SPECTRAL ESTIMATES OF PLANET ALBEDO AND COMPARISON TO SKYLAB OBSERVATIONS

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ABSTRACT

Since the Skylab and ERTS (LANDSAT) missions of recent years, information on the radiation field of the earth has become available for comparison with values calculated using some of the various solutions to the equation of transfer. An application of the "adding" or "doubling" method of van de Hulst has been made for two homogeneous atmospheric layers composing an inhomogeneous atmosphere. The lower layer was assumed to be an aerosol composed of silicate particles of complex refractive index 1.55-0i corresponding to a wavelength of 0.5 μm. The particle size distribution was measured on-site. The upper layer was assumed to be a gaseous layer scattering light in accordance with Rayleigh's phase function. Radiance values were computed for a planetary system composed of the described composite atmospheric layer overlying a Lambert reflecting ground. These values showed fairly good agreement with Skylab observations given a reasonable knowledge of the scattering (or reflecting) properties of the atmosphere and ground.

INTRODUCTION

The problem of planetary radiative transfer processes involves a variety of solutions to the equation of transfer (Chandrasekhar, 1960).

These solutions are basically of two kinds: those which give the radiation field at various points in a planetary system (atmosphere and ground) and those which provide normal or slant flux at various points in a planetary system.

Irvine (1965, 1968) reviewed several solutions including an exact solution, known as the Neumann series, for the radiation field. This solution is an iterative process giving a series in increasing orders of scattering. He also reviewed some well known approximations for albedo of a layer, which are equivalent to flux type solutions to the equation of transfer. These approximations are commonly known as the Eddington, two stream and modified two stream.

Another approach, called the "invariant imbedding" method, was employed by Bellman and Ueno (1972) who derived expressions for the S and T functions of Chandrasekhar by an application of the principles of invariance (Chandrasekhar, 1960). A solution similar to the "invariant imbedding" method, called the "adding" or "doubling" method (van de Hulst, 1963; Hansen, 1969) and also based on the principles of invariance, was used by Hansen (1971) to study polarization of sunlight in water aerosols, and by Lacis and Hansen (1974) to obtain a parameterization for absorption of solar radiation by atmospheric ozone as a function of ozone depth (cm NPT). Here we see an application of a radiation field model to estimate more accurately the upward and downward normal fluxes at points in the earth's atmosphere.

Braslau and Dave (1973; parts I and II) used a solution similar to the Neumann solution to obtain the azimuth independent term of the Fourier series representation of the intensity function. Fluxes and albedos were then calculated from this term.

Plass, Kattawar and Catchings (1973) describe a matrix operator theory for exact solutions to the equation of transfer given homogeneous or inhomogeneous atmospheres and Lambert reflecting ground surfaces. This theory includes, as a special case, the "doubling" method. Examination of the methods used by Lacis and Hansen reveals great similarity to those employed by Plass, Kattawar and Catchings.

Several specialized applications have been made by Shettle and Green (1974), who examined fluxes of the middle ultraviolet light at the ground; Weinman and Guetter (1972), who studied radiative transfer relationships for plant canopies (corn leaves); Wang and Domoto (1974), who studied radiative effects of aerosols on solar radiation.

The present study makes use of the "adding" algorithm, both to "double" optically thin, homogeneous layers to a desired optical depth, and to "add" two homogeneous layers to construct the radiative transfer functions for a composite layer. The ground albedo effects are then added into the atmospheric effects by the analysis of Chandrasekhar (1960) also used by Bellman and Ueno (1972) and Braslau and Dave (1973). This approach assumes a Lambert reflecting ground and that scattering between the ground and sky occurs only once. The later assumption is in direct opposition to the principles intrinsic to the "adding" method.

Polarization of scattered radiation has been neglected to simplify calculations which produces errors in the intensities of the radiation field reaching a maximum for scattering angles approaching 90 degrees. According to Lacis and Hansen (1974), errors in radiance values are approximately 10% for Rayleigh scattering and about 1% for clouds. They also note that this error is much less when fluxes are calculated from the radiance calculated for the unpolarized case. Hansen (1971) gives a more exhaustive analysis of the errors introduced by neglecting polarization in computations of intensities and fluxes.

