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Effects of Liquid Saturated Porosities on Sound Transmission in an Ocean Subbottom

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EFFECTS OF LIQUID SATURATED POROSITIES ON SOUND TRANSMISSION IN AN OCEAN SUBBOTTOM

by

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NOMENCLATURE

A_o, B_o - arbitrary constants employed in differential equations, concerning liquid medium

A_L, A_T - arbitrary constants employed in differential equations, concerning viscoelastic medium

E - the modulus of elasticity

E^{ik}_{mn} - coefficient matrix of viscoelasticity

$\hat{e}$ - unit vector

g^{ik} - general matrix

K_L - longitudinal wave number

K_T - transverse wave number

C_L - longitudinal sound velocity

C_T - transverse sound velocity

r - radial coordinate direction and distance in cylindrical coordinates

t - time

u_i - the components of the displacement vector

z - longitudinal coordinate direction and distance in cylindrical coordinates

θ - circumferential coordinate direction in cylindrical coordinates

λ, μ - complex Lamé's constants for viscoelasticity

λ', μ' - elastic parameters of Lamé's constants

λ'', μ'' - viscous parameters of Lamé's constants

$\bar{\lambda}$, $\bar{\mu}$ - Lamé's constants in Fourier frequency domain

ρ_o - mass density of liquid medium

ρ, ρ_{sed} - mass density of viscoelastic medium

δ - Kronecker delta

ε^{mn} - strain tensor

σ_{ik} - stress components

ω - frequency - radians per second

∂_i - denotes $\frac{\partial}{x_i}$

Subscript or superscript L - denotes components corresponding to longitudinal waves

Subscript or superscript T - denotes components corresponding to transverse waves

Subscripts r, θ, z - denotes components in the radial, circumferential and longitudinal directions

ABSTRACT

Wave propagation properties in a liquid saturated, ocean subbottom are investigated by making use of theoretical field theory formalism. Consolidated subbottom soils, which included porous voids completely saturated with liquid, change the medium characteristics of the subbottom, and such perturbations (porosities) behave like small scattering centers disrupting onsetting waves. Due to these scattering centers, it is furthermore shown that short wave propagation becomes severely limited. This is an analogous situation to well known multiple scattering processes.

I. INTRODUCTION

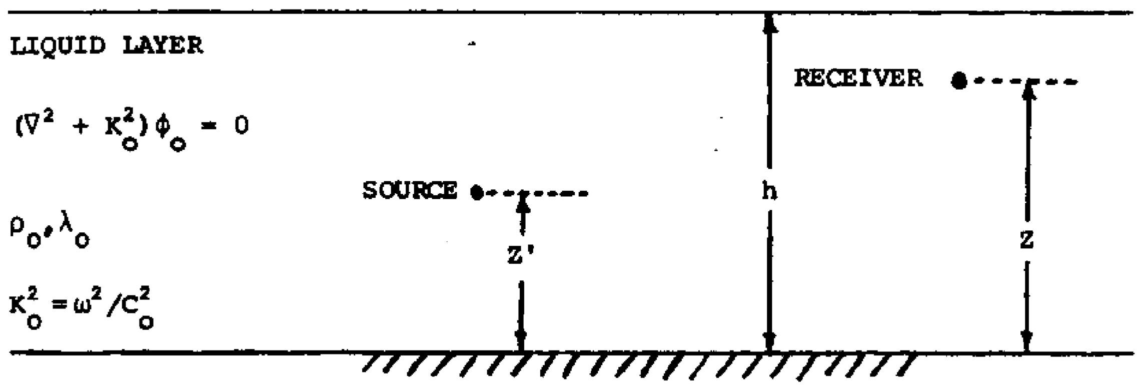
Presently, most subbottom studies have focused on an elastic model approach. Correction due to liquid saturated porosities in soils in wave propagation considerations are, in most situations, neglected or disregarded. Such discussions are emphasized largely by geophysicists when discussing subbottom material classification and soil mechanics data evaluation.

When considering the ocean subbottom identification by utilizing acoustic probing techniques, the acoustic signals in the subbottom are attenuated not only by the Voigt model-type dampings (internal damping), which are closely related to the particle size and the heterogeneity of the medium, but also by voids, which are completely saturated with the liquid, and act as small scattering centers. In this thesis, we introduce a viscoelastic model and also take into consideration in our discussions the effects of liquid saturated porosities. It is shown that the attenuations of the propagating waves in the subbottom can be and largely are due to the porosity effects.

II. METHOD OF APPROACH

In order to discuss the LSP (liquid saturated porosity) effects, we shall first introduce a realistic model of a semi-infinite ocean subbottom covered by a finite layer of a liquid medium and make use of the analytical formalism, which was developed by the UNH theoretical group in conjunction with the UNH-Raytheon Sea Grant Project.

It is assumed that a sound source and receiver are imbedded in the ocean (liquid layer). See figure 1.



VISCOELASTIC LAYER (SEMI-INFINITE)

$$(\nabla^2 + K_L^2)\phi_L = 0$$

$$(\nabla^2 + K_T^2)\phi_T = 0$$

$$K_L^2 = \omega^2/C_L^2; \quad K_T^2 = \omega^2/C_T^2$$

$$\rho_1, \quad \lambda_1, \quad \mu_1$$

FIG. 1

The ocean subbottom is assumed to be a semi-infinite medium. The Green's function formalism for a unit source is analogous to the reflection coefficient in the theoretical field theory approach, which corresponds to the Rayleigh reflection coefficient in the ray theory approach, [4].

A unit impulse (Green's function) function is an expression which characterizes the function and response for an arbitrary source and at arbitrary locations. The integral form of such a Green's function for our particular configuration is

$$G(r,r',z,z',\omega) = \int_0^\infty \frac{\sinh a_o(h - z')}{a_o} \frac{N(\zeta^2,z)}{D(\zeta^2,h)} J_o(\zeta r)\zeta d\zeta \tag{1}$$

where

$$N(\zeta^2,z) = \frac{\rho_1}{\rho_o} \cosh(a_o z)[(2\zeta^2 - K_T^2)^2 - 4a_L a_T \zeta^2] + a_L K_T^4 \sinh(a_o z) \tag{2-a}$$

and

$$D(\zeta^2,z) = N(\zeta^2,z) \Big|_{z = h} \tag{2-b}$$

We shall concentrate largely on the LSP effects and make analytical as well as computer comparisons of results which neglect LSP effects. Considerations will also be made of an approximation of Green's function obtained by using stationary phase techniques. The analytical development for stationary phase approximation of our Green's function [16] yields the following results where if

$$G = \int F(\zeta) e^{ixf(\zeta)}$$

then the Green's function becomes

$$G(r,r',z,z',\omega) = \frac{1}{\sqrt{(z + d)^2 + r^2}} \left[\frac{N^{-1}}{N^{+1}}\right] e^{ik_o \sqrt{(z + d)^2 + r^2}} \tag{3}$$

where

$$\frac{N^{-1}}{N^{+1}} = \frac{\rho_1 C_L \cos\theta [(2\frac{C_T^2}{C_o^2}\sin^2\theta - 1)^2 + \frac{4C_T^3}{C_o^2 C_L}\sqrt{1 - \frac{C_L^2}{C_o^2}\sin^2\theta}\sqrt{1 - \frac{C_T^2}{C_o^2}\sin^2\theta}] -}{\rho_1 C_L \cos\theta [(2\frac{C_T}{C_o}\sin^2\theta - 1)^2 + \frac{4C_T^3}{C_o C_L}\sqrt{1 - \frac{C_L^2}{C_o}\sin^2\theta}\sqrt{1 - \frac{C_T^2}{C_o}\sin^2\theta}] +}$$

$$\frac{\rho_o C_o \sqrt{1 - \frac{C_L^2}{C_o^2}\sin^2\theta}}{\rho_o C_o \sqrt{1 - \frac{C_L^2}{C_o^2}\sin^2\theta}} \qquad (4)$$

which can be considered as a quasi-ray theory. Indeed, equations (1) and (3) are advantageous to our analysis, in that they include shear wave terms which are not otherwise available in other ray theory formulisms.

