FINAL REPORT

The

VARIABILITY

of the

UNIT HYDROGRAPH

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CONTENTS

TABLES

FIGURES

APPENDICES

1. References
2. Equations
3. Notation
4. Program Abstract
5. Rainfall-Runoff Relations

VARIABILITY OF THE UNIT HYDROGRAPH

1. INTRODUCTION

The objectives of this investigation are two-fold: to collect statistics on the variability of the unit hydrograph from storm to storm, and to evaluate the factors responsible for the variation. The first objective should provide information on safety factors to assign to design or forecasting computations based on a constant unitgraph. The second objective should lead to refinements of such computations through the use of a variable unitgraph.

Ordinary rainfall and streamflow data were used to derive unitgraphs for each storm analysed within each basin selected for study. It is expected from recent researches (1) that deficiencies of this type of data are a major cause of variation between the derived unitgraphs for a particular basin. Therefore, to distinguish real from apparent effects, it is necessary to study a large number of storms per basin. Similarly, physical and climatic features will be factors, thereby dictating the need to consider a variety of study basins.

The general technique employed here is to express the unitgraph shape in terms of a function containing two or three parameters; and evaluate these parameters by the method of moments. This technique is economical computation-wise, and yields numbers representing unitgraph shape that can be handled statistically, or used as predictands in correlations with various predictors.

2. SURFACE RUNOFF MODELS

The functions selected for unitgraph derivation are based on the well-known surface runoff models conceived by Clark, Nash and Dooge (2,3,4,5). The Clark model consists of an idealized system of channels with pure translation characteristics discharging into a reservoir with linear storage characteristics. The proportion of watershed area discharging during any time period is defined by an area-concentration curve. This curve is developed by plotting contour lines of equal river distance from the outlet on a map of the basin, and planimetering the areas between the contour lines. By assuming a uniform stream velocity the area-concentration curve can be converted into a time-concentration curve. The parameters of the Clark function are T, total translation time, and K, the storage-discharge ratio of storage delay time.

The Nash model consists of a series of equal linear reservoirs with zero translation time. The parameters are K, the storage delay time of each reservoir, and n, the total number of reservoirs. The Dooge model consists of linear channels discharging into a series of linear reservoirs. In this study equal reservoirs are hypothesized, so that the parameters of the Dooge function consist of T, total translation time, n, the number of reservoirs, and K, the storage delay time of each reservoir. As in the Clark model, a time-concentration curve of the basin is used to define the time distribution of reservoir inflow.

The Hydrograph produced by routing a unit impulse of excess rainfall through either of the three models discussed here represents an instantaneous unit hydrograph (IUH). The IUH can then be converted to a unitgraph of any desired time unit by application of the S-curve procedure. The IUH of the Nash model does not involve watershed characteristics, and its function can be expressed analytically, as shown in Equation 1 of Appendix 2. Equations 2 and 3 provide the maximum value of this function, or unit peak discharge, and the time of peak, respectively.

3. METHOD OF MOMENTS

Various numerical methods can be employed to derive a unitgraph from the rainfall and streamflow data associated with a stream rise. This unitgraph can then be fitted by least squares to one or more of the functional forms described previously to obtain the desired unitgraph parameters. The method of moments is simpler because it enables direct acquisition of parameter values from the data. Let us assume that the rainfall and streamflow data have been reduced to proper form by abstraction of losses and separation of base flow. This means that an excess rainfall hyetograph (ERH) and a direct runoff hydrograph (DRH) are available. The first step in the procedure is the computation of the first three moments of the ERH and DRH, followed by their substitution into Equations 4,5, and 6 to obtain the corresponding moments of the IUH. Relationships linking these moments with unitgraph parameters, as shown in Equations 7,8, and 9, can then be applied. These equations may be applied to all three models by substituting one for n in the case of the Clark model, and zero for T in the case of the Nash model.

4. STUDY BASINS

The drainage basins selected for study totalled 12 in number, and are located in states of convenient data access, namely Missouri, Illinois, and Louisiana. The selections, shown in Table 1, were based on consideration of size of drainage area, length and quality of automatic recording rainfall and streamflow records, and absence of significant stream regulation or urbanization. The selected drainage areas are within the size range of basin subdivisions ordinarily used in forecasting or design computations, and best meet the desired standards within the three states.

All of the streamgaging stations under study are equipped with recorders that have been converted from graphic to digital type in recent years. There was insufficient time to analyze the complete record at all stations, and it was decided to subdivide the stations into primary and secondary classes. The primary stations received the complete treatment utilizing both graphic and digital data covering 36 to 51 storms. Analysis of data for the secondary stations was confined to the digital record covering 9 to 24 storms. The object of subdivision into two classes is to concentrate the correlation analysis on the extensive data collected for the primary basins, while utilizing all the data, both primary and secondary, for the variability analysis.

The study basins differ considerably in shape of watershed and drainage pattern, as shown in Figure 1. There are also wide differences in topography and climate between basins. The group of four basins in Illinois and two basins in Louisiana are flat, while the six Missouri basins have relatively steep topography, accounting for the low stream velocities and high ground water storage of the first group relative to the second group of basins. Vegetation becomes dormant during the winter season in the Missouri and Illinois basins, and seasonal changes in surface cover are greater in those basins than in the Louisiana group.

5. DATA REDUCTION

To achieve an unbiased set of storms for analysis it was decided to study all stream rises within a selected interval of water years yielding flows exceeding the base values specified by the USGS for peak flow publication, as listed in Table 1. Some of the storms were omitted later because of missing gage-height data or snowmelt complications.

TABLE 1. PERTINENT DATA, STUDY BASINS

PRIMARY AREAS

	AREA SQ. MI.	PERIOD OF STUDY	BASE CFS	NO. OF STORMS	RAINFALL REC	STATIONS TOTAL
Whisky Chitto Creek near Oberlin, La.	510	1953-68	3000	38	2	8
Bundick Creek near De Ridder, La.	120	1956-67	900	50	1	3
Big River near De Soto, Mo.	718	1949-68	8000	49	3	4
Meramec River near Steelville, Mo.	781	1949-68	6500	43	3	4
Salt Creek near Rowell, Ill.	334	1950-68	1300	36	3	4
Spring Creek near Springfield, Ill.	107	1957-68	600	40	2	2
			Total=	256		

SECONDARY AREAS

	AREA SQ. MI.	PERIOD OF STUDY	BASE CFS	NO. OF STORMS	RAINFALL REC	STATIONS TOTAL
North River at Palmyra, Mo.	373	1964-69	5000	17	2	5
Bourbeuse River at Union, Mo.	808	1965-69	8500	9	3	5
Big Piney River near Big Piney, Mo.	560	1965-70	6800	9	3	5
Maries River at Westphalia, Mo.	257	1965-70	6000	24	3	5
Sangamon River at Mahomet, Ill.	356	1963-70	1500	13	5	7
Beaucoup Creek near Matthews, Ill.	291	1967-69	2300	11	3	6
			Total=	83		

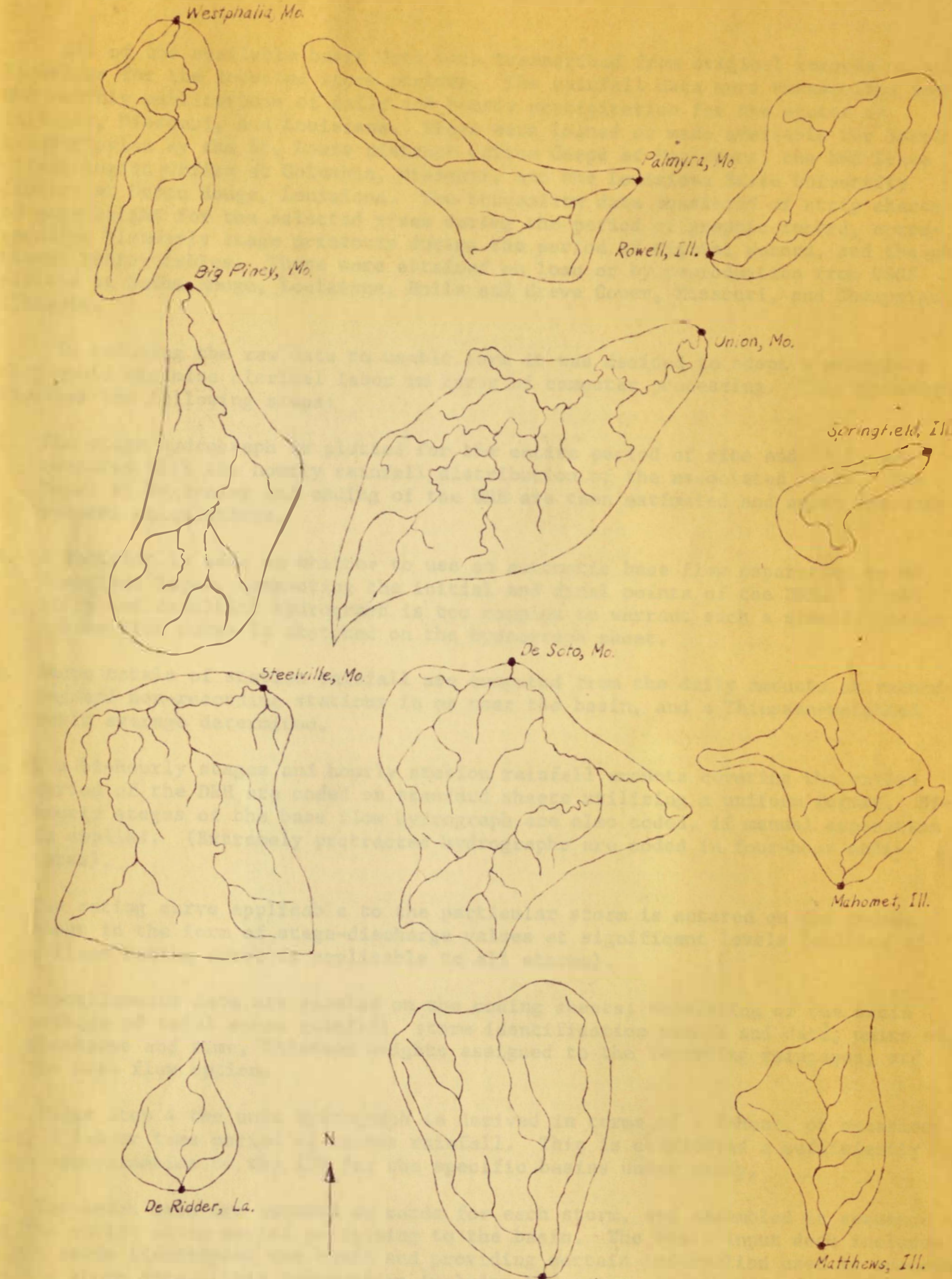

Figure 1. Drainage Maps, Study Basins

All of the available basic data were transcribed from original records or publications for the selected storm periods. The rainfall data were transcribed from NWS monthly publications of daily and hourly precipitation for the states of Illinois, Missouri, and Louisiana. These were loaned or made available for local transcription by the St. Louis District Office Corps of Engineers, the NWS State Climatologist Office at Columbia, Missouri, and the Louisiana State University library at Baton Rouge, Louisiana. The streamflow data consisted of strip charts of gage-height for the selected rises during the period of graphic record, corresponding bi-hourly stage printouts during the period of digital record, and the pertinent rating tables. These were obtained on loan or by reproduction from USGS offices at Baton Rouge, Louisiana, Rolla and Creve Couer, Missouri, and Champaign, Illinois.