METHOD

The problem of describing the process of the transfer of solar radiation in a planetary system involves the complete description of the scattering and transmission properties of the atmosphere, and secondly, the determination of the effects of inserting a ground reflecting according to a given law. Description of the transfer properties of the atmosphere may be facilitated by the introduction of quantities which characterize the power of a medium (i.e. the atmosphere) to scatter and transmit radiation.

The atmospheric radiative transfer process may be given in terms of scattering and transmission functions with respect to a plane parallel layer of given optical properties such as optical depth and single scattering phase function (cf. Chandrasekhar, 1960; ch. I).

Consider a plane parallel layer of optical depth τ with an incident parallel beam of radiative flux πF per unit area perpendicular to the direction of propogation. Let μ be the cosine of the angle subtented by the scattered ray and the normal to the layer and ϕ be the azimuth measured about the normal to the atmosphere from some fixed reference. The mathematical representations of the intensity diffusely reflected from the top of the layer and that diffusely transmitted to the bottom are, respectively,

$$I(0;\mu,\phi) = \frac{F}{4\mu} S(\tau;\mu,\phi;\mu_o,\phi_o)$$

and (1)

$$I(\tau;\mu,\phi) = \frac{F}{4\mu} \bar{T}(\tau;\mu,\phi;\mu_o,\phi_o)$$

where $S(\tau;\mu,\phi;\mu_o,\phi_o)$ is the scattering function and $\bar{T}(\tau;\mu,\phi;\mu_o,\phi_o)$ is the transmission function (cf. Chandrasekhar, 1960; ch. I). The notation (μ_o, ϕ_o) denotes the direction of incidence of solar radiation.

Another form for equations (1) follows that given by Hansen (1969) which uses, for the reflection function, a modified form of Chandrasekhar's R function and an analogous form for transmission, namely

$$I(0;\mu,\phi) = \mu_o F R(\tau;\mu,\phi;\mu_o,\phi_o)$$

and (2)

$$I(\tau;\mu,\phi) = \mu_o F T(\tau;\mu,\phi;\mu_o,\phi_o)$$

where

$$R(\tau;\mu,\phi;\mu_o,\phi_o) = \frac{1}{4\mu\mu_o} S(\tau;\mu,\phi;\mu_o,\phi_o)$$

and (2a)

$$T(\tau;\mu,\phi;\mu_o,\phi_o) = \frac{1}{4\mu\mu_o} \bar{T}(\tau;\mu,\phi;\mu_o,\phi_o)$$

The method used to obtain R and T for any optical depth τ was first derived from the principles of invariance (Chandrasekhar, 1960) by van de Hulst (1963) and was reduced to a relatively simple algorithm by Hansen (1969, 1971). This method, known as the "adding" method, has been shown by Hansen (1969) to compare very satisfactorily with other exact methods, in particular with the so-called Neumann series, but with considerably greater computer speed.

Application of the "adding" method involves steps in which an optical depth is chosen for initial computations which is small enough that only single scattering need be considered. The R and T functions are, therefore, determined by the single scattering phase function in the following way:

$$R(\tau_o;\mu,\phi;\mu_o,\phi_o) = \frac{1}{4(\mu+\mu_o)} \{1-\exp\left[\tau_o\left(\frac{1}{\mu}+\frac{1}{\mu_o}\right)\right]\}\, p(\mu,\phi;-\mu_o,\phi_o)$$

and

$$T(\tau_o;\mu,\phi;\mu_o,\phi_o) = \frac{1}{4(\mu-\mu_o)} \{\exp(-\tau_o/\mu_o) - \exp(-\tau_o/\mu)\}\, p(\mu,\phi;\mu_o,\phi_o)$$

(cf. Chandrasekhar, 1960; ch. VII, eq. 35). The function $p(\mu,\phi;\mu_o,\phi_o)$ is the single scattering phase function and τ_o is the optical depth of the starting layer. τ_o was approximately 2^{-25} for all computations.

The next step is to make a special application of the "adding" method to layers of equal optical depth and phase function (i.e. the so-called "doubling" method), doubling the layer until the desired optical depth is reached.

The steps of this "doubling" method are carried out for each of two phase functions corresponding to layers of optical depth τ_r, scattering light in accordance with Rayleigh's phase function, and τ_a, scattering according to a generalized phase function. In the present case, τ_a refers to a layer composed of suspended particles (i.e. an aerosol). Then the two layers are "added". First, the layer τ_r is "added" to the top of the layer τ_a and then the reverse operation is performed to obtain R* and T* for illumination of the composite layer from below.

