy kinetic energy present in the middle tropospheric layer as well. The mean radial pressure gradient and the mean tangential flow are much stronger at this level, however, so that the asymmetric portion of the flow is a smaller percentage of the mean flow. The strong, symmetric pressure gradient force at this level is undoubtedly resisting the development of the azimuthal perturbations to a greater extent than the weaker pressure field in the outflow layer.

Figure 13 shows radial profiles of $\overline{Z}$, $\overline{\zeta}_a$, and $\overline{\zeta}_a\overline{Z}$ for selected times at level $1\frac{1}{2}$. All the profiles have a minimum in absolute vorticity, so that criterion (25) is also satisfied during the forecast. We next investigate the generation of the negative vorticity near the center. The vorticity equation in σ-coordinates may be written

$$\frac{d\zeta_a}{dt} = D + T + F \tag{26}$$

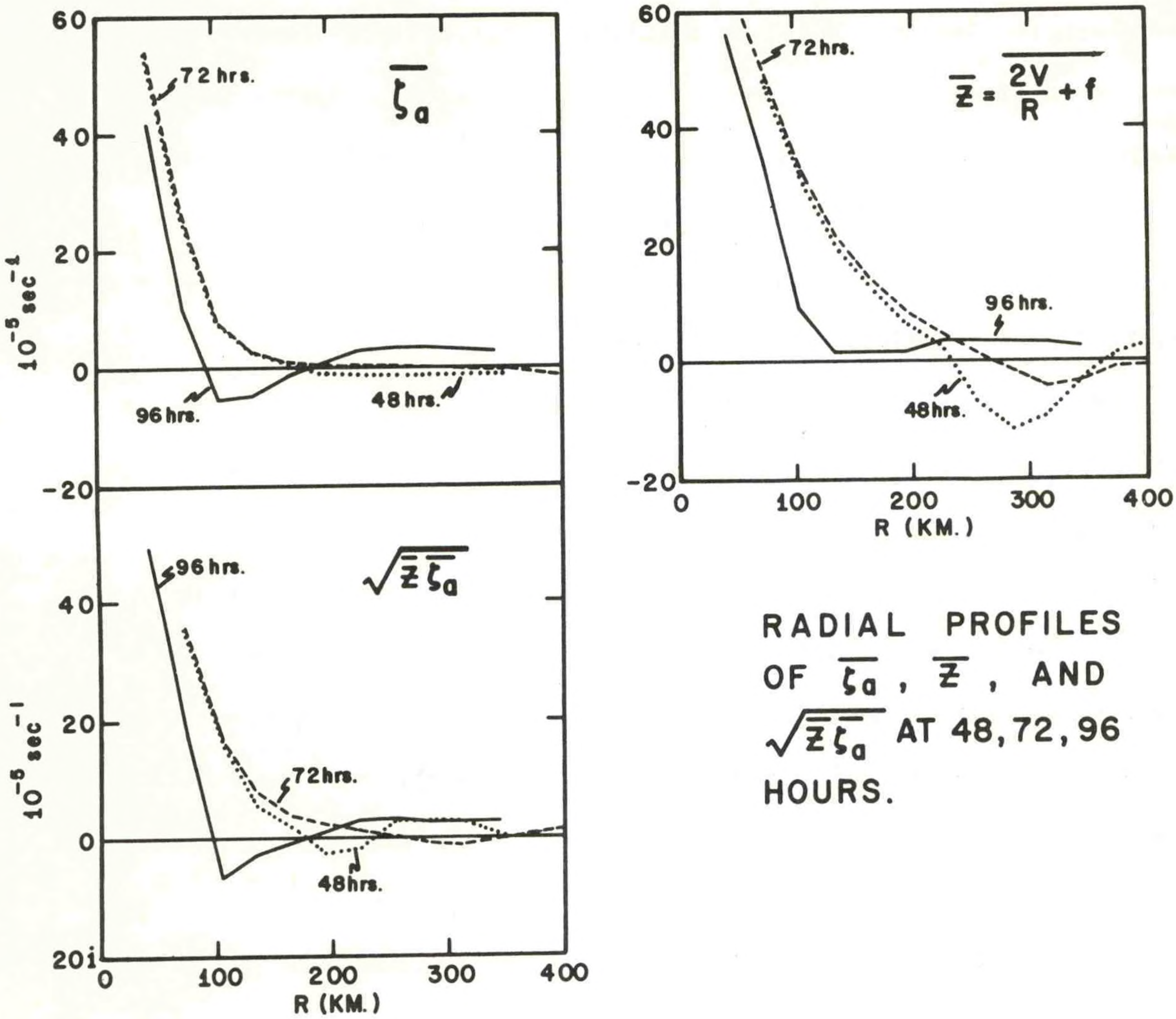


Figure 13. Radial profiles of $_a$, Z, and $\sqrt{\overline{\zeta_a Z}}$ at 48, 72, and 96 hours of Experiment 6.