In reducing the raw data to usable form it was decided to adopt a procedure that would minimize clerical labor in favor of computer processing. This procedure involves the following steps:

1. The stage hydrograph is plotted for the entire period of rise and fall, and compared with the hourly rainfall distribution of the associated storm. The hours of beginning and ending of the DRH are then estimated and noted for subsequent calculations.

2. A decision is made on whether to use an automatic base flow separation to be described later, connecting the initial and final points of the DRH. If the storm and resultant hydrograph is too complex to warrant such a simplification a base flow curve is sketched on the hydrograph sheet.

3. Storm totals of station rainfall are computed from the daily amounts at recording and non-recording stations in or near the basin, and a Thiessen-weighted basin average determined.

4. The bi-hourly stages and hourly station rainfall amounts covering the entire period of the DRH are coded on standard sheets utilizing a uniform format. Bi-hourly stages of the base flow hydrograph are also coded, if manual separation is applied. (Extremely protracted hydrographs are coded in four-hour ordinates).

5. The rating curve applicable to the particular storm is entered on the coding sheet in the form of stage-discharge values at significant levels (omitted if a fixed rating curve is applicable to all storms).

6. Miscellaneous data are entered on the coding sheets; consisting of the basin average of total storm rainfall, storm identification number and dates, units of discharge and time, Thiessen weights assigned to the recording raingages, and the base flow option.

Under Step 4 the unit hydrograph is derived in terms of a 2-hour, or occasionally, a 4-hour time period of excess rainfall. This is considered a sufficiently close approximation to the IUH for the specific basins under stody.

The coded data are punched on cards for each storm, and assembled in sequence for the entire storm series pertaining to the basin. The basin input deck includes header cards identifying the basin and providing certain information used in processing the storm data. This information includes the area-concentration data of the basin, the type of rainfall-runoff relation, coefficients of the rainfall-runoff relation, coefficients of the normal ground water recession curve, and values of the fixed rating curve, if a single rating curve is applicable to the entire storm series. The rainfall-runoff and ground water coefficients will be discussed later. The area-concentration data were obtained by measurements made on USGS 1:250,000 scale topographic charts, and the results in terms of cumulative river mileages and areas are listed for each study basin in Table 2.

6. COMPUTER PROGRAMS

The June 1970 Phase I report described a Fortran IV program developed for an IBM 360 computer, labelled UNITCOM, used to derive and evaluate parameters of the Clark and Nash unitgraph functions. This program has been superseded by an improved program, called UNITGRAF, that executes more accurate computations, extends unitgraph derivations to the Dooge model, and provides additional types of error output. However, some of the UNITCOM output is still useful, and will be described later. A description of UNITGRAF is furnished in the abstract of Appendix 4. Three versions of this program are available, corresponding to the three unitgraph models. Only one set of unitgraph results can be obtained at one time, but data for several basins can be processed in a single computer run.

The automatic base flow separation in the UNITGRAF program consists of two segments: a normal ground water recession from the initial point on the hydrograph to a point underneath the peak, followed by a straight-line connection to the end-point of the hydrograph. The recession curve is based on the 2-coefficient formula of Equation 16. The coefficients applicable to a particular stream flow station are obtained by plotting changes in flow during a specific period for selected recessions against initial flow, and fitting a straight line to the plotted points. Coefficients obtained in this manner are listed in Table 3 for each basin.

The ERH is developed from an inverse solution of the rainfall-runoff relation operating on the rainfall and streamflow data. There are three alternate subroutines to accomplish this job, one each for the co-axial relations developed at the Ft. Worth and Kansas City River Forecast Centers, and one for the Phi Index, or constant loss rate method. The three runoff subroutines are labelled ROFORT, ROKANS, and ROFILT respectively, and are appended to the main unitgraph routine. The basin input to the main routine specifies the appropriate subroutine, and in the case of the RFC subroutine, furnishes the applicable set of coefficients of the rainfall-runoff equation. Material used in programming the co-axial subroutines were furnished by the Ft. Worth and Kansas City RFC's, and are reproduced in Appendix 5. The necessary coefficients applicable to each study basin were also furnished by the two RFC's, and are listed in Table 4.

Other computer programs, shown schematically in Figure 2, are used to evaluate the output of UNITGRAF. These programs are either of a simple nature, a minor modification of UNITGRAF, or based on a library subprogram, and do not seem to require detailed explanation. UNITCOR takes the punched output of UNITGRAF and produces multiple linear regressions between storm and unitgraph parameters for preselected combinations of variables. UNITEST, a modified form of UNITGRAF, takes selected regression equations produced by UNITCOR, or fixed values of unitgraph parameters, and backtests the results against the original data. UNITSUM takes the punched output of this program or UNITGRAF, and lists a summary of storm data and overall performance indices by basin. UNITPEAK takes the same punched output and produces a statistical summary of parameters of the unitgraph function, both in original form and in the more familiar form of unitgraph peak discharge and time of peak. The UNITEST and UNITPEAK programs are limited to computations made with the Nash model.

TABLE 2. AREA-CONCENTRATION DATA

AREA, SQ. MILES

RIVER MILEAGE	OBER-LIN	DE-RIDDER	DE-SOTO	STEEL-VILLE	ROWELL	SPRING-FIELD	PAL-MYRA	UNION	BIG PINEY	WEST-PHALIA	MAHO-MET	MAT-THEWS
4.0	13.0	12.0				12.0	12.0					
7.9	37.0	43.0	36.5	17.7	21.7	31.1	28.0	22.0	27.0	26.0	29.0	34.0
11.9	86.0	82.0				52.3	59.0					
15.8	147.0	100.0	131.7	35.6	63.6	82.7	95.0	50.0	77.0	64.0	93.0	86.0
19.8	206.0	115.0				104.4	135.0					
23.7	280.0	120.0	267.0	83.8	107.2		191.0	73.0	144.0	106.0	166.0	135.0
27.7	373.0						224.0					
31.6	440.0		355.0	137.9	229.4		251.0	108.0	205.0	173.0	252.0	244.0
35.6	498.0						288.0					
39.5			461.0	239.6	323.6		308.0	140.0	300.0	226.0	343.0	
43.4							325.0					
47.3			490.1	372.3			356.0	173.0	386.0			
51.3							370.0					
55.2			555.6	472.2				184.0	467.0			
63.1			640.6	594.3				213.0	526.0			
71.0			717.1	639.2				291.0				
78.9				681.2				363.0				
86.8				713.9				429.0				
94.7				778.6				509.0				
102.6								596.0				
110.5								707.0				
118.4								783.0				
TOTALS												
DIST.	37.9	23.7	71.8	96.7	41.4	20.9	52.5	120.7	683.0	45.0	45.8	36.7
AREA	510.0	120.0	718.3	782.4	334.3	107.2	373.0	808.0	560.0	257.0	356.0	291.0

TABLE 3. GROUND WATER RECESSION COEFFICIENTS

	a	b
Oberlin	0.960	0
De Ridder	0.985	6.40×10^{-4}
De Soto	0.910	0
Steelville	1.000	7.14×10^{-5}
Rowell	1.000	1.00×10^{-4}
Springfield	1.036	7.25×10^{-4}
Union	1.000	2.05×10^{-4}
Palmyra	1.000	6.68×10^{-4}
Mahomet	0.974	9.80×10^{-4}
Matthews	1.000	1.19×10^{-3}
Big Piney	0.958	3.90×10^{-5}
Westphalia	0.926	3.34×10^{-4}

TABLE 4. RAINFALL-RUNOFF COEFFICIENTS

FT. WORTH RFC

	A	I	WN	WX	E1	E2	K	M	POW
Oberlin	3	10.5	8	33	0.55	0.70	0.75	20	1.25
De Ridder	3	10.5	8	33	0.55	0.70	0.75	20	1.25

KANSAS CITY RFC

	C1	C2	C3	C4	C5	C6
De Soto	.747791	.436800	3.531763	-.612710	-1.058400	.007526
Steelville	.747791	.436800	3.531763	-.612710	-1.058400	.007526
Rowell	.852832	.426400	3.305032	-.623867	-1.592893	.066308
Springfield	.852832	.426400	3.305032	-.623867	-1.592893	.066308
Union	.747791	.436800	3.531763	-.612710	-1.058400	.007526
Palmyra	.882700	.197200	3.316680	-.734400	-1.508000	.120000
Mahomet	.852832	.426400	3.305032	-.623867	-1.592893	.066308
Matthews	.852832	.426400	3.305032	-.623867	-1.592893	.066308
Big Piney	.747791	.436800	3.531763	-.612710	-1.058400	.007526
Westphalia	.747791	.436800	3.531763	-.612710	-1.058400	.007526