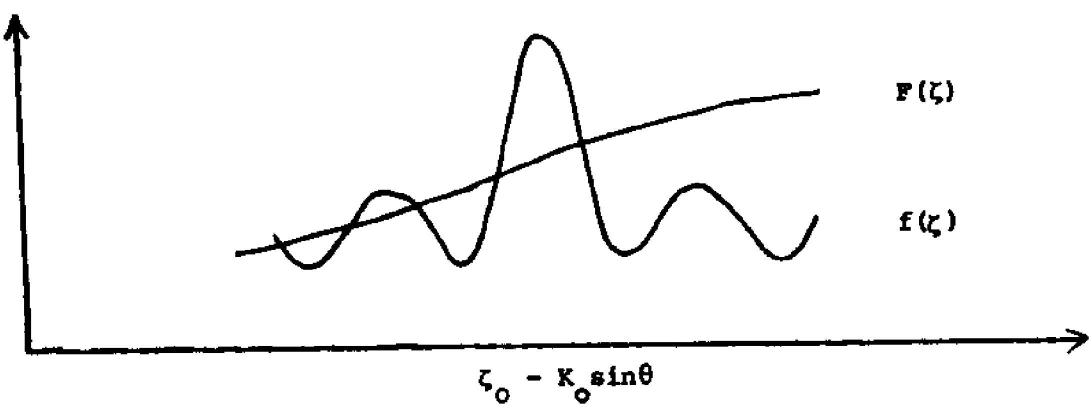


FIG. 2

III. LIQUID SATURATED POROSITY EFFECTS IN AN OCEAN SUBBOTTOM

We will now discuss the effects of liquid saturated porosities (LSP) in a liquid saturated subbottom. We will make use of a well known soil mechanics relationship representing porosity [8], namely

$$\rho_{sed} = n\rho_o + \rho_s(1 - n) \tag{5-a}$$

where the individual terms are defined as

ρ_{sed} = saturated bulk density of the actual sediment

n = porosity

ρ_o = density of the liquid in the voids

ρ_s = bulk density of the mineral soils

Rearranging equation (5-a) into a more useful form yields

$$\rho_{sed} = n\rho_o + \rho_s(1 - n)$$

$$\rho_{sed} = n\rho_o + \rho_s - n\rho_s$$

$$\rho_{sed} = \rho_s + n(\rho_o - \rho_s)$$

$$\rho_{sed} = \rho_s[1+n(\frac{\rho_o - \rho_s}{\rho_s})] \tag{5-b}$$

Using equations (B-12a,b) of appendix B and (C-14,15) of appendix C will enable us to consider and apply the effects or porosity in our theoretical development of the viscoelastic medium, namely

$$K_L^2 = \frac{\omega^2\rho}{\lambda + 2\mu} \tag{6-a}$$

$$K_T^2 = \frac{\omega^2\rho}{\mu} \tag{6-b}$$

and

$$a_L = \sqrt{\zeta^2 - K_L^2} \qquad (7\text{-a})$$

$$a_T = \sqrt{\zeta^2 - K_T^2} \qquad (7\text{-b})$$

By substituting equations (6-a,b) into equations (7-a,b) we yield

$$a_L = \sqrt{\zeta^2 - \frac{\omega^2 \rho}{2\mu + \lambda}} \qquad (8\text{-a})$$

$$a_T = \sqrt{\zeta^2 - \frac{\omega^2 \rho}{\mu}} \qquad (8\text{-b})$$

We can now substitute equation (5-b) for the ρ term in equations (8-a,b), since they are identically equal to each other, thus

$$a_{L_{(sed)}} = \sqrt{\zeta^2 - \frac{\omega^2}{2\mu + \lambda}\,\rho_s[1 + n(\frac{\rho_o - \rho_s}{\rho_s})]} \qquad (9\text{-a})$$

$$a_{T_{(sed)}} = \sqrt{\zeta^2 - \frac{\omega^2}{\mu}\,\rho_s[1 + n(\frac{\rho_o - \rho_s}{\rho_s})]} \qquad (9\text{-b})$$

where $a_{L_{(sed)}}$ and $a_{T_{(sed)}}$ denote a state of complete saturation of the sub-bottom with the liquid. Similarly we can redefine our wave velocity and wave number expressions, or

$$C^2_{L_{(sed)}} = \frac{\lambda + 2\mu}{\rho_{sed}} \qquad (10\text{-a})$$

$$C^2_{T_{(sed)}} = \frac{\mu}{\rho_{sed}} \qquad (10\text{-b})$$

and

$$K^2_{L_{(sed)}} = \frac{\omega^2}{C^2_{L_{(sed)}}} \qquad (11\text{-a})$$

$$K^2_{T_{(sed)}} = \frac{\omega^2}{C^2_{T_{(sed)}}} \qquad (11\text{-b})$$

where again, (sed), denotes a state of complete saturation.

Substituting equation (10-a,b) and (11-a,b) into equation (9-a,b) respectively, we yield

$$a_{L_{(sed)}} = \sqrt{\zeta^2 - K^2_{L_{(sed)}}} \qquad (12\text{-a})$$

$$a_{T_{(sed)}} = \sqrt{\zeta^2 - K^2_{T_{(sed)}}} \qquad (12\text{-b})$$

At this time a useful check can be made by letting porosity, n, equal to zero in equations (9-a,b), which will lead to our previous results, namely equations (7-a,b), or

$$a_{L_{(sed)}} = \sqrt{\zeta^2 - \frac{\omega^2 \rho_s}{2\mu + \lambda}} \quad \sqrt{\zeta^2 - \frac{\omega^2}{C^2_{L_{(sed)}}}}$$

$$a_{L_{(sed)}} = \sqrt{\zeta^2 - \frac{\omega^2}{C^2_L}} = a_L$$

and

$$a_{T_{(sed)}} = \sqrt{\zeta^2 - \frac{\omega^2 \rho_s}{\mu}} = \sqrt{\zeta^2 - \frac{\omega^2}{C^2_{T_{(sed)}}}}$$

$$a_{T_{(sed)}} = \sqrt{\zeta^2 - \frac{\omega^2}{C^2_T}} = a_T$$

since $\rho_{sed} = \rho = \rho_s$, when $n = 0$.

Let us now consider equations (9-a,b),

$$a_{L_{(sed)}} = \sqrt{\zeta^2 \quad \frac{\omega^2}{2\mu + \lambda} \rho_s \; [1{+}n \; (\frac{\rho_o - \rho_s}{\rho_s})]}$$

$$a_{T_{(sed)}} = \sqrt{\zeta^2 \quad \frac{\omega^2}{\mu} \rho_s \; [1{+}n \; (\frac{\rho_o - \rho_s}{\rho_s})]}$$

which can be written as

$$a_{L(sed)} = \sqrt{\zeta^2 - K^2_{L(s)} \quad [1 + n(\frac{\rho_o - \rho_s}{\rho_s})]} \tag{13-a}$$

$$a_{T(sed)} = \sqrt{\zeta^2 - K^2_{T(s)} \quad [1 + n(\frac{\rho_o - \rho_s}{\rho_s})]} \tag{13-b}$$

where

$$K^2_{L(s)} = \frac{\omega^2 \rho_s}{2\mu + \lambda} \tag{14-a}$$

$$K^2_{T(s)} = \frac{\omega^2 \rho_s}{\mu} \tag{14-b}$$

Expanding equations (13-a,b) yields

$$a_{L(sed)} = \sqrt{\zeta^2 - K^2_{L(s)} - K^2_{L(s)} \quad n(\frac{\rho_o - \rho_s}{\rho_s})}$$

$$a_{T(sed)} = \sqrt{\zeta^2 - K^2_{T(s)} - K^2_{T(s)} \quad n(\frac{\rho_o - \rho_s}{\rho_s})}$$

or

$$a_{L(sed)} = \sqrt{\zeta^2 - K^2_{L(s)} - K^2_{L(s)} \quad \Omega}$$

$$a_{T(sed)} = \sqrt{\zeta^2 - K^2_{T(s)} - K^2_{T(s)} \quad \Omega}$$

where

$$\Omega = n(\frac{\rho_o - \rho_s}{\rho_s}) \tag{15}$$

or finally

$$a_{L(sed)} = \sqrt{a^2_{L(s)} - K^2_{L(s)} \quad \Omega} \tag{16-a}$$

$$a_{T(sed)} = \sqrt{a^2_{T(s)} - K^2_{T(s)} \quad \Omega} \tag{16-b}$$

where

$$a^2_{L_{(s)}} = \zeta^2 - K^2_{L_{(s)}} \tag{17-a}$$

$$a^2_{T_{(s)}} = \zeta^2 - K^2_{T_{(s)}} \tag{17-b}$$

Let us now consider equations (16-a) and (16-b). We will assume that there exists two possibilities, that is, Case 1: $a_{L_{(s)}} > K^2_{L_{(s)}}\,\Omega$; $a^2_{T_{(s)}} > K^2_{T_{(s)}}\,\Omega$; and Case 2: $a^2_{L_{(s)}} < K^2_{L_{(s)}}\,\Omega$; $a^2_{T_{(s)}} < K^2_{T_{(s)}}\,\Omega$.

