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MICROCOMPUTER MODELING OF COLLISION TOLERANT PILE STRUCTURES DYNAMICS

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ABSTRACT

Computer programs have been written for simulating the dynamics of Collision Tolerant Pile Structures (CTPS's) under storm and collision conditions. CTPS systems are intended for the deployment of navigation aids and have the ability to sustain collisions by marine traffic (mainly barges). A CTPS consists of the aid itself mounted to the top of a rigid pile which is hinged just above the mud line. The hinge is omnidirectional and allows the pile to fold down in the event of the collision. The hinge also provides a restoring moment to return the structure to the vertical position.

To serve as a research, development and design tool, computer models have been developed for predicting pile motion, major forces and force impulses. Computer programs which have been modified for use on IBM compatible microcomputer systems are presented in this report. The programs in addition have the ability to account for the effect of buoyancy chambers which have been incorporated into the most recent CTPS designs. Separate programs for storm conditions, barge collision knockdowns and pile recovery are included. In each program the CTPS is modeled as a rigid beam - flexible hinge system with loadings and resulting motions confined to a vertical plane. Wind is considered steady; wave action is represented by a regular, sinusoidal wave, and current is taken to be uniform with depth. During the collision processes, the barge is assumed to maintain constant speed. The analysis is based principally on the time rate of change of angular momentum equation applied about the hinge pivot point. In the case of barge - pile contact, the interaction geometry also serves a fundamental role in the mathematical model.

This report presents both a detailed theoretical development and a "how to use" section for each program. Documentation includes sample runs and listings (written in BASIC). Utility programs used to support the effective implementation of the main programs are also presented.

NOMENCLATURE

A_b	=	area of boards
B_w	=	bell width
C_a	=	drag coefficient of pile in air
C_m	=	inertia (added mass) coefficient of pile
C_w	=	drag coefficient of pile in water
d	=	depth to hinge
d_b	=	draft of barge
d_p	=	diameter of pile
d_t	=	total depth
F_b	=	barge force on pile
f_b	=	barge freeboard
F_c	=	current force on pile
g	=	gravitational constant
h_w	=	wave height
I_m	=	pile mass moment of inertia about hinge
I_h	=	pile system moment of inertia with added mass
k	=	wavenumber
k_1	=	initial stiffness constant
k_2	=	large angle stiffness constant
l	=	barge length
L	=	wavelength
l_c	=	length from hinge to point of barge contact
l_p	=	pile length
l_w	=	length to center of gravity
l_s	=	pile submerged length

l_1	=	length to bottom of buoyant chamber
l_2	=	length to top of buoyant chamber
l_3	=	length to centroid of submerged volume
M	=	applied moment about hinge axis
m_a	=	added mass
M_b	=	moment exerted by barge
M_c	=	moment due to relative water movement
M_g	=	weight/buoyant moment
M_h	=	hinge moment
m_p	=	mass of pile
M_w	=	wind moment
R	=	reaction force
R_h	=	horizontal base reaction force
R_v	=	vertical base reaction force
s	=	coordinate measured parallel to pile
T	=	wave period
t	=	time
U_a	=	wind velocity
U_b	=	barge velocity
U_c	=	current velocity
u_r	=	water velocity relative to pile
$\dot{u}_r$	=	water acceleration relative to pile
u_w	=	horizontal component of wave fluid velocity
v_w	=	vertical component of wave fluid velocity
W	=	weight of pile
x	=	horizontal coordinate measured from hinge position
y	=	vertical coordinate measured from the equilibrium waterline

θ	=	angle of pile with respect to the vertical
θ_b	=	hinge moment breakpoint angle
θ_c	=	angle between pile and barge force
ρ_a	=	air density
ρ_w	=	water density
ω	=	angular velocity
σ	=	wave radian frequency
ϕ	=	angle of friction

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I. INTRODUCTION

BACKGROUND

Rigid pile structures currently used by the Coast Guard to support navigation aids in shallow channels are susceptible to collision by marine vehicles especially towed barges. The expense involved in replacing damaged markers can be reduced by replacing existing rigid pile supports with compliant Collision tolerant Pile Structures (CTPS's). A CTPS is a single pile, hinged at the mud line, on which the navigation aid can be mounted as shown in Fig. 1. The hinge allows the pile to fold down during the vessel/pile collision yet retains a restoring moment to return the pile to an upright position. Detailed descriptions of CTPS design concepts under development are provided by Swift and Baldwin (1985, 1986) and Baldwin et al. (1987).

To support the initial design development efforts, computer models were developed to predict CTPS dynamics during storm conditions, collision knockdown and pile recovery. The original computer simulations, described in the above references, were vertically 2-dimensional in which the CTPS is modeled as a rigid beam - flexible hinge system. These early programs were written in BASIC and implemented on an Apple IIe microcomputer.

Model predictions were compared with experimental observations with reasonably good correspondence as reported by Swift and Baldwin (1985, 1986), Mielke (1987) and Baldwin et al. (1987). The programs, however, were slow to run. Also, recent CTPS design changes, including the adoption of the central stay hinge concept and the use of buoyancy, required that the models be updated.

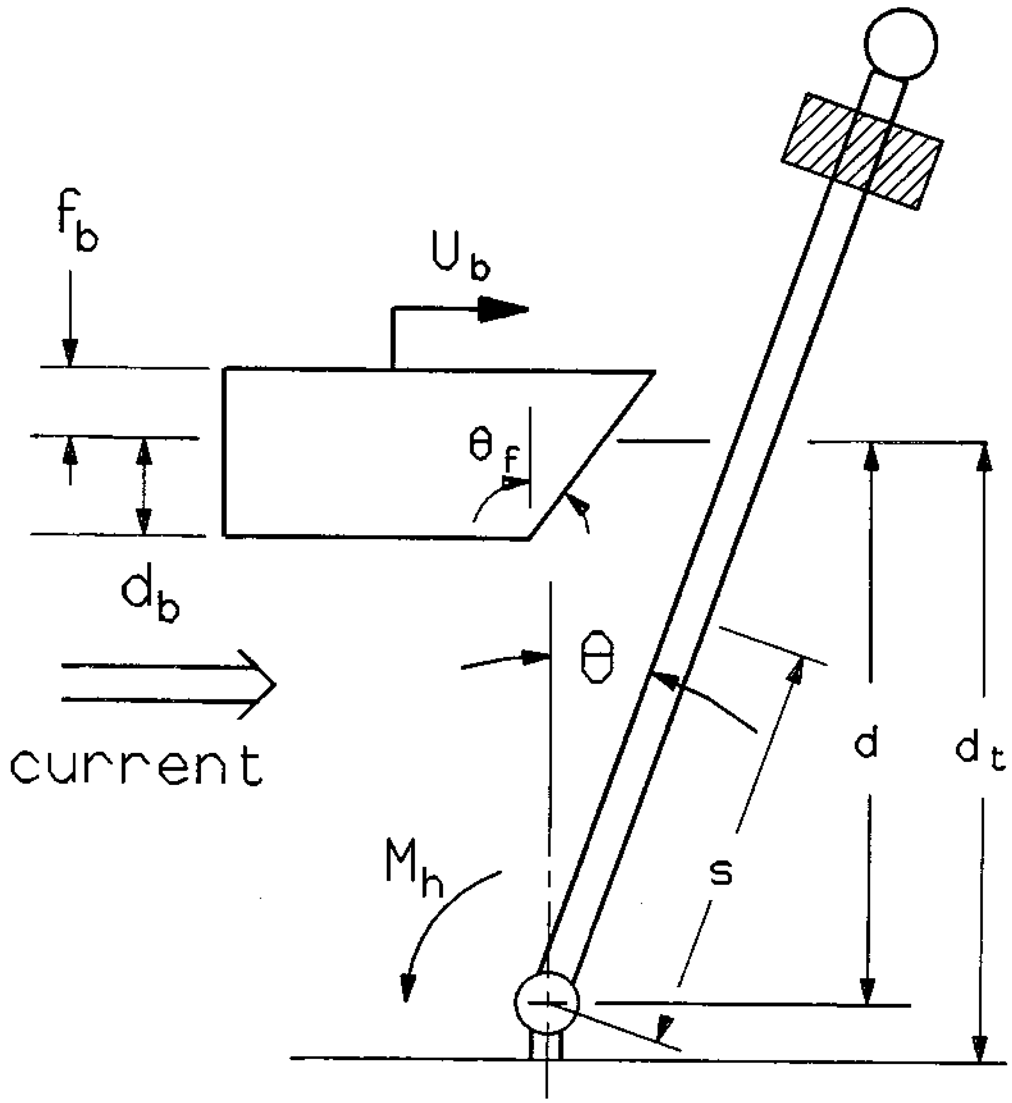

Fig. 1. The CTPS concept with barge and pile nomenclature.

In the computer models presented here, the necessary program modifications have been incorporated. The coding has been re-written for use on IBM compatible microcomputers. Trials using an AT&T 6300 indicate that run times are significantly reduced. Methods for handling input and output data are more flexible, and an output plotting capability has been added.

MODELING APPROACH

Three different principal programs are developed and correspond to the following situations: large amplitude motion during storms, the knockdown phase of a collision and pile recovery after a hit. In all models the CTPS is considered a rigid beam - flexible hinge system (as shown in Fig. 2) in which the hinge possesses restoring moment stiffness. Loadings and resulting motions are restricted to a vertical plane. The general governing equation is the time rate of change of angular momentum equation applied about the hinge pivot point.

$$I_m \ddot{\theta} = M_{applied} \, , \tag{1}$$

where I_m = mass moment of inertia about the hinge and θ = inclination angle with respect to the vertical. (Terminology is summarized in the NOMENCLATURE section). Applied moments may be due to the hinge, gravity/buoyancy, wind, fluid motion relative to the pile (due to steady current, waves and pile movement) and barge contact depending on model application. In the wave forcing and recovery programs, Eq. 1 is solved numerically for θ using a Runge-Kutta approach. In the collision knockdown program, it is assumed that barge speed remains constant and contact is maintained with the pile. Inclination angle is found independently from the barge/pile geometry so that Eq. 1 can be used to find the barge forcing moment and, with more geometrical and dynamic analysis, the barge contact force and base reactions.

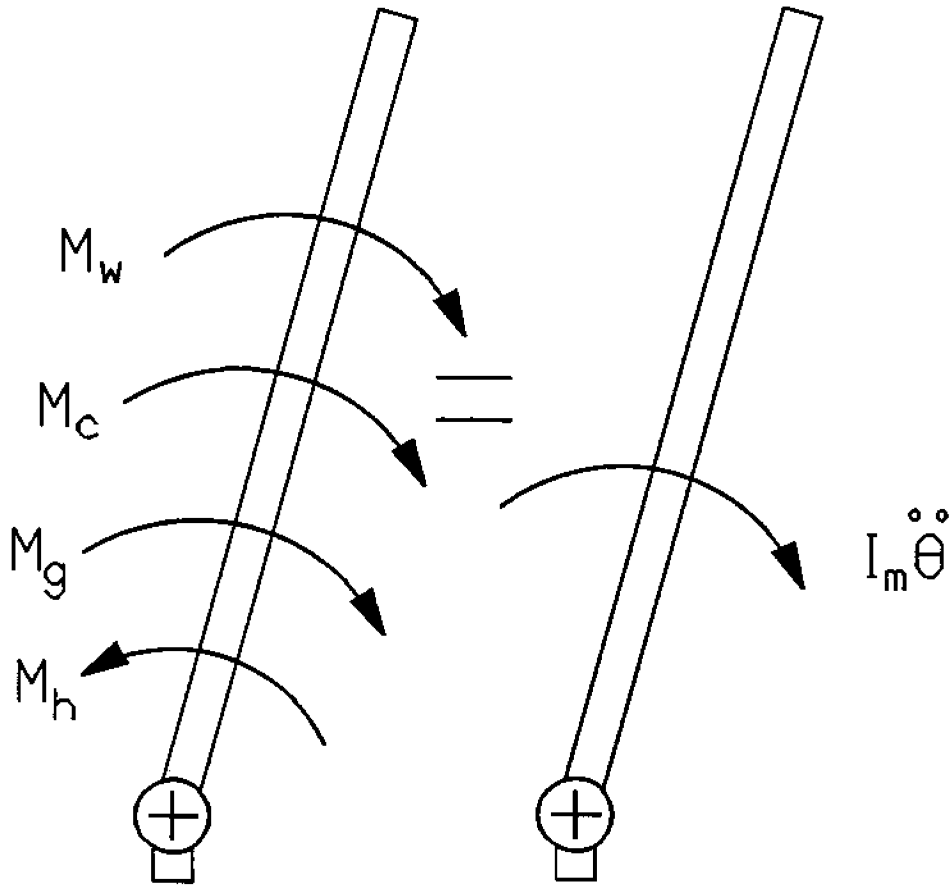

Fig. 2. Applied moments set equal to time rate of change of angular momentum about the hinge pivot point.

The three principal programs - STORM, COLLISION and RECOVERY - are each developed in a separate chapter. The chapters contain a detailed presentation of the mathematical theory. Following the theoretical development, there is a "how to use" section describing the steps required for running the program. Documentation includes sample runs and listings. It is structured so that the reader can go directly to the "how to" sections and make use of the programs.

II. STORM CONDITIONS

PURPOSE

The principal purpose of the STORM model is to predict pile motion as a function of time during severe wind, wave and current forcing. Large amplitude inclination angles are assumed to exist. Most of this chapter is devoted to developing the mathematical theory governing the pile dynamics. The program STORM based on these equations is described in the last sections. Instructions on how to use the program and the program listing are provided at the end of the chapter.

MAJOR ASSUMPTIONS

Important assumptions for the STORM model beyond the general modeling aspects described previously are:

1. Pile motion is restricted to a vertical plane. The worst case situation is considered in which wind, wave and current directions are collinear. This restriction vastly reduces the complexity of the analysis since there is only one degree of freedom (the inclination angle).
2. The light daymark boards are taken to be sacrificial. These boards are relatively inexpensive to repair or replace and are therefore meant to snap off the pile during a hurricane.
3. Wave motion consists of a regular, sinusoidal, hydrodynamic surface wave.
4. Current is assumed uniform with depth.

APPROACH

The time rate of change of angular momentum equation, Eq. 1, is applied at the hinge of the pile to relate the pile's angular acceleration to the externally applied moments ($I_M\ddot{\theta} = M_{applied}$). Eq. 1 is then numerically integrated using the Runge-Kutta method (see APPENDIX C) to determine the angle (θ) at different times. Moment contributions are due to the current and wave motion (M_c), the weight/buoyancy moment of the pile (M_g), the hinge moment (M_h) and the wind moment (M_v). The total applied moment ($M_{applied}$) at the hinge is then

$$M_{applied} = -M_h + M_g + M_c + M_v \quad . \tag{2}$$

The directions of these moments are indicated in Fig. 2.

MOMENT CALCULATIONS

Objective

The objective of this section is to derive expressions for the pile moment loadings in terms of given parameters and the dependent inclination angle variable. The four moments of interest are the hinge moment, the current moment, the wind moment and the weight/buoyancy moment.

Hinge Moment

The hinge restoring moment is only a function of the pile's angle (θ), and is assumed to be piece-wise linear as shown in Fig. 3. The hinge's spring constant at small angles has a value of k_1 in the region $-\theta_b < \theta < \theta_b$, where θ_b is the hinge breakpoint angle. For $\theta > \theta_b$ or $\theta < -\theta_b$, the spring constant is reduced to a value of k_2. For each region in Fig. 3, equations are found relating the hinge moment to the pile's angle. The three equations are:

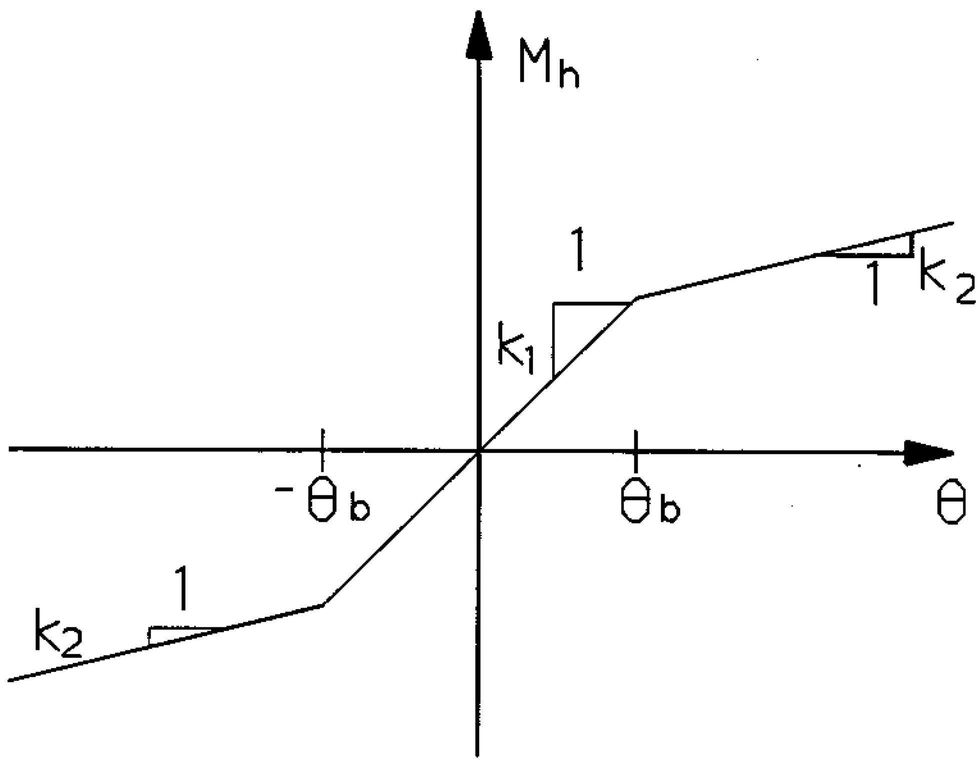

Fig. 3. Piece-wise linear hinge moment behavior. Samall angle stiffness is k_1; large angle stiffness is k_2, and the breakpoint angle is θ_b.

$$M_h = k_1\ (\theta) \qquad \text{for } -\theta_b < \theta < \theta_b$$

$$M_h = k_2\ (\theta - \theta_b) + k_1\ (\theta_b) \qquad \text{for } \theta > \theta_b \tag{3}$$

$$M_h = -k_1\ (\theta_b) + k_2\ (\theta + \theta_b) \qquad \text{for } \theta < -\theta_b\ .$$

From these three equations, M_h can be found given a value of θ.

Weight/Buoyancy Moment

The weight/buoyancy moment M_g is found hydrostatically as a function of θ only. The expression for M_g is determined by separately adding the moment due to weight (M_{weight}) to the buoyancy moment ($M_{buoyancy}$). See Fig. 4 for the central stay design; for the central universal joint configuration, $B_w = 0$.

The moment due to weight is evaluated from the cross product between the position vector $\bar{r}_{ow}$ and the weight vector $\bar{W}$ as

$$\bar{M}_{weight} = \bar{r}_{ow} \times \bar{W}\ . \tag{4}$$

From Fig. 4, $\bar{r}_{ow}$ is

$$\bar{r}_{ow} = (-B_w \cos\ (\theta)\ \text{sgn}\ (\theta) + l_w \sin\ (\theta))\bar{i}$$

$$+ (B_w|\sin(\theta)| + L_w \cos\ (\theta))\bar{j} \tag{5}$$

where B_w = base width and l_w = length to pile center of gravity. The weight vector is $\bar{W} = -\ m_p\ g\ \bar{j}$. Therefore the moment due to weight from Eq. 4 becomes

$$\bar{M}_{weight} = (B_w \cos(\theta)\ \text{sgn}(\theta) - L_w \sin(\theta))\ W\ \bar{k} \tag{6}$$

The buoyant force acting on the pile, F_b is the weight density of the water times the volume displaced resulting in

$$F_b = \gamma\ V \tag{7}$$

where γ = specific weight of water and V = submerged volume of the pile. It acts upward through the centroid of the submerged volume. The buoyant moment

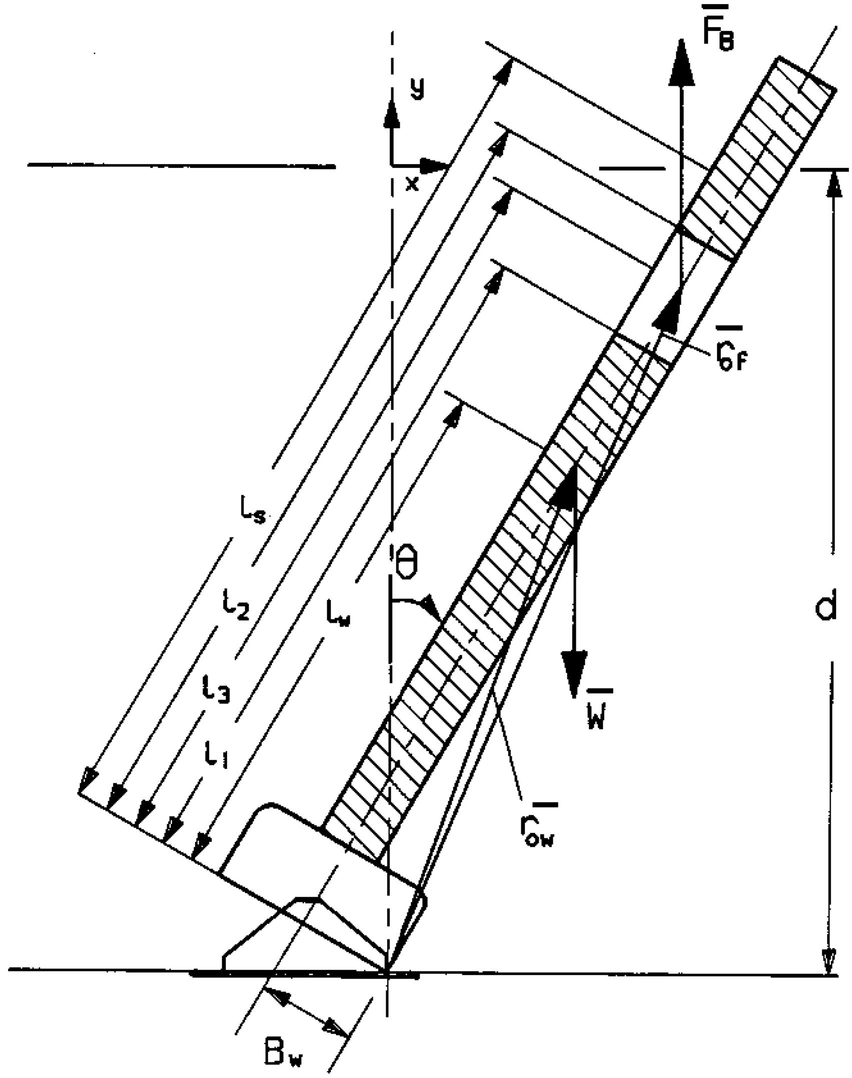

Fig. 4. STORM model nomenclature illustrated for the central stay design concept.

is then the cross product of the centroid position vector $\bar{r}_{of}$ and the buoyant force $\bar{F}_b$ so that

$$\bar{M}_{buoyant} = \bar{r}_{of} \times F_b \, \bar{j} \tag{8}$$

It will further be assumed that the displaced volume is negligible except for a foam filled buoyancy section extending from l_1 to l_2 (see Fig. 4). The lengths l_1, l_2 are distances from the hinge to the bottom, top of the chamber, respectively. To determine the buoyant force F_b, the pile's submerged length l_s first has to be known. Distance l_s is found as a function of the pile angle θ using the geometry shown in Fig. 5. The results are:

$$l_s = d \,/\, \cos\theta \quad \text{for} \quad l_s < l_p \tag{9}$$

$$l_s = l_p \qquad\qquad \text{for} \quad l_s > l_p$$

where l_p = length of pile and d = depth to hinge.

Three cases of the buoyant moment are considered depending on the submerged length. For $l_s < l_1$, the chamber is above water, therefore $F_b = 0$ and there is no buoyant moment contribution.

For $l_s > l_2$, the chamber is fully submerged as shown in Fig. 6. The distance to the centroid of the submerged chamber is

$$l_3 = (l_1 + l_2) \,/\, 2,$$

and the submerged volume is

$$V = (l_2 - l_1)\pi d_p^{\,2} \,/\, 4. \tag{10}$$

For $l_1 > l_s > l_2$, the chamber is partially submerged and

$$l_3 = (l_1 + l_s) \,/\, 2. \tag{11}$$

From Fig. 7 the submerged volume is found as

$$V = (l_s - l_1)\pi d_p^{\,2} \,/\, 4. \tag{12}$$

Having determined F_b and l_3, the buoyant moment given by Eq. 8 can be evaluated. By Fig. 4 geometry, the position vector $\bar{r}_{of}$ is

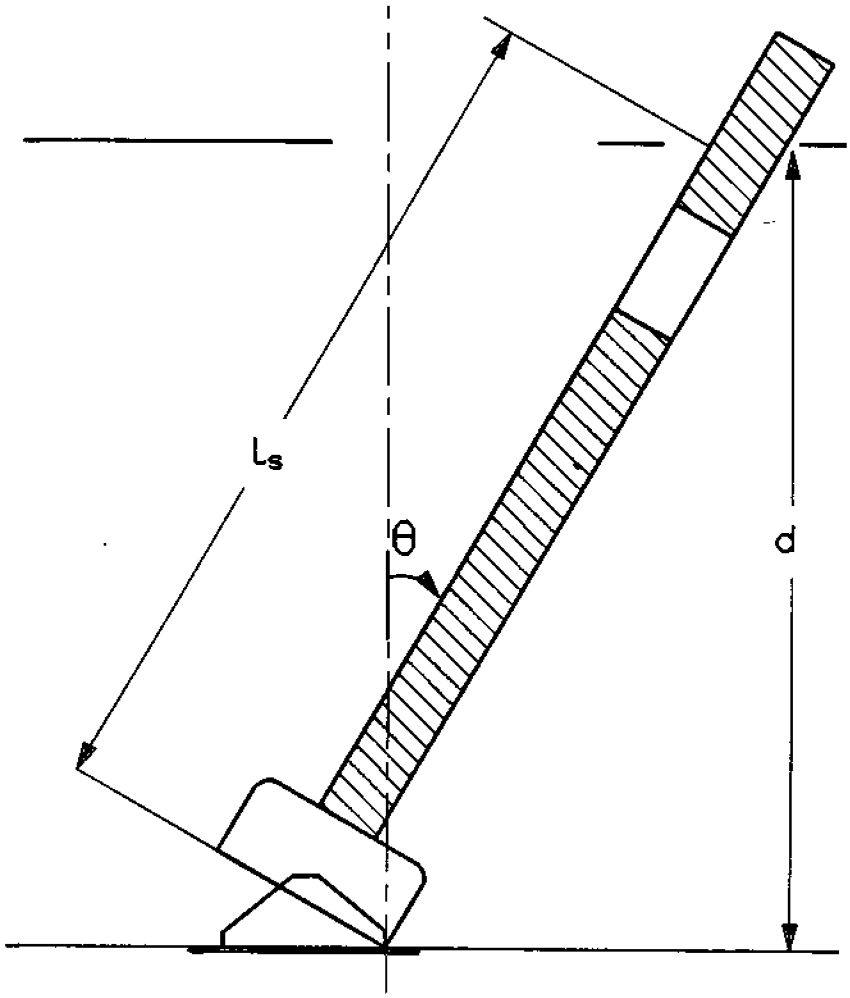

Fig. 5. Pile geometry to find the submerged length l_s.

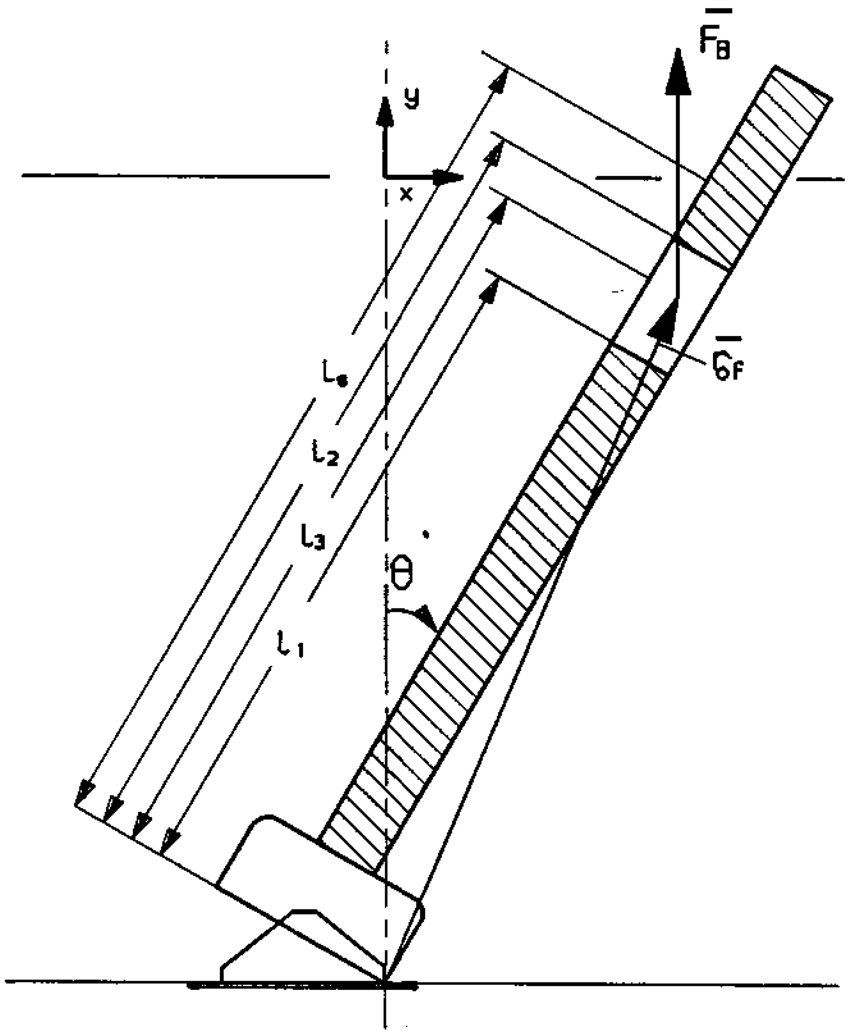

Fig. 6. Case where the buoyant section is fully submerged ($l_s > l_2$).