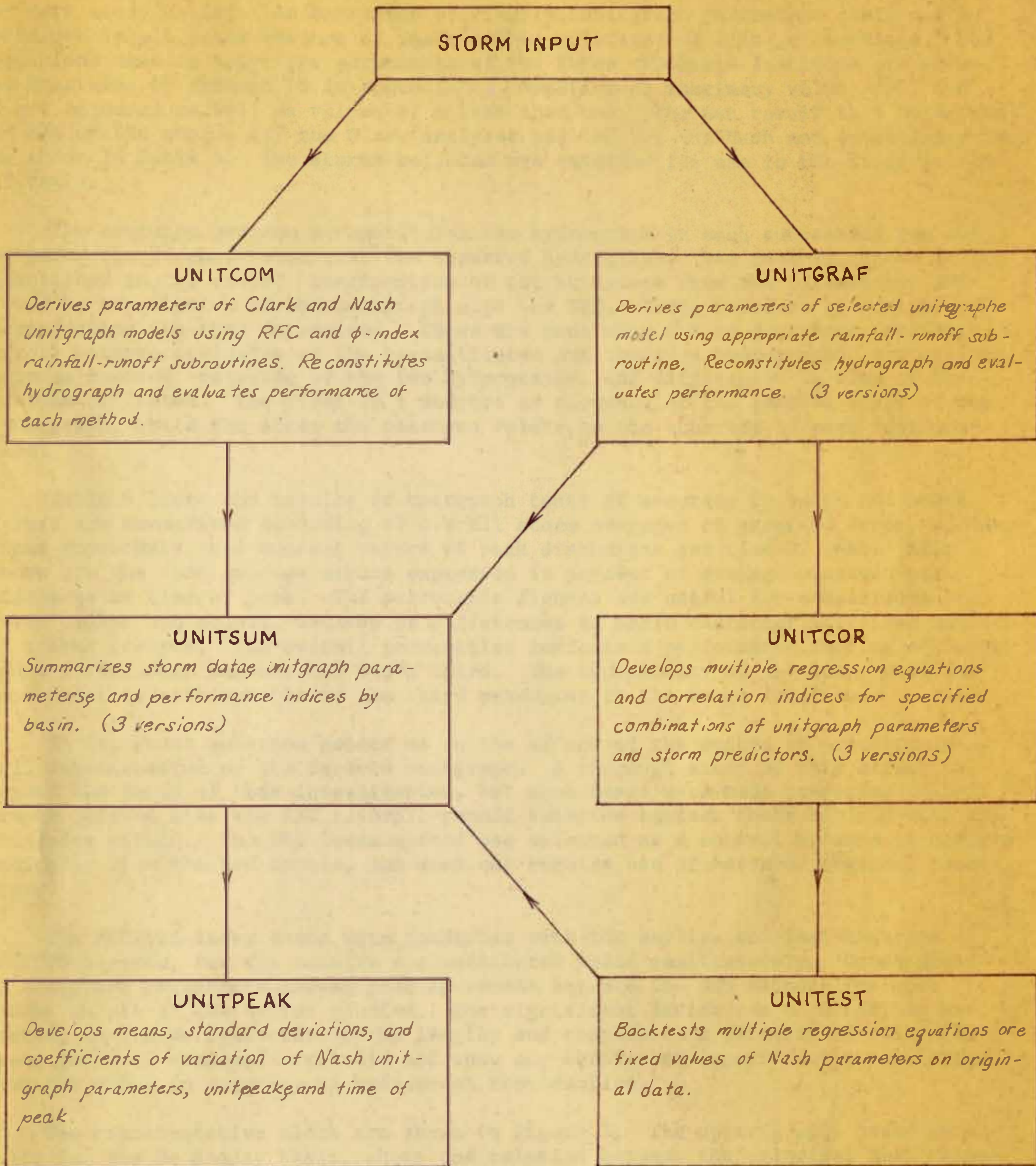

Figure 2. Schematic Diagram, Computer Processing

7. PRIMARY STORM ANALYSIS

A total number of 258 storms were available for processing through the six primary study basins. As mentioned previously, unitgraph parameters could not be obtained in all cases because of inadmissible solutions of linkage equations. The equations used to solve for parameters of the three unitgraph functions are shown as Equations 10 through 15 in Appendix 2. Negative or imaginary values of T and K are rejected as well as values of n less than one. The net result is a rejection of 27% of the storms for the Clark analyses and 14% for the Nash and Dooge analysis, as shown in Table 5. The storms rejected are retained for use in the final series of tests.

The computer program reconstitutes the hydrograph in each successful run and compares the reconstituted with the observed hydrograph. The reconstitution is accomplished in two steps: construction of the unitgraph from the parameters, followed by convolution of the unitgraph with the ERH. Three measures of quality of reproduction are then determined. These are root mean square departure (standard error) between ordinates of the reconstituted and observed hydrographs, differences between maximum ordinates of the two hydrographs, and differences in time of the maximum ordinates. The first is a measure of accuracy of the general shape of the hydrograph, while the other two measures relate to the accuracy of peak reproduction.

Table 6 lists the results of unitgraph tests of accuracy by basin and model. Errors are summarized according to overall storm averages of standard error, as defined previously, and average errors of peak discharges and time of peak. Also shown are the same average errors expressed in percent of average observed peak discharge or time of peak. The percentile figures are useful for comparisons between basins and models, because of differences in basin characteristics and number of storms treated. The overall percentiles indicate a performance ranking of Dooge model first, Nash second, and Clark third. The superiority of the Dooge over the Nash model is attributable to the third parameter in the Dooge function.

An important question arises as to the effect of the method of excess rainfall determination on the derived unitgraph. A thorough study of this effect is beyond the scope of this investigation, but some tests were made comparing unitgraphs derived with the RFC rainfall-runoff relation against those derived with the Phi-Index method. The Phi Index method was selected as a control because it differs radically from the RFC method, and does not require use of basin or regional parameters.

The RFC-Phi Index tests were conducted with the earlier and less accurate UNITCOM program, but the results are considered valid qualitatively. Cross plots of unitgraph parameters showed good agreement between the two methods for most storms in all of the basins studied. The significant deviations occurred, as expected, in storms characterized by lengthy and complex time patterns of rainfall. Cross plots of standard error did not show any consistent superiority of one method over the other in accuracy of hydrograph reproduction.

Two representative plots are shown in Figure 3. The upper graph, based on results for the De Ridder basin, shows the relation between the principal Nash parameter, K, obtained using the Ft. Worth RFC rainfall-runoff relation and that obtained using the Phi Index method. The lower graph shows the corresponding results in terms of standard error. The analysis of Storm 22, which produced the greatest difference in values of K between the two methods is plotted in Figure 4. This

TABLE 5. SUMMARY OF UNITGRAPH DERIVATIONS

BASIN	TOTAL NO. OF STORMS	NO. ANALYSED (1)	NO. ANALYSED (2)
Oberlin	38	32	36
De Ridder	51	30	37
De Soto	48	30	42
Steelville	45	29	38
Rowell	36	33	33
Springfield	40	34	36
Grand Total	258	188	222
	%	72.8	86.0

(1) Clark model
(2) Nash and Dooge models

TABLE 6. ERROR SUMMARY

PARA-METERS	DRAINAGE BASIN	NO. OF STORMS	PEAK DISCHARGE CFS	ERROR	%	TIME OF PEAK HRS	ERROR	%	STD. ERROR CFS	%
Clark	Oberlin	32	12487	3609	28.9	91	22	24.2	1909	15.3
	De Ridder	30	2273	478	21.0	51	6	11.8	286	12.6
	De Soto	30	15433	2114	13.7	26	3	11.5	1719	11.1
	Steelville	29	11383	1693	14.9	46	8	17.4	1622	14.3
	Rowell	33	4061	840	20.7	70	14	20.0	528	13.0
	Springfield	34	1545	315	20.4	31	8	25.8	197	12.8
	TOTAL	188	AVERAGE		19.9			18.5		13.2
Nash	Oberlin	36	12043	3433	28.5	93	20	21.5	1803	15.0
	De Ridder	37	2305	420	18.2	51	6	11.8	309	13.4
	De Soto	42	14243	2274	16.0	28	3	10.7	1657	11.6
	Steelville	38	12039	1449	12.0	45	8	17.8	1590	13.2
	Rowell	33	4061	856	21.1	70	16	22.9	477	11.7
	Springfield	36	1510	238	15.8	38	9	23.7	170	11.3
	TOTAL	222	AVERAGE		18.6			18.1		12.7
Dooge	Oberlin	36	12043	2889	24.0	93	21	22.6	1671	13.9
	De Ridder	37	2305	436	18.9	51	6	11.8	302	13.1
	De Soto	42	14243	1729	12.1	28	3	10.7	1510	10.6
	Steelville	38	12039	1352	11.2	45	8	17.8	1622	13.5
	Rowell	33	4061	639	15.7	70	16	22.9	451	11.1
	Springfield	36	1510	216	14.3	38	10	26.3	173	11.4
	TOTAL	222	AVERAGE		16.0			18.7		12.3