Let us consider Case 1.

$$a^2_{L_{(s)}} > K^2_{L_{(s)}} \tag{18-a}$$

$$a^2_{T_{(s)}} > K^2_{T_{(s)}} \tag{18-b}$$

Substituting equations (17-a,b) into equations (18-a,b) respectively, yields

$$\zeta^2 > K^2_{L_{(s)}}\,[1 + \Omega] \tag{19-a}$$

$$\zeta^2 > K^2_{T_{(s)}}\,[1 + \Omega] \tag{19-b}$$

Thus it follows that if conditions (19-a,b) hold, equations (16-a,b) can be written as

$$a_{L_{(sed)}} = a_{L_{(s)}} \sqrt{1 - \frac{K^2_{L_{(s)}}}{a^2_{L_{(s)}}}} \tag{20-a}$$

$$a_{T_{(sed)}} = a_{T_{(s)}} \sqrt{1 - \frac{K^2_{T_{(s)}}}{a^2_{T_{(s)}}}} \tag{20-b}$$

We can alternatively express equations (20-a,b) in a power series expansion of the form

$$(1 - x)^{\frac{1}{2}} = 1 - \frac{x}{2} - \frac{x^2}{8} - \ . \ . \ . \ ,\ \text{or}$$

$$a_{L(sed)} \simeq a_{L(s)} \left[1 - \frac{1}{2} \frac{K^2_{L(s)} \Omega}{a^2_{L(s)}}\right] \qquad (21\text{-a})$$

$$a_{T(sed)} \simeq a_{T(s)} \left[1 - \frac{1}{2} \frac{K^2_{T(s)} \Omega}{a^2_{T(s)}}\right] \qquad (21\text{-b})$$

which can be written as

$$a_{L(sed)} = a_{L(s)} - \frac{1}{2} \frac{K^2_{L(s)} \Omega}{a_{L(s)}} \qquad (22\text{-a})$$

$$a_{T(sed)} = a_{T(s)} - \frac{1}{2} \frac{K^2_{T(s)} \Omega}{a_{T(s)}} \qquad (22\text{-b})$$

Now let us consider Case 2:

$$a^2_{L(s)} < K^2_{L(s)} \Omega \qquad (23\text{-a})$$

$$a^2_{T(s)} < K^2_{T(s)} \Omega \qquad (23\text{-b})$$

Substituting equations (17-a,b) into equations (23-a,b) respectively, yields

$$\zeta^2 < K^2_{L(s)} [1 + \Omega] \qquad (24\text{-a})$$

$$\zeta^2 < K^2_{T(s)} [1 + \Omega] \qquad (24\text{-b})$$

Thus it follows that if equations (24-a,b) hold, then equations (16-a,b) can be written as

$$a_{L(sed)} = iK_{L(s)} \Omega^{\frac{1}{2}} \sqrt{1 - \frac{a^2_{L(s)}}{K^2_{L(s)} \Omega}} \qquad (25\text{-a})$$

$$a_{T(sed)} = iK_{T(s)} \Omega^{\frac{1}{2}} \sqrt{1 - \frac{a^2_{T(s)}}{K^2_{T(s)} \Omega}} \qquad (25\text{-b})$$

Equations (25-a,b) can alternatively be expressed in power series expansion as

$$a_{L_{(sed)}} \simeq iK_{L_{(s)}} \Omega^{\frac{1}{2}} - \frac{i}{2} \frac{a^2_{L_{(s)}}}{K_{L_{(s)}} \Omega^{\frac{1}{2}}} \qquad (26\text{-a})$$

$$a_{T_{(sed)}} \simeq iK_{T_{(s)}} \Omega^{\frac{1}{2}} - \frac{i}{2} \frac{a^2_{T_{(s)}}}{K_{T_{(s)}} \Omega^{\frac{1}{2}}} \qquad (26\text{-b})$$

A comparison of Case 1 and Case 2 results, namely equations (22-a,b) and (26-a,b), can be made. Equations (22-a,b) are analogous to a "long wavelength approximation" consideration of a_L and a_T; whereas equations (26-a,b) correspond to a "short wavelength approximation" analogy. For our particular investigation, due to the characteristic properties of an ocean subbottom medium, the "long wavelength approximation" is of much more interest and significance than that of a "short wavelength approximation." Alternatively, equations (22-a,b) are expressions exhibiting "free" (undamped) oscillatory motion, whereas equations (26-a,b) are "confined" (damped) in their oscillatory motion. Said differently, "short wavelength approximation" is an insensitive means of investigating "LSP" effects.

IV. LSP EFFECTS ON THE WAVE PROPAGATION IN AN OCEAN SUBBOTTOM

So far we have developed the exact relationships which will effect the general formalism of our Green's function expressions for our particular configuration under investigation. There are two Green's function expressions which will be considered, namely one representing the exact integral representation [equation (1)], and another representing the stationary phase approximation [equation (3)], which have been previously developed. The theoretical development of these Green's function expressions have been developed by the UNH analytical group. Presently we shall make comparative discussions, by applying the LSP effects directly to the previously developed formalisms that are available, that is, equations (1) and (3). This can be easily accomplished, since equations (1) and (3) contain such terms as a_L, a_T, C_L, C_T, K_L and K_T which can, and have been redefined to include LSP effects, namely equations (9-a,b), (10-a,b), and (11-a,b) respectively. Thus by direct substitution of the above equations, where appropriate, into equations (1) and (3), we can obtain qualitative and quantitative results depicting the influence of LSP effects on our theoretical model.

V. COMPUTER SIMULATION RESULTS OF LSP EFFECTS

Our investigation of the LSP effects on our two Green's function formalisms, equations (1) and (3), has been accomplished through the utilization of a numerical computer analysis developed by Dr. A. K. Newman. Due to the complexity of our integral representations, the computer representation and analysis of these expressions proved indispensable. The actual computer development and program are discussed in Appendix E. For our investigations, an idealized soil from E. L. Hamilton (ref. 9) was used, namely that of a "silty-clay" sediment, identified by the following parameters: density = 1.42 gm/cm , porosity = 76% longitudinal velocity = 1519 m/sec, and the transverse velocity = 287 m/sec. For purposes of exhibiting the effects of the liquid saturated porosities (LSP), a 2.5% change in the porosity value is investigated. Also included in our experimental work is the variation of our "gain" (Green's function values) with respect to the angle of incidence of our unit source impulse (see figures 3 and 4). Our results of this illustrative example of LSP effects can be represented by a set of graphs; one representing the exact integral formula with and without LSP effects and another representing the stationary phase approximation with and without LSP effects (see figures 3 and 4). Upon investigation of our resulting curves, we see that there exists a marked difference in our output "gain" curves for two identical sediments varying by only a 2.5% change in porosity. Curve A represents a Hamilton "silty-clay" sediment (see ref. 9). Curve B then represents the same sediment differing by only a decrease of 2.5% in porosity. The "gain delta," or numerical percent difference between corresponding gains for each case, attains its smallest value in the case of normal incidence, and generally increases with an increase in the angle of incidence. In figure (3), the case for the exact integral formula, the gain delta is fairly constant, ranging from 13.5% at normal incidence to a value of 23.3% at an angle of incidence of 50°. However, as the angle of incidence increases up to 85°, the gain delta increases up to 112%. Although in real life an 85° angle of incidence is highly impractical, it is of importance from a theoretical point of view. In figure (4), the case for a stationary-phase approximation, we observe similar patterns as the gain delta is 13.2% at normal incidence, and ranges up to 23.6% for an angle of incidence of 50°. A maximum gain delta is 128%, occurring at an angle of incidence of 75°. An overlapping of curves occurs after 75°, which can be attributed to the characteristic behavior of the stationary phase approximation. As the curves in figures (3) and (4) clearly show, there is a marked significance of LSP effects encountered when acoustically identifying ocean subbottom sediments.