where the divergence term, D, is

$$D = -\zeta_a \nabla \cdot \vec{V} \tag{27}$$

the tilting term, T, is

$$T = \frac{\partial\dot{\sigma}}{\partial y}\frac{\partial u}{\partial \sigma} - \frac{\partial\dot{\sigma}}{\partial x}\frac{\partial v}{\partial \sigma} \tag{28}$$

and the friction term, F, is

$$F = \frac{\partial F(v)}{\partial x} - \frac{\partial F(u)}{\partial y} \tag{29}$$

In (29) F(v) and F(u) represent the vertical and horizontal friction terms for the v and u components respectively. Given an initially positive absolute vorticity pattern, the divergence term by itself cannot produce negative ζ_a. Production of negative ζ_a must come from the tilting term, T. Figure 14 shows minimum azimuthal averages of D, T, and the sum, (D + T). The friction term, F, was also computed, and was much smaller in magnitude than either D or T. The terms in (27) through (29) were computed by interpolating the velocity components to the σ-levels and approximating the derivatives with centered differences. Figure 14 shows that both the divergence and tilting term contribute to the negative tendency. The spatial distributions of the minimums of T and D reveal the minimum divergence term to occur very close to the vortex center where the vorticity is large and positive and the divergence is maximum. The tilting term minimum occurs farther out, beyond the radius of maximum upward motion, where $\frac{\partial\dot{\sigma}}{\partial r} > 0$. (For a symmetric vortex, $T = -\frac{\partial\dot{\sigma}}{\partial r}\frac{\partial v_\lambda}{\partial \sigma}$ and since $\frac{\partial v_\lambda}{\partial \sigma}$ is positive in the hurricane, the negative contribution occurs with $\frac{\partial\dot{\sigma}}{\partial r} > 0$.)

The production of negative ζ_a may be physically interpreted from angular momentum considerations, where the angular momentum, M, is defined by

$$M = rv_\lambda + \tfrac{1}{2}fr^2 \tag{30}$$

and

$$\overline{\zeta_a} = \frac{\partial \overline{M}}{\partial r} . \tag{31}$$

In the early stages of cyclone development, M increases radially outward. The reversal of the radial gradient of M (production of negative ζ_a) may occur as air at large distances (high M) is advected inward in the boundary layer. When this air is ultimately carried upward in the narrow ring of ascent near the center of the vortex, it may, in spite of some frictional loss to the sea surface, retain a higher value of M than air at the same level but at a larger radial distance. The air at larger distances, in the middle levels, experiences little radial or vertical motion as shown by the middle level trajectory in figure 5.

In summary, substantial regions of negative absolute vorticity are produced in the middle and upper tropospheric layers through the tilting term in the vorticity equation. Both criterion for the development of azimuthal perturbations (in horizontal, non-divergent flow) are satisfied throughout the integration. Preference for the growth of longer waves (wave numbers one and two) are probably the result of the selective effects of static stability in the presence of horizontal divergence (Houghton and Young, 1970). In the next section we investigate the eddy kinetic energy budget.

3.7 Eddy Kinetic Energy Budget

The eddy kinetic energy equation may be derived by forming the equation for the azimuthal mean kinetic energy and subtracting this equation from the total kinetic energy equation. The resulting equation is

$$\frac{\partial \overline{k_e}}{\partial t} = C_H + C_V + B + F \tag{32}$$

where the eddy kinetic energy is defined

$$\overline{k_e} \equiv \frac{\overline{u'^2} + \overline{v'^2}}{2} \tag{33}$$

and

$$C_H = -\overline{v_r'^2}\frac{\partial \overline{v_r}}{\partial r} + \overline{v_r'v_\lambda'}\left(\frac{2\overline{v_\lambda}}{r} - \frac{\partial r\overline{v_\lambda}}{r\partial r}\right) - \frac{\overline{v_r}\,\overline{v_\lambda'^2}}{r} \tag{34}$$

$$C_V = -\overline{\dot{\sigma}'v_r'}\frac{\partial \overline{v_r}}{\partial \sigma} - \overline{\dot{\sigma}'v_\lambda'}\frac{\partial \overline{v_\lambda}}{\partial \sigma} \tag{35}$$

$$B = -\overline{\frac{v_\lambda'}{r}\frac{\partial \phi'}{\partial \lambda}} - \overline{v_r'\frac{\partial \phi'}{\partial r}} - \overline{\left(\frac{RT}{p}\right)}\;\overline{\left(v_r'\frac{\partial p'}{\partial r} + \frac{v_\lambda'}{r}\frac{\partial p'}{\partial \lambda}\right)}$$
$$-\overline{\left(\frac{RT}{p}\right)'\left(v_r'\frac{\partial p'}{\partial r} + \frac{v_\lambda'}{r}\frac{\partial p'}{\partial \lambda}\right)} - \overline{\left(\frac{RT}{p}\right)'u'}\frac{\partial \overline{p}}{\partial r} \tag{36}$$

$$F = \overline{v_r'F_r'} + \overline{v_\lambda'F_\lambda'}\,. \tag{37}$$

The term C_H represents the conversion of mean to eddy kinetic energy through horizontal, barotropic processes, and should be the predominant source of energy for the eddies if dynamic instability is the important mechanism in the eddy generation. The term C_V represents the effect of vertical eddies. The term B represents baroclinic effects and would be predominant if baroclinic instability were important. (The last two terms in B are several orders of magnitude smaller than the first three terms.) Finally, F represents the effects of asymmetries in the parameterization of the sub-grid scale eddy stresses.