NOTE: Percentages are expressed in torms of peak discharge or time of peak

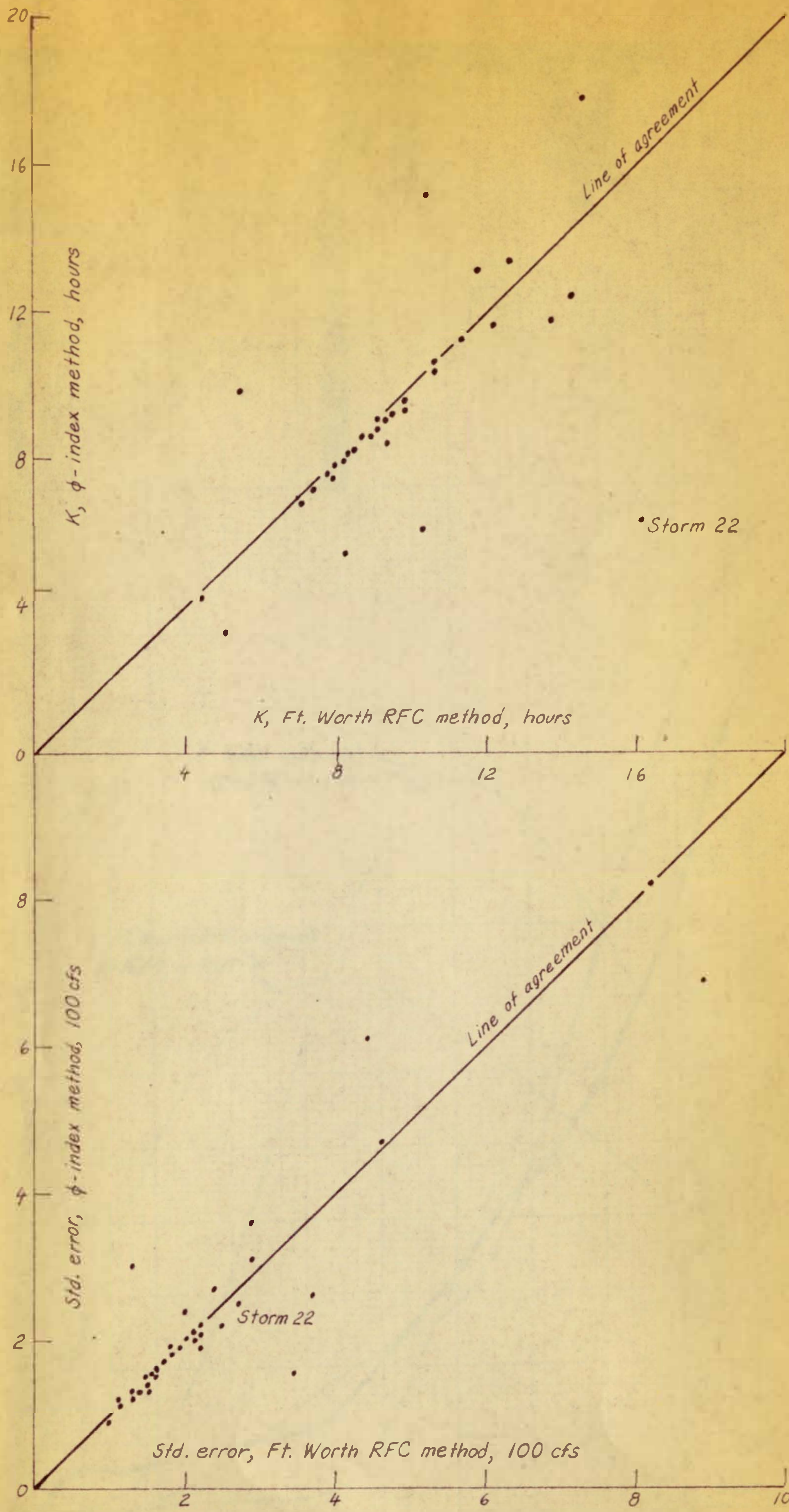

Figure 3. Comparative Results, RFC & Φ-Index Methods, De Ridder Basin

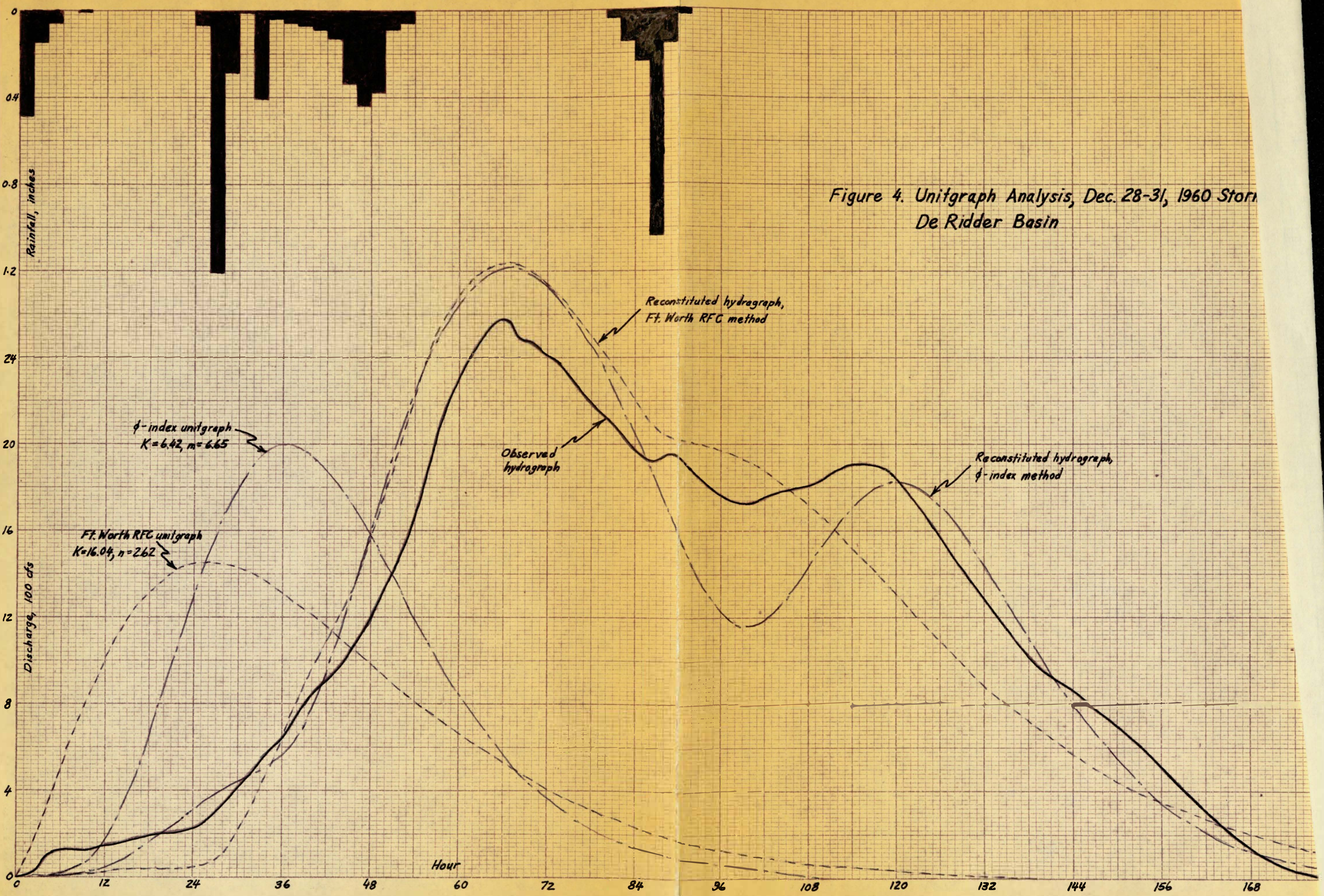

Figure 4. Unitgraph Analysis, Dec. 28-31, 1960 Storm
De Ridder Basin

storm lasted four days, and consisted of three periods of heavy rain separated by lulls of 18 and 24 hours. Figure 4 shows a large difference between unitgraphs for the two methods, but only a minor difference is evident in the reconstituted hydrographs. This illustrates the occasional ambiguity associated with unitgraph derivations because of the uncertainties of excess rainfall estimation.

8. CORRELATION ANALYSIS

A multiple regression analysis was undertaken to develop relations for predicting changes in unitgraph shape for a given basin under varying storm and antecedent conditions. Two important factors are recognized in selecting independent variables for the regressions: nonlinear response of runoff to rainfall, and seasonal and long-term changes in surface cover of the watershed. Causative variables selected to represent these factors are total volumes of storm rainfall and direct runoff, the first and second moments of excess rainfall distribution, year of storm occurence, and calendar date expressed in number of weeks since January 1, or week number. Since the effect is periodic, the actual terms used in the regression equation are the sine and cosine transformations of week number expressed as an angle in radians of the 52-week cycle. Standard practice is to use two sinusoidel terms because the angular phase relative to January 1 as well as the amplitude of the seasonal factor must be considered (10). The cross products of the transformed week number terms and the remaining variables are used as additional independent variables.

The basic dependent variables in the regression analysis are the K, T, and n parameters of the three unitgraph functions. Other dependent variables are formed from dimensionally compatible combinations of these parameters, such as K + T, nK, and T + nK. In addition, since the relationships might be curvilinear, logarithms of all these variables were also included, making a total of 12 dependent variables to be tested as possible predictands.

The 17 independent and 12 dependent variables described thus far provide a great variety of relationships that can be tested. These were reduced to a reasonable number of 384 per basin by selecting the groupings of variables shown in Table 7. The results of tests made with all of these equations for all basins would be difficult to present, and is unnecessary because many of the trials yielded insignificant correlations. Table 8 lists the values of the Fisher ratio (F) and coefficient of determination (R-square) of the best correlation attained for each basin and model for trials I and II, representing equations containing all the single and all the product independent variables, respectively. These results indicate good correlation in every case for at least one of the predictands. The confidence levels of these correlations, based on the values of F and degrees of freedom and utilizing Snedecor's tables (11), are 95% or higher.

The final step in the analysis is to obtain the optimum equations by testing a promising set of equations on the original data, and comparing the results in terms of the reconstituted hydrographs. This step is necessary for three reasons. One, the combination of equations yielding the highest individuals correlations of unitgraph parameters does not necessarily produce the most accurate hydrographs. Two, data not used in the development of the equations are available, and could affect the choice of the final equations. Three, it is desirable to check the performance of the variable unitgraph against that of the conventional fixed unitgraph.

Because of time limitations the final testing process was confined to the

TABLE 7. MULTIPLE REGRESSION PATTERNS

INDEPENDENT VARIABLES: SINGLE

TRIAL	X1	X2	X3	X4	X5	X6	X7
1	*	*	*	*	*	*	*
2		*	*	*	*	*	*
3		*	*		*	*	*
4		*		*	*	*	*
5		*			*	*	*
6		*				*	*
7		*	*	*	*		
8		*		*	*		
9		*		*			
10		*			*		

INDEPENDENT VARIABLES: PRODUCT

TRIAL	X1X6	X1X7	X2X6	X2X7	X3X6	X3X7	X4X6	X4X7	X5X6	X5X7
11	*	*	*	*	*	*	*	*	*	*
12			*	*	*	*	*	*	*	*
13			*	*	*	*			*	*
14			*	*			*	*	*	*
15			*	*					*	*
16			*	*						

DEPENDENT VARIABLES

	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10	Y11	Y12
Clark	*	*		*			*	*		*		
Nash	*		*		*		*		*		*	
Dooge	*	*	*	*	*	*	*	*	*	*	*	*

SYMBOLS

X1= Total storm rainfall
X2= " " runoff
X3= 1st moment rainfall excess
X4= 2nd ".. " "
X5= Year of storm occurrence
X6= Sin(Pi x Week No./26)
X7= Cos(Pi x Week No./26)

Y1= K
Y2= T
Y3= n
Y4= K + T
Y5= nK
Y6= T + nK

Y7= Log Y1
Y8= Log Y2
Y9= Log Y3
Y10= Log Y4
Y11= Log Y5
Y12= Log Y6

TABLE 8. BEST CORRELATIONS

BASIN	TRIAL	CLARK SYSTEM VAR	F	R-SQ	NASH SYSTEM VAR	F	R-SQ	DOOGE SYSTEM VAR	F	R-SQ
Oberlin	1	Y1	3.0	.465	Y1	3.7	.478	Y1	2.5	.387
	11	Y1	3.1	.597	Y1	4.7	.654	Y1	3.3	.566
De Ridder	1	Y8	4.7	.601	Y9	3.7	.474	Y6	4.8	.534
	11	Y2	3.2	.624	Y11	2.4	.483	Y12	2.2	.460
De Soto	1	Y8	13.4	.811	Y1	7.4	.603	Y1	3.4	.410
	11	Y8	12.4	.867	Y1	4.9	.614	Y12	2.5	.446
Steelville	1	Y1	3.1	.506	Y1	2.8	.393	Y7	2.9	.406
	11	Y1	3.2	.638	Y1	5.9	.687	Y1	6.8	.717
Rowell	1	Y2	2.5	.414	Y3	3.7	.510	Y1	2.8	.443
	11	Y7	3.2	.592	Y5	2.8	.561	Y5	7.9	.781
Springfield	1	Y1	2.1	.364	Y1	4.2	.513	Y10	4.7	.538
	11	Y7	2.4	.513	Y1	5.0	.667	Y1	5.4	.684

Nash unitgraph model and a selected small number of regression equations. The Nash model was used because it only involves computation of two parameters and is superior, as previously noted, in hydrograph reproduction and number of valid solutions to the other 2-parameter model. The correlation output was screened to determine the most consistently effective pairs of relations for determining parameters. The choice was narrowed to four combinations based on trials 2 and 12 in Table 8, each one omitting the total rainfall variable because of strong intercorrelation with runoff. Two of the combinations relate K and Log n separately to either six single variables or eight product variables. The correlation indices for these equations are listed in Table 9, and the corresponding regression coefficients are listed in Tables 10 and 11.

Two additional sets of relationships are formed by substituting a cross correlation between unitgraph parameters in place of the multiple regression involving n. The best overall cross correlation is that between log n and log K, as shown by the pairwise correlation coefficients listed in Table 12. The corresponding regression coefficients are shown in Table 13.

The final testing consists of a determination of hydrograph errors resulting from the use of the four different sets of regression equations just described, and the use of a constant unitgraph. The constant unitgraph can be derived by averaging the unitgraphs of all the storms studied or of selected major storms. The latter approach simulates the common procedure of selecting a small number of major storms for derivation of a representative unitgraph.

The results of the six variable and constant unitgraph tests are summarized in Table 14. The constant unitgraph tests are based on all-storm averages of parameters, and averages of parameters of the five maximum runoff-producing storms. Four measures of error are shown for each type of unitgraph and basin, expressed both in absolute values and percentages, as in Table 6. Three of these measures have been described previously. The fourth measure, labelled SE2, represents the standard error of the entire hydrograph record. Minimum values of error for each category and in each basin are underlined. It may be noted that the variable unitgraphs generally produce substantial reductions in peak discharge error, but the reductions in other categories of error are less clear-cut. Using peak discharge as a criterion the best overall system for predicing the shape of the unitgraph consists of the pair of six-variable equations listed as test no. 3 in Table 14.