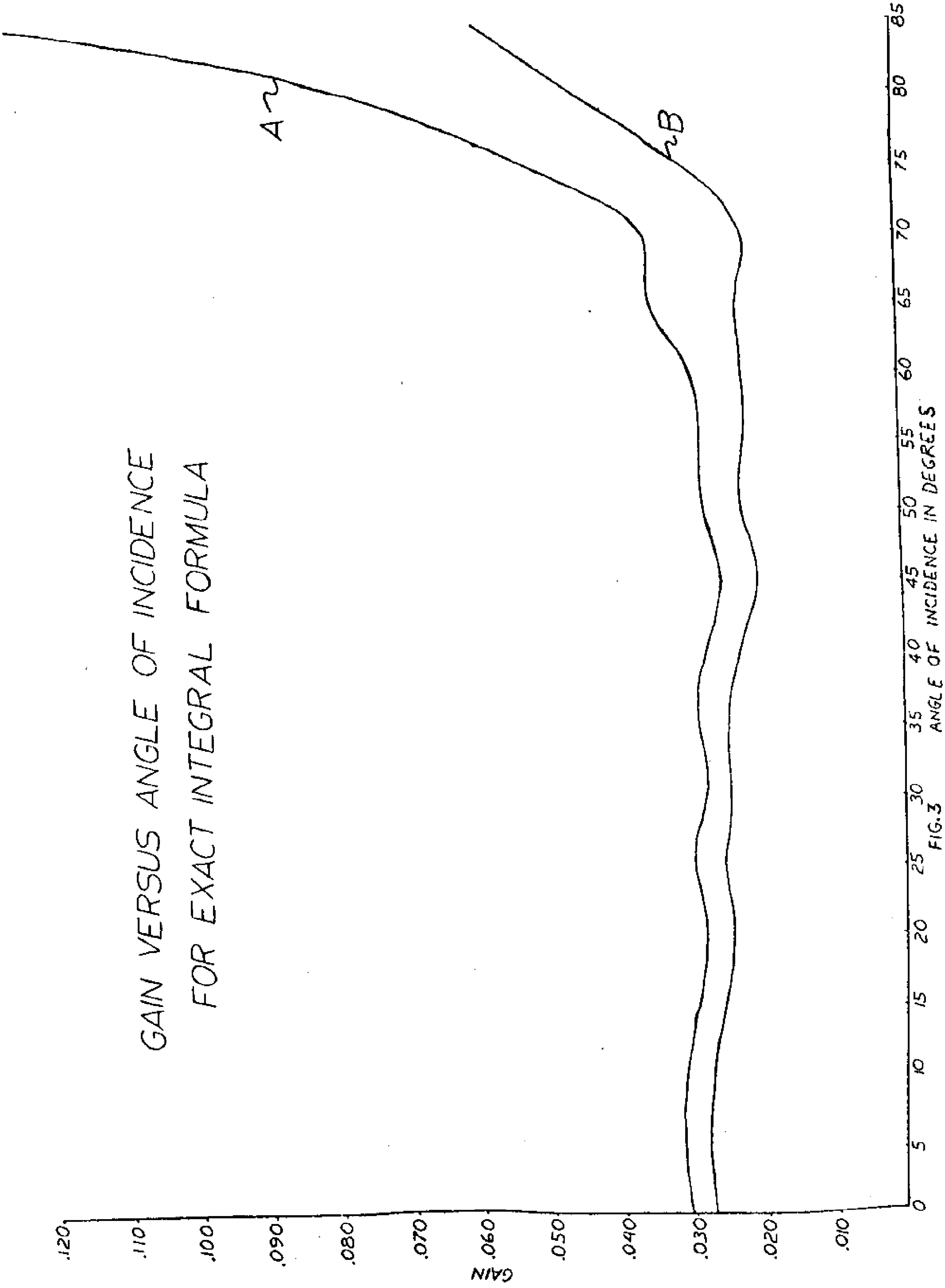

GAIN VERSUS ANGLE OF INCIDENCE
FOR EXACT INTEGRAL FORMULA
A
B
GAIN
.120
.110
.100
.090
.080
.070
.060
.050
.040
.030
.020
.010
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
ANGLE OF INCIDENCE IN DEGREES

FIG.3

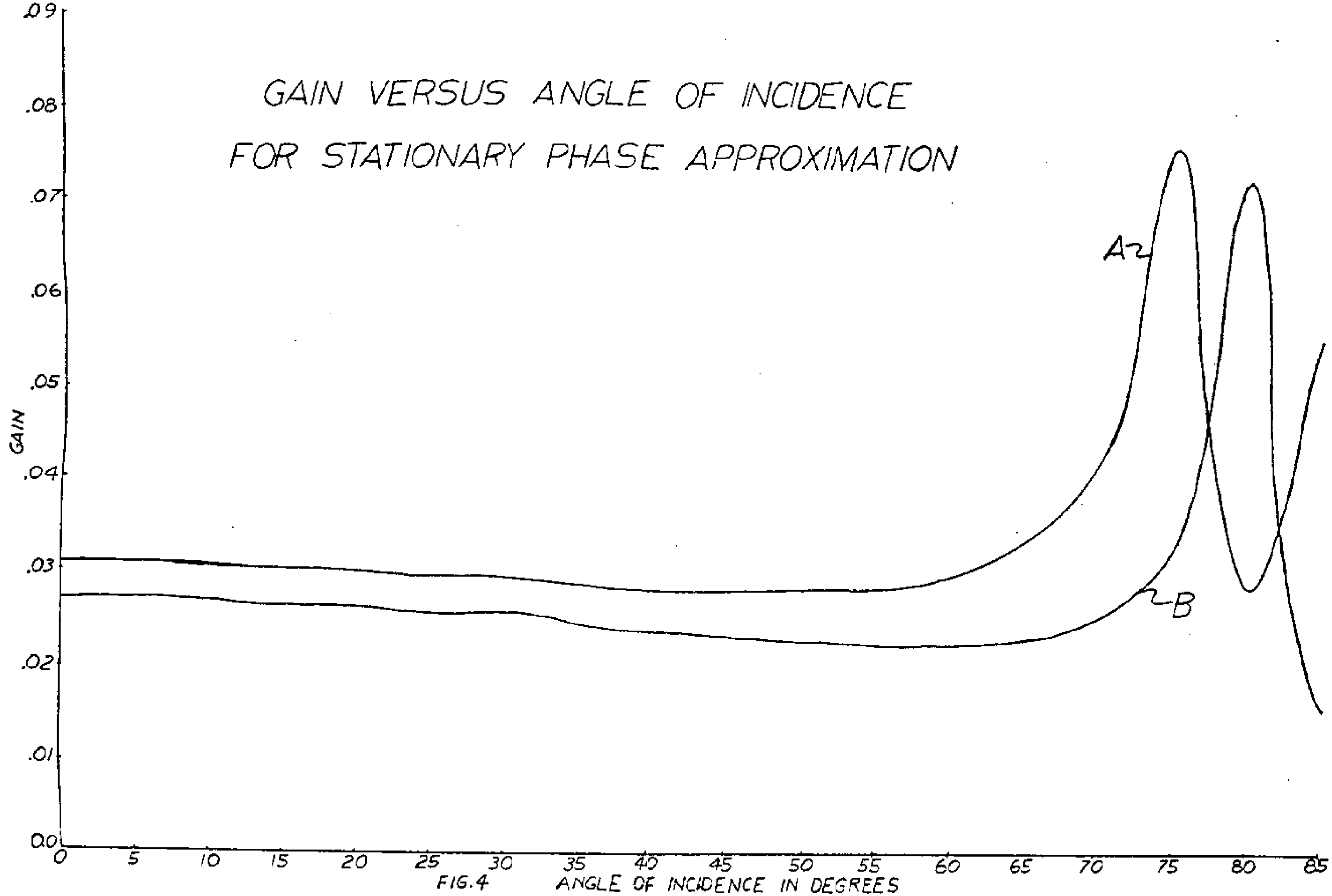

GAIN VERSUS ANGLE OF INCIDENCE
FOR STATIONARY PHASE APPROXIMATION
GAIN
.09
.08
.07
.06
.05
.04
.03
.02
.01
0.0
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
ANGLE OF INCIDENCE IN DEGREES
A
B

FIG.4

VI. RESULTS AND CONCLUSIONS

From our computer analysis, we have attained significant results, which can be clearly and easily represented. During the last ten or fifteen years, researchers (Hamilton, Breslau, and others) have proven the validity of using porosity considerations in the accurate identification of ocean subbottom sediments. Countless research programs have concentrated on geophysical classification of closely related soils, where only slight changes in the bulk density, sound velocity and porosity can account for large discrepancies in soil classification (see ref. 8). In our analysis we have not only considered perturbations in porosity, but have also incorporated this perturbation simultaneously in other sediment parameters, namely bulk density, and longitudinal and transverse velocity. Augmented to this classification, we have proceeded to show how our LSP effects have affected the acoustic signature, by introducing such LSP effects empirically into our theoretical model, which attempts to identify the acoustic signature or experimental "gain." This has been accomplished by introducing LSP effects in longitudinal and transsound velocities, longitudinal and transverse wave numbers, longitudinal and transverse exponential coefficients, and the bulk density. However, only through this dual consideration of LSP effects, both from a soil mechanics point of view and from that of an experimental acoustician, have we gained a keen insight in the ability to correlate the interdependency between an identifying acoustic output (gain) and the physical sediment to which it relates, namely, we can write the following as important results obtained, both directly and indirectly, from our investigations:

1. Up to approximately an angle of incidence of 65°, there is an agreement to within 7% between the results of our

geometrical stationary phase approximation and theoretical exact integral formulism. However, between a 65° and 85° angle of incidence, there are grave differences that exist between computed values of the exact integral and stationary phase approximation values (up to 92%).

2. LSP effects are of prime importance when one is trying to use electronic acoustic identification as a means of sediment classification. Experimental results have shown that a 2.5% change in porosity results in "gain delta" values of 13.5% at normal incidence and ranging up to 128% at an 85° angle of incidence.

3. Due to inherent characteristics of viscoelastic, subbottom sediments, considerations in the longitudinal direction, that is, longitudinal sound velocity, are of more interest and have greater quantitative relevancy than those in the transverse direction, that is, transverse sound velocity.

4. Due to inherent characteristics of ocean subbottoms, it is more advantageous and necessary to consider acoustic sediment models from a viscoelastic point of view, than from a purely elastic point of view, which leaves the model too far from reality.

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APPENDIX A

In Appendix A, we are concerned with the theoretical development of the hydrodynamic field equations. We will derive a set of differential equations which will define the specific problem to be solved. It is far more advantageous and realistic to consider the three dimensional theory of wave propagation. The field description of the present problem consists of two parts: a hydrodynamic field and a viscoelastic field; therefore, let us consider the hydrodynamic field first.