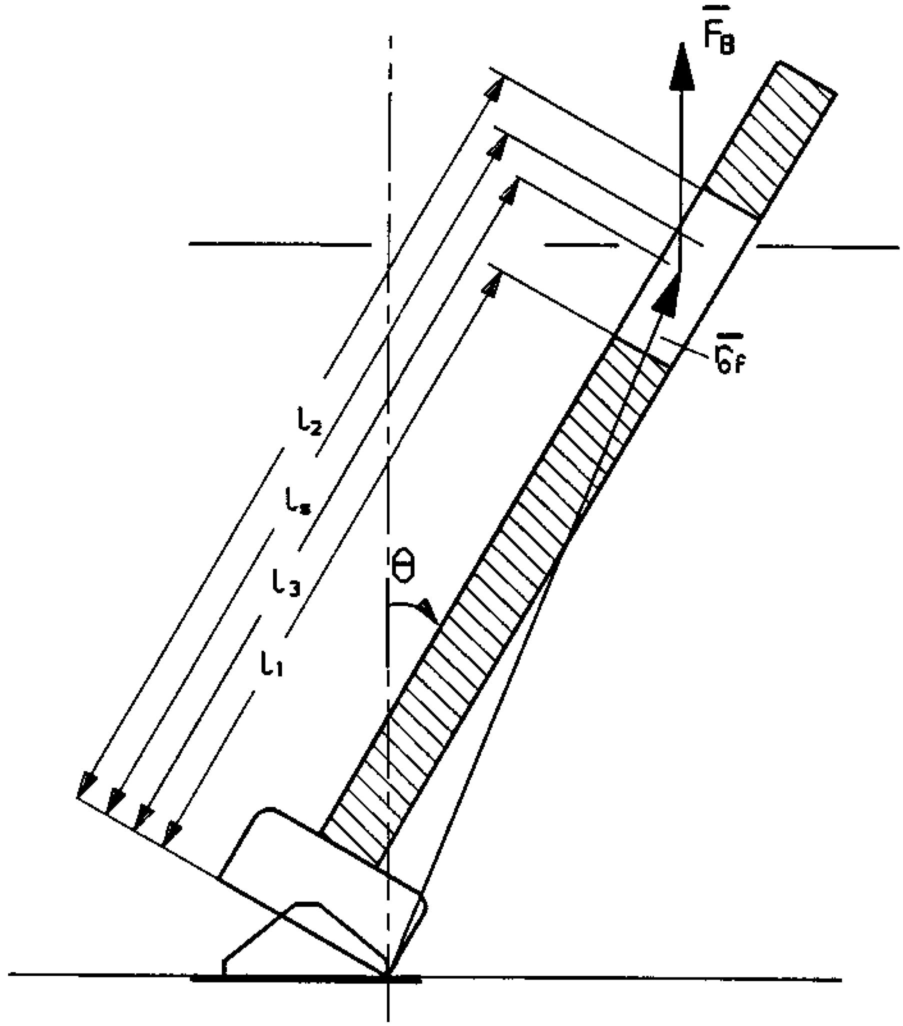

Fig. 7. Case where the buoyant section is partially submerged ($l_1 < l_s < l_2$).

$$\bar{r}_{of} = (-B_w \cos\theta \, \mathrm{sgn}(\theta) + l_3 \sin\theta)i +$$

$$(B_w |\sin\theta| + l_3 \cos\theta)\bar{j} \; . \qquad (13)$$

Evaluating the buoyant moment given by Eq. 8 by crossing the position vector $\bar{r}_{of}$ with the buoyant force $\bar{F}_b = F_b\bar{j}$ yields

$$\bar{M}_{buoyant} = (-B_w \cos\theta \, \mathrm{sgn}(\theta) + l_3 \sin\theta)\,(F_b)\,\bar{k} \; . \qquad (14)$$

The total weight/buoyant moment M_g is the sum of the weight moment and the buoyant moment. Summing Eq. 14 and Eq. 6 yields the expression

$$\bar{M}_g = ((l_3 F_b - l_w W)\sin\theta - (F_b - W)\, B_w \cos\theta \, \mathrm{sgn}\,(\theta))\bar{k} \; . \qquad (15)$$

Wind Moment

In this section an expression for the overturning wind moment as a function of θ is determined. The analysis is based on projected area and the use of a drag coefficient approach.

Additional assumptions are that the wind is steady and is much larger than the maximum pile velocity. The wind moment M_w is found as the product of the wind drag force resultant F_w and the vertical distance from the hinge to drag force resultant (see Fig. 8).

It should be noticed that when the pile tip submerges, the wind moment M_w becomes zero. From Fig. 5

$$M_w = 0 \qquad\qquad \text{for } l_p \cos\theta < d. \qquad (16)$$

The pile tip is above water when $l_p \cos\theta > d$. From fluid mechanics, the resultant wind drag force can be expressed as

$$(\text{Force})_{wind} = \text{area } C_a \, \rho_a \, U_a^{\,2} \,/\, 2 \qquad (17)$$

where C_a = drag coefficient of the pile in air, ρ_a = density of air, U_a = air velocity and area = pile area normal to the wind. From Fig. 8, the moment arm for the drag force is

$$(\text{moment arm})_{wind} = (l_p \cos\theta + d) \,/\, 2 \; . \qquad (18)$$

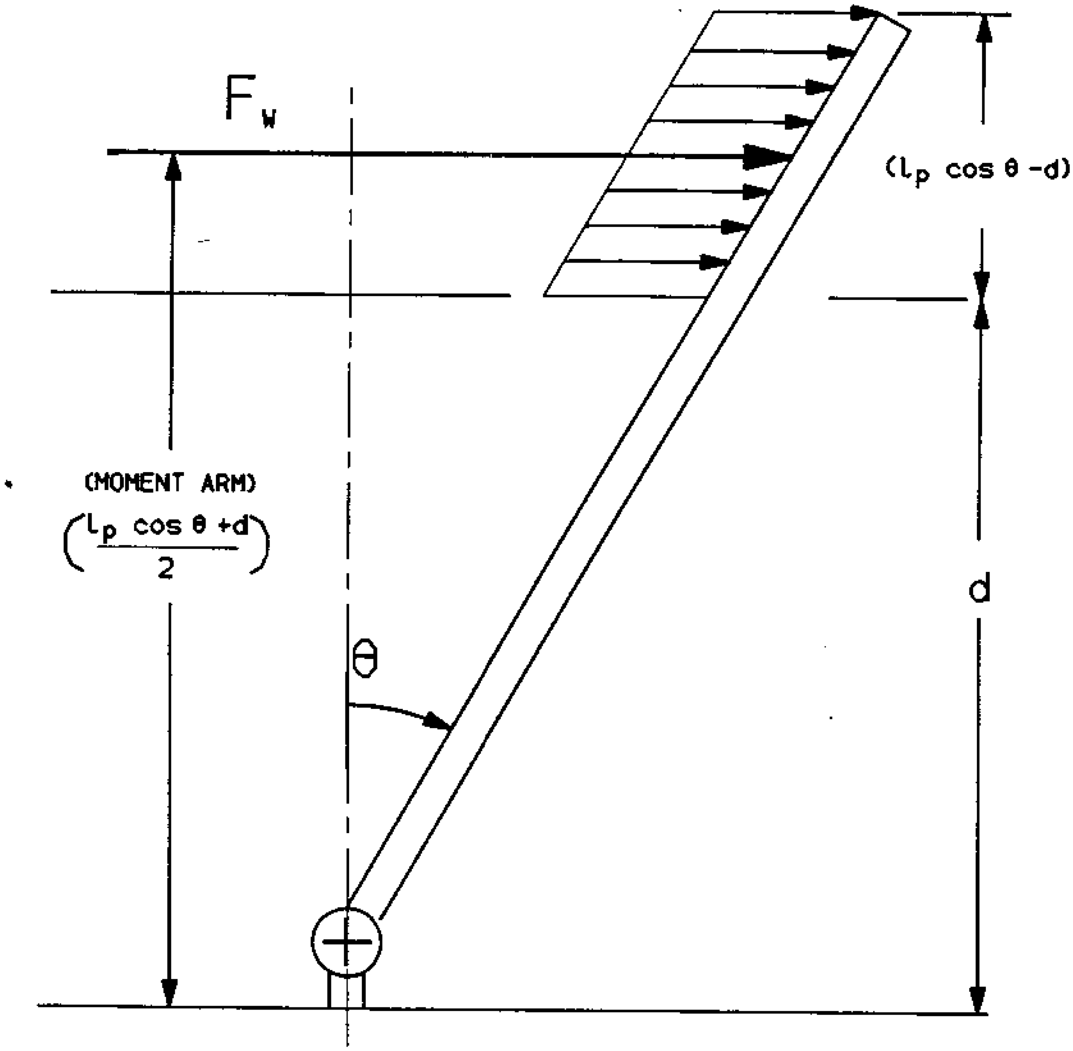

Fig. 8. Location of the resultant wind drag force.

Also the pile's exposed area to the wind is

$$\text{area} = d_p\ (l_p \cos\theta - d) \tag{19}$$

where d_p = pile diameter. The wind moment is the product of Eq. 17,18,19 resulting in

$$M_w = (l_p^{\ 2} \cos^2\theta - d^2)\ \rho_a\ C_a\ d_p\ U_a^{\ 2} / 4\ . \tag{20}$$

Current Moment/Force

In this section the total current moment (M_c) in terms of θ, $\dot{\theta}$, $\ddot{\theta}$, $\bar{U}_c$, u_w, v_w, $\dot{u}_w$, $\dot{v}_w$ and L is developed. U_c is the current velocity, u_w and v_w are the horizontal and vertical components of water velocity due to wave motion, $\dot{u}_w$ and $\dot{v}_w$ are horizontal and vertical components of fluid acceleration due to wave motion and L = wavelength. The approach due to Morrison is used in which the fluid load on a differential length of the pile is divided into drag and inertial contributions. Total moment forcing is found by integrating along the pile, and "added mass" (inertia) effects are combined with the $I_m\ \ddot{\theta}$ term in the governing dynamic equation. In this presentation the fluid velocity field is considered first, then drag force/moments and inertial force/moments are developed.

From small amplitude wave theory, the vertical and horizontal components of the fluid at a height y and a distance x from a reference point on the equilibrium surface position (see Fig. 9) are

$$u_w = (\sigma H_w/2) \cos(kx - \sigma t) \cosh\ (k(d_t + y))/\sinh(k\ d_t) \tag{21}$$

and $$v_w = (\sigma H_w/2) \sin(kx - \sigma t) \sinh\ (k(d_t + y))/\sinh(k\ d_t) \tag{22}$$

where σ = wave radian frequency, H_w = wave height, d_t = total water depth and $k = 2\pi/L$.

Resolving v_w and u_w into a normal velocity to the pile yields

$$u_\perp = (u_w \cos\theta - v_w \sin\theta)\ . \tag{23}$$

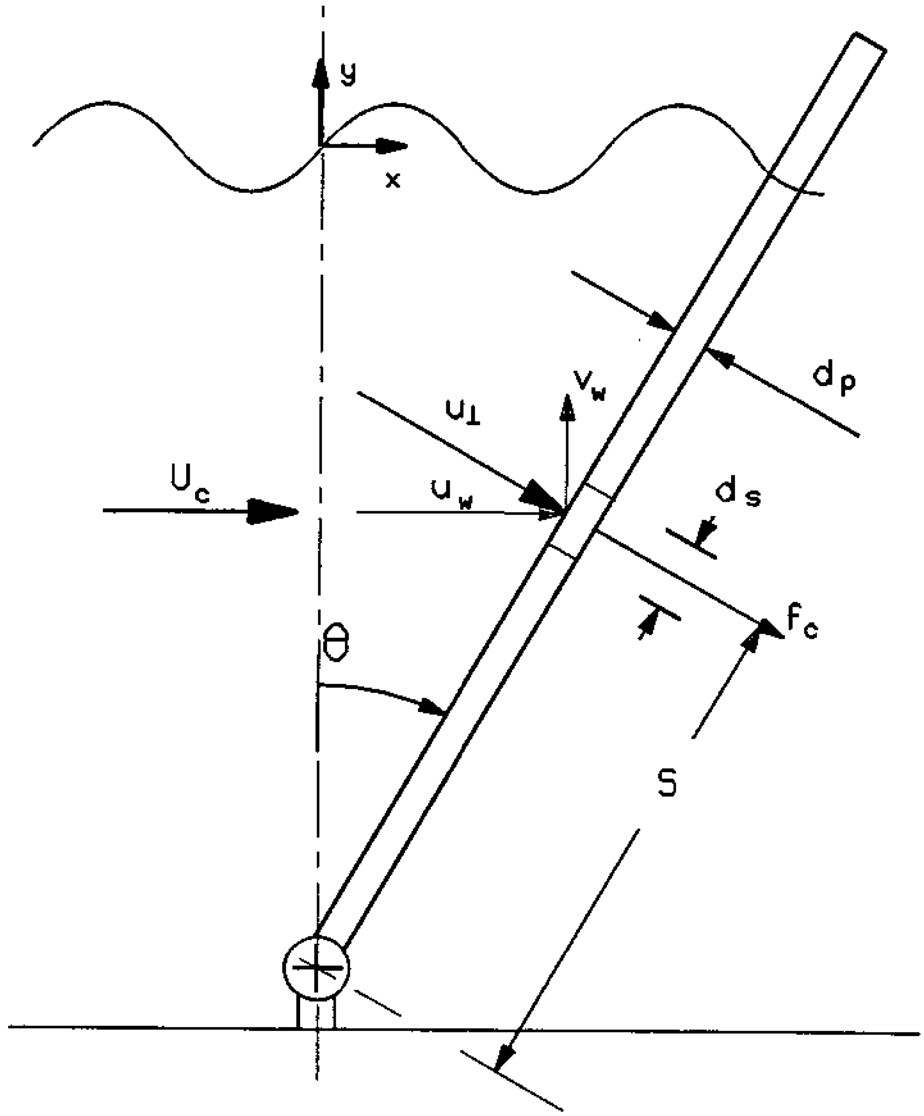

Fig. 9. Wave fluid velocity components and differential water force f_c acting on pile.

To find the fluid acceleration due to wave motion, the time derivatives of Eqs. 21 and 22 are taken resulting in

$$\dot{u}_w = (\sigma^2 H_w/2)(\cosh k(d_t+y)/\sinh k\, d_t)\, \sin(kx - \sigma t) \tag{24}$$

and

$$\dot{v}_w = -(\sigma^2 H_w/2)(\cosh k(d_t+y)/\sinh k\, d_t)\, \cos(kx - \sigma t)\ . \tag{25}$$

Resolving these components into a normal acceleration to the pile yields

$$\dot{u}_\perp = \dot{u}_w \cos\theta - \dot{v}_w \sin\theta\ . \tag{26}$$

The relative velocity of the water with respect to the moving pile a distance s away from the hinge (see Fig. 9) is

$$U_r = (u_\perp + U_c \cos\theta - s\,\dot{\theta}) \tag{27}$$

where U_c = current velocity. Substituting Eq. 23 into Eq. 27 gives the result

$$U_r = (u_w \cos\theta - v_w \sin\theta + U_c \cos\theta - s\,\dot{\theta})\ . \tag{28}$$

The relative acceleration of the fluid a distance s away form the hinge is found by taking the time derivative of Eq. 27. The relative fluid acceleration then becomes

$$\dot{U}_r = (\dot{u}_\perp - U_c (\sin\theta)\dot{\theta} - s\,\ddot{\theta}) \tag{29}$$

Substituting Eq. 26 into Eq. 29 gives the result

$$\dot{U}_r = \dot{u}_w \cos\theta - \dot{v}_w \sin\theta - U_c (\sin\theta)(\dot{\theta}) - s\,\ddot{\theta} \tag{30}$$

The differential drag force acting on the pile a distance s away from the hinge is

$$f_{cd} = (\rho_w/2)\, C_w\, U_r^2\, \mathrm{sgn}\,(U_r) dA \tag{31}$$

where the differential area $dA = ds\, d_p$, U_r = total relative velocity of the fluid with respect to the pile a distance s away from the hinge, and C_w = drag coefficient of the pile in water. To find the total current drag force, integrate Eq. 31 from s = 0 to l_s. This gives the equation for the total drag force F_{cd} as

$$F_{cd} = \int_0^{l_s} (\rho_w/2)\ C_w\ U_r^2\ d_p\ \mathrm{sgn}(U_r)\ ds\ , \tag{32}$$

where U_r is specified by Eq. 27.

The differential drag moment on the pile due to fluid motion is the differential drag force times the lever arm s. Therefore, the total drag moment on the pile is the integral of s times Eq. 31 integrated from s = 0 to l_s. This yields

$$M_{cd} = \int_0^{l_s} (s/2)\ \rho_w\ C_w\ d_p U_r^2\ \mathrm{sgn}(U_r)\ ds\ . \tag{33}$$

The differential inertial force on the pile due to relative fluid acceleration U_r is

$$f_{ci} = \pi(d_p^2/4)\ C_m\ \rho_w\ \dot{U}_r\ ds \tag{34}$$

where C_m = inertial coefficient of the pile in water. The total inertial force of the pile in water is the integral of the differential inertial force integrated from s = 0 to l_s. this gives

$$F_{ci} = \int_0^{l_s} fci = \int_0^{l_s} \pi(d_p^2/4)\ C_m\ \rho_w\ \dot{U}_r\ ds. \tag{35}$$

The relative velocity term $\dot{U}_r$ is substituted from Eq. 30 into Eq. 35 giving

$$F_{ci} = \int_0^{l_s} fci = \int_0^{l_s} \pi C_m\ \rho_w\ (d_p^2/4)(\dot{u}_w \cos\theta - \dot{v}_w \sin\theta - U_c (\sin\theta)\ \dot{\theta} - s\ \ddot{\theta})\ ds\ . \tag{36}$$

The term in Eq. 36 involving $\ddot{\theta}$ is separated from the others as

$$F_{ci} = \int_0^{l_s} C_m\ \rho_w \pi (d_p^2/4)(\dot{u}_w \cos\theta - \dot{v}_w \sin\theta - U_c (\sin\theta)\ \dot{\theta})\ ds - \int_0^{l_s} C_m\ p_w \pi (d_p^2/4)(s\ \ddot{\theta})\ ds. \tag{37}$$

The first integral in Eq. 37 is called $F_{ci'}$, while the second integral is referred to as F_{cia}. Therefore

$$F_{ci} = F_{ci'} + F_{cia} \tag{38}$$

where

$$F_{ci'} = \int_0^{l_s} C_m \rho_w \pi (d_p^{\,2}/4)(\dot{u}_w \cos\theta - \dot{v}_w \sin\theta - U_c \sin(\theta)\dot{\theta})ds \tag{39}$$

and

$$F_{cia} = -\int_0^{l_s} C_m \; \rho_w \pi (d_p^{\,2}/4)(s\;\ddot{\theta})\;ds. \tag{40}$$

Eq. 40 is next integrated to give the total added mass force as

$$F_{cia} = -(\pi/4)\;(C_m\;\rho_w\;d_p^{\,2}\;l_s)(l_s/2)\ddot{\theta}\;. \tag{41}$$

The term in Eq. 41, $(\pi/4)(C_m\;\rho_w\;d_p^{\,2}\;l_s)$ is equal to the mass of the displaced water times C_m and is termed the "added mass effect". This added mass can be regarded as a point mass located a distance $l_s/2$ away from the hinge and having a magnitude of

$$m_a = (\pi/4)\;C_m\;\rho_w\;d_p^{\,2}\;l_s. \tag{42}$$

The differential force contributions can likewise be separated according to

$$f_{ci} = f_{ci} + f_{cia} \tag{43}$$

where $f_{ci'}$, f_{cia} are the integrands of $F_{ci'}$, F_{cia} (respectively) times ds. The differential inertial moment, m_{ci} due to relative pile acceleration of the fluid a distance s away form the hinge, is next found by multiplying the differential inertial force by the its lever arm s. Thus m_{ci} is

$$m_{ci} = s\;f_{ci} = m_{ci'} + m_{cia} \tag{44}$$

where

$$m_{ci'} = s\;f_{ci'} \tag{45}$$

and

$$m_{cia} = s\;f_{cia}\;. \tag{46}$$

To find the total inertial moment, M_{ci}, Eq. 44 is integrated from s = 0 to $s = l_s$:

$$M_{ci} = \int_0^{l_s} s\ f_{ci} = \int_0^{l_s} s\ f_{ci'} + \int_0^{l_s} s\ f_{cia}$$

$$= M_{ci'} + M_{cia}\ . \tag{47}$$

The M_{cia} term is then to yield

$$M_{cia} = -(\pi/12)\ C_m\ \rho_w\ d_p^{\ 2}\ l_s^{\ 2}\ \ddot{\theta} \tag{48}$$

where M_{cia} is termed the "added inertia effect". The term in Eq. 48, $(\pi/12)$ $d_p^{\ 2}\ \rho_w\ C_m\ l_s^{\ 3}$, is equal to the mass moment of inertia of the column of water displaced times C_m and is called the added inertia (I_{added}) where

$$I_{added} = (\pi/12)\ d_p^{\ 2}\ \rho_w\ C_m\ l_s^{\ 3}\ . \tag{49}$$

This term may be combined with the left hand side of Eq. 1 by changing the pile effective moment of inertia to

$$I_h = I_m + I_{added} \tag{50}$$

where I_h = total effective moment of inertia. Since M_{cia} is separated from Eq. 44 and added to the pile's moment of inertia, only inertial contributions of M_{ci}' re used as direct forcing on the pile. The differential moment, $m_{ci'}$, is found by comparison of Eqs. 39, 43, 45 and 47 to yield

$$m_{ci'} = s\ C_m\ \rho_w \pi (d_p^{\ 2}/4)(\dot{u}_w\ \cos\theta - \dot{v}_w\ \sin\theta - U_c\ (\sin\ \theta)\dot{\theta})ds. \tag{51}$$

The total inertial forcing moment is therefore

$$M_{ci'} = \int_0^{l_s} m_{ci'} \tag{52}$$

where $m_{ci'}$ is given above in Eq. 51.

The total current moment acting on the pile is sum of the drag moment in Eq. 33 and the inertial moment in Eq. 52, namely

$$M_c = M_{cd} + M_{ci'}\ . \tag{53}$$

Likewise the total current force acting on the pile is the sum of the drag force in Eq. 32 and the inertial force in Eq. 39, namely

$$F_c = F_{ci'} + F_{cd}\ . \tag{54}$$

The trapezoidal rule is used to complete the current force and moment integrals. To numerically integrate the current force expression or the current moment equation along the pile's submerged length, the pile's submerged length must first be found by use of Eq. 9. In the analysis the pile is broken up into 10 equal segments with a length per segment of $l_s/10$. The trapezoidal rule formula applied to the force integration is then

$$F_c = \int_0^{l_s} f_c(s)\, ds = \left(\tfrac{1}{2}\, f_c(s_o) + \sum_{n=1}^{9} f_c(s_n) + \tfrac{1}{2}\, f_c(s_{10})\right) l_s/10 \qquad (55)$$

where $s_n = n(l_s/10)$. Moment integrations are done similarly.

STATIC EQUILIBRIUM ANGLE

At time = 0, the pile is started at a small nonzero angle θ_{static} where θ_{static} is the equilibrium angle the pile would maintain if the time dependent wave forcing was omitted. This approach is used to help reduce transient effects. To calculate the static equilibrium angle, all steady moment contributions are summed and set equal to zero. A small angle approximation is employed so that $\sin\theta_{static} = \theta_{static}$ and $\cos\theta_{static} = 1$. With these restrictions, the static moment equation becomes

$$M_c - M_h + M_w + M_g = 0. \qquad (56)$$

The hinge moment M_h is for small angles given by

$$M_h = k_1\, \theta_{static}\;. \qquad (57)$$

The weight/buoyant moment is expressed by

$$M_g = (-l_3\, F_b + l_w\, W)\, \theta_{static} + (F_b - W)\, B_w\;. \qquad (58)$$

The wind moment reduces to

$$M_w = (l_p^{\,2} - d^2)\, U_a^{\,2}\, d_p C_a\, \rho_a/4\;. \qquad (59)$$

The current moment under static conditions is

$$M_c = \int_0^{l_s} (s/2)\ \rho_w\ C_w\ d_p\ U_c^{\,2} d_s = l_s^{\,2}\ \rho_w\ U_c^{\,2}\ C_w\ d_p/4\ . \tag{60}$$

Solving for the equilibrium angle by using Eq. 56 in combination with Eqs. 57 - 60 gives

$$\theta_{static} = ((l_p^{\,2} - d^2)\ \rho_a\ C_a\ d_p\ U_a^{\,2}\ /\ 4 + d^2\ \rho_w\ C_w\ d_p\ U_c^{\,2}\ /\ 4$$
$$- (W - F_b)\ B_w)\ /\ (k_1 + l_3\ F_b - l_w\ W). \tag{61}$$

SOLUTION PROCESS

Due to the added inertia effects of M_{cia}, the mass moment of inertia is replaced by I_h which is angle dependent. The time rate of change of angular momentum, Eq. 1, then becomes

$$\ddot{\theta} = (1\ /\ I_h)\ (-M_h + M_g + M_w + M_c)\ . \tag{62}$$

Eq. 62 is numerically integrated using the Runge-Kuta method (described in Appendix C) to solve for the pile angle as a function of time. Other dependent variables, such as M_h are evaluated in the solution process and can be printed out if wanted.

HOW TO USE THE "STORM" PROGRAM

The solution approach has been implemented in a program called STORM. Input can come either from an input file named "sin" or from the terminal. Input must be entered into this file before execution of the STORM program. Data can be entered into this file by the use of a word processor. Variables that are to be entered are named on the first page of the STORM program which is listed in Table 1. Input variable definitions are provided, but Figs. 1, 3 and 4 may also be helpful. List one variable per line as shown.

Since the program is written in BASIC, switch from the operating system to BASIC before execution. To load the STORM program, put the disk in drive b and type load "b:storm.bas". After loading STORM, the user can execute the program by typing run. A prompt will appear saying

Input from a file type f

or from terminal type t

Type t or f _______.

Another prompt will appear asking where to send the output. A message on the terminal is printed saying

If output is to a file type f

or else type 0

Type f or 0 _____.

If output is not desired to go to the output file "sout", then type a zero.

A prompt will then appear on the screen saying

If output is to the terminal type t

or else type 0

type t or 0 _____.

If output is not desired to dump out on the screen type a zero.

If input was requested to come from the terminal, then input the data when prompted. After all data has been entered, the main portion of the program will be executed. Output will be sent to the output file "sout" and/or to the terminal. This output consists of the pile angle, the hinge moment and the elapsed time as shown in Fig. 10.

For plotting purposes, information about the angle versus time will be stored in a file name "sth". Likewise, information on the hinge moment versus time will be stored in the file "smom". To plot the results within "sth" and "smom", see Appendix B. Output plots are shown in Figs. 11 and 12. Fig. 11 represents the pile angle versus time, and Fig. 12 is the pile hinge moment versus time.

```
static equil angle =  1.322281

time (s) =  0  theta (deg) =  1.322281 hinge moment (ft-lbs) =  3863.292
time(s) =  .2  theta(deg) =  1.559746  hinge moment (ft-lbs) =  4557.09
time(s) =  .4  theta(deg) =  2.142777  hinge moment (ft-lbs) =  6260.525
time(s) =  .6  theta(deg) =  2.879089  hinge moment (ft-lbs) =  8411.799
time(s) =  .8  theta(deg) =  3.589827  hinge moment (ft-lbs) =  10488.35
time(s) =  1   theta(deg) =  4.119949  hinge moment (ft-lbs) =  12037.2
time(s) =  1.2 theta(deg) =  4.346276  hinge moment (ft-lbs) =  12698.46
time(s) =  1.4 theta(deg) =  4.184139  hinge moment (ft-lbs) =  12224.75
time(s) =  1.6 theta(deg) =  3.592898  hinge moment (ft-lbs) =  10497.33
time(s) =  1.8 theta(deg) =  2.580043  hinge moment (ft-lbs) =  7538.081
time(s) =  2   theta(deg) =  1.202828  hinge moment (ft-lbs) =  3514.287
time(s) =  2.2 theta(deg) = -.4341804  hinge moment (ft-lbs) = -1268.54
time(s) =  2.4 theta(deg) = -1.74282   hinge moment (ft-lbs) = -5091.975
time(s) =  2.6 theta(deg) = -2.397519  hinge moment (ft-lbs) = -7004.802
time(s) =  2.8 theta(deg) = -2.322828  hinge moment (ft-lbs) = -6786.579
time(s) =  3.0 theta(deg) = -1.475946  hinge moment (ft-lbs) = -4312.254
time(s) =  3.2 theta(deg) =  .13912    hinge moment (ft-lbs) =  406.4651
time(s) =  3.4 theta(deg) =  2.007154  hinge moment (ft-lbs) =  5864.279
time(s) =  3.6 theta(deg) =  3.829402  hinge moment (ft-lbs) =  11188.32
time(s) =  3.8 theta(deg) =  5.542743  hinge moment (ft-lbs) =  16194.16
time(s) =  4.0 theta(deg) =  7.098201  hinge moment (ft-lbs) =  20738.73
time(s) =  4.2 theta(deg) =  8.466496  hinge moment (ft-lbs) =  24736.46
time(s) =  4.4 theta(deg) =  9.638084  hinge moment (ft-lbs) =  28159.47
time(s) =  4.6 theta(deg) =  10.61801  hinge moment (ft-lbs) =  31022.5
time(s) =  4.8 theta(deg) =  11.41695  hinge moment (ft-lbs) =  33356.75
time(s) =  5   theta(deg) =  12.04128  hinge moment (ft-lbs) =  35180.87
time(s) =  5.2 theta(deg) =  12.48507  hinge moment (ft-lbs) =  36477.48
time(s) =  5.4 theta(deg) =  12.72592  hinge moment (ft-lbs) =  37181.16
time(s) =  5.6 theta(deg) =  12.7253   hinge moment (ft-lbs) =  37179.35
time(s) =  5.8 theta(deg) =  12.43275  hinge moment (ft-lbs) =  36324.61
time(s) =  6   theta(deg) =  11.79307  hinge moment (ft-lbs) =  34455.67
time(s) =  6.2 theta(deg) =  10.7558   hinge moment (ft-lbs) =  31425.1
time(s) =  6.4 theta(deg) =  9.286594  hinge moment (ft-lbs) =  27132.53
time(s) =  6.6 theta(deg) =  7.379894  hinge moment (ft-lbs) =  21561.75
time(s) =  6.8 theta(deg) =  5.071445  hinge moment (ft-lbs) =  14817.18
time(s) =  7   theta(deg) =  2.44767   hinge moment (ft-lbs) =  7151.326
time(s) =  7.2 theta(deg) = -.3519593  hinge moment (ft-lbs) = -1028.315
time(s) =  7.4 theta(deg) = -2.704104  hinge moment (ft-lbs) = -7900.547
time(s) =  7.6 theta(deg) = -4.257643  hinge moment (ft-lbs) = -12439.5
time(s) =  7.8 theta(deg) = -4.910317  hinge moment (ft-lbs) = -14346.41
time(s) =  8.0 theta(deg) = -4.595206  hinge moment (ft-lbs) = -13425.76
time(s) =  8.2 theta(deg) = -3.296808  hinge moment (ft-lbs) = -9632.243
time(s) =  8.4 theta(deg) = -1.060432  hinge moment (ft-lbs) = -3098.25
time(s) =  8.6 theta(deg) =  1.771558  hinge moment (ft-lbs) =  5175.939
time(s) =  8.8 theta(deg) =  4.594225  hinge moment (ft-lbs) =  13422.89
time(s) =  9.0 theta(deg) =  7.264001  hinge moment (ft-lbs) =  21223.14
```

Fig. 10. Format of STORM output.

To view the output file "sout" when in the operating system (MSDOS), type type b:sout. To print the results in "sout" to the printer type copy b:sout prin.

A listing for STORM is provided in Table 1.