Figure 15 shows the time variation of the volume integral of k_e, defined by

$$k_e = -\frac{2\pi}{g}\int_0^{R_m} r \int_1^0 \frac{\partial k_e}{\partial t} p^* \, d\sigma \, dr \tag{38}$$

where R_m is the edge of the domain (about 440 km). Because the computation of the various terms in (34) through (37) requires interpolation in the vertical, as well as from the cartesian grid to a polar coordinate grid, the values shown in figure 15 should be considered approximate.

Figure 15 shows that the significant positive contribution to the eddy kinetic energy is C_H, representing the barotropic conversion of mean to eddy kinetic energy, and provides strong support for the hypothesis that dynamic instability is present in the model. The vertical eddy term, C_V, is slightly positive at first, then slightly negative toward the end of the forecast. The baroclinic term (B) and the friction term (F) are negative throughout the forecast.

The eddy kinetic energy budget establishes dynamic, or inertial instability as the mechanism for the breakdown in symmetry and the development of large scale eddies in the outflow layer. These eddies continually extract kinetic energy from the mean circulation. The mean kinetic energy is maintained through the low-level cross isobar flow associated with the mean meridional circulation and the upward transport of kinetic energy by the rising motion near the center of the storm.

Figure 16 shows radial profiles of the various terms in the eddy kinetic energy equation at a typical time (138 hours) during the forecast. The terms are less reliable near the origin where the errors

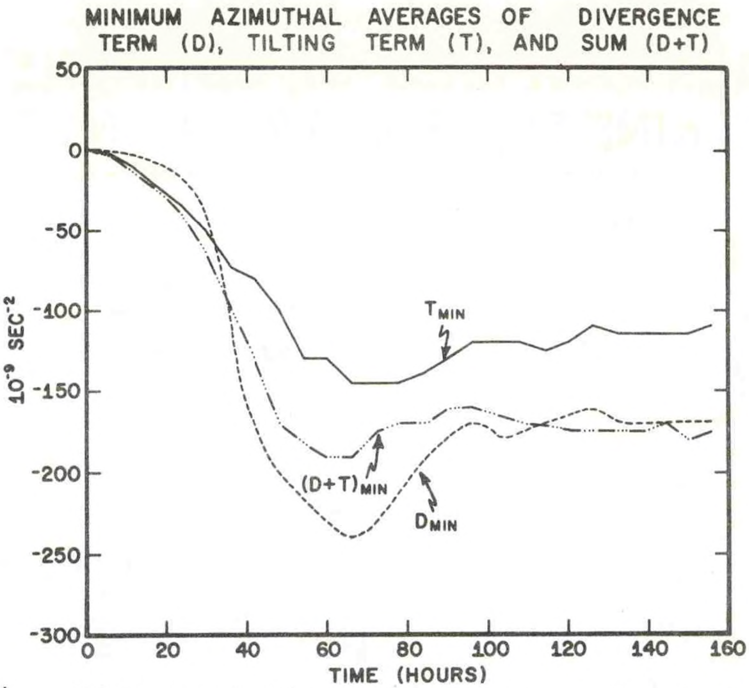


Figure 14. Time variation of minimum azimuthal averages of divergence term (d), tilting term (T), and sum (D+T) in the vorticity equation at level 2 for Experiment 6.

