A physics based routing model for modular green roof systems

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Notation List

h	depth of saturated water within soil layer (m)
h_l	head loss due to soil (m)
i	rainfall intensity (m/s)
k	saturated hydraulic conductivity (m/s)
n	reservoir routing parameter (-)
q_{out}	outflow per unit area (mm/min)
s	storage (L)
t	time (s)
A_M	module area (m^2)
A_O	orifice area (m^2)
C_D	orifice discharge coefficient (-)
H	module depth (m)
L	module length (m)
Q_{in}	inflow volume flow rate (m^3/s)
Q_{out}	outflow volume flow rate (m^3/s)
R	characteristic path length of flow (m)
U	representative bulk velocity (m/s)
$\forall$	volume of free water in module (m^3)

W	module width (m)
Φ	air-filled soil porosity (-)

Abstract

Much research has looked at the ability of green roof systems to impact stormwater runoff from buildings. However, a lot of these studies have looked at long term retention while less work has focused on how to model a green roof's response to larger design storms. Work that has examined flow routing for individual rainfall events has focused on empirical routing models that are tuned to the specific roof being modeled. This paper presents a new physics based model for flow routing based on the green roof module geometry and soil properties that requires only a single discharge coefficient to be measured. This model is compared to results of a series of experiments to quantify a modular green roof system's hydraulic response to drawdown and steady rainfall.

Introduction

Climate change and a warming atmosphere will lead to increased moisture content in the atmosphere and an accelerated hydrologic cycle (National Research Council of the National Academies 2011). This in turn will lead to increased frequency of high intensity rainfall events. This additional annual rainfall and more intense design storm events will stress current stormwater infrastructure that was designed for historic rather than projected rainfalls. New land development (the USDA estimates that approximately 6000 acres of open space in the US is lost to various land development activities each day (USDA Forest Service 2007)) will increase runoff and reduce water quality. New land development leads to local reductions in rainfall infiltration and increases in rainfall runoff volume due to the replacement of permeable vegetated surfaces with impermeable surfaces such as buildings, roadways, and parking lots

(Akan and Houghtalen 2003). Land development can also increase the mass of pollutants deposited on the land due to industrial activity, vehicle traffic, and other human activity (Heaney et al. 1977). Many of these pollutants attach to sediment particles that become suspended in the increased runoff and transported to downstream receiving waters (Lee et al. 2005, Li and Zuo 2013) leading to a deleterious impact on public health (Gaffield et al. 2003).

Low impact development (LID), also known as sustainable drainage systems (SUDS), incorporates technologies such as green roof systems which mimic the natural hydrologic cycle by replacing traditional impervious surfaces and pipes with surfaces and channels that allow infiltration and evaporation and retard the flow of runoff. This increases the post-development time of concentration and reduces the peak runoff. Green roof systems retain rainfall in the soil and act as small distributed detention ponds. Some of the water is absorbed by the plants, some evaporates, and the remainder is released over time (Morgan et al. 2013). The reduction in peak and total runoff compared to traditional development stormwater management designs means that the post-development site hydrology more closely matches the pre-development hydrology. As such, green roof systems can increase the resiliency of a stormwater system (Birgani and Yazdandoost 2016, Lamond et al. 2015) and have the potential to reduce the required size of downstream storm sewers and detention facilities (effectively increasing the land area that can be serviced by existing infrastructure) by reducing both the peak runoff rate and total runoff.

Literature review

Most of the current work in the area of green roof systems is focused on water quality or the long term water balance (Morgan et al. 2013, Sherrard and Jacobs 2012, Stovin et al. 2013) rather than the shorter duration (24-hour) larger depth design event storms that are commonly used for stormwater management design. Case studies that monitor water quantity, e.g. (Voyde et al. 2010), typically report an overall or average percent reduction of rainfall. Many researchers have acknowledged it is not very meaningful for the larger design storm event modeling because the percent reductions for the smaller, more frequent storms captured by case studies are typically much larger than what would be seen for the larger design storm events (Stovin et al. 2013). However, Stovin et al. (2013) does present a robust initial abstraction model that is key to modeling the response to any rainfall event including extreme design storms.

Longer term studies of the rainfall – runoff relationship for green roof systems typically quantify the behavior in terms of a curve number. For example, Fassman-Beck et al. (2016) found that a curve number of 84 was appropriate for storm depths that exceeded the soil moisture storage capacity. This approach is also applied to more intense storms in which the green roof system becomes saturated. For example, Loiola et al. (in press) studied the behavior of modular green roof systems under heavy rainfall conditions typical of a tropical climate and found higher curve numbers (88-97) were appropriate. Villarreal & Bengtsson (2005) developed an empirical unit hydrograph (UH) based on test bed measurements for a particular green roof system. The UH indicates that the vast majority of a given depth of rainfall will runoff in approximately 6 minutes. This short detention time significantly limits the capacity of the green roof system to reduce the roof top peak discharge compared to an impervious roof. In fact, from a design viewpoint, the 6 minute drawdown is similar to the minimum time of concentration used in many stormwater manuals for impervious rooftops. Therefore, a flow routing analysis of the green roof would likely result in a runoff hydrograph very similar to that of a standard impervious roof.

