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NOAA Technical Memorandum NOS NGS 90

The Mathematical Relation between IFVM2022 as Expressed in ITRF2020 with IFVM2022 as Expressed in the Four Terrestrial Reference Frames of the Modernized NSRS with Dependence on EPP2022

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October 2021

National Oceanic and Atmospheric Administration • National Geodetic Survey

Versions

Date	Changes
October 28, 2021	Original Release

1 Introduction

As part of the proposed modernization of the National Spatial Reference System (NSRS), the National Geodetic Survey (NGS) has committed to providing tools by which our users may

1. transform coordinates between the four national frames (NATRF2022, PATRF2022, CATRF2022, and MATRF2022) and the International Terrestrial Reference Frame (ITRF) (NGS, 2021), and
2. estimate coordinates at epochs different from the epoch of observations, by accounting for motion from perennial sources such as: rigid rotation of tectonic plates and intraplate deformation due to faulting, glacial isostatic adjustment, etc.

As described in NGS (2021) each of the national frames listed in item 1 above is an Earth-centered Earth-fixed (ECEF) frame from which the corresponding plate motion has been removed (e.g., NATRF2022 is an ECEF frame from which rigid North American plate motion has been removed). Thus, to facilitate transformation between the ITRF and these four frames, NGS will provide Euler Pole Parameters (EPPs) that describe the rigid plate motion of each associated plate with respect to the ITRF (ibid)[1]. These EPPs will allow users to mathematically transform coordinates from the ITRF to any of the four national frames and back again. These parameters will be made available in a model called EPP2022.

To accomplish item 2 above, NGS is developing an intra-frame velocity model (IFVM)[2] which will provide a description of point motion for every mark in the NSRS dating back to at least 1994. The model will be called IFVM2022. The goal is for users of the NSRS to be able to move coordinates for mark(s) of interest through time by applying the motion model provided. Mark motion in IFVM2022 will be given with respect to the most recent ITRF and will account for motion due to rigid plate rotation as well as intraplate deformation due to faulting and glacial isostatic adjustment. However, since IFVM2022 will include motion due to rigid plate rotation expressed in the ITRF and the four national frames will be defined such that their associated rigid plate rotation has been removed via EPP2022, users will not be able to apply IFVM2022 directly to coordinates expressed in one of the four national frames without first transforming their coordinates to the ITRF (see Figure 1). Thus, it would be convenient if there were four additional versions of IFVM2022, one for each of the national frames, wherein NSRS mark

[1] The determination of the EPPs requires stations with ITRF2020 velocities. While NGS relies upon the ITRF as the foundation for the NSRS, there are some deeper, non-mathematical details which surround that decision. The International Terrestrial Reference System (ITRS) has been agreed upon through several UN Resolutions to provide a consistent system for comparison of geospatial data. In the Americas, additional UN Agreements have adopted the SIRGAS densification of the ITRS. The United States and broader SIRGAS community of Americas Nations will be adopting ITRF2020 as the realization once it is available. In turn, each of the Nations will further densify this SIRGAS framework to meet national needs. In the U.S., the Geospatial Data Act of 2018 codified the need for this modernization by ensuring that U.S. geospatial data assets can conform to the accepted international standards.

[2] The term IFVM is tentative, as the model intends to capture all motions of all geodetic control marks, not just simple velocities. However, the global geodetic and geophysics communities are split on what term is best suited to be used. Replacement candidates for “velocity” include “deformation” (primarily because the model would likely be derived from crustal deformation) and “motion” (primarily because the model will include all motions of marks). When a final term is decided, NGS will make an announcement and use the final term in future documentation.

motion would be described with respect to each corresponding frame. However, rather than maintain five versions of IFVM2022 (one for each of the four national frames plus the ITRF) it would be even more convenient if there were a mathematical relationship between IFVM2022 motions (expressed in ITRF2020) and those same motions relative to an NSRS frame that would allow "on the fly" computation of mark motion expressed in the user's frame of interest. The purpose of this work is to derive this proposed mathematical relationship. The result (equation 20) is a convenient and compact relationship which relies upon EPP2022.