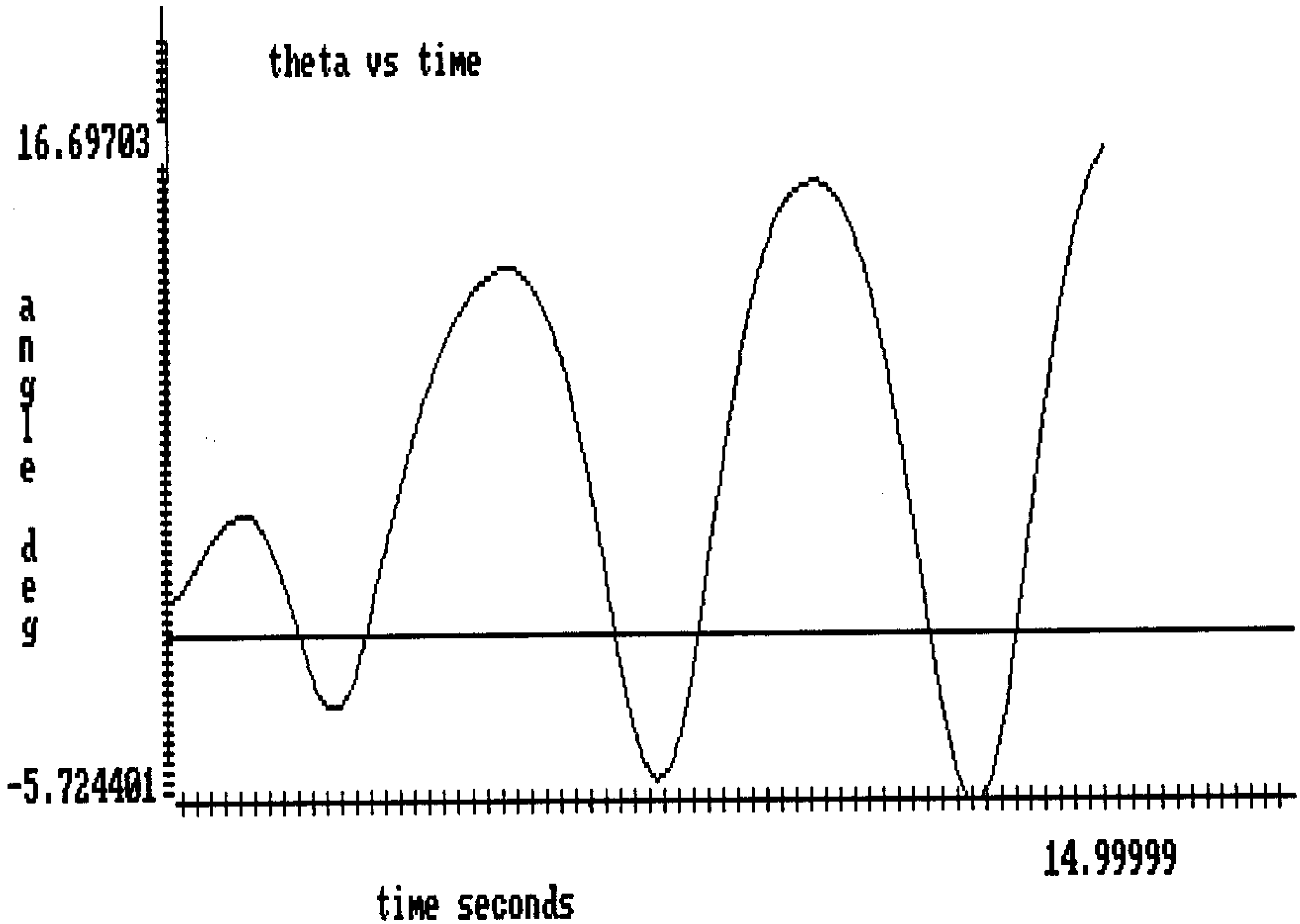

Fig. 11. Example pile inclination angle (θ) response calculated by STORM.

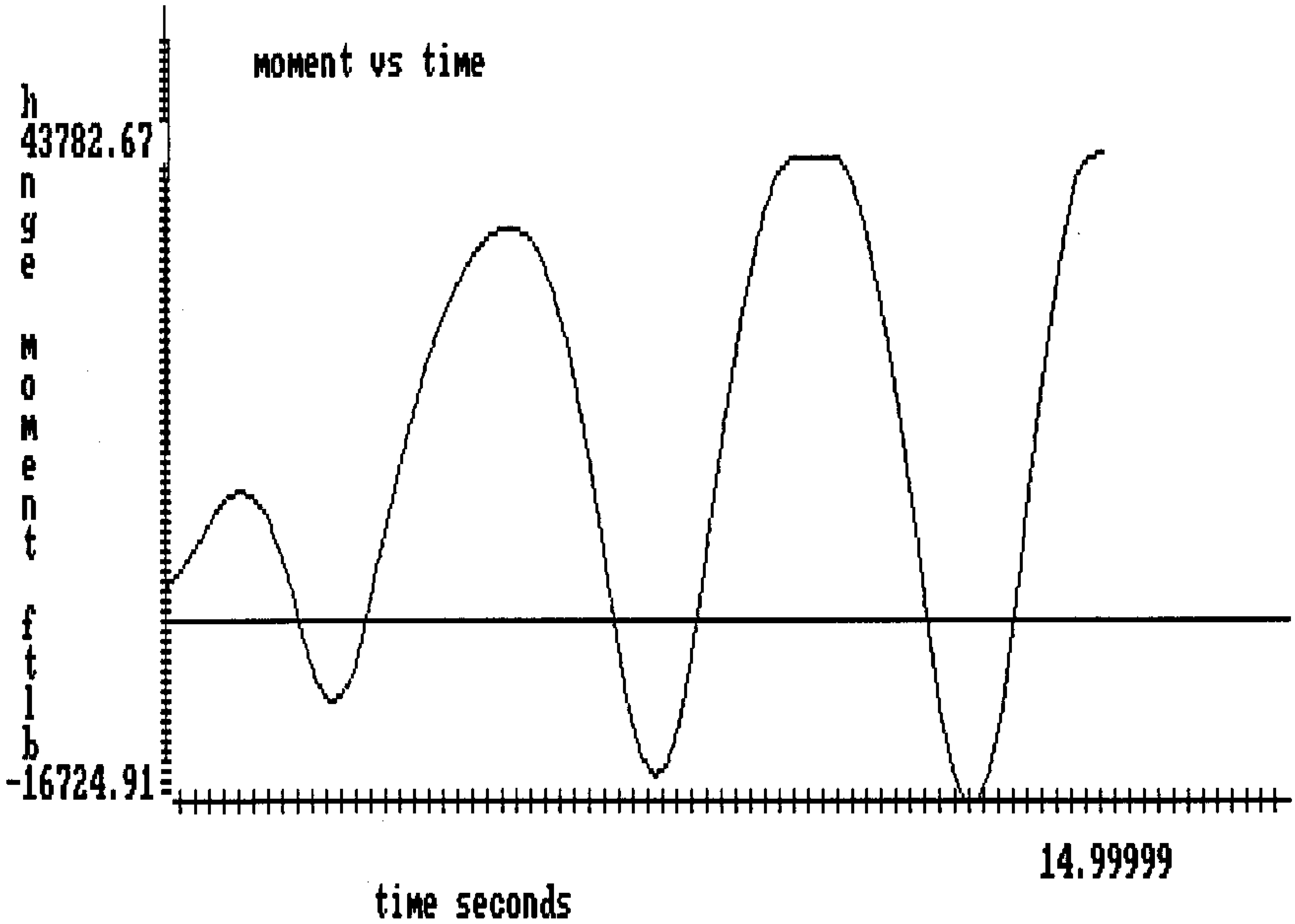

Fig. 12. Example hinge moment response calculated by STORM.

"STORM" LISTING

Table 1. Listing of STORM program.

```
5      REM  storm  models the central stay system (with buoyancy)
during extreme
10 REM   wind/wave conditions.
15 REM -----------------------
20 REM input variables are
25 REM  h       stepsize (s)
30 REM  max     maximum time (s)
35 REM  k1      1st stiffness constant (ft-lbs/rad)
40 REM  k2      2nd stiffness constant (ft-lbs/rad)
45 REM  ba      breakpoint angle (deg)
50 REM  im      mass moment of inertia about hinge (ft ^2-slugs)
55 REM  lp      pile length (ft)
60 REM  dp      pile diameter (ft)
65 REM  lw      length to center of gravity (ft)
70 REM  w       total weight (lbs)
75 REM  dt      total water depth (ft)
80 REM  d       depth to hinge (ft)
85 REM  l1      length to buoyancy section base (ft)
90 REM  l2      length to buoyancy section top (ft)
95 REM  bw      bell width (ft)
100 REM ua      wind velocity (ft/s)
105 REM uc      current velocity (ft/s)
110 REM hw      wave height (ft)
115 REM per     wave period (s)
116 PRINT "-----------------"
120 REM lambda wave length (ft)
125 REM itrvl$ number of time steps for output (integer)
130 REM -------------------------------------------
135 REM other variables are
140 REM btheta     breakpoint angle (rad)
145 REM dlta       trapezoidal integration step
150 REM fb         barge force (lbs)
155 REM fi         pile inertial force in water (lbs)
160 REM i          time counter for arrays
165 REM ih         mass moment of inertia + added mass
170 REM j          runge kutta  integration counter
175 REM l%         number of output values
180 REM lb         length to boyant centroid (ft)
185 REM ls         submerged length (ft)
190 REM m          angular acceleration for runge kutta
195 REM mw         wind moment array
200 REM mc         current moment array
205 REM mg         weight / boyant moment array
210 REM mh         spring moment
215 REM n          trapezoidal integration loop counter
220 REM omega      angular velocity array
225 REM s          length to integration on pile
230 REM sum        trapezoidal integration sum
235 REM fd         drag force
240 REM t          time array
245 REM theta      angle array
250 REM u          x dir water velocity due to wave
255 REM udot       x dir water acceleration due to wave
260 REM v          y dir water velocity due to wave
265 REM vdot       y dir water acceleration due to wave
270 REM x          x component of s
275 REM y          y component of s
280 REM ------------------------------------------------
285 REM preset variables are
286 L = LO
287 CLS 0
290 REM ca         drag coefficient in air
295 REM cm         added mass coefficient of the pile
296 IF ABS (F) < E THEN GOTO 315
300 REM cw         drag coefficient of pile in water
```

```
305 REM ka
310 REM ra        density of air
315 REM rw        density of water
320 REM ab
325 REM ----------------------------------------------
330 REM determine form of input
335 PRINT "if input is from file type f"
340 PRINT " or if input is form the terminal type t"
345 INPUT "type t or f";IN$
350 PRINT"----------------------------------------"
355 REM determine form of output
360 PRINT "if output is to a file  type f"
365 PRINT " else type 0"
370 INPUT "type f or 0";F$
375 PRINT "------------------------------"
380 PRINT "if output is to the terminal type t"
385 PRINT " else type 0"
390 INPUT "type t or 0";T$
395 REM ----------------------------------------------
400 REM open files for input and output
405 OPEN "sout" FOR OUTPUT AS #1    'general output file
410 OPEN "sth" FOR OUTPUT AS #2     ' theta output file
415 OPEN "smom" FOR OUTPUT AS #3    'moment file
420   OPEN "sin" FOR INPUT AS #4   ' input file
425 REM ----------------------------------------------
430 REM input variables
431 CLS 0
435 IF IN$ = "f" THEN GOTO  660
440 PRINT
445 INPUT "stepsize (s) ";H
450 PRINT
455 INPUT "max t (s) ";MAX
460 PRINT
465 INPUT "1st stiffness constant (ft-lbs/rad) ";K1
470 PRINT
475 INPUT "2nd stiffness constant (ft-lbs/rad) ";K2
480 PRINT
485 INPUT "breakpoint angle (deg) ";BA
490 PRINT
495 INPUT "mass moment of inertia about base (ft^2-slugs) ";IM
500 PRINT
505 INPUT "pile length (ft) ";LP
510 PRINT
515 INPUT " pile diameter (ft)";DP
520 PRINT
525 INPUT "length to center of gravity (ft) ";LW
530 PRINT
535 INPUT "total weight";W
540 PRINT
545 INPUT "total depth (ft)";DT
550 PRINT
555 INPUT "depth to hinge";D
560 PRINT
565 INPUT "length to buoyancy section base (ft)";L1
570 INPUT "length to buoyancy section top (ft) ";L2
575 PRINT
580 INPUT " bell width (ft)";BW
585 PRINT
590 INPUT "wind velocity (ft/s)";UA
595 PRINT
600 INPUT "current velocity (ft/s) ";UC
605 PRINT
610 INPUT "wave height (ft)";HW
615 PRINT
620 INPUT "wave period (s)";PER
625 PRINT
```

```
630 INPUT "wave length (ft) "; LAMBDA
635 PRINT
640 INPUT "number of steps for output";ITRVL%
645 PRINT
650 REM --------------------------------------------------------

655 GOTO 765
660 INPUT#4,H
665 INPUT#4,MAX
670 INPUT#4,K1
675 INPUT#4,K2
680 INPUT#4,BA
685 INPUT#4,IM
690 INPUT#4,LP
695 INPUT#4,DP
700 INPUT#4,LW
705 INPUT#4,W
710 INPUT#4,DT
715 INPUT#4,D
720 INPUT#4,L1
725 INPUT#4,L2
730 INPUT#4,BW
735 INPUT#4,UA
740 INPUT#4,UC
745 INPUT#4,HW
750 INPUT#4,PER
755 INPUT#4,LAMBDA
760 INPUT#4,ITRVL%
765
REM------------------------------------------------------------
766 PRINT#2,"theta vs time"
767 PRINT#2,"time seconds"
768 PRINT#2,"angle deg"
769 PRINT#3,"moment vs time"
770 PRINT#3,"time seconds"
771 PRINT#3,"hinge moment ftlbs"
775 KA = 2 * 3.1416 / LAMBDA
780 RA = .077 / 32.2
785 RW = 64! / 32.2
790 AB = 36
795 CA = 1!
800 CW = 1!
805 CM = 2!
810 L% = INT (MAX / H)
815 BTHETA = BA * 3.1416 / 180
816 SIGMA = 2 * 3.1416 / PER
820                                                          REM
-------------------------------------------------------     825
REM dimensionalize arrays
830 DIM T(L% + 2,3)
835 DIM THETA(L% + 2,3)
840 DIM OMEGA(L% + 2,3)
845 DIM K(3)
850 DIM M(3)
855 DIM MH(3)
860 DIM MG(3)
865 DIM MW(3)
870 DIM MC(3)
875 REM ---------------------------------------------
880 I% = 0 'set i = 0
885 T(0,0) = 0 'set time = 0
890 REM-----------------------------------------------
895 REM  we want  to set  the initial  theta equal  to the static
angle 900 REM static angle calculation
905 REM find buoyant force
910 IF D > = L2 THEN GOTO  935
```

```
915 FB = 64! * 3.1416 * .25  *  (DP  ^  2)  *  (D  -  L1) 'volume
partially submerged
920 IF FB < 0 THEN FB = 0 ' non negative buoyant force
925 LB = (D + L1) / 2 ' volume partially submerged
930 GOTO  945
935 FB  = 64!  * 3.1416  * .25  * (DP  ^ 2)  * (L2  - L1) 'volume
totally submerged
940 LB = (L1 + L2) / 2 ' volume totally submerged
945 REM set static angle = Theta(0,0)
950 THETA(0,0) = (.25 * (LP ^2 - D ^ 2) * RA * CA * DP * UA ^ 2 +
.25 *              D * RW * CW * DP * D * UC ^ 2 - (W - FB) * BW)
/(K1 + LB * FB - LW *              W)
951 IF THETA(0,0) < 0 THEN THETA(0,0) = 0
955 REM---------------------------------------
960 REM print out results
965 IF T$ = "O" THEN GOTO  985
970 PRINT "static equil. angle = ";THETA(0,0) * 180 / 3.1416
975 PRINT
980 PRINT " time (s) = "; T(I%,0);"theta (deg) =  ";THETA(I%,0) *
180 / 3.1416;
985 IF F$ = "O" THEN GOTO  1005
990 PRINT#1,"static equil angle = ";THETA(0,0) * 180 / 3.1416
995 PRINT#1,
1000 PRINT#1,"time  (s) = ";T(I%,0);" theta (deg) = ";THETA(I%,0)
* 180 / 3.1416;
1005 REM -------------------------------------------------------
1010 REM start main program
1015            REM            i             loop          begins
**************************************************
1020 T(I% + 1,0) = T(I%,0) + H  ' update base time
1025 IF T(I% + 1,0) > MAX THEN GOTO  1460
1030    T(I%,1) = T(I%,0) + .5 * H 'time at j = 1
1035    T(I%,2) = T(I%,1)          ' time at j=2
1040    T(I%,3) = T(I%,0) + H      ' time at j=3
1045 REM --------------------------------------
1050 J% = 0  'set j = 0 before j loop
1055            REM            j             loop          begins
//////////////////////////////////////////////////
1060 REM this loop calculates k(0...3)  and m(0...3)  which later
will be
1065 REM used in the i loop in the runge kutta recurrence formula
to
1070 REM calculate preceding theta and omega values.
1075 K(J%) = OMEGA(I%,J%)  'set k(0...3) = omega(i%,0...3)
1080 REM ----------------------------------------------------
1085 REM calculate the spring moment mh(1...3)
1090 IF THETA (I%,J%) > (BTHETA) THEN GOTO  1110
1095 IF THETA (I%,J%) + BTHETA < 0 THEN GOTO  1120
1100 MH(J%) = K1 * THETA(I%,J%) '-btheta < theta < btheta
1105 GOTO  1125
1110 MH(J%) = K1 * BTHETA + K2 * (THETA(I%,J%)  - BTHETA) 'btheta
< theta
1115 GOTO  1125
1120 MH(J%) = -K1 * BTHETA + K2 * (THETA(I%,J%) + BTHETA)'theta <
-btheta
1124 REM ----------------------------------------
1125 IF J% <> 0 THEN GOTO 1130
1126 IF T$ = "O" THEN GOTO 1128
1127 PRINT "hinge moment(ft-lbs) = ";MH(0)
1128 IF F$ = "O" THEN GOTO 1130
1129 PRINT#1,"hinge moment (ft-lbs) = ";MH(0)
1130 REM calculate  the  submerged  length  and  the  wind moment
mw(1...3)
1135 IF LP * COS (THETA(I%,J%)) < D THEN GOTO  1155
1140 MW(J%)  = .25 * ((LP * COS (THETA(I%,J%))) ^ 2 - D ^ 2) * RA
* CA * DP *         (UA ^ 2)   'tip not submerged
```

```
1145 LS = D / COS (THETA(I%,J%)) 'tip not submerged
1150 GOTO  1165
1155 MW(J%) = 0 'tip submerged
1160 LS = LP 'tip submerged
1165 REM ---------------------------------------------------
1170 IH = IH + (3.1416 / 12) * (DP ^ 2) * RW * CM * (LS ^ 3)
1175 REM ------------------------------------------------
1180 REM calculate the current moment mc(1...3)
1185 DLTA = LS / 10
1190 SUM = 0
1195 REM beginning of loop ....................
1200 FOR N = 1 TO 10
1205 S = DLTA * N
1210 X = S * SIN (THETA(I%,J%))
1215 Y = -D + S * COS (THETA(I%,J%))
1220 U = .5 * SIGMA * HW * (( EXP (KA * DT + KA * Y) + EXP  ( -KA
* DT  -                  KA * Y)) / ( EXP (KA * DT) - EXP ( -KA *
DT))) * COS (KA * X - SIGMA               * T(I%,J%))
1225 V = .5 * SIGMA * HW * (( EXP (KA * DT + KA * Y) - EXP  ( -KA
* DT  -                   KA * Y)) / (EXP (KA * DT) - EXP ( -KA *
DT))) * SIN (KA * X - SIGMA               * T(I%,J%))
1230 UDOT = .5 * (SIGMA ^ 2) * HW * (( EXP (KA * DT  + KA  * Y) +
EXP (  -KA                    * DT - KA * Y)) / (EXP (KA * DT) -
EXP ( -KA * DT))) * SIN ( KA                       * X  - SIGMA *
T(I%,J%))
1235 VDOT  = -.5 * (SIGMA ^ 2) * HW * (( EXP (KA * DT + KA * Y) -
EXP ( -KA                 * DT - KA * Y))  / (EXP  ( KA  * DT) -
EXP (  -KA *  DT))) *  COS (KA                       * X - SIGMA *
T(I%,J%))
1236 Z = COS (THETA (I%,J%))
1240 FD = .5 * RW * CW * DP * ((U * Z  - V * SIN (THETA(I%,J%)) +
UC *  Z                   -S * OMEGA(I%,J%)) ^2) * SGN(U * Z -V *
SIN (THETA(I%,J%)) +                           UC  *  Z  -  S *
OMEGA(I%,J%))
1245  FI  =  .25  *  3.1416  *  RW  * CM * (DP ^ 2) * (UDOT * COS
(THETA(I%,J%)) -              VDOT * SIN (THETA(I%,J%)) -  UC *
( SIN (THETA(I%,J%))) * OMEGA(                IX,J%))
1250 IF N > 9.899999  THEN  GOTO  1265
1255 SUM = SUM + S * (FD + FI)
1260 GOTO 1270
1265 SUM = SUM + .5 * S * (FD + FI)
1270 NEXT N
1275 REM end of loop .......................................
1280 MC(J%) = DLTA * SUM
1285 REM end of current calculation-------------------
1290  REM  calculate  buoyant  force  and  centroid  to calculate
weight/buoyant
1295 REM moment mg(1...3)
1300 REM ----------------------------------------
1305 IF LS > = L2 THEN GOTO    1330
1310 FB = 64!  * 3.1416  * .25  * (DP  ^ 2)  * (LS  - L1)' volume
partially submerged
1315 IF FB < 0 THEN FB = 0 'non negative buoyant force
1320 LB = (L1 + LS) / 2 'volume partially submerged
1325 GOTO 1340
1330  FB  =  64!  *  3.1416  *  .25 * (DP ^ 2) * (L2 - L1)'volume
totally submerged
1335 LB = (L1 + L2) / 2 'volume totally submerged
1340 MG(J%) = (-LB * FB + LW * W) * SIN (THETA(I%,J%)) + (FB - W)
*  BW  *                                COS (THETA(I%,J%)) * SGN
(THETA(I%,J%))
1345 REM end of mc ---------------------------------------
1350 REM calculate angular acceleration m(0...3)
1355 M(J%) = (-MH(J%) + MG(J%) + MW(J%) + MC(J%)) / IH
1360 IF J% = 3  THEN GOTO  1395
1365 REM ========================================
```

```
1370 THETA(I%,J% + 1) = THETA(I%,0) + (T(I%,J% + 1)  - T(I%,0)) *
K(J%)
1375 OMEGA(I%,J%  + 1) = OMEGA(I%,0) + (T(I%,J% + 1) - T(I%,0)) *
M(J%)
1380 REM ==========================================
1385 J% = J% + 1 ' update j
1390 GOTO   1075
1395          REM          end          of          j          loop
//////////////////////////////////////////////
1400 REM runge kutta recurrence formula to calculate preceding
1405 REM theta and omega values theta(i% + 1,0),omega(i% + 1,0)
1410 THETA(I% + 1,0) = THETA(I%,0) + (H / 6) * (K(0) + 2 * K(1) +
2 * K(2) +               K(3))
1415 OMEGA(I% + 1,0) = OMEGA(I%,0) + (H / 6) * (M(0) + 2 * M(1) +
2 * M(2) +               M(3))
1420 REM ------------------------------
1425 I% = I% + 1 'update i
1430          REM          print          out          results
'''''''''''''''''''''''''''''''''''''''''''''
1435 IF T$ = "O" THEN GOTO 1445
1440 PRINT "time (s) = ";T(I%,0);"theta  (deg) =  ";THETA(I%,0) *
180 / 3.1416;
1445 IF F$ = "O" THEN GOTO 1455
1450 PRINT#1,  "time(s) = ";T(I%,0);"theta(deg) = ";THETA(I%,0) *
180 / 3.1416;
1455 GOTO  1020
1460          REM          end          of          i          loop
*****************************************************
1465 REM calculate the spring moment for a given time step
1470 I% = 0'set counter = 0
1475 REM beginning of  loop ++++++++++++++++++++++++++++
1480 IF THETA(I%,0) > (BTHETA) THEN GOTO  1500
1485 IF THETA(I%,0) + BTHETA < 0 THEN GOTO  1510
1490 MH(0) = K1 * THETA(I%,0) '-btheta < theta < btheta
1495 GOTO 1515
1500 MH(0)  =   K1 * BTHETA + K2 * (THETA(I%,0) - BTHETA)'theta >
btheta
1505 GOTO 1515
1510  MH(0)  =  -K1  *  BTHETA  +  K2  *  (THETA(I%,0)  + BTHETA)
'theta<-btheta
1515 REM print out results to moment and theta files
1520 REM -----------------------------------------
1525 PRINT#2, T(I%,0);THETA(I%,0)*180 / 3.1416
1530 PRINT#3, T(I%,0);MH(0)
1535 REM ------------------------------------------
1540 I% = I% + ITRVL% ' update i%
1545 IF I% > L% THEN GOTO   1555
1550 GOTO  1480
1555 REM +++++++++++++++++++++++++++++++++++++++++++++++
1560 CLOSE #1
1565 CLOSE #2
1570 CLOSE #3
1575 END
```

III. COLLISION

PURPOSE

The purpose of the collision model is to determine as a function of time:

1) the inclination angle
2) the pile's hinge moment
3) the barge force and moment
4) the pile's vertical and horizontal reaction force components at the base.

Also during impact between the barge and the pile, the barge force/moment impulse and the reaction force impulse are found.

MAJOR ASSUMPTIONS

1. Wave and steady current motion are neglected.
2. There is a negligible wind drag force and moment.
3. Light daymark boards on the pile are sacrificial.
4. The pile remains in contact with the barge whose speed remains constant.
5. The pile tip starts above water, that is, $l_p > d + f_b$.

GENERAL APPROACH

Collision modeling was grouped into the sequence of 5 processes shown in Fig. 13. These events include:

1) Impact at A
2) Pivoting/sliding about A and sliding on front of barge bow for $0 < \theta < \theta_f$

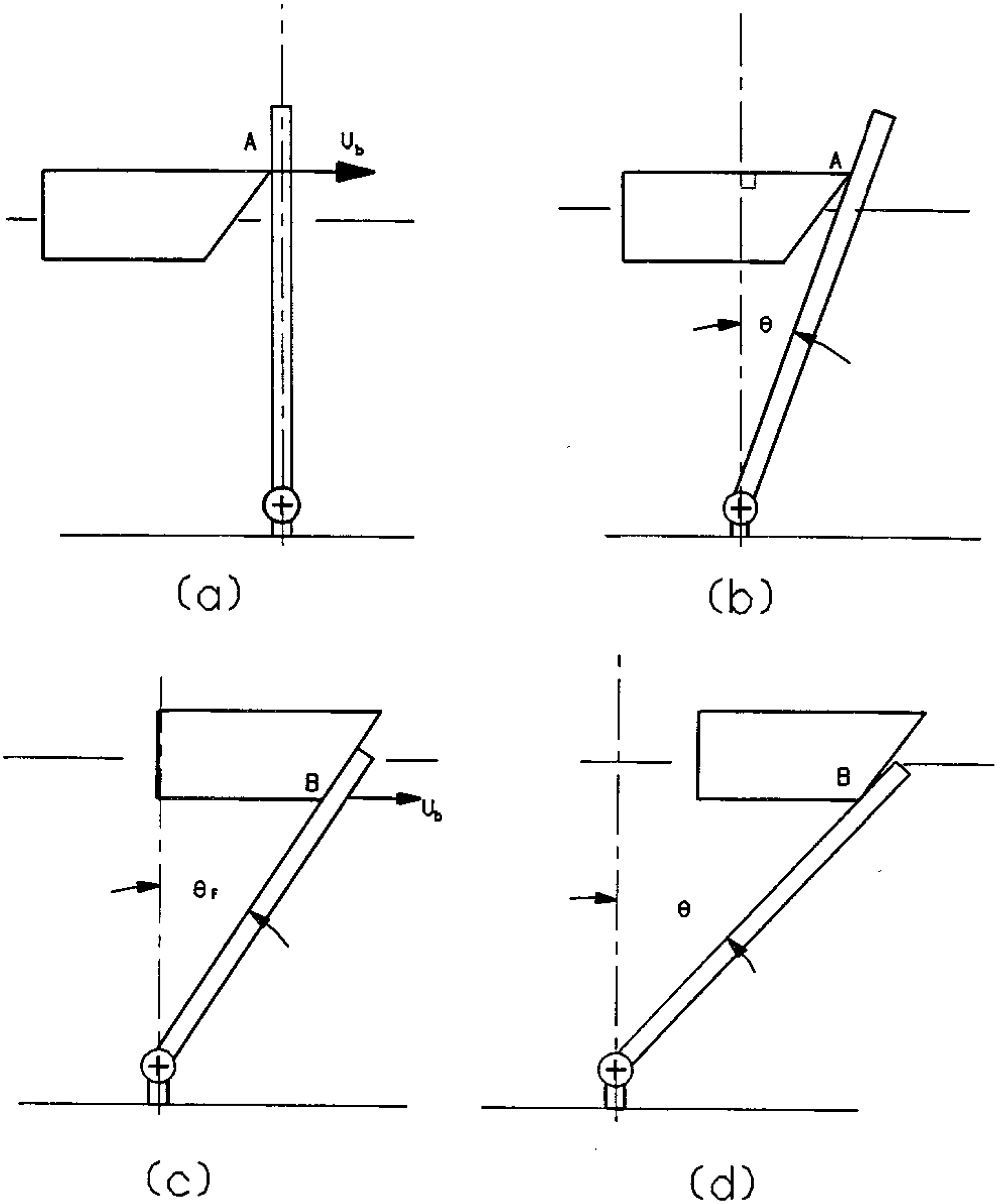

Fig. 13. Sequence of collision processes. Following (d) the pile tip slides along the barge bottom.

3) Impact at B for $\theta = \theta_f$

4) Pivoting/sliding about B for $\theta_f < \theta < \theta_k$

5) Sliding of pile under barge with $\theta = \theta_k$

These processes are analyzed in chronological order. Collision nomenclature is summarized on Fig. 14.

Throughout the collision modeling, the barge speed is taken to be a constant since the barge is so massive. Hence the pile kinematics can be analyzed independently of the forces involved. Because the horizontal velocity component of the contact point must equal barge speed, θ, $\dot{\theta}$ and $\ddot{\theta}$ can be calculated from the problem geometry. Then forces/moments can be determined using the dynamical equations.

For CASES 1 and 3 where there is impact, linear and angular principles of impulse and momentum are applied to determine desired impulsive forces and moments. For CASES 2 and 4, linear and angular equations of motion are incorporated with kinematic relations to obtain equations for solving for desired forces and moments. The time limits for CASES 2 and 4 are also found so that each case can be applied within the proper time constraints.

Pile forces include the barge contact force F_b, the vertical and horizontal reaction forces on the pile R_v and R_h, the force due to water F_c and the force due to the pile's weight and buoyancy F_g. Pile moment contributions are due to the hinge moment M_h, the moment due to water M_c, the weight and buoyant moment M_g and the moment due to the barge contact force M_b.

