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GENERIC PERFORMANCE OF SMALL SCALE TIDAL POWER PLANTS

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Summary

Tidal power systems providing small power output (100 KW to 10 MW) may have proportionally less environmental impact and possibly superior economic advantages compared to the very large systems which have received the most attention. In eastern New England there exist numerous potential sites for generating marginal electric power on this scale to replace the consumption of fossil fuels. These sites have mean tidal ranges of 3m to 5m.

The characteristics of current low-head turbines suitable for such use were reviewed and found to be insensitive to the design head (as measured by the ratio of design head to runner diameter). For the purpose of this study, the part load performance of a typical low head turbine was described by a simple function.

A mathematical model of the operation of a tidal power plant was constructed, the operational variables and design parameters being defined in dimensionless form. The principal variables of interest determined from the model were time-averaged power, turbine head and tidal pond level. The major design parameters which effect the values of these performance variables were the ratios of sluiceway flow to turbine flow and turbine flow to tidal flow. Of these two, the former more significantly determined the value of the average power which, for a single effect plant, was approximately one half of the design power when sufficient sluiceway flow was provided.

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NOMENCLATURE

A = Surface area of tidal pond
A_o = Value of A at mean tidal level
A_s = Sluiceway effective flow area
D = Turbine runner tip diameter
H = Head
H_o = Turbine design head
H_t = Tidal half-amplitude
M = Turbine part load performance parameter (Eq. 5)
P = Turbine ideal power (Eq. 12)
P_o = Turbine design power
Q_o = Turbine design flow rate
Q_s = Sluiceway flow rate (Eq. 10)
Q_t = Turbine flow rate
T = Tidal period
V = Turbine part load performance parameter (Eq. 5)
Z = Tidal pond surface elevation above mean tidal level
Z_t = Sea surface elevation above mean tidal level

f = Dimensionless turbine flow rate as a function of head (Eq. 5)
g = Acceleration of gravity
h_s = Sluiceway flow function (Eq. 11)
h_t = Turbine flow function (Eq. 11)
ℓ = dA/dZ (Eq. 9)
t = time
β = Dimensionless turbine/tidal flow rate parameter (Eq. 13)
γ = Dimensionless sluiceway/turbine flow rate parameter (Eq. 13)
λ = Dimensionless tidal pond volume parameter (Eq. 13)
ϕ = Dimensionless turbine head parameter (Eq. 1)
ψ = Ratio of tidal amplitude to turbine design head (Eq. 13)
ρ = Density of sea water
σ = Dimensionless turbine flow rate parameter (Eq. 2)
Ω_o = Turbine design angular speed

Superscript

$\hat{}$ = Dimensionless variable defined in Eq. (13)

1. INTRODUCTION

Tidal power has not proceeded much beyond the demonstration stage primarily because of unfavorable economics. The largest operational facility (La Rance, France) has a capacity of 240 MW of electric power, but plants ten times as large have been studied (Ref. 1 and 2). The primary emphasis has been on large scale plants producing several hundred to several thousand megawatts and requiring tidal pools of several thousand to nearly one hundred thousand hectares in area. Given the necessity of a large tidal range and a considerable area to be enclosed by a dike, there exist relatively few locations in the U.S. (e.g., Cobscook Bay, ME and Cook Inlet, AK) which have the potential for generating power on a scale comparable with a modern central electric generating station (1,000 MW). None of these locations is close to a major market for power.

The resurgence of interest in producing power from renewable resources has concentrated on small scale sources (e.g., less than 1 MW). Solar electric, wind and small scale hydropower plants are currently being developed for use in supplying power to isolated or special purpose users or to be fed into an electric power distribution network. Where the energy is not stored but delivered when available, it replaces energy from fossil or nuclear fuels.

Small scale tidal power plants could serve the same purposes as wind, solar and hydropower sources of comparable size. More importantly, small scale greatly increases the possible number of sites, since only small tidal pond areas will be needed, and makes it more likely that sites which minimize the cost of dike construction can be found. Adverse environmental impacts may be more tolerable for small scale plants. Furthermore, electric power demand on a comparable small scale is likely to be found nearby, reducing the need for and diseconomies of long distance transmission. Finally, despite the economies of scale, it may turn out that small scale tidal power plants will be more economical than the large ones which have been almost exclusively considered to date.

The mean tidal range along the New England coast north of Cape Cod varies between 3 and 5 m, becoming highest in eastern Maine near the mouth of the Bay of Fundy. For this range the ideal tidal power output is 10 to 25 KW per hectare of tidal pond surface but only a fraction of this power can be expected to be extractable. For power plants with installed capacities of 10 KW to 1 MW, tidal ponds of one to 100 hectares would be required. While there are many natural coves of this size in New England, especially along the coast of Maine, only a few could be expected to meet criteria for limited environmental and land/water use impacts.

Because of the potential for preliminary consideration of a large number of sites, we have sought to characterize the design of the plants by a limited number of parameters (e.g., design power, head, sluiceway flow, etc.) and their performance by a few time averaged properties (e.g., average power, tidal pool elevation, etc.). Our goal is to relate the performance variables to the design parameters, presenting the results in dimensionless form. This is a significant departure from previous analyses (Ref. 3 and 4), which makes these results more generally useful. In other respects, however, our dynamical model of plant operation is not greatly different than that used by Swales and Wilson (Ref. 4) and Bernshtein (Ref. 3).