To overcome this problem, so called green-blue roofs have been developed. A blue roof is simply an impervious roof system that includes rooftop runoff storage. This has the advantage of adding site detention volume that would otherwise require underground storage or the allocation of land to a pond. A green-blue roof is a blue roof located beneath a modular green roof system. See, for example, Shafique et al. (2016a). The performance of a green-blue roof system was evaluated by Shafique et al. (2016b) in a field study. The researchers measured rainfall and runoff and found that the peak discharge was reduced by 65% for a 60 mm/hr rainfall intensity. However, the study did not provide a detailed routing model that could be used in design.

A routing model that calculates the change in storage over time as a function of the inflow and outflow was presented by Yio et al. (2013) in which the outflow per unit area (q) from the system was given by

$$q_{out} = kh^n \tag{1}$$

where h is the water depth and k and n are fitted parameters. Fitted values of k and n were presented based on an optimization algorithm applied to a series of laboratory experiments. The pre-factor k was found to decrease with increasing substrate depth but showed no systematic variation with the substrate saturated hydraulic conductivity. A version of the model in which $n = 1.5$ was fixed and k fitted also performed well.

This modeling approach was extended to look at green-blue roof systems in which the green roof outflow was used as the lower storage layer inflow and the outflow from both components was modeled using the same non-linear empirical outflow model structure though with storage (s) replacing depth (h) in equation (1) above such that

$$Q_{out} = ks^n \tag{2}$$

in which Q_{out} is the outflow volume flow rate. See Vesuviano & Stovin (2013) and Vesuviano et al. (2014). In these studies, the exponent values (n) ranged from $2 < n < 2.8$ which is consistent with the value of $n = 1.5$ in Yio et al. (2013) under the assumption that the storage increases approximately linearly with depth.

Such a routing approach is of great value to design engineers as it enables the green or green-blue roof system to be included in site hydrologic models and the retention and detention capacities of the system can be accounted for. The main drawback is that the outflow model, i.e. equations (1) or (2) rely on empirically fitted parameters rather than on the underlying physical behavior of the system. It is, therefore, possible that the model is only valid for the range of parameters for which it was tested for the purposes of fitting the model. The aim of this paper is to present a physics based routing model that relates the outflow to the geometric properties of the physical modules and the hydraulic properties of the soil (substrate). A green-blue roof system is fundamentally a modular green roof system placed on top of a storage volume with an outlet control, i.e. a detention pond. The hydraulics of detention ponds are well understood so herein the focus is on quantifying the hydraulics of the overlying green roof. As such, a series of experiments were run to establish the hydraulic behavior of modular green roof systems. The goals were to establish a generic stage-discharge function that could be used in a site hydrologic model.

Model development

The goal of this section is to develop a simple physics based model for the hydraulic behavior of a modular green roof system during a design rainfall event. That is, the model is for the 24 hours of the rainfall event and the post event drawdown. The focus is on modeling the immediate drawdown of the free water in the system and, as such, only relatively rapid processes are modeled. Therefore, evapotranspiration and unsaturated flow seepage out of the base of the module are ignored as they are observed to have negligible impact on the total volume of water removed from the module over the time scale of a few days when compared to the initial drawdown. Further, as many stormwater regulations require that stormwater retention and detention facilities return to their design initial conditions within 48-72 hours of peak storage, it is also assumed that the initial condition for the module is with the soil at field capacity where there is no free water stored in the voids between the soil particles. We also seek to develop a model that can be easily solved using standard numerical ODE solvers such as can be found in MATLAB or Python. Therefore, the three dimensional porous media flow models are avoided. Instead a simple length scale analysis is used to develop a conceptual model with parameters that can be measured and a single empirically derived parameter, namely an outflow discharge coefficient. There have been numerous attempts to quantify the flow of water through green roof systems using the 1-D and 2-D porous media flow equations (see for example Liu & Fassman-Beck 2018, Palla et al. 2009, and Palla et al. 2011). However, these approaches require the solution of a set of partial differential equations. The focus herein is to develop a physics based model that approximates this behavior but results in only ordinary differential equations that are more straight forward to solve numerically.

As discussed in the introduction, a green roof module is effectively a storage volume filled with soil with one or more outlet orifices. For the sake of simplicity for the model derivation we assume a rectangular module with an orifice at the center of the base. The module has a depth $H\ (m)$ and horizontal dimensions of length $L\ (m)$ and width $W(m)$. The horizontal dimensions are assumed constant with height such that the module area is $A_M = W \times L\ (m^2)$ though the model is easily extended to more complex geometries. The orifice is centrally located and has an area denoted by $A_o\ (m^2)$ with a discharge coefficient $C_D\ (-)$. The soil has an air-filled porosity $\phi\ (-)$ and saturated hydraulic conductivity $k\ (m/s)$. See figure 1 for a schematic diagram of the model.