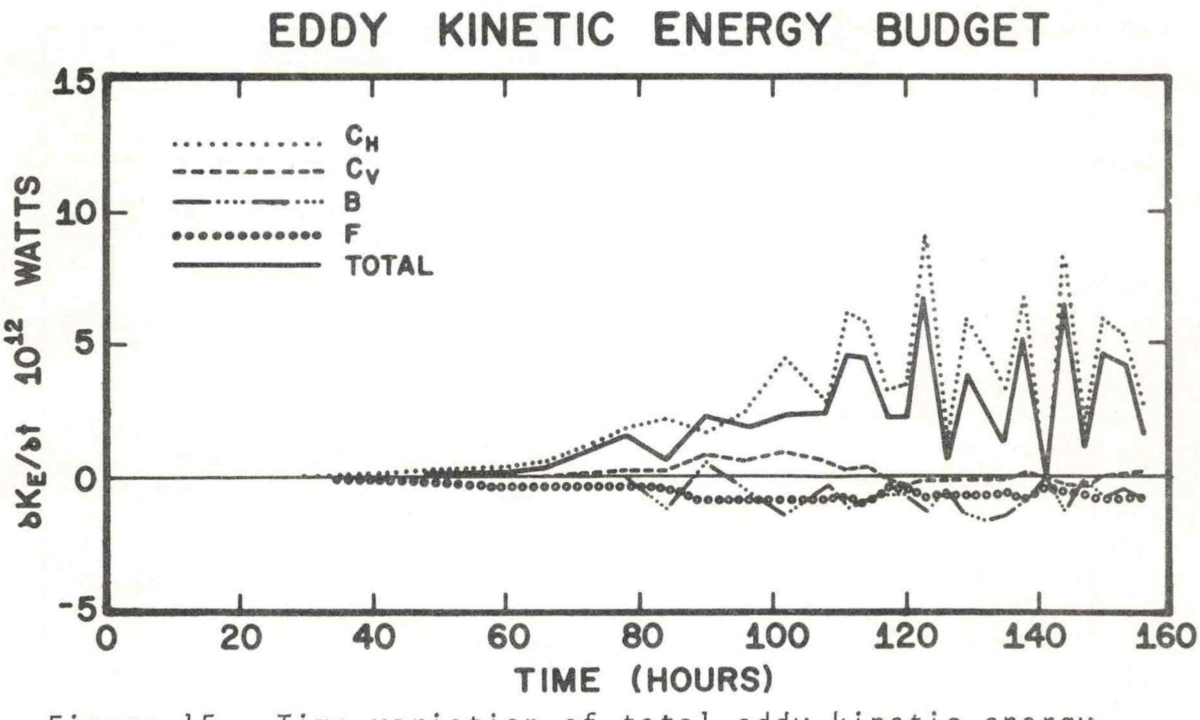


Figure 15. Time variation of total eddy kinetic energy budget. The term C_H represents the conversion of azimuthal mean to eddy kinetic energy through horizontal, barotropic processes; C_V represents the effects of vertical eddies; B represents baroclinic effects; and F represents frictional effects. See section 3.7 for discussion of these terms.