GROUND REFLECTANCE

The treatment of ground reflectance follows the analysis found in Chandrasekhar (1960; ch. X). His equations for total radiance reflected from a planetary system (atmosphere and ground) and total radiance transmitted to the ground, after modification using equations (2a), are respectively

$$I(0;\mu,\phi) = \mu_o F\{R(\tau;\mu,\phi;\mu_o,\phi_o) + \frac{\rho_g}{1-\rho_g \bar{s}^*}\gamma_1^*(\mu)\gamma_1(\mu_o)\}$$

and (3)

$$I(\tau;\mu,\phi) = \mu_o F\{T(\tau;\mu,\phi;\mu_o,\phi_o) + \frac{\rho_g}{1-\rho_g \bar{s}^*}\gamma_1(\mu_o)\frac{s^*(\mu)}{\mu}\}$$

where ρ_g is Lambert ground reflectance, and where

$$S_o(\mu,\mu') = \frac{1}{2\pi}\int_0^{2\pi} S(\tau;\mu,\phi;\mu',\phi')d\phi', \quad s(\mu) = \frac{1}{2}\int_0^1 S_o(\mu,\mu')d\mu',$$

$$T_o(\mu,\mu') = \frac{1}{2\pi}\int_0^{2\pi} T(\tau;\mu,\phi;\mu',\phi')d\phi', \quad t(\mu) = \frac{1}{2}\int_0^1 T_o(\mu,\mu')d\mu',$$

$$\bar{s} = 2\int_0^1 s(\mu)d\mu \quad , \qquad \bar{t} = 2\int_0^1 t(\mu)d\mu$$

$$\gamma_1(\mu) = \exp(-\tau/\mu) + \frac{t(\mu)}{\mu} \quad , \qquad \bar{\gamma} = \int_0^1 \mu\gamma_1(\mu)d\mu. \qquad (3a)$$

The starred quantities in equations (3) are derived in an analogous fashion, except that the functions $S^*(\tau;\mu,\phi;\mu',\phi')$ and $T^*(\tau;\mu,\phi;\mu',\phi')$ for illumination of the atmosphere from below are used instead.

PLANETARY ALBEDO AND ATMOSPHERIC TRANSMISSION

The expressions of planet albedo and atmospheric transmission, which we define as fractions of the total incident flux πF, are derived from the expressions for upward and downward fluxes normal to the atmosphere respectively. Upward normal flux at the top of the atmosphere is:

$$\pi F\uparrow = \int_0^1 \int_0^{2\pi} \mu' I(0;\mu',\phi')d\mu' d\phi'.$$

Substituting appropriate expressions for $I(0;\mu',\phi')$ and its component functions (cf. eq. 3a), then integrating, we obtain

$$\frac{\pi F\uparrow}{\pi F} = s(\mu_o) + \frac{2\rho_g \bar{\gamma}^*}{1-\rho_g \bar{s}^*} \mu_o \gamma_1(\mu_o) \tag{4}$$

We use a development for downward normal flux indentical to that of Chandrasekhar, except that notation is used to clarify the expressions for the case of the illumination of the composite layer from below.

Downward normal flux at the ground $\pi F\downarrow$ is composed of three component fluxes. The first is the reduced incident flux $\pi\mu_o F \exp(-\tau/\mu_o)$. The second component is the light diffusely transmitted by the atmosphere:

$$\frac{F}{4} \int_o^1 \int_o^{2\pi} \bar{T}(\tau;\mu,\phi;\mu_o,\phi_o) d\mu \, d\phi = \pi F t(\mu_o)$$

(cf. eq. 3a). The final component is the flux diffusely reflected by the ground, then again by the sky back to the ground. This flux is represented by

$$\int_o^1 \int_o^{2\pi} \mu I_g\uparrow(\mu) d\mu \, d\phi \tag{5}$$

where

$$I_g\uparrow(\mu) = \frac{1}{4\pi\mu} \int_o^1 \int_o^{2\pi} S^*(\tau;\mu,\phi;\mu',\phi') I_g \, d\mu' \, d\phi' = I_g \frac{s^*(\mu)}{\mu}$$

and where I_g is the radiance of the light reflected diffusely from the ground. Thus, the flux of expression (5) becomes