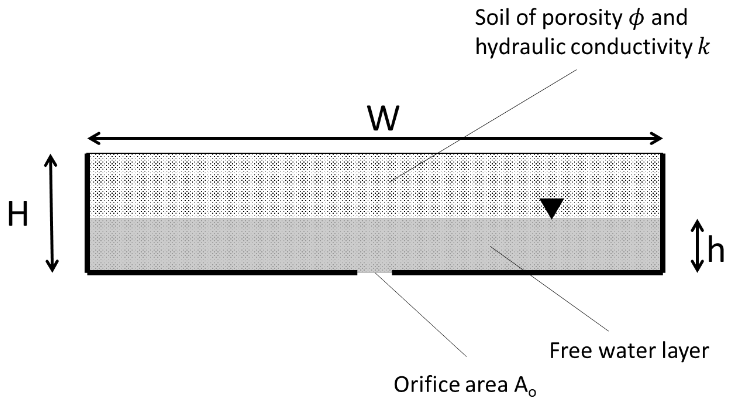


Fig. 1: Schematic diagram of a green roof module showing the soil layer, free water layer, and outlet orifice centrally located at the base of the module.

The outflow through the orifice at the base is given by

$$Q_{out} = C_D A_o \sqrt{2g}(h - h_l)^{\frac{1}{2}} \tag{3}$$

where h is the depth of the saturated water within the soil layer and h_l is the head loss due to the flow of water through the soil. The head loss due to the saturated flow through the soil layer is given by Darcy's equation that can be written as

$$h_l = \frac{R}{k}U \tag{4}$$

Where R is a characteristic path length of the flow from the free surface to the orifice, k is the saturated soil hydraulic conductivity and U is a representative bulk velocity within the soil layer (Nield & Bejan, 2006). As stated earlier, this is an oversimplification that avoids the need to use a three dimensional porous media flow model that would require solving a set of partial differential equations. However, the oversimplification does capture the underlying physics of the flow head loss in the systems and, as is shown in the results section, proves remarkably effective in capturing the drawdown behavior of the test modules.

Assuming that the orifice is centrally located the mean path length from the free surface to the orifice is the distance from the orifice to the center of area of the rectangles that form the 4 quadrants of the module that surround the orifice at the height of the free surface. We take this to be the representative path length R which is given by

$$R = \sqrt{h^2 + \left(\frac{W}{2}\right)^2 + \left(\frac{L}{2}\right)^2} \tag{5}$$

The representative velocity scale for the flow is taken to be the vertical velocity of the water in the saturated layer. Conservation of volume for the free water in the module is given by

$$\frac{d\forall}{dt} + Q_{out} - Q_{in} = 0 \tag{6}$$

Where Q_{out} is given by the orifice equation (3), Q_{in} is given by the rainfall intensity (i) multiplied by the plan area of the module. That is, $Q_{in} = iA_M$. The volume of free water in the module is given by

$$\forall = h\phi A_M \tag{7}$$

Draining flow model

We consider first the case of an initially full module that is draining with no rainfall. In this case $Q_{in} = 0$, the velocity of the water in the saturated layer is taken to be the rate at which the free surface is dropping (i.e. $\mathrm{U} = -\mathrm{dh}/\mathrm{dt}$) and

$$\frac{d}{dt}(h\phi A_M) = -C_D A_o\sqrt{2g}\left(h + \frac{R}{k}\frac{dh}{dt}\right)^{\frac{1}{2}} \tag{8}$$

Denoting $\Gamma = C_D A_o\sqrt{2g}/\phi A_M$ the previous equation can be written as

$$\frac{dh}{dt} = -\Gamma\left(h + \frac{R}{k}\frac{dh}{dt}\right)^{\frac{1}{2}} \tag{9}$$

Squaring both sides results in a quadratic equation in dh/dt that can be solved to give

$$\frac{dh}{dt} = \frac{1}{2}\left(\frac{\Gamma^2 R}{k} - \sqrt{\frac{\Gamma^4 R^2}{k^2} + 4\Gamma^2 h}\right) \tag{10}$$

Full routing model with inflows and outflows

For the case where there is rain falling on the module the soil head loss term must be modified to include the source term, i (m/s) in the representative velocity. The appropriate velocity scale is given by

$$U = i - \frac{dh}{dt}. \tag{11}$$

Including this and the source term in the conservation of volume equation for the free water in the module leads to

$$\frac{d}{dt}(h\phi A_M) - iA_M = -C_D A_o \sqrt{2g}\left(h - \frac{R}{k}\left(i - \frac{dh}{dt}\right)\right)^{\frac{1}{2}} \tag{12}$$