This paper considers some of the generic features of small scale tidal power plants. In the first part we review the principal design variables of existing low-head hydropower turbogenerators to illustrate the ranges of these variables found in existing hydropower installations and to see how different these might be for very low-head uses, of which tidal power is one of the most prominent. We also consider the variable load, constant speed characteristics of such turbines. In the second part we examine the performance characteristics of single pool tidal power systems operating in three modes: outflow, inflow and double effect, expressing the results in terms of the principal design parameters of the turbine, sluiceway and tidal pond. In both cases we express results in terms of dimensionless variables and parameters so that they may be applied to any particular site without further detailed analysis.

2. DESIGN CHARACTERISTICS OF LOW-HEAD HYDROTURBINES

The principle design parameters of a hydroturbine are the runner tip diameter D and angular speed Ω_o, the design head H_o, the design volume flow rate Q_o and power output P_o. While the values of these dimensional quantities are quite different for different installations, their ratios, appropriately non-dimensionalized, should not differ greatly.

Two dimensionless ratios will be found to be convenient to use. The first relates the design head to the runner tip speed:

$$\phi \equiv (gH_o)^{1/2}/\Omega_o D \qquad (1)$$

where g is the acceleration of gravity. In the second, we refer the ideal flow rate needed to produce the design power $(P_o/\rho g H_o)$ to a volume flow rate characterized by speed $\Omega_o D$ times flow area D^2:

$$\sigma \equiv P_o/\rho g H_o \Omega_o D^3 \qquad (2)$$

In Eq. (2), we use $P_o/\rho g H_o$ in place of Q_o because the latter is seldom specified in the references we have consulted.

We choose as a measure of "lowness" of head the ratio H_o/D. For what is generally described as low-head turbines, H_o/D is less than five.

We have examined the published properties of low-head hydroturbines (Ref. 5 and 6) and determined the values of ϕ and σ for Kaplan, bulb and tube turbines. In Figs. 1 and 2 we plot the values of ϕ and σ, respectively, as a function of the ratio H_o/D As can be seen from these figures, there is no descernable trend of either of these parameters with the ratio H_o/D In other words, no significant changes in turbine design appear to have been made for the very low head applications. The mean values and standard deviation of ϕ and σ are given by:

$$\bar{\phi} = 0.356 \pm 0.0638 \qquad (3)$$

$$\bar{\sigma} = 0.881 \pm 0.178 \qquad (4)$$

If H_o and P_o are specified, then Eqs. (1) - (4) can be solved to determine values of Ω_o and D based upon average values of ϕ and σ.

3. PARTIAL LOAD CHARACTERISTICS

Tidal power plants inherently provide a large range of head. It would be desirable to produce power at the rated power of the generator over as wide a range of head as possible. This can be accomplished by controllable wicket gates (and variable runner blade pitch where design permits) but only up to a point where increased flow at lowering head is no longer possible because the entrance flow area has reached a maximum. For lower head, the flow and hence power will be reduced. For either synchronous or induction generators, turbine speed will remain constant while head changes, limiting the range of head for which rated power can be achieved and limiting the turbine efficiency to non-optimum values for a given head.

In analyzing the performance of tidal power plants in a generic way it is necessary to specify the off-design performance so that the turbine power and flow may be determined as a function of the varying head. Obviously, such plant load characteristics will depend upon the turbine design and control system. For the purposes of our analysis, we will model the variation of volume flow Q_t with head H according to the following relation:

$$Q_t/Q_o = f(H/H_o)$$

$$\text{where} \quad f = \frac{H_o}{H} \quad \text{if } \frac{H}{H_o} \geq V \leq 1$$

$$= V^{-3/2}(H/H_o)^{1/2} \quad \text{if } M < \frac{H}{H_o} < V$$

$$= 0 \quad \text{if } 0 < \frac{H}{H_o} < M \qquad (5)$$

where V and M are constants of the turbine design, with $0 < M < V < 1$. For $H/H_o > V$, $Q_tH = Q_oH_o$ and so the ideal power is constant and equal to the rated power. When $H/H_o \leq V$, no further increase in Q_t is possible because the wicket gates are wide open and volume flow must decline as $H^{1/2}$. Finally, at low enough head ($H/H_o = M$) the power output will be so small as the free-wheeling condition is approached that the wicket gates will be closed and the turbine stopped.

In Fig. 3 we show two flow curves for two tidal power plant turbine designs illustrating the nature of the Q_t vs H relationship. Also shown in Fig. 3 is the model relation of Eq. (5) for values of V and M which provide a rough approximation to the two designs.

4. MODEL OF TIDAL POWER PLANT OPERATION

Our purpose in studying the generic characteristics of tidal power plants is to determine several performance parameters of interest (average power, mean tidal pool level and pool amplitude) as a function of the principal design parameters (tidal pool surface area, design power, sluice gate size). For a specific site, the performance parameters may be sufficient for a preliminary site analysis and plant design specification. For a detailed study of a selected site, a more complete analysis would be required. It is therefor sufficient for our present purposes to use a simplified model of the plant operation.

