FIRST IMPRESSIONS

Analytical Description of Ground-Water Seepage across a Lake Floor

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ANALYTICAL DESCRIPTION

OF GROUND-WATER SEEPAGE

ACROSS A LAKE FLOOR

A First Impressions Report*

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I am very grateful to Akira Okubo for reviewing the mathematics represented in this paper and for his valuable suggestions.

AUTHOR'S NOTE

This paper was presented orally at the eastern meeting of the American Geophysical Union in Toronto, Canada on 23 May 1980. I have added a few important references and made minor editorial changes to create a manuscript in the proper form. Otherwise, the text is unchanged.

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ABSTRACT

Recent measurements have shown the importance of ground-water seepage across the floors of lakes and lagoons. The subaqueous discharge is not only a source of water but also supplies dissolved chemicals across the sediment-water interface. To help define this process, an analytical solution to the steady-state Richards' equation was studied in two-dimensions. This solution was used to derive an expression for the vertical flow of ground water across the floor of a large, shallow lake. The lake is assumed to rest on an aquifer of uniform thickness, l. The aquifer is homogeneous but not necessarily isotropic (its vertical and horizontal hydraulic conductivities are K_h and K_v respectively); it is unconfined and it overlies an impermeable surface. The distance from the lake shore to the ground-water divide is s and the hydraulic gradient, i, is assumed to be a constant over this distance. When $\pi sk/4l > 3$, the discharge, $q = K_v i \ln(\coth \pi xk/4l)/k\pi$ where $k^2 = K_v/K_h$ and x is the distance offshore. The width of the zone

of subaqueous discharge is effectively $4l/k$. The hypothetical lake must be wide compared to this distance. The solution fails at the shore (and also at the groundwater divide) because the boundary condition is not differentiable there. As a result, the solution can not be applied when $x < 0.06\ l/k$.

INTRODUCTION

People are becoming more and more interested in the flow of ground water offshore. Biologists and environmental scientists, as well as hydrologists, are recognizing the importance of ground-water seepage out of submerged sediments to supply nutrients and contaminants to lakes, marshes, and lagoons. Many excellent studies have been done in glacial lakes in Canada and the United States (e.g. Lee, 1977) and in the coastal zone of the United States, Australia, New Zealand, and the Bahamas, and work is continuing in this field. At the Marine Sciences Research Center we have studied the submarine discharge in a large coastal lagoon in New York called Great South Bay (Bokuniewicz, 1980; Bokuniewicz and Zeitlin, 1980; Zeitlin, 1980).

I will present an expression that describes the distribution of ground-water seepage across the floor of a large, shallow body of water. Analytical solutions have severe limitations. They can usually only be done for relatively simple systems and my solution is no exception. Nevertheless, if the solution can be written in a simple form, it can be useful for estimating the seepage in situations where it is not practical to do a numerical solution perhaps because of limited time, inadequate data, or insufficient funds.

The solution is developed for the specific configuration of the aquifer and the lake that is shown in Figure 1. There is an unconfined aquifer sitting on a flat impermeable basement. The aquifer is homogeneous but not necessarily isotropic. A wide shallow lake covers part of the aquifer and the other part contains an elevated water table with the ground-water divide some distance from shore. The expression that I will present here describes the flow of ground water up across the lake floor. To simplify the boundary conditions we will assume that the picture has a great vertical exaggeration, that is, that the surface relief is small compared to the thickness of the aquifer. If this is the case we can ignore the surface topography and bathymetry. The coordinate system is shown in Figure 2.

The origin is at the shoreline. The variable x increases to the right away from the land, and z, the depth variable is positive downward. The distance from the shoreline to the ground-water divide is s. Because we are assuming the surface relief to be small, the only other parameter we need to define the geometry of the aquifer is the thickness, l, which is a constant.

The boundary conditions (Fig. 3) are:

1. there is no flux of water through the bottom of the aquifer (at $z = l$);
2. there is no flux of water across the ground-water divide;
3. the right boundary is open but the solution must give finite values for the discharge as the distance from shore becomes infinitely large.
4. at the top of the aquifer the fluid potential is zero over the lake and increases linearly landward to the ground-water divide.