Again denoting $\Gamma = C_D A_o \sqrt{2g}/\phi A_M$ the previous equation can be written as

$$\frac{dh}{dt} - \frac{i}{\phi} = -\Gamma\left(h - \frac{R}{k}\left(i - \frac{dh}{dt}\right)\right)^{\frac{1}{2}} \tag{13}$$

Squaring both sides results in a quadratic equation in dh/dt that can be solved to give

$$\frac{dh}{dt} = \frac{1}{2}\left(\left(2\frac{i}{\phi} + \Gamma^2\frac{R}{k}\right) - \sqrt{\left(2\frac{i}{\phi} + \Gamma^2\frac{R}{k}\right)^2 + 4\left(\left(\frac{i}{\phi}\right)^2 - \Gamma^2 h + \Gamma^2\frac{R}{k}i\right)}\right) \tag{14}$$

Both (10) and (14) can be solved numerically given appropriate initial conditions, module geometry, and soil properties.

For a module in which the cross sectional area varies with height the area A_M is replaced by $A(h)$ in Γ. The rainfall intensity also needs to be corrected as the rain falls on the area at the top of the module $A(H)$ but is filling a section of module of area $A(h)$. The routing equation then becomes

$$\frac{dh}{dt} = \frac{1}{2}\left(\left(2\frac{i}{\phi}\frac{A(H)}{A(h)} + \Gamma^2\frac{R}{k}\right) - \sqrt{\left(2\frac{i}{\phi}\frac{A(H)}{A(h)} + \Gamma^2\frac{R}{k}\right)^2 + 4\left(\left(\frac{i}{\phi}\frac{A(H)}{A(h)}\right)^2 - \Gamma^2 h + \Gamma^2\frac{R}{k}i\frac{A(H)}{A(h)}\right)}\right). \tag{15}$$

In order to validate the model presented above a series of tests were conducted to measure the response of a modular green roof system to various inflow conditions.

Experimental method

A series of experiments was conducted in which the change in storage was measured over time for two test cases. The first case was a drawdown test in which an initially full green roof module was allowed to drain until the free water was fully removed. The second case tested the response of the system to two high intensity steady rainfall events.

Test module properties

The green roof modules tested were supplied by Green Roof Outfitters of Charleston, SC, U.S.A. (http://greenroofoutfitters.com/). The modules were 30.5 cm wide by 61.0 cm long and 10.2 cm deep (12 inches by 24 inches by 4 inches). The lower half of each module was divided into four submodules by ridges running along the two axes of symmetry. Each submodule has a set of 5 outlet orifices raised slightly above the base of the module. The outlet orifices had a diameter of 0.79 cm (5/16 in). There were additional outlets of the same diameter along the upper ridges. See figure 2 for images and dimensions of the green roof modules tested. The internal dimensions of the module were measured to establish the cross sectional area of the modules as a function of height above the lower outlets. Although the module side walls are slightly sloped away from vertical the module internal area could be very well approximated by $A=0.125\ m^2$ ($1.35\ ft^2$) up to a height of 3.6 cm (1.4 in) and $0.180\ m^2$ ($1.94\ ft^2$) above that.

When installed, each module is filled with soil and planted with geographically appropriate sedum. As the goal of these tests is to understand the stage-discharge relationship for the modules it is important to quantify the properties of the soil. In particular, it is important to understand the saturated hydraulic conductivity of the soil and the air-filled soil porosity. The air-filled soil porosity was calculated as part of the drawdown tests described below. The saturated hydraulic conductivity was measured using a constant head permeameter similar to that described by Sobolewski (2005).

A sample of soil from one of the modules, including roots from the plants, was placed in a cylinder with a porous base and porous cap, a cross sectional area $A=80.0\ cm^2$ ($12.4\ in^2$), and the soil sample height was h=11.6 cm (4.58 in). A series of five tests were run which gave an average saturated hydraulic conductivity of 0.96 mm/s with a standard deviation of 0.4 mm/s.