We measure the surface height Z of the tidal pool and that of the sea Z_t with respect to mean sea level. The head H available to the turbine (or sluiceway) is:

$$H = |Z - Z_t| \qquad (6)$$

If H_t is the tidal amplitude and T the tidal period, we represent the tidal level Z_t by:

$$Z_t = H_t \sin\left(\frac{2\pi t}{T}\right) \qquad (7)$$

where t is time. Although tidal amplitude and phase will change with day of the lunar month, it is not necessary to include such changes in Eq. (7) since our calculations will be concerned only with a single tidal period.

The surface area A of the tidal pond will generally increase with surface level Z, depending upon the specific site characteristics. For simplicity, and as suggested by Swales and Wilson (Ref. 4), we assume that A increases linearly with Z:

$$A = A_o + \ell Z \qquad (8)$$

in which A_o is the pond surface area at mean tidal level and ℓ is a characteristic length scale determined by

$$\ell = (dA/dZ)_{Z = 0} \qquad (9)$$

Obviously, a more complex and non-linear relationship than Eq. (8) might be needed for some sites.

For the volume flow Q_t through the turbine, a function of the form given in Eq. (5) is used. For the sluiceway volume flow Q_s we use:

$$Q_s = A_s(gH)^{1/2} \tag{10}$$

in which A_s is the effective flow area of the sluiceway.

The dynamical condition for flow of water into the pond is then given by:

$$A\frac{dz}{dt} = Q_t h_t + Q_s h_s \tag{11}$$

in which h_t and h_s are either 0, 1 or -1, depending upon whether the turbine (sluiceway) is closed or open for inflow or outflow, respectively. Thus Eq. (11) provides for both single effect turbine outflow or inflow and double effect operation.

The ideal power from the turbine, P, is simply

$$P = (\rho Q_t)(gH) \tag{12}$$

where ρ is the density of sea water.

For a generic study, it is desirable to express the principal design parameters and performance variables in dimensionless form. Somewhat arbitrarily, we introduce the following dimensionless parameters and variables:

variables	parameters
$\hat{A} = A/A_o$	$\beta = Q_oT/A_oH_t$
$\hat{Z} = Z/H_t$	$\psi = H_t/H_o$
$\hat{Z}_t = Z_t/H_t$	$\gamma = A_s(gH_t)^{1/2}/Q_o$
$\hat{t} = t/T$	$\lambda = \ell H_t/A_o$
$\hat{H} = H/H_t$	
$\hat{Q}_t = Q_t/Q_o$	
$\hat{Q}_s = Q_s/Q_o$	
$\hat{P} = Q_tH/Q_oH_o$	

(13)

Using these definitions in Eqs. (6) through (12) above, their dimensionless form becomes

$$\frac{d\hat{z}}{d\hat{t}} = \beta\left|\frac{\hat{Q}_t h_t + \hat{Q}_s h_s}{1 + \lambda\hat{z}}\right|$$

$$\hat{H} = |\hat{Z} - \sin 2\pi\hat{t}|$$

$$\hat{Q}_t = f\{\psi\hat{H}\}h_t$$

$$\hat{Q}_s = \gamma(\psi\hat{H})^{1/2}h_s$$

$$\hat{P} = \psi\hat{Q}_t\hat{H} \tag{14}$$

Eq. (14) can be integrated on time, beginning with an arbitrary initial condition (e.g., $Z = 0$ at $t = 0$). After about ten tidal periods, the solution becomes periodic, and the following time-averaged quantities can be calculated:

$$
\begin{aligned}
\langle\hat{P}\rangle &= \int_0^1 \hat{P}\,d\hat{t} \\
\langle\hat{H}\rangle &= \langle\hat{P}\rangle \Big/ \int_0^1 \hat{Q}_t h_t\,d\hat{t} \\
\langle\hat{Z}\rangle &= \int_0^1 \hat{Z}\,d\hat{t}
\end{aligned}
\qquad (15)
$$

The first of these is the average ideal power output of the turbine which, except for the variation of turbine efficiency with head, would also equal the average fraction of rated power which is generated. The second is a volume-flow averaged turbine head, which is subsequently used to specify the most desirable rated head for a given mode of operation. The third defines the mean tidal pool level, which is of interest in assessing the environmental effects of disturbing the natural flow in the tidal pool.

A typical solution of Eq. (14), for nominal values of the design parameters, is shown in Fig. 4.

There are four principal parameters which determine the performance of the power plant: β,ψ,γ and λ . The ratio of turbine flow rate to undisturbed tidal flow rate, β, might be expected to lie between about 0.1 and 1. The ratio of tidal amplitude to design head, ψ, should probably be between 2 and 0.5. It seems likely that the sluiceway flow rate should be 1 to 5 times the turbine flowrate,or $5 > \gamma > 1$.In order to avoid negative pond areas at low tide, $\lambda < 1$. Thus the reasonable range of these four parameters can be estimated and a matrix of trial values established.

We have made a set of computations using the following approach. First, we set $M = 0.3$ and $V = 0.8$ (see Eq. 5 and Fig. 3) as representative of conventional turbine design. Next, we set $\lambda = 0$ (prismatic tidal volume) and select a set of values of β and γ. For each calculation, we adjust the value of ψ until the average head $\langle H\rangle$is ψ^{-1}, i.e., we choose the design head H_0 to be equal to the turbine volume-flow averaged head as defined (dimensionlessly) by Eq. (15). We then calculate $\langle\hat{P}\rangle$ and $\langle\hat{Z}\rangle$ as the principal performance parameters.