$$\pi I_g \bar{s}^* .$$

The ground is assumed to be a Lambert reflector, therefore, $\pi I_g = \rho_g \pi F\downarrow$. Thus by combining the three fluxes above, we have the relation

$$\rho_g \pi F\downarrow = \rho_g \{\mu_o \pi F \exp(-\tau/\mu_o) + \pi F t(\mu_o) + \pi I_g \bar{s}^*\}$$

or, by substituting $\rho_g \pi F\downarrow$ for πI_g and rearranging terms we get

$$\frac{\pi F\uparrow}{\pi F} = \frac{\mu_o \gamma_1(\mu_o)}{1-\rho_g \bar{s}^*} \quad . \tag{6}$$

Figure 1 shows the metamorphosis of planet albedo with changes in the scattering characteristics of the atmosphere. Figure 1a displays, for six ground reflectances, the planet albedo (vs. solar zenith angle) for a plane parallel atmosphere reflecting radiation in accordance with Rayleigh's phase function. Figure 1b shows the same as the previous one, except that scattering is according to an aerosol (Mie) phase function (eg. Fig. 2). Figure 1c shows reflection from a layer composed of a Rayleigh scattering layer overlying an aerosol of Mie scattering layer.

DATA ACQUISITION

Observations of spectral radiance were made using an infrared spectrometer (Skylab experiment S191). These observations were made by instruments mounted in the Skylab satellite and in a helicopter flown concurrently over selected target areas. The S191 instrument is a manually aimed, two-band spectrometer with a one milliradian field-of-view whose output has been smoothed and calibrated to radiance units, watts/cm^2/micrometer/steradian. (The field-of-view of the helicopter mounted instrument was set to 22 degrees in order to sample areas with approximately the same reflective properties as the image observed from space.) The instrument samples a shortwave band for monochromatic radiance from 0.4 to 2.4 micrometers (with a median resolution of approximately 4 X 10^{-2} micrometers), as well as an infrared band not pertinent to this study (NASA, 1971).

Targets of the S191 were continental in nature and were therefore, assumed to have an aerosol composed of silicate particles with complex index of refraction m = 1.55-0i (Reeser and Marlatt, 1975). These sites were chosen for their relative uniformity of reflective properties and their levelness. The targets are summarized in Table 1.

Table 1. Summary of target sites.

Nr.	Site	Latitude	Longitude	Description
1.	White Sands, New Mexico	33.48	106.15	Black lava 12 Aug. 1973, a.m.
2.	White Sands, New Mexico	32.88	106.27	White sand dunes 12 Aug. 1973, a.m.
3.	Rainbow Valley Airport Phoenix, Ariz.	33.19	112.43	Sandy desert 6 Sept. 1973, p.m.
4.	Old Verde Canal Phoenix, Ariz.	33.67	111. 98	Grassy desert 6 Sept. 1973, p.m.

Aerosol particle counts by size classes were taken using an aircraft mounted Bausch and Lomb particle counter. Profiles were flown over the target areas and from these data profiles, mean particle size distributions were computed in order to provide one homogeneous aerosol layer.

Ground observation of sky parameters include estimates of Lambert ground reflectance ρ_g using average incoming and reflected fluxes in the 0.5 to 0.7 micrometer radiation band (Reeser and Marlatt, 1975). Eppley pyranometers with appropriate filters were used for these observations. The albedos obtained represent an average value for the band.

Optical depth measurements were made with a hand held sun photometer which measures a voltage proportional to radiant intensity. Beer's law is applied to the voltage output of the photometer, so that we now have the equivalent expressions

$$J = J_o e^{-\tau m}$$

and

$$\tau = \frac{1}{m} \ell n(\frac{J_o}{J})$$

for optical depth. J_o is the voltage output of the photometer extrapolated to zero air mass and J is the voltage output for air mass m. Reeser (1972) gives an analysis of the error for this technique. For example, the error in optical depth for light of wavelength 0.5 μm (near an optical depth 0.4) is about 12%.

The aerosol optical depth may be extracted from the measured value of τ by subtracting the Rayleigh optical depth and any contribution by gaseous absorption. Rayleigh optical depth is computed after the manner of Elterman (1964). The selection of a wavelength of 0.5 μm virtually eliminates the need to account for gaseous absorption. This may be justified by observing that for a mid-latitude ozone depth of 0.4 cm (NPT) and ozone absorption coefficient, for wavelength 0.5 μm, equal to 0.0384 cm^{-1} (Vigroux, 1953), the total contribution to optical depth made by ozone is only 0.0015.