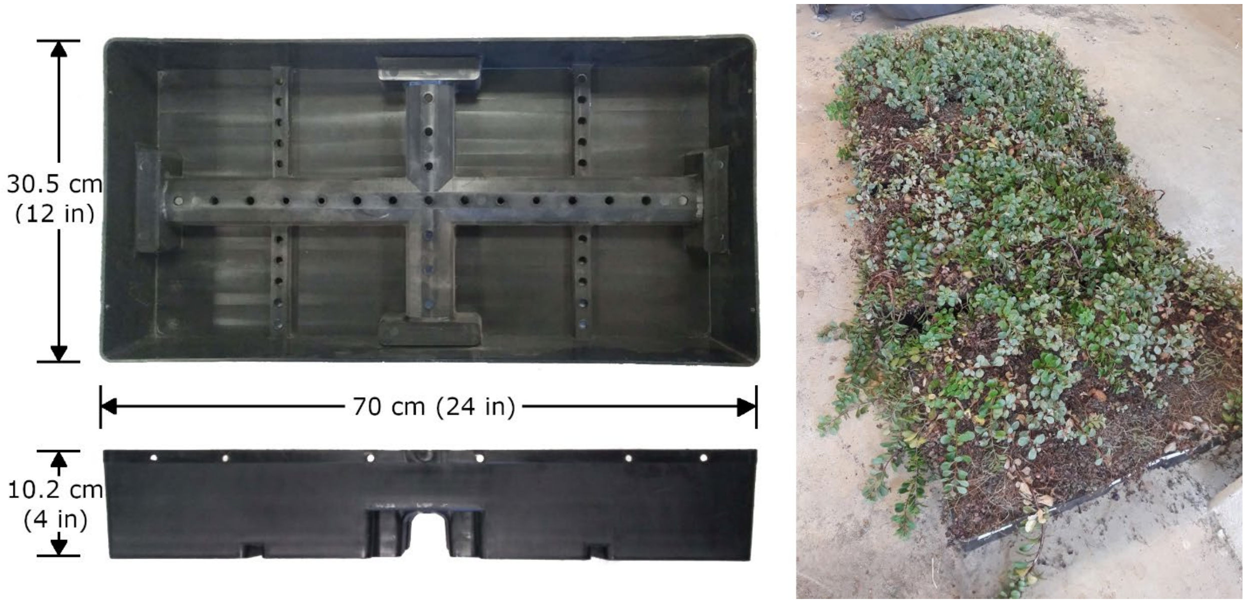


Fig. 2. Left: Dimensions of module, shown empty. Right: Photograph of module with plants ready for testing. Note that in all tests all the upper orifices (along the centerline ridges) were plugged as were all but one of the lower orifices in each quadrant.

Test Rig design and development

Tests were run to measure the time variation of water volume within the modules under different water loading conditions. In all cases the volume of water was calculated based on the weight of water in the modules. A test rig was designed and constructed to continuously weigh the modules during each test. A steel frame capable of supporting four modules was constructed and mounted at the corners on four load cells (Phidgets Micro Load Cell 0-50kg - CZL635). The load cells were connected to a Data Acquisition board (DAQ) that had a USB connection to a PC that logged the weight as a function of time.

A rainfall simulation system was mounted above the module frame but connected directly to the table such that its weight was not supported by the load cells. The rainfall system consisted of a

61.0 cm wide by 121.9 cm long and 61.0 cm high (24 inch by 48 inch by 24 inch) metal frame. The top of the frame supported a 61.0 cm wide by 121.9 cm long (24 inch by 48 inch) expanded metal sheet on top of which was laid some standard gardening porous soaker hose. The hose was tied to the expanded metal frame with thin cable ties and the entire system was shrouded in plastic sheeting on the top and down the sides. The plastic sheeting prevented water spraying away from the modules. The soaker hose was connected to a faucet via a Cole Parmer rotamater flow rate meter. The flow rate meter had a built in needle valve to control the flow rate and had a scale range of 1-20 L/min. A schematic diagram of the experimental test rig is shown in figure 3.

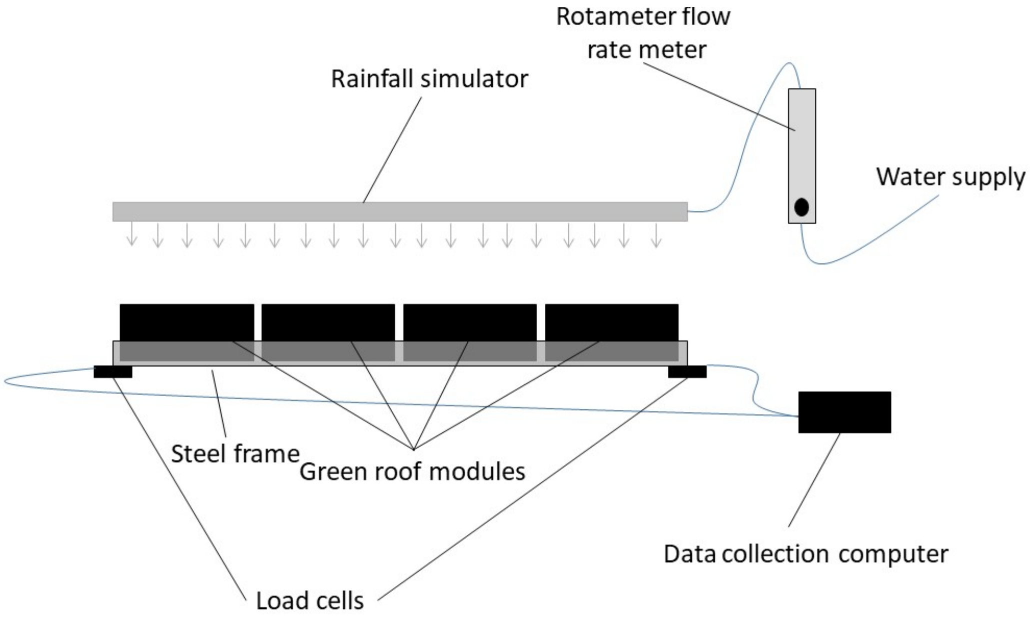


Fig. 3. Schematic diagram of the experimental setup.