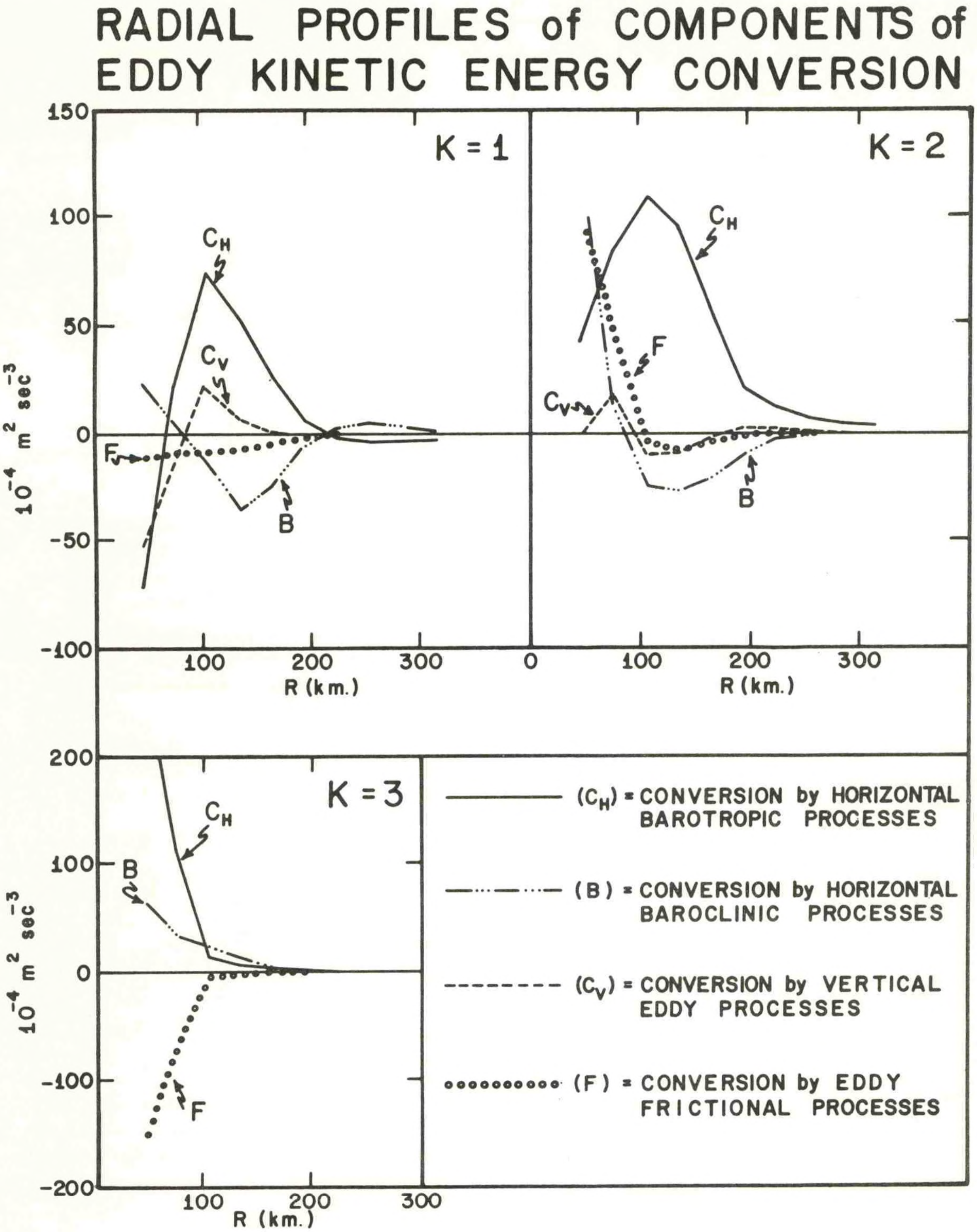


Figure 16. Radial profiles of terms in eddy kinetic energy budget at 138 hours in Experiment 6.

associated with the interpolation to the polar grid are maximum. From figure 16, the maximum conversion occurs around 100 km. The baroclinic processes consume eddy kinetic energy in the middle and upper troposphere. The slight positive contribution by baroclinic processes in the boundary layer may be associated with the generation of eddy kinetic energy on the scale of the spiral bands.

3.8 Increase of Sea Temperature in One Quadrant

Empirical results (Palmén, 1948; Miller, 1957; Perlroth, 1962) and symmetric hurricane model calculations (Ooyama, 1968) indicate a strong relationship between sea surface temperature and hurricane intensity. Figure 17 shows the effect of varying the sea surface temperature over a 2-degree range for 20 hours on the symmetric analog. The immediate response and the total range of 26 m sec^{-1} in the maximum wind speed confirms the sensitivity of the hurricane to small variations in sea surface temperature.

In addition to the recognized importance of the sea surface temperature to the intensity of the storm, it is sometimes suggested that horizontal sea-temperature gradients may also affect the storm's motion, perhaps through a differential enhancement of convective latent heat release. Although the looping motion of the asymmetric model storm is small, and poorly understood, it does afford the opportunity for at least a crude experiment to investigate the effect, if any, of sea surface temperature differences on the storm's motion.

Figure 18 shows the effect of raising the sea temperature from 302 to 303°K in the "northeast" quadrant of the grid over the last 30 hours

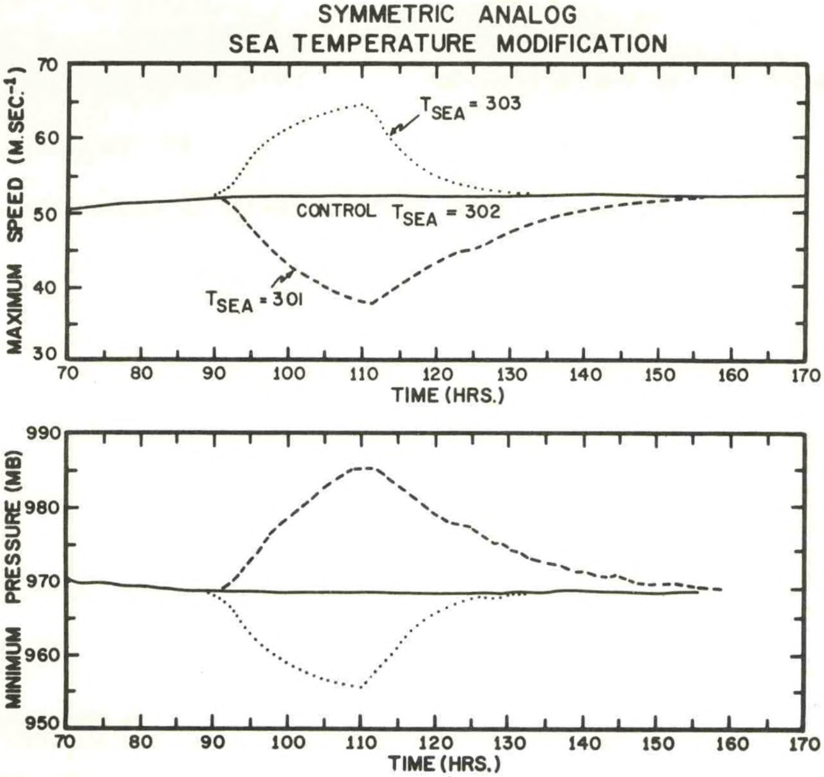


Figure 17. Effect of increasing and decreasing sea temperature by 1°C on maximum wind speed and minimum pressure.