This choice of design head is not an optimization technique, such as that of Bernshtein (Ref. 3) or Swales and Wilson (Ref. 4), but merely a rational method of ensuring that the average operation of the turbine is close to its design point. Off-design performance of the turbine is represented by the schedule of Eq. (5) (see also Fig. 3), although this may not be the optimum as explained by Bernshtein (Ref. 3). While it is possible that further improvements in performance would result from optimizing the mode of turbine operation, we defer such investigations until after the effects of major design parameters have been determined.

The most significant parameter determining the performance of the power plant is γ, the ratio of sluiceway flow rate to turbine flow rate (see Eq. 13). As can be seen from Fig. 5, the time-averaged power increases rapidly with increasing γ until $\gamma = 5$, after which there is very little gain in power from enlarging the sluiceway flow. The average power for $\gamma > 5$ is about 50% of the rated power, and the duration of power production is somewhat more than half the tidal cycle. This conclusion is only slightly affected by the value of the turbine flow parameter β. Similar dependence of average turbine flow head on γ is shown in Fig. 6, although it is more sensitive to the value of β. Fig. 6 shows that the design head should be approximately equal to the tidal amplitude for $\gamma > 5$.

In Fig. 7 is plotted the time-averaged power as a function of the turbine flow parameter β for several values of γ. This figure illustrates how insensitive is the average power to the ratio turbine flow/tidal flow over the range calculated. However, for large enough values of β the average turbine power/design power must decline since the turbine power cannot exceed the tidal power available.

5. CONCLUSIONS

Very low head hydroturbines suitable for tidal power plants possess design parameters (Eqs. 1 and 2) which are not significantly dependent upon the design head. Similarly, performance characteristics under variable head conditions appear to scale with the design conditions (Fig. 3).

Performance of the tidal power plant (turbine, sluiceway and tidal pond) can be determined as a function of several dimensionless parameters (Eq. 13). Of these, the most important (γ) measures the ratio of sluiceway flow rate to turbine flow rate. For $\gamma \geq 5$, no major increase in time-averaged power is achieved by adding sluiceway area. Time-averaged power is quite insensitive to the value of β, the ratio of turbine flow rate to tidal flow rate, at least for the values examined.

For single-effect tidal power plants an average of about fifty percent of the design power can be produced if proper selection of design parameters is made.

6. ACKNOWLEDGEMENT

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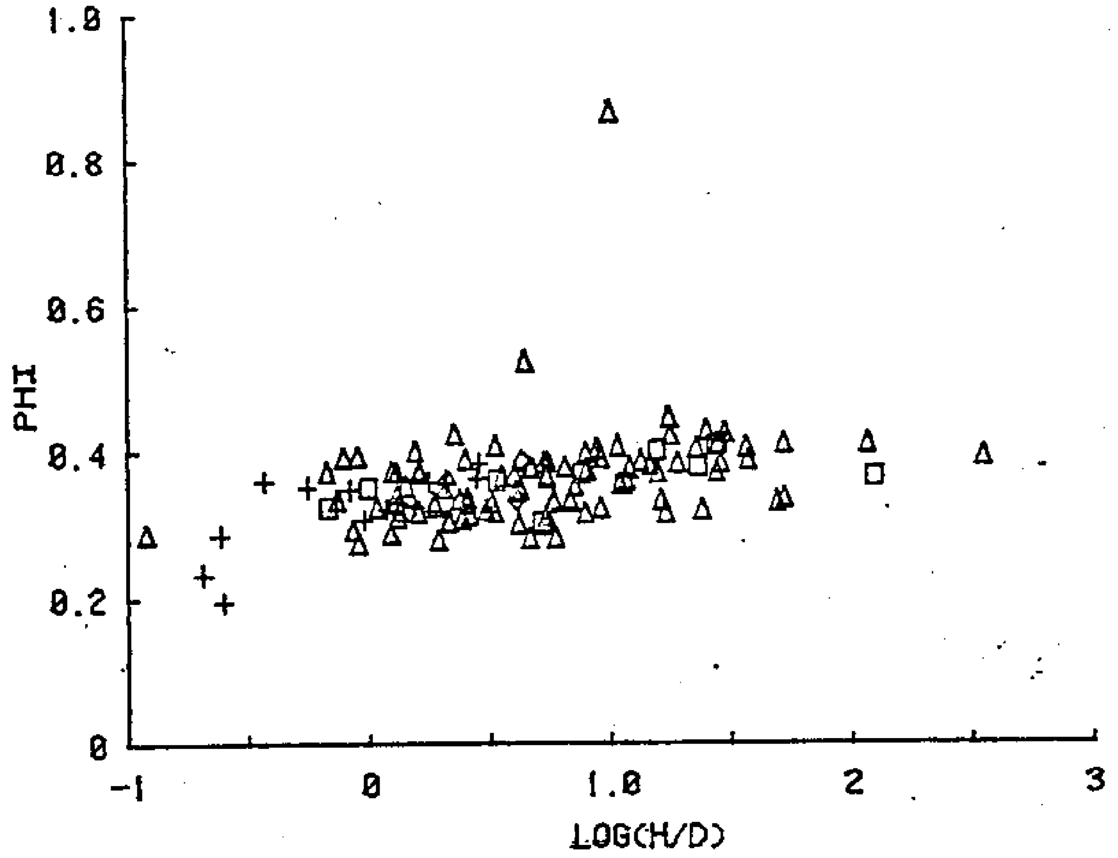

Fig. 1 Dimensionless head parameter φ (Eq. 1) as a function of head/diameter ratio. Turbine types are bulb Δ, tube □, Kaplan + and vertical propeller ◇ .