Test procedures

Two sets of tests were run on the green roof module systems. First, drawdown tests were run in which the modules were flooded and then their weight was tracked as water drained out of the base. In the second set of tests the initially empty (unsaturated) modules were exposed to a steady rainfall rate and their weight was tracked until it reached a steady state. Preliminary drawdown tests showed very rapid drawdown. To slow the draining all but four of the outlets were plugged. Tests were run to see if the drawdown was significantly impacted by the choice of which outlet to leave unplugged and no differences were found.

Before each test the same initial conditions were established using the following procedure. The rainfall simulator was removed from the test rig and a number of modules were placed on the steel frame. A hose with a spray nozzle was used to soak and fill the module. Water was added until it overflowed the top of the module. During this fill time the data logger was turned on and the module weight was logged every second and plotted on the screen. Once the water was overflowing the hose was turned off and the module was allowed to drawdown until no more water was dripping out of the base of the module and the plot of weight against time on the DAQ software showed no change in weight with time. The data logging was then stopped and the scale tared (zeroed) so that weight was recorded as zero at the start of each test. Therefore, the weight measurement on the scale during testing corresponded to the weight of free water in the modules.

Each drawdown test was run in the same manner. First the initial conditions were established as described above. Then the hose was turned on and the module was filled until water was overflowing the top of the module at which point the data logging was turned on and the hose was then turned off. The data was recorded until water was no longer observed to be dripping out of the base of the module. Drawdown tests were run for two different sets of open outlets and each configuration was run 3 times. To reduce the outflow and increase the detention time a test was run with pinhole outlets in the base of the modules. However, the pinholes filled with soil and the pinholes immediately clogged so no results are presented for those tests.

The initial conditions for the constant rainfall tests were established as described above. Once established the rainfall simulator was placed over the modules and the data logger turned on. The faucet was turned on and the needle valve was adjusted on the rotameter to establish the desired flow rate. The test was run until the system reached a steady-state in which the rate of rainfall onto the module was balanced by the rate at which it drained out of the base of the module. Tests were run for two different flow rates of 4 litres per minute and 12 litres per minute corresponding to rainfall intensities of 32.3 cm per hour (12.7 in per hour) and 96.8 cm per hour (38.1 in per hour) respectively. The selected rainfall intensities are high compared to typical design storm intensity values. They were chosen in order to give steady-state water depths that were a substantial fraction of the module depth in order to provide a better data for model validation.

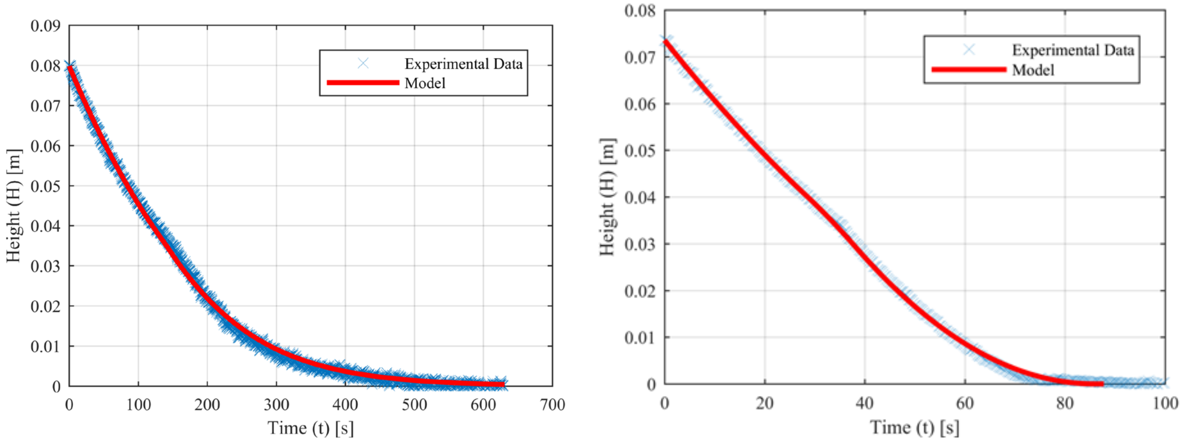
Experimental results

The modeling approach presented was validated using the draining flow and rainfall tests described above. To apply the model, the soil properties (air-filled porosity and saturated hydraulic conductivity) and module properties (geometry and discharge coefficient) are required. The soil air-filled porosity was calculated by dividing the volume of water measured at the start of each drawdown test by the module volume. The six drawdown tests for the module containing soil gave an average air-filled porosity for the soil of 21.7% with a standard deviation of 0.5%.