For a hydrodynamic medium, we can assume adiabatic conditions, therefore the entropy conservation equation is not necessarily needed to develop the wave equation. In a hydrodynamic medium, the equation of motion can be written as

$$\rho(\partial_t v^k + v^i \partial_i v^k) + \partial_i p^{ik} + F^k = 0 \tag{A-1}$$

where $\partial_t = \frac{\partial}{\partial t}$; $\partial_i = \frac{\partial}{\partial x_i}$; F^k is the external force; v^k is the velocity vector; and the pressure tensor, which does not contain a deviatoric part. Thus p^{ik} = $p\delta^{ik}$, where δ^{ik} is the kronecker delta. The conservation of mass density or continuity equation can be written as

$$\partial_t \rho + \partial_k (\rho v^k) = 0 \tag{A-2}$$

The equation of state or constitutive relation is written from $p = p(\rho)$ as

$$\partial^k p = \left(\frac{\partial p}{\partial \rho}\right)_T \partial^k \rho \tag{A-3a}$$

or

$$\partial^k p = C_o^2 \partial^k \rho \tag{A-3b}$$

where

$$C_o^2 = \left(\frac{\partial p}{\partial \rho}\right)_T$$

Since we have a set of non-linear differential equations, we will linearize equations (A-1,2) and (A-3) by using well-known perturbation technique which follows as

$$v^k = v_o + \underset{\sim}{v}^k(\bar{r},t) \tag{A-4a}$$

$$p = p_o + \underset{\sim}{p}(\bar{r},t) \tag{A-4b}$$

$$\rho = \rho_o + \underset{\sim}{\rho}(\bar{r},t) \tag{A-4c}$$

Assuming a non-flow regime ($v_o = 0$) and noting that underscripts ~ refer to the fluctuating part of the spacetime variable functions, we can substitute the above identities into the field equations where appropriate, thus yielding in linearized form

$$\rho_o \partial_t \underset{\sim}{v}^k + \partial^k \underset{\sim}{p} = S^k_{01} - F^k \tag{A-5a}$$

$$\partial_t \underset{\sim}{\rho} + \rho_o \partial^k \underset{\sim}{v}^k = -\underset{\sim}{\rho} \quad \partial_k v^k \tag{A-5b}$$

$$\partial^k p - C_o^2 \partial^k \underset{\sim}{\rho} = 0 \tag{A-5c}$$

where terms on the right-hand sides are force terms or equivalent source terms composed of higher order non-linear terms, which are responsible for turbulence. In our considerations they will be omitted, and disregarding the non-linear underscript terms, we can express equations (A-5) in vector notation

$$\rho_o \partial_t \vec{v} + \ p = 0 \tag{A-6a}$$

$$\partial_t \rho + \rho_o \nabla \cdot \vec{v} = 0 \tag{A-6b}$$

$$\nabla p - C_o^2 \nabla \rho = 0 \tag{A-6c}$$

Having obtained equations (A-6,a,b,c), substitute in $C_o^2 \nabla \rho$ for ∇p in equation (A-6a), obtained from equation (A-6c); and taking the divergence, $(\nabla \cdot)$, we obtain

$$\rho_o \partial_t \nabla \cdot \bar{v} + C_o^2 \nabla^2 \rho = 0 \qquad \text{(A-7a)}$$

Taking the time derivative of equation (A-7a) yields

$$\rho_o \partial_t^2 \nabla \cdot \bar{v} - C_o^2 \nabla^2 \rho = 0 \qquad \text{(A-7b)}$$

In this form we can substitute for $\partial_t \rho$ from equation (A-6b) yielding

$$\rho_o \partial_t^2 \nabla \cdot \bar{v} - C_o^2 \rho_o \nabla^2 \nabla \cdot \bar{v} = 0 \qquad \text{(A-7c)}$$

At this time it is necessary to define the acoustic field potential as

$$\nabla \cdot \bar{v} = \phi_a \qquad \text{(A-8)}$$

where ϕ_a is the acoustic potential term. Rearranging terms, and substituting equation (A-8) into (A-7c) yields

$$(\nabla^2 - \frac{1}{C_o^2} \frac{\partial^2}{\partial t^2}) \phi_a = 0 \qquad \text{(A-9)}$$

We will now introduce the standard Fourier Transform pair, which is given by

$$\chi(\omega) = \int_{-\infty}^{+\infty} \chi(t) e^{-i\omega t} dt \qquad \text{(A-10a)}$$

$$\chi(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \chi(\omega) e^{i\omega t} d\omega \qquad \text{(A-10b)}$$

The transformation (A-10b) alternatively states that any time derivative operators transform into algebraic forms in the frequency domain, that is

$$(\partial t)^n = (- i\omega)^n$$

where n indicates the order of the derivative.

Applying the Fourier Transform to equation (A-9) yields

$$(\nabla^2 + K_a^2) \phi_a = 0 \qquad \text{(A-11)}$$

where

$$K_a = \frac{\omega}{C_a} \qquad \text{(A-12)}$$

and C_a can be written through an analogy from an elastic point of view as

$$C_a = \frac{\lambda}{\rho_a} \tag{A-13}$$

which is the constant propagation velocity in the liquid. A substitution of subscripts a to subscripts o, so as to represent the case of a hydrodynamic medium, is made, thus equation (A-11) becomes

$$(\nabla^2 + K_o^2)\phi_o = 0 \tag{A-14}$$

which is known as the Helmholtz wave equation.

APPENDIX B

In this appendix a set of wave equations, analogous to equation (A-14), will be derived to describe the field potential for a viscoelastic medium. These specific differential equations can be derived from isotropic, homogeneous viscoelastic field equations. It is possible to equate D'Alembert's force, obtaining

$$\rho\partial_t^2 u^i = \partial_k \sigma^{ik} \tag{B-1}$$

which completely describes the dynamic field equation, where u^i is the displacement, σ^{ik} is the stress tensor. This is a simple nonrelativistic (Newtonian regime) expression and deserves no further comment. A derivation of the general constitutive relationship yields a stress-strain relation for linear viscoelastic materials, in the Hookean regime

$$\sigma^{ik} = E_{mn}^{ik}\varepsilon^{mn} \tag{B-2}$$

where for an inhomogeneous and isotropic medium, the constitutive tensor can be expressed as

$$E_{mn}^{ik} = \lambda(g^{ik}g_{mn}) + \mu(g_m^i g_n^k + g_n^i g_m^k) \tag{B-3}$$

where λ and μ are complex Lamé constants which can be written as

$$\lambda = \frac{E}{(1 + \nu)(1 - 2\nu)} \tag{B-4a}$$

$$\mu = \frac{E}{2(1 + \nu)} \tag{B-4b}$$

where E and ν are complex elastic modulus and Poisson's ratio, respectively.

For the linear regime, the strain tensor, ε^{mn}, can be expressed as

$$\varepsilon^{mn} = \partial^m u^n \tag{B-5a}$$

or

$$\varepsilon^{mn} = \frac{1}{2}(\partial^m u^n + \partial^n u^m) \qquad \text{(B-5b)}$$

where

$$\partial^m = \frac{\partial}{\partial x_m}$$

Substitution of equations (B-2,3) and (B-5b) into equation (B-1) yields

$$\rho\partial_t^2 u^i = \lambda\partial^i\partial^m u_m + \mu\partial^i\partial_k u^k + \mu\partial_k\partial^k u^i \qquad \text{(B-6a)}$$

In a Cartesian coordinate system, there need not be no distinction between a contravariant and covariant vector, thus equation (B-6a) can be written as

$$\rho\partial_t^2\bar{u} - \mu\nabla^2\bar{u} - (\lambda + \mu)\nabla(\nabla\cdot\bar{u}) = 0 \qquad \text{(B-6b)}$$

Thus equation (B-6b) is the governing equation describing dynamic behavior for an isotropic, homogeneous, viscoelastic medium.