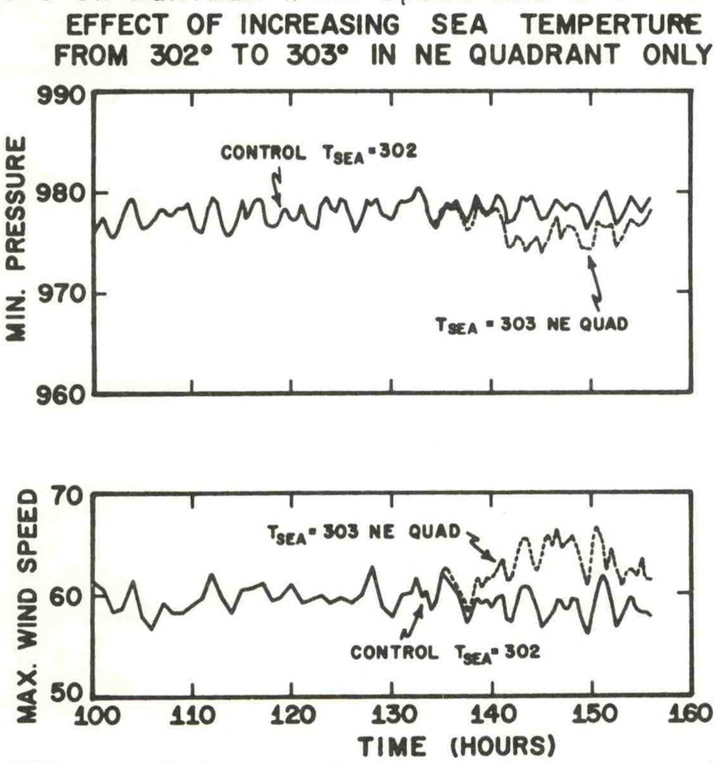


Figure 18. Effect of increasing sea temperature by 1°C in NE Quadrant alone for asymmetric model, Experiment 6.

of Experiment 6. The sea temperature over the rest of the grid remains unchanged. The position of the storm center at 136 hours, shown in figure 8, is in the "northwest" quadrant where the temperature is constant. As the storm moves over the warmer water (at about 140 hours) an increase in intensity occurs, reaching a maximum at about 146 hours when the wind speed is about 6 m sec^{-1} greater than that of the control. As the storm moves back over colder water, the differences from the control become less.

If the differing sea temperature field affected the storm motion, one would expect at least a small change in speed or direction as the storm moved over the warmer water. There was, however, no discernable change in the motion of the storm, in spite of a considerable (10%) change in the storm's intensity. This result suggests, although by no means proves, that storm <u>motion</u> is not appreciably affected by horizontal temperature gradients in the sea temperature.

4. SUMMARY AND CONCLUSIONS

Additional experiments with an improved version of an isolated, asymmetric model of the tropical cyclone are described. Increased horizontal resolution is achieved through the horizontal staggering of the variables. An explicit water vapor cycle is included which enables the study of the hurricane's water vapor budget and the simulation of non-convective heat release. The modeling of the horizontal diffusion of heat, water vapor, and momentum is considered more realistic in the current version of the model.

The development of the asymmetric structure of the model hurricane is examined in considerable detail. The asymmetries in the outflow layer are shown to result from dynamic instability, with the source of eddy kinetic energy being the mean azimuthal flow.

Well-defined spiral bands of convective heat release occur in these experiments. These bands propagate outward at a speed of about 24 knots, and compare favorably in structure with outer rain bands in nature.

An anticyclonic looping of the vortex center is observed during the later stages of these experiments. Although the mechanism for this motion is not clear, the looping seems to be closely associated with the asymmetries in the upper level circulation.

The intensity of the model is found to be sensitive to the sea surface temperature, in agreement with previous results. However, the looping motion of the storm is not noticeably affected as the storm passes over water of varying temperature.

The structure of the model storm, and the components of the energy budget agree quite well with empirical results and with previous symmetric model results, in spite of the coarse vertical and horizontal resolution.

ACKNOWLEDGMENTS

Rapid progress with NHRL's asymmetric hurricane model would not have been possible without the firm background of work with the symmetric model by the head of the theoretical studies group, Dr. Stanley L. Rosenthal. His support and suggestions concerning this work are gratefully acknowledged.

Mr. James Trout aided substantially in the programming of various aspects of the current model. Mr. Robert Carrodus and Charles True were responsible for the figures, and Mrs. Mary Jane Moss typed the manuscript.