The routing model presented uses the water depth rather than storage (see equations 10 and 14). The depth of water was calculated based on the weight of free water in the module measured by the scale. This was divided by the water specific weight to give the volume of water in the module that was converted to depth by inverting the stage-storage function. The discharge coefficient for the module is the only parameter that was not measured independently of the drawdown experiments. It is unclear what a typical value for the discharge coefficient should be as the outlet orifice is surrounded by soil on one side that will significantly reduce the effective area of the outlet. Therefore, the model was optimized using C_D as the only free parameter. Note that in the case of a green roof system with multiple outlet orifices at the same elevation the total area can be used. In this case, the optimized C_D will also account for any orifice interaction effects.

Drawdown Tests

Results for a sample drawdown test are shown in figure 4 along with the optimized model results. Also shown is a plot of the drawdown for a module containing no soil or plants. The addition of soil to the module significantly impacts its hydraulic behavior. The soil contributes additional head loss to the system which slows the discharge. This additional loss is due to the head loss as the water flows through the soil (see equation 9) and due to blocking of the outlet orifice by the soil which significantly reduces the orifice discharge coefficient. While the empty module drained in about 80 seconds, the module with soil took approximately 10 minutes to completely drain. This delay was most pronounced at lower depths where it took half the total time (5 minutes) to empty the last 8 mm of water with the soil whereas for the empty module it took less than a quarter of the total time (10 seconds) to empty the same depth. This slower drawdown is due to the head loss in the soil becoming a greater fraction of the free water head as h gets small. For example, for the result shown in figure 4(a) the initial head loss in the soil is approximately 1.5 cm or slightly less than 20% of the total head. However, this percentage increases over time such that when the water depth is 1 cm the head loss is 0.45 cm or 45% of the total head. One other impact that is not visible from the figures is that there is significantly less water stored in the module due to the water only being stored in the soil pore spaces.



(a) (b)

Fig. 4: (a) Example of drawdown test result showing water depth versus time along with the model prediction for an optimized discharge coefficient of $C_D = 0.10$. (b) Drawdown results for a test run on an empty module along with the model prediction for $C_D = 1.0$.

For the six runs of the module filled with soil and vegetation, the average Cd value was 0.10 with a standard deviation of 0.012 as determined by fitting the model to the drawdown data using the least-squares technique. The reduction of the Cd value can be partially attributed to the soil physically blocking the orifice, effectively reducing the orifice area, but it might be expected that the area would be reduced proportionally to the soil air-filled porosity, 21.7% giving $C_D = 0.22$. However, the actual C_D is less than half of this so there are other factors contributing to this reduction. A detailed theoretical analysis of the cause of this reduced discharge coefficient is beyond the scope of this paper.

The sensitivity of the model to uncertainty in the soil saturated hydraulic conductivity was assessed by simulating the drawdown in a simple green roof module with the same total dimensions of the test modules but with one centrally located outlet with total area equivalent to four of the test module outlets. Equation (10) was integrated numerically for values of the measured mean k and for $2k$, $3k/2$, $2k/3$, and $k/2$. The time taken for the water depth in the tank to drop from 10 cm to 1 cm was recorded. These data are shown in table 1 below. The data clearly shows that the results are quite sensitive to the value of k. For example, a 33% reduction in k resulted in a 38% increase in the drawdown time while a 50% increase in k lead to a 24% decrease in the drawdown time. Therefore, care should be taken in accurately characterizing the soil properties when using this model.

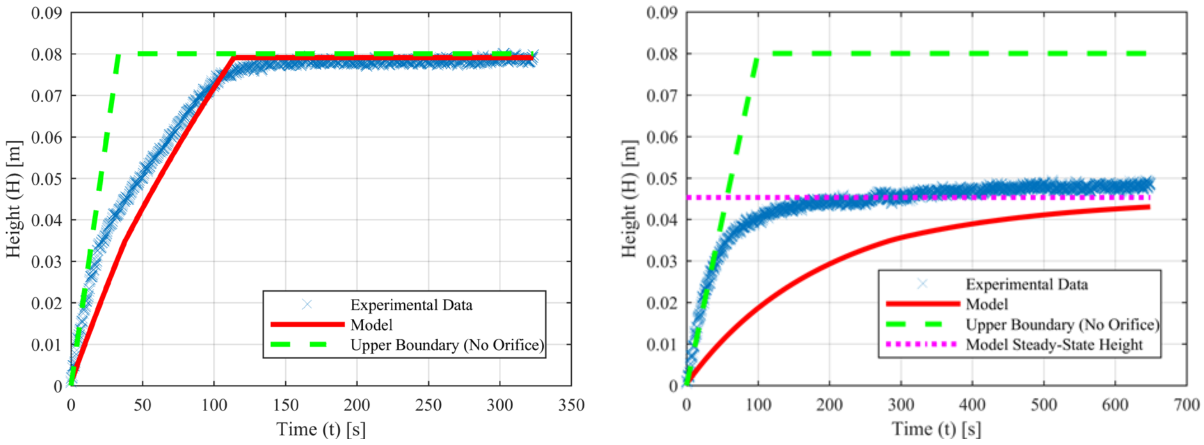
Table 1. Time taken for the simulated module to draw down from 10 cm to 1 cm for different fractional values of k.