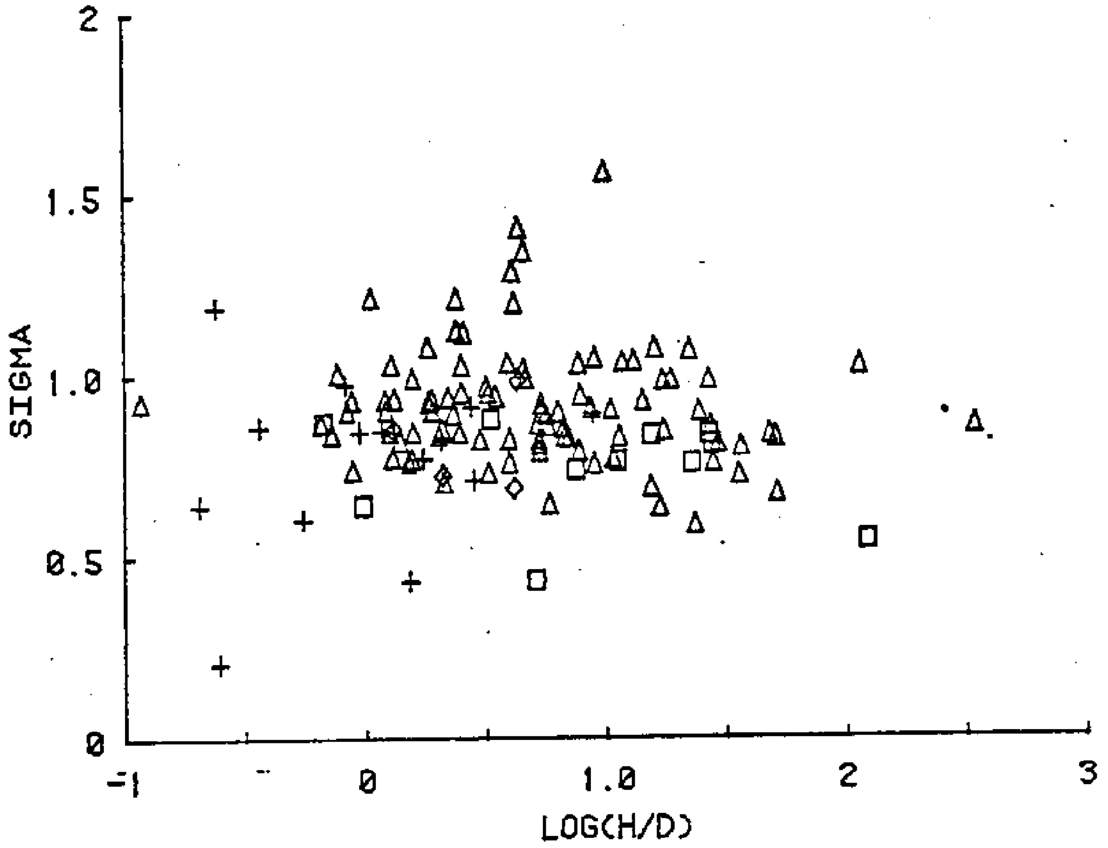

Fig. 2 Dimensionless flow parameter σ (Eq. 2) as a function of head/diameter ratio. Symbols are identical to those of Fig. 1.