APPENDIX A - FINITE DIFFERENCE EQUATIONS

A.1: Finite Difference Equations for Staggered Grid S-1

In Appendix A, the following notation will be useful (Shuman and Stackpole, 1968):

$$\alpha_x \equiv \frac{\alpha_{i,j+\frac{1}{2}} - \alpha_{i,j-\frac{1}{2}}}{\Delta x}$$

$$\overline{\alpha}^x \equiv \frac{\alpha_{i,j+\frac{1}{2}} + \alpha_{i,j-\frac{1}{2}}}{2}$$

$$\alpha_y \equiv \frac{\alpha_{i+\frac{1}{2},j} - \alpha_{i-\frac{1}{2},j}}{\Delta y} \qquad \text{A.1.1}$$

$$\overline{\alpha}^y \equiv \frac{\alpha_{i+\frac{1}{2},j} + \alpha_{i-\frac{1}{2},j}}{2}$$

We also introduce the following 4-point operators:

$$\widetilde{\alpha}^y \equiv \frac{\alpha_{i+1,j} + 2\alpha_{ij} + \alpha_{i-1,j}}{4}$$

$$\qquad \text{A.1.2}$$

$$\widetilde{\alpha}^x \equiv \frac{\alpha_{i,j+1} + 2\alpha_{ij} + \alpha_{i,j-1}}{4}$$

For vertical differences and averages, we define

$$\overline{\alpha}^\sigma \equiv (\alpha_{k+\frac{1}{2}} + \alpha_{k-\frac{1}{2}})/2$$

$$\qquad \text{A.1.3}$$

$$\delta\alpha \equiv (\alpha_{k+\frac{1}{2}} - \alpha_{k-\frac{1}{2}})$$

The finite difference equations associated with staggered grid S-1 for the u- and v-component equations of motion are:

$$\frac{\partial p^*u}{\partial t} \approx -\left(\overline{\overline{u}^{x}\ \widetilde{p^*u}^{y}}^{x}\right)_x - \left(\overline{\overline{u}^{y}\ \widetilde{p^*v}^{x}}^{y}\right)_y - \frac{\overline{\delta\dot{\sigma}}^{xy}\ \overline{p^*u}^{\sigma}}{\delta\sigma}$$

$$- \overline{p^*}^{xy}\ \overline{\overline{\phi}^{\sigma}_x}^{y} - R\overline{T}^{xy}\ \overline{p^*_x}^{y} + fp^*v \qquad \text{A.1.4}$$

$$+ [K_H(p^*u)_x]_x + [K_H(p^*u)_y]_y + F_V(u),$$

$$\frac{\partial p^*v}{\partial t} \approx -\left(\overline{\overline{v}^{x}\ \widetilde{p^*u}^{y}}^{x}\right)_x - \left(\overline{\overline{v}^{y}\ \widetilde{p^*v}^{x}}^{y}\right)_y - \frac{\overline{\delta\dot{\sigma}}^{xy}\ \overline{p^*v}^{\sigma}}{\delta\sigma}$$

$$- \overline{p^*}^{xy}\ \overline{\overline{\phi}^{\sigma}_y}^{x} - R\overline{T}^{xy}\ \overline{p^*_y}^{x} - fp^*u \qquad \text{A.1.5}$$

$$+ [K_H(p^*v)_x]_x + [K_H(p^*v)_y]_y + F_V(v)$$

where the finite difference analogs for the vertical friction terms, $F_V(u)$ and $F_V(v)$ have been presented earlier (6). In section A.1, the terms (p^*u) and (p^*v) represent $(u\overline{p^*}^{xy})$ and $(v\overline{p^*}^{xy})$ respectively.

The analogs for the continuity and thermodynamic equations are:

$$\frac{\partial p^*}{\partial t} \approx -\overline{(p^*u)_x}^{y} - \overline{(p^*v)_y}^{x} - p^*\ \frac{\delta\dot{\sigma}}{\delta\sigma}, \qquad \text{A.1.6}$$

and

$$\frac{\partial p^*T}{\partial t} \approx -\left(\overline{p^*u}^{y}\ \overline{T}^{x}\right)_x - \left(\overline{p^*v}^{x}\ \overline{T}^{y}\right)_y - p^*\ \frac{\delta\dot{\sigma}\widetilde{T}^{\sigma}}{\delta\sigma}$$

$$\text{A.1.7}$$

$$+ \frac{RT\omega}{C_p\sigma} + \frac{p^*}{C_p}\dot{Q} + p^*\ [K_T(T)_x]_x + p^*\ [K_T(T)_y]_y,$$

with the finite difference form for ω given by

$$\omega = \frac{dp}{dt} = p^*\dot{\sigma} + \sigma \frac{dp^*}{dt}$$

$$\approx p^*\overline{\dot{\sigma}}^{\sigma} + \overline{\sigma}^{\sigma} \left(\frac{\partial p^*}{\partial t} + \overline{u}^{xy}\, \overline{p^*_x}^{x} + \overline{v}^{xy}\, \overline{p^*_y}^{y}\right). \qquad \text{A.1.8}$$

As in paper A, the notation $\widetilde{T}^{\sigma}$ in (A.1.7) signifies that potential temperature is linearly interpolated between σ-levels rather than temperature itself.

The finite difference analogs for the horizontal derivatives in the water vapor forecast are analogous to the equation for temperature and are not given.