Hydraulic conductivity (mm/s)	2	1.5	1	.67	.5
Time (s)	341	401	527	729	945

Steady State Tests

The drawdown tests were used to establish the soil's air-filled porosity and the orifice discharge coefficient. The constant rainfall tests were run to evaluate how well the model was able to predict the filling of the module in response to rainfall using the model parameter values established in the drawdown tests. The rainfall rates simulated were very large (32.3 cm per hour (12.7 in per hour) and 96.8 cm per hour (38.1 in per hour)) to allow the water depth to reach measurable values. If more realistic (i.e. smaller) rainfall rates had been used, the water depths would have been too small to accurately measure and compare to the model. The results of 2 steady-state tests are shown in figure 5.

The 96.8 cm per hour (38.1 in per hour) rainfall intensity was large enough to fill the module. See Figure 5a. In this test, the model lags behind the experimental data initially, but as the water level reaches the top of the module at around 2 minutes, they converge. This behavior is largely an artifact from data collection and analysis method. The experimental water depth is calculated from the weight of water in the model. When the rainfall begins, there is a period of time when the wetting front is moving vertically through the module but has not reached the base of the module. During this time there is no water in the bottom of the module, no saturated layer depth, and no outflow. However, the weight of free water in the module is logged and, therefore, the experimentally inferred measure of height, is not zero and is increasing. Therefore, the model predicts the outflow to begin instantaneously whereas outflow only starts once the wetting front reaches the base of the module. To capture the time scale over which this occurs the upper bound of height was also included in figure 5. This represents the height of water in a soil filled module with no outlets. As can be seen in the figure, the experimental data follows this line for the first 20 seconds and then diverges. The point at which it diverges is when the water reaches the base of the module and discharge begins. Because this travel time was small and it doesn't have a significant impact on the model's prediction of filling time, it was felt that ignoring the travel time in the model was an acceptable simplification.



(a) (b)

Fig. 5: (a) Height of water in module for a rainfall intensity of 96.8 cm per hour (38.1 in per hour). (b) Height of water in module for a rainfall intensity of 32.3 cm per hour (12.7 in per hour).

The lower flow rate, 32.3 cm per hour (12.7 in per hour), was not large enough for the module to fill. See Figure 5b. In this test, similar behavior to the higher flow rate test can be seen. The experimental height data initially rises following the upper bound, which means there is no discharge until the water reaches the bottom of the model. The experimental data reaches a steady state height of 4.79 cm while the model reaches a steady state height of 4.53 cm, a model under-prediction of 5.4%. Though initially there is a significant difference between the experimental data and the model, this initial period only lasts for approximately 10 minutes. Also in practice, it would be extremely rare to see such a sudden jump from no rain to a rainfall intensity of 12.7 inches per hour. In most rain events, the intensity would step up more gradually and the adjustment time would be further reduced. Regardless of the nature of the rainfall hyetograph, the time lag (of the order of 6 minutes) is comparable to the standard integration time step in many hydrologic models.

Discussion and conclusions

A series of experiments was run and a physics based model developed to predict the hydraulic behavior of modular green roof systems. Our results indicate that a simple routing model for the module stage-storage and stage-discharge relationships can be established using the module geometry, soil properties (air-filled porosity and saturated hydraulic conductivity), and a discharge coefficient. All these parameters can be easily established for any green roof module using standard soil tests and a simple drawdown test in which the module is flooded and then the rate of draining measured. This routing model can then be included in land development hydrologic models so that the detention benefits of the modular green roof system can be included in the site model and drainage design. However, it is important to note that the model developed herein is for an individual green roof module and does not account for the subsequent flow of the module discharge over the roof structure.

Typical modular green roof systems need to have outlet holes at their base to drain excess water and prevent long term soil saturation and damage to plants. These holes need to be large enough that they do not get clogged by the soil above. As a result, the draining time for modular green roof systems is typically quite short, of the order of 10 minutes. Further, because the modules are filled with soil they have very little storage volume for free water. The net results is that, for larger design storm events, standard modular green roof systems offer very little detention benefit and minimal reduction in peak runoff compared to a standard impermeable roof system. To enhance the hydraulic benefit of a modular green roof system, while still maintaining the other benefits of green roof systems, it is necessary to increase the storage capacity of the module and reduce the outlet area so as to increase the detention time. This can be achieved by adding an empty storage layer below the green roof module (Shafique et al. 2016a,b and Vesuviano et al. 2014). Adding a storage layer and forming a so-called green-blue roof has the advantage of allowing the use of very small outlet areas to control the storage layer outflow as there is little risk of clogging and significantly increasing the detention time of the system. The model developed herein can be used to route the flow through the green roof module into the lower storage layer.

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