CASE 1 IMPACT AT A

In this analysis, the reaction force impulses and the barge force/moment impulses are calculated for the initial hit. Additional assumptions include neglecting spring, fluid, friction and weight forces in

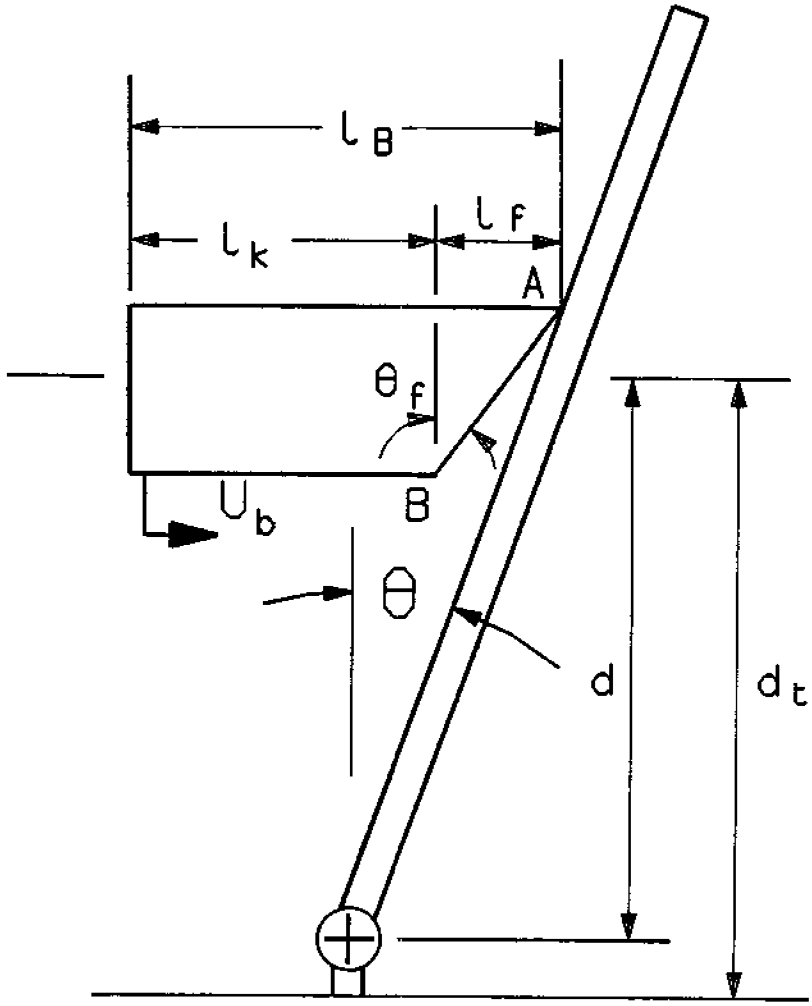

Fig. 14. Major dimensional parameters used in the collision analysis.

comparison to the impulsive forces. Also the pile is taken to be vertical ($\theta = 0$) at time = 0, and because the pile is vertical, the vertical reaction impulse is negligible. The linear and angular principle of impulse and momentum equations are used to solve for the desired force/moment impulses.

Using the angular principle of impulse and momentum applied about the pile hinge in Fig. 15 results in

$$\int M_b \, dt = (d + f_b) \int F_b \, dt = I_h \, (\dot{\theta}_2 - \dot{\theta}_1) \tag{63}$$

where $\dot{\theta}_1$ = pile angular velocity just before impact, $\dot{\theta}_2$ = the pile angular velocity just after impact, $\int M_b \, dt$ = barge moment impulse, $\int F_b \, dt$ = barge force impulse and f_b = barge freeboard. Assuming the pile was motionless before impact requires that $\dot{\theta}_1 = 0$.

From the pile kinematics shown in Fig. 16,

$$\dot{\theta}_2 = V_2/r_2 \tag{64}$$

In this equation V_2 is the tangential pile velocity which is equal to U_b, the barge velocity and r_2 is $d + f_b$. Making these substitutions into Eq. 64 yields

$$\dot{\theta}_2 = U_b/(d + f_b) \tag{65}$$

Substituting Eq. 65 and $\dot{\theta}_1 = 0$ into Eq. 63 yields the barge moment impulse as

$$\int M_b \, dt = I_h \, U_b/(d + f_b) \; . \tag{66}$$

Solving for the barge force impulse in Eq. 63 results in

$$\int F_b \, dt = I_h \, U_b/(d + f_b)^2 \; . \tag{67}$$

To solve for the reaction force impulse, the linear principle of impulse and momentum theory is used. From Fig. 15 this principle becomes

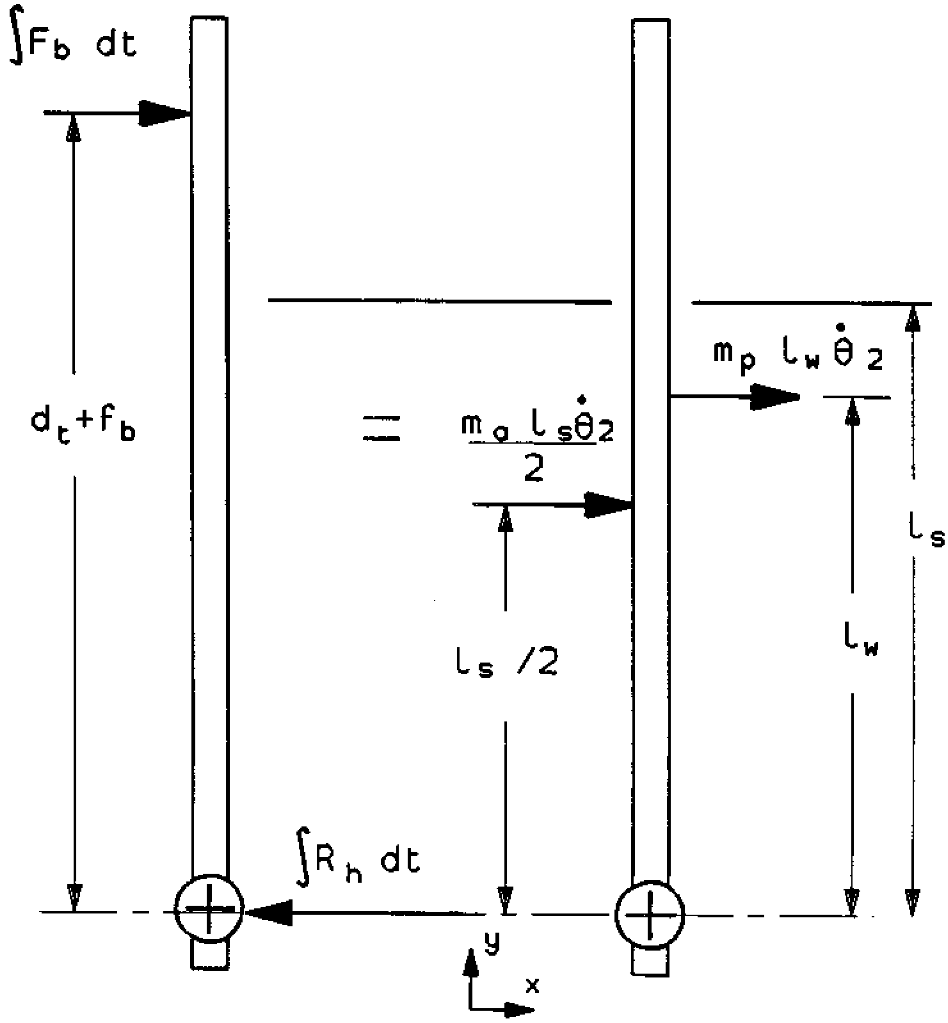

Fig. 15. Impulse and momentum diagram for impact at A.

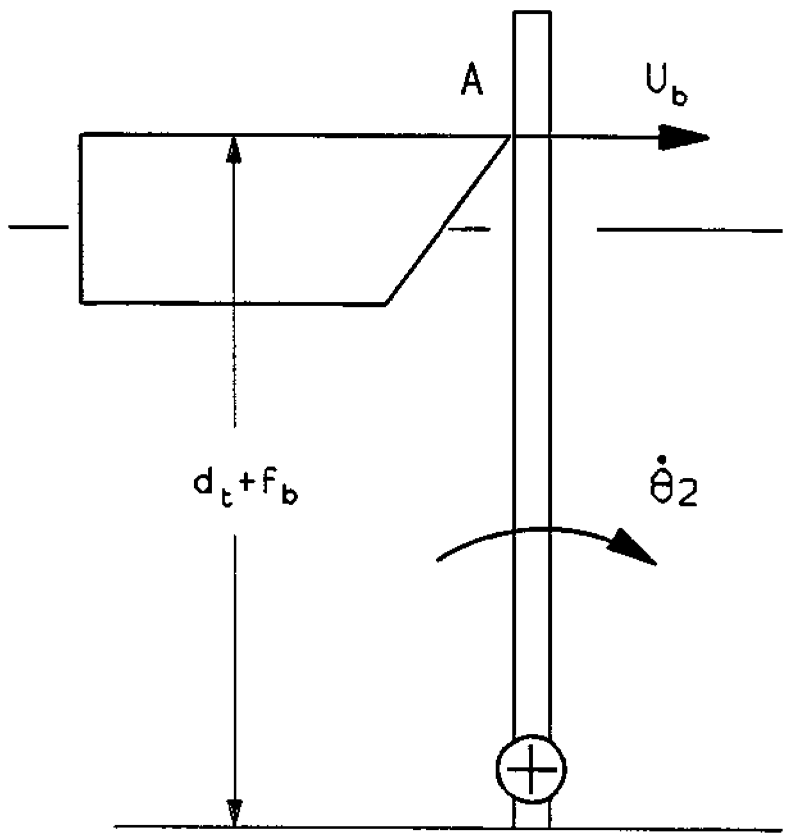

Fig. 16. Kinematic geometry for impact at A. After impact the tangential velocity of the pile at the point of contact is U_b.

$$\int F_b\, dt - \int R_h\, dt = m\, v_2 - m\, v_1 \tag{68}$$

where $\int R_h\, dt$ = reaction force impulse due to the pile foundation. From Fig. 15,

$$m\, v_2 = (m_a\, l_s/2 + m_p\, l_w)\, \dot{\theta}_2 \tag{69}$$

where l_s is d. Since $\dot{\theta}_1$ is zero, v_1 in Eq. 68 must be zero. Substituting this simplification and Eq. 69 into Eq. 68 results in

$$\int F_b \int dt - R_h\, dt = (m_a\, l_s/2 + m_p\, l_w)\, \dot{\theta}_2\ . \tag{70}$$

Substituting Eq. 67 and Eq. 65 into Eq. 70 and solving for the horizontal reaction impulse yields

$$\int R_h\, dt = (I_h\, U_b/(d + f_b)^2) - (m_p l_w + m_a l_s/2)(U_b/(d + f_b)) \tag{71}$$

CASE 2 PIVOTING AT A

General Approach

In this section the pile's angle and the desired forces/moments are analyzed as a function of time when the pile is in contact with the top of the bow rake. CASE 2 starts with pivoting about A and continues with sliding on the barge bow if the tip of the pile reaches point A.

First the pile kinematic equations are derived. Next the dynamic equations are developed to determine M_b, F_b, M_h and the reaction forces as a function of time. This case is completed when the pile stops sliding on the bow face, lower bow rake impact occurs and the pile stars pivoting about B. Termination takes place when the pile angle reaches θ_f at a time t_f.

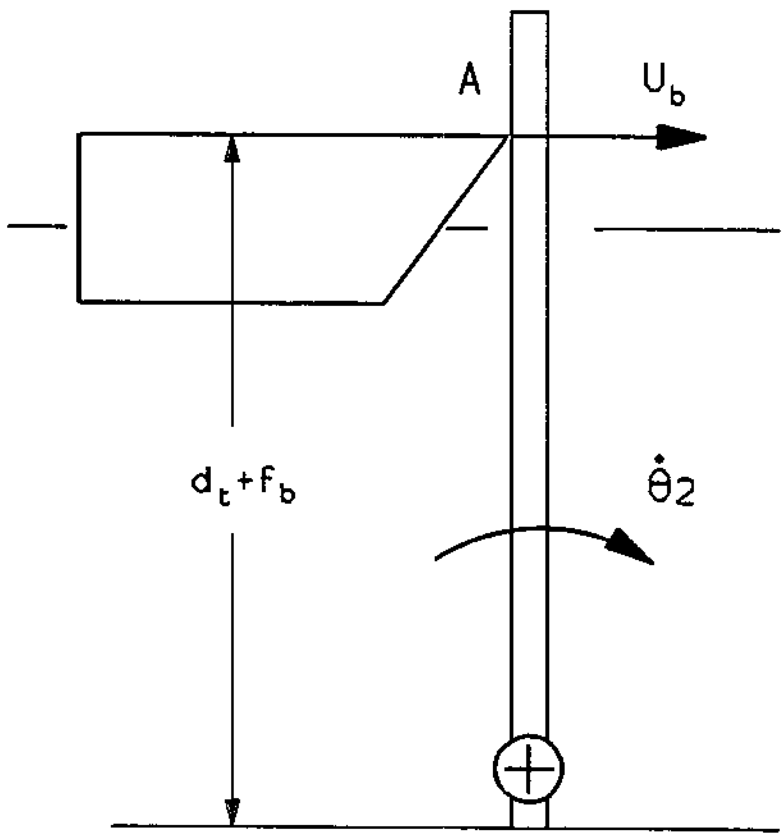

Fig. 16. Kinematic geometry for impact at A. After impact the tangential velocity of the pile at the point of contact is U_b.

$$\int F_b \, dt - \int R_h \, dt = m \, v_2 - m \, v_1 \tag{68}$$

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$$m \, v_2 = (m_a \, l_s/2 + m_p \, l_w) \, \dot{\theta}_2 \tag{69}$$

where l_s is d. Since $\dot{\theta}_1$ is zero, v_1 in Eq. 68 must be zero. Substituting this simplification and Eq. 69 into Eq. 68 results in

$$\int F_b \int dt - R_h \, dt = (m_a \, l_s/2 + m_p \, l_w) \, \dot{\theta}_2 \, . \tag{70}$$

Substituting Eq. 67 and Eq. 65 into Eq. 70 and solving for the horizontal reaction impulse yields

$$\int R_h \, dt = (I_h \, U_b/(d + f_b)^2) - (m_p l_w + m_a l_s/2)(U_b/(d + f_b)) \tag{71}$$

CASE 2 PIVOTING AT A

General Approach

In this section the pile's angle and the desired forces/moments are analyzed as a function of time when the pile is in contact with the top of the bow rake. CASE 2 starts with pivoting about A and continues with sliding on the barge bow if the tip of the pile reaches point A.

First the pile kinematic equations are derived. Next the dynamic equations are developed to determine M_b, F_b, M_h and the reaction forces as a function of time. This case is completed when the pile stops sliding on the bow face, lower bow rake impact occurs and the pile stars pivoting about B. Termination takes place when the pile angle reaches θ_f at a time t_f.

Kinematics

The objective here is to obtain expressions for the pile's angle, angular velocity and angular acceleration so that they can later be used with pile dynamics equations to calculate pile forces/moments.

By the use of Fig. 17,

$$\tan \theta = U_b\ t/(d + f_b), \tag{72}$$

so that the pile's angle as a function of time simplifies to

$$\theta = \tan^{-1} (U_b t/(d + f_b)). \tag{73}$$

Noting that $\tan \theta = \sin \theta \cos^{-1}(\theta)$, Eq. 73 becomes

$$\cos^{-1}(\theta) \sin \theta = U_b\ t/(d + f_b). \tag{74}$$

Taking the time derivative of Eq. 74 using the product rule of calculus yields

$$(\cos(\theta) \cos^{-1}(\theta) - \sin(\theta)(\cos\theta)^{-2}(-\sin\theta))\ \dot{\theta} = U_b/(d+f_b)\ . \tag{75}$$

Simplifying Eq. 75 by multiplying common terms yields

$$\dot{\theta}\ (1 + \sin^2\theta/\cos^2\theta) = U_b/(d + f_b)\ . \tag{76}$$

Multiplying both sides of Eq. 76 by $\cos^2\theta$ and realizing that $\cos^2\theta + \sin^2 \theta = 1$ yields

$$\dot{\theta} = (U_b \cos^2\theta)/(d + f_b). \tag{77}$$

Taking the time derivative of Eq. 77 to obtain the pile angular acceleration results in

$$\ddot{\theta} = (U_b/(d + f_b))2 \cos \theta\ (-\sin \theta)\ \dot{\theta}\ . \tag{78}$$

Substituting $\dot{\theta}$ from Eq. 77 into Eq. 78 results in

$$\ddot{\theta} = (U_b/(d + f_b))2 \cos^3(\theta) \sin (\theta)\ . \tag{79}$$

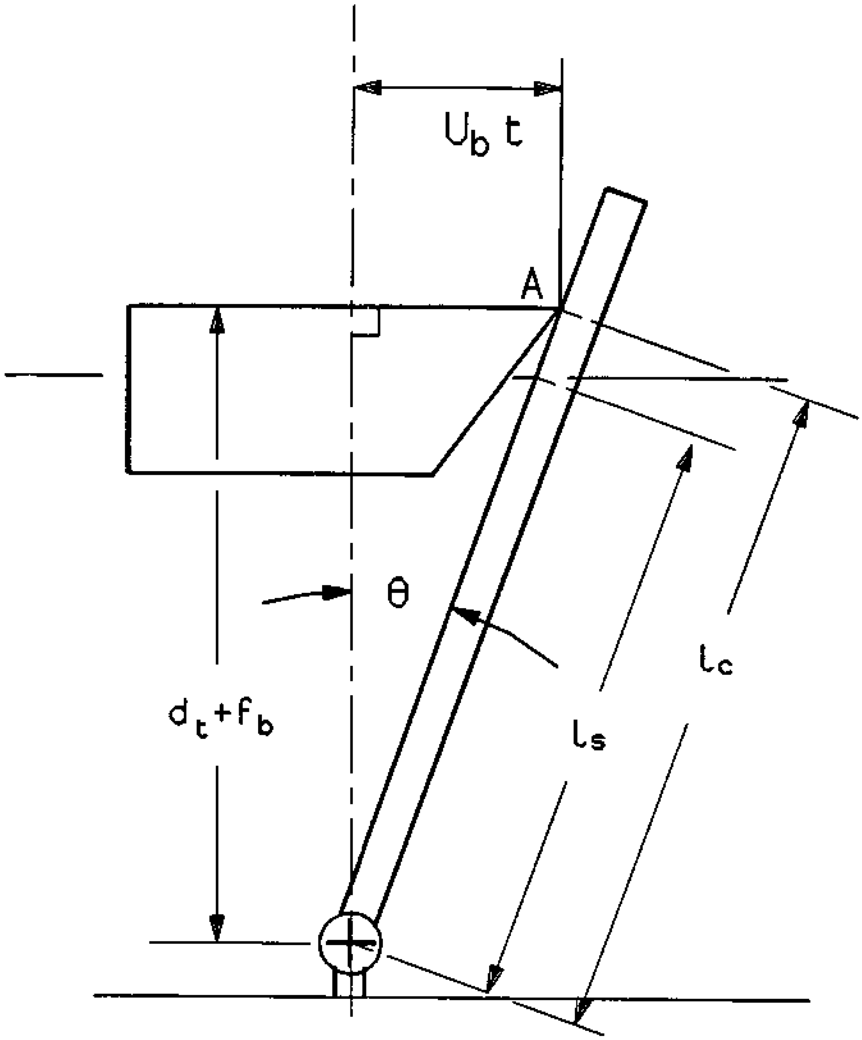


Fig. 17. Kinematic geometry for CASE 2 contact at A.

From the geometry shown in Fig. 17, the contact and submerged lengths are

$$l_c = (d + f_b)/\cos\theta \qquad (80)$$

and

$$l_s = d/\cos\theta\ . \qquad (81)$$

Pile Dynamics

Knowing the pile kinematics, the pile dynamic equations of motion are used to solve for F_b, M_b, R_h and R_v. See Fig. 18 for the force/moment diagram. To solve for the barge moment M_b at a given time t, use the time rate of change of angular momentum equation resulting in

$$M_b = I_h\,\ddot{\theta} + M_h - M_g - M_c. \qquad (82)$$

As in the STORM program, M_h can be found from Eq. 3, M_g from Eq. 15 and M_c from Eq. 53 omitting wave and steady current effects. Therefore in Eq. 53, with u_v, u_w, $\dot{u}_v$, $\dot{u}_w$, U_c equal to zero, M_{ci}' = zero and M_{cd} is simplified from Eq. 33 to

$$M_{cd} = \int_0^{l_s} (s/2)\ \rho_w\ C_w\ d_p(-s\ \dot{\theta})^2\ \mathrm{sgn}\ (-s\dot{\theta})\ ds. \qquad (83)$$

where $\dot{\theta}$ is known from the kinematic analysis.

To solve for the barge force, F_b is related to M_b according to Fig. 18 which yields

$$M_b = N_b\ l_c \qquad (84)$$

where N_b is the contact force normal to the pile. Noting that

$$N_b = F_b\ \cos\ (\phi)\ l_c \qquad (85)$$

where ϕ is the angle of friction, Eq. 84 becomes

$$F_b = (M_b/(l_c\ \cos\ (\phi))\ . \qquad (86)$$

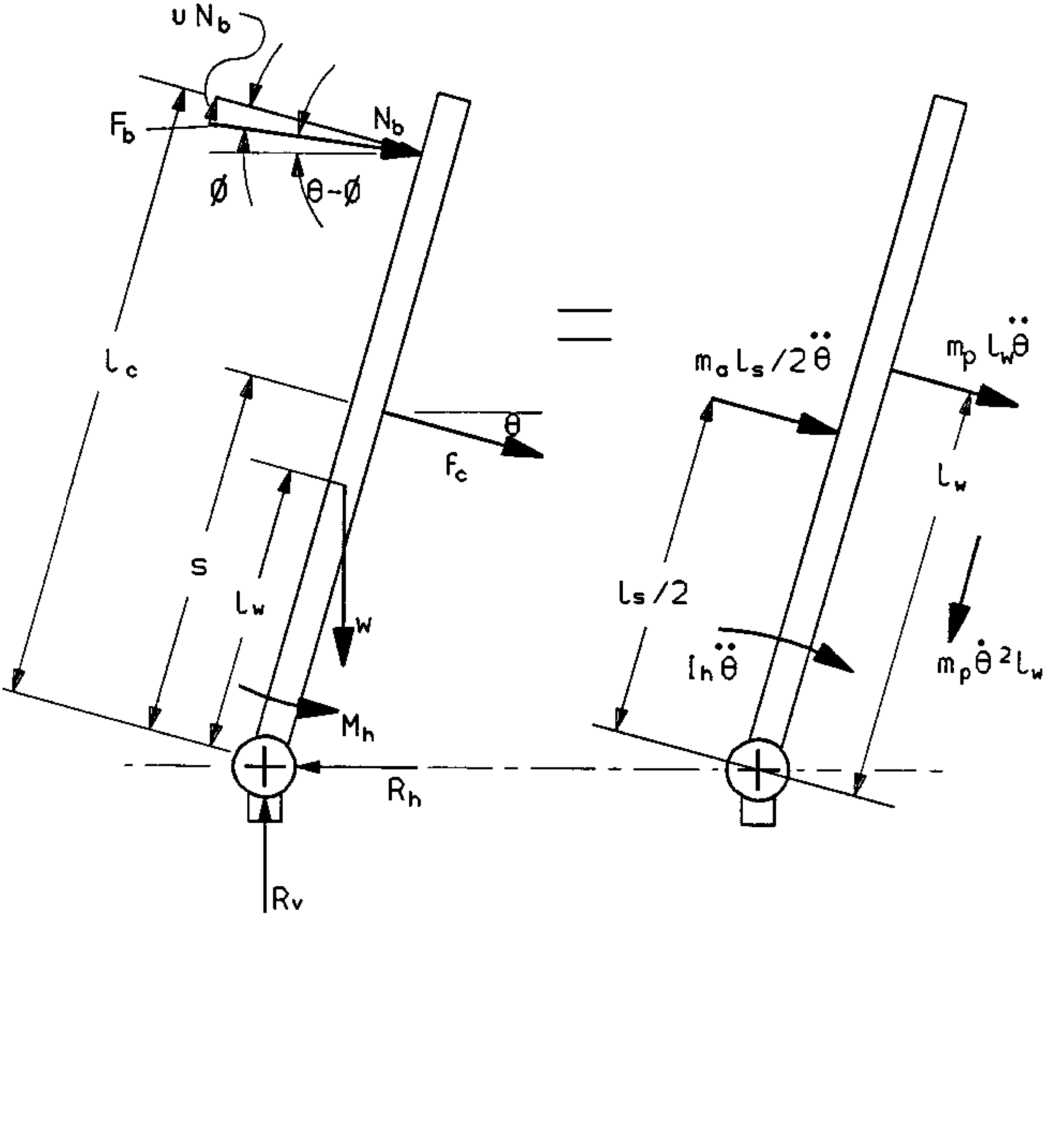


Fig. 18. Free body and inertial diagrams for CASE 2 contact at A.

Since the contact moment has been determined by use of Eq. 82 and the contact length is known by Eq. 80, the barge force can then be found from Eq. 85.

Once the barge force F_b has been found, Newton's 2nd law can be used to calculate the reaction forces. By use of Fig. 18, Newton's law in the x direction can be expressed as

$$F_b(\cos\theta - \phi) + F_c \cos\theta - R_h = (ma\, l_s/2 + m_p\, l_w)\, \ddot{\theta} \cos\theta - m_p\, l_w(\dot{\theta})^2 \sin\theta. \tag{87}$$

Solving Eq. 87 for the horizontal reaction force results in

$$R_h = F_b \cos(\theta - \phi) + F_c \cos\theta + m_p\, l_w\, ((\dot{\theta})^2 \sin\theta - \ddot{\theta}\cos\theta) - ma\, l_s/2\, \ddot{\theta} \cos\theta \tag{88}$$

where F_b, F_c, θ, $\dot{\theta}$, $\ddot{\theta}$ are all known.

Newton's law in the y direction requires

$$-F_b \sin(\theta - \phi) - F_c \sin\theta - W + R_v = -(ma\, l_s/2 + m_p\, l_w)\ddot{\theta} \sin\theta - m_p\, l_w\, (\dot{\theta})^2 \cos\theta\ . \tag{89}$$

Solving Eq. 89 for the vertical reaction force yields

$$R_v = F_b \sin(\theta - \phi) + F_c \sin\theta + W - (m_p\, l_w)\, (\ddot{\theta} \sin\theta + (\dot{\theta})^2 \cos\theta) - ma\, l_s/2\, \ddot{\theta} \sin\theta \tag{90}$$

The pivoting about A process ends when either $\theta = \theta_f$ (in which case impact at B occurs) or when the pile tip starts to slide down the bow face. Which event ends pivoting about A depends on pile length. If the pile is sufficient by short, pile tip sliding occurs before $\theta = \theta_f$. From the geometry shown in Fig. 19, bow face sliding is initiated when

$$l_p = (d + f_b)/\cos\theta. \tag{91}$$

for $\theta < \theta_f$.

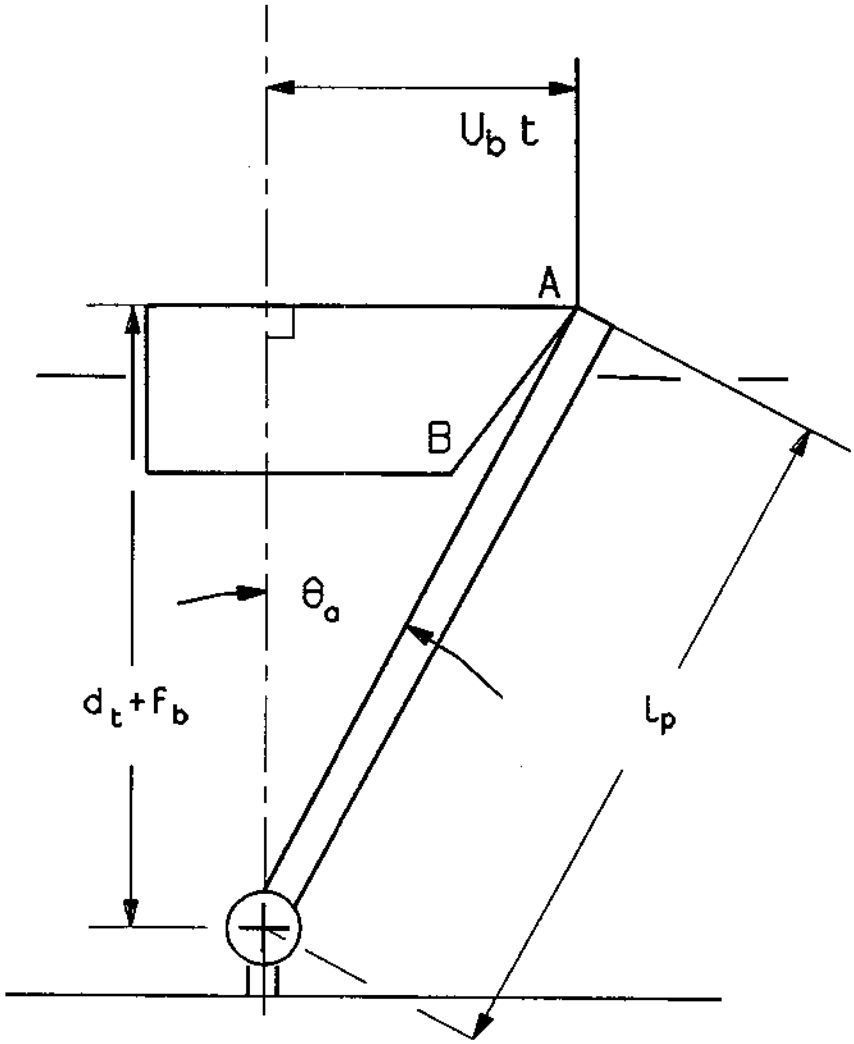


Fig. 19. The limiting position for CASE 2 contact at A. At this time $\theta = \theta_a$. The pile tip then proceeds to slide along the barge bow.

CASE 2 SLIDING ON BOW FACE

Kinematics

As in the case of pivoting about A, the objective is to obtain relations for θ, $\dot{\theta}$ and $\ddot{\theta}$ which will be later used in the barge dynamic equations to solve for M_b, F_b, R_v and R_h.