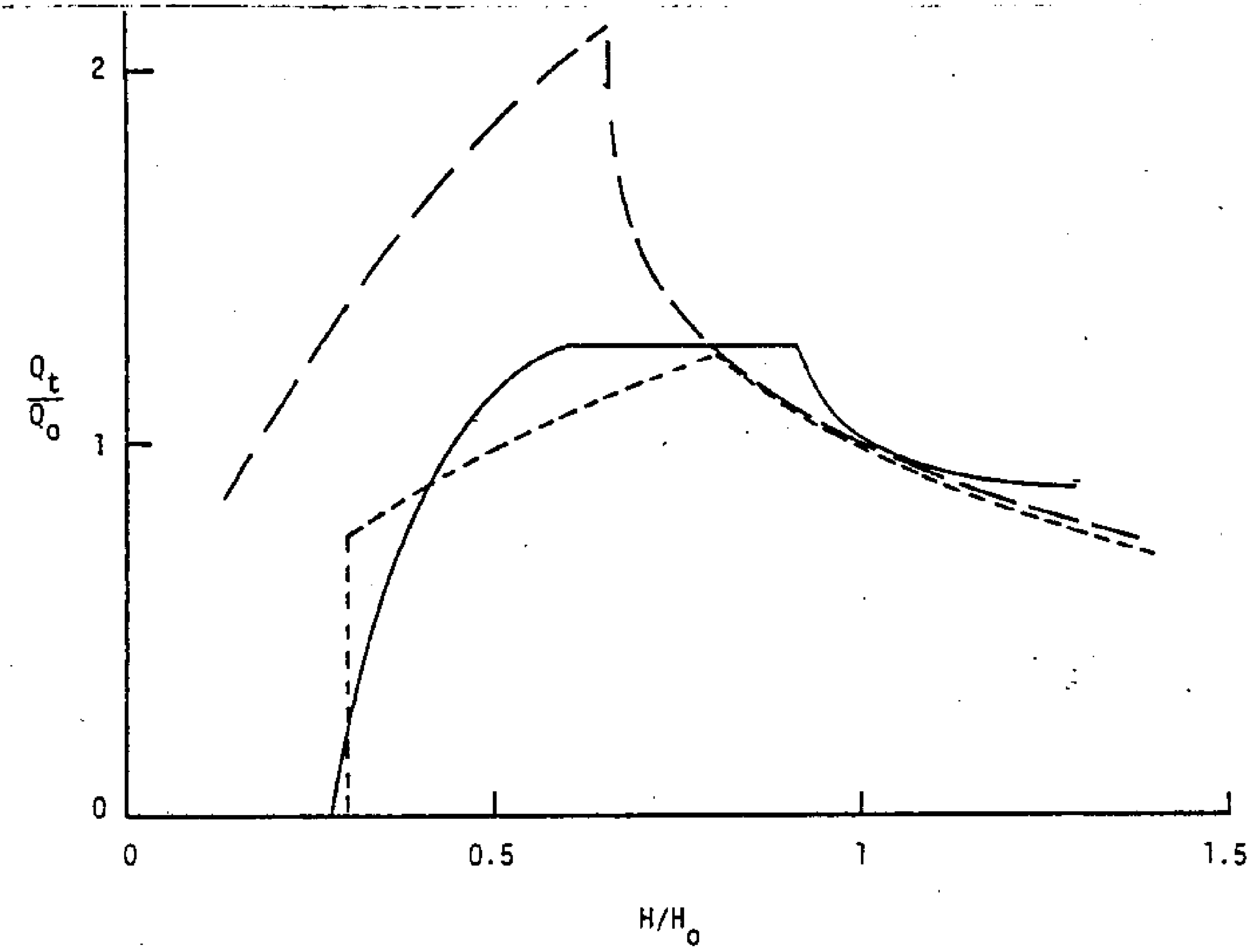

Fig. 3 Normalized flow Q_t/Q_o versus head H/H_o for partial load. – – – Eq. 5 with M = 0.3, V = 0.8; —— Annapolis Royal tidal power Plant (Ref. 7); —— —Rance turbine (Ref. 3).

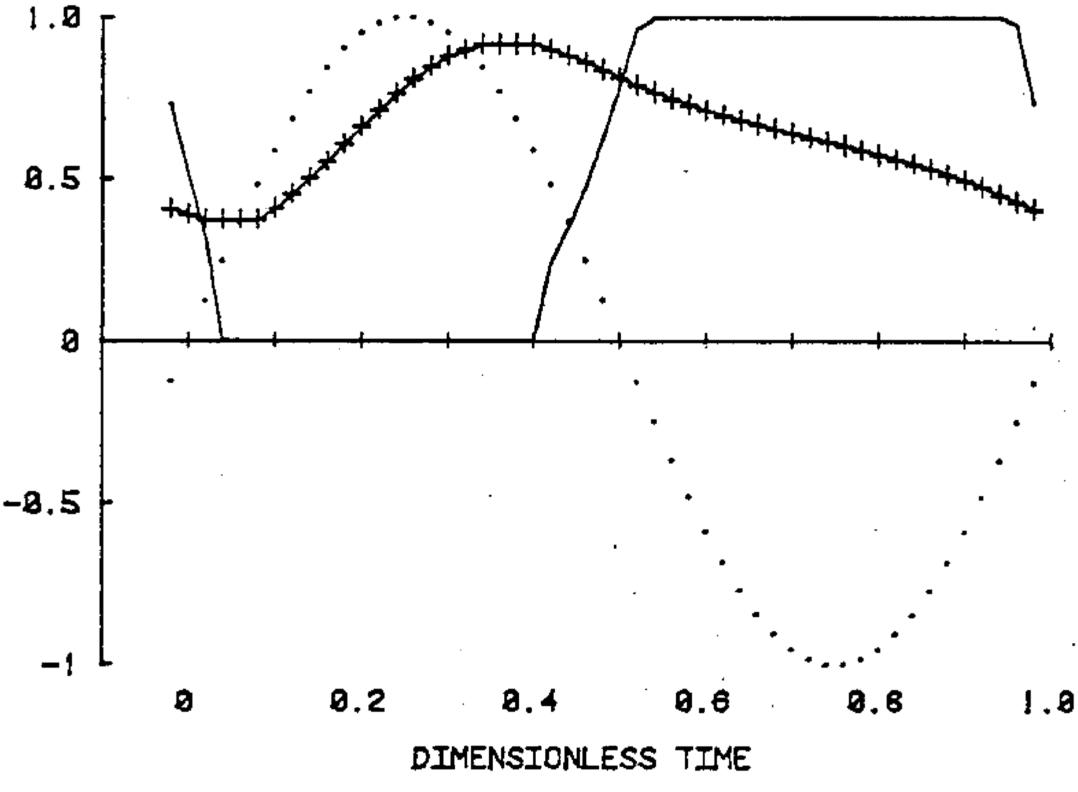

Fig. 4 Dimensionless variables $\hat{P}$(——), $\hat{Z}_t$(....), and $\hat{Z}$(++++) as a function of time $\hat{t}$ for a single effect outflow tidal power plant when $\beta = 1$, $\lambda = 0$ and $\gamma = 5$.