This arrangement is similar to the one used by Tóth (e.g. 1962) in his study of small drainage basins. Tóth solved an isotropic equation and his solution for the fluid potential was an infinite series. I have an open-ended aquifer and obtained a simple solution by confining my attention to only the vertical flux at its surface.

METHODS

The solution was done as follows. Start with the steady-state, but anisotropic, Richards' equation and boundary conditions (Richards, 1931). Apply a cosine transform and solve the transformed equations. Then write down the solution to the transformed equations (Carslaw and Jaeger, 1959). At this point there is a problem; the inverse transform of this solution is a definite integral that can not be integrated in closed form. Nevertheless, the derivative of this integral evaluated at $z = 0$ (the top of the aquifer) is proportional to the vertical, subaqueous discharge and this form can be integrated and I can now write the surface discharge as an analytical function by applying Darcy's law.

RESULTS

Figure 5 is a schematic of the characteristics of the solution. There is an area of recharge near the ground-water divide and an area of discharge near the shore. This distribution had been found in previous studies (Tóth, 1962). Offshore there is another area of discharge and the discharge rate decreases rapidly away from shore. The solution, however, has an unfortunate characteristic. The seepage fluxes get very large both near the ground-water divide and near the shore. This is an artifact of the simplified boundary condition which is discontinuous at both places. While this behavior is not fatal, it is troublesome because the predicted discharge rate is not continuous across the shoreline. In practice, this means that the solution can not be applied too near to the shore. I will return to this condition in a moment.

The analytical expression for this solution is:

$$q = \frac{K_v i}{\pi k} \ln \left(\frac{\coth \frac{\pi |x| k}{4l} \quad \coth \frac{\pi (x + 2s) k}{4l}}{\coth^2 \frac{\pi (x + s) k}{4l}} \right)$$

where:

q is the discharge. I have been expressing it as ℓ/day-m^2 after appropriate conversion.

i is the fluid potential gradient away from the shore (recall that we have assumed it to be constant).

K_v is the vertical hydraulic conductivity at the surface and

k is the square root of the ratio of the vertical to the horizontal hydraulic conductivity.

x is the distance from shore.

s is the distance to the ground-water divide, and

l is the thickness of the aquifer.

π is pi.

although K_v/k is the bulk hydraulic conductivity, I have left the ratio K_v/k in the expression because in practice it might be more appropriate to use typical values of the conductivity for K_v.

This does not look very simple, but for most cases it can be simplified to study the offshore discharge with no detectable loss of accuracy. If s is large so that the argument of the hyperbolic cotangent is greater than 3, that is, if $\pi sk/4l > 3$, then two of the coth-terms have the value of 1 and the solution can be rewritten as:

$$q = \frac{K_v i}{\pi k} \ln \left[\coth \frac{\pi x k}{4l}\right]$$

In the same way, if $x > 4l/\pi k$, then the discharge is effectively zero. The width of the zone of subaqueous discharge is therefore $4l/\pi k$. For the inner limit, I have chosen the case where ln (coth) = π so that the discharge is the bulk conductivity times the gradient. This is the form that is usually used to calculate underflows past the shoreline.

DISCUSSION

To illustrate this solution I will use some hydrogeologic parameters from the glacial outwash aquifer on Long Island, New York. These are:

$$K_v = 8 \text{ m/day}$$

$$i = 0.002$$

$$k = 0.316 \ (1{:}10)$$

$$l = 30 \text{ m}$$

The predicted subaqueous discharge is in $\ell/\text{day m}^2$, shown in Figure 6. Most of the discharge is confined to a narrow zone nearshore about 100 m wide. This is quite reasonable; similar results were found numerically and by direct measurement of other investigators. If we increase l from 30 m to 300 m, the discharge increase and the region where the solution should not be applied also increases. Alternatively, if we increase k to 1 (the isotropic case) the flows are drastically reduced.

I would like to conclude by comparing the predictions to some data from a large coastal lagoon on the south shore of Long Island, N.Y., called Great South Bay. As part of a study of Great South Bay, we have made some direct measurements of the subaqueous discharge by embedding water collection chambers into the lagoon floor. This is a method that has been devised and tested by David Lee (1977) who is using it to study glacial lakes.