ANALYSIS AND RESULTS

For this analysis, a wavelength of 0.5 μm was selected only because computer time for calculation of phase functions and the various coefficients in the azimuthal expansion of the reflection and transmission functions is relatively long (cf. Hansen, 1969). The methods presented here serve to outline the process by which spectral -- as well as integrated -- planet albedo might be estimated.

The phase function for aerosols for both White Sands and Phoenix sites were calculated from the particle size distributions measured over those general areas. These phase functions were computed by means of the equations for Mie scattering given in papers by Diermendjian, Clasen, and Viezee (1961) and Diermendjian (1964) and were then expanded in a series in Legendre polynomials. The coefficients of this series were used in the "adding" algorithm to generate all of the functions defined in previous sections.

Estimated radiance at the satellite is computed using the first expression of equations (3):

$$I(0;\mu,\phi) \cong \mu_o F\{R(\tau;\mu,\phi;\mu_o,\phi_o) + \frac{\rho_g}{1-\rho_g \bar{s}^*}\gamma_1^*(\mu)\gamma_1(\mu_o)\} \qquad (7)$$

where μ, ϕ, μ_o and ϕ_o are calculated from the position of the satellite and the sun with respect to the target. $R(\tau;\mu,\phi;\mu_o,\phi_o)$ is the reflection function and $\gamma_1^*(\mu)$ and $\gamma_1(\mu_o)$ are functions defined in the set of equations (3a). F is incident solar radiance calculated from tables of πF (Thekaekara and Drummond, 1970).

We may solve equation (7) for an estimation of Lambert ground albedo:

$$\rho_g \cong \frac{\eta}{1+\eta \bar{s}^*}$$

$$\eta \cong \frac{1}{\mu_o F \gamma_1^*(\mu)\gamma_1(\mu_o)}\{I_{obs} - \mu_o F R(\tau;\mu,\phi;\mu_o,\phi_o)\} \qquad (8)$$

where ρ_g is the estimated Lambert ground albedo and I_{obs} is the radiance observed at the satellite.

Estimated radiance for low altitudes of the helicopter profiles, where the effects of the "path radiance" are assumed to be negligible compared to the total radiance, may be given by

$$H \simeq \frac{\mu_o F \gamma_1(\mu_o)\rho_g}{1-\rho_g \bar{s}*} \tag{9}$$

We have, therefore, defined H to be the outward radiance at the Lambert ground — that is, Lambert albedo times inward normal flux at the ground divided by π.

Inversely, the estimated Lambert ground reflectance given the radiance observed at the helicopter H, is

$$\rho_g \simeq \frac{H}{F\mu_o\gamma_1(\mu_o) + H\bar{s}*} \tag{10}$$

Table 2 displays the results of the analysis for one scan of the spectrometer for each of the four sites. Where applicable, equations used in the calculations are indicated.

Table 2. Results of analysis

Line Nr.	Description	Site 1	Site 2	Site 3	Site 4
1.	Look angle (θ)	13.8	31.2	1.7	15.7
2.	Solar zenith angle (θ_o)	61.0	61.0	38.5	38.5
3.	Relative azimuth ($\phi-\phi_o$)	117.2	47.3	88.2	0.7
4.	Aerosol optical depth	0.180	0.180	0.230	0.230 1/
5.	Rayleigh optical depth	0.121	0.121	0.139	0.139
6.	Measured Lambert reflectance	0.4	0.07	0.16	0.16 1/
	Radiance				
7.	S191 observation	0.0053	0.0031	0.0091	0.0077
8.	Computed S191 estimate	0.0122	0.0044	0.0098	0.0096 2/
9.	Helicopter observation	--	--	0.0083	0.0060
10.	Helicopter estimate	0.0106	0.0018	0.0070	0.0070 3/
	Computed Reflectance				
11.	From satellite	0.123	0.013	0.143	0.113 4/
12.	From helicopter	--	--	0.187	0.137 5/

1/Reeser and Marlatt (1975
2/cf. equations (3)
3/cf. equation (9)
4/cf. equation (8)
5/cf. equation (10)

The units of values presented in Table 2 are degrees (lines 1-3), dimensionless in lines 4-5, watts/cm^2/micrometer/steradian (lines 7-10), and dimensionless in lines 11-12.