Some of the predictors used in the optimum equations are more important than others. The coefficients of these equations, given in Table 10, are not indicative of relative importance because of differences between variables in scaling and variance. However, the relative weights of variables can be obtained by converting these coefficients into the dimensionless form of Beta coefficients (12). This has been done for the equation determining the principal parameter, K, after combining the two harmonic terms into a single week number term. The resultant series of five Beta coefficients are listed for each basin in Table 15. This table indicates the predominant importance of the first moment of excess rainfall, a variable representative of storm duration. The coefficients are all positive, signifying an increasing K, or flattening tendency of the unitgraph, with increasing length of storm. Coefficients of the annual term are consistently small, indicating little or no tendency toward long-term change of unitgraph shape. This is not unexpected considering that one criterion for basin selection was minimum man-made change in the basin environment.

TABLE 9. COEFFICIENTS OF DETERMINATION, EQUATIONS 2 AND 12, NASH MODEL

BASIN	EQUATION	K	Log K	n	Log n
Oberlin	2	.473	.326	.111	.303
	12	.580	.425	.154	.328
De Ridder	2	.456	.321	.243	.471
	12	.178	.151	.154	.248
De Soto	2	.600	.520	.333	.398
	12	.609	.500	.373	.474
Steelville	2	.389	.362	.301	.386
	12	.588	.482	.413	.494
Rowell	2	.415	.438	.508	.458
	12	.311	.404	.386	.369
Springfield	2	.511	.486	.092	.178
	12	.608	.575	.195	.337

TABLE 10. REGRESSION COEFFICIENTS, EQUATION 2, NASH MODEL

PARAMETER	BASIN	b_0	b_1	b_2	b_3	b_4	b_5	b_6
K	Oberlin	20.7	-1.11	.386	-.0135	-.0710	4.28	-1.18
	De Ridder	4.8	0.91	.297	-.0229	-.0014	2.07	2.28
	De Soto	5.0	3.21	.057	-.0134	-.0361	0.40	-0.14
	Steelville	10.2	0.23	.171	.0155	-.0848	-1.02	-0.44
	Rowell	10.4	0.88	.463	-.0297	.0640	0.30	-0.11
	Springfield	7.7	0.91	.431	-.0114	.0193	1.94	2.14
Log n	Oberlin	.292	.014	-.0077	.0003	.0059	-.079	.051
	De Ridder	.828	-.026	-.0184	.0011	.0006	-.139	-.115
	De Soto	.507	-.154	-.0087	.0021	.0037	.008	.063
	Steelville	.407	.064	-.0160	-.0002	.0051	.070	.066
	Rowell	.282	-.081	-.0076	-.0009	.0044	.090	-.008
	Springfield	-.216	.029	-.0150	.0006	.0109	-.081	-.022

TABLE 11, REGRESSION COEFFICIENTS,
EQUATION 12, NASH MODEL

PARA-METER	BASIN	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
K	Oberlin	18.9	-3.11	-1.86	.683	0.35	-.0000	-.0171	-.0036	-.013
	De Ridder	10.1	1.35	1.71	.097	0.01	-.0050	-.0204	-.0375	-.029
	De Soto	6.1	1.46	-2.59	.422	-0.05	-.0466	.0033	-.0405	.037
	Steelville	6.4	-.050	0.40	-.247	0.25	.0415	-.0931	.0136	-.024
	Rowell	18.3	0.38	2.34	.009	-0.36	-.0194	.0429	.0579	-.024
	Springfield	13.7	4.73	2.24	-.604	-1.16	.0381	.0481	.0315	.123
Log n	Oberlin	.591	.047	.046	-.0143	-.0060	.0003	.0003	.0004	.0000
	De Ridder	.569	-.054	-.066	-.0154	.0058	.0007	.0003	.0029	.0006
	De Soto	.545	-.124	.075	-.0298	.0094	.0019	-.0033	.0045	.0002
	Steelville	.640	.205	-.243	.0042	-.0027	-.0021	.0063	-.0009	.0033
	Rowell	.438	-.074	.001	-.0019	.0059	.0007	-.0013	.0014	.0001
	Springfield	.401	-.196	.048	.0260	.0384	-.0021	-.0028	-.0009	-.0038

TABLE 12, CROSS-CORRELATION COEFFICIENTS,
K AND n PARAMETERS

	OBER-LIN	DE RIDDER	DE SOTO	STEEL-VILLE	ROWELL	SPRING-FIELD
n vs. K	-.646	-.775	-.724	-.811	-.694	-.415
n vs. Log K	-.828	-.857	-.807	-.912	-.703	-.524
Log n vs. K	-.891	-.841	-.833	-.883	-.741	-.533
Log n vs. Log K	-.905	-.819	-.843	-.931	-.684	-.585

TABLE 13, REGRESSION COEFFICIENTS,
K-n EQUATIONS

BASIN	a	b
Oberlin	1.680	-.904
De Ridder	1.530	-.986
De Soto	1.278	-.934
Steelville	1.522	-1.090
Rowell	1.297	-.685
Springfield	1.133	-.690

TABLE 14, BACKTESTS, FINAL SET OF EQUATIONS

BASIN	TEST	K	n	ERQP	%	ERTP	%	SE1	%	SE2	%
Oberlin	1	20.22	4.55	4930	41.6	29.0	28.7	3025	25.5	6480	54.7
	2	14.56	3.86	4547	38.4	25.6	25.4	2698	22.8	4400	37.2
N= 38	3	v	v	2638	22.5	25.3	25.1	1854	15.7	2660	22.5
QP= 11833	4	v	v	3972	33.6	28.7	28.4	2283	19.3	3490	29.5
TP= 101	5	v	v	3348	27.4	25.1	24.9	2644	22.3	5640	47.6
	6	v	v	4127	34.8	25.7	25.5	2708	22.9	5250	44.4
De Ridder	1	9.95	4.27	433	18.9	12.0	21.8	383	16.8	588	25.7
	2	12.02	3.62	479	21.0	12.3	22.4	373	16.3	562	24.6
N= 50	3	v	v	444	19.4	11.8	21.4	420	18.4	588	25.7
QP= 2285	4	v	v	600	26.2	11.4	20.7	426	18.6	715	31.3
TP= 55	5	v	v	393	17.2	12.2	22.2	412	18.0	516	22.6
	6	v	v	630	27.6	11.6	21.1	436	19.1	583	25.5
De Soto	1	6.43	3.80	2364	16.5	5.0	16.1	2291	16.0	3080	21.6
	2	9.57	2.58	2457	17.2	5.0	16.1	2085	14.6	2520	17.6
N= 49	3	v	v	2264	15.8	4.8	15.5	1986	13.9	2420	16.9
QP= 14295	4	v	v	3320	23.3	5.0	16.1	2335	16.3	3510	24.6
TP= 31	5	v	v	2730	19.1	4.7	15.2	2241	15.7	2550	17.9
	6	v	v	2938	20.6	4.5	14.5	2383	16.7	2880	20.2
Steelville	1	6.97	4.80	2427	20.3	7.0	15.2	1732	14.5	2040	17.1
	2	6.75	5.73	2816	23.6	6.0	13.0	2248	18.8	2510	21.0
N= 43	3	v	v	1976	16.5	9.7	21.1	1834	15.3	2240	18.7
QP= 11958	4	v	v	2122	17.8	9.3	20.2	2125	17.8	2680	22.4
TP= 46	5	v	v	1796	15.0	10.6	23.0	2038	17.1	2440	20.4
	6	v	v	1874	15.7	10.7	23.2	2198	18.4	2650	22.2
Rowell	1	18.97	3.09	1297	31.5	26.0	37.1	777	18.9	1072	26.1
	2	19.48	2.62	1173	28.5	27.5	39.3	740	18.0	1182	28.8
N= 36	3	v	v	1084	26.4	26.9	38.4	686	16.7	882	21.4
QP= 4114	4	v	v	888	21.6	26.7	38.1	691	16.8	903	21.9
TP= 70	5	v	v	1210	29.4	27.2	38.8	754	18.3	1001	24.3
	6	v	v	1216	29.6	27.4	39.1	767	18.6	1025	24.9
Springfield	1	12.89	2.93	463	31.0	21.0	51.2	330	22.1	396	26.5
	2	12.40	1.93	452	30.3	17.8	43.4	289	19.3	347	23.2
N= 40	3	v	v	392	26.2	20.6	50.3	289	19.3	354	23.7
QP= 1493	4	v	v	385	25.8	18.8	45.8	289	19.3	371	24.8
TP= 41	5	v	v	403	27.0	20.2	49.3	271	18.1	323	21.6
	6	v	v	399	26.7	20.2	49.3	271	18.1	330	22.1

TABLE 14, (CONTINUED)

Meaning of symbols:

N= Number of storms
QP= Average peak discharge in cfs
TP= Average time of peak in hours
ERQP= Average error in peak discharge in cfs
ERTP= Average error in time of peak in hours
SE1= Unweighted average of standard errors in cfs
SE2= Standard error of ordinates of entire record analysed in cfs
v= Variable

Description of tests:

1. Fixed K, n based on average of all storms
2. Fixed K, n based on average of five maximum runoff-producing storms
3. K, Log n based on equation 2.
4. K, Log n based on equation 12.
5. K based on equation 2, Log n related to Log K
6. K based on equation 12, Log n related to Log K

TABLE 15. BETA COEFFICIENTS, EQUATION 2, PARAMETER K

PREDICTOR	OBER-LIN	DE RIDDER	DE SOTO	STEEL-VILLE	ROWELL	SPRING-FIELD
Total Storm Runoff	-.224	.258	.730	.038	.096	.224
1st Moment Excess Rainfall	.628	.857	.107	.328	.698	.614
2nd Moment Excess Rainfall	-.275	-.800	-.132	.193	-.580	-.379
Year of Storm Occurrence	-.030	-.001	-.088	-.179	.049	.014
Week Number	.285	.443	.096	-.208	.043	.372

TABLE 16. UNITGRAPH PARAMETER STATISTICS

SD= Standard Deviation CV= Coefficient of Variation

BASIN	NO. OF STORMS	K, HRS. MEAN	K, HRS. SD	K, HRS. %CV	n MEAN	n SD	n %CV
Oberlin, La.	36	20.22	10.88	53.8	4.55	3.75	82.4
De Ridder, La.	37	9.96	3.82	38.4	4.27	2.24	52.5
De Soto, Mo.	42	6.43	2.58	40.0	3.80	1.44	38.0
Steelville, Mo.	38	6.98	3.09	44.2	4.80	2.06	42.9
Rowell, Ill.	33	18.98	7.48	39.4	3.09	1.49	48.2
Springfield, Ill.	36	12.90	5.01	38.8	2.93	2.59	88.3
Union, Mo.	13	8.47	3.60	42.5	7.44	3.64	49.0
Palmyra, Mo.	8	6.41	2.59	40.4	3.10	1.17	37.9
Mahomet, Ill.	9	18.97	5.35	28.2	2.83	0.61	21.4
Matthews, Ill.	17	12.25	3.89	31.7	5.14	1.39	27.0
Big Piney, Mo.	8	6.95	3.14	45.2	5.13	1.81	35.3
Westphalia, Mo.	10	5.28	1.55	29.4	3.21	1.00	31.1