A viscoelastic material, by definition, exhibits both elastic and viscous properties. Ideally, a linear elastic element is a spring, and a linear viscous element is a dashpot. A variety of spring and dashpot combinations have been used to represent a mathematical model of a viscoelastic material. Well known examples are the Maxwell and Voigt models. A detailed description of these models can be found in Bland [1]. For purposes in this model, the Lamé parameters can be represented as time operators.

$$\lambda = \lambda' + \lambda''\frac{\partial}{\partial t} \qquad \text{(B-7a)}$$

$$\mu = \mu' + \mu''\frac{\partial}{\partial t} \qquad \text{(B-7b)}$$

Taking the Fourier Transform in time, namely equation (A-10b), of equation (B-6b), we yield

$$(\bar{\lambda} + \bar{\mu})\nabla(\nabla\cdot\bar{u}) + \bar{\mu}\nabla^2\bar{u} + \rho\omega^2\bar{u} = 0 \qquad \text{(B-8)}$$

where the complex Lamé parameters in the frequency domain take the following form

$$\bar{\lambda} = \lambda' + i\omega\lambda'' \tag{B-9a}$$

$$\bar{\mu} = \mu' + i\omega\mu'' \tag{B-9b}$$

Dividing through by $\bar{\mu}$ and rearranging terms yields

$$(\nabla^2 + \frac{\rho\omega^2}{\bar{\mu}})\bar{u} - [1 - \frac{\rho\omega^2(\bar{\lambda} + 2\bar{\mu})}{\bar{\mu}\rho\omega^2}]\nabla(\nabla\cdot\bar{u}) = 0 \tag{B-10}$$

or

$$(\nabla^2 + K_T^2)\bar{u} - (1 - \frac{K_T^2}{K_L^2})\nabla(\nabla\cdot\bar{u}) = 0 \tag{B-11}$$

where we define the shear and dilitational wave numbers, respectfully as

$$K_T = \omega(\frac{\rho}{\bar{\mu}})^{\frac{1}{2}} = \frac{\omega}{C_T} \tag{B-12a}$$

$$K_L = \omega(\frac{\rho}{\bar{\lambda} + 2\bar{\mu}})^{\frac{1}{2}} = \frac{\omega}{C_L} \tag{B-12b}$$

where C_T and C_L are the shear and dilatational propagation velocities, respectively. If we use equations (B-9a,b), we can express the shear and dilitational velocities exclusively as

$$C_T^2 = \frac{\bar{\mu}}{\rho} = \frac{\mu' + i\omega\mu''}{\rho} \tag{B-13a}$$

$$C_L^2 = \frac{\bar{\lambda} + 2\bar{\mu}}{\rho} = \frac{\lambda' + 2\mu' + i\omega(\lambda'' + 2\mu'')}{\rho} \tag{B-13b}$$

Since a vector field can be expressed as the sum of the gradient of a scalar potential and the curl of a vector potential, the former is called the irrotational vector and the latter is called the solenoidal vector. However, there can be a great deal of arbitrariness in the designation of these potentials. But in order to obtain the particular wave potentials which will satisfy equation (B-11), the potentials must be chosen such that one of the three scalars specifying the vector field is the scalar potential and the other two are uniquely specifying the vector potential. This concept of "duality" will lead to the solution of equation (B-11).

Propagating waves traveling in the interior of a homogeneous rigid body have two distinct velocities, called irrotational or dilatational and equivoluminal or distortional waves, respectively. Since these terms are somewhat misleading, we use the terms transverse and longitudinal. The volume change accompanying deformation can be represented by the sum of the diagonal terms in the strain tensor, that is, $\partial_i u^i = \nabla \cdot \bar{u}$, and the magnitude of the angular deformation, given by the vorticity tensor, $\varepsilon_{ijk}\partial_j u_k = \nabla \times \bar{u}$. Or mathematically

$$\bar{u} = \bar{u}_T + \bar{u}_L \tag{B-14}$$

where $\bar{u}_T$ and $\bar{u}_L$ are the transverse and longitudinal displacements, respectively, and

$$\nabla \cdot \bar{u}_T = 0 \tag{B-15a}$$

$$\nabla x \bar{u}_L = 0 \tag{B-15b}$$

For dilatational and shear waves, the respective displacements in terms of the field potentials are

$$\bar{u}_L = \nabla \phi_L \tag{B-16a}$$

$$\bar{u}_T = \nabla x \nabla x \hat{e}_z \phi_T \tag{B-16b}$$

where $\hat{e}_z$ is the unit vector in the axial (z) direction and ϕ_L and ϕ_T are the scalar potentials, which obey the individual components of the Helmholtz equations in the form

$$(\nabla^2 + K_L^2)\phi_L = 0 \tag{B-17a}$$

$$(\nabla^2 + K_T^2)\phi_T = 0 \tag{B-17b}$$

Thus the problem is essentially reduced to solving equations (A-14) and (B-17a,b) for the appropriate boundary conditions.

APPENDIX C

The approach that is taken in the solution of the governing equations, equations (A-14) and (B-17a,b) will now be discussed. Due to the isotropy of the liquid layer and the semi-infinite viscoelastic medium, there exists an axial symmetry in the problem. This suggests that the proper coordinate system is the cylindrical coordinate system. While in general the problem can be solved in any coordinate system (physical laws being independent of the choice of the coordinate system), this specific choice of coordinates makes our calculations considerable simpler. Thus equations (A-14) and B-17a, b) can be written in cylindrical coordinates; equation (A-14) becomes

$$\left(\frac{\partial}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} + K_o^2\right)\phi_o = 0 \qquad \text{(C-1)}$$

where, due to axial symmetry, $\frac{\partial}{\partial \theta}$ will vanish, thus reducing equation (C-1) to

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} + K_o^2\right)\phi_o = 0 \qquad \text{(C-2)}$$

Similarly, due to axial symmetry, equations (B-17a,b) can be expressed in cylindrical coordinates as

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} + K_L^2\right)\phi_L = 0 \qquad \text{(C-3)}$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} + K_T^2\right)\phi_T = 0 \qquad \text{(C-4)}$$

We assume a general solution to these homogeneous equations by an application of a standard technique of separation of variables. These same homogeneous equations take the form of Bessel's differential equations. The solution of these equations consist of terms of these same Bessel functions of zeroth order. Thus the general solution of equations (C-2,3) and (C-4) are of the form

$$\phi(r,z) = AJ_o(\zeta r)e^{-az} + BJ_o(\zeta r)e^{az} \qquad \text{(C-5)}$$

where

$$a = \sqrt{\zeta^2 - K^2} \tag{C-6}$$

and ζ is the transformation parameter and A and B are amplitude functions, which will be characteristic of their particular medium. In equation (C-5), the first term represents the downward traveling wave, and the second term represents the upward traveling (reflected) wave. We will now identify the hydrodynamic and viscoelastic fields in analogous forms of equation (C-5). The hydrodynamic or liquid medium can be written with appropriate predefined subscripts, as

$$\phi_o = A_o J_o(\zeta r) e^{-a_o z} + B_o J_o(\zeta r) e^{a_o z} \tag{C-7}$$

where

$$a_o = \sqrt{\zeta^2 - K_o^2} \tag{C-8}$$

and the subscripts, o, are characteristic of the liquid medium. Our hydrodynamic field is considered from an elastic point of view, where our Lamé parameters will take into account the absence of viscosity (no shear waves) and damping, thus enabling us to write the acoustic field equation (in the liquid medium) from the viscoelastic wave equations by setting $\mu = 0$, and considering only the real parts of λ. Equation (C-7) represents the "natural" state of the hydrodynamic field, but does not include and define the excitation source, which is present in our model. In representing our source, we use a previously developed formulism, that is

$$\phi_s = \frac{e^{-iK_o R}}{R} \tag{C-9}$$

where

$$R = \sqrt{r^2 + (z - z_s)^2} \tag{C-10}$$

and Z is the position of the receiver (or observer) and Z_s is the position of the excitation source.

Substituting equation (C-9) into equation (C-7) yields

$$\phi_o(r,z) = A_o e^{-a_o z} + B_o e^{a_o z} + \frac{e^{-iK_o R}}{R} \tag{C-11}$$

In the semi-infinite, viscoelastic medium, there is a marked absence of an upward (reflected) traveling waves, as they do not return, resulting in a modified form of equation (C-5), or

$$\phi_L(r,z) = A_L e^{-a_L z} J_o(\zeta r) \tag{C-12}$$

$$\phi_T(r,z) = A_T e^{-a_T z} J_o(\zeta r) \tag{C-13}$$

where

$$a_L = \sqrt{\zeta^2 - K_L^2} \tag{C-14}$$

$$a_T = \sqrt{\zeta^2 - K_T^2} \tag{C-15}$$

and A_L and A_T are amplitude functions characteristic of the viscoelastic medium.

With the acoustic field general solutions defined for each medium, we will apply boundary conditions which are concerned with perpendicular planes to the z-axis. Due to symmetry, there are no boundaries in the radial direction enabling us to transform r-dependent parameters into constants, by making use of the following standard transformation pair

$$\chi(\zeta) = \int_0^{+\infty} \chi(r) J_o(\zeta r) r dr \tag{C-16a}$$

$$\chi(r) = \int_0^{+\infty} \chi(\zeta) J_o(\zeta r) \zeta d\zeta \tag{C-16b}$$

where ζ is the previously introduced transformation parameter. By making use of this transformation pair, we can transform our source term in the liquid field, which in cylindrical coordinates is located at $Z_s = -(h - h_s)$ and $r = 0$, yielding

$$\phi_s = \int_0^\infty J_o(\zeta\, r)e^{-a_o\left|Z + (h - h_s)\right|}\zeta d\zeta \tag{C-17}$$

Using the results of equation (C-17) along with the transformation (C-16a,b) of equation (C-11), we yield

$$\phi_o(\zeta,z,\omega) = A_o(\zeta,\omega) + B_o(\zeta,\omega)e^{a_o z} + \frac{e^{-a_o\left|Z + (h - h_s)\right|}}{a_o}S(\omega) \tag{C-18}$$

where A_o and B_o are functions of ζ and ω, and the expression $S(\omega)$ is defined as

$$S(\omega) = \int_0^\infty e^{-i\omega t} S(t)dt \tag{C-19}$$

which is the Fourier Transform of the time-dependant source S(t).