For the mathematical prediction I used hydrogeologic data from the principal aquifer under Long Island called the Magothy. In applying the solution there are some problems because in the literature the important parameters are specified by ranges rather than by specific values. So, depending upon your choices, you can generate several different predicted curves. Figure 7 shows the data and some predictions.

The data points represent over 300 measurements at five locations on the lagoon shore. They decrease slowly offshore from about 50 ℓ/day-m^2 to about 30 ℓ/day-m^2 at a distance of 100 m. There are three predicted curves shown. The uppermost shows the greatest discharges that I could obtain using the reported values in the literature. The discharges decrease from 100 to about 70 ℓ/day-m^2. The smallest discharges I could get were all below about 10 ℓ/day-m^2. These two extremes bracket the data nicely. The center curve is one of about eight others that lie between the two extremes. This one seems to fit the data best.

The agreement here is encouraging because the predictions are made completely independently from the data. There are no adjustable parameters. Furthermore, there are several reasons why it should not work. This agreement also suggests that the discharge at least near shore is not sensitive to the salinity in Great South Bay because density differences have not been included in the equation.

I hope that this equation will be useful to other investigators in this field.

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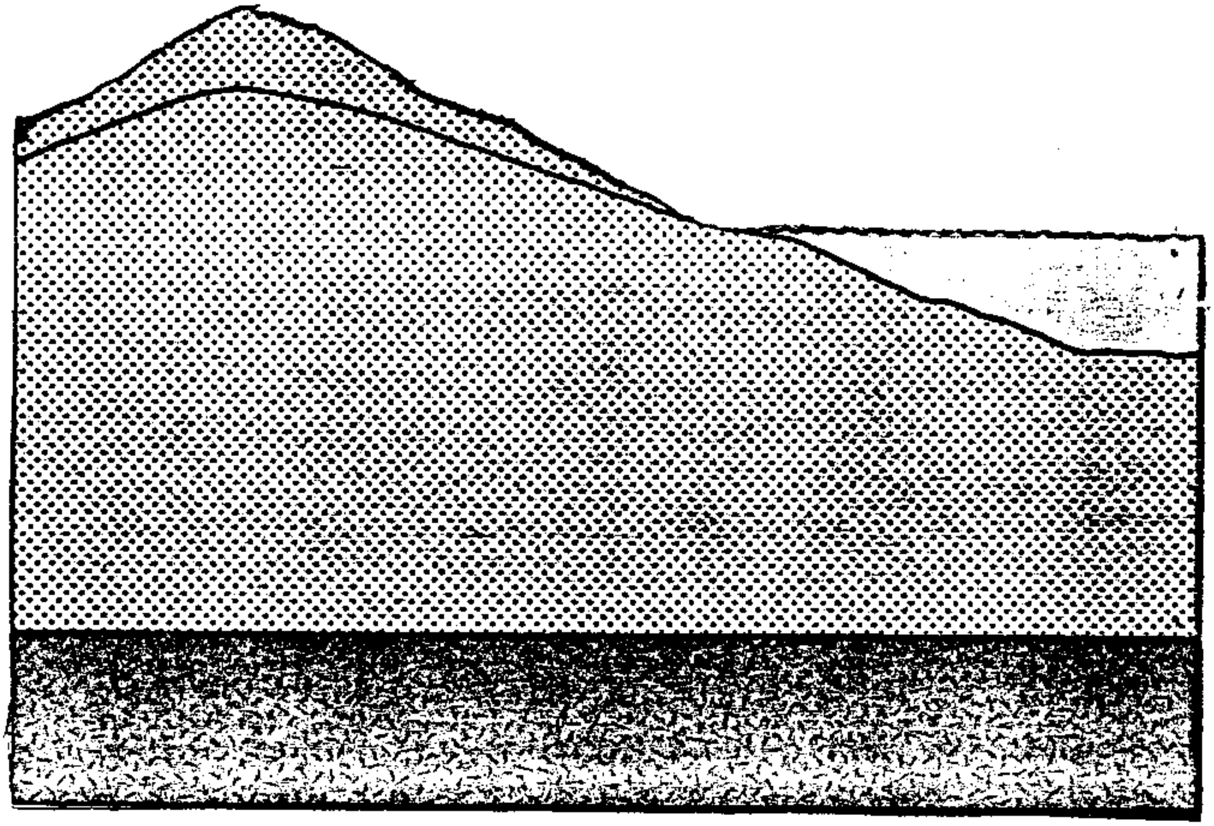
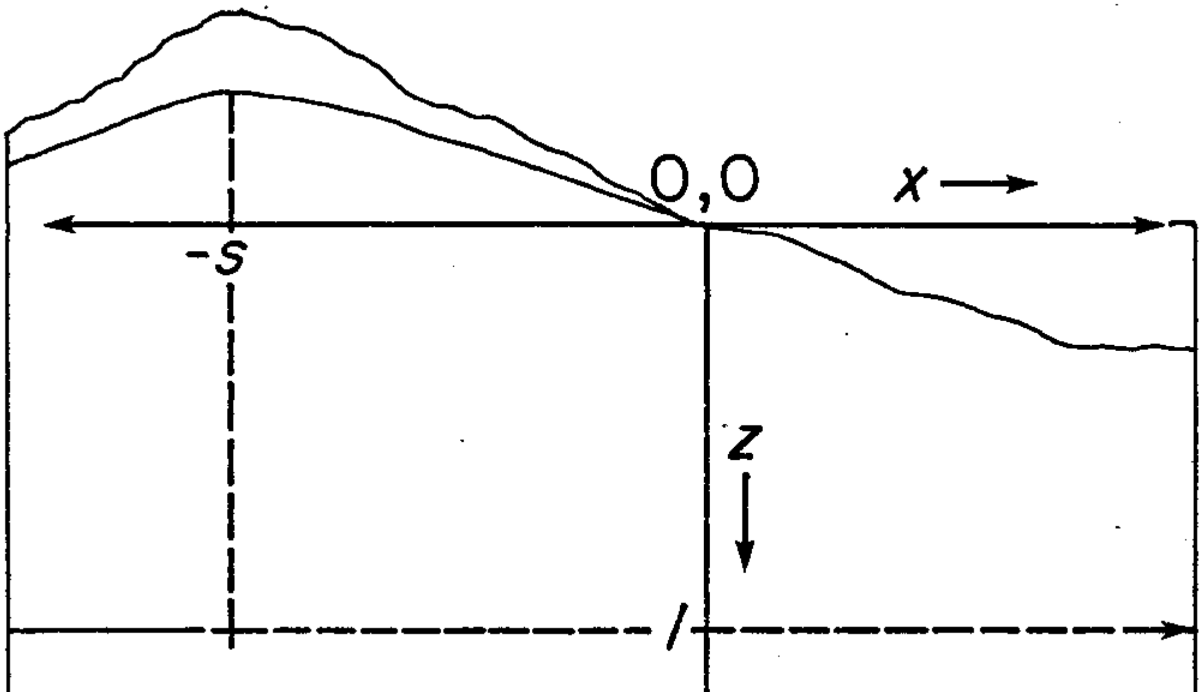