By the use of Fig. 20 geometry,

$$l_p \sin\theta + d_1 = U_b\, t \qquad (92)$$

and

$$d_2 = d + f_b - (l_p \cos\theta)\ . \qquad (93)$$

A physical constraint for the system is the bow angle θ_f which results in

$$d_1 = d_2 \tan\theta_f\ . \qquad (94)$$

Substituting Eq. 93 into Eq. 94 for d_2 yields

$$d_1 = (d + f_b - l_p \cos\theta) \tan\theta_f \qquad (95)$$

Substituting Eq. 95 into Eq. 92 for d_1 yields

$$l_p \sin\theta = U_b\, t - ((d + f_b) - l_p \cos\theta) \tan\theta_f \qquad (96)$$

which can be rearranged in the form

$$\sin\theta - \tan\theta_f \cos\theta = U_b\, t/l_p - (d + f_b) \tan\theta_f/l_p \qquad (97)$$

or equivalently

$$\tan\theta = \tan\theta_f + U_b\, t/(l_p \cos\theta) - (d + f_b) \tan\theta_f/(l_p \cos\theta). \qquad (98)$$

Eq. 98 is not in a useful form because there are still θ terms on the right hand side of the equation. To eliminate this problem, θ is estimated by the use of Fig. 21 where θ is approximately equal to θ_a. The angle between θ and θ_a is assumed negligible. Approximating $\theta = \theta_a$ in Eq. 98 yields the useful relation for the pile angle

$$\tan\theta = \tan\theta_f + U_b\, t/(l_p \cos\theta_a) - \tan\theta_f\, (d + f_b)/(l_p \cos\theta_a)\ . \qquad (99)$$

Solving for θ in Eq. 99 results in

$$\theta = \tan^{-1}(\tan\theta_f + U_b\, t/(l_p \cos\theta_a) - \tan\theta_f\, (d + f_b)/(l_p \cos\theta_a). \qquad (100)$$

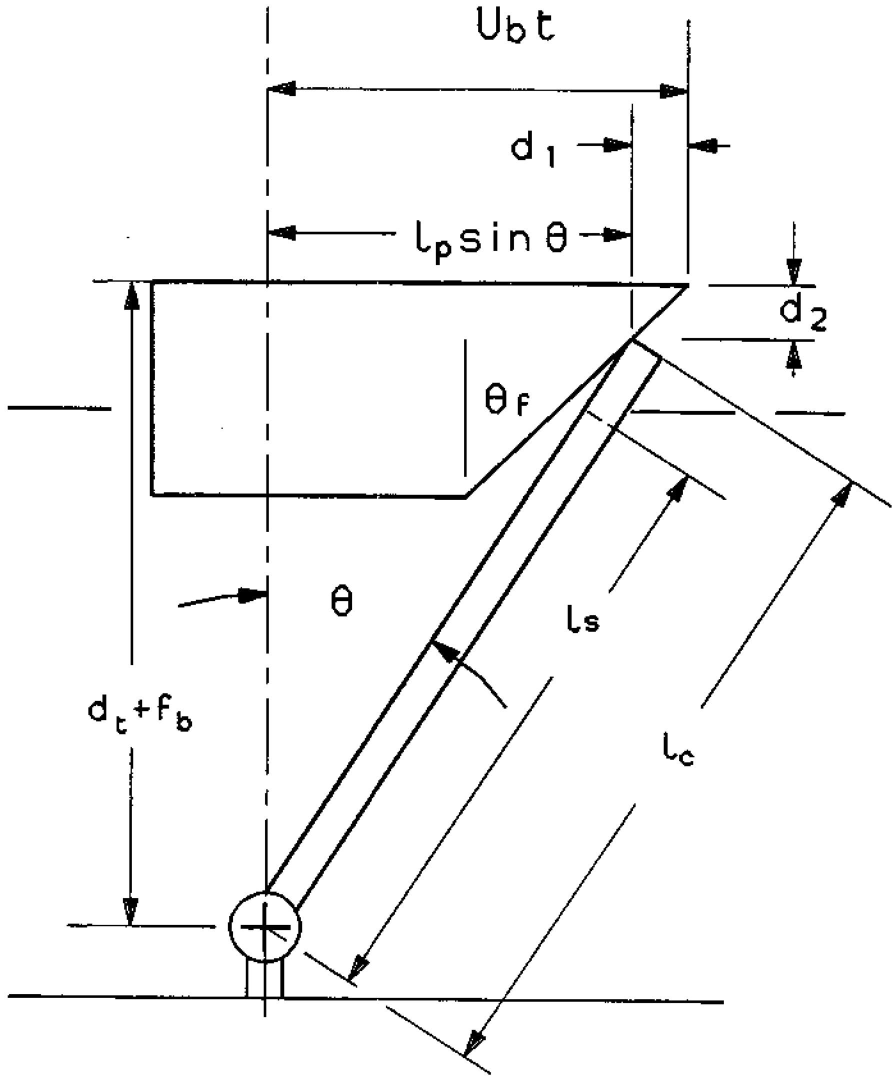


Fig. 20. Kinematic geometry for CASE 2 sliding on the bow face. The limiting position occurs when the pile angle equals the bow angle θ_f.

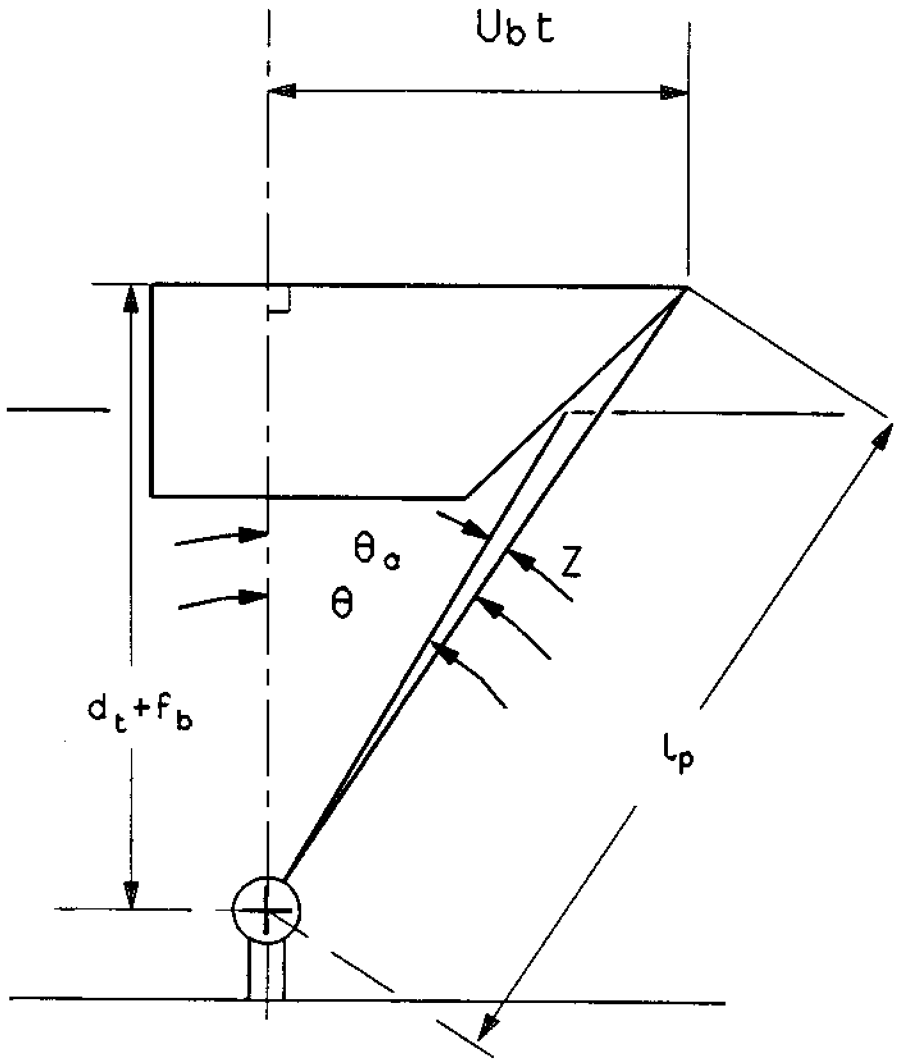


Fig. 21. Pile angle approximation during sliding on the bow face. The enclosure angle z is small so that θ is approximately θ_a.

From Fig. 21 geometry, θ_a is found as

$$\theta_a = \tan^{-1}(U_b\ t/(d + f_b)). \tag{101}$$

To find $\dot{\theta}$, take the time derivtive of Eq. 98 which yields

$$\dot{\theta}(\cos(\theta) - \tan\theta_f\ (-\sin\theta)) = U_b/l_p. \tag{102}$$

Grouping the $\dot{\theta}$ terms in Eq. 102 and solving for $\dot{\theta}$ gives

$$\dot{\theta} = U_b/(l_p(\cos\theta + \tan\theta_f\ \sin\theta)). \tag{103}$$

To find $\ddot{\theta}$, put all θ terms in Eq. 103 to the left hand side of the equation,

$$\cos\theta\ (\dot{\theta}) + \tan\theta_f\ \sin\theta\ (\dot{\theta}) = U_b/l_p, \tag{104}$$

and take the time derivative (by the use of the chain rule of calculus) to find

$$-\sin\theta\ (\dot{\theta})^2 + \cos\theta\ (\ddot{\theta}) + \tan\theta_f\ \cos\theta\ (\dot{\theta})^2 + \tan\theta_f\ \sin(\theta)\ \ddot{\theta} = 0. \qquad (105)$$

Grouping the $\dot{\theta}$ and $\ddot{\theta}$ terms in Eq. 105 yields

$$(\cos\theta + \tan\theta_f\ \sin\theta)\ \theta = (\sin\theta - \tan\theta_f\ \cos\theta)\ \dot{\theta}^2. \qquad (105)$$

Substituting $\dot{\theta}$ from Eq. 103 into Eq. 106 and solving for $\ddot{\theta}$ yields the equation for the pile's angular acceleration

$$\ddot{\theta} = ((\sin\theta - \tan\theta_f\ \cos\theta)/(\cos\theta + \tan\theta_f\ \sin\theta)^3)(U_b/l_p)^2. \tag{107}$$

It should be noted that contact length is always equal to the pile length l_p, that is

$$l_c = l_p. \tag{108}$$

The submerged length l_s may be found from Eq. 9.

Pile Dynamics

Knowing the pile kinematic relations, the dynamic equations of motion are used to solve for F_b, M_b, R_h and R_v. These dynamic equations are nearly identical with those from point A contact developed previously with the exception of the contact force.

The barge moment M_b is solved the same way as in point A contact by the use of Eq. 82. However, the direction of the contact force, F_b, differs from that in point A contact. From Fig. 22 geometry, the barge force is related to the barge moment by

$$F_b = M_b/(l_p \cos(\theta - \theta_f + \phi)) \tag{109}$$

where θ_f is the bow angle and ϕ is the angle of friction. Once the barge moment M_b has been found, the barge force is found from Eq. 109.

From Fig. 22 geometry, the x and y components of the barge force are

$$F_{bx} = F_b \cos(\theta_f - \phi) \tag{110}$$

and

$$F_{by} = -F_b \cos(\phi + 90 - \theta_f) = F_b \sin(\phi - \theta_f). \tag{111}$$

To solve for the reaction forces R_v and R_h, simply replace $F_b \cos(\theta - \phi)$ in Eq. 88 by $F_b \cos(\theta_f - \phi)$. Also for the vertical reaction force, replace the $F_b \sin(\theta - \phi)$ term in Eq. 90 by $F_b \sin(\theta_f - \phi)$.

Timing

The time limit for CASE 2 termination is reached when the pile inclination angle becomes θ_f. That is,

$$t_f = t\ (\theta = \theta_f) \tag{112}$$

where t_f is the CASE 2 termination time. Time t_f is found by solving for time in Eq. 72 when $\theta = \theta_f$ which gives

$$t_f = \tan\theta_f\ (d + f_b)/U_b. \tag{113}$$

CASE 3 IMPACT AT B

The purpose of this section is to develop expression for barge force/moment impulses and reaction force impulses when impact occurs at point B. In this analysis the spring, fluid, friction, and weight forces are considered negligible in comparison to the impulsive forces. An impulse and

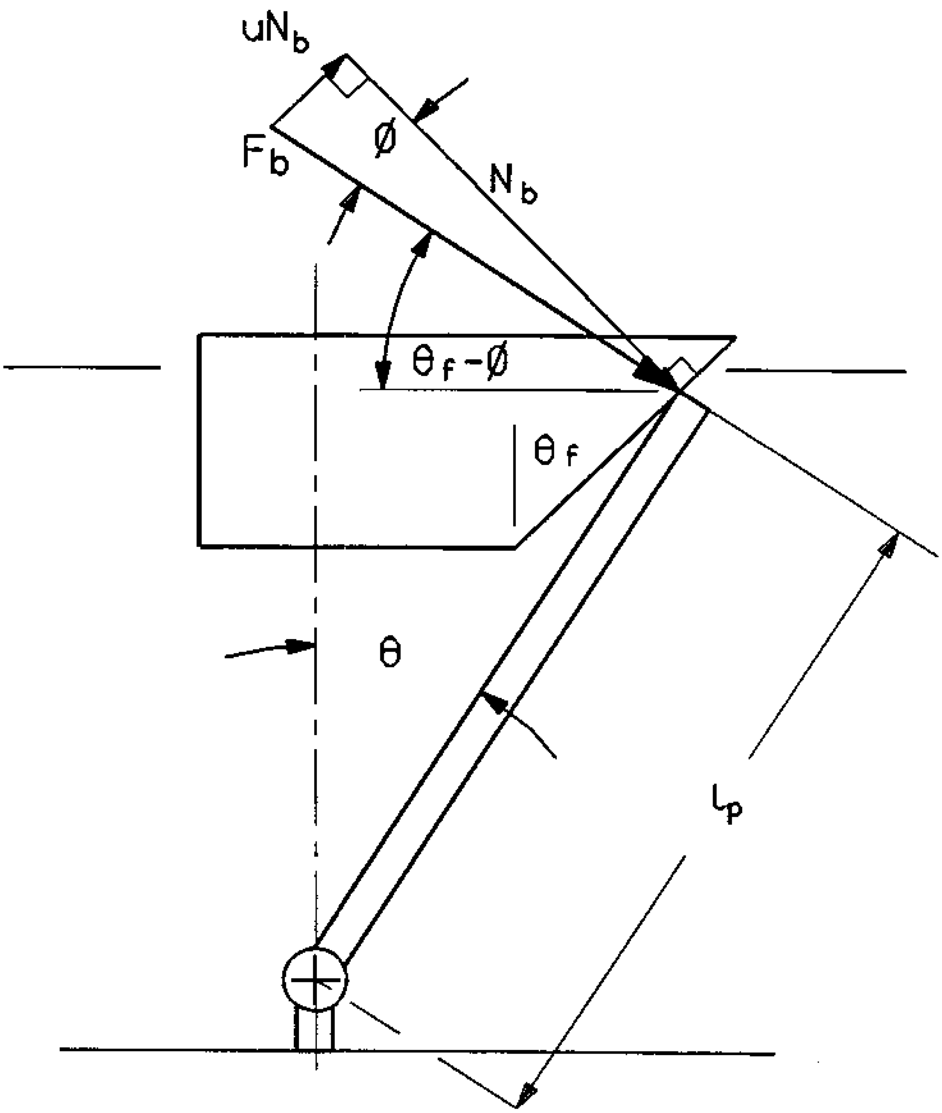


Fig. 22. Contact force and friction geometry for CASE 2 sliding on the bow face.

momentum approach is used to evaluate the remaining important collision loads associated with the barge and pile base.

The angular velocity of the pile just before the collision is equal to the final angular velocity of the pile in CASE 2 where $\theta = \theta_f$. The angular velocity of the pile $\dot{\theta}_1$ from Eq. 77 at $\theta = \theta_f$ is

$$\dot{\theta}_1 = U_b \cos^2 \theta_f/(d + f_b). \tag{114}$$

The angular velocity of the pile just after impact is illustrated by Fig. 23. It can be seen from the figure that $\dot{\theta}_2$ can be found by replacing $(d + f_b)$ in Eq. 114 by h_b to obtain

$$\dot{\theta}_2 = U_b \cos^2 \theta_f/h_b \tag{115}$$

where h_b = distance from pile hinge to barge bottom.

From the principle of moment impulse and angular momentum combined with the use of Fig. 24, the relation

$$I_h \dot{\theta}_1 + l_c \int F_b \, dt = I_h \dot{\theta}_2 \tag{116}$$

is obtained. In this equation the length to contact point is given by

$$l_c = h_b/\cos\theta_f \tag{117}$$

Solving for the barge force impulse in Eq. 116 yields

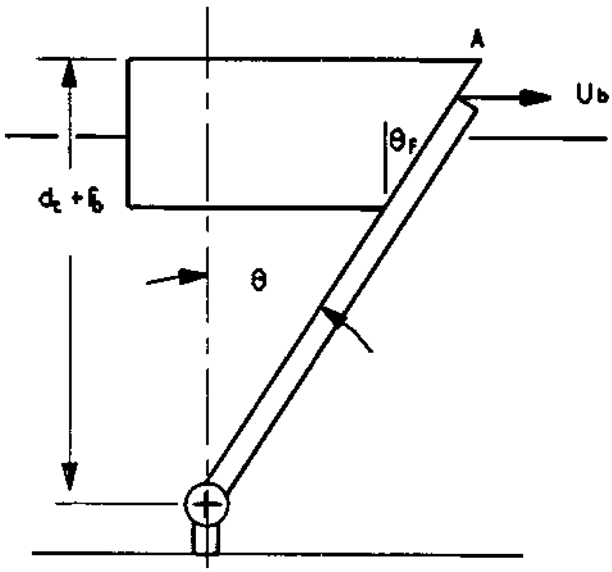
$$\int F_b \, dt = (I_h/l_c)(\dot{\theta}_2 - \dot{\theta}_1) \tag{118}$$

where l_c is provided by Eq. 117, $\dot{\theta}_2$ is given by Eq. 115 and $\dot{\theta}_1$ by Eq. 114.

Linear impulse/momentum is applied to solve for the reaction force impulses. Applying this principle in the x direction and by the use of Fig. 24, the relation

$$(m_a l_s/2 + m_p l_w) \dot{\theta}_1 \cos \theta_f + \int F_b \, dt \cos \theta_f - \int R_h \, dt = (m_a l_s/2 + m_p l_w) \dot{\theta}_2 \cos \theta_f \tag{119}$$

is found. Solving for the horizontal reaction impulse by grouping the m_p and m_a terms and substituting the barge force impulse solution from Eq. 120 yields



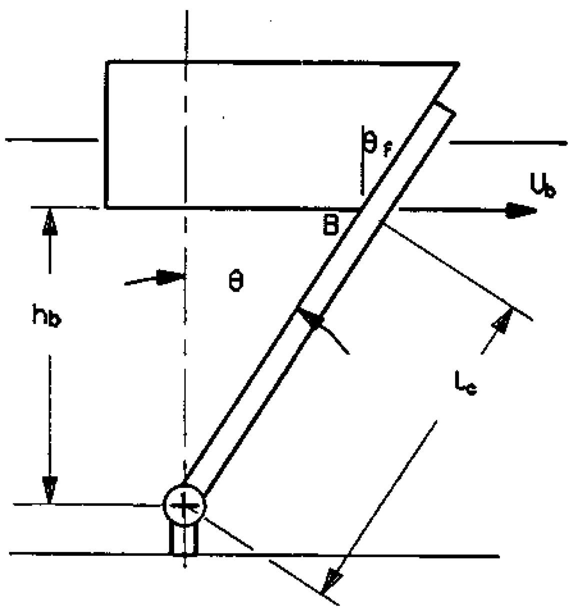


Fig. 23. Pile kinematics just before (top) and just after (bottom) impact at B. Immediately after lower bow rake impact, the contact point is B.

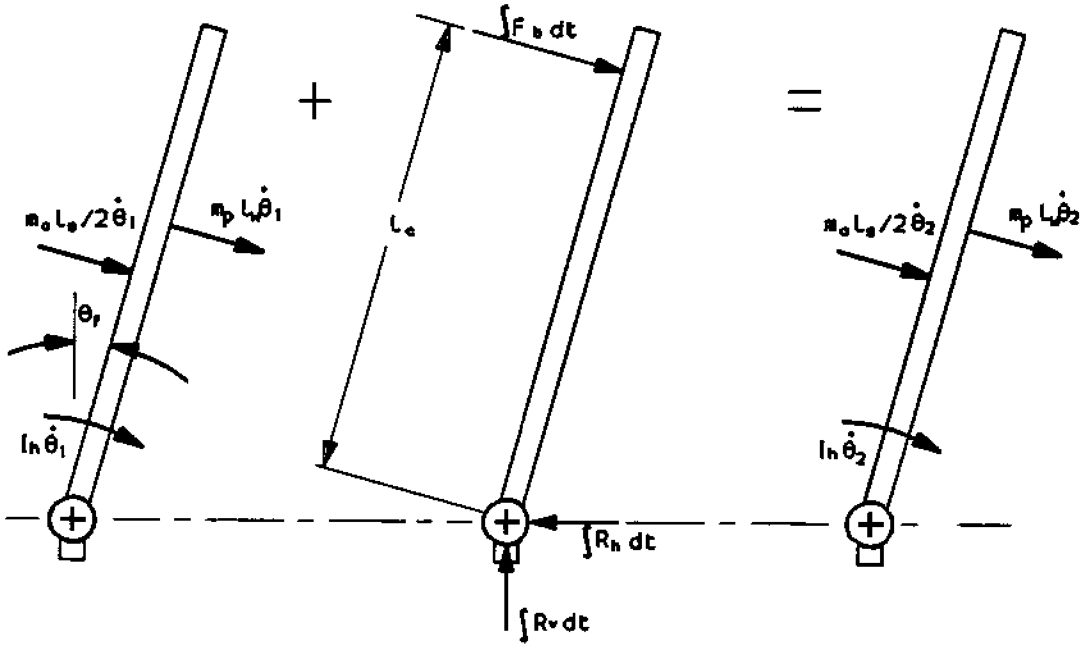


Fig. 24. Impulse and momentum diagrams for lower bow rake impact at B (CASE 3).

$$\int R_h \, dt = (-(m_a \, l_s/2 + m_p \, l_w) + I_h/l_c)(\dot{\theta}_2 - \dot{\theta}_1)\cos\theta_f \tag{120}$$

From linear impulse and momentum in the y direction with the use of Fig. 24, the expression

$$-(m_a \, l_s/2 + m_p \, l_w) \, \dot{\theta}_1 \sin \theta_f - \int F_b \, dt \sin \theta_f + \int R_v \, dt = - (m_a \, l_s/2 + m_p \, l_w) \, \dot{\theta}_2 \sin \theta_f \tag{121}$$

is obtained. Grouping the m_a and m_p terms, substituting the relation for the barge force impulse and solving for the vertical reaction impulse results in

$$\int R_v \, dt = (I_h/l_c - m_a \, l_s/2 + m_p \, l_w) \, (\dot{\theta}_2 - \dot{\theta}_1) \sin \theta_f \; . \tag{122}$$

CASE 4 PIVOTING AT B

General Approach

In this section pile angle, barge force and moment, and reaction force are evaluated as the pile is pushed down by lower bow rake contact at B. As in CASE 2 theory, kinematic relations are first found, then pile dynamics considerations are employed.

The sliding at B is similar to sliding at A except the position of the contact point is different. The similarity allows previous results to be used by simply replacing the contact point height. CASE 4 kinematic relations are found by replacing $(d + f_b)$ by h_b in CASE 2 relations. Similarly CASE 2 dynamic equations are used in CASE 4 by replacing $(d + f_b)$ by h_b. This case terminates when the tip of the pile starts sliding on the barge bottom at a time $t = t_k$.

Kinematics

From Fig. 25 geometry the pile angle is such that

$$\tan \theta = (U_b \, t - l_f)/h_b \; . \tag{123}$$

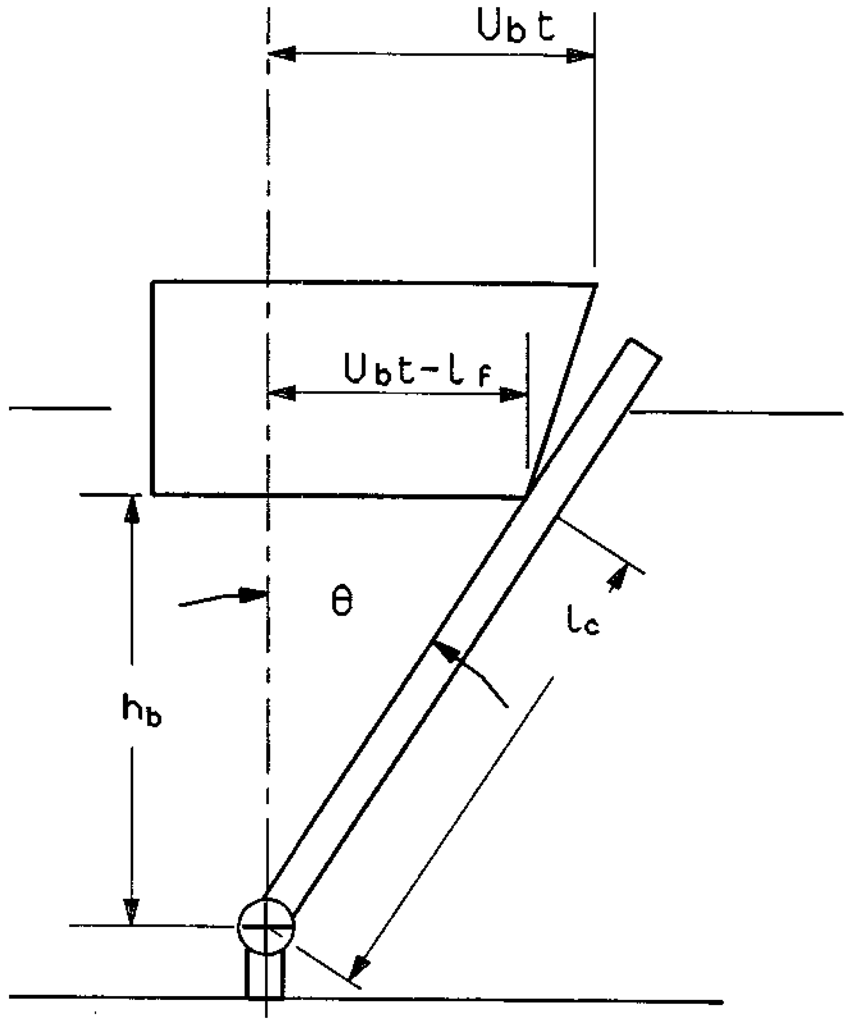


Fig. 25. Kinematic geometry for CASE 4 pivoting about point B.

Solving for θ in Eq. 123 gives

$$\theta = \tan^{-1} ((U_b\ t - l_f)/h_b)\ . \tag{124}$$

Replacing $(d + f_b)$ by h_b in the angular velocity relation of Eq. 77 gives

$$\dot{\theta} = U_b \cos^2\theta / h_b\ . \tag{125}$$

Similarly replacing $(d + f_b)$ in the angular acceleration relation of Eq. 79 by h_b yields

$$\ddot{\theta} = -2\ (U_b/h_b)^2 \cos^3 \theta \sin \theta \tag{126}$$

The contact length is found from geometry by the use of Fig. 25 resulting in

$$l_c = h_b / \cos \theta. \tag{127}$$

Pile Dynamics

The pile dynamic situation is exactly analogous to that of CASE 2 pivoting about A. M_b is found by use of Eq. 82, F_b is determined from Eq. 86, R_h from Eq. 88 and R_v from Eq. 90. The modified kinematic expressions, Eqs. 123-127, are used, however, when evaluating these moments and forces.

Timing

CASE 4 is terminated at time $t = t_k$, which from Fig. 26, occurs when $\theta = \theta_k$. From Fig. 26 geometry, the angle θ_k is found as

$$\theta_k = \cos^{-1}(h_b/l_p). \tag{128}$$

Solving for t in Eq. 125 and substituting $\theta = \theta_k$ yields

$$t_k = (h_b \tan \theta_k + l_f)\ U_b^{-1}. \tag{129}$$

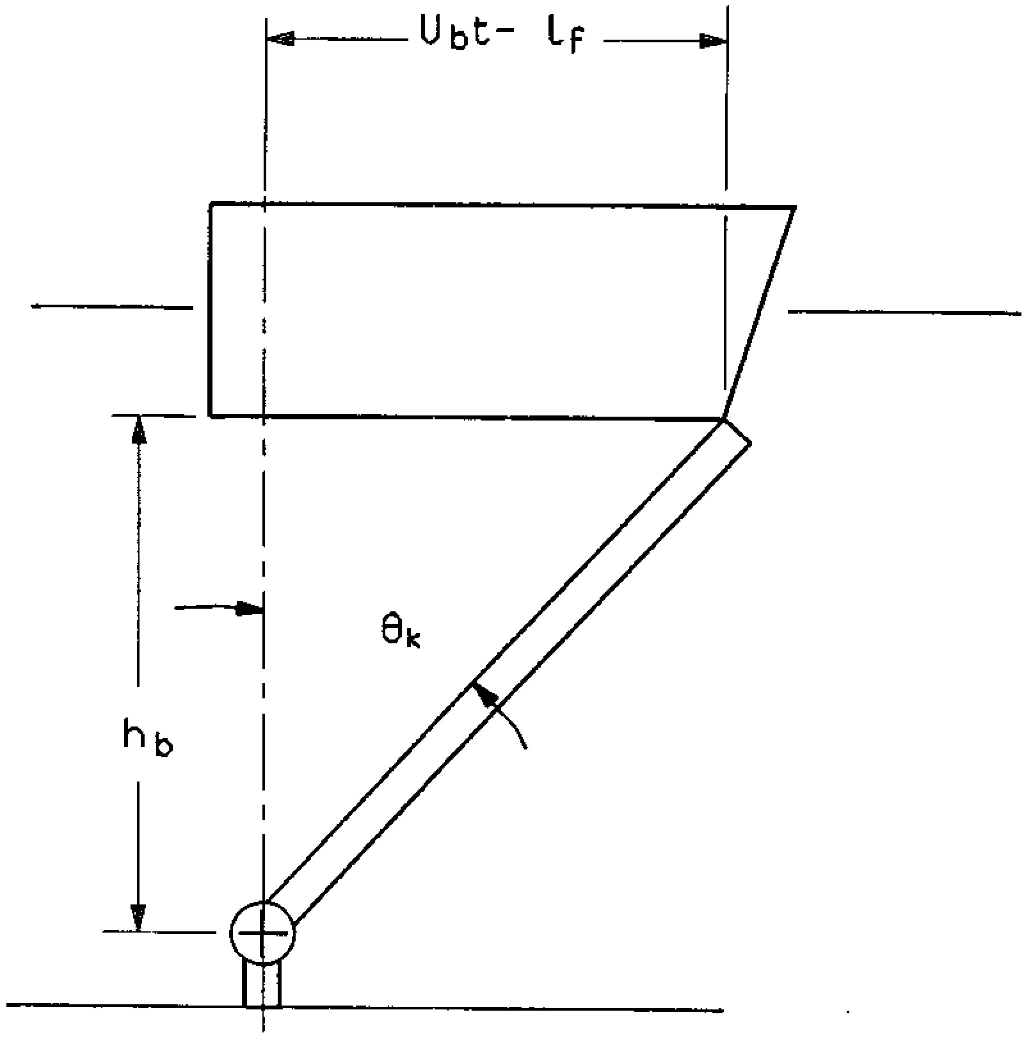


Fig. 26. Limiting position for CASE 4 for which θ equals θ_k.

CASE 5 SLIDING UNDER BARGE BOTTOM

Between the time the pile tip goes under point B on the lower bow rake and the time it is released from the bottom at the stern, pile movement is negligible. Static analysis is used to determine the barge force/moment and the pile's reaction force components. The time to recovery t_r is calculated from considerations of the barge/pile geometry.