Similarly applying the transformation pair to equations (C-12) and (C-13), we get

$$\phi_L(\zeta,z,\omega) = A_L(\zeta,\omega)e^{-a_L z} \tag{C-20}$$

$$\phi_T(\zeta,z,\omega) = A_T(\zeta,\omega)e^{-a_T z} \tag{C-21}$$

where A_L and A_T are functions of ζ and ω.

Thus the problem has been essentially reduced to solving equations (C-18,20) and C-21) for the appropriate boundary conditions.

APPENDIX D

Four independent boundary conditions are needed to determine the four integration parameters A_o, B_o, A_L, and A_T. Included in the set of boundary conditions is an expression for the pressure at the liquid surface. The boundary conditions are as follows:

$$\sigma_{zz}^{(o)}\Big|_{z = -h} = 0 \tag{D-1a}$$

$$\tau_{rz}^{(1)}\Big|_{z = 0} = 0 \tag{D-1b}$$

$$\sigma_{zz}^{(o)}\Big|_{z = 0} = \sigma_{zz}^{(1)}\Big|_{z = 0} \tag{D-1c}$$

$$V_z^{(o)}\Big|_{z = 0} = V_z^{(1)}\Big|_{z = 0} \tag{D-1d}$$

where superscripts (0) and (1) represent the acoustic hydrodynamic and visco-elastic medium, respectively, and V_z is the velocity. Equation (D-1a) represents the stress condition at any given point $r = r$, $z = -h$, in the free space; and also that there is no source excitation above the surface of the liquid. These conditions represent the continuity of the stress components at the liquid-solid interface; and the last expression (D-1d) represents the continuity of momentum at the interface.

We will now develop an analytical expression which will represent the relationship between the input and output pressure fields. For convenience of the mathematical calculations, the stress field components in the liquid and viscoelastic media will be expressed in terms of the scalar potential equations (C-2,3) and (C-4). In the hydrodynamic medium we have

$$\sigma_{rr} = \lambda_o \nabla^2 \phi_o \tag{D-2a}$$

$$\sigma_{zz} = \lambda_o \nabla^2 \phi_o \tag{D-2b}$$

$$\tau_{rz} = \tau_{r\theta} = \tau_{z\theta} = 0 \tag{D-2c}$$

Similarly in the viscoelastic medium, we have

$$\sigma_{rr} = \bar{\lambda}\nabla^2\phi_L + 2\bar{\mu}\frac{\partial}{\partial r}[\frac{\partial\phi_L}{\partial r} + \frac{\partial^2\phi_T}{\partial r\partial z}] \tag{D-3a}$$

$$\sigma_{zz} = \bar{\lambda}\nabla^2\phi_L + 2\bar{\mu}\frac{\partial}{\partial}[\frac{\partial\phi_L}{\partial z} - \nabla^2\phi_T + \frac{\partial^2\phi_T}{\partial z^2}] \tag{D-3b}$$

$$\tau_{rz} = \bar{\mu}\frac{\partial}{\partial r}[2\frac{\partial\phi_L}{\partial z} - \nabla^2\phi_T + 2\frac{\partial^2\phi_T}{\partial z^2}] \tag{D-3c}$$

$$u_z = \frac{\partial\phi_L}{\partial z} + K_T^2\ \phi_T + \frac{\partial^2\phi_T}{\partial z^2} \tag{D-3d}$$

It is worthwhile to mention that when the second Lame parameter, , becomes zero in equations (D-3a,b,c) we obtain corresponding equations (D-2a,b,c).

We can now apply equations (C-18,20) and (C-21) to our boundary conditions, equations (D-1a,b,c,d); and making use of our analyticity relations, equations (D-2a,b,c) and (D-3a,b,c,d); thus yielding us four simultaneous equations, containing four unknowns namely A_o, B_o, A_L, and A_T. These four simultaneous equations may be written, after considerable algebra, in a matrix form as

$$\begin{bmatrix} -\rho_o\omega^2 e^{a_o h} & -\rho_o\omega^2 e^{-a_o h} & 0 & 0 \\ -\rho_o\omega^2 & -\rho_o\omega^2 & \bar{\mu}(K_T^2-2\zeta^2) & 2\bar{\mu}a_T\zeta^2 \\ 0 & 0 & -2a_L & 2\zeta^2-K_T^2 \\ -a_o & a_o & a_L & -\zeta^2 \end{bmatrix} \begin{bmatrix} A_o \\ B_o \\ A_L \\ A_T \end{bmatrix} = \zeta S(\omega) \begin{bmatrix} -\frac{e}{a_o}^{-a_o h_s} \\ \frac{\rho_o\omega^2 e^{-a_o(h-h_s)}}{a_o} \\ 0 \\ e^{-a_o(h-h_s)} \end{bmatrix} \tag{D-4}$$

However, by applying equation (C-18) to equation (D-2a), we -an eliminate B_o from the hydrodynamic field expression, thus yielding

$$\phi_o' = - 2A_o e^{a_o h} \sinh a_o(z + h) \tag{D-5a}$$

$$\phi_o'' = -2\{A_o e^{a_o h} \sinh a_o(z + h) + \frac{\sinh}{a_o} [z + (h - h_s)]\} \tag{D-5b}$$

which are expressions representing the potentials above the below the excitation source, respectively. Thus a complete description of an acoustic response in the hydrodynamic medium can be achieved with an appropriate determined value of only A_o. We can now reduce equation (D-4) by applying our new expression for the field potential below the source, equation (D-5b), to our boundary condiditons, yielding

$$\begin{vmatrix} 2\rho_o\omega^2 e^{a_o h} \sinh(a_o h) & -\bar{\mu}(2\zeta^2-K_T^2) & 2\bar{\mu}a_T\zeta^2 \\ 0 & -2a_L & 2\zeta^2-K_T^2 \\ -2e^{a_o h} a_o \cosh(a_o h) & a_L & -\zeta^2 \end{vmatrix} \begin{vmatrix} A_o \\ A_L \\ A_T \end{vmatrix} = \begin{vmatrix} \frac{-2\rho_o\omega^2 \sinh a_o(h-h_s)}{a_o} \\ 0 \\ 2\cosh a_o(h-h_s) \end{vmatrix} \tag{D-6}$$

We can now solve our matrix formulism equation (D-6) for A_o by utilizing the well known Cramer's Rule. Using Cramer's technique we can represent A_o, which describes the amplitude function in the liquid medium, as

$$A_o = - \frac{1}{a_o\Delta S}\{\frac{a_L\rho_o\omega^4}{a_o C_T^2}\sinh a_o(h-h_s) + \rho_1 C_T^2[(2\zeta^2-K_T^2)^2-4a_La_T\zeta^2]\cosh a_o(h-h_s)\} \tag{D-7}$$

where we define ΔS as the determinant of the three-by-three matrix in equation (D-6). Equation (D-7) is only valid, if and only if, ΔS is not equal to zero. However, if we let S equal to zero, we will obtain the characteristic equation (Eigen value solution) for our system, thus

$$\frac{\rho_o\omega^4}{C_T^2} a_L \sinh(a_o h) + \rho_1 C_T^2 a_o[(2\zeta^2-K_T^2)^2-4a_La_T\zeta^2] \cosh(a_o h) = 0 \tag{D-8}$$

We will now relate the output pressure to the input source strength. We know that the pressure is related to the potential by the following expression

$$\bar{P}_{out}(\zeta,\omega,z) = \rho_o \omega^2 \phi(\zeta,\omega,z) \qquad \text{(D-9)}$$

After considerable algebra and manipulation, it is possible to represent our acoustic output as an integral representation of Green's function, or

$$G(r,r',z,z',\omega) = \int_0^\infty \frac{\sinh[a_o(h_o - z')]}{a_o} \frac{N(\zeta^2,z)}{D(\zeta^2,h)} J_o(\zeta r)\zeta d\zeta \qquad \text{(D-10)}$$

where $N(\zeta^2,z) = \frac{\rho_1}{\rho_o} \; a_o \cosh(a_o z)[(2\zeta^2 - K_T^2)^2 - 4a_L a_T \zeta^2] + a_L K_T^4 \sinh(a_o z)$ (D-11)

and

$$D(\zeta^2,z) = N(\zeta^2 z)\Big|_{z = h} \qquad \text{(D-12)}$$

APPENDIX E

This appendix will be concerned with the application of our theoretical modeling developments to computer analysis to ascertain quantitive, as well as qualitive, numerical results. Due to the complexity and length of our mathematical expressions, computer simulation was mandatory. For purposes of computer adaptation, the physical problem can be represented as