In (A.1.4) and (A.1.5), the use of the 4-point averaging operator is necessary if the finite difference equations are to conserve kinetic energy. The kinetic energy equation for this system may be written

$$\frac{\partial \overline{p^*}^{xy} k}{\partial t} + k\left(\frac{\partial \overline{p^*}^{xy}}{\partial t} + \nabla\cdot\overline{p^*}^{xy}\, \vec{V}\right) + \nabla\cdot\overline{p^*}^{xy}\, \vec{V}k + \frac{\vec{V}_i \cdot \overline{p^*}^{xy}\, \delta\dot{\sigma}\overline{\vec{V}}_i}{\delta_i \sigma}. \qquad \text{A.1.9}$$

For conservation of kinetic energy, the part of the finite difference analogs to the horizontal momentum flux terms that yield the $\nabla\cdot\overline{p^*}^{xy}\, \vec{V}$ term in (A.1.9) must cancel with $\frac{\partial \overline{p^*}^{xy}}{\partial t}$ as computed from the finite difference analog to the continuity equation. Consider the u-component equation (in one-dimension for simplicity)

$$\frac{\partial p^* u}{\partial t} = - \frac{\partial p^* uu}{\partial x}, \qquad \text{A.1.10}$$

which may be written, analytically,

$$p^* \frac{\partial u}{\partial t} + u \frac{\partial p^*}{\partial t} = - p^* u \frac{\partial u}{\partial x} - u \frac{\partial p^* u}{\partial x}, \qquad \text{A.1.11}$$

In the exact, differential equation, the cancellation of $u\frac{\partial p^*}{\partial t}$ with $-u\frac{\partial p^* u}{\partial x}$ by the continuity equation is necessary for the conservation of kinetic energy. Considering any finite differerence analog to (A.1.10)

$$\frac{\delta \overline{p^*}^{xy}\, u}{\delta t} = - \frac{\delta \overline{p^*}^{xy}\, uu}{\delta x} \quad , \qquad \text{A.1.12}$$

it is also necessary that the part of $\frac{\delta \overline{p^*}^{xy}\, uu}{\delta x}$ corresponding to $u\frac{\delta \overline{p^*}^{xy}\, u}{\delta x}$ cancel with the part of $\frac{\delta \overline{p^*}^{xy}\, u}{\delta t}$ corresponding to $u\frac{\delta \overline{p^*}^{xy}}{\delta t}$ as computed from the continuity equation. It may be verified by substitution that the 4-point averaging operator yields this necessary cancellation with the continuity equation.

A.2: Finite Difference Equations for Staggered Grid S-2

The finite difference analogs associated with staggered grid S-2 are derived by Lilly (1965) and tested as Scheme C by Grammeltvedt (1969).

For the equations of motion, the analogs are,

$$\begin{aligned} \frac{\partial p^* u}{\partial t} \approx & - [\overline{u}^x \overline{(p^* u)}^x]_x - [\overline{u}^y \overline{(p^* v)}^x]_y - p^* \frac{\delta \overline{\dot{\sigma}}^{x\,\sigma}\, u}{\delta \sigma} \\ & - \overline{p^*}^x\, \overline{\phi_x}^\sigma - R\overline{T}^x\, p^*_x + f\overline{(p^* v)}^{xy}, \end{aligned} \qquad \text{A.2.1}$$

$$\begin{aligned} \frac{\partial p^* v}{\partial t} \approx & - [\overline{v}^x \overline{(p^* u)}^y]_x - [\overline{v}^y \overline{(p^* v)}^y]_y - p^* \frac{\delta \overline{\dot{\sigma}}^{y\,\sigma}\, v}{\delta \sigma} \\ & - \overline{p^*}^y\, \overline{\phi_y}^\sigma - R\overline{T}^y\, p^*_y - f\overline{(p^* u)}^{xy}. \end{aligned} \qquad \text{A.2.2}$$

The friction terms are the same as those in (A.1.4) and (A.1.5) and are not repeated. In this section, the terms (p^*u) and (p^*v) represent $\overline{p^*}^x u$ and $\overline{p^*}^y v$ respectively.

The continuity and thermodynamic equations in this scheme are,

$$\frac{\partial p^*}{\partial t} \approx - (p^*u)_x - (p^*v)_y - p^* \frac{\delta\dot{\sigma}}{\delta\sigma} \qquad \text{A.2.3}$$

and

$$\frac{\partial p^*T}{\partial t} \approx - [\overline{T}^x (p^*u)]_x - [\overline{T}^y (p^*v)]_y + \frac{RT\omega}{C_p\sigma} + \cdot\cdot\cdot\cdot\cdot \qquad \text{A.2.4}$$

where

$$\omega \approx p^*\overline{\dot{\sigma}}^{\sigma} + \overline{\sigma}^{\sigma} \left(\frac{\partial p^*}{\partial t} + \overline{u}^x \, \overline{p^*_x}^x + \overline{v}^y \, \overline{p^*_y}^y\right) . \qquad \text{A.2.5}$$

The remaining terms in (A.2.4) are the same as those in (A.1.7). This scheme also conserves total energy (Lilly, 1965).

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