Figure 1



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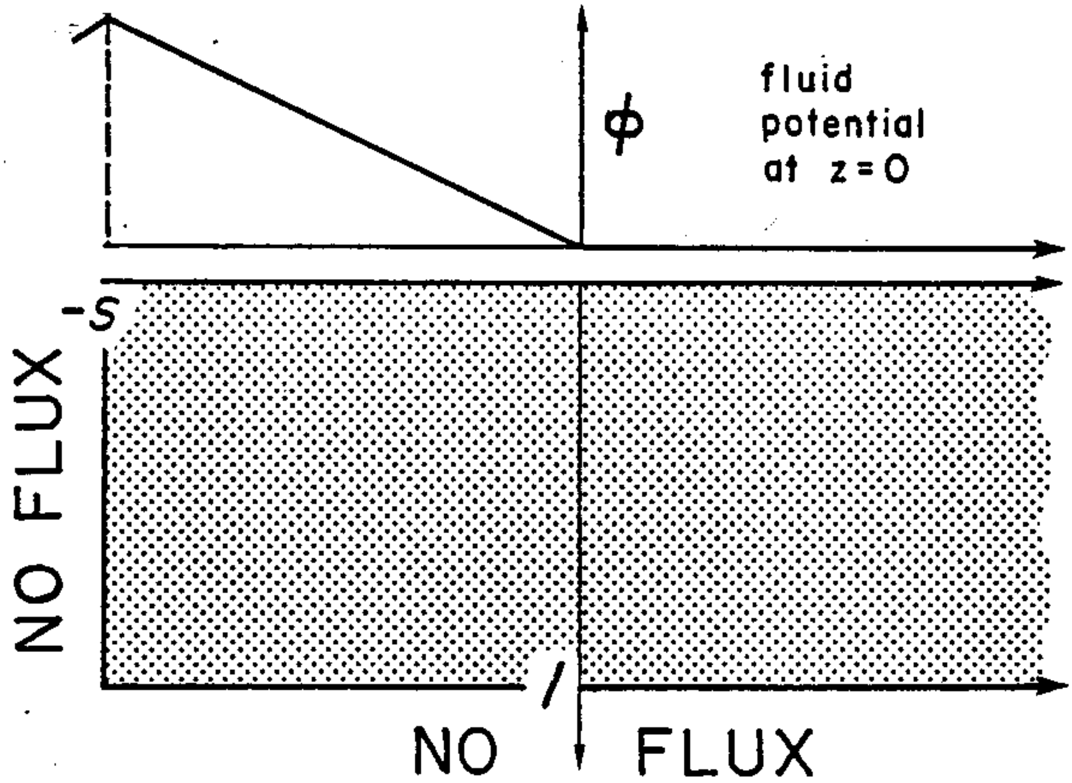
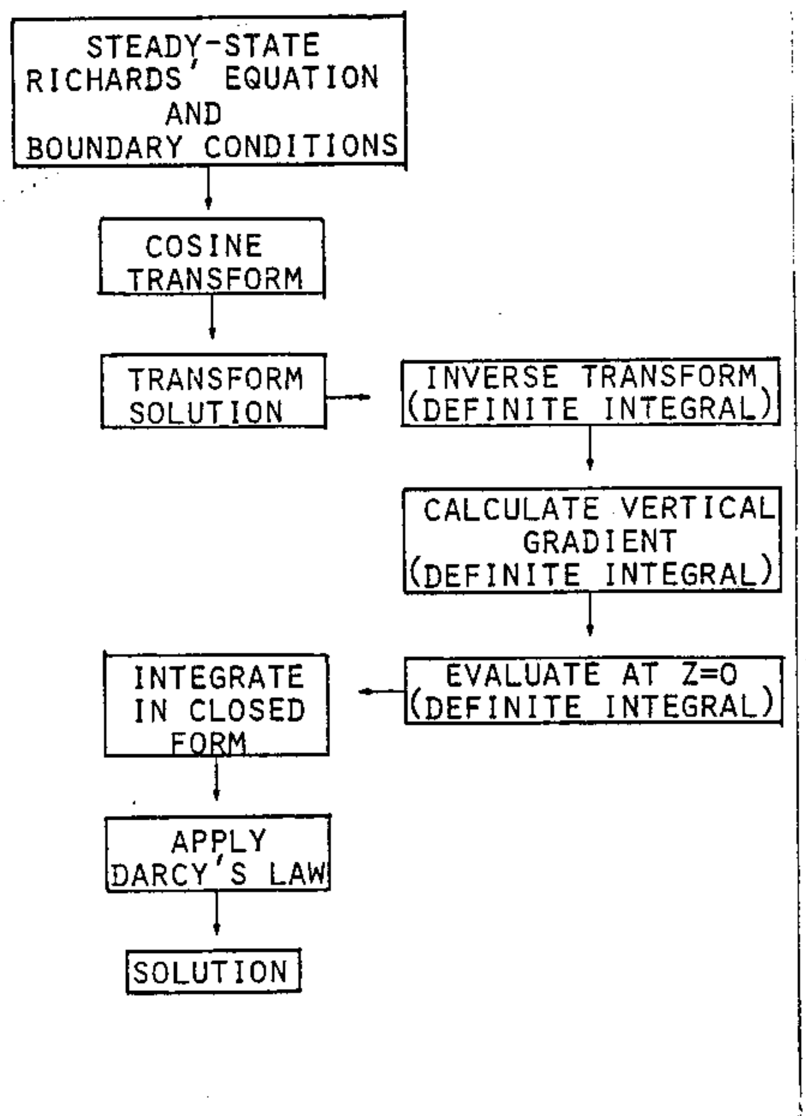