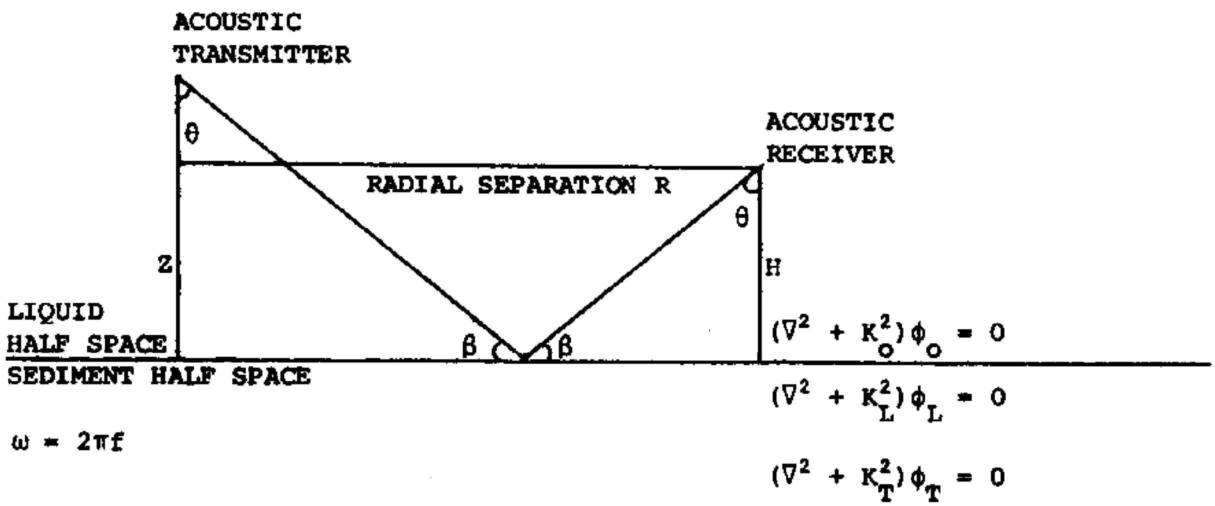

FIG. 5

where is the angle of incidence, ω is the frequency at which the acoustic impulse is driven; with the remainder of the physical set up already having been described. It should be mentioned that the "bounce distance," that is the total path traveled by an acoustic signal from transmitter to receiver, is held constant for all values of incident angles.

Our computer analysis was performed on an IBM 360/50 computer. The major portion of the program was written CSMP (Continuous System Modeling Program) language, while subroutines were written in Fortran language. Utilization was made of complex double precision intergrand computations with single precision integrator registers. Subroutines were developed to compute the real and imaginary parts of the intergrand in double precision. The IBM CSMP supplied, single-precision, Milne 5th order variable interval predictor-corrector integration subroutine was used. Assumed attenuations for sea water, sediment, and compressional and shear wave damping were taken accepted geophysical tables. See references 20 and 22. Some difficulties were experienced in maintaining the accuracy of the absolute integral value. This occurs because there is abnormal "peaking" of the integral in which approximately 35% of the total area of the integral exists over a very short range with respect to its total integration. The program appears as follows:

```
                ****CONTINUOUS SYSTEM MODELING PROGRAM****

                     ***PROBLEM INPUT STATEMENTS***

TITLE    THEORETICAL SEDIMENT ANALYSIS   PROFESSOR ASIM YILDIZ

TITLE    POROSITY PERTURBATION STUDY   MARCH 9, 1972

TITLE    THIS RUN FOR ETA = .1

PARAM    SEDP = 1.0

PARAM    DFEET = 20.0

PARAM    RMEAS = 1.42, CLMEAS = 1519.0, CTMEAS = 287.0, ETA = .1
         FREQ = 5000.0

PARAM    ANCE=(0.0,5.0)

         RANCE=ANCE*3.141593/180.0

         TB=TAN (RANCE)

         RFEET+TB*DFEET/STBB

         R=RFEET*0.3048

         STBB=SQRT(TB*TB+1.0)

         ZFEET+0.5*DFEET?STBB

         WHFEET=ZFEET

         Z=FEET*0.3048
```

```
        AFREQ=FREQ
        W=FREQ*2.0EO*3.141593EO
        EO=C(*2.88463E 4*.1151293/(W*W)
        EL=CL*2.605773E-1*.1151293/(W*W)
        ET=CT*2.605773E-1*.1151293/(2.0*W*W)
        X=TIME
        Y=0.0EO
        CO=1501.0
        RO=1.025EO
*  IF ETA=0 THEN RS=RME AS,CL=CLMEAS,CT=CTMEAS AND FACTOR=1
        RS=(1.0/(1.0-ETA)*(RMEAS-ETA*1.025)
        R1=RS/SEDP
        FACTOR=SQRT(RS/RMEAS)
        CL=SEDP*FACTOR*CLMEAS
        CT FACTOR*CTMEAS
        U1=R1*CT*CT
        QPDQ=SQRT(R*R+(Z+WH)*(Z+WH))
        TDFT=QPDQ/0.3048
        QTAN=ATAN(R/(Z+WH))
        SQU=ABS(TWREAL)
        HUGH=1.0/VALEXP
FINISH  HUGH=1.0E6
METHOD  MILNE
TERMINAL
        A,B=AKNPP(TIME, GAIN,ALGAIN,PLNRFC,DEGREE,ANCE,ETA,FREQ)
TIMER   FINTIM=30.0,DELT=.01,PRDEL=.02,OUTDEL=08,DELMIN=1.0E-72
```

```
PRINT     GAIN,DEGREE,PLNGN,PLNPHS,VALINT,EER,ENUM,DELT
PRTPLT    VALINT(PLNRFC,ALGAIN,ANGINC)GAIN(TWREAL,QPDQ,BLGAIN)
PRTPLT    DRATIO(CL,CT,R1)
END
STOP
ENDJOB
```

APPENDIX F

The results obtained for the "stationary phase" approximation to the integral expression of equation (1) are very similar to those obtained by a ray theoretical approach, but contain additional terms including the shear wave velocity. Indeed, for $C_T = 0$ in equation (4) of the text, we obtain the classical Rayleigh reflection coefficient for an incident plane wave,

$$\frac{N^{-1}}{N^{+1}} = \frac{\rho_1 C_L \cos\theta - \rho_o C_o \left(1 - \frac{C_L^2}{C_o^2}\sin^2\theta\right)^{\frac{1}{2}}}{\rho_1 C_L \cos\theta + \rho_o C_o \left(1 - \frac{C_L^2}{C_o^2}\sin^2\theta\right)^{\frac{1}{2}}} . \qquad \text{(F-1)}$$

The reflection coefficient of equation (4) is implicitly a function of the compressional and shear wave damping parameters as indicated by the following complex expressions for wave velocities in a viscoelastic medium:

$$C_L^2 = C_{LE}^2(1+i\omega b_L) \qquad C_T^2 = C_{TE}^2(1+i\omega b_T), \qquad \text{(F-2)}$$

where

$$C_{LE}^2 = \frac{\lambda'+2\mu'}{\rho} \qquad C_{TE}^2 = \frac{\mu'}{\rho} \qquad \text{(F-3)}$$

are the elastic wave velocities, and

$$b_L = \frac{\lambda''+2\mu''}{\lambda'+2\mu'} \qquad b_T = \frac{\mu''}{\mu'} \qquad \text{(F-4)}$$

are the damping parameters. These forms are consistent with equations (B-13) in Appendix B.

Several interesting points regarding the "stationary phase" solution of equation (1) can be studied. For normal incidence of the acoustic signal, i.e., when $\theta = 0°$, equation (F-1) assumes the form

$$\frac{N^{-1}}{N^{+1}} = \frac{\rho_1 C_L - \rho_o C_o}{\rho_1 C_L + \rho_o C_o} \qquad \text{(F-5)}$$

where the terms containing shear wave velocity vanish. Further inspection of the form in equation (4) reveals that shear wave information will be conserved for angles of incidence in the range $0° < \theta < 90°$. For the limiting case where $\theta = 90°$, i.e., when the grazing angle is parallel to the liquid-solid interface, we find that

$$\frac{N^{-1}}{N^{+1}} = -1, \qquad \text{(F-6)}$$

indicating that the incident wave is totally reflected.

These analytical results are valuable in that they provide insight into the problem of measuring the compressional and shear wave velocities, and the respective damping values, for porous, saturated marine sediments. A feasible method of accurately determining these sediment characteristic quantities by remote acoustic sensing techniques consists of a procedure where measurements are made at two separate receiver points, one on the vertical z-axis with the source, and a second receiver some radial distance from the z-axis. The first receiver collects only the information from longitudinal (compressional) waves since it corresponds to the normal incidence configuration discussed above (equation (F-5)). The second receiver collects both transverse (shear) and longitudinal wave information since waves arrive at this receiver from an oblique angle of incidence (equation 4). Assuming that the density and compressional wave velocity of sea water, as well as the density of subbottom sediment, are known, this information can be arranged in the form of two equations in the four unknowns C_{LE}, C_{TE}, b_L and b_T, or equivalently, λ', μ', λ'' and μ''. This system of equations can be solved, however, if data is taken at two different pulse frequencies, thus providing two additional equations. These extra equations are required to solve for the two unknown attenuation parameters in the viscoelastic sediment. Therefore, we have devised a system of four equations in four unknowns completely characterizing the parameters of the subbottom.