2 Functional Application of IFVM2022 and EPP2022

From a functional standpoint, Figure 1 below shows how changing either an *epoch* or a *frame* must rely either upon IFVM2022 or EPP2022. In this example, the left third of the diagram is in PATRF2022, the central third in ITRF2020 and the right third in NATRF2022. The top half of the diagram is epoch 2028.048 and the bottom half of the diagram is epoch 2039.704.

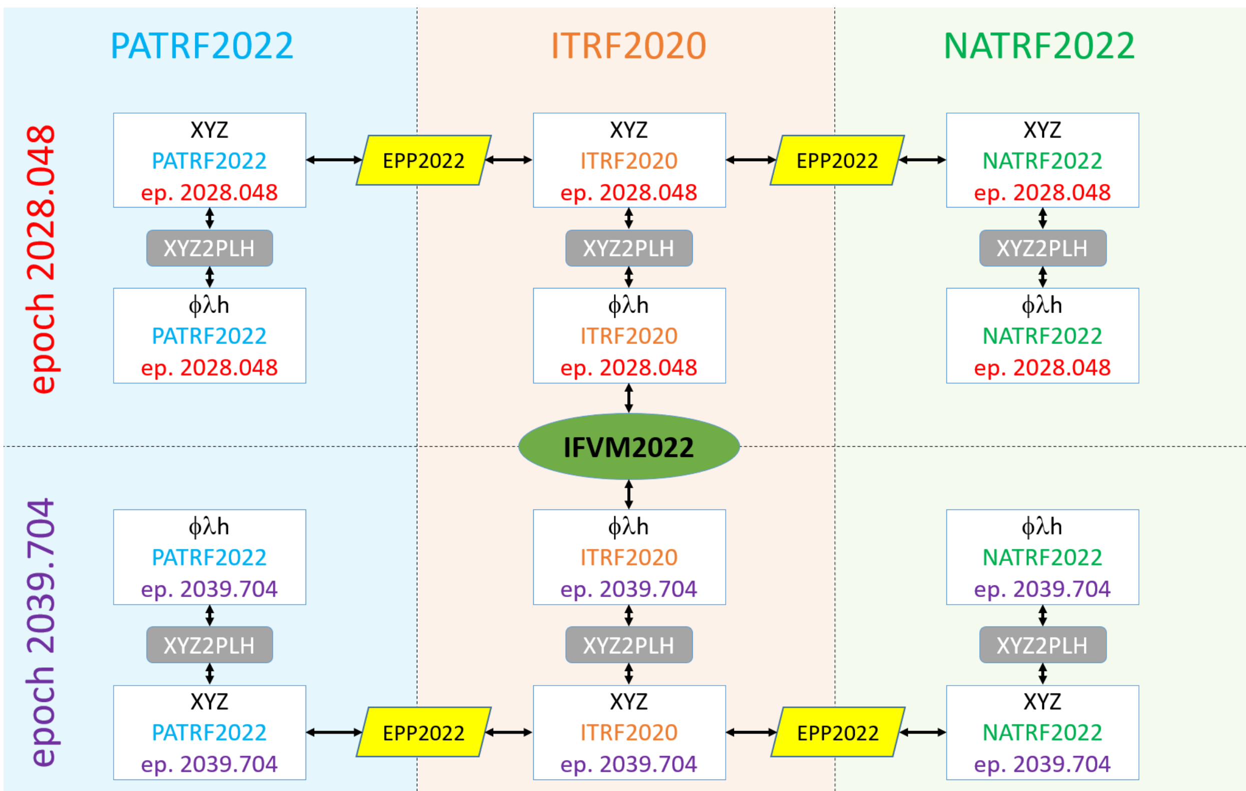

Figure 1: How IFVM2022 and EPP2022 are used to change epochs and frames

To put it in a rule of thumb:

- IFVM2022 changes the epoch, not the frame
- EPP2022 changes the frame, not the epoch

A few things might be noticed in Figure 1. First, NGS regularly transforms ECEF XYZ coordinates to geodetic latitude, longitude and ellipsoid height (ϕλh), and back, using program "XYZ2PLH". In the modernized NSRS this will continue, with continued reliance upon the GRS-80 ellipsoid. This program has little bearing on the following discussion, but is mentioned because it appears in Figure 1 quite a few times.

Second, the connection between frames using EPP2022 is made between XYZ coordinates, but the connection between epochs using IFVM2022 is made between ϕλh coordinates. This difference is important, because it complicates the math necessary to relate EPP2022 and IFVM2022. To avoid this complication NGS might have chosen to define the connection between NATRF2022 (et al.) frames and ITRF2020 using ϕλh coordinates, rather than XYZ. Or, NGS might have chosen to define IFVM2022 as changes in XYZ, rather than changes to ϕλh. However, neither of these choices is worth the difficulties they raise[3].

The third thing to note in Figure 1 is subtler, but is the key issue of this paper. Consider a surveyor interested in comparing coordinates in PATRF2022 at epoch 2028.048 to those in PATRF2022 in epoch 2039.704. To convert and transform those coordinates across that epoch change requires seven steps:

$$XYZ2PLH \rightarrow EPP2022 \rightarrow XYZ2PLH \rightarrow IFVM2022 \rightarrow XYZ2PLH \rightarrow EPP2022 \rightarrow XYZ2PLH$$

This is because IFVM2022 will be stored as a model of absolute motions within ITRF2020. The reason for this is *uniqueness*. NGS will not develop five independent IFVM2022 models, one for each frame, since this has the potential for providing non-uniqueness to our customer base.

However, it is inefficient to have to always transform every coordinate into ITRF2020 for the application of IFVM2022. Therefore, it is attractive to consider *virtual* versions of IFVM2022, as shown in Figure 2. In Figure 2, two *virtual* models, labeled IFVM2022(P) and IFVM2022(N) operate within PATRF2022 and NATRF2022 (rather than ITRF2020), but they *are not independent models* and *will not actually exist (thus "virtual")*[4]. Rather they should be considered as *identical to* IFVM2022 except expressed in PATRF2022 or NATRF2022. Taking the one-step path "through" IFVM2022(P) to compare coordinates in PATRF2022 at epoch 2028.048 to those in PATRF2022 in epoch 2039.704 will be proven mathematically identical to taking the seven-step "long way round" in Figure 1.

The final thing of note, but which does not appear in Figure 1, is that the actual motions saved in IFVM2022 are likely to be stored in length units (e.g. millimeters) in the East, North and Up directions. This is identical to how NGS's Horizontal Time-Dependent Positioning (HTDP) tool

[3] For EPP2022 to relate sets of ϕλh across different frames would require significant complexity to the simple relationship between ITRF2022 and NATRF2022 et al. (see equation 10). On the other hand, IFVM2022 is likely to reflect the deformation of the Earth's surface, and this deformation has radically different space and time signatures in the horizontal (latitude/longitude) direction than in the vertical (ellipsoid height). If transformed into XYZ, IFVM2022 would have significant bleeding of very different signals into all three Cartesian coordinates and would frankly make the model incredibly difficult to interpret.

[4] Two additional virtual IFVM2022 models can also be hypothesized: IFVM2022(C) for CATRF2022 and IFVM2022(M) for MATRF2022.

operates today. Thus, while IFVM2022 is shown connecting two different epochs of ϕ, λ and h, there is a hidden conversion from δE, δN, δU to δϕ, δλ, δh. This paper will account for that conversion in its derivations.

In summary, this paper concerns itself with deriving the equations which relate IFVM2022 to its *virtual* versions, which we have called IFVM2022(N), IFVM2022(P), IFVM2022(C) and IFVM2022(M).

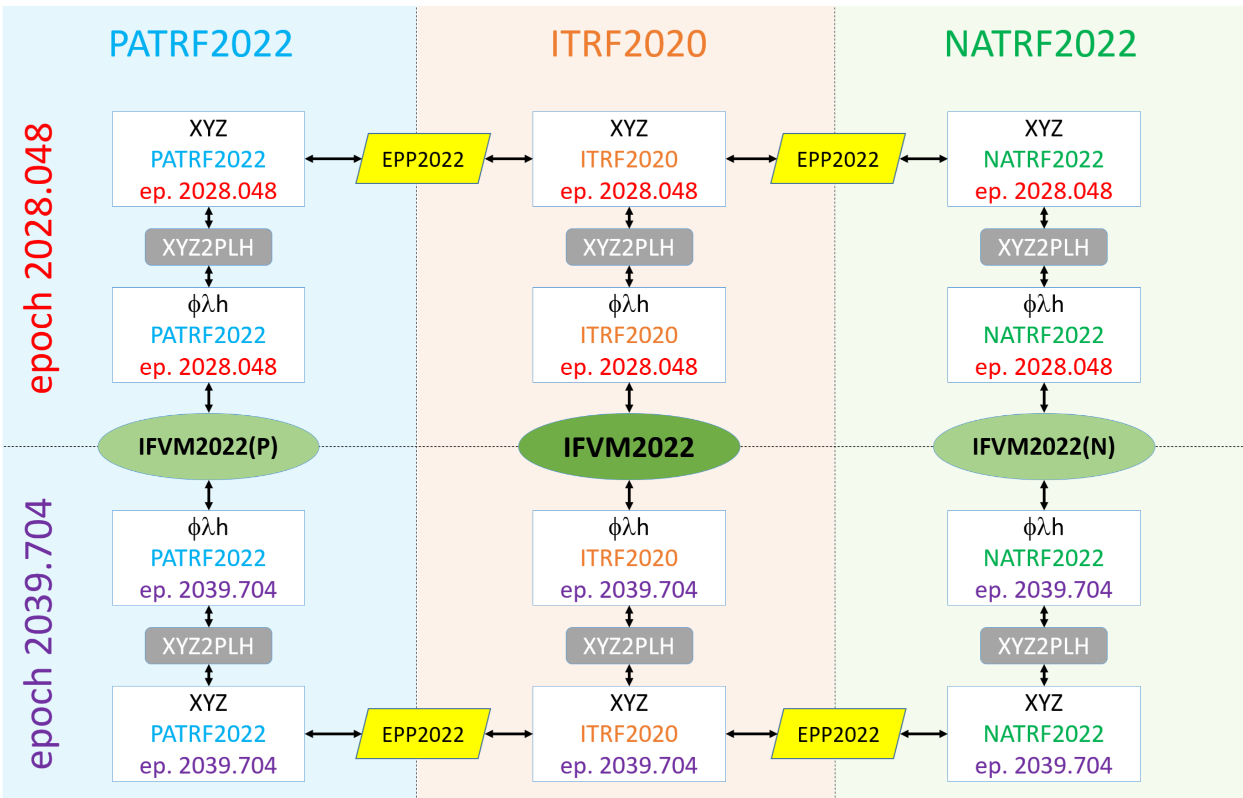

Figure 2: Inserting *virtual* IFVM2022 models (labeled IFVM2022(P) and IFVM2022(N)) into PATRF2022 and NATRF2022

3 Conventions and common equations

Before addressing the derivation proper, we adopt certain conventions and well-known equations.

3.1 Rotation matrices

We designate three rotation matrices R_1, R_2 and R_3 to represent a rotation of a Cartesian coordinate frame about the X, Y or Z axes of that frame, respectively. These rotation matrices are consistent with a positive rotation in the counterclockwise direction of a right-handed coordinate system, when viewed down the axis from the viewpoint of its positive end (Leick and van Gelder, 1975). Thus, for some rotation angle ω:

$$R_1(\omega) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix} \quad (1)$$

$$R_2(\omega) = \begin{bmatrix} \cos\omega & 0 & -\sin\omega \\ 0 & 1 & 0 \\ \sin\omega & 0 & \cos\omega \end{bmatrix} \quad (2)$$

$$R_3(\omega) = \begin{bmatrix} \cos\omega & \sin\omega & 0 \\ -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

3.2 Left- and Right-Handed Coordinate Frames

Geodesists have tended, by and large, to use right-handed coordinate frames for global positioning (XYZ), but have a strong tendency to switch to a left-handed frame for local positioning (North/East/Up). This adds an unnecessary level of complexity, especially when the right-handed East/North/Up system is equally viable as a local positioning frame.

Therefore, whenever this paper references any frame, it will be a right-handed one.

3.3 Converting δX,δY,δZ to δE,δN,δU

We adopt, without derivation, the following well-known formulae:

$$\begin{bmatrix} \delta E \\ \delta N \\ \delta U \end{bmatrix} = R_1\left(\frac{\pi}{2} - \phi\right) R_3\left(\frac{\pi}{2} + \lambda\right) \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} \quad (5)$$

Which can be fully expanded as:

$$\begin{bmatrix} \delta E \\ \delta N \\ \delta U \end{bmatrix} = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\cos\lambda\sin\phi & -\sin\lambda\sin\phi & \cos\phi \\ \cos\lambda\cos\phi & \sin\lambda\cos\phi & \sin\phi \end{bmatrix} \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} \quad (6)$$

But which we will shorten to:

$$\begin{bmatrix} \delta E \\ \delta N \\ \delta U \end{bmatrix} = Q_{\phi,\lambda} \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} \quad (7)$$

3.4 Converting δE,δN,δU to δϕ, δλ, δh

Again, without derivation, we adopt these well-known formulae (where δϕ and δλ are in radians, *N*, *M* and *h* are the radius of curvature in the prime vertical, radius of curvature in the meridian, and the ellipsoid height, all in meters):

$$\begin{bmatrix} \delta \lambda \\ \delta \phi \\ \delta h \end{bmatrix} = \begin{bmatrix} (N+h)\cos\phi & 0 & 0 \\ 0 & M+h & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \delta E \\ \delta N \\ \delta U \end{bmatrix} \quad (8)$$

Which we will shorten to:

$$\begin{bmatrix} \delta\lambda \\ \delta\phi \\ \delta h \end{bmatrix} = S_{\phi,h}^{-1} \begin{bmatrix} \delta E \\ \delta N \\ \delta U \end{bmatrix} \tag{9}$$

4 Derivation

We begin with a target equation, inspired by the definition of IFVM2022 and IFVM2022(N) as presented in NGS (2021). Although we use IFVM2022(N) in this derivation, an identical derivation exists for IFVM2022(P), IFVM2022(C) and IFVM2022(M). From (ibid):

This singular nature of IFVM2022 is best exemplified by considering that the movement of any geodetic control point anywhere on Earth can, with perfect equality, be described as any of these five things:

1) Movement in ITRF2020
2) (a) Movement in NATRF2022 plus (b) the rotation of NATRF2022 relative to ITRF2020
3) (a) Movement in PATRF2022 plus (b) the rotation of PATRF2022 relative to ITRF2020
4) (a) Movement in CATRF2022 plus (b) the rotation of CATRF2022 relative to ITRF2020
5) (a) Movement in MATRF2022 plus (b) the rotation of MATRF2022 relative to ITRF2020

In the following derivation "IFVM2022" represents all motions of a point in ITRF2020. However, the movement of that same point *in NATRF2022* is represented by "IFVM2022(N)". To put it into equation form, the following definition must hold, not only for NATRF2022, but for each of the four plate-fixed frames of the modernized NSRS. In these equations N means NATRF2022 while I means ITRF2020:

$$\begin{bmatrix} \delta X(\phi,\lambda,t_1,t_2) \\ \delta Y(\phi,\lambda,t_1,t_2) \\ \delta Z(\phi,\lambda,t_1,t_2) \end{bmatrix}_I^{IFVM2022} \equiv \begin{bmatrix} \delta X(\phi,\lambda,t_1,t_2) \\ \delta Y(\phi,\lambda,t_1,t_2) \\ \delta Z(\phi,\lambda,t_1,t_2) \end{bmatrix}_N^{IFVM2022(N)} + \begin{bmatrix} \delta X(\phi,\lambda,t_1,t_2) \\ \delta Y(\phi,\lambda,t_1,t_2) \\ \delta Z(\phi,\lambda,t_1,t_2) \end{bmatrix}_{N,I}^{EPP2022} \tag{10}$$

Equation 10 is correct, but not entirely useful as-is, since only EPP2022 is easily expressed in X, Y, Z coordinates. The IFVM2022 model will be stored in a way that expresses changes in E, N and U coordinates. So, without loss of generality, we can also write:

$$\begin{bmatrix} \delta E(\phi,\lambda,t_1,t_2) \\ \delta N(\phi,\lambda,t_1,t_2) \\ \delta U(\phi,\lambda,t_1,t_2) \end{bmatrix}_I^{IFVM2022} \equiv \begin{bmatrix} \delta E(\phi,\lambda,t_1,t_2) \\ \delta N(\phi,\lambda,t_1,t_2) \\ \delta U(\phi,\lambda,t_1,t_2) \end{bmatrix}_N^{IFVM2022(N)} + \begin{bmatrix} \delta E(\phi,\lambda,t_1,t_2) \\ \delta N(\phi,\lambda,t_1,t_2) \\ \delta U(\phi,\lambda,t_1,t_2) \end{bmatrix}_{N,I}^{EPP2022} \tag{11}$$

Equations 10 and 11 are both true, but what we have easily available are the 3rd term in (10), from EPP2022, and the first term of (11), from IFVM2022. What we need is a simple expression for the 2nd term of (11).

We therefore modify (11), using (7):

$$\begin{bmatrix} \delta E(\phi,\lambda,t_1,t_2) \\ \delta N(\phi,\lambda,t_1,t_2) \\ \delta U(\phi,\lambda,t_1,t_2) \end{bmatrix}_I^{IFVM2022} \equiv \begin{bmatrix} \delta E(\phi,\lambda,t_1,t_2) \\ \delta N(\phi,\lambda,t_1,t_2) \\ \delta U(\phi,\lambda,t_1,t_2) \end{bmatrix}_N^{IFVM2022(N)} + Q_{\phi,\lambda} \begin{bmatrix} \delta X(\phi,\lambda,t_1,t_2) \\ \delta Y(\phi,\lambda,t_1,t_2) \\ \delta Z(\phi,\lambda,t_1,t_2) \end{bmatrix}_{N,I}^{EPP2022} \tag{12}$$

It then falls upon us to find an expression for δX, δY and δZ, from time t_1 to time t_2 in terms of the EPPs. Thankfully these are simple values based upon the angular rotation of the plate alone, and so represent nothing more than a simple distance = velocity × time equation.

That is[5]:

$$\begin{bmatrix} \delta X(\phi,\lambda,t_1,t_2) \\ \delta Y(\phi,\lambda,t_1,t_2) \\ \delta Z(\phi,\lambda,t_1,t_2) \end{bmatrix}_{N,I}^{EPP2022} = (t_2 - t_1) \begin{bmatrix} \dot{X}(\phi,\lambda,t_1,t_2) \\ \dot{Y}(\phi,\lambda,t_1,t_2) \\ \dot{Z}(\phi,\lambda,t_1,t_2) \end{bmatrix}_{N,I}^{EPP2022} \tag{13}$$

The velocities represent the rotation of the North American (N) plate (or frame) relative to the ITRF.

$$\begin{bmatrix} \dot{X}(\phi,\lambda,t_1,t_2) \\ \dot{Y}(\phi,\lambda,t_1,t_2) \\ \dot{Z}(\phi,\lambda,t_1,t_2) \end{bmatrix}_{N,I}^{EPP2022} = \begin{bmatrix} 0 & \dot{\omega}_Z & -\dot{\omega}_Y \\ -\dot{\omega}_Z & 0 & \dot{\omega}_X \\ \dot{\omega}_Y & -\dot{\omega}_X & 0 \end{bmatrix}_{N,I} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_I \tag{14}$$

So that

$$\begin{bmatrix} \delta E(\phi,\lambda,t_1,t_2) \\ \delta N(\phi,\lambda,t_1,t_2) \\ \delta U(\phi,\lambda,t_1,t_2) \end{bmatrix}_I^{IFVM2022} \equiv \begin{bmatrix} \delta E(\phi,\lambda,t_1,t_2) \\ \delta N(\phi,\lambda,t_1,t_2) \\ \delta U(\phi,\lambda,t_1,t_2) \end{bmatrix}_N^{IFVM2022(N)} + (t_2 - t_1) Q_{\phi,\lambda} \begin{bmatrix} 0 & \dot{\omega}_Z & -\dot{\omega}_Y \\ -\dot{\omega}_Z & 0 & \dot{\omega}_X \\ \dot{\omega}_Y & -\dot{\omega}_X & 0 \end{bmatrix}_{N,I} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_I \tag{15}$$

Or:

$$\begin{aligned} \begin{bmatrix} \delta E(\phi,\lambda,t_1,t_2) \\ \delta N(\phi,\lambda,t_1,t_2) \\ \delta U(\phi,\lambda,t_1,t_2) \end{bmatrix}_I^{IFVM2022} &\equiv \begin{bmatrix} \delta E(\phi,\lambda,t_1,t_2) \\ \delta N(\phi,\lambda,t_1,t_2) \\ \delta U(\phi,\lambda,t_1,t_2) \end{bmatrix}_N^{IFVM2022(N)} \\ &+ (t_2 - t_1) \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\cos\lambda\sin\phi & -\sin\lambda\sin\phi & \cos\phi \\ \cos\lambda\cos\phi & \sin\lambda\cos\phi & \sin\phi \end{bmatrix} \begin{bmatrix} 0 & \dot{\omega}_Z & -\dot{\omega}_Y \\ -\dot{\omega}_Z & 0 & \dot{\omega}_X \\ \dot{\omega}_Y & -\dot{\omega}_X & 0 \end{bmatrix}_{N,I} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_I \end{aligned} \tag{16}$$

[5] EPP2022 is a model of rotations, and a point which rotates is technically accelerating, not moving with a linear velocity. However, considering the incredibly slow nature of the rotation, one may assume that a point is moving linearly (in a tangent direction to the rotation) and not actually rotating, over centuries before the assumption of a linear velocity breaks down. Thus, we show the position changes induced by rotation in (13) using the simple distance = velocity × time equation.

Finally, if we assume that points are rotating on a sphere, we may replace X, Y and Z with their spherical equivalences:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_I \equiv \begin{bmatrix} r_e \cos\phi \cos\lambda \\ r_e \cos\phi \sin\lambda \\ r_e \sin\phi \end{bmatrix}_I \tag{17}$$

This leads to:

$$\begin{bmatrix} \delta E(\phi,\lambda,t_1,t_2) \\ \delta N(\phi,\lambda,t_1,t_2) \\ \delta U(\phi,\lambda,t_1,t_2) \end{bmatrix}_I^{IFVM2022} \equiv \begin{bmatrix} \delta E(\phi,\lambda,t_1,t_2) \\ \delta N(\phi,\lambda,t_1,t_2) \\ \delta U(\phi,\lambda,t_1,t_2) \end{bmatrix}_N^{IFVM2022(N)} + (t_2 - t_1) \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\cos\lambda\sin\phi & -\sin\lambda\sin\phi & \cos\phi \\ \cos\lambda\cos\phi & \sin\lambda\cos\phi & \sin\phi \end{bmatrix} \begin{bmatrix} 0 & \dot{\omega}_Z & -\dot{\omega}_Y \\ -\dot{\omega}_Z & 0 & \dot{\omega}_X \\ \dot{\omega}_Y & -\dot{\omega}_X & 0 \end{bmatrix}_{N,I} \begin{bmatrix} r_e \cos\phi \cos\lambda \\ r_e \cos\phi \sin\lambda \\ r_e \sin\phi \end{bmatrix}_I \tag{18}$$

And finally:

$$\begin{bmatrix} \delta E(\phi,\lambda,t_1,t_2) \\ \delta N(\phi,\lambda,t_1,t_2) \\ \delta U(\phi,\lambda,t_1,t_2) \end{bmatrix}_I^{IFVM2022} \equiv \begin{bmatrix} \delta E(\phi,\lambda,t_1,t_2) \\ \delta N(\phi,\lambda,t_1,t_2) \\ \delta U(\phi,\lambda,t_1,t_2) \end{bmatrix}_N^{IFVM2022(N)} + (t_2 - t_1) r_e \begin{bmatrix} \dot{\omega}_X \cos\lambda\sin\phi + \dot{\omega}_y \sin\lambda\sin\phi - \dot{\omega}_z \cos\phi \\ -\dot{\omega}_X \sin\lambda - \dot{\omega}_y \cos\lambda \\ 0 \end{bmatrix}_{N,I} \tag{19}$$

Or, re-arranging, under the assumption that we have IFVM2022 (in ITRF2020) and the EPP2022 parameters ($\dot{\omega}_X$, $\dot{\omega}_Y$ nd $\dot{\omega}_Z$) as well as ϕ, λ, t_1 and t_2, we can find the changes in coordinates in IFVM2022* in NATRF2022 as a function of IFVM2022 and EPP2022 as:

$$\begin{bmatrix} \delta E(\phi,\lambda,t_1,t_2) \\ \delta N(\phi,\lambda,t_1,t_2) \\ \delta U(\phi,\lambda,t_1,t_2) \end{bmatrix}_N^{IFVM2022(N)} = \begin{bmatrix} \delta E(\phi,\lambda,t_1,t_2) \\ \delta N(\phi,\lambda,t_1,t_2) \\ \delta U(\phi,\lambda,t_1,t_2) \end{bmatrix}_I^{IFVM2022} - (t_2 - t_1) r_e \begin{bmatrix} \dot{\omega}_X \cos\lambda\sin\phi + \dot{\omega}_y \sin\lambda\sin\phi - \dot{\omega}_z \cos\phi \\ -\dot{\omega}_X \sin\lambda - \dot{\omega}_y \cos\lambda \\ 0 \end{bmatrix}_{N,I} \tag{20}$$

Identical equations to (20) exist for PATRF2022, CATRF2022 and MATRF2022 by simply substituting out N for P, C or M respectively.

5 Conclusions

The four national frames of the modernized NSRS will be defined, relative to the ITRF2020, according to three EPPs per frame. Application of these EPPs will change the frame, but not the epoch. Within the ITRF2020, NGS will define an intra-frame velocity model (IFVM2022), which will describe coordinates at different epochs, within the one frame. However, because users of the national frames might wish to have similar relationships between epochs in those frames, the concept of a *virtual* IFVM2022 model, functioning in each of the four frames was created.

This paper provides a mathematical relationship between the IFVM2022 model (built for ITRF2020) and *virtual* models, each of which function in one of the four national frames. Although the relationship was already anecdotally assumed to be "removing the rotation of the frame", this paper formalizes the equations which perform that step.

6 Bibliography

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