Figure 3

IV



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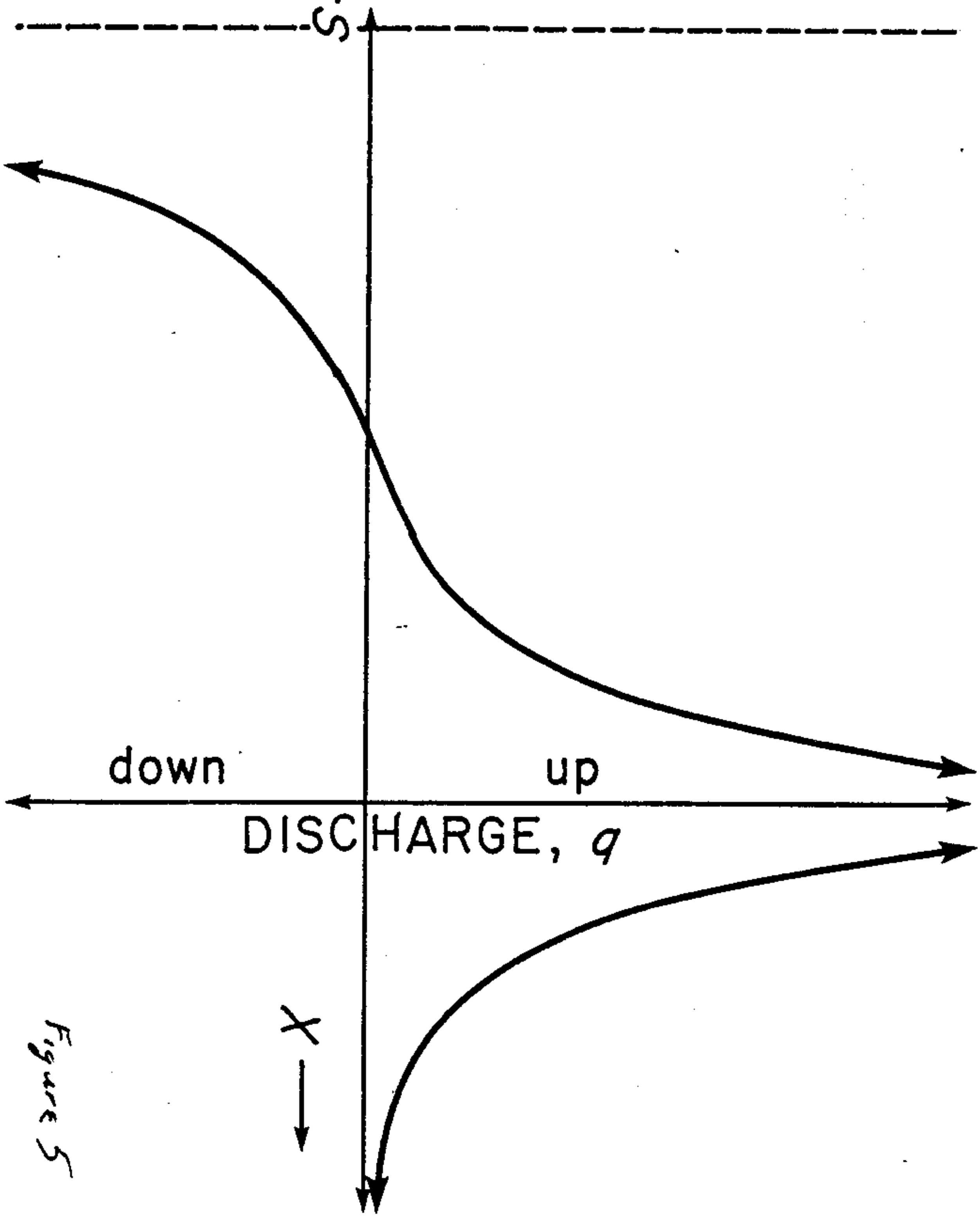