During this phase the pile angle is constant at $\theta = \theta_k$ as shown in Fig. 26 and given by Eq. 128. The pile is essentially in static equilibrium where $\ddot{\theta} = \dot{\theta} = 0$. Also the pile contact length is the pile length l_p, that is,

$$l_c = l_p \ . \tag{130}$$

CASE 5 moment and force equations are the same as that of the CASE 2 equations with $\theta = \theta_k$ and $\ddot{\theta} = \dot{\theta} = 0$. But the direction of the contact force is changed. The barge moment is found using Eq. 82. From Fig. 27 geometry, the barge force is related to the barge moment by

$$F_b = M_b/(l_p \cos(\theta_k + \phi - 90)) = M_b/(l_p \sin(\theta_k + \phi)). \tag{131}$$

The x and y components of the barge contact force are found from Fig. 27 geometry as

$$F_{bx} = F_b \sin \phi$$

and

$$F_{by} = F_b \cos \phi. \tag{132}$$

Thus to find the horizontal reaction force for CASE 5, replace $F_b \cos(\theta - \phi)$ in Eq. 88 by $F_b \sin \phi$. To find the vertical pile reaction force, replace $F_b \sin(\theta - \phi)$ in Eq. 90 by $F_b \cos \phi$. Of course $\theta = \theta_k$ and $\dot{\theta} = \ddot{\theta} = 0$ in these equations as noted above.

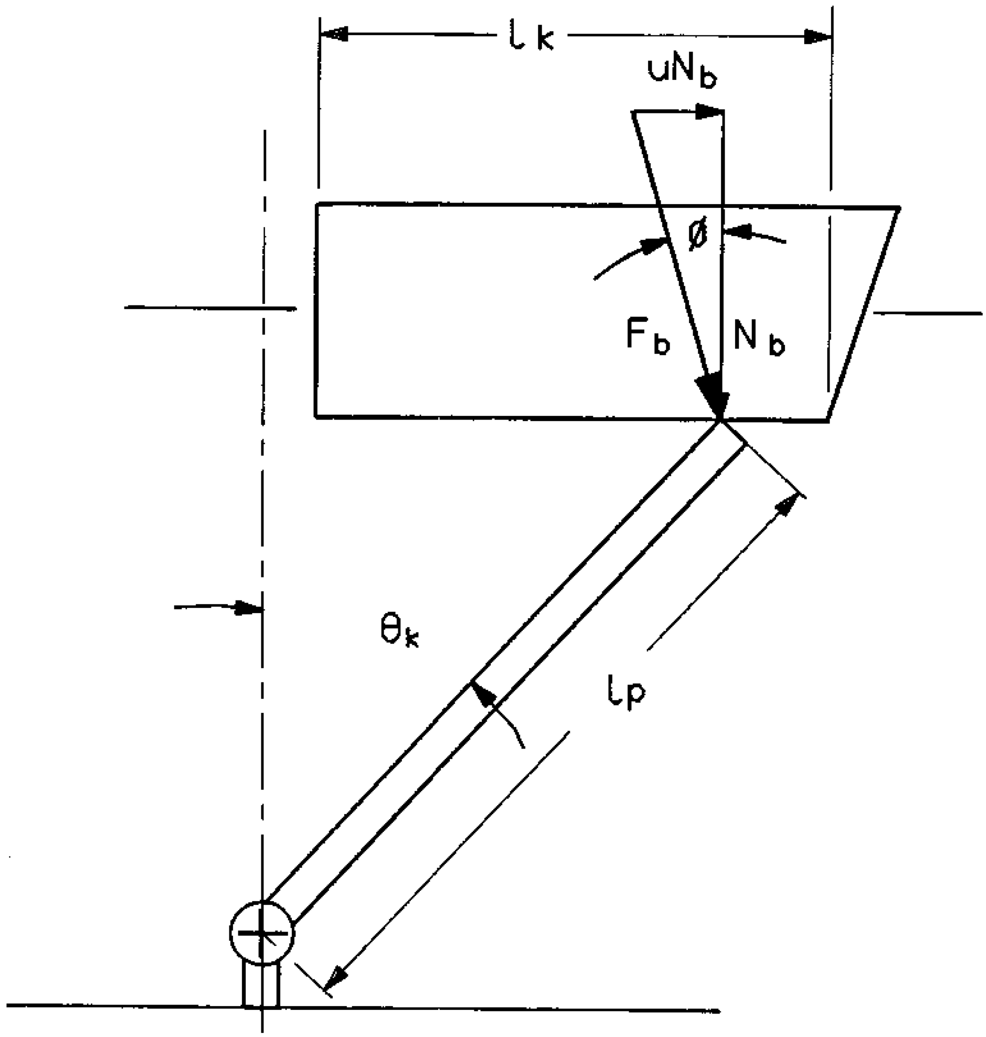


Fig. 27. Pile/barge geometry for CASE 5 sliding under the barge bottom.

The time to recovery t_r is found by adding the time to the start of CASE 5 (t_k) to the duration of CASE 5. The duration of CASE 5 is equal to the barge bottom length l_k divided by the barge speed. Time t_k is given by Eq. 129 so that

$$t_r = (h_b \tan \theta_k + l_f)\, U_b^{-1} + l_k/U_b. \qquad (133)$$

HOW TO USE THE "COLLISION" PROGRAM

The solution approach has been implemented in a program called COLLISION. Input can come either from an input file named "col.i" or from the terminal. Input must be entered into this file before execution of the COLLISION program. Variables that need to be entered are identified on the first page of the COLLISION program which is listed in Table 2. Input parameter definitions are provided in the program listing, but Figs. 1,3,4 and 14 may be helpful. An input file can be created using a line editor or a word processor. In the file, variable values are to be provided one per line in order. When entering from the terminal, simply respond to the prompts.

Since the program is written in BASIC, switch from the operating system to BASIC before execution. To load the COLLISION program, put the program disk in drive b and type load "b:col.bas". After loading COLLISION, the user can execute the program by typing run. A prompt will then appear saying

Input from terminal type t

or else from file type f

Type t or f______.

Another prompt will appear asking where to send the output. A message on the terminal is printed saying

If output is to a file type f

or else type 0

Type f or 0_____.

If output was not desired to go to the output file "res", then type a zero.

A prompt will appear on the screen saying

If output is to the terminal type t

or else type 0

Type t or 0_____.

If output was not desired to dump out on the screen then type a zero.

If input was requested to come from the terminal, then input the data when prompted. After all data has been entered, the program will be executed. Output will either be sent to the output file "res" and/or sent to the terminal. This output is in the form shown in Fig. 28.

For plotting purposes, a file named "cth" is created to store information on the pile's angle. Likewise, a file named "cmo" contains hinge moment information. To plot the contents of these two files see Appendix B. Sample plots are shown in Fig. 29 and Fig. 30.

To view the output file "res" when in MSDOS, type type b:res. To print the results in "res" at the line printer, type copy b:res prn. To copy a file from a hard disk to a file on a floppy disk in drive b, type copy b:hardfile diskfile.

A listing of the COLLISION program is shown in Table 2.

```
-----------------
impact at top  of bow
time (s) = 0  theta (deg) = 0
barge moment impulse (ft-lbs-s) =  65045.05
barge force impulse (ft-lbs-s) =  3252.252
horiz reaction impulse (lb-s) = -487.0426
vert reaction impulse (lbs-s) =  0
-----------------
time (s) =  .25  theta (deg) =  11.91462
barge moment (ft-lbs) =  66967.73  barge force (lbs) =  3341.132
hinge moment (ft_lbs) =  34810.79
horiz react (lbs) =  2933.951  vert react (lbs) =  3865.232
-------------------------
time (s) =  .5  theta (deg) =  22.87969
barge moment (ft-lbs) =  54008.05  barge force (lbs) =  2537.213
hinge moment (ft_lbs) =  45461.71
horiz react (lbs) =  3146.553  vert react (lbs) =  4955.317
-------------------------
impact at bottom of barge bow
time (s) =  .5524986  theta (deg) =  25
barge moment impulse (ft-lb-s) =  90357.62
barge force impulse (lb-s) =  10236.47
horiz reaction impulse (lb-s) =  4662.055
vert reaction impulse (lb-s) =  2173.958
------------------------
time (s) =  .75 theta (deg) =  41.44559
BARGE MOMENT (FT-LBS) =  83016.71
BARGE FORCE (LBS) =  7932.54
HINGE MOMENT (FT-LBS) =  50503.73
horiz reaction (lbs) =  13245.23
vert reaction (lbs) =  10777.63
 ----------------------
time (s) =  1 theta (deg) =  54.66514
BARGE MOMENT (FT-LBS) =  102814.8
BARGE FORCE (LBS) =  7580.095
HINGE MOMENT (FT-LBS) =  54093.81
horiz reaction (lbs) =  9285.595
vert reaction (lbs) =  14000.04
 ----------------------
time (s) =  1.25 theta (deg) =  62.70675
BARGE MOMENT (FT-LBS) =  44141.4
BARGE FORCE (LBS) =  2580.195
HINGE MOMENT (FT-LBS) =  56277.7
horiz reaction (lbs) =  3756.732
vert reaction (lbs) =  10636.7
 ----------------------
time (s) =  1.5 theta (deg) =  67.92281
BARGE MOMENT (FT-LBS) =  24897.58
BARGE FORCE (LBS) =  1192.893
HINGE MOMENT (FT-LBS) =  57694.24
horiz reaction (lbs) =  1830.262
vert reaction (lbs) =  8790.208
 ----------------------
time (s) =  1.75 theta (deg) =  71.5249
BARGE MOMENT (FT-LBS) =  25732.85
BARGE FORCE (LBS) =  1039.496
HINGE MOMENT (FT-LBS) =  58672.48
horiz reaction (lbs) =  1178.139
vert reaction (lbs) =  7999.291
 ----------------------
time (s) =  2 theta (deg) =  74.14276
BARGE MOMENT (FT-LBS) =  30032.53
BARGE FORCE (LBS) =  1046.069
HINGE MOMENT (FT-LBS) =  59383.42
horiz reaction (lbs) =  870.0916
```

Fig. 28. COLLISION output format.

```
vert reaction (lbs) =  7544.928
 ----------------------
time (s) =  2.25 theta (deg) =  76.12368
BARGE MOMENT (FT-LBS) =  34159.84
BARGE FORCE (LBS) =  1044.324
HINGE MOMENT (FT-LBS) =  59921.39
horiz reaction (lbs) =  688.9994
vert reaction (lbs) =  7234.831
 ----------------------
pile in contact with barge bottom
time duration (s) is from  2.471593  to  12.8036
theta (deg) =  77.51293
barge moment (ft-lbs) =  48834.48
barge force (lbs) =  1320.129
hinge moment (ft-lbs) =  60298.67
horiz reaction (lbs) =  258.8987
vert reaction (lbs) =  6734.493
pile recovery is initiated at t(s) = 12.8036
-----------------------
```

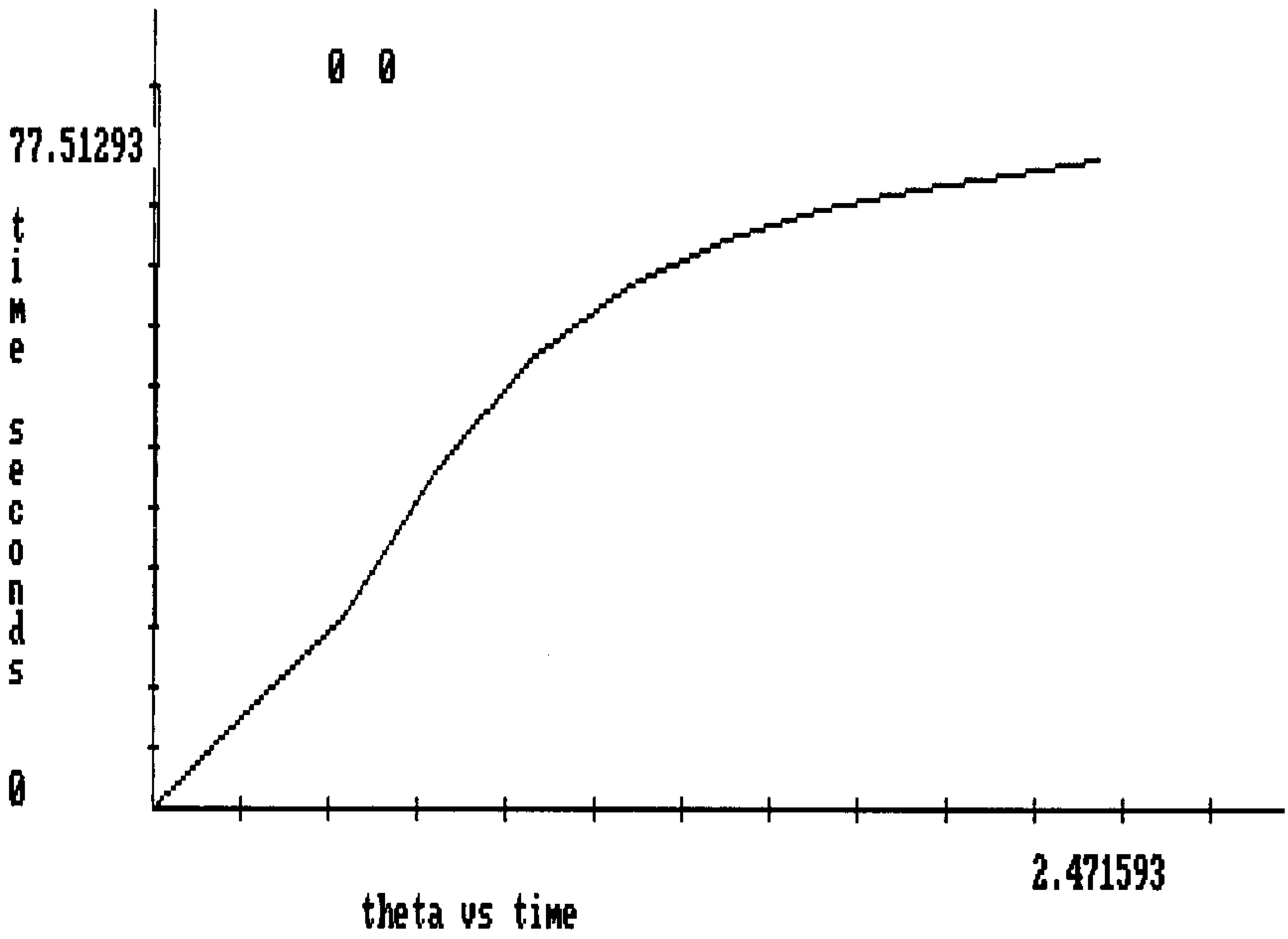


Fig. 29. Example pile angle (θ) response calculated using COLLISION.

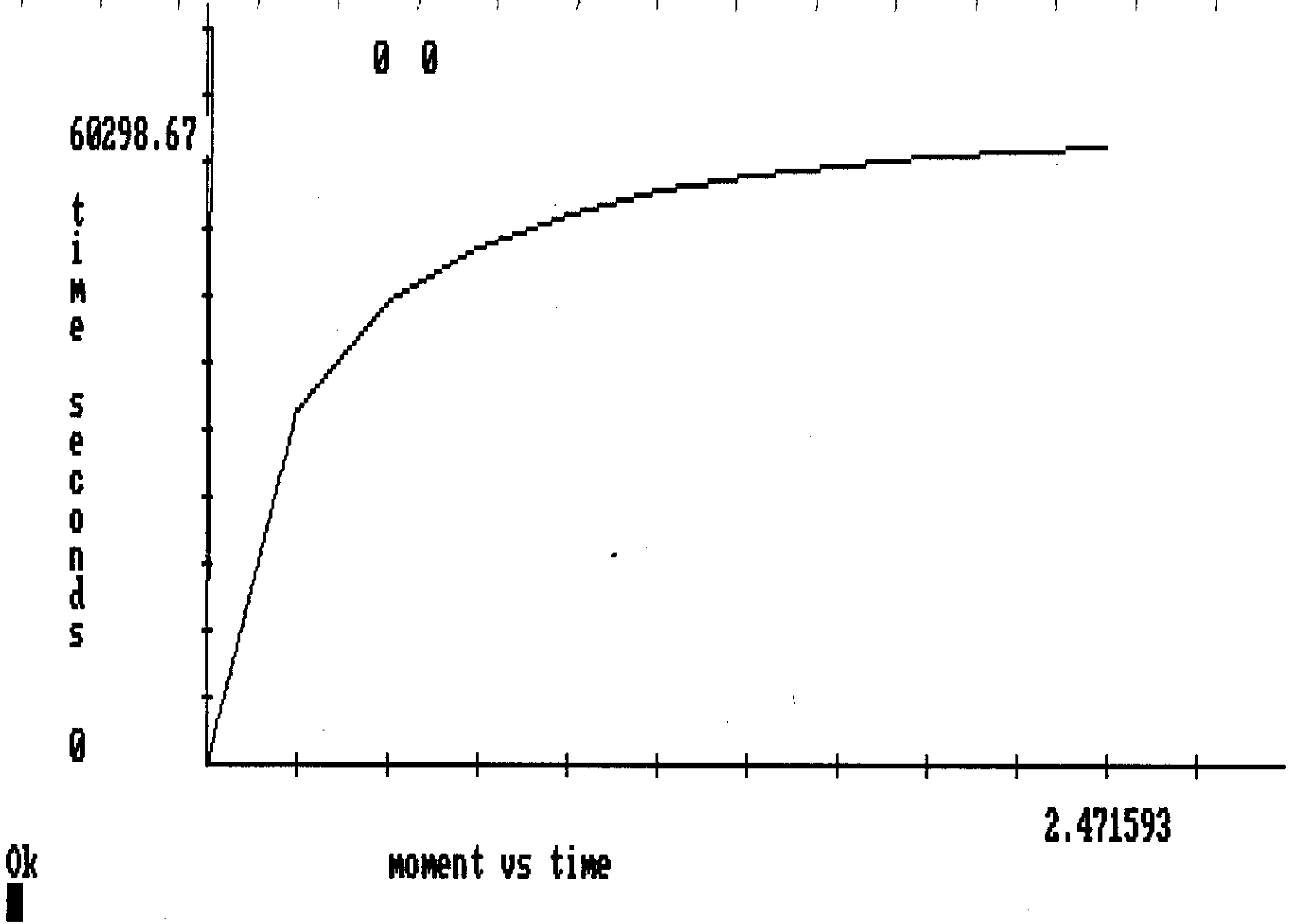


Fig. 30. Example hinge moment response calculated by COLLISION.

"COLLISION" LISTING

Table 2. COLLISION listing.

```
5 REM  collision evaluates  the dynamic  equations of motion at a
users
6 CLS 0
10 REM   specified time interval starting at initial impact.  At
15 REM  each time step the kinematic equations are used to find
20 REM  angular position,velocity and acceleration while the
25 REM  dynamic equations are used to determine barge and hinge
30 REM  forces and moments.  Time, position, barge forces and
35 REM  moments and hinge forces/moments are printed as time
40  REM   evolves.  Impact  loads   are  treated   in  terms  of
force/moment
45 REM  impulses.
50 REM
55 REM  input can be either from the terminal or from a file.
60 REM  output can be directed to the terminal, or to an output
65 REM  file and later can be plotted out to the screen.
70 REM
75 REM  the variables that need to be imputed are
80 REM  h,  input size step  (seconds)
85 REM  k1,  1st stiffness constant  (ft-lbs/rad)
90 REM  k2,  2nd stiffness constant  (ft-lbs/rad)
95 REM  ba,  breakpoint angle        (degrees)
100 REM w,   weight                  (lbs)
105 REM lw,  length to center of mass   (ft)
110 REM lp,  pile length                (ft)
115 REM dp,  pile diameter              (ft)
120 REM l1,  length to buoyancy section base (ft)
125 REM l2,  length to buoyancy section top  (ft)
130 REM d,   depth to hinge                 (ft)
135 REM im,  mass moment of inertia      (slugs-ft^2)
140 REM bw,  bell width                  (ft)
145 REM mu,  barge coefficient of friction
150 REM fb,  barge freeboard                (ft)
155 REM db,  barge draft                    (ft)
160 REM fa,  barge bow angle               (deg)
165 REM lb,  barge length                  (ft)
170 REM ub,  barge speed                 (ft/s)
175 REM --------------------------------------------
180 REM  preset variables are
185 REM  cm    inertial coefficient
190 REM  cw    drag coefficient
195 REM  rw    water density
200 REM  pi    3.1416
205 REM ----------------------------------------------
210 REM variables contained in the main program
215 REM alpha     angular accel of pile     (rad/s)
220 REM btheta    spring breakpoint angle  (rad)
225 REM f         barge force              (lbs)
230 REM fc        water current force      (lbs)
235 REM fdt       barge force impulse      (lbs-s)
240 REM ftheta    barge bow angle          (rad)
245 REM hb        vertical height from hinge to barge  (ft)
250 REM hdt       horizontal reaction impulse         (lbs-s)
255 REM ih        effective mass moment of inertia    (slug-ft^2)
260 REM ktheta    final pile angle           (rad)
265 REM lc        barge to hinge contact length   (ft)
270 REM lf        point a to b horizontal length  (ft)
275 REM lk        point b to c horizontal length  (ft)
280 REM ls        submerged pile length           (ft)
285 REM m         barge moment                  (ft-lbs)
290 REM ma        added mass                    (lbs)
295 REM mc        current moment                (ft-lbs)
300 REM mg        weigh and buoyant moment      (ft-lbs)
305 REM mdt       barge moment impulse         (ft-lbs-s)
310 REM ml        load mass                    (slugs)
315 REM mp        pile mass                    (slugs)
```

```
320 REM omega    angular velocity of pile      (rad/s)
325 REM phi      barge angle of friction
330 REM rh       horizontal reaction force      (lbs)
335 REM rv       vertical reaction force        (lbs)
340 REM t        current time from impact       (s)
345 REM tf       time duration of process 2     (s)
350 REM theta    current angle of the pile      (rad)
360 REM tk       time duration of process 5     (s)
365 REM tnkt     tangent of the final base anglr ktheta
370 REM tr       total time duration to recovery   (s)
375 REM vdt      vertical reaction impulse        (lbs-s)
380 REM rdt      reaction hinge impulse           (lbs-s)
385 REM -------------------------------------------------------
390 REM open input and output files
395  OPEN"i",#4,"col.i"  'collision input file
400  OPEN "res" FOR OUTPUT AS #1 'collision output file
401 PRINT#1, "hello"
405  OPEN "cth" FOR OUTPUT AS #2 'theta output file
410  OPEN "cmo" FOR OUTPUT AS #3 'moment output file
411 PRINT#1, "where are you"
415 REM ------------------------------------------------------
420 REM determine form of input
425 PRINT "input from terminal or file"
430 PRINT "input from terminal type t"
435 PRINT "or input from file type f"
440 PRINT "type t or f"
445 INPUT IN$
450 REM ------------------------------------------------------
455 REM determine form of output
460 PRINT "output to terminal type t"
465 PRINT "or else type O"
470 INPUT "print t or O";T$
475 PRINT "output to file print f"
480 PRINT "or else type O"
485 INPUT "type f or O";F$
490 REM --------------------------------------------------------
495 REM input data
500 IF IN$ = "f" THEN GOTO 705
505 PRINT
510 INPUT "input stepsize (s)";H
515 PRINT
520 INPUT "input 1st stiffness constant k1 (ft-lbs/rad)";K1
525 PRINT
530 INPUT "input 2nd stiffness constant k2 (ft-lbs/rad)";K2
535 PRINT
540 INPUT "input breakpoint angle (deg)";BA
545 PRINT
550 INPUT "input weight (lbs)";W
555 PRINT
560 INPUT "input length to center of mass (ft)";LW
565 PRINT
570 INPUT "input pile length (ft)";LP
575 PRINT
580 INPUT "input pile diameter (ft)";DP
585 PRINT
590 INPUT "input length to buoyancy section base (ft)";L1
595 PRINT
600 INPUT "input length to buoyancy section top (ft)";L2
605 PRINT
610 INPUT "input depth to hinge (ft)";D
615 PRINT
620 INPUT "input mass moment of inertia (slugs-ft^2)";IM
625 PRINT
630 INPUT "input bell width (ft)";BW
635 PRINT
640 INPUT "input barge coefficient of friction";MU
```

```
645 PRINT
650 INPUT "input barge freeboard (ft)";FB
655 PRINT
660 INPUT "input barge draft(ft)";DB
665 PRINT
670 INPUT "input barge bow angle (deg)";FA
675 PRINT
680 INPUT "input barge length (ft)";LB
685 PRINT
690 INPUT "input barge speed (ft/s)";UB
695 PRINT
700 GOTO 810
705 REM input from file -----------------------------------
710 INPUT#4,H
715 INPUT#4,K1
720 INPUT#4,K2
725 INPUT#4,BA
730 INPUT#4,W
735 INPUT#4,LW
740 INPUT#4,LP
750 INPUT#4,DP
755 INPUT#4,L1
760 INPUT#4,L2
765 INPUT#4,D
770 INPUT#4,IM
775 INPUT#4,BW
780 INPUT#4,MU
785 INPUT#4,FB
790 INPUT#4,DB
795 INPUT#4,FA
800 INPUT#4,LB
805 INPUT#4,UB
810 REM -----------------------------------
815 REM preset variables
820 CW = 1!
825 RW = 64!/ 32.2
830 CM = 2!
831 PI = 3.1416
835 BTHETA = BA * PI / 180 'convert to radians
840 FTHETA = FA * PI / 180 'convert to radians
845 LF = (DB + FB) * TAN (FTHETA)
850 PHI = ATN (MU)
855 LK = LB - LF
860 HB = D - DB
865 TF = ((D + FB) / UB) * TAN (FTHETA)
870 TNKT = SQR ((LP ^ 2 / HB ^ 2) - 1)
875 KTHETA = ATN (TNKT)
880 TK = (HB * TNKT + LF) / UB
885 TR = TK + LK / UB
890 REM ---------------------------------------------------------
895 T=0   ' set time
900                                                          REM
**********************************************************
905 REM      case 1 impact at a
910                                                          REM
**********************************************************
915 IH = IM + RW * CM  * PI  * (DP  ^ 2)  * (D  ^ 3)  / 12 'added
inertia
920 MDT = IH * UB / (D + FB)
925 FDT = (IH * UB) / ((D + FB) ^ 2)
930 MA = CM * RW * PI * (DP ^ 2) * D / 4
940 HDT  = (IH / (D + FB) - (LW * W / 32.2 + .5 * D * MA)) * UB /
(D + FB)
945 VDT = 0
946 PRINT#2,0;0
947 PRINT#3,0;0
```

```
950 REM print out results ------------------------
951 CLS
955 IF T$ = "O" THEN GOTO  1005
960     PRINT
965     PRINT "---------------------"
970     PRINT "impact at top of barge bow"
975     PRINT "time (s) = 0      theta (deg) = 0"
980     PRINT "barge moment impulse (ft-lbs-s) = ";MDT
985     PRINT " barge force impulse (lbs-s) = ";FDT
990     PRINT " horiz react impulse (lbs-s) = ";HDT
995     PRINT " vert react impulse (lbs-s) =  ";VDT
1000    PRINT "---------------"
1005  REM   ----------------
1010 REM output to file
1015 IF F$ = "O" THEN GOTO 1055
1017 PRINT#1,"-----------------"
1020  PRINT#1," impact at top  of bow"
1025 PRINT#1," time (s) = 0  theta (deg) = 0 "
1030 PRINT#1," barge moment impulse (ft-lbs-s) = ";MDT
1035 PRINT#1," barge force impulse (ft-lbs-s) = ";FDT
1040 PRINT#1," horiz reaction impulse (lb-s) = ";HDT
1045 PRINT#1," vert reaction impulse (lbs-s) = ";VDT
1050 PRINT#1, "------------------"
1055 PRINT#2,"theta vs time"
1056 PRINT#2,"time seconds"
1057 PRINT#2,"theta deg"
1058 PRINT#3,"moment vs time"
1059 PRINT#3,"time seconds"
1060 REM update theta file
1061 PRINT#3,"momt ftlbs"
1065 PRINT#2,T;T
1070 REM ****************************************************
1075 REM end of case 1
1080 REM case 2 if , contact at point a
1081 IF  (T + H) > = TF THEN GOTO 1490
1085 REM  case 2 loop
1090 REM ****************************************************
1095  T = T + H ' update time
1100 REM kinematic analysis _______________
1105 REM -------------
1110 THETA = ATN (UB * T / (D + FB))  ' for point a contact
1115 IF (D + FB) / COS (THETA) < = LP THEN GOTO 1180
1120 REM  pile tip contact
1125 THETA = ATN  ((1 -  (D +  FB) /  (LP *  COS (THETA))  * TAN
(FTHETA) +                        UB * T / (LP * COS (THETA)))
1140   OMEGA =  UB /  (LP *  ( COS  (THETA) +  TAN (FTHETA) * SIN
(THETA)))
1145 ALPHA = ( SIN (THETA) - TAN (FTHETA) * COS (THETA))  * (( UB
/ LP)  ^ 2)                / (( COS (THETA) + TAN (FTHETA) * SIN
(THETA)) ^ 3)
1165  LC = LP   'contact length
1170  GOTO 1200
1175 REM   analysis for point a contact ---------------------
1180  OMEGA = UB * (( COS (THETA)) ^ 2) / (D + FB)
1185 ALPHA = - 2 * ((UB / (D + FB)) ^ 2) * (( COS (THETA)) ^ 3) *
SIN(THETA)
1195  LC = (D+ FB) / COS (THETA)
1200 REM end of kinematic analysis ----------------------------
1205 REM print out theta to cth file
1210 PRINT#2,    T;   THETA*180/3.1416
1215 REM -----------
1220 REM submerged length calculations
1225 IF LP * COS (THETA) < D THEN GOTO  1240
1230    LS = D / COS (THETA) 'tip not submerged
1235    GOTO 1245
1240  LS = LP  ' tip submerged
```

```
1245 REM ----------------------------
1250 IH = IM + RW * CM * PI * (DP ^ 2) * (LS ^ 3 ) / 12
1255 MA = CM * RW * PI * (DP ^ 2) * LS / 4
1260  GOSUB 2565   ' spring moment subroutine
1265  GOSUB 2640   ' current moment subroutine
1270  GOSUB 2870   ' weight/buoyancy moment subroutine
1275  M = IH * ALPHA + MH - MG - MC
1280  GOSUB 2750   ' current force subroutine
1285  REM barge force, reaction force calculations
1290  REM ---------------------------------------
1295  IF (D + FB) / COS (THETA) > LP THEN GOTO  1350
1300 REM  pile contact at point a
1305 F = M / (LC * COS (PHI))
1310 RH  = F  * COS  (THETA - PHI) + FC * COS (THETA) + (LW * W /
32.2) * (                (OMEGA ^  2) *  SIN (THETA)  - ALPHA *
COS (THETA)) - MA * .5 * LS                     * ALPHA * COS (THETA)
1325 RV =  F * SIN (THETA - PHI) + FC * SIN (THETA) + W - (LW * W
/ 32.2)                        * (ALPHA * SIN (THETA) + (OMEGA ^ 2) *
COS (THETA)) -MA * .5 * LS               * ALPHA * SIN (THETA)
1340 GOTO 1390
1345 REM ---------------------------------------
1350 REM pile tip contact on barge
1355 F = M / (LP * COS (THETA - FTHETA + PHI))
1360 RH = F * COS ( FTHETA - PHI) + FC * COS (THETA) + (LW *  W /
32.2) *                   ((OMEGA ^ 2) * SIN (THETA) - ALPHA * COS
(THETA)) - MA * .5 * LS                  * ALPHA * COS (THETA)
1375 RV = F * SIN (FTHETA - PHI) + FC * SIN (THETA) + W - (LW * W
/ 32.2)                        * (ALPHA * SIN (THETA) + (OMEGA ^ 2) *
COS (THETA)) - MA * .5 *                 LS * ALPHA * SIN (THETA)
1390 REM end of force moment calculations
1395 REM ------------------------------------------
1400 REM print out moment to moment file
1405  PRINT#3, T;MH
1410 REM print out results to terminal/output file
1415 REM ------------------------------------------
1420   IF T$ = "O" THEN GOTO 1450
1425 PRINT "time (s) = ";T;"  theta (deg) = ";THETA * 180 / PI
1430 PRINT "barge moment (ft-lbs) = ";M;" barge force (lbs) = ";F
1435 PRINT "hinge moment (ft-lbs) = ";MH
1440 PRINT "horiz react (lbs) = ";RH;" vert react (lbs) = ";RV
1445 PRINT
1450   IF F$ = "O" THEN GOTO 1480
1455 PRINT#1, "time (s) = ";T;" theta (deg) = ";THETA * 180 / PI
1460 PRINT#1, "barge moment (ft-lbs) = ";M;" barge  force (lbs) =
";F
1465 PRINT#1, "hinge moment (ft_lbs) = ";MH
1470 PRINT#1, "horiz react (lbs) = ";RH;" vert react (lbs) = ";RV
1475 PRINT#1,"------------------------"
1480 REM ----------------------------------------------
1485 GOTO 1081
1490 REM end of case 2 loop
1500 REM *******************************************************
1505 REM  case 3 impact at b
1510 REM *******************************************************
1515 REM calcukate the submerged length
1520 IF LP * COS (FTHETA) < D THEN GOTO 1535
1525  LS = D / COS (FTHETA)   ' tip not submerged
1530  GOTO 1540
1535  LS = LP 'tip submerged
1540 REM ----------------------------
1545 REM calculate contact length, added mass/moment of inertia
1550 IH = IM + RW * CM * PI * (DP ^ 2) * (LS ^ 3) / 12
1555 MA = CM * RW * PI * (DP ^ 2) * LS / 4
1560 LC = HB / COS (FTHETA)
1565 REM -----------------------------
1570 REM calculate force and moment impulses
```

```
1575 MDT = IH * UB * (( COS (FTHETA)) ^ 2) * (1 / HB - (1  / (D +
FB)))
1580 FDT = MDT / LC
1585 RDT  = (IH  / LC - (LW * W / 32.2 + .5 * LS * MA)) * UB * ((
COS (FTHETA           )) ^ 2) * ((1 / HB) - (1 / (D + FB)))
1900 HDT = RDT * COS (FTHETA)
1905 VDT = RDT * SIN (FTHETA)
1910 REM print out results
1915 REM -----------------------------
1920 IF T$ = "O" THEN GOTO 1950
1921 PRINT "impact at bottom of barge bow"
1925 PRINT "time (s) = ";TF;" theta (deg) = ";FTHETA * 180 / PI
1930 PRINT "barge moment impulse (ft-lb-s) = ";MDT
1935 PRINT "barge force impulse (lb-s) = ";FDT
1940 PRINT "horiz react impulse (lb-s) = ";HDT
1945 PRINT "vert react impulse (lb-s)  =";VDT
1946 PRINT
1950 IF F$ = "O" THEN GOTO  1985
1955 PRINT#1, "impact at bottom of barge bow"
1960 PRINT#1, "time (s) = ";TF;" theta (deg)  = ";FTHETA  * 180 /
PI
1965 PRINT#1, "barge moment impulse (ft-lb-s) = ";MDT
1970 PRINT#1, "barge force impulse (lb-s) = ";FDT
1975 PRINT#1, "horiz reaction impulse (lb-s) = ";HDT
1980 PRINT#1, "vert reaction impulse (lb-s) = ";VDT
1981 PRINT#1, "-----------------------"
1985 REM ---------------------------
1990 REM   END OF PRINT OUT
1995 REM   END OF CASE 3
2000 REM  CASE 4 IF
2005 IF (T + H) > = TK THEN GOTO   2300
2010
REM***********************************************************
2015 REM case 4 loop  piviting/sliding at b
2020                                                        REM
**********************************************************
2025 T = T + H  'update time
2030 REM kinematic analysis
2035 THETA = ATN ((UB * T - LF) / HB)
2040 OMEGA = (UB / HB) * (( COS (THETA)) ^2)
2041 ALPHA = -2 * ((UB / HB)  ^ 2)  * ((COS  (THETA)) ^  3) * SIN
(THETA)
2045 LC = HB / COS (THETA)
2050 REM calculate submerged length
2055 REM ------------------------
2060 IF LP * COS (THETA) < D THEN GOTO 2075
2065 LS = D / COS (THETA)  'tip not submerged
2070 GOTO 2080
2075 LS = LP   'tip submerged
2080 REM -------------------------
2085 IH = IM + RW * CM * PI * (DP ^ 2) * (LS ^ 3 ) / 12
2090 MA = CM * RW * PI * (DP ^ 2) * LS / 4
2095 REM call spring moment subroutine to get mh
2100 GOSUB 2565 'spring moment subroutine
2105 REM call current moment subroutine to get mc
2110 GOSUB 2640
2115 REM call weight /buoyant subroutine to get mg
2120 GOSUB 2870
2125 M = IH * ALPHA + MH - MG - MC
2130 F = M / (LC * COS (PHI))
2135 REM  CALL CURRENT  SUBROUTINE TO GET MC
2140 GOSUB 2750
2145 REM calculate reaction forces
2150 REM --------------------------
2155 RH  = F  * COS  (THETA - PHI) + FC * COS (THETA) + (LW * W /
32.2) * (                 (OMEGA ^  2) *  SIN (THETA)  - ALPHA *
```

```
COS (THETA)) - MA * .5 * LS *              ALPHA * COS (THETA)
2175 RV  = F * SIN (THETA - PHI) + FC * SIN (THETA) + W - (LW * W
/ 32.2)                   * (ALPHA * SIN  (THETA) +  (OMEGA ^  2) *
COS (THETA)) - MA * .5 * LS              * ALPHA * SIN (THETA)
2190 REM ----------------------------
2195 REM print out results to moment and theta files
2200 PRINT#3, T;MH
2205 PRINT#2,     T;THETA*180/3.1416
2210 REM ------------------------------------------
2215 REM print out results
2220 IF T$ = "O" THEN GOTO 2255
2221 PRINT "time (s) = ";T; "theta (deg) = ";THETA * 180 /PI
2225    PRINT "barge moment (ft-lbs) = ";M
2230    PRINT "barge force (lbs) = ";F
2235    PRINT "hinge moment (ft-lbs) = ";MH
2240    PRINT "horiz reaction (lbs) = ";RH
2245    PRINT "vert reaction (lbs) = ";RV
2250    PRINT
2255 IF F$ = "O" THEN GOTO 2290
2256 PRINT#1, "time (s) = ";T;"theta (deg) = ";THETA * 180 / PI
2260    PRINT#1, "BARGE MOMENT (FT-LBS) = ";M
2265    PRINT#1, "BARGE FORCE (LBS) = ";F
2270    PRINT#1, "HINGE MOMENT (FT-LBS) = ";MH
2275    PRINT#1, "horiz reaction (lbs) = ";RH
2280    PRINT#1, "vert reaction (lbs) = ";RV
2285    PRINT#1," ---------------------"
2290 REM ---------------------------------------------
2295 GOTO 2005
2300 REM end Of case 4 loop
2305 REM ***********************************************
2310 REM case 5 contact with barge bottom
2315 REM ***********************************************
2320 THETA = KTHETA   'constant
2325 OMEGA = 0       'static
2330 ALPHA = 0       'static
2335 LC = LP
2340 LS = LP
2345 REM call spring moment subroutine to get mh
2350 GOSUB 2565
2355 REM call current moment subroutine to get mc
2360 GOSUB 2640
2365 REM call weight/buoyant subroutine to get mg
2370 GOSUB 2870
2375 M = MH - MG - MC
2380 F = M / (LP * SIN (KTHETA + PHI))
2385 REM  cALL SURRENT FORCE SUBROUTINE TO GET FC
2390 GOSUB 2750
2395 REM calculate reaction forces
2400 RH = F * SIN (PHI) + FC * COS (THETA)
2405 RV = F * COS (PHI) + FC * SIN (THETA) + W
2410 REM print out results to theta,moment files
2415 REM --------------------
2420 PRINT#2,TK,THETA*180/3.1416
2425 PRINT#3,         TK  ; MH
2430 REM --------------------
2435 REM print out results
2440 IF T$ = "O" THEN GOTO 2490
2445 PRINT "pile in contact with barge bottom"
2450 PRINT "time duration (s) is from ";TK; " to ";TR
2455 PRINT "theta (deg) = ";KTHETA * 180 / PI
2460 PRINT "barge moment (ft-lbs) = ";M
2465 PRINT "hinge moment (ft-lbs) = ";MH
2466 PRINT "hinge moment (ft-lbs) = ";MH
2470 PRINT "horiz reaction (lbs) = ";RH
2475 PRINT "vert reaction (lbs) = ";RV
2480 PRINT "pile recovery is initiated at t(s) = ";TR
```

```
2485 PRINT
2490 IF F$ = "O" THEN GOTO 2545
2495 PRINT#1," pile in contact with barge bottom"
2500 PRINT#1, "time duration (s) is from ";TK;" to ";TR
2505 PRINT#1, "theta (deg) = ";KTHETA * 180 / PI
2510 PRINT#1, "barge moment (ft-lbs) = ";M
2515 PRINT#1, "barge force (lbs) = ";F
2520 PRINT#1, "hinge moment (ft-lbs) = ";MH
2525 PRINT#1, "horiz reaction (lbs) = ";RH
2530 PRINT#1, "vert reaction (lbs) = ";RV
2535 PRINT#1, "pile recovery is initiated at t(s) =";TR
2540 PRINT#1,"------------------------"
2541 REM close output files
2542 CLOSE #2
2543 CLOSE #1
2544 CLOSE #3
2545 REM ***********************************************
2550 REM    END OF CASE 5 AND PROGRAM
2555 END
2560 REM ***********************************************
2565 REM subroutine spring moment
2570 REM this subroutine is given a value for theta and
2575 REM gives back a value for mh the spring moment
2580 REM btheta is the breakpoint angle (rad)
2585 REM k1 is the 1st stiffness constant
2590 REM k2 is the 2nd stiffness constant
2595 IF THETA > (BTHETA) THEN GOTO 2615
2600 IF THETA < ( - BTHETA) THEN GOTO  2625
2605     MH = K1 * THETA
2610     GOTO 2630
2615     MH = K1 * BTHETA + K2 * (THETA - BTHETA)
2620     GOTO 2630
2625     MH = -K1 * BTHETA + K2 * (THETA + BTHETA)
2630  RETURN
2635 REM ***********************************************
2640 REM subroutine current moment
2645  REM  this  subroutine  is  given  a  set  of  parameters to
integrate
2650 REM by the use of the trapezoidal rule. It returns a
2655 REM a value for the current moment,mc.
2660 REM delta is  the integration step
2665 REM ls is the submerged length
2670 REM sum is a counter
2675 REM fd is the drag force
2680 REM fi is the inertial force
2685  DLTA = LS / 10 'pile broken into 10 integration steps
2690  SUM = 0
2695  FOR N = 1 TO 10
2700     S = DLTA * N
2705 FD = .5 * RW * CW * DP * ((  -S *  OMEGA) ^  2) *  SGN (-S *
OMEGA)
2710     IF N > 9.899999 THEN GOTO  2725
2715      SUM = SUM + S * (FD )  'update sum
2720      GOTO   2730
2725      SUM = SUM + .5 * S * (FD )  'update sum
2730   NEXT N
2735 MC = DLTA * SUM
2740 RETURN
2745 REM ***************************************************
2750 REM subroutine current force
2755 REM this subroutine uses the trapezoidal rule to
2760 REM numerically integrate values for the current
2765 REM force along the length of the pile. it returns
2770 REM a value for the current force.
2775 REM delta, integration step
2780 REM sum,  counter
```

```
2785 REM fd ,  drag force
2790 REM fi , inertial force
2795 REM fc , current force
2800 DLTA = LS / 10
2805 SUM = 0
2810 FOR N = 0 TO 10  ' integration loop
2815     S = DLTA * N
2820 FD  = .5  * RW  * CW * DP * (( -S * OMEGA) ^ 2) * SGN ( -S *
OMEGA)
2825     IF N < .9 THEN GOTO  2845
2830     IF N > 9.899999 THEN GOTO  2845
2835     SUM = SUM + .5 * (FD)  ' update sum
2840     GOTO  2850
2845     SUM = SUM + .5 * (FD )
2850  NEXT N  ' end of integration loop
2855 FC = DLTA * SUM
2860 RETURN
2865 REM ****************************************************
2870 REM subroutine weight/buoyancy
2875 REM this subroutine gives back a value for the
2880 REM buoyant/weight moment
2885 REM the inputs are
2890 REM dp, diameter of pile
2895 REM l1, length to buoyant base
2900 REM l2, length to buoyant top
2905 REM ls, submerged pile length
2910 REM other variables are
2915 REM by buoyant force
2920 REM l3 centroidal length
2925 REM mg, weight & buoyant moment
2930 IF LS > = L2 THEN GOTO 2955      'submerged tip if
2931      REM       volume       not       totally      submerged
-----------------------------
2935    BY  = 64!  * PI  * (DP  ^ 2)  * (LS - L1) / 4 'volume not
totally submergd
2940     IF BY < 0 THEN BY = 0  'no negative buoyancy force
2945   L3 = (L1 + LS) / 2    'volume centroidal length
2950   GOTO  2975
2955 REM -------------------------------------------
2960 REM volume totally submerged
2965 BY = 64! * PI * (DP ^ 2) * (L2 - L1) / 4
2970 L3 = (L1 + L2) / 2 ' length to centroid of volume
2975 MG = ( -L3 * BY + LW * W) * SIN (THETA)  + (BY  - W)  * BW *
COS (THETA)                 * SGN (THETA)
2990 RETURN
3000                                                         REM
*********************************************************
```

IV. RECOVERY

PURPOSE

The principal purpose of the RECOVERY model is to determine the pile's angle response as a function of time after a collision as the pile returns to an upright position. In this analysis the pile is initially displaced an angle θ_k which is the pile angle as the tip is released from the bottom of the barge. The initial angle θ_k can be obtained from the COLLISION model output.

MAJOR ASSUMPTIONS

1. Pile motion is restricted to a vertical plane so that there is one degree of freedom characterized by the inclination angle.
2. The light daymark boards are sacrificed in the collision. This reduces the fluid drag and inertial forces on the pile.
3. Wave motion is neglected.
4. Wind is neglected.
5. At time = 0, $\dot{\theta}_o = 0$ and $\theta_o = \theta_k$.

APPROACH

The RECOVERY model is essentially the STORM model without wind and waves. The initial position is specified to correspond to the pile when released from under the barge bottom. The basic dynamic equation is the time rate of change of angular momentum equation applied to the base of the pile. Moment contributions are due to the weight/buoyant moment, the hinge moment and the current moment. Thus

$$\ddot{\theta} = (-M_h + M_c + M_g)/I_H \tag{134}$$

The dynamic equation is then numerically integrated using the Runge-Kutta fourth order method. Initial conditions are that the pile is released at angle θ_k with no angular velocity so that $\theta_o = \theta_k$ and $\dot{\theta}_o = 0$.

THEORY

The analysis is based on Eq. 134 in which the applied moments are evaluated as in the STORM model. Thus the hinge moment is found by using Eq. 3. The weight/buoyant moment can be found by Eq. 15. The current moment is evaluated using Eq. 5 and assuming no wave motion. Expressions for inertial and drag contributions are trapezoidally integrated. Once all the moments are found, the pile angle is numerically integrated using the Runge-Kutta fourth order method.

HOW TO USE THE "RECOVERY" PROGRAM

The solution approach has been implemented in a program named RECOVERY. Input can come either from an input file name "rin" or from the terminal. Input must be entered into the "rin" file before execution of the RECOVERY program. Data can be entered into this file by the use of a word processor. Variables that need to be entered are provided on the first page of the RECOVERY program which is listed in Table 3. List one variable per line in order.

Since the program is written in BASIC, switch from the operating system to BASIC before attempting to run. To load the RECOVERY program, put the program disk in drive b and type <u>load "b:rec.bas"</u>. After loading RECOVERY, the user can execute the program by typing <u>run</u>. A message will say

If input is from a file type f

or input from the terminal type t

Type t or f______.

Another prompt will appear asking where to send the output. A message on the screen is printed

If output is to a file type f

or else type 0

type f or 0______.

If output is not desired to go to the output file "rout", then type a zero.

A prompt will appear on the screen saying

If output is to the terminal type t

or else type 0

type t or 0______.

If output is not desired to dump out on the screen, type a zero.

If input was requested to come from the terminal, then input the data when prompted. After all the data has been entered, the main part of the program will be executed. Output will be sent to the terminal and/or sent to the output file "rout". This output consists of time, the pile angle and hinge moment as shown in Fig. 31.

For plotting purposes, information on the pile's angle will be stored in a file named "rth". Likewise, hinge moment output will be stored in the file name "rm". To plot the contents of these files, see Appendix B. Sample plots shown in Figs. 32 and 33.

To view the output file "rout" when in the operating system (MSDOS), type type b:rout. To print the results in rout to the printer, type copy b:rout prn.

A listing of the RECOVERY program is provided in Table 3.

```
t(s) =   0 theta(deg) =  90       hinge moment (ft-lbs) =  63689.83
t(s) =  .2 theta(deg) =  89.75014 hinge moment (ft-lbs) =  63621.98
t(s) =  .4 theta(deg) =  89.01696 hinge moment (ft-lbs) =  63422.87
t(s) =  .6 theta(deg) =  87.84554 hinge moment (ft-lbs) =  63104.74
t(s) =  .8 theta(deg) =  86.29958 hinge moment (ft-lbs) =  62684.9
t(s) =   1 theta(deg) =  84.44963 hinge moment (ft-lbs) =  62182.5
t(s) = 1.2 theta(deg) =  82.36333 hinge moment (ft-lbs) =  61615.92
t(s) = 1.4 theta(deg) =  80.09962 hinge moment (ft-lbs) =  61001.15
t(s) = 1.6 theta(deg) =  77.70643 hinge moment (ft-lbs) =  60351.22
t(s) = 1.8 theta(deg) =  75.22096 hinge moment (ft-lbs) =  59676.23
t(s) =   2 theta(deg) =  72.67104 hinge moment (ft-lbs) =  58983.74
t(s) = 2.2 theta(deg) =  70.07699 hinge moment (ft-lbs) =  58279.27
t(s) = 2.4 theta(deg) =  67.45333 hinge moment (ft-lbs) =  57566.75
t(s) = 2.6 theta(deg) =  64.81032 hinge moment (ft-lbs) =  56848.98
t(s) = 2.8 theta(deg) =  62.15503 hinge moment (ft-lbs) =  56127.87
t(s) =   3 theta(deg) =  59.49235 hinge moment (ft-lbs) =  55404.76
t(s) = 3.2 theta(deg) =  56.82557 hinge moment (ft-lbs) =  54680.53
t(s) = 3.4 theta(deg) =  54.1225  hinge moment (ft-lbs) =  53946.45
t(s) = 3.6 theta(deg) =  51.28966 hinge moment (ft-lbs) =  53177.12
t(s) = 3.8 theta(deg) =  48.24288 hinge moment (ft-lbs) =  52349.69
t(s) =   4 theta(deg) =  44.91146 hinge moment (ft-lbs) =  51444.97
t(s) = 4.2 theta(deg) =  41.27253 hinge moment (ft-lbs) =  50456.73
t(s) = 4.4 theta(deg) =  37.33703 hinge moment (ft-lbs) =  49387.95
t(s) = 4.6 theta(deg) =  33.09641 hinge moment (ft-lbs) =  48236.31
t(s) = 4.8 theta(deg) =  28.52311 hinge moment (ft-lbs) =  46994.32
t(s) =   5 theta(deg) =  23.57287 hinge moment (ft-lbs) =  45649.96
t(s) = 5.2 theta(deg) =  18.1879  hinge moment (ft-lbs) =  44187.55
t(s) = 5.4 theta(deg) =  12.3023  hinge moment (ft-lbs) =  35943.47
t(s) = 5.6 theta(deg) =  6.047133 hinge moment (ft-lbs) =  17667.84
t(s) = 5.8 theta(deg) = -.2300744 hinge moment (ft-lbs) = -672.2056
t(s) =   6 theta(deg) = -5.774936 hinge moment (ft-lbs) = -16872.56
t(s) = 6.2 theta(deg) = -10.20689 hinge moment (ft-lbs) = -29821.35
t(s) = 6.4 theta(deg) = -13.44051 hinge moment (ft-lbs) = -39268.99
t(s) = 6.6 theta(deg) = -15.43222 hinge moment (ft-lbs) = -43439.17
t(s) = 6.8 theta(deg) = -16.20868 hinge moment (ft-lbs) = -43650.04
t(s) =   7 theta(deg) = -15.81897 hinge moment (ft-lbs) = -43544.21
t(s) = 7.2 theta(deg) = -14.27898 hinge moment (ft-lbs) = -41718.73
t(s) = 7.4 theta(deg) = -11.67296 hinge moment (ft-lbs) = -34104.74
t(s) = 7.6 theta(deg) = -8.217073 hinge moment (ft-lbs) = -24007.72
t(s) = 7.8 theta(deg) = -4.182929 hinge moment (ft-lbs) = -12221.21
t(s) =   8 theta(deg) =  .1441303 hinge moment (ft-lbs) =  421.1038
t(s) = 8.2 theta(deg) =  4.052518 hinge moment (ft-lbs) =  11840.19
t(s) = 8.4 theta(deg) =  7.165084 hinge moment (ft-lbs) =  20934.14
t(s) = 8.6 theta(deg) =  9.387996 hinge moment (ft-lbs) =  27428.79
t(s) = 8.8 theta(deg) =  10.66388 hinge moment (ft-lbs) =  31156.51
t(s) =   9 theta(deg) =  10.96374 hinge moment (ft-lbs) =  32032.62
```

Fig. 31. RECOVERY output format.

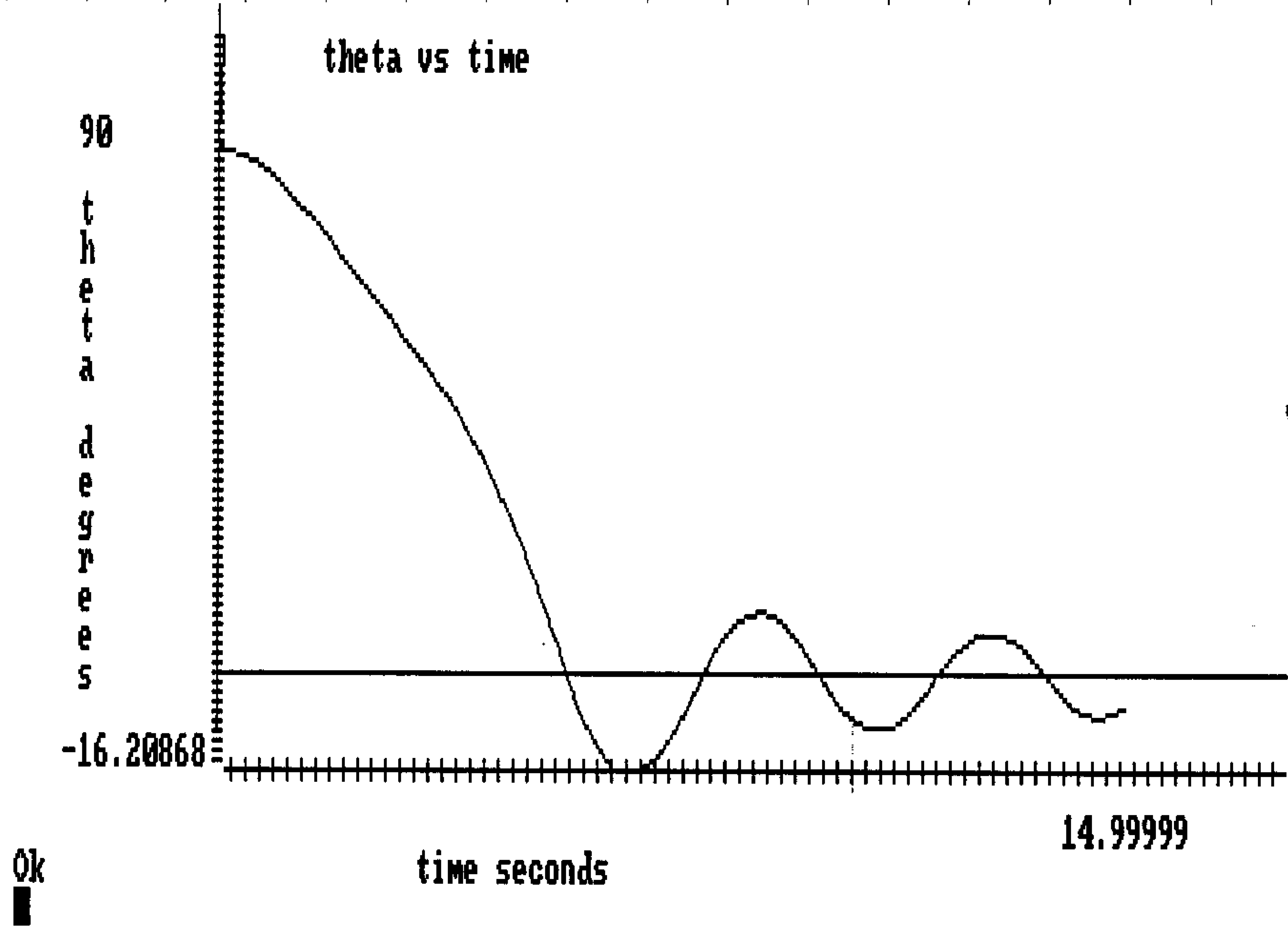


Fig. 32. Example of angle response (θ) calculated by RECOVERY.

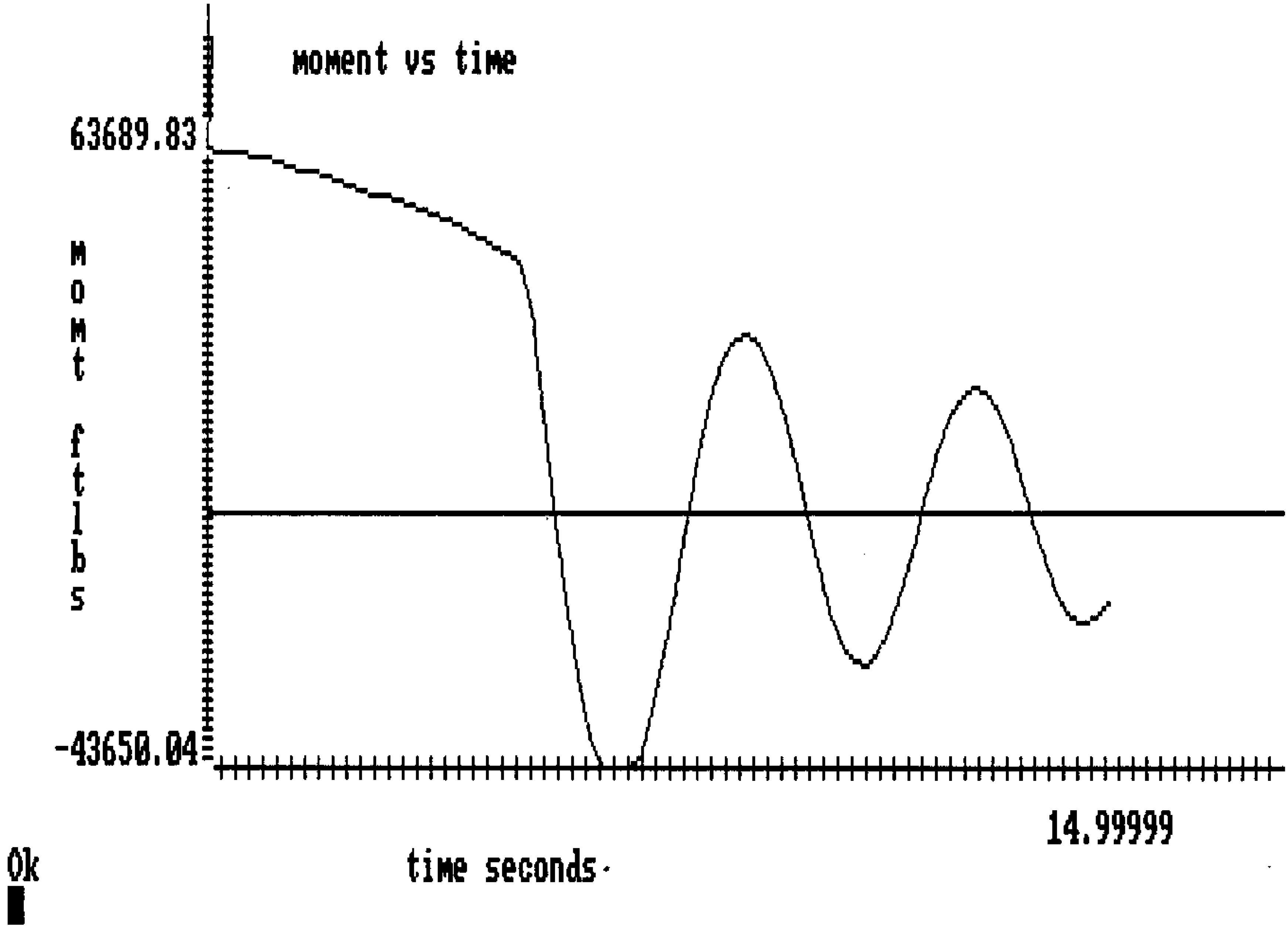


Fig. 33. Example of hinge moment response calculated using RECOVERY.