BASIN	PEAK, CFS MEAN	PEAK, CFS SD	PEAK, CFS %CV	LAG, HRS. MEAN	LAG, HRS. SD	LAG, HRS. %CV
Oberlin, La.	4931	2108	42.7	46.2	20.0	43.4
De Ridder, La.	2120	867	40.9	26.1	9.3	35.6
De Soto, Mo.	19396	4111	21.2	15.4	4.7	30.9
Steelville, Mo.	17139	2616	15.3	21.5	5.7	26.6
Rowell, Ill.	3914	2171	55.5	32.2	13.8	42.9
Springfield, Ill.	2022	689	34.1	19.7	17.8	90.6
Union, Mo.	11270	2377	21.1	44.3	5.5	12.4
Palmyra, Mo.	11944	3371	28.1	11.0	3.4	30.6
Mahomet, Ill.	3708	845	22.8	32.5	7.4	22.8
Matthews, Ill.	3246	852	26.2	46.9	11.4	24.3
Big Piney, Mo.	11722	1535	13.1	23.8	7.3	30.5
Westphalia, Mo.	8926	1663	18.6	10.7	3.6	33.8

9. VARIABILITY ANALYSIS

The data obtained from the Nash unitgraph analyses of both primary and secondary study basins were used to determine means, standard deviations, and co-efficients of variation of the K and n parameters. This statistical analysis was extended to the more familiar unitgraph characteristics of peak discharge and time of peak, utilizing Equations ~~1~~2 and 3 to convert from one form of parameters to the other. The resultant statistics are presented for each of the 12 basins in Table 16. Using peak discharge as a criterion there is considerable variation in unitgraph shape between basins, the Missouri group showing the greatest peakedness. The variability factor, as evidenced by the coefficient of variation of unit peak, also differs greatly between basins, and shows an inverse tendency with unitpeak. This indicates that the unitgraph tends to be more stable in watersheds characterized by higher peak discharges.

A study was made of possible interrelationships between unitgraph parameter statistics and physical basin factors. Snyder (13) and Taylor-Schwarz (14) factors were determined for each basin from measurements made from topographic charts. These measurements, listed in Table 17, include lengths of the main channel from the outlet to the divide, L, and from the outlet to the centroid of the basin, LC, and the weighted mean slope of the main channel, S. A composite term, $A/(LxLC)^{.3}$, used to derive the Snyder synthetic unitgraph, is also listed. Plots of this term against mean unitpeak, Figure 5, show a wide scatter that cannot be explained by variations of the Taylor-Schwarz slope factor. However, straight lines can be fitted satisfactorily to two sets of points, one set representing the Missouri basins, and the other the basins in Illinois and Louisiana. As was noted previously, these groups correspond to respective areas of relatively steep and flat topography. It appears from these data that the general topography has a more important bearing on peak discharge characteristics than the slope of the main channel. Unitgraph peak discharges per square mile and lag plotted in Figure 6 show the well-known inverse relationship.

Attempts were made to correlate variability of unitgraph parameters, as defined by standard deviation and coefficient of variation, to physical basin factors, but no definite relationships were discovered. However, when the standard deviation is plotted against the mean value of the unitpeak, as in Figure 7, a trend line is indicated. This line can be converted to a safety factor curve defining the probable limits of error of a peak discharge computation based on the use of a constant unitgraph. Such a curve, calculated from twice the standard deviation and shown in Figure 7, defines the upper limit of error of a peak discharge computation in terms of 95% probability. The range of error covers all influences producing variations of the unitgraph, including storm factors, seasonal and long-term effects, and normal data errors. Thus, the curve can only be a rough approximation because these influences will vary from basin to basin.

10. CONCLUSIONS

Conclusions based on results of analysis to date of 12 study basins in Missouri, Illinois and Louisiana are as follows:

1. The 2-parameter Nash function provides a satisfactory fit of the unit hydorgraph, and is superior to the Clark function.

2. The shapes of unitgraphs derived from storm data are affected considerably at times by the method used in determining the time distribution of excess rainfall.

TABLE 17. PHYSICAL BASIN CHARACTERISTICS

BASIN	L MILES	L_c MILES	A SQ. MI.	S FT./MI.	$A/(LL_c)^{.3}$
Oberlin, La.	37.9	22.1	510	6.12	67.8
De Ridder, La.	23.7	9.5	120	6.36	23.7
De Soto, Mo.	71.8	50.9	718	5.54	61.4
Steelville, Mo.	96.7	51.4	781	7.33	61.1
Rowell, Ill.	41.4	27.6	334	3.48	40.5
Springfield, Ill.	20.9	11.0	107	5.52	20.9
Union, Mo.	120.7	86.9	808	3.38	50.2
Palmyra, Mo.	52.5	25.7	373	6.44	42.9
Mahomet, Ill.	45.8	23.7	356	2.90	43.8
Matthews, Ill.	36.7	22.5	291	3.22	38.8
Big Piney, Mo.	68.3	36.7	560	6.49	53.3
Westphalia, Mo.	45.0	26.1	257	9.18	30.8