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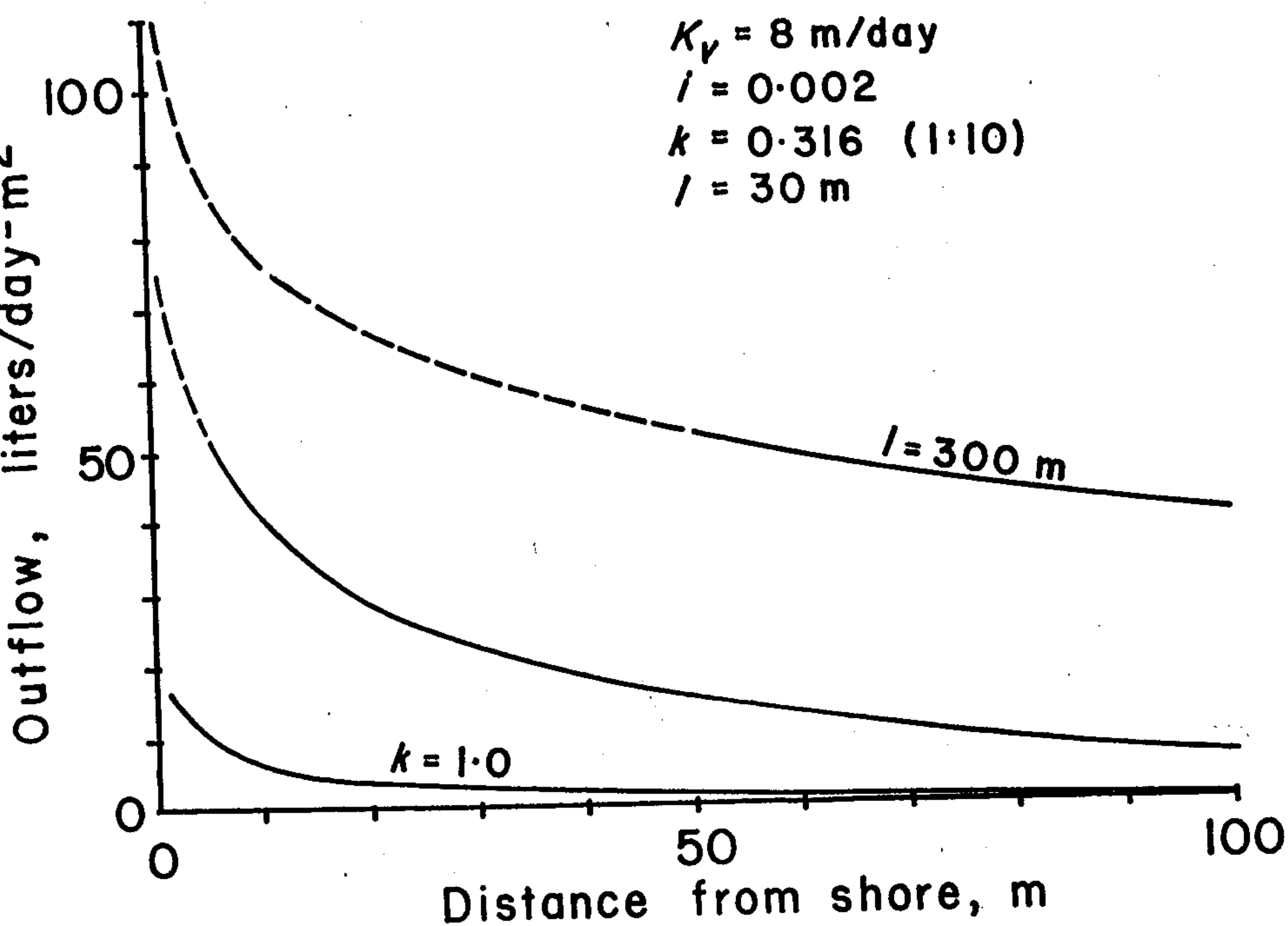