DISCUSSION AND CONCLUSIONS

Computed intensities for the top of the atmosphere are fairly sensitive to the accuracy of measurements of Lambert ground reflectance. An error of 12.5% in Lambert reflectance, for example, produces a 7.7% error in scattered intensity over the Rainbow Valley site (site 3). The error may be seen to be a function of the actual reflectance as well as the percent deviation of the reflectance from some given value.

Table 3. Errors in output reflecting errors in measurement of the Lambert ground reflectance

Site	Input error (%)	Output error (%)
Site 1	69.3	56.5
Site 2	85.7	42.1
Site 3	12.5	7.7
Site 4	31.2	24.4

The values of reflectance given by Reeser and Marlatt (1975) used in this study, as stated before, represent an average value of the spectral reflectance for a band B of radiation, i.e.

$$\bar{\rho}_g = \frac{\int_B I\uparrow(\lambda)d\lambda}{\int_B I\downarrow(\lambda)d\lambda}$$

where $\bar{\rho}_g$ is the average spectral reflectance and the functions $I\uparrow(\lambda)$ and $I\downarrow(\lambda)$ are, respectively, the upward and downward normal fluxes. The integrals represent Eppley pyranometer measurements.

On the other hand, spectral reflectance, given by $I\uparrow(\lambda)/I\downarrow(\lambda)$ with λ belonging to B, clearly does not generally equal the quantity $\bar{\rho}_g$ given above. Only for uniformly reflecting surfaces does equality of spectral and average reflectance occur. The wavelength 0.5 μm is near a chlorophyll absorption band, so some reduction in spectral albedo may be expected there. This possibility is displayed for the Phoenix examples (sites 3 and 4). S191 observations there are larger for the less densely vegetated ground at Rainbow Valley (site 3) than for the grassy background at Old Verde Canal (site 4).

Some question arises concerning the observations at White Sands, New Mexico. Observations seem to be lower than would be expected given the spectral reflectance measurements of Reeser and Marlatt. Brighter targets were observed later in the Skylab overpass of this area which would better correspond with the ground measurements of Reeser and Marlatt, but these targets were not near enough their ground measurement sites to be used.

Helicopter observations were much higher than would be expected for a Lambertian reflector. This is due, in part, to the enlarged field-of-view. This enlarged field-of-view allows more light to enter the instrument and, if the target is not isotropic, simple division by π (implicit in the calibration of the S191 instrument) may not suffice to resolve the upward radiance. Salomonson and Marlatt (1971) showed that ground surfaces, as might be intuitively guessed, display rather marked anisotropy. This is particularly true for back-scattered light. Their analysis showed that anisotropy of ground targets deviates most from Lambert surfaces for heavily vegetated backgrounds and least for level desert surfaces devoid of vegetation. Greatest differences for all cases occurs for relatively large deflections from the nadir in the direction of the solar back-azimuth.

It may be noted that when the calculations of upward radiance at the helicopter are carried out using the albedo calculated from satellite observations (line 11, Table 2) instead of those values taken from Reeser and Marlatt (line 6, Table 2), the values computed are still lower than would be expected if path radiance can be neglected for helicopter heights of 500 feet or less. For instance, using equation (9) to recompute H, we obtain values 0.0063 and 0.0049 watts/cm^2/micrometer/steradian for sites 3 and 4 respectively. In conclusion, we may assert that even for such low helicopter altitudes as 500 feet, the path radiance may not be a negligible quantity. This may be especially true, if we may infer from the results of Salomonson and Marlatt that the field-of-view of the helicopter mounted instrument is not large enough to assume a significant departure of the observed image from a Lambert reflector.