"RECOVERY" LISTING

Table 3. RECOVERY Listing

```
10 REM  RECOVERY MODELS  THE CENTRAL  STAY SYSTEM (WITH BUOYANCY)
DURING RECOVERY
11 CLS 0
15 REM from a collision.   It  is  essentially  recpile  with the
combined
20 REM weight/buoyancy term.
25 REM input variables are
30 REM ------------------
35 REM h  stepsize (s)
40 REM max   maximum time (s)
45 REM k1    1st stiffness constant (ft-lbs/rad)
50 REM k2    2nd stiffness constant (ft-lbs/rad)
55 REM ba    breakpoint angle (deg)
60 REM im    mass moment of inertia (ft^2-slugs)
65 REM lp    pile length (ft)
70 REM dp    pile diameter (ft)
75 REM lw    length to center of gravity (ft)
80 REM w     total weight (lbs)
85 REM d     depth to hinge (ft)
90 REM l1    length to buoyancy section base (ft)
95 REM l2    length to buoyancy section top (ft)
100 REM bw   bell width
105 REM uc   current velocity (ft/s)
110 REM ti   initial theta (deg)
115 REM intrvl$   output time step (integer)
120 REM -----------------------------------
125 REM preset variables are
130 REM rw    density of water
135 REM cw    drag coefficient of pile in water
140 REM cm    added mass coefficient
145 REM -----------------------------------
150 REM other variables are
155 REM BTHETA       BREAKPOINT ANGLE (RAD)
160 REM dlta         trapezoidal integration step
165 REM fb           barge force (lbs)
170 REM fi           pile inertial force (lbs)
175 REM i            time counter for arrays
180 REM ih           mass moment of inertia + added mass
185 REM j            runge kutta integration counter
190 REM k            angular velocity for runge kutta
195 REM ktheta       initial angle (rad)
200 REM lX           number of output values
205 REM lb           length to buoyant centroid (ft)
210 REM ls           submerged pile length (ft)
215 REM m            angular accel for runge kutta
220 REM mc           current moment array
225 REM mg           weight /buoyant moment array
230 REM mh           spring moment array
235 REM n            trapezoidal integration loop counter
240 REM omega        angular velocity  array (rad/sec)
245 REM s            length to integration on pile
250 REM sum          trapezoidal integration sum
255 REM fd           drag coefficient of pile
260 REM t            time array from recovery
265 REM theta        angle array
270 REM ------------------------------------------------
275 REM  determine form of input
280 PRINT "if input is from a file type f"
285 PRINT "or if input is from the terminal type t"
290 INPUT "type t or f";IN$
295 REM ------------------------------------------------
303 REM determine form of output
305 PRINT "----------------------"
310 PRINT "if output is to a file type f"
315 PRINT "else type O"
320 INPUT "type f or O";F$
```

```
325 PRINT "----------------------"
330 PRINT "if output is to the terminal type t"
335 PRINT "else type O"
340 INPUT "type t or O";T$
345 REM ------------------------------------------------
350 REM open files for input and output
355 OPEN "rout" FOR OUTPUT AS #1   ' general output file
360 OPEN "rm" FOR OUTPUT AS #2     ' moment output
365 OPEN "rt" FOR OUTPUT AS #3     'theta output
370  OPEN "rin" FOR INPUT AS #4 'input file
371  PRINT#3,"theta vs time"
372  PRINT#3,"time seconds"
373  PRINT#3,"theta degrees"
374  PRINT#2,"moment vs time"
375 REM ---------------------------------------------
376  PRINT#2,"time seconds"
377  PRINT#2,"momt ftlbs"
380 REM input variables
385 IF IN$ = "f" THEN GOTO 575
390 INPUT "stepsize (s) " ;H
395 PRINT
400 INPUT "input max t";MAX
405 PRINT
410 INPUT " 1st stiffness constant k1 (ft-lbs/rad) ";K1
414 PRINT
420 INPUT " 2nd stiffness constant k2 (ft-lbs/rad) ";K2
425 PRINT
430 INPUT " breakpoint angle";BA
435 PRINT
440  INPUT  "  input  mass  moment  of  inertia  about  base
(ft^2-slugs)";IM
445 PRINT
450 PRINT
455 INPUT "input pile length (ft) ";LP
460 PRINT
465 INPUT " input pile diameter (ft) ";DP
470 PRINT
475 INPUT "length to center of gravity (ft)";LW
480 PRINT
485 INPUT "input total weight (lbs)";W
490 PRINT
495 INPUT " input depth to hinge (ft) ";D
500 PRINT
505 INPUT "length to buoyancy section base (ft) ";L1
510 PRINT
515 INPUT "length to buoyancy section top (ft)";L2
520 PRINT
525 INPUT "input bell width (ft)";BW
530 PRINT
535 INPUT "input current velocity (ft/s) ";UC
540 PRINT
545 INPUT " initial theta (deg)";TI
550 PRINT
555 INPUT "input OUTPUT TIME STEP (INTEGER)";ITRVLX
560 PRINT
565 REM ---------------------------------------------
570 GOTO 660
575 INPUT#4, H
580 INPUT#4, MAX
585 INPUT#4, K1
590 INPUT#4, K2
595 INPUT#4, BA
600 INPUT#4, IM
605 INPUT#4, LP
610 INPUT#4, DP
615 INPUT#4, LW
```

```
620 INPUT#4, W
625 INPUT#4, D
630 INPUT#4, L1
635 INPUT#4, L2
640 INPUT#4, BW
645 INPUT#4, UC
650 INPUT#4, TI
655 INPUT#4, ITRVL%
660 REM ------------------------------------------------
665 REM preset variables
666 BTHETA =    BA * 3.1416 / 180
670 KTHETA = TI * 3.1416 / 180
675 RW = 64! / 32.2
680 CW = 1!
685 CM = 2!
690 LX = INT (MAX / H) ' number of output values
695 REM --------------------------------------------------
700 REM dimensionalize arrays
705 DIM T(LX + 2,3)
710 DIM THETA(LX + 3,3)
715 DIM OMEGA(LX + 2,3)
720 DIM K(3)
725 DIM M(3)
730 DIM MH(3)
735 DIM MG(3)
740 DIM MC(3)
745 REM ---------------------------------------------------
750 I% = 0 'set i = 0
755 T(0,0) = 0 'initialize array item t(0,0) = 0
760 THETA(0,0) = KTHETA
761 MH(0) = K1 * BTHETA + K2 * (THETA(0,0) - BTHETA)
765 REM ---------------------------------------------------
770 REM print out results
775 IF T$ = "0" THEN GOTO 785
780 PRINT "t (s) = ";T(I%,0);"theta(deg) = ";THETA(I%,0)  * 180 /
3.1416;
785 IF F$ = "0" THEN GOTO 795
790 PRINT#1,  "t(s) = ";T(I%,0);" theta(deg) = "THETA(I%,0) *180/
3.1416;
795 REM ----------------------------------------------------
800 REM start main program
805           REM          i           loop           begins
*************************************************
810 T(I% + 1,0) = T(I%,0) + H ' update base time
815 IF T(I% + 1,0) > MAX THEN GOTO 1205 'main program if
820    T(I%,1) = T(I%,0) + .5 * H   ' time at j=1
825    T(I%,2) = T(I%,1)            ' time at j=2
830    T(I%,3) = T(I%,0) + H        ' time at j=3
835 REM----------------------------------------------
840 J% = 0 'set j=0 before j loop
845 REM j loop begins ////////////////////////////
850 REM this loop calculates k(0...3) and m(0...3) which later
855 REM will be used in the i loop in the runge kutta recurrence
860 REM formula to calculate preceding theta and omega values
865 K(J%) = OMEGA(I%,J%) ' set k(0...3) = omega(i%,0...3)
870 REM calculate the spring moment mh(1...3)
871 REM -----------------------------------
875 IF THETA (I%,J%) > (BTHETA) THEN GOTO 895
880 IF THETA (I%,J%) + BTHETA < 0 THEN GOTO 905
885 MH(J%) = K1 * THETA(I%,J%) '-btheta<theta<btheta
890 GOTO 910
895 MH(J%)  =  K1  *  BTHETA  +  K2  *  (THETA(I%,J%)  - BTHETA)'
btheta<theta
900 GOTO 910
905 MH(J%) = -K1 * BTHETA + K2 * (THETA(I%,J%) + BTHETA)' theta <
-btheta
```

```
909 REM -----------------------------------
910 IF J% < >  0  THEN GOTO 915
911 IF T$ = "O" THEN GOTO 913
912 PRINT "hinge moment (ft-lbs) = ";MH(0)
913 PRINT#1, "hinge moment (ft-lbs) = ";MH(0)
915 REM submerged length calculations
920 IF LP * COS (THETA(I%,J%)) < D THEN GOTO 935
925 LS = D / COS (THETA(I%,J%))   ' tip not submerged
930 GOTO 940
935 LS = LP  ' tip submerged
940 REM ---------------------------------------
945 IH = IH + (3.1416 / 12) * (DP ^ 2) * RW * CM * (LS ^ 3)
950 REM ---------------------------------------
955  REM  calculate  the  current  moment  mc(1...3)  using  the
trapezoidal
960 REM numerical integration method
965 DLTA = LS / 10 'integration step
970 SUM = 0 'set sum = 0
975 FOR N = 1 TO 10        ' integration loop for 10 steps
976  REM .................................................
980 S = DLTA * N
985 FD  = .5  * RW  * CW  * DP  * ((UC * COS (THETA(I%,J%)) - S *
OMEGA(I%,J%            )) ^ 2) * SGN (UC *  COS (THETA(I%,J%)) -
S * OMEGA(I%,J%))
990 FI = -.25 * 3.1416 * RW * CM * (DP ^ 2) * UC * OMEGA(I%,J%) *
SIN  THETA(I%,J%))
995 IF N > 9.899999 THEN GOTO 1010
1000 SUM = SUM +  S * (FI + FD) ' update sum
1005 GOTO 1015
1010 SUM = SUM + .5 * S * (FD + FI) 'last time thru
1015 NEXT N   ' end of integration loop
1016 REM ...............................................
1020 MC(J%) = DLTA * SUM
1025 REM -------------------------------------------
1030 REM calculate buoyant force and moment mg(1...3)
1035 IF LS > = L2 THEN GOTO 1060
1040 FB = 64! * 3.1416 * (DP ^ 2) * (LS  - L1)  / 4  'chamber not
fully submg
1045 IF FB < 0 THEN FB = 0   ' chamber above ground, non negative
force
1050 LB = (L1 + LS) / 2 ' chamber not fully submg
1055 GOTO 1070
1060 FB = 64! * 3.1416 * (DP ^ 2) * (L2 - L1) / 4 ' chamber fully
submg
1065 LB = (L1 + L2) / 2     ' chamber fully submg
1070 MG(J%)  = ( - LB * FB + LW * W) * SIN (THETA(I%,J%)) + (FB -
W) * BW *           COS (THETA(I%,J%)) * SGN (THETA(I%,J%))
1075 REM ---------------------------------------------
1080 REM calculate angular acceleration m(0...3)
1085 M(J%) = ( - MH(J%) + MG(J%) + MC(J%)) / IH
1095 IF J% = 3 THEN GOTO 1135
1100 REM calculate theta(i%,1...3),omega(i%,1...3)
1105 REM ====================================
1110 THETA(I%,J% + 1) = THETA(I%,0) + (T(I%,J% + 1)  - T(I%,0)) *
K(J%)
1115 OMEGA(I%,J%  + 1) = OMEGA(I%,0) + (T(I%,J% + 1) - T(I%,0)) *
M(J%)
1120 REM ====================================
1125 J% = J% + 1 ' update j
1130 GOTO 845
1135 REM end of j loop //////////////////////
1140 REM ---------------------------------------------------
1145 REM runge kutta recurrence formula to calculate preceding
1150 REM theta and omega values theta(i% + 1,0),omega(i% + 1,0)
1155 THETA(I% + 1,0) = THETA(I%,0) + (H / 6) * (K(0) + 2 * K(1) +
2 * K(2) +          K(3))
```

```
1160 OMEGA(I% + 1,0) = OMEGA(I%,0) + (H / 6) * (M(0) + 2 * M(1) +
2 * M(2) +         M(3))
1165 REM -----------------------------------------------------
1170 I% = I% + 1
1171 YY =  INT(100*(T(I%,0)) + .1)/100
1175           REM           PRINT           OUT           RESULTS
''''''''''''''''''''''''''''''''''''''''''''''''''
1180 IF T$ = "0" THEN GOTO 1190
1185 PRINT  "t (s)  = ";YY  ;"THETA (DEG) = ";THETA(I%,0) * 180 /
3.1416;
1190 IF F$ = "0" THEN GOTO 1200
1195 PRINT#1, "t(s) = ";YY  ;"theta(deg) = ";THETA(I%,0) *  180 /
3.1416;
1200 GOTO 810
1205          REM          end          of          i          loop
**************************************************
1210 REM calculate the spring moment for a given time step
1215 I% = 0 ' set counter = 0
1216 REM beginning of loop ++++++++++++++++++++++++
1220 IF THETA(I%,0) > (BTHETA) THEN GOTO 1240
1225 IF THETA(I%,0) + BTHETA < 0 THEN GOTO 1250
1230 MH(0) = K1 * THETA(I%,0)  '  -btheta<theta<btheta
1235 GOTO 1255
1240 MH(0)  =  K1    *  BTHETA  +  K2  *  (THETA(I%,0)  - BTHETA)
'theta>btheta
1245 GOTO 1255
1250  MH(0)  =  -  K1  *  BTHETA  +  K2  * (THETA(I%,0) + BTHETA)
'theta<-btheta
1255 REM print out results to theta and moment files
1260 REM -----------------------------------------
1265 PRINT#2, T(I%,0);MH(0)
1270 PRINT#3, T(I%,0);THETA(I%,0)*180 / 3.1416
1275 REM ------------------------------------------
1280 I% = I% + ITRVL%   'update i%
1285 IF I% > L% THEN GOTO 1295
1290 GOTO 1220
1291 REM ++++++++++++++++++++++++++++++++++++++++++
1295 CLOSE #1
1300 CLOSE #2
1305 CLOSE #3
1310 END
```

V. DISCUSSION

During the development of the models presented here, the theory was rechecked for accuracy and no mistakes were found. New programs were verified by comparison of test runs with runs generated by the Swift and Baldwin (1985, 1986) programs. Numerical results are consistent. New programs implemented on an AT&T 6300 execute in nearly half as much time as compared to previous CTPS computer models operating on an Apple IIe.

Programs are now much more easier to read and are structured more efficiently. Greater facility will increase program effectiveness as a design tool.

VI. REFERENCES

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Swift, M.R. and K.C. Baldwin (1986) "The Development of a Collision Tolerant Pile Structure Concept", Ocean Engineering, Vol. 13, No. 2, pp. 131-156.

VII. APPENDIX A: WAVELENGTH

THEORY

The STORM program input requires that the wave parameters be provided - height, period and wavelength. Height must be small with respect to wavelength but is otherwise unrestricted. Period (T) and wavelength (L), on the other hand, must satisfy the small amplitude wave dispersion relation,

$$L = (gT^2/2\pi)\ \tanh(2\pi d_t/L). \tag{135}$$

Thus the STORM user can choose height and period, but the wavelength value entered must be consistent with Eq. 135. Because this is a transcendental equation, the utility program WAVELENGTH was developed to calculate the necessary L for a user specified T.

The solution approach is based on the Newton-Raphson method. First a wavelength parameter L_o is defined by

$$L_o = gT^2/2\pi \tag{136}$$

and a function of L defined by

$$F(L) = L/L_o - \tanh(2\pi d_t/L). \tag{137}$$

a sketch of F(L) is shown in Fig. 34.

The value of L which makes F(L) = 0 is the solution to Eq. 135. Thus the problem is to find the root of Eq. 137.

The root is located using an interative procedure. The analysis starts with a trial L. The trial L is next used to calculate an L closer to the root. The process is repeated until F(L) is sufficiently small.

Consider the nth estimate for L ($=L_n$). As shown in Fig. 34, the n+1 estimate (L_{n+1}) is obtained by linear extrapolation. From the geometry of the figure,

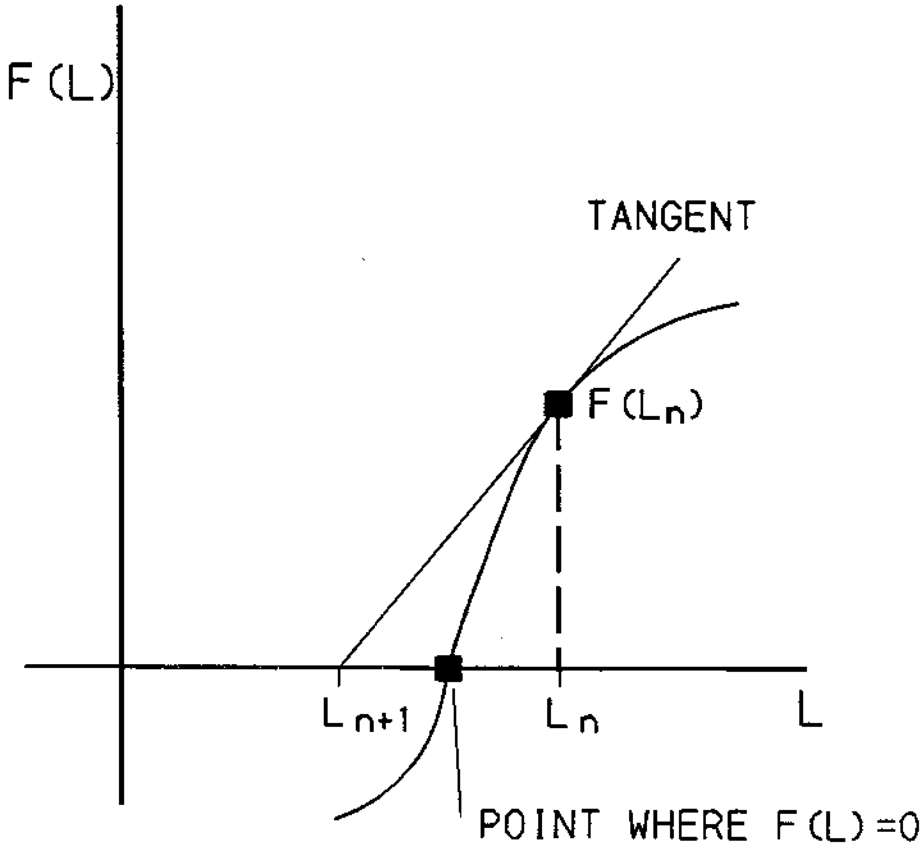

Fig. 34. The function F(L) showing iterative solution procedure.

$$\frac{F(L_n)}{L_n - L_{n+1}} = F'(L_n) \quad (138)$$

which can be rearranged to obtain

$$L_{n+1} = L_n - F(L_n)/F'(L_n) \quad (139)$$

In the WAVELENGTH implementation of this solution, Eq. 139 is applied repeatedly (starting with a trial value) until F is less than a (small) prescribed error limitation.

HOW TO USE THE "WAVELENGTH" PROGRAM

Since WAVELENGTH is written in BASIC, switch from the operating system to BASIC before attempting to execute. To load this program, put the disk in drive b and type load "b:wav.bvas". To run this program, type run. A message will then appear asking where the user wants to supply input. The message will say

Input from terminal type t

or from file type f

type t or f_____.

A message will then appear asking if output is desired to go to the terminal. The message will say

Output to terminal type t

or else type 0

type t or 0_____.

If output is not desired to go to the terminal type 0.

Next a message will appear asking if the output is to go to a file.

The screen will state

Output to a file type f

or else type 0

type f or 0

If output is not desired to go to the output file "wout", then type a zero.

If terminal input is selected a prompt will then appear asking for the water depth. After typing in the water depth a prompt will appear asking for a wave period. After typing in the wave period the program will execute. Output consists of a single value of the wavelength. See Fig. 35 for output format. A listing of the WAVELENGTH program is provided in Table 4.

Table 4. WAVELENGTH listing

```
5 REM wavelength
10 REM wavelength determines the wavelength of a small amplitude
15 CLS 0
20 REM imputed variables are
25 REM --------------------
30 REM per  wave period (s)
35 REM d    water depth (ft)
40 REM --------------------
45 REM preset variables
50 REM pi = 3.1416
55 REM g    gravitational acceleration (ft/s^2)
60 REM e    output tolerance
65 REM ----------------------
70 REM rem other variables are
75 REM df     derivative of the function
80 REM f      the function that we set = 0
85 REM l      desired wavelength  (ft)
90 REM lo     constant
95 REM ----------------------
100 REM determine form of input
105 PRINT "input from terminal type t"
110 PRINT "or from a file type f"
115 INPUT "type t or f"; IN$
116 PRINT "----------------"
120 REM ------------------------
125 REM determine form of output
130 PRINT "output to terminal type t"
135 PRINT "or else type 0"
140 INPUT "type t or 0";T$
145 PRINT "---------------"
150 PRINT " output to a file type f"
155 PRINT " or else type 0"
160 INPUT "  type f or 0";F$
165 REM ----------------------------
170 REM open input/output files
175 OPEN "wout" FOR OUTPUT AS #1 'output file
180 REM open "win" for input as #2 ' input file
185 REM -------------------------------
190 REM input data
195 IF IN$ = "F" THEN GOTO 245
200 PRINT "wavelength calculation"
205 PRINT " for a regular wave"
210 PRINT
215 INPUT "input wave period (s) ";PER
220 PRINT
225 INPUT " input depth (ft) ";D
230 PRINT
235 GOTO 260
245 INPUT#2, PER
250 INPUT#2,D
260 REM ------------------------------------
265 REM preset variables
270 PI = 3.1416
275 G = 32.2
280 E = .01
285 LO = G * (PER ^2) / (2 * PI)
286 L = LO
287 CLS 0
290 REM ------------------------------------
295  REM start newton raphson loop
300 F  = (L  / LO)  - (EXP (2 * PI * D / L) - EXP ( -2 * PI * D /
L)) / (                  EXP (2 * PI * D / L) + EXP ( -2 * PI * D
/ L))
301 IF ABS (F) < E THEN GOTO 315
305 DF = (1 / LO) + (( EXP (2 * PI * D / L) + EXP ( -2 * PI * D /
L)) ^                 ( -2)) * 8 * PI * D / (L ^ 2)
```

```
309 L = L - F / DF
310 GOTO 295
315                                                                    REM
------------------------------------------------------------
320 REM print out results
325 IF T$ = "O" THEN GOTO 350
330 PRINT  "wavelength =  ";L ;"for a period of "PER;"and a depth
of ";D
350 IF F$ = "O" THEN GOTO 370
355 PRINT#1,"wavelength = ";L;"for a period of ";PER;"and a depth
of ";D
366 INPUT "another calculation y/n";Y$
367 IF Y$ = "y" THEN GOTO 190
370 CLOSE #1
375 CLOSE #2
380 END
^Z
```

VIII. APPENDIX B; PLOTTING

DESCRIPTION

The plotting program plots the contents of a specified output data file. In general, this program plots the pile's angle and hinge moment time response. For plotting angle vs time, the maximum and minimum angles are printed as is the maximum time. For plotting hinge moment vs time, the maximum and minimum hinge moments are listed as is the maximum time.

To use the plotting program, first switch to BASIC and load the program type typing load "b:plot.bas". To run the program, type run. A prompt asking for the output file to be plotted will then appear. To plot the contents of the file, type b:filename. Once the file name is specified, results will appear on the screen. To print the results on the screen on the printer, type lcopy. To get back into text mode, hit the F10 control key.

A listing of the PLOT program appears in Table 5.

Table 5. PLOTTER listing

```
5 CLS 0
10 DIM X(100)
15 DIM Y(100)
20 PRINT "what is the file to be plotted"
25 INPUT "type the filename";FILE$
30 OPEN FILE$ FOR INPUT AS #1
35 CLS 0
40 REM ----------------------
45 REM input title for graph
50 SCREEN 2
55 INPUT#1, TITLE$
60 INPUT#1, X$,Y$
65 REM -----------------------------
70 REM input data
75 K = 0
80 ENTRIES = 0
85 MAX = 0
90 YMIN = 100000!
95 YMAX = 0
100 XMAX = 0
105 IF EOF(1) THEN GOTO   145
110 K = K + 1
115 INPUT#1, X(K),Y(K)
120 IF X(K) > XMAX THEN XMAX = X(K)
125 IF Y(K) > YMAX THEN YMAX = Y(K)
130 IF Y(K) < YMIN THEN YMIN = Y(K)
135 ENTRIES = ENTRIES + 1
140 GOTO 105
145 REM ------------------------
150 REM draw axes
155 XORGIN = 100
160 YORGIN = 199 - 40
165 PRESET (XORGIN,YORGIN)
170 LINE -STEP (639 - XORGIN,0)
175 PRESET (XORGIN,YORGIN)
180 LINE -STEP (639 - XORGIN,0)
185 PRESET (XORGIN,YORGIN)
190 LINE -STEP(0,0 - YORGIN)
195 REM
200 REM label the ordinate
205 CENTER = INT(( YORGIN - 0)/2/8) - 2
210 START = CENTER - INT(.5 * LEN(Y$)) + 4
215 FOR L = 1 TO LEN(Y$)
220 LOCATE START + L,5
225 PRINT MID$ (Y$,L,1)
230 NEXT L
235 REM label the abscissa
240 CENTER = INT((600 - XORGIN)/2/8)
245 START = CENTER - INT(.5 * LEN(X$))
250 FOR M = 1 TO LEN(X$)
255 LOCATE 23,START + M
260 PRINT MID$(X$,M,1)
265 NEXT M
270 REM    PRINT OUT TITLE
275 CENTER = 25
280 START = CENTER - INT(.5 * LEN(TITLE$))
285 FOR N = 1 TO LEN(TITLE$)
290 LOCATE 2,INT(START + N)
295 PRINT MID$(TITLE$,N,1)
300 NEXT N
305 REM ----------------
310 REM calculate x,y interval
315  XINT = FIX(600 - XORGIN) / ENTRIES
320 YINT = FIX (YORGIN - 10)/ENTRIES
325 REM ----------------------
330 REM      PUT TICK MARKS
```

```
331 LOCATE 22,65:PRINT XMAX
335 PRESET (XORGIN,YORGIN + 2)
340 FOR O = 1 TO ENTRIES
345 PRESET STEP(XINT,-4)
350 LINE -STEP(0,4)
355 XDELTA = INT(XMAX/ENTRIES)
360 NEXT O
365 PRESET (XORGIN+2,YORGIN)
370 FOR P = 1 TO ENTRIES
375 YDELTA = INT(YMAX - YMIN)/ENTRIES
380 IF P = 1 THEN GOTO 395
385 IF P = ENTRIES THEN GOTO 400
390 GOTO 405
395 LOCATE 20,4 :PRINT YMIN
396 GOTO 405
400 LOCATE 4,4 :PRINT YMAX
405 PSET     STEP (-4, -YINT)
410 LINE -STEP (4,0)
415 NEXT P
420 REM plot points
425 FOR Q = 1 TO ENTRIES
430 XDATA = X(Q)
435 YDATA = Y(Q)
440 GOSUB 475
445 LINE -(XSCREN,YSCREN)
450 NEXT Q
455 CLOSE #1
456 KEY OFF
460 LOCATE 23,1
461   LINE   (XORGIN,YORGIN   +  INT(YMIN/CY))  -  (650,YORGIN  +
INT(YMIN/CY))
465 END
470 REM subroutine to convert data to graph values
475 CY = (YMAX - YMIN) / 129
480 CX = XMAX / 450
485 XSCREN = XORGIN + INT(XDATA/CX)
490 YSCREN = YORGIN - INT((YDATA - YMIN)/CY)
495 RETURN
```

IX. APPENDIX C: RUNGE-KUTTA NUMERICAL SOLUTION

In the STORM and RECOVERY models, differential/integral dynamic equations must be solved for θ as a function of time. Because the equations are nonlinear, the Runge-Kutta numerical approach is taken. The Runge-Kutta extrapolation is a "marching" solution in which the initial conditions and derivatives (rate of changes) provided by the dynamic equations are used to estimate the variables a short time step away. The process is repeated, time step by (equal) time step, until the desired termination time t_{max} is reached. (In the COLLISION program, on the other hand, θ, $\dot{\theta}$ and $\ddot{\theta}$ are found directly by geometrical analysis, and the dynamic equations are solved algebraically for forces and moments).

The Runge-Kutta approach applied to the pile motion problem requires restating the dynamic equations in terms of 2 first order differential equations:

$$\frac{d\theta}{dt} = f(t, \theta, \omega) \tag{140}$$

and

$$\frac{d\omega}{dt} = g(t, \theta, \omega) \tag{141}$$

where (omega) $\omega = \frac{d\theta}{dt}$. The function $g = (1/I_H) \sum M_{applied}$ in which the M's have been described previously in the STORM and RECOVERY theoretical developments. The function f is simply $f = \omega$ in this application. The 2 fundamental dependent variables θ and ω are extrapolated from the nth time step to the next according to

$$\theta_{n+1} = \theta_n + (\overline{\frac{d\theta}{dt}})\ \Delta t \tag{142}$$

and

$$\omega_{n+1} = \omega_n + (\overline{\frac{d\omega}{dt}})\ \Delta t \quad . \tag{143}$$

The derivatives $(\overline{\frac{d\theta}{dt}})$ and $(\overline{\frac{d\omega}{dt}})$ are "weighted averages" of f and g, respectively (from Eqs. 140 and 141), and Δt is the time step. Since expressions for f and g are known, the "average" derivatives can be evaluated, and the solution proceeds to the next time step. Explicitly the "averages" are:

$$(\overline{\frac{d\theta}{dt}}) = 1/6(k_o + 2k_1 + 2k_2 + k_3) \tag{144}$$

and

$$(\overline{\frac{d\omega}{dt}}) = 1/6\ (m_o + 2m_1 + 2m_2 + m_3)\ , \tag{145}$$

where according to standard Runge-Kutta nomenclature,

$$\begin{aligned} k_o &= f(t_n,\ \theta_n,\ \omega_n) \\ k_1 &= f\ (t_n + \Delta t/2,\ \theta_n + k_o\ \Delta t/2,\ \omega_n + m_o\ \Delta t/2 \\ k_2 &= (t_n + \Delta t/2,\ \theta_n + k_1\ \Delta t/2,\ \omega_n + m_1\ \Delta t/2) \\ k_3 &= f(t_n + \Delta t,\ \theta_n + k_2\ \Delta t,\ \omega_n + m_2\ \Delta t) \end{aligned} \tag{146}$$

and

$$\begin{aligned} m_o &= g\ (t_n,\ \theta_n,\ \omega_n) \\ m_1 &= g\ (t_n + \Delta t/2,\ \theta_n + k_o\ \Delta t/2,\ \omega_n + m_o\ \Delta t/2) \\ m_2 &= g\ (t_n + \Delta t/2,\ \theta_n + k_1\ \Delta t/2,\ \omega_n + m_1\ \Delta t/2) \\ m_3 &= g\ (t_n + \Delta t,\ \theta_n + k_2\ \Delta t,\ \omega_n + m_2\ \Delta t)\ . \end{aligned} \tag{147}$$

Of course in this application where $f = \omega$, the k's reduce to:

$$\begin{aligned} k_o &= \omega_n \\ k_1 &= \omega_n + m_o\ \Delta t/2 \\ k_2 &= \omega_n + m_2\ \Delta t/2 \\ k_3 &= \omega_n + m_3\ \Delta t. \end{aligned} \tag{148}$$

This choice of extrapolation coefficients is equivalent to a 4th order Taylor Series expansion (as developed in most standard Differential Equations and/or Numerical Analysis textbooks).

Within the programs, a loop is set up with index I% corresponding to the time step number. Each pass through the loop generates θ_n and ω_n for which n = I%. Nested within this loop is a second loop with index J% which is used to compute the k's and m's contributing to the "averages". During each pass through this loop, $k_{J\%}$ and $m_{J\%}$ are computed. The upper limit on I% is the total number of time steps, which J% always varies from 0 to 3.

For computational convenience, important variables are doubled subscripted. The first subscript is I% which indicates the time step, and J% pertains to the k or m calculation taking place. Thus for example, THETA (19,0) = $\theta_{n=19} = \theta(t = t_{initial} + 19\Delta t)$, while THETA (19,2) = $\theta_{n=19} + k_1\Delta t/2$ for k_1 of time step 19.