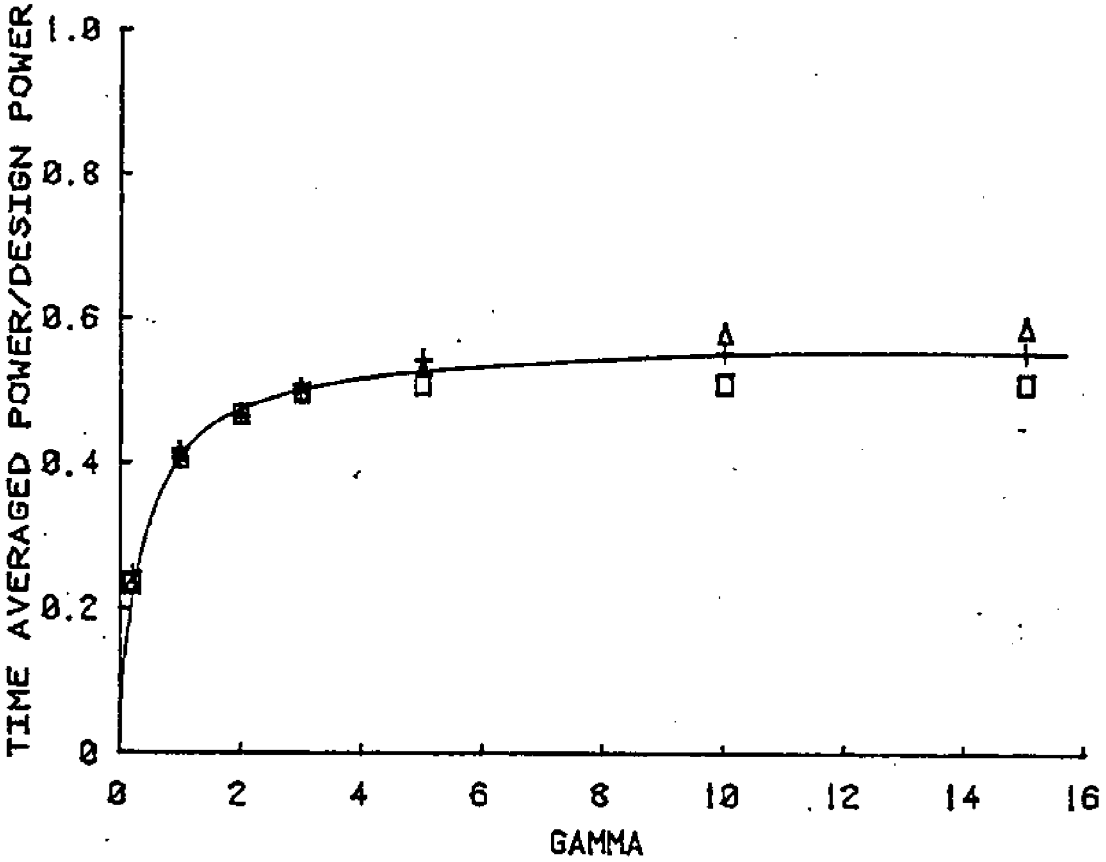

Fig. 5 Time-averaged power as a function of the sluiceway area parameter γ (Eq. 13). Symbols identify the values of β: + = 1 , Δ = 0.5 , □ = 2.