Table 4. Reflection function, observed and computed radiance at the satellite, and estimated Lambert reflectance at the ground

Site	θ	θ_o	$\phi-\phi_o$	$R(\tau;\delta;\delta_o)$	S191 Obs.	Est. S191	Est. ρ_g
Site 3	1.73	38.5	88.15	0.0691	0.0091	0.0098	0.143
	1.86	38.5	59.29	0.0690	0.0092	0.0098	0.146
	2.21	38.5	38.96	0.0689	0.0090	0.0098	0.142
	2.90	38.5	26.73	0.0686	0.0090	0.0098	0.142
	3.67	38.5	17.73	0.0683	0.0092	0.0097	0.148
Site 4	15.74	38.5	0.73	0.0672	0.0077	0.0096	0.113
	16.43	38.5	0.06	0.0672	0.0075	0.0096	0.109
	17.23	38.5	0.56	0.0673	0.0075	0.0096	0.109
	18.02	38.5	1.02	0.0674	0.0075	0.0096	0.107
	18.80	38.5	1.43	0.0674	0.0075	0.0096	0.108
	19.57	38.5	1.82	0.0675	0.0074	0.0096	0.107

Table 4 shows the variation of the reflection function $R(\tau;\delta=(\theta,\phi);\delta_o=(\theta_o,\phi_o))$ and the estimated intensity at the top of the atmosphere and estimated Lambert reflectance of the ground. The angles θ and θ_o correspond to the cosines μ and μ_o defined in previous sections. This table, although the data are extremely scarce, may also suggest something of the nature of ground anisotropy (for reflection). The large error in estimated radiance for the Old Verde Canal site (site 4) is apparently due to the high values of the ground albedo given by Reeser and Marlatt. It may be noted that the values for Rainbow Valley (site 3) are in much better agreement with Skylab observations, since the ground albedo measured there is near the value estimated from satellite data.

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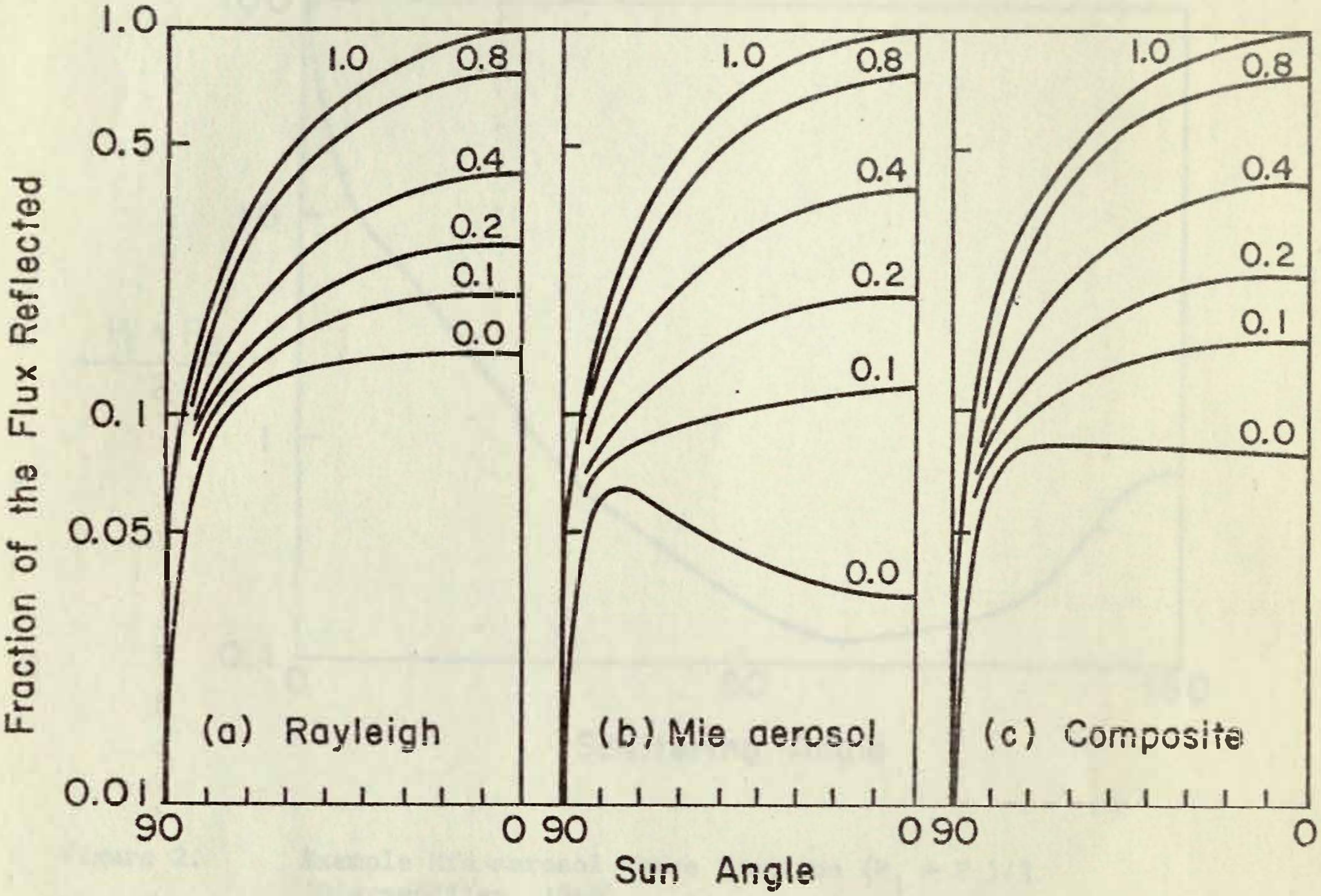

Figure 1. Fraction of the incident flux πF reflected for three planetary systems. Total optical depth in each case is 0.369. The numbers on the curves are Lambert ground reflectances.

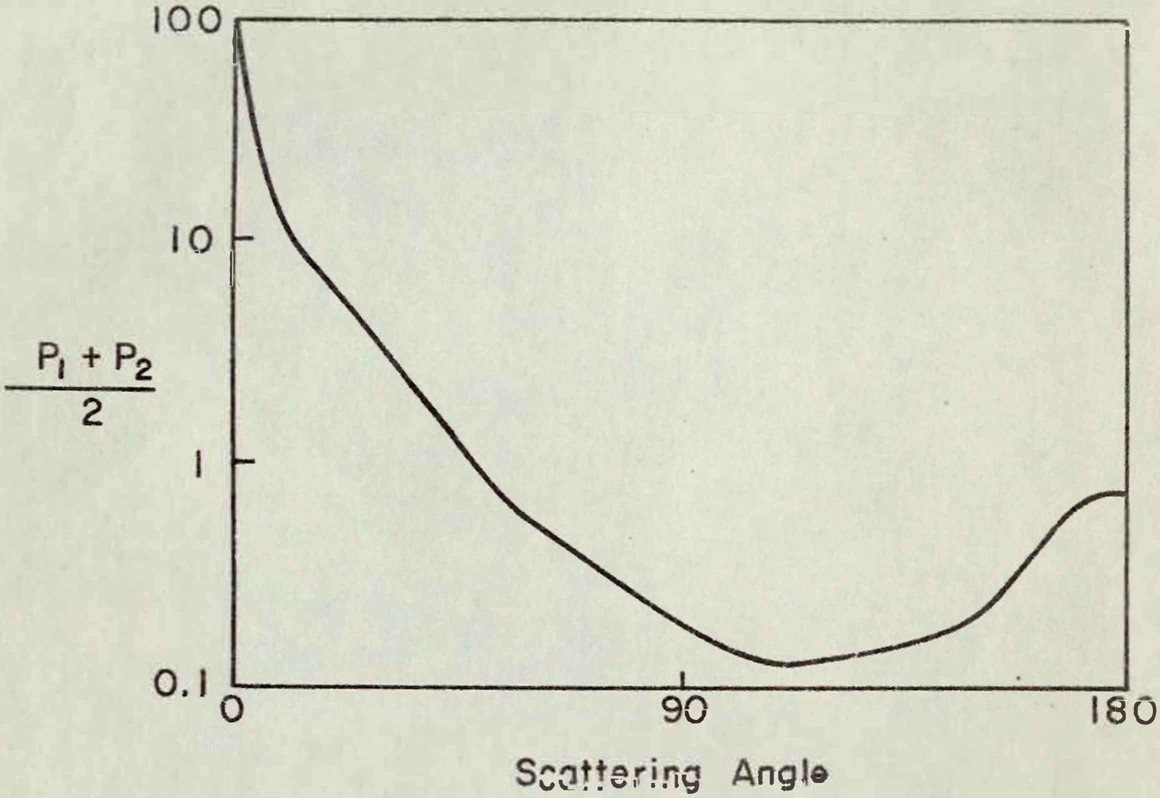

Figure 2. Example Mie aerosol phase function $(P_1 + P_2)/2$ (Diermendjian, 1964).

(Phoenix, Ariz., 6 Sep. 1973, 2:00 PM)