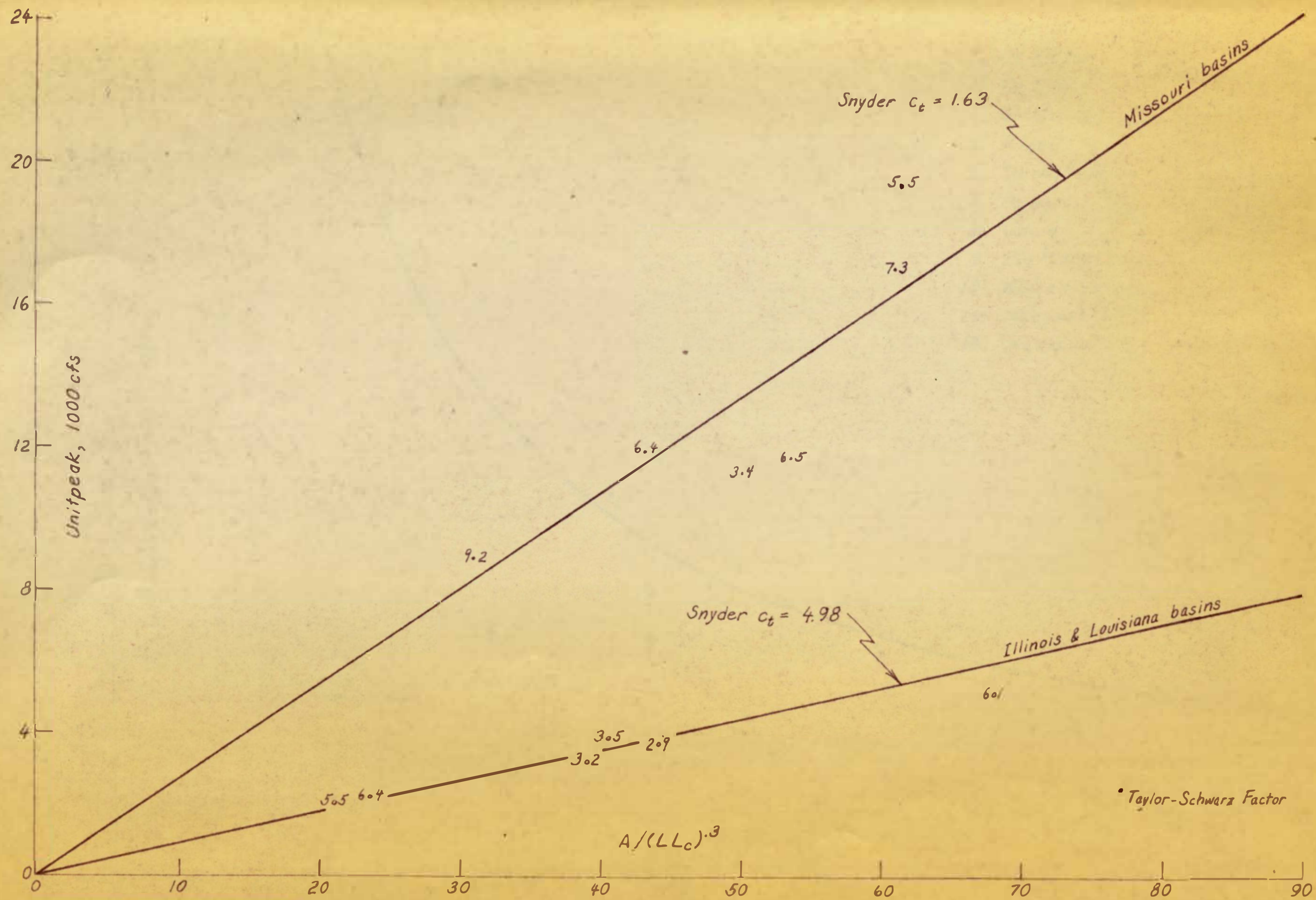

Figure 5. Effect of Physical Characteristics on Unitpeak

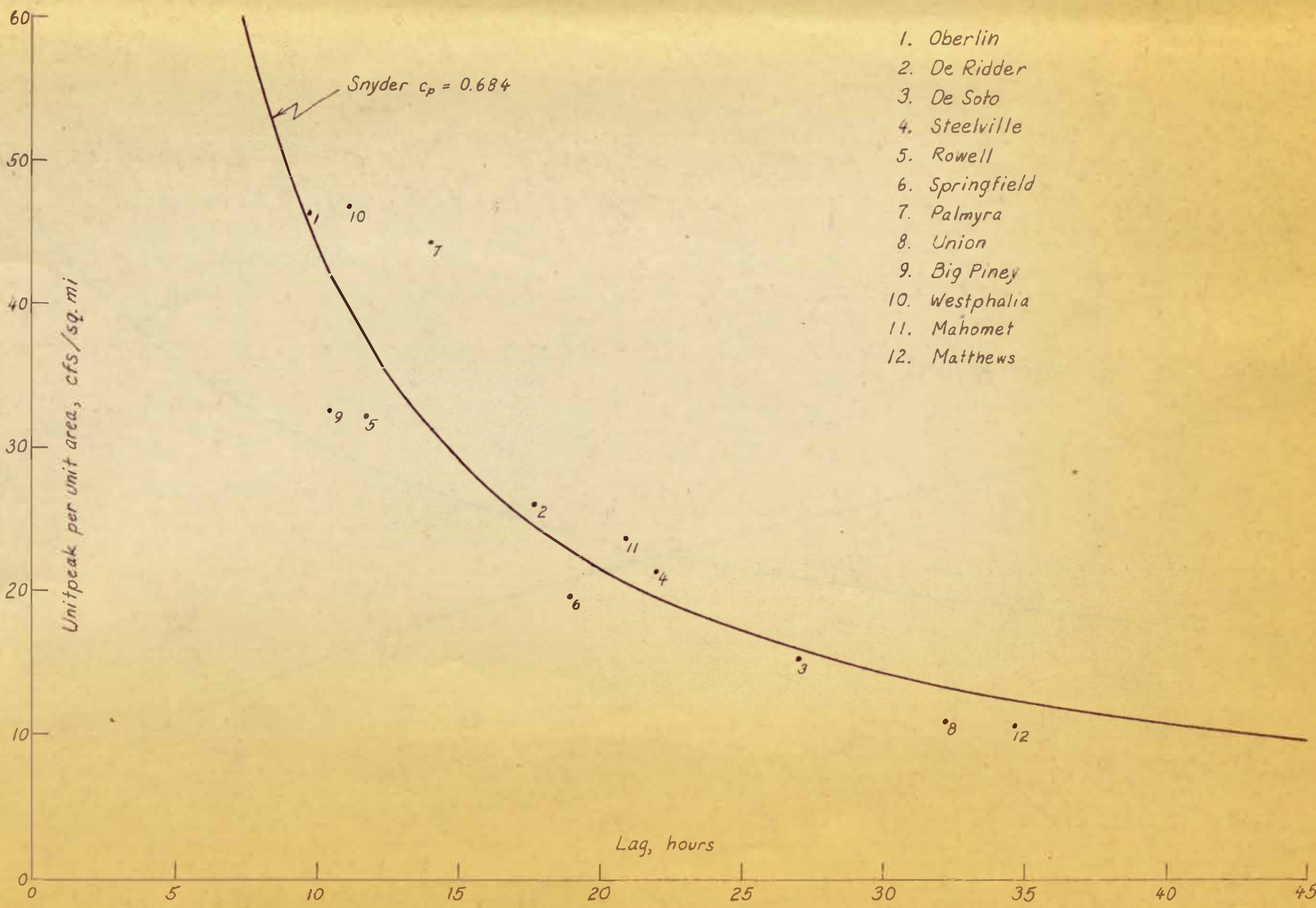

Figure 6. Unitpeak - Lag Relation

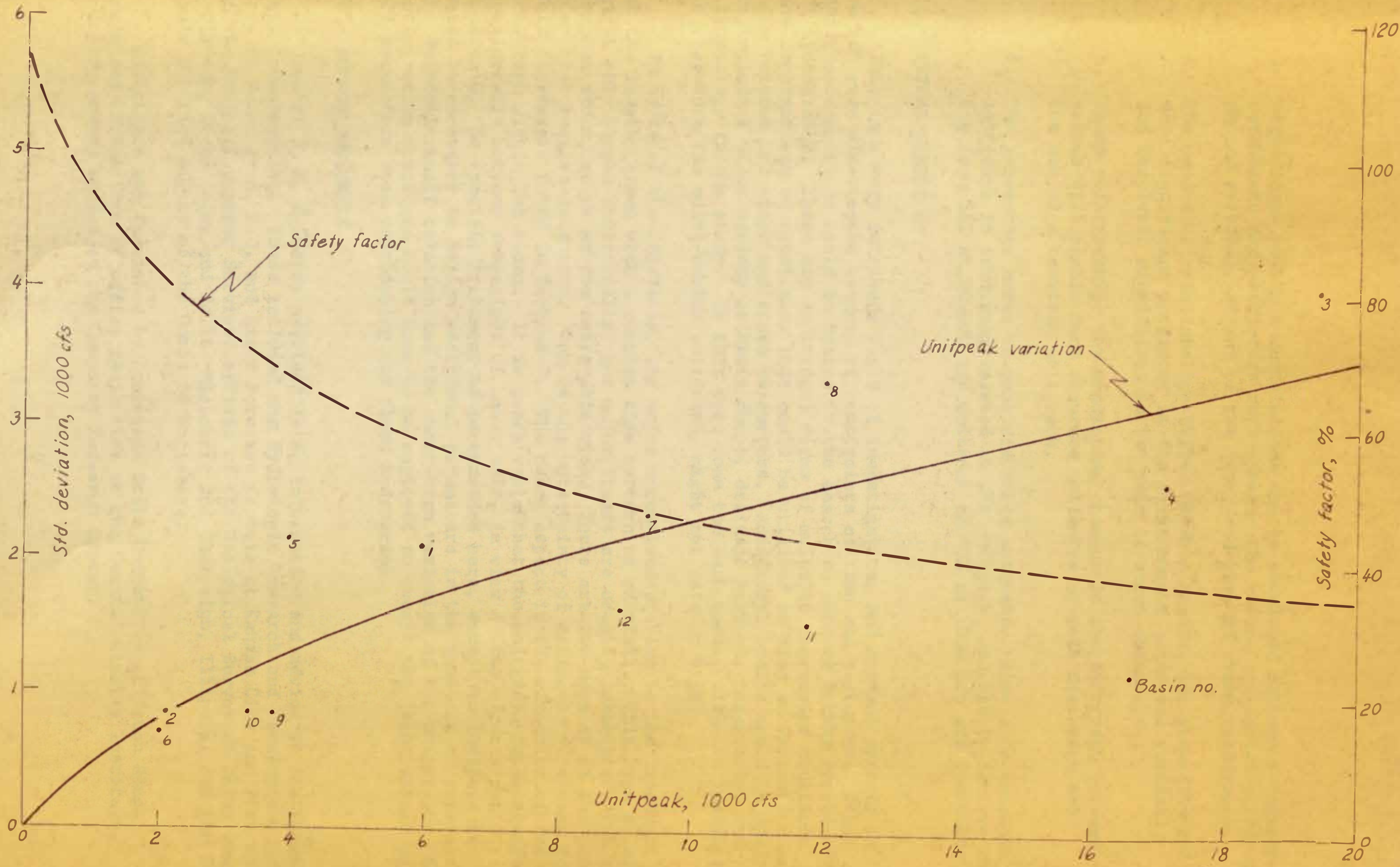

Figure 7. Unitpeak Variation and Reliability

3. Significant multiple correlations can be developed between unitgraph parameters and predictors representing the volume and distribution of excess rainfall, season of the year, and year of storm occurrence.

4. The parameter relationships differ between basins, but generally the most significant predictor is the first moment of excess rainfall, and the least significant is the year of storm occurrence.

5. These relationships for predicting changes in the unitgraph between storms will enable more accurate estimates of peak discharge than the use of a constant unitgraph.

6. The percentage error in peak discharge estimates based on a constant unitgraph is inversely related to the unitpeak, and safety factors ranging from 35% at 20,000 cfs unitpeak to 90% at 1000 cfs are indicated.

11. FUTURE RESEARCH

This is a very intricate field of investigation, and several avenues of research were sidestepped because of constraints of time and resources. Other unitgraph functions could be tested in the search to achieve better reproduction of hydrographs. There are additional forms of multiple regression equations and combinations of predictors that could be utilized to find an improved correlation between unitgraph and storm parameters. Additional basins could be analysed to include a wider variety of basin shape, drainage pattern, topography, and climatology in the study. If this were done the Nash model, with its inability to reproduce the multi-peaked unitgraph, might not rate so high.

In spite of the aforesaid, the major unresolved problem is the treatment of the lengthy storm with a complex time pattern of rainfall. This type of storm might only happen occasionally, but major floods are usually associated with a complex storm, so it merits extra attention. There are two aspects of the problem that require attention. One is the uncertainty of estimates of excess rainfall increments noted in Section 7. The other aspect is the variation of the unitgraph within the storm. It is conceivable that the unitgraph could change considerably between subperiods of heavy rain, in view of the effects noted in Section 8. No specific treatment is recommended here, except to indicate it would be necessary to assign additional parameters in the analysis to represent the rainfall-runoff relation and the short-term variation of the unitgraph. An optimization procedure would have to be employed to obtain the joint values of all parameters best reproducing the flood hydrograph.

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Assistance was furnished by Louisiana State University at Baton Rouge, Louisiana in the form of office facilities at the Coastal Studies Institute, and programming advice at the Computer Research Center.

APPENDIX 1. REFERENCES

1. Laurenson, E.M. and O'Donnell, T. Data Error Effects in Unit Hydrograph Derivation, Proc. ASCE 6887, Nov. 1969

2. Clark, C.O. Storage and the Unit Hydrograph, Trans. ASCE, Vol. 110, pp. 1419-1488, 1945

3. Nash, J.E. The Form of the Instantane ous Unit Hydrograph, IASH Pub. 45, Vol. 3, pp 114-121, 1957

4. Nash, J.E. Systematic Determination of Unit Hydrograph Parameters, JGR, Vol. 64, No. 1, Jan. 1959

5. Dooge, J.C.I. A General Theory of the Unit Hydrograph, JGR, Vol. 64, No. 2, Feb. 1959

6. Diskin, M.H. A Laplace Transform Proof of the Theorem of Moments for the Instantaneous Unit Hydrograph, WRR, Vol. 3, No. 2, 1967

7. Diskin, M.H. and Boneh, A. Moments of the Input, Output, and Impulse Response Moments of Linear Systems about Arbitrary Points, WRR, Vol. 4, No. 4, Aug. 1969

8. Hydrologic Engineering Center. Unit Hydrograph and Loss Rate Optimization, U.S. Army Engineer District Bulletin, Sacramento, Calif., Aug. 1966

9. Snyder, W.M. Hydrograph Analysis by the Method of Least Squares, Proc. ASCE, Vol. 81, Separate no. 793, Sept. 1955

10. Yevjevich, V. Stochastic Processes in Hydrology, pp 14-15, Water Resources Publications, Ft. Collins, Colo., 1972

11. Snedecor, G.W. Statistical Methods, Iowa State College Press, 5th ed. 1956

12. Chow, V.T. Handbook of Applied Hydrology, Section 8-II, McGraw-Hill Book Co., 1964

13. Snyder, F.F. Synthetic Unit Hydrographs, Trans. AGU, Vol. 19, Pt. 1, pp 447-454, 1938

14. Taylor, A.B. and Schwarz, H.E. Unit-Hydrograph Lag and Peak Flow Related to Basin Characteristics, Trans. AGU, Vol. 33, pp 235-246, April 1952

APPENDIX 2. EQUATIONS

Nash unitgraph function:

$$q(t)= (V/K\Gamma(n))(t/K)^{n-1}\exp(-t/K) \qquad (1)$$

Peak discharge, Nash function:

$$q_p= (V/K\Gamma(n))(n-1)^{n-1}\exp(1-n) \qquad (2)$$

Time of peak, Nash function:

$$t_p= (n-1)K \qquad (3)$$

Moment equations:

$$U_1'= Q_1'- P_1' \qquad (4)$$

$$U_2 = Q_2- P_2 \qquad (5)$$

$$U_3 = Q_3- P_3 \qquad (6)$$

Linkage equations, Dooge model:

$$U_1'= A_1'T + nK \qquad (7)$$

$$U_2 = A_2T^2 + nK^2 \qquad (8)$$

$$U_3 = A_3T^3 + 2nK^3 \qquad (9)$$

Note: substitute n= 1 for Clark model and T= 0 for Nash model

Solution of Clark equations:

$$T= (U_1'A_1' \pm \sqrt{U_2A_2'- U_1'^2A_2})/A_2' \qquad (10)$$

$$K= U_1'- A_1'T \qquad (11)$$

Solution of Nash equations:

$$K= U_2/U_1' \qquad (12)$$

$$n= U_1'/K \qquad (13)$$

APPENDIX 3. NOTATION

a= ground-water recession coefficient
A_r= r^{th} moment of area-concentration curve with respect to c.m.
A_r'= r^{th} moment of area-concentration curve with respect to origin.
b= ground-water recession coefficient
K= storage-discharge ratio
n= number of reservoirs in series in Nash and Dooge models
P_r= r^{th} moment of excess rainfall hyetograph with respect to c.m.
P_r'= r^{th} moment of excess rainfall hyetograph with respect to origin
q= discharge ordinate of unitgraph
q_p= peak discharge of unitgraph
q_o= discharge of recession segment at zero hour
q_t= discharge of recession segment t hours later
Q_r= r^{th} moment of direct runoff hydrograph with respect to c.m.
Q_r'= r^{th} moment of direct runoff hydrograph with respect to origin
t= time from beginning of direct runoff
t_p= time of peak discharge of unitgraph
T= total translation time in Clark and Dooge models
U_r= r^{th} moment of unitgraph with respect to c.m.
U_r'= r^{th} moment of unitgraph with respect to origin
V= volume of runoff per unit depth
Γ= Gamma function

Approximate solution of Dooge equations:

$$K = (U_2 - A_2T^2)/(U_1' - A_1'T) \qquad (14)$$

$$n = (U_1' - A_1'T)/K \qquad (15)$$

Note: use iterative process increasing T successively starting with T= 0, and solving for K and n at each trial. Select values of T, K, and n minimizing differences between third moments computed by equations (6) and (9).