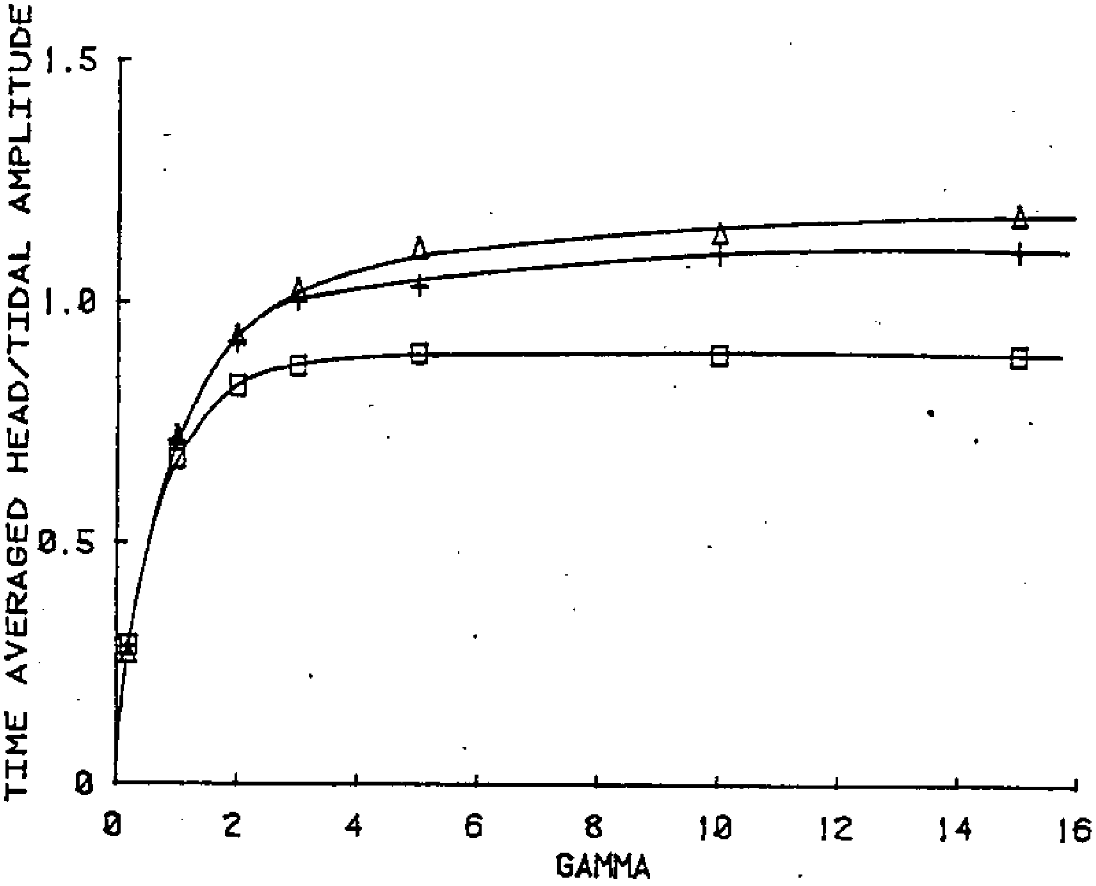

Fig. 6 Time-averaged head as a function of the sluiceway area parameter γ (Eq. 13). Symbols as in Fig. 5.

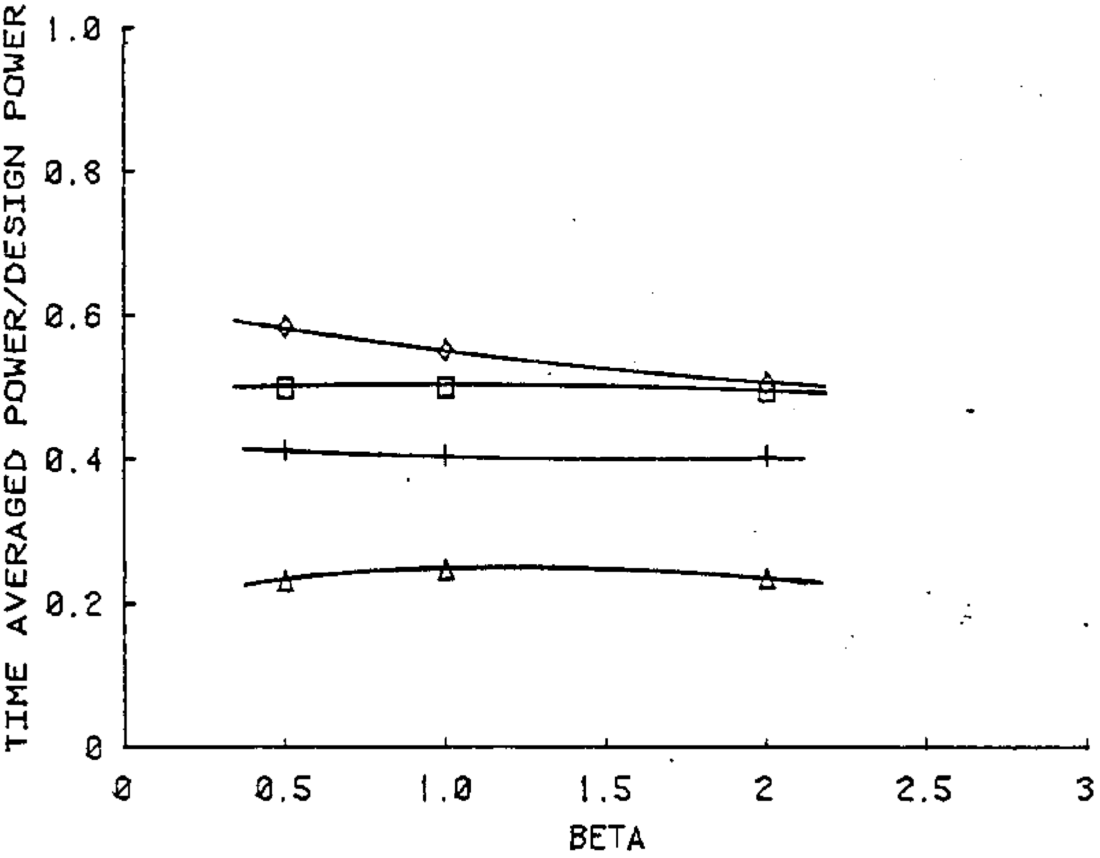

Fig. 7 Time-averaged power as a function of the parameter β (turbine flow/tidal flow, Eq. 13). Symbols identify the parameter γ: ◇ = 15, □ = 3, + = 1, △ = 0.2.

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