Kv = 8 m/day
i = 0·002
k = 0·316 (1:10)
l = 30 m
l = 300 m
k = 1·0
100
50
0
0
50
100
Outflow, liters/day-m2
Distance from shore, m

Figure 6

IX

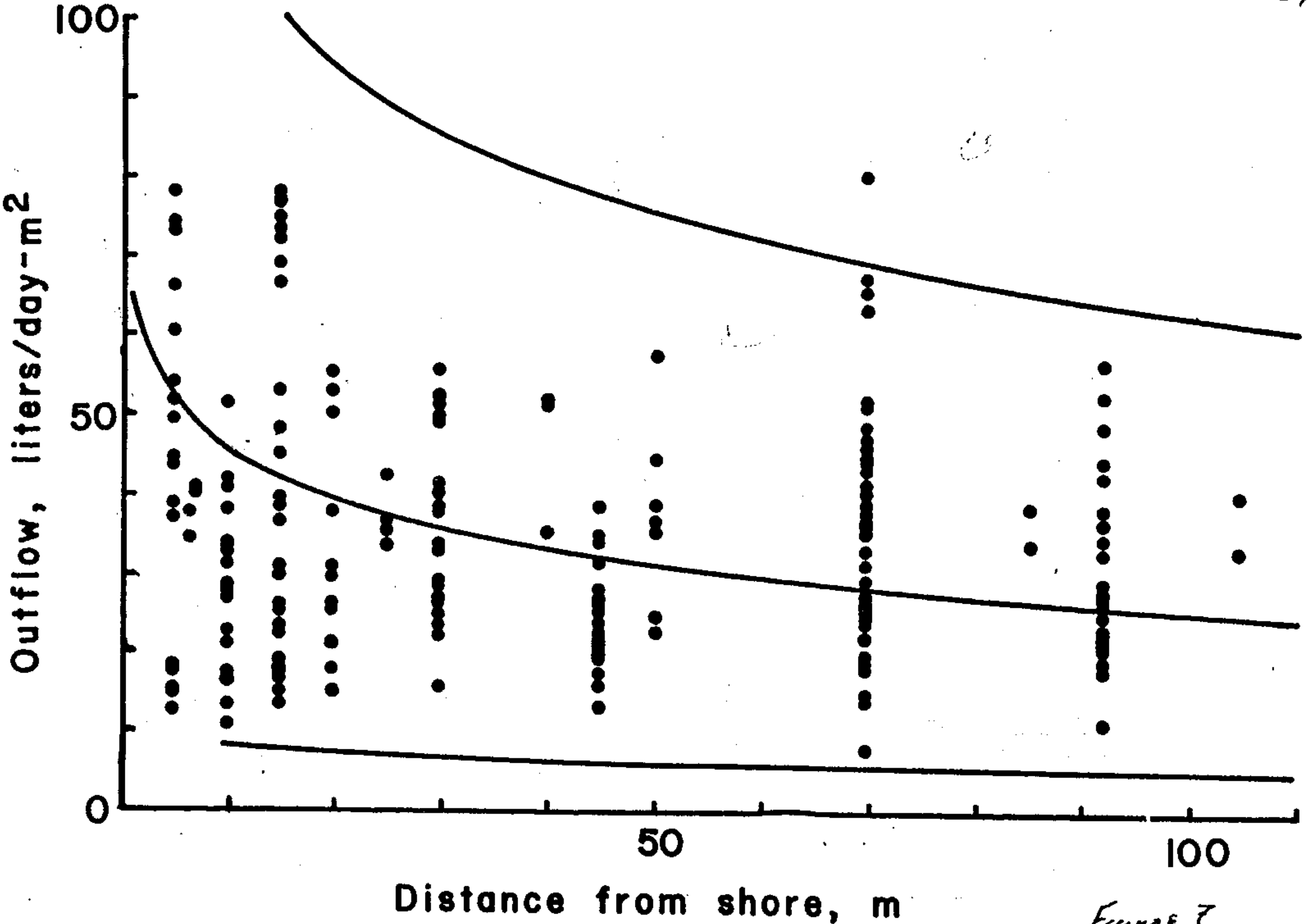

Figure 7