Normal ground-water recession:

$$q_t/q_o = (a - bq_o)^{t/12} \qquad (16)$$

APPENDIX 4. ABSTRACT: PROGRAM UNITGRAF

GENERAL DESCRIPTION

Derives unit hydrograph parameters for each storm from rainfall, gage-height, and cartographic data, using the RFC rainfall-runoff relation appropriate to the basin. Reconstitutes the direct runoff hydrograph and evaluates the performance of the surface runoff model employed in the analysis. Three versions are available, depending on whether the Clark, Nash, or Dooge model is employed. Results are assembled by basin, and the process repeated for a series of basins.

FACILITY

Fortran IV adapted to IBM 360 Model 65 complex located at the Louisiana State University Computer Research Center in Baton Rouge, Louisiana.

INPUT

Punchcards are assembled first by storm, then for a series of storms by basin preceded by header cards, and finally the basin decks are combined to form the input deck. The basin headers consist of the following cards:

1. Basin identification, control number for fixed or variable rating table, control number for rainfall-runoff method, and number of rainfall-runoff coefficients.
2. Rainfall-runoff coefficients
3. Fixed rating table (optional-same form as storm card no. 4)
4. Area-concentration table
5. Ground-water recession coefficients.

Each storm deck consists of the following cards:

1. Month, day, and year of peak discharge, storm number, discharge unit, control number for method of base-flow separation, time unit
2. Number of recording raingages, total basin rainfall, number of rain intervals at each gage
3. Hour of beginning and ending of rain and amount of rain each hour, repeated for each time interval at each gage
4. Number of points selected on rating curve, and gage-height and discharge at each point
5. Number of tabulated points on hydrograph, and corresponding gage-height and discharge at each point.
6. Number of points on base-flow hydrograph, and corresponding gage-height and discharge at each point.

EXECUTION (Main Program)

The basin hyetograph is computed from station-weighted averages of hourly rainfall adjusted to agree with total storm precipitation. The total gage-height hydrograph is converted to a discharge hydrograph using linear interpolation between tabulated levels of the rating table. Base flow is deducted from the discharge hydrograph to obtain the direct runoff hydrograph (DRH) using the pre-selected automatic or manual separation method. The automatic method applies the ground-water recession formula from the initial point of the hydrograph to the time of peak, and a straight-line connection from that point to the endpoint of the hydrograph. Manual base flow separation is obtained from the base flow gage-height input and application of the rating table.

Rainfall-runoff subroutines are used to derive the excess rainfall hyetograph (ERH) from the basin hyetograph. These subroutines require input of week number and volume of direct runoff as well as the basin hyetograph. Week number is obtained from storm date by computing the number of weeks since January 1, taking account of leap year. Runoff volume is obtained by summation of the DRH and use of a formula involving drainage area and the units of time and discharge.

The first three moments of the ERH, DRH, and area-concentration curve are computed, and used to determine the corresponding moments, parameters, and form of the unitgraph. The methods and formulas applied vary between models, as follows:

Clark version: Equations (10) and (11) of Appendix 2 are used to compute the T and K parameters. Two sets of values are obtained, and negative or imaginary values are rejected. If both sets are valid solutions, the set yielding the closest approximation to the third moment is accepted. If both sets are invalid, a notice to that effect is printed, and execution branches to the next storm. The unit inflow hydrograph is computed from the area-concentration curve and the parameter, T, and routed through storage using the parameter, K, to derive the unitgraph.

Nash version: Equations (12) and (13) of Appendix 2 are used to compute the K and n parameters. Only one set of parameters is obtained, and solutions involving zero or negative K or n less than one are rejected. Equation (1) is used to derive the unitgraph from the two parameters.

Dooge version: Trial values of K and n are obtained using Equations (12) and (13) of Appendix 2, and invalid solutions rejected as before. A search is made for that value of T yielding the closest approximation of the third moment when substituted successively into Equations (14), (15), and (9). The unit inflow hydrograph is derived in the same manner as the Clark version, utilizing the parameter, T. The unitgraph is obtained by convoluting this hydrograph with the function derived by substituting K and n into Equation (1).

The remainder of the program consists of reconstituting the DRH by convoluting the ERH with the unitgraph, and testing the reconstituted against the observed DRH.

Three measures of error are derived for each storm: mean root square residual between the two hydrographs, labelled standard error, error in peak discharge, and error in time of peak. A Fourth measure is derived through a computation of the error variance for the totality of hydrographs of all storms analysed for the basin.

EXECUTION (Subroutines)

Two alternate subroutines, ROKANS and ROFORT, are available for calculating the ERH, depending on whether the basin of interest is located in the Kansas City or Ft. Worth RFC region. Each subroutine receives data from the main program of rainfall-runoff coefficients pertaining to the basin, and the week number, total direct runoff, and hyetograph data pertaining to the storm. In turn the subroutine, utilizing a search procedure, develops the values of the ERH satisfying the data and the rainfall-runoff equations, and supplies those values to the main program. Standard rules for computing API and duration are followed.

OUTPUT

Output consists of printed listings and punchcards. The first page of the print-out identifies the basin and the rainfall-runoff method, and lists the coefficients of the rainfall-runoff and ground-water recession equations and the area-concentration table. This is followed by a series of pages listing the essential storm data and results of storm analyses, one page for each storm. The storm listing consists of storm number, date of peak, week number, moments of the input, output, and unitgraph, and a table furnishing basin rainfall amount, ERH, unitgraph ordinates, and computed and observed DRH by two-hour increments from 2 to 200 hours, or four- hour increments from 4 to 400 hours. The bottom of the page shows total rain, total runoff, and standard error. A terminating page for the basin provides the total error variance.

The punchcard deck consists of a separate card for each storm. Entries on each card consist of a number signifying the basin sequence in the data assembly, rainfall-runoff identification number, storm number, unitgraph parameters, total storm rainfall and runoff, first and second moments of the ERH, year of storm occurrence, magnitude and time of peak discharge, errors in peak discharge and time, number of discharge ordinates, and standard error.

APPENDIX 5. FT. WORTH RFC RAINFALL-RUNOFF RELATION

UNITED STATES DEPARTMENT OF COMMERCE
WEATHER BUREAU
RIVER FORECAST CENTER
819 Taylor Street, Room 10A02
Fort Worth, Texas 76102
November 14, 1969

Mr. Phillip Light
Room 210, Coastal Studies Institute
Louisiana State University
Baton Rouge, Louisiana 70803

Dear Mr. Light:

Here is the latest version of equations for the season quadrant of the rainfall runoff relation currently used at the Fort Worth RFC.

$RI1 = (A+BY)\cdot(C)^{API}$

A is the intercept of WN on the RI1 axis.
I is the intercept of WX on the RI1 axis.
WN is the wettest week.
WX is the driest week.
W is the week number of event.
E1 is curvature constant for WN.
E2 is curvature constant for WX.
G1 determines the rate at which E1 approaches E2.
G2 determines the rate at which E2 approaches E1.
$B = (I-A)/2$
CP determines distribution of week curves. With CP = 1.0 curves are distributed evenly between WX and WN. As CP approaches zero the week curves tend to be more closely packed around WX and WN, and as CP increases above 1 the week curves tend to cluster midway between WX and WN.

For weeks between WN and WX

$Y = 1-(\cos((W-WN)(Pi/(WX-WN))))^{CP}$

$C = E1+G1((W-WN)/(WX-WN))$

For weeks between WX and 52

$Y = 1+(\cos((W-WX)(Pi/(52+WN-WX))))^{CP}$

$C = E2+G2((W-WX)/(52+WN-WX))$

For weeks between 52 and WN

$Y = 1+(\cos((W+52=WX)(Pi/(52+WN-WX))))^{CP}$

$C = E2+G2((52+W-WX)/(52+WN-WX))$

API is the Antecedent Precipitation Index using a 0.9 regression factor.

NOTE: At present all our season quadrants use CP = 1.0

Very truly yours,

Victor W. Hoffman

Victor W. Hoffman
Hydrologist in Charge

Previous Communication 10-16-69

The Second Quadrant (Precipitation)

$RI2 = P \cdot ((P/(P+1))^{RI1})$
P is observed precipitation

The Third Quadrant (Duration)

$RO = RI2 \cdot (K)^{FD}$
$FD = ((DUR(RI1+1))/(6+M(RI2)^{POW})$

DUR is storm duration in hours.
M, POW, K, are constants, (K is less than 1).

APPENDIX 5. EXCERPT FROM PROGRAM RUNOFF, KANSAS CITY RFCR (6)

```
                                        3200  FORTRAN    (2.2)R              03/04/68
C      SUBPROGRAM Q2.  TRI-QUADRANT CO-AXIAL RELATION
       FUNCTION QCAL(WK,AP ,DUR,PCPN,C,K,I)
       DIMENSION WK(600), AP(600), DUR(600), PCPN( 600), C(8)
       REAL COSF
       DURDA=DUR(I)/24.
       X = WK(I) * .1208303846
       FX = COSF(X-C(1)) + C(2) * COSF(2. * X)R
       API=AP(I)
       APISQ = API * API
       AI = C(3) + C(4) * API + .07 * APISQ + C(5) * FX + C(6) * FX * API
       FI = AI + .20 * DURDA
       IF(FI) 3, 3, 4
     3 EN = .89
       D = 0.0
       GO TO 5
     4 EN = .89 + .63 * FI
       D = .306 * FI ** 1.865
     5 QCAL     = ( PCPN( I) ** EN + D ** EN ) ** ( 1. / EN ) - D
       IF(QCAL) 1,2,2
     1 QCALR= 0.0
     2 RETURN
       END

         3200 FORTRAN DIAGNOSTIC RESULTS - FOR   QCAL

NO ERRORS
EQUIP,1=60R
  IP,2=61R
 AD,56
RUN
```

KANSAS CITY RFC RAINFALL-RUNOFF EQUATIONS

WK = Week Number
AP = Antecedent Precipitation Index
DUR = Duration, hours
PCPN = Cumulative Rainfall, inches
QCAL = ,, Runoff, inches