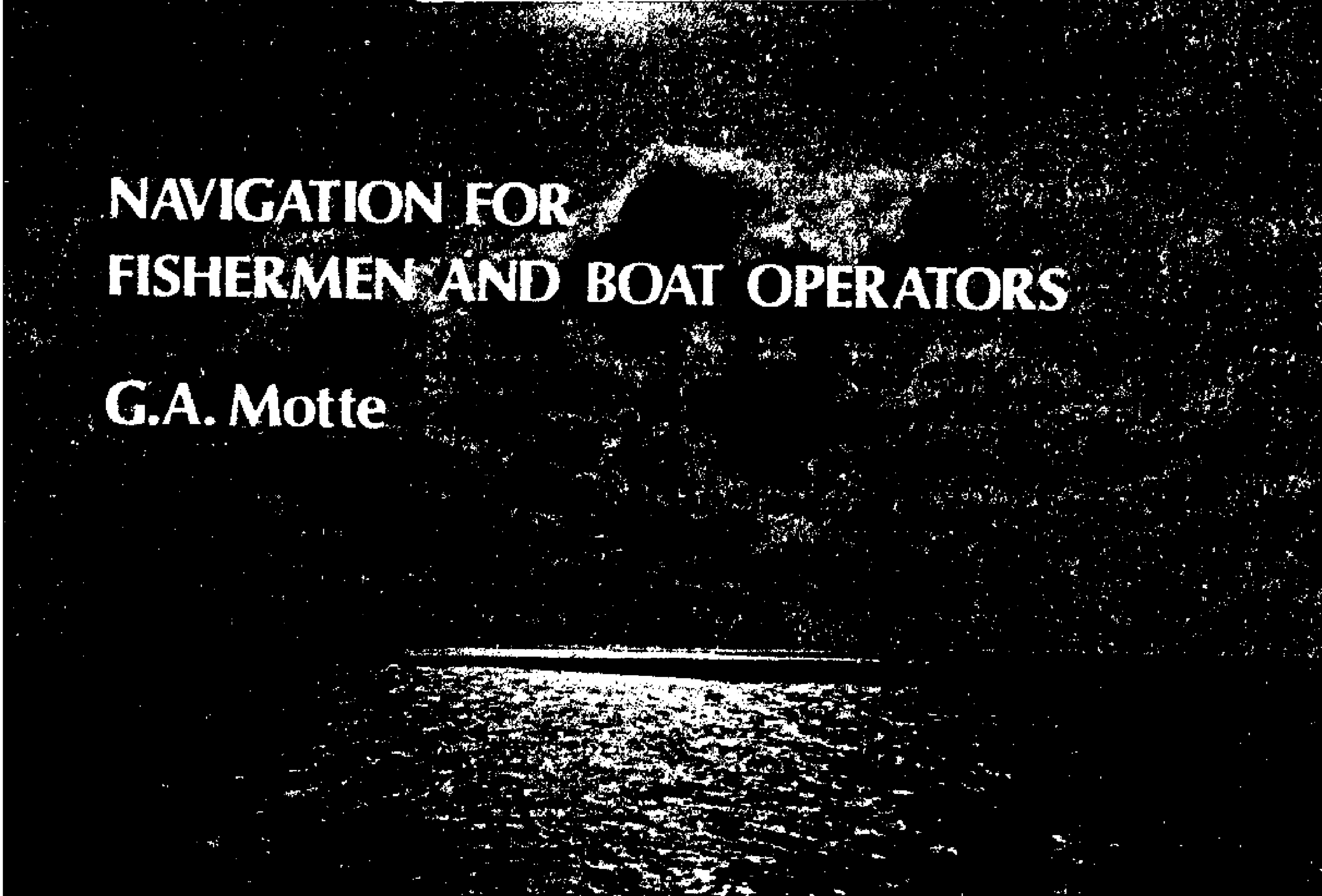

NAVIGATION FOR FISHERMEN AND BOAT OPERATORS

G.A. Motte

FISHERIES AND MARINE TECHNOLOGY
SEA GRANT

University of Rhode Island
Marine Bulletin Number 10

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College of Resource Development
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Dept. 3

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THE AUTHOR

Captain Motte served 11 years in the British Merchant Marine and holds Part A of the Extra Masters Certificate in Marine Law, the Foreign Going Masters Certificate of Competency and three pilotage licenses. He is presently assistant professor in the Department of Fisheries and Marine Technology at the University of Rhode Island where he teaches navigation, meteorology and marine transportation. He also holds the Master of Science Degree in experimental statistics.

Additional copies of this publication at $3.50 each may be obtained from the Marine Advisory Service, University of Rhode Island, Narragansett Bay Campus, Narragansett, Rhode Island 02882. The other book in this set, *Chartwork for Fishermen and Boat Operators*, Marine Bulletin Number 7, is available for $3.00 from MAS. Make checks payable to the University of Rhode Island.

Contents

EXERCISES

H.O. Publication 214, Vol. IV, and a set of nautical tables are needed to complete the exercise following each section. The necessary parts of the 1968 Nautical Almanac *are in the back of this book and a table of increments from any year's* Nautical Almanac *may be used to supplement these. The use of* Ageton's Short Method Tables, *H.O. Publication 211, is optional.*

APPENDICES

Introduction

This text is intended to continue where Marine Bulletin Number 7, *Chartwork for Fishermen and Boat Operators,* stops.* The only presupposed knowledge is either a grounding in trigonometry and chartwork or satisfactory completion of the exercises in *Chartwork* on parallel, plane and Mercator sailing. A study of these three sailing methods should help you make the transition between chartwork—plotting a ship's course in sight of land—and "deep sea" navigation—plotting a course out of sight of land. And, of course, many of the principles used in *Chartwork* can be applied to both celestial and electronic methods of navigation. (The principle of the transferred position line is one example.)

Celestial navigation is still the primary method of ocean navigation despite predictions of its replacement by electronic and satellite devices. However, professional navigators never cease to seek faster and better methods of sight reduction—the process of deriving from observations of a celestial body the information needed for establishing a line of position or series of possible positions of a vessel. It is doubtful that many of the operations at sea give the mariner the satisfaction that quickly and successfully reducing a number of star sights to a final observed position does, especially after navigating for a considerable time on dead reckoning in overcast conditions.

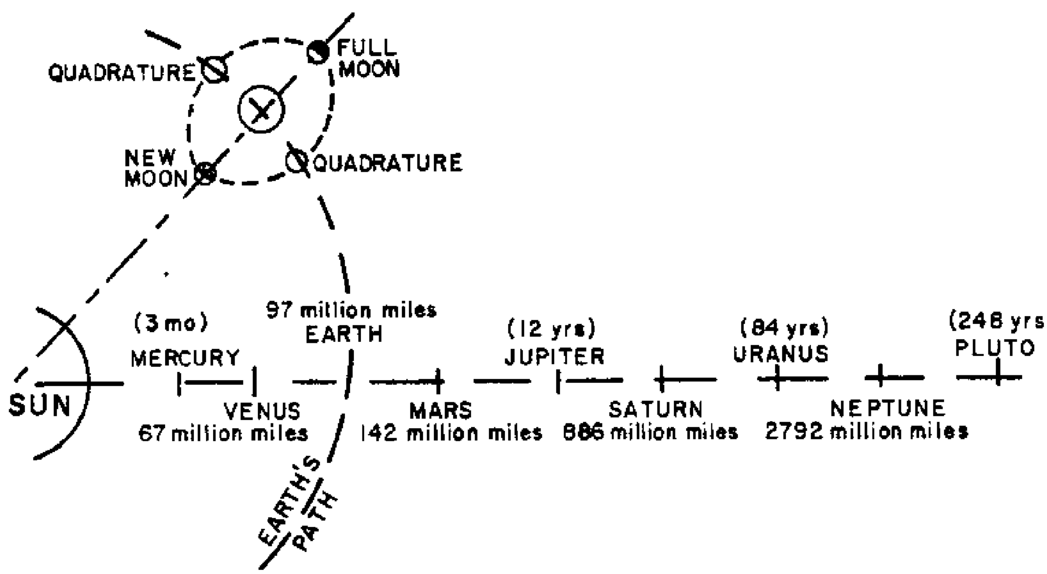
But, before you can attack such problems as sight reduction, you must first study the relevant associated theory. Thus, this text has been divided into two parts. The first part is confined to principles of navigation and the second, to practical problems, including calculations and the use of necessary nautical tables. Both this book and *Chartwork* have been pre-tested in classrooms and field work in the Department of Fisheries and Marine Technology at the University of Rhode Island over the last three years.

*Available for $3.00 from the Marine Advisory Service, University of Rhode Island, Narragansett, Rhode Island, 02882. Make checks payable to the University of Rhode Island.

PART 1. Principles of Navigation

The Solar System

The solar system consists of the Sun, its planets and their individual satellites, all of which shine by means of the Sun's reflected light. The planets spin on their axes and move in elliptical orbits around the Sun with their orbital planes inclined at various angles. The farther away a planet is from the Sun, the longer it takes to complete one orbit. The relative positions of the planets, their distance from the Sun and the approximate time for one complete orbit of the Sun is shown in the following diagram:



Bode's Law indicates the approximate relative distances of the planets from the Sun by adding four to each number in the series 3, 6, 12, 24, 48, 96, 192.

When the Earth and a given planet are in line on opposite sides of the Sun they are said to be in opposition. When they are in line on the same side of the Sun they are said to be in conjunction. If the Earth and a planet are 90 degrees from each other they are said to be in a position of quadrature. Only the planets Venus, Mars, Jupiter and Saturn are used for purposes of practical navigation.

The planets have a number of satellites, or moons, revolving around them while they themselves rotate on their axes as they orbit the Sun. The Earth has only one moon, but other planets have a number of moons. For example, Jupiter has eight.

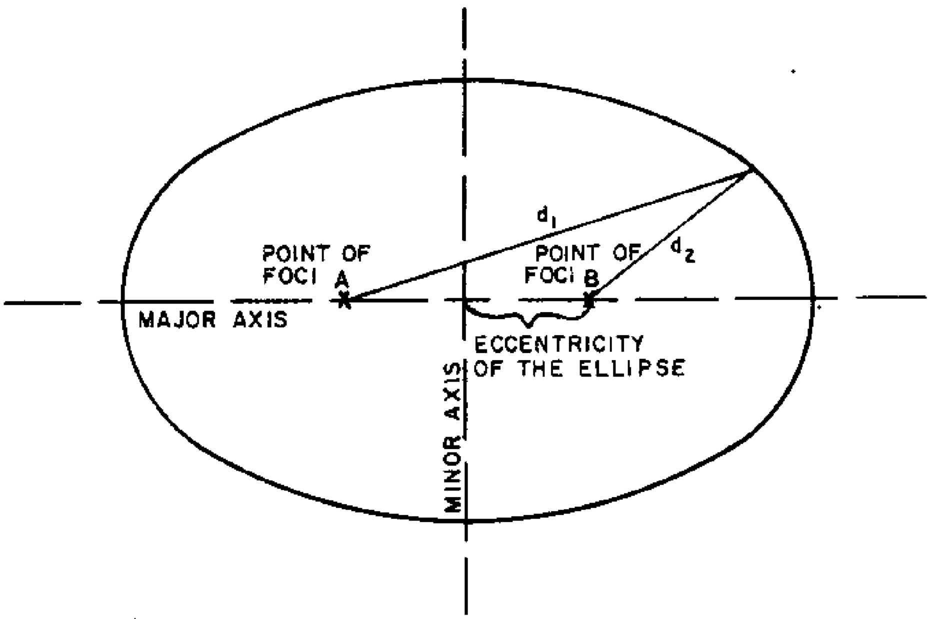
The planets Mercury and Venus, which are nearer to the Sun than the Earth, are termed inferior planets. All other planets are called superior planets.

GENERAL MOTION OF THE PLANETS

The Earth in its path about the Sun obeys three basic rules, known as Kepler's Laws. These rules which apply to all planets are described below.

Rule 1

Every planet moves in an orbit which is an ellipse with the Sun at one of the points of foci. Similarly the track of a moon or satellite about its parent planet is also an ellipse with the planet at one of the points of foci. In the illustration, $d_1 + d_2$ is always constant, and the eccentricity of an ellipse is the distance of a point of foci from the center of the major axis.



Rule 2

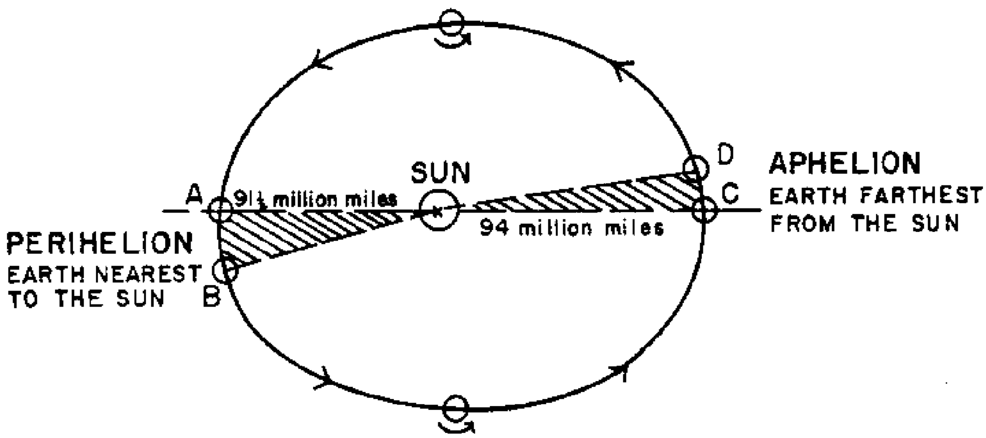
A straight line joining the Sun to the center of a planet, i.e. the planet's radius vector, sweeps out equal areas in equal intervals of time.

Rule 3

The square of the time taken for a planet to orbit the Sun is directly proportional to the cube of its distance from the Sun.

The Earth's Motion

TRUE MOTION OF THE EARTH



The Earth rotates on its axis through 360 degrees every 23 hours, 56 minutes and 04 seconds as it revolves around the Sun in a counterclockwise direction once every 365.2422 days. That is, the Earth rotates once on its axis in about a day as it orbits the Sun in about 365¼ days. The Earth's path is an ellipse and the Sun is at one of the points of foci of that ellipse. The Sun's diameter is about 100 times that of the Earth.

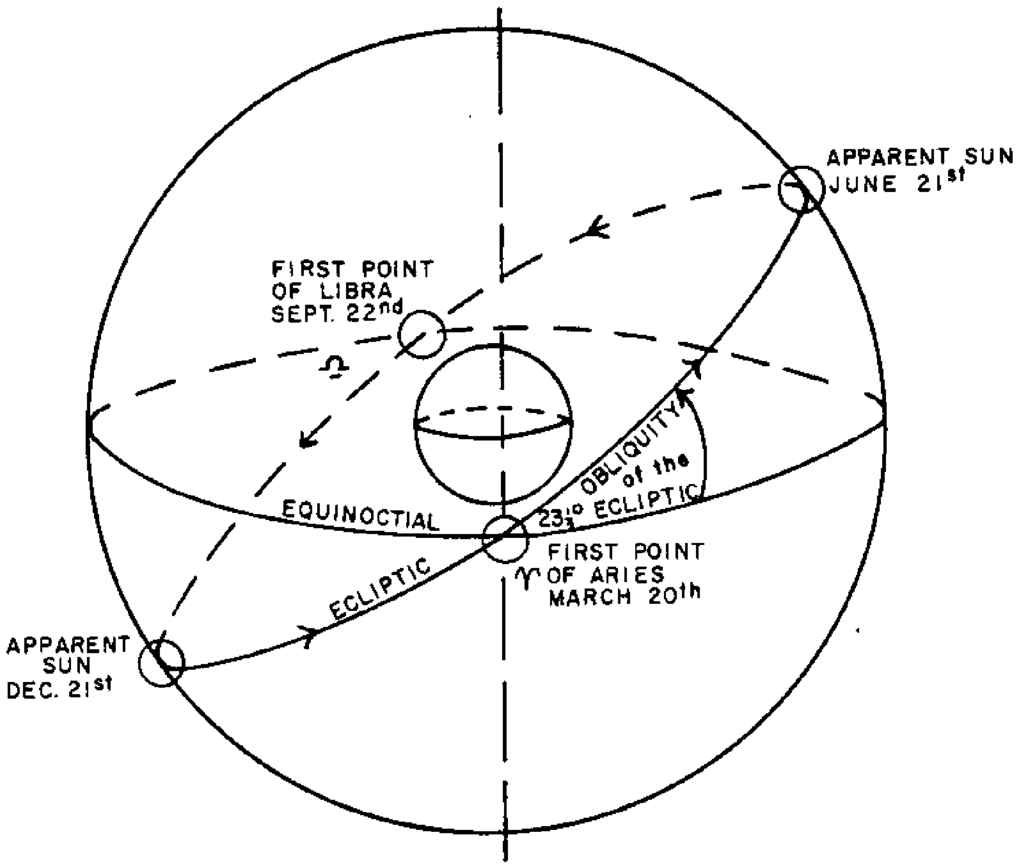
It is easily seen from the diagram that in order for Kepler's Second Law to be obeyed the two shaded triangles must be equal in area, provided the time taken for the Earth to move from *A* to *B* is equal to the time taken to move from *C* to *D*. Clearly, however, *AB* is a greater distance than *CD*, and, in fact, the Earth's velocity must be faster at *AB* than *CD*.

The Earth moves more quickly at perihelion (when it is closest to the Sun) than at aphelion (when it is farthest from the Sun).

The Earth's equator is inclined at an angle of about 23½ degrees to the orbital plane of the Earth.

APPARENT MOTION OF THE SUN AND THE SEASONS

On Earth we tend to imagine that we are the center of the solar system. Our true motion is taken up in the apparent motion of the Sun as illustrated below. But for navigational purposes we disregard distances in space and consider all heavenly bodies projected onto the inner surface of a huge sphere concentric with Earth. This sphere of infinite radius is referred to as the celestial sphere.

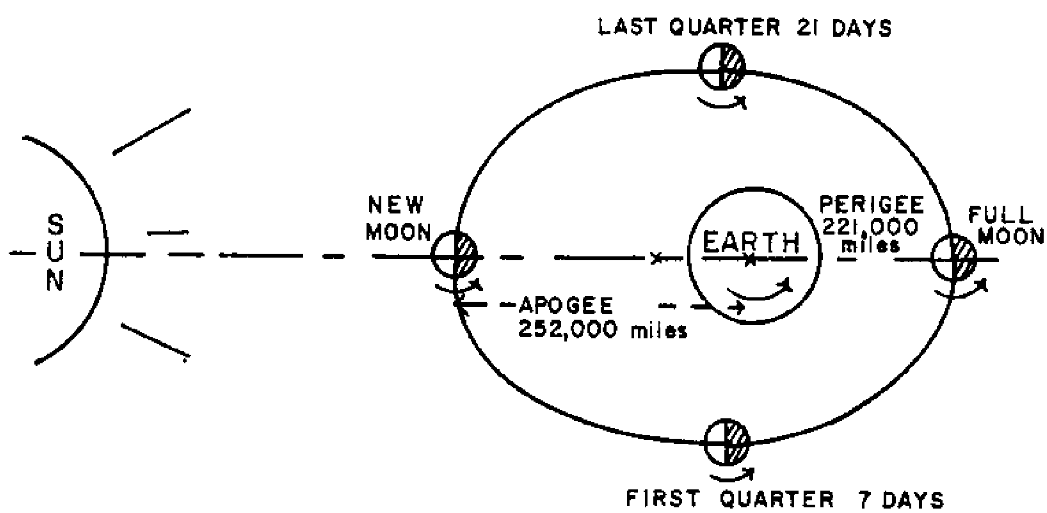


The path tracked out by the apparent Sun on the celestial sphere is called the ecliptic. The equator extended to the celestial sphere is termed the celestial equator, or equinoctial. The angle between the planes of the ecliptic and equinoctial is about 23½ degrees and is referred to as the obliquity of the ecliptic.

This feature gives rise to the Earth's seasons. The northern hemisphere spring begins when the apparent Sun in its northerly path along the ecliptic crosses the equinoctial. This occurs on March 20, and is known as the vernal equinox and is also referred to as the first point of Aries, indicated by the sign ♈. Summer begins when the apparent Sun reaches its most northerly point on June 21 at the summer solstice. Summer ends when the apparent Sun, moving south, crosses the equinoctial on September 22 at the fall equinox. This point is also referred to as the first point of Libra, indicated by the sign ♎. The northern hemisphere winter begins when the apparent Sun attains its most southerly point on the ecliptic on December 21 at the winter solstice.

The Moon's Motion

The plane of the Moon's orbit around Earth is inclined at an angle of about five and one-half degrees to the plane of the Earth's orbit about the Sun. The Moon is only about one-quarter million miles from Earth and its diameter is approximately a quarter that of the Earth.



A lunation, the time interval between two successive new moons, takes about 29½ days. During this period, the Moon itself turns once on its axis and so always presents the same side to the Earth. Note that the Moon will complete a 360-degree circuit of Earth in about 27½ days, but that during this time the Earth has also moved in its path about the Sun, so that it will take about another two days for the Sun, Moon and Earth to come back in line and the next new moon to occur.

The Stars

THE PRINCIPAL BRIGHT STARS

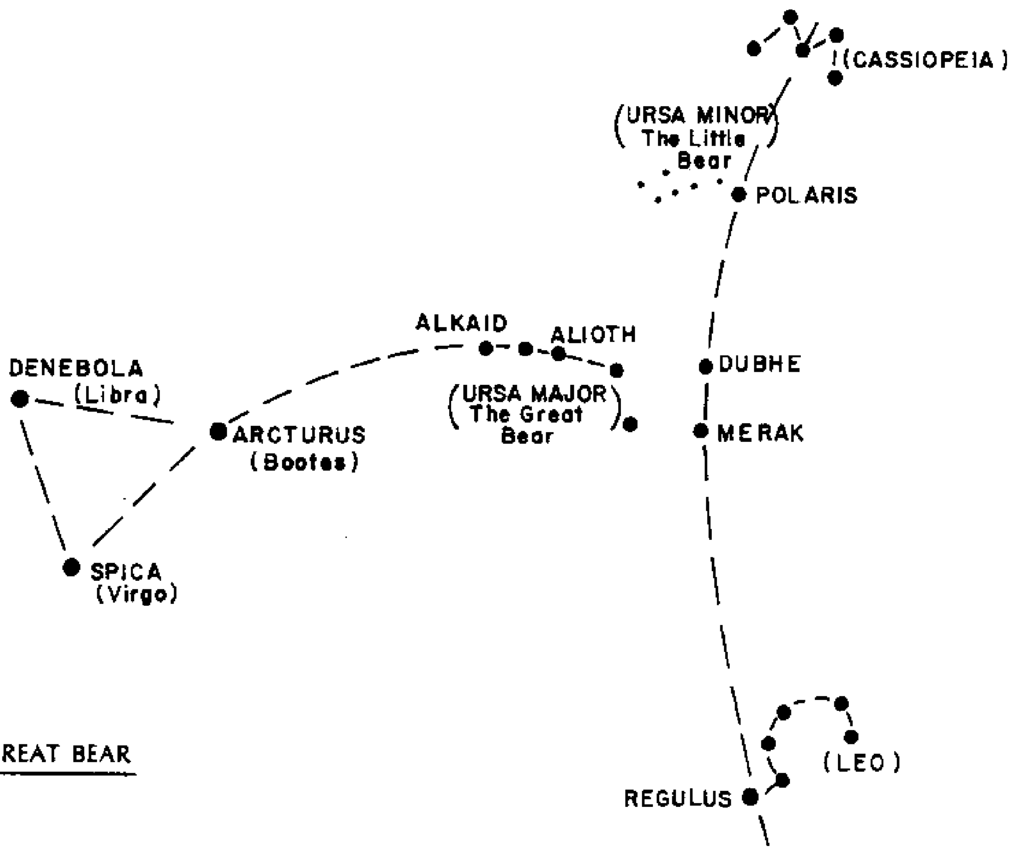
The multitude of stars in the heavens appears highly mysterious and complex to the student navigator. Stars have individual motion and many also move within a group just as Earth does within the solar group. However, because even the closest stars are such a great distance from Earth, this motion appears negligible and the pattern of the heavens changes but little over hundreds of years. The nearest star is Proxima Centauri, which is three-and-a-half light years from Earth. A light year is the distance that light travels in a year at the constant speed of light, which is about 186,000 miles per second. Therefore, one light year is about six million, million miles. Obviously, when dealing with such fantastic distances the light year is a far more expressive and easily handled unit of measure.

For purposes of navigation, these tremendous distances mean little. All stars are considered projected onto the inner surface of the celestial sphere and, therefore, of infinite and equal distance.

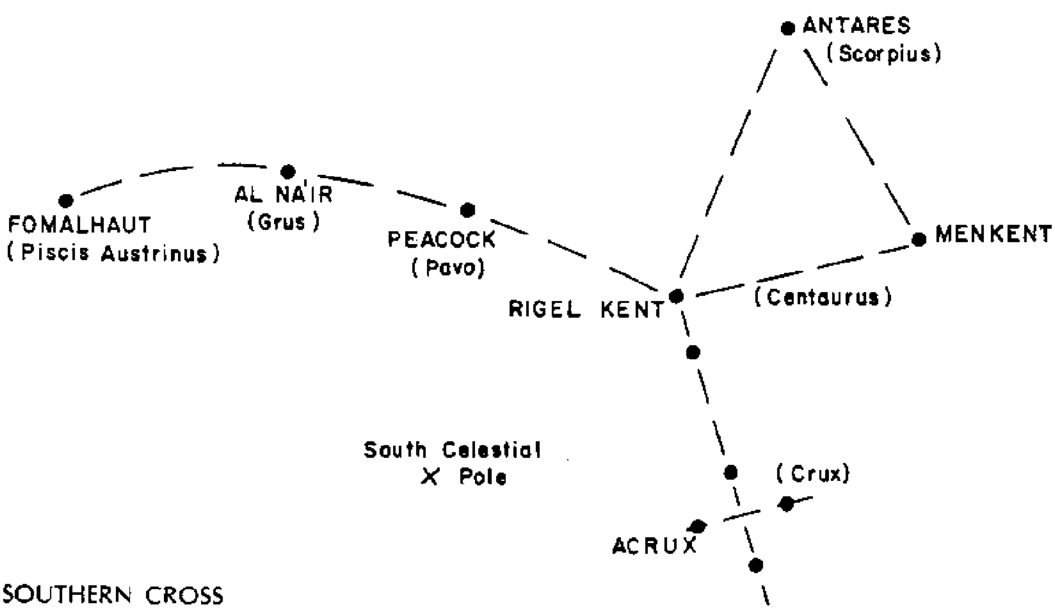
Observing the sky on a clear night, you will notice that the stars maintain the same configurations relative to each other, but the surface of the celestial sphere appears to be rotating slowly toward the west. Thus, stars to the west are approaching the horizon to eventually set, while stars to the east are climbing in the sky with other stars rising beneath them. This apparent motion is due to the Earth's rotating within the celestial sphere. Stars near the projected axis of the Earth will appear almost stationary. The star Polaris, commonly called the Pole Star, is very near the north celestial pole and remains almost fixed. All other stars will appear to describe small circles about the Pole Star.

As previously stated, the Earth turns through 360 degrees in about 23 hours and 56 minutes. Therefore, the presentation of stars on the celestial sphere will appear to rotate once in the same time. Because time on Earth is kept with a 24-hour day, stars will rise and set about four minutes earlier each day. Thus, the overall configuration of the heavens will move to the west about four minutes of time or about one degree of arc at the same clock time on succeeding nights.

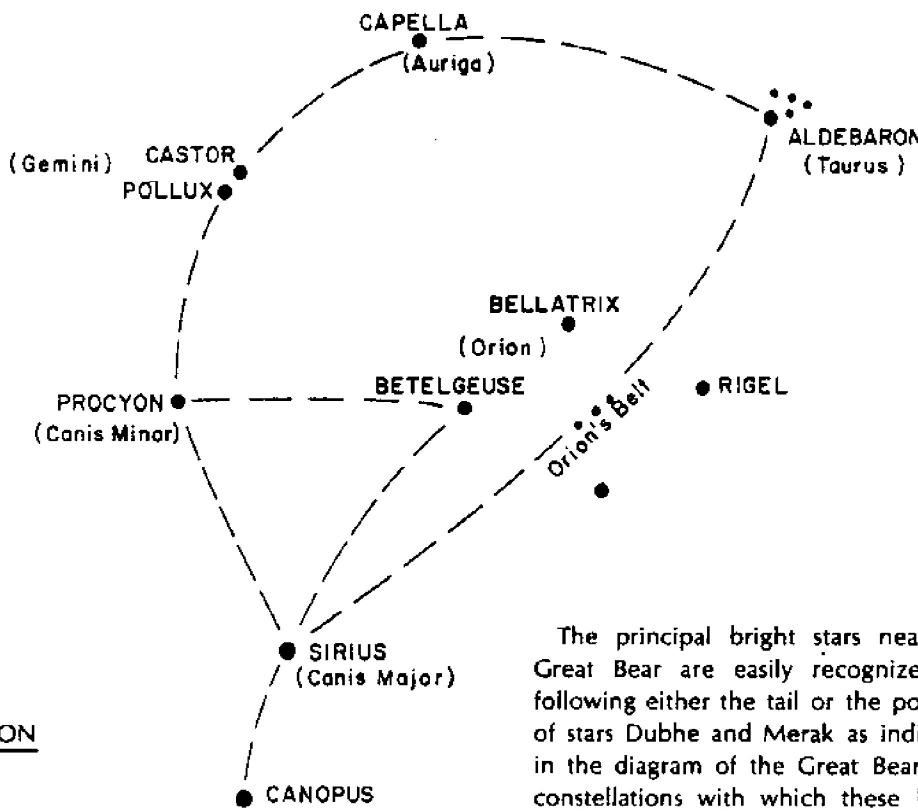
Individual stars are most easily recognized by their relative position within their group or constellation. Some of the more important stars used for navigation are shown in the following diagrams. They are shown in the position that they occupy in some easily recognized pattern with adjacent stars and constellations. The two principal constellations of the northern hemisphere are the Great Bear, or Plough, and Orion the Hunter. Most of the northern stars important to navigation can be recognized when related to these two well-known groups. Of the some 3000 stars visible to the naked eye, only about 30 are used commonly by most navigators.



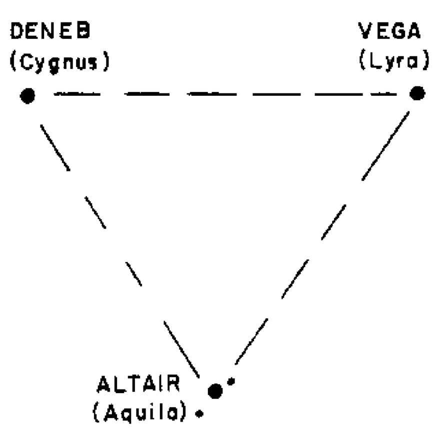
GREAT BEAR



SOUTHERN CROSS



ORION



ALTAIR, VEGA and DENEB

The principal bright stars near the Great Bear are easily recognized by following either the tail or the pointers of stars Dubhe and Merak as indicated in the diagram of the Great Bear. The constellations with which these bright stars are associated are shown in parentheses.

The stars most often used for navigation near Orion are best located by tracing the three stars that make up Orion's Belt. Go one way to locate Aldebaron, and go the other way to locate Sirius, which is the brightest star in the heavens. The other bright stars then lie along a broad arc drawn between these two stars.

Other important stars of the northern hemisphere include Altair, Vega and Deneb. These three stars are linked in an isosceles triangle pattern as shown in the diagram. Altair is easily recognized, having two small stars, one on each side of it, which line up to point toward Vega.

The principal constellation of the southern hemisphere is the well-known Southern Cross. The nearby bright stars are shown in the diagram of it.

MAGNITUDE OF THE STARS

Stars vary a great deal in size and distance from Earth and their brightness is affected by these two factors. For example, the well-known bright star Capella is some 36 light years from Earth but, because its diameter is calculated to be as large as the Earth's orbital plane about the Sun, it is one of the brightest stars in the sky.

The system of grading stars according to their apparent brightness to the observer on earth was established about 800 B.C. by Hipparchus and Ptolemy. A star just visible to the naked eye is said to be of the sixth magnitude, and a star from which the Earth receives 100 times as much light as one of the sixth magnitude is said to be of the first magnitude. Thus,

$$\frac{S^6}{S^1} = 100$$

$$\text{Therefore, } S^5 = 100$$

$$\text{Therefore, } S = \sqrt[5]{100}$$

$$S = 2.51$$

Therefore, a rise of one magnitude of star indicates a 2.5 times increase in brightness.

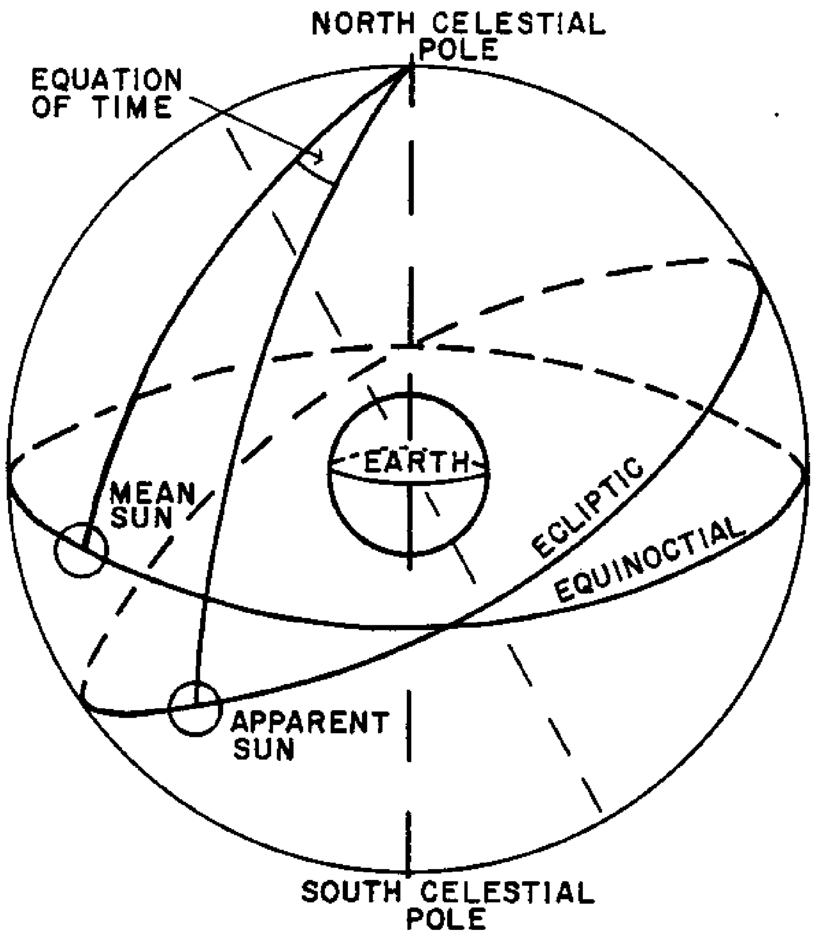
The stars Sirius and Canopus are so bright that they require negative magnitudes. In fact, Sirius has a magnitude of -1.6 and Canopus, -0.9.

If we compare Sirius -1.6 with the star Regulus 1.3, we note that there are 2.9 intervening magnitudes. Therefore Sirius would appear $(2.51)^{2.9}$, or nearly 16 times, brighter than Regulus.

The magnitudes of selected stars are listed in the *Nautical Almanac*, a publication which will be discussed later in the text.

Time

When discussing the true motion of the Earth we noted that, according to Kepler's Second Law, the Earth moves at varying speeds along its orbital path. When this motion is applied to the relative motion of the apparent Sun, its speed also must vary as it moves around the ecliptic.



On Earth we keep solar time which uses a 24-hour day based on the movement of Earth around the Sun. Obviously the essence of time is a constant base, but since the apparent Sun does not provide this, we use a theoretical Sun. This mean, or astronomical mean, Sun is conceived to move along the Equinoctial at a uniform rate and is the Sun on which our time is based.

We know that the apparent Sun will be moving faster at perihelion than at aphelion in order to sweep out equal areas of orbital plane in the same time. The mean Sun, however, moves at a constant speed. It is obvious, therefore, that at times the apparent Sun will be ahead of the mean Sun and at other times will be behind it.

The apparent Sun that we actually see and the mean Sun that we imagine are only in the same position at the times of perihelion and aphelion. In 1968 the mean and apparent Suns coincided on April 15 and September 1.

The origin of our time system is Greenwich mean time (G.M.T.). The G.M.T. day begins when the mean Sun crosses the Greenwich *midnight* meridian and progresses one hour for each 15 degrees of the mean Sun's westerly motion beyond this meridian. Greenwich noon occurs when the mean Sun reaches the Greenwich, or prime, meridian from which longitude is measured. However, we are only able to observe the apparent Sun and, therefore, require some link to establish the position of the mean Sun. This link is computed for each day and listed in the *Nautical Almanac* as the equation of time.

TIME MEASUREMENT AND THE EQUATION OF TIME

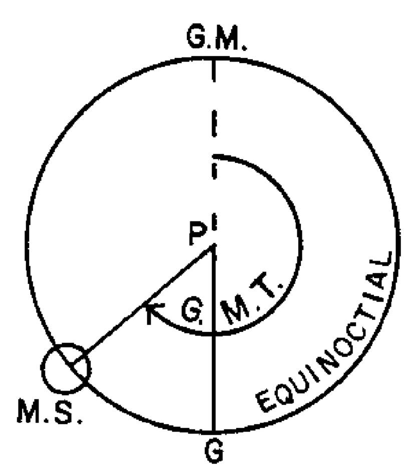
The equation of time is the excess of mean time over apparent time. The equation of time has a positive value when the mean Sun is ahead of the apparent Sun and a negative value when the mean Sun is behind the apparent Sun. When the positions of the mean and apparent Suns coincide, at perihelion and aphelion, the value of the equation of time is zero.

Values for the equation of time are listed in the *Nautical Almanac* without signs because the sign conventions of the United States and the United Kingdom differ in respect to it.

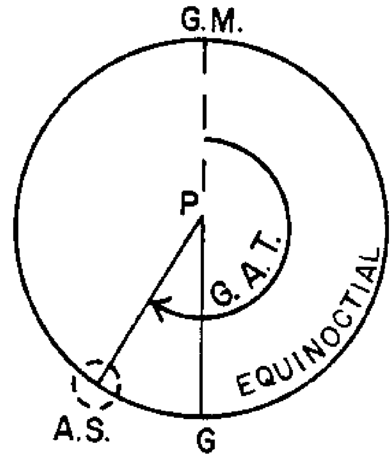
The following diagrams are in the plane of the equinoctial with the north celestial pole at the center.

G. Greenwich, or prime, meridian
G.M. Greenwich midnight, or 180 degree, meridian
O. observer's meridian
O.M. observer's lower, or midnight, meridian
M.S. mean Sun
A.S. apparent Sun

Greenwich mean time (G.M.T.) is the angle at the celestial pole subtended by the Greenwich midnight meridian and the meridian passing through the mean Sun, measured westward from the Greenwich midnight meridian from 0-24 hours. Since 24 hours equals 360 degrees, one hour equals 15 degrees.

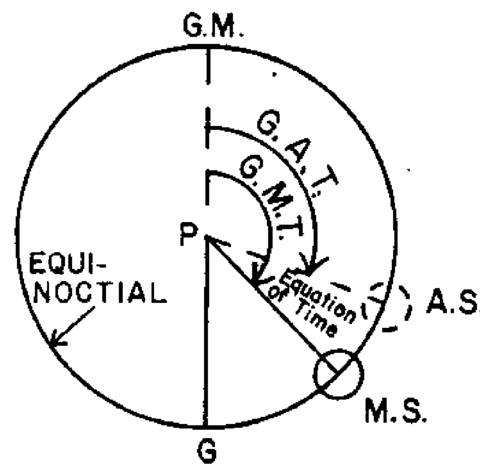


Greenwich apparent time (G.A.T.) is the angle at the celestial pole subtended by the Greenwich midnight meridian and the meridian passing through the apparent Sun, measured westward from G.M. from 0-24 hours.

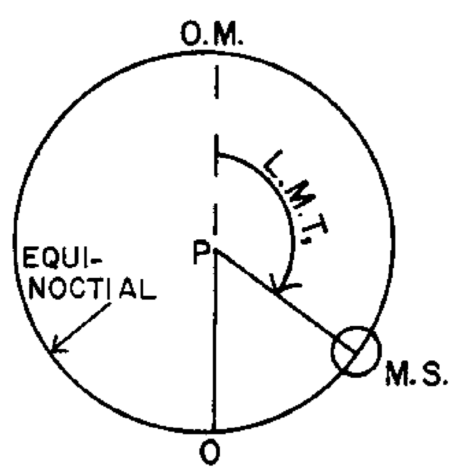


Equation of time (Eq. of time), the excess of mean over apparent time, is *positive* when the mean Sun is ahead of the apparent Sun. Thus,

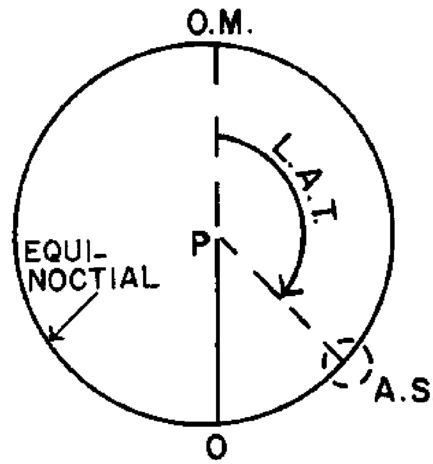
G.A.T. ± Eq. of time = G.M.T.



Local mean time (L.M.T.) is the angle at the celestial pole subtended by the observer's midnight meridian and the meridian passing through the mean Sun, measured westward from the observer's midnight meridian from 0-24 hours.

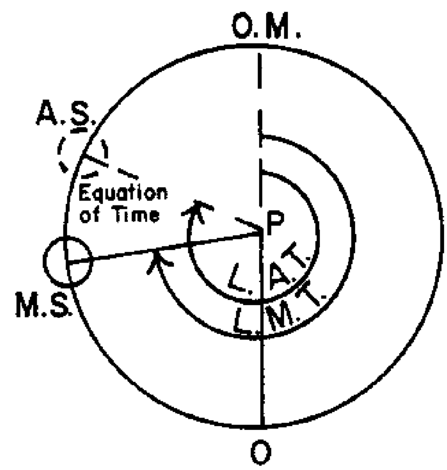


Local apparent time (L.A.T.) is the angle at the celestial pole subtended by the observer's midnight meridian and the meridian passing through the apparent Sun, measured westward from the observer's midnight meridian from 0-24 hours.



Equation of time, the excess of mean over apparent time, is *negative* when the mean Sun is behind the apparent Sun.

L.A.T. ± Eq. of time = L.M.T.



THE CIVIL CALENDAR

The Mean Solar Day

The mean solar day is the time taken for two successive transits of a stationary observer's meridian with the mean Sun. This results in a constant 24-hour day which is the mean of all the apparent solar days of the year.

The Sidereal Day

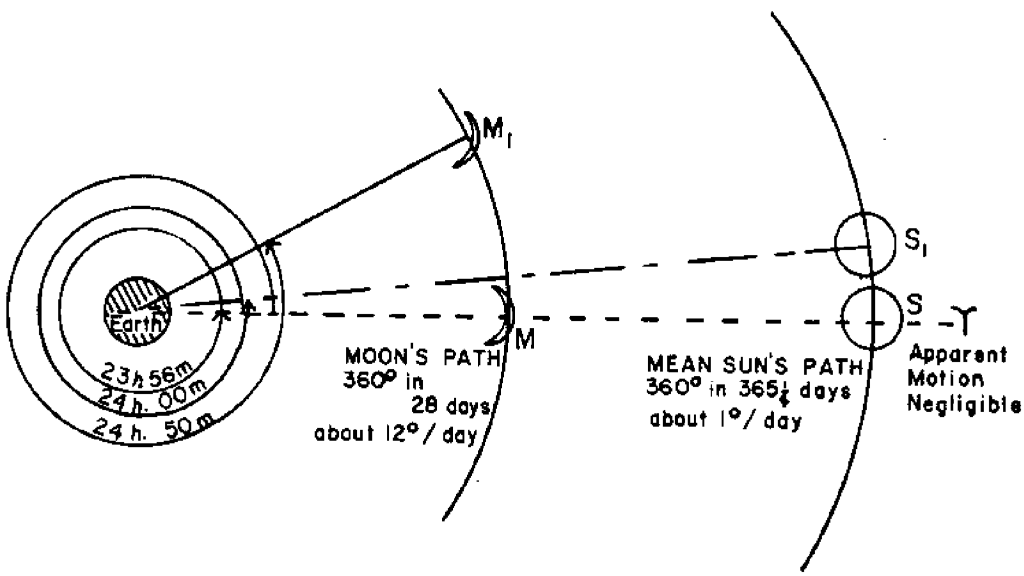
The sidereal day is the time taken for two successive transits of a stationary observer's meridian with Aries or any distant star. It is the time taken for a given meridian to turn through 360 degrees and is a constant 23 hours 56 minutes 04 seconds.

The Lunar Day

The lunar day is the time taken for two successive transits of a stationary observer's meridian with the Moon. While the Earth rotates once, the Moon moves about 12 degrees along its orbital path. Taking this motion into consideration, we find a lunar day of about 24 hours and 50 minutes.

The Calendar

Our calendar system was devised by Pope Gregory in about 1600. The Earth takes exactly 365.2422 days to orbit the Sun. This necessitates a calendar year of 365 days with an extra day added to give a 366-day leap year every fourth year when the last two figures of the year are divisible by four. For example, 1972 was a leap year. This brings the resultant year to 365¼ days. To further refine this, there is no leap year at the turn of a century unless the first two figures are divisible by four. For example, the year 1900 was not a leap year, but the year 2000 will be.



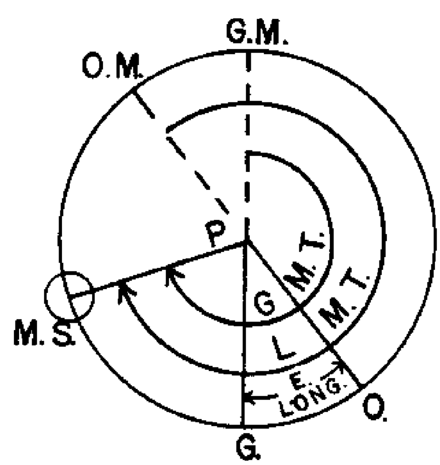
LONGITUDE AND TIME

The following diagrams in the plane of the equinoctial illustrate the longitude relationship between local and Greenwich time.

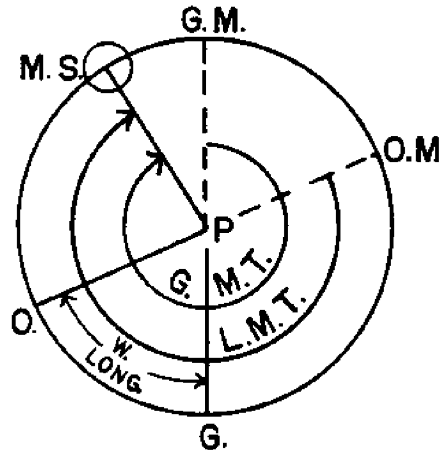
G.M.T. + Long. East = L.M.T.

When converting longitude to time remember:

15° = 1 hr. Therefore, 15′ = 1 min.
1° = 4 min. Therefore, 1′ = 4 sec.

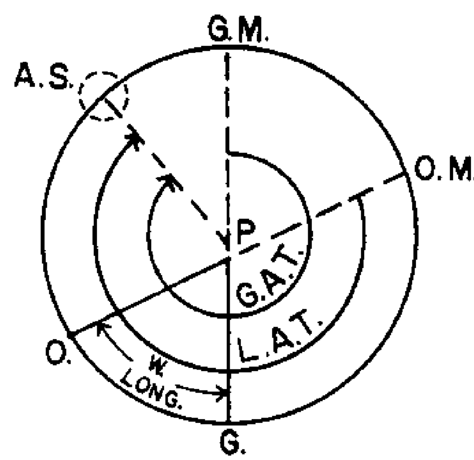


G.M.T. - Long. West = L.M.T.

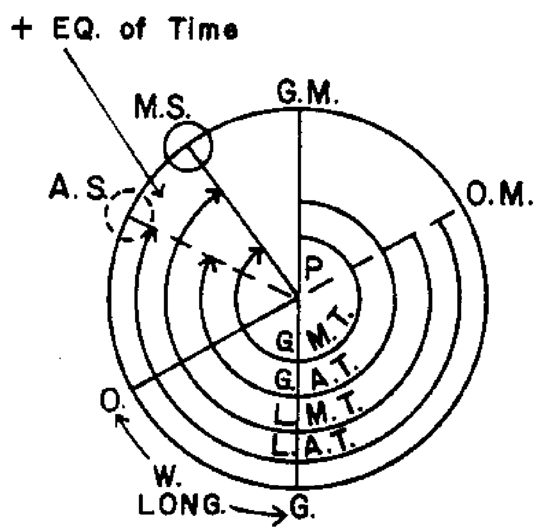


An easily remembered rule which establishes the longitude relationship of Greenwich to local time is:

Longitude west Greenwich time best; longitude east Greenwich time least.

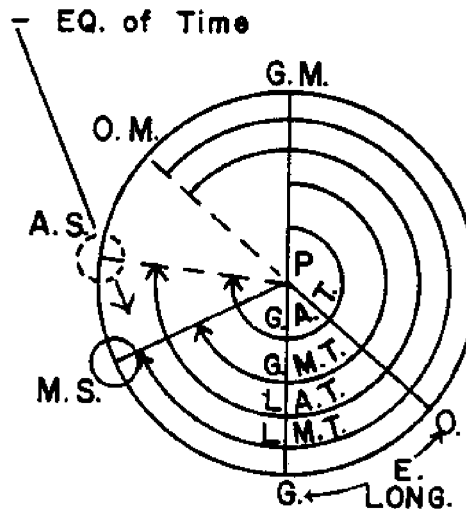


G.A.T. - Long. West = L.A.T.



L.A.T. + Eq. Time + W. Long. = G.M.T.
G.A.T. + Eq. Time - W. Long. = L.M.T.

L.A.T. -Eq. Time -E. Long. = G.M.T.
G.A.T. -Eq. Time + E. Long. = . L.M.T.



ZONE TIME

In order to keep the Sun somewhere near the meridian at local noon time, it is necessary to lag the time of noon behind 1200 at Greenwich in westerly longitudes and advance the time of noon ahead of 1200 at Greenwich in easterly longitudes.

Because 15 degrees of longitude represent one hour of time, all longitudes within seven and one-half degrees east and west come within the Greenwich Zone. Between seven and one-half degrees and twenty-two and one-half degrees west longitude, a zone time of *plus* one hour is in effect, so that one hour is to be added to local time to obtain Greenwich time. Conversely, between the longitudes of seven and one-half degrees and twenty-two and one-half degrees east a zone time of *minus* one hour exists, so that one hour is to be subtracted from local time to obtain Greenwich time.

For each successive 15 degrees west, the zone time is an additional hour behind Greenwich time and for each successive 15 degrees east the zone time is one additional hour ahead of Greenwich time. Clearly then a vessel approaching the 180-degree meridian going west would be keeping time 12 hours behind Greenwich time, while a vessel approaching the same meridian going east would be keeping time 12 hours ahead of Greenwich. To account for the 24-hour time difference, the international date line has been established in the vicinity of the 180-degree meridian and the vessel's calendar gains a day going eastward and loses a day going westward.

Some countries are vast enough to contain many time zones; for instance, Russia has 11. India simplifies matters by keeping a mean zone time of minus five and one-half hours.

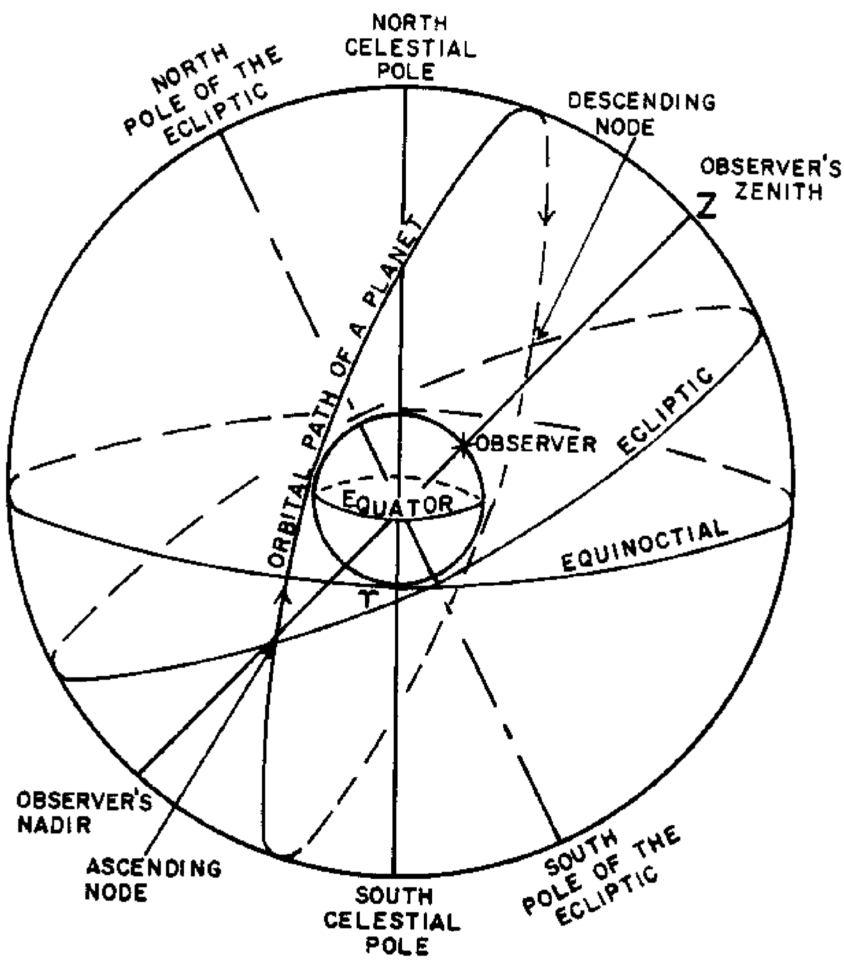
Exercise 1. TIME

A diagram in the plane of the equinoctial should accompany each calculation. Remember that (1) the equation of time is the excess of mean time over apparent time and (2) if longitude west, Greenwich time is best.

1. Given G.M.T. 0530, state the L.M.T. of an observer in longitude 45°00′ W.
2. Given L.M.T. 1624, state the G.M.T. of an observer in longitude 73°00′ E.
3. Given G.A.T. 0220 and eq. of time -3m12s, state G.M.T.
4. Given L.M.T. 2317 and eq. of time +6m43s, state L.A.T.
5. Given G.A.T. 11h27m24s and longitude of observer 23°44′ E, find L.A.T.
6. If L.A.T. is 13h02m20s and eq. of time is -7m49s, find L.M.T.
7. An observer in longitude 78°12′ W has L.A.T. 1622. If the eq. of time is +3m18s find the G.M.T.
8. If the G.M.T. of an observer in longitude 23°12′ E is 0729 and the eq. of time is -6m21s, find the L.A.T.
9. If an observer in longitude 69°18′ E has an L.A.T. of 0214 on July 20, calculate the G.M.T. if the equation of time is +3m12s.
10. An observer in longitude 123°12′ W has G.A.T. 0523 on December 22. Calculate the L.M.T. of the observer if the eq. of time is -3m40s.

Celestial Sphere

CELESTIAL SPHERE DEFINITIONS



Observer's Zenith and Nadir

The point where a straight line drawn from the center of the Earth through the observer's position on Earth meets the celestial sphere is called the observer's zenith. The point on the celestial sphere directly opposite the observer's zenith is called the observer's nadir. The great circle on the celestial sphere, whose plane is perpendicular to the line joining the observer's zenith and nadir, is known as the observer's celestial, or rational, horizon. The importance of the rational horizon will become evident in studying the section on sextant altitude correction.

Geographical Position

This is the point on the earth's surface cut by a straight line joining a particular body to the center of the earth. The position on Earth directly beneath the Sun is known as a sub-solar point and the position on Earth directly beneath a star is termed a sub-stellar point.

Nodes

When the path of a planet is tracked out on the celestial sphere it will cut the ecliptic in two places. The point of intersection of the planet's orbital path, going from south to north, is the ascending node, while the point of intersection, going from north to south, is the descending node.

Celestial Poles

The points where the earth's axis, when projected, cuts the celestial sphere are known as the north and south celestial poles. Semi-great circles which pass through the celestial poles and correspond with the terrestrial meridians are called celestial meridians.

ROLE OF THE NAUTICAL ALMANAC

In order to calculate a position from observations of celestial bodies, it is first necessary to know the exact position of those bodies on the celestial sphere at the instant required.

The *Nautical Almanac* tabulates the precomputed positions of the Moon, Sun, planets and principal stars on the celestial sphere for each hour G.M.T. of the year. The almanac also contains a table of increments so that the position of any of the bodies may be obtained for any particular second of the year.

Certain corrections and simple calculations reduce the angular distance of a body above the observer's horizon, as obtained by sextant observation, to a position circle on the celestial sphere. This celestial position circle is centered on the body observed; the observer's earth location, when projected onto the celestial sphere, will be somewhere along this circle. The intersection of two such celestial position circles will provide the projected position of the observer on the celestial sphere, i.e. the observer's zenith.

The latitude of a body on the celestial sphere, or angular distance north or south of the equinoctial, is known as the declination of that body. Greenwich hour angle (G.H.A.) is used instead of longitude to position a body on the celestial sphere. The G.H.A. is the angular distance of a body west of the Greenwich or prime meridian.

The *Nautical Almanac* provides the declination and G.H.A. for all the heavenly bodies normally used for navigational purposes for each hour of the year. Because the motion of the stars relative to each other appears negligible to us on Earth, it is only necessary to catalog the G.H.A. of one star for each hour. The star used is Aries; other selected stars are referred to Aries by their sidereal hour angle (S.H.A.). The S.H.A. of a star is its angular distance west of Aries. The S.H.A. and declination of the stars are listed on every other page of the *Almanac*, which means, in fact, at six-day intervals. Very little change will be seen in the S.H.A. or declination of any star from week to week.

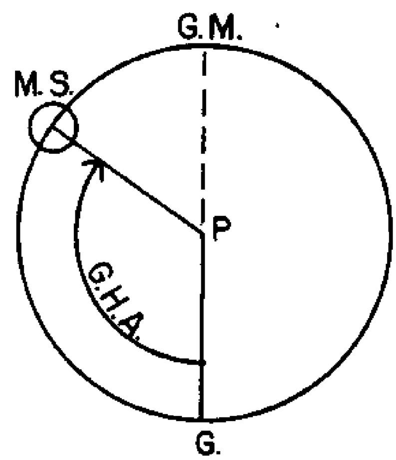
HOUR ANGLES

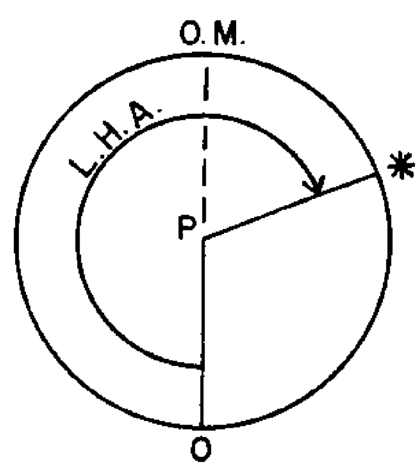
The following diagrams are in the plane of the equinoctial with the north celestial pole at the center.

G. Greenwich, or prime, meridian
G.M. Greenwich lower, or midnight, meridian
O. observer's meridian
O.M. observer's lower meridian
M.S. mean sun
* star
♈ Aries

Greenwich hour angle (G.H.A.) of a body is the angle at the celestial pole subtended by the Greenwich meridian and the meridian which passes through the body concerned, measured westward from Greenwich from 0-360 degrees. Note that

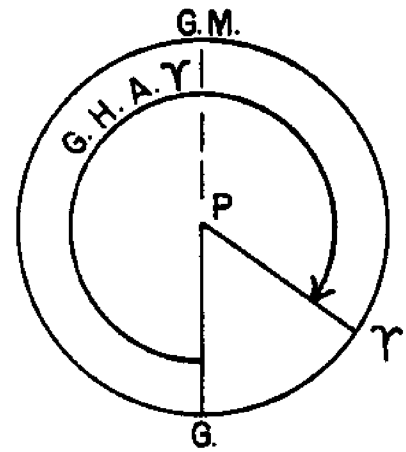
G.H.A. of mean Sun ± 12 hr. = G.M.T.





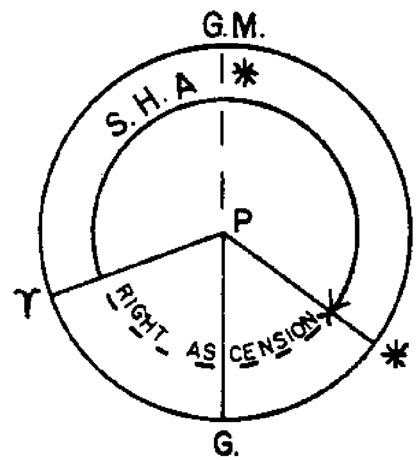
Local hour angle (L.H.A.) of a body is the angle at the celestial pole subtended by the observer's meridian and the meridian which passes through the body, measured westward from observer 0-360 degrees.

Greenwich hour angle of Aries (G.H.A.♈) is the angle at the celestial pole between the Greenwich meridian and the meridian which passes through Aries, measured westward from Greenwich from 0-360 degrees.



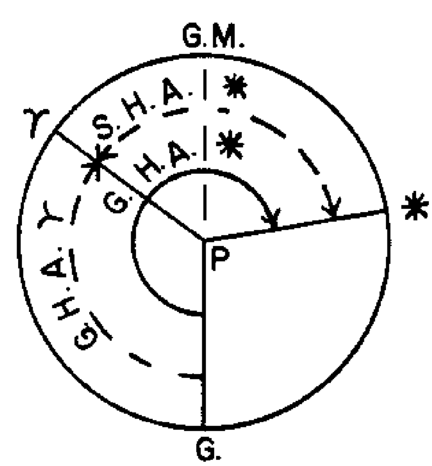
Sidereal hour angle of a star (S.H.A.*) is the angle at the celestial pole between the meridian which passes through Aries and the meridian which passes through the star, measured westward from Aries from 0-360 degrees.

Right ascension of a star is the angle at the celestial pole between Aries and the star, measured eastward from Aries from 0-24 hours.

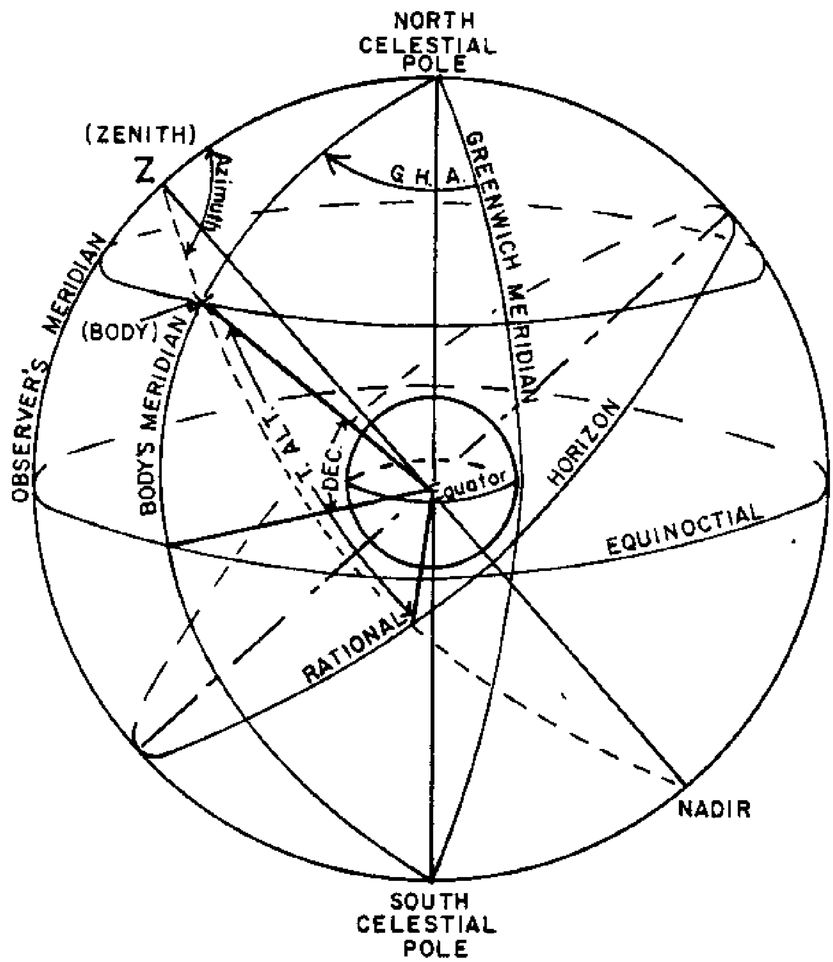


Greenwich hour angle of a star (G.H.A.*) is the angle at the celestial pole between the Greenwich meridian and the meridian which passes through the star, measured westward from Greenwich from 0-360 degrees.

G.H.A.* = G.H.A.♈ + S.H.A.*



LOCATING A BODY ON THE CELESTIAL SPHERE



The position of a body at any instant can be found by extracting its declination and G.H.A. for that particular moment of time from the *Nautical Almanac.* The above diagram illustrates how this system fixes a certain body *X* on the celestial sphere. Clearly the G.H.A. is the angle at the celestial pole subtended by the Greenwich meridian and the meridian which passes through the body, while declination is the angle at the center of the earth subtended by the equinoctial and the body.

A great circle on the celestial sphere which passes through the observer's zenith and nadir is known as a vertical circle. In particular, the vertical circle that passes through the east and west points of the observer's rational horizon is known as the prime vertical. The corrected true altitude of a body above the observer's rational horizon is the angular distance between the rational horizon and the body measured along the vertical circle through the body. The arc of that same vertical circle contained between *Z* and *X* in the diagram is known as the zenith distance of the body; clearly the sum of the true altitude and zenith distance of a body is 90 degrees.

A body may also be positioned on the celestial sphere by its true bearing, or azimuth, from the observer's zenith plus its zenith distance. From the diagram it can be seen that the azimuth of a body is defined as the angle at the observer's zenith contained between the observer's true meridian and the vertical circle which passes through the body. Obviously, if the exact position of a body can be established from the *Nautical Almanac*, then the position of the observer's zenith can be found by laying back the azimuth and zenith distance from the body.

Note that when a body bears due north or south from the observer, the observer's meridian becomes a vertical circle.

Exercise 2 HOUR ANGLES

1. If the G.H.A. of star Sirius was 195°27′, what would be the L.H.A. of that star to an observer in 57°13′ W?

2. State the G.H.A. of star Procyon if its L.H.A. was 284°18′ to an observer in 113°18′ W.

3. What is the L.H.A. of star Arcturus if its G.H.A. is 12°57′ and the observer is in 18°22′ E longitude?

4. Find the G.H.A. of the Sun if its L.H.A. was 36°42′ to an observer in 57°38′ E.

5. If the G.H.A. of star Canopus was 342°18′ and its L.H.A. was 297°42′, what is the observer's longitude?

6. If the L.H.A. of the Sun is 357°22′ and its G.H.A. 18°16′, what is the observer's longitude?

7. What is the right ascension of star Capella if its S.H.A. is 127°15′?

8. If the G.H.A. of Aries is 117°52′ and the S.H.A. of star Spica 206°4′, what is the G.H.A. of Spica?

9. What is the L.H.A. of star Vega to an observer in longitude 42°18′ W when G.H.A. of Aries is 217°8′ and S.H.A. of Vega is 87°42′?

10. Find the S.H.A. of star Rigel if its L.H.A. was 182°15′ to an observer in 169°18′ E when G.H.A. of Aries was 342°17′.

Instruments and Sextant Angles

THE CHRONOMETER

It is a relatively easy procedure to determine latitude by observation of the Sun when it crosses the observer's meridian or by determining the sextant altitude of the Pole Star. Both of these methods will be examined later in the text. But it has only been in the last 200 years that sufficiently accurate and durable time pieces have been available to facilitate the calculation of longitude at sea.

And longitude determination is really a matter of accurate timekeeping. For example, an observer had the Sun overhead at noon in London, and then sailed west for a number of days with one clock set on London time. When an observation of the Sun as it crossed the observer's meridian indicated that there was exactly one hour's difference between the ship's time and London time he had altered his position 15 degrees.

Recognizing the importance of accurate timekeeping a Board of Longitude was instituted in England in 1714 with a prize of 20,000 pounds offered for solving the longitude problem. John Harrison, a Lincolnshire carpenter, devoted his entire lifetime to producing a chronometer to meet the board's requirements. In 1761, when Harrison was 68, his fourth version of the chronometer easily met all the accuracies demanded by the Board. On a voyage from England to Jamaica on the ship *Deptford*, the chronometer was only five seconds in error after a two-month time span. This wonderful achievement by an uneducated carpenter confounded most scientists of the day, but was marred by the fact that the prize money was not awarded until Harrison was 80-years-old. All of Harrison's original chronometers are still in good working order and can be seen at Britain's National Maritime Museum.

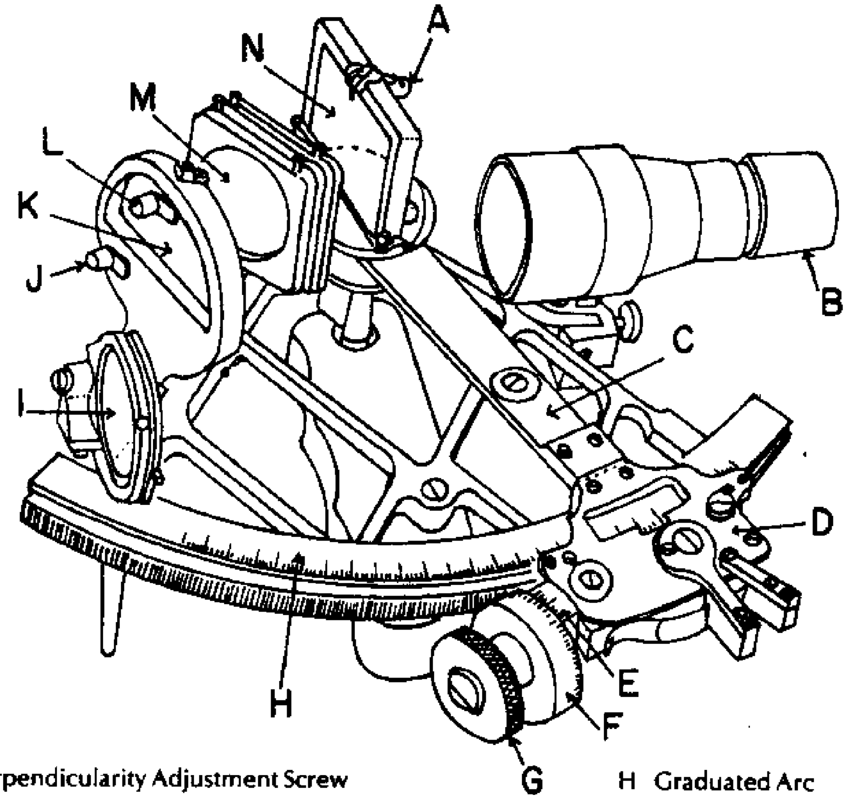
Today's chronometers are little advanced from Harrison's, but radio time checks allow constant daily checks on their accuracy. If the daily rate of time loss or gain is constant, then the accumulated error of the chronometer can be reliably computed by multiplying the daily rate by the number of days since the last radio time check was obtained. Chronometers are slung in gimbals and placed in well padded boxes to protect them against vibration, temperature changes and dampness. Temperature change is compensated for in the balance wheel of the chronometer. It is good practice to wind a chronometer at the same time each day in order to use the same part of the spring and also help maintain a steady "daily rate."

The chronometer is set on G.M.T., while the ship's clock is altered to coincide with zone time or some local time determined by the meridian the ship is on at noon.

The importance of the accuracy of the chronometer can be realized when making the time-to-longitude comparison. Twenty-four hours of time represents 360 degrees of longitude; thus, one hour of time represents 15 degrees of longitude and one minute of time represents 15 minutes of longitude. Therefore, a four-second error in chronometer time results in a one-minute error in longitude.

THE MARINE SEXTANT

The principal tool of trade of the "deep sea" navigator is the marine sextant. The sextant's main purpose is to measure the altitude of heavenly bodies above the visible horizon at sea in order to compute the vessel's position.

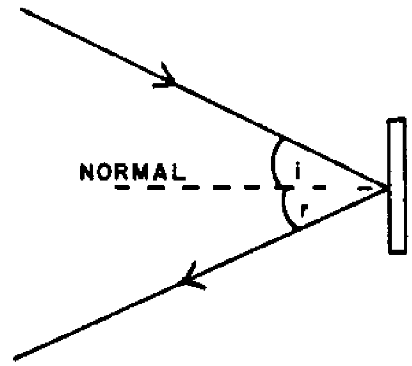


A Perpendicularity Adjustment Screw
B Telescope
C Index Bar
D Arc Clamp
E Vernier
F Micrometer
G Adjusting Wheel
H Graduated Arc
I Shades
J Parallelism Adjusting Screw
K Horizon Glass
L Side Error Adjustment Screw
M Shades
N Index Mirror

The sextant consists of a framework bearing a radial index bar. One end of the bar moves along a graduated arc and the other end pivots about the center of curvature of the arc. The telescope is in line with the horizon glass, which is in two halves. The half nearest the plane of the instrument is a plane mirror which shows the double reflection of an object from the index mirror. The other half is clear to allow the observer to look directly at an object. Thus, the reflected image of one object can be brought into line with another object by moving the index bar along the arc.

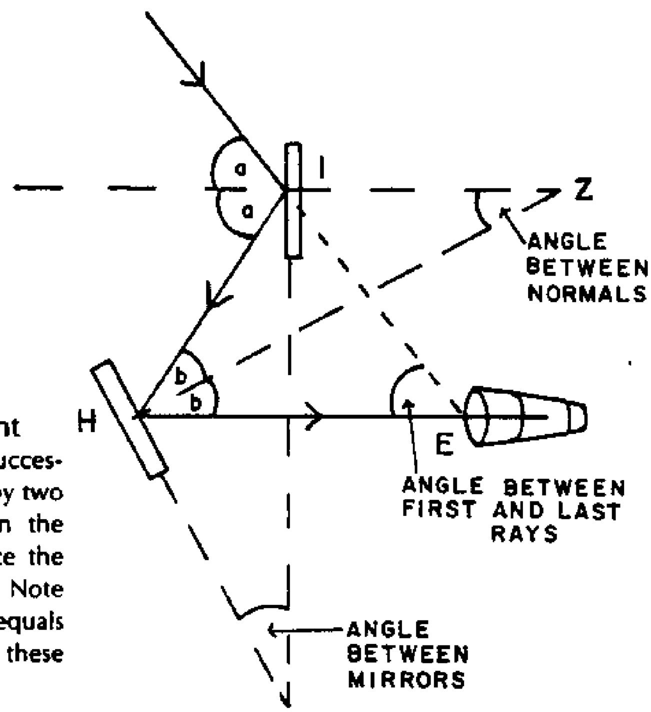
The sextant arc is about one-sixth of a circle but because the process of double reflection will result in a measured angle of only half the size of the true angle, the arc is graduated to about 120 degrees. This second principle of the sextant is explained in the diagram below.

A micrometer allows readings at an accuracy within one minute of arc, and a small vernier attached to the micrometer facilitates readings down to ten seconds of arc.



First Principle of the Sextant

When a ray of light strikes a plane mirror, the angle of incidence is always equal to the angle of reflection.



Second Principle of the Sextant

When a ray of light suffers two successive reflections in the same plane by two plane mirrors, the angle between the first and last rays is equal to twice the angle between the two mirrors. Note that the angle between the mirrors equals the angle between the normals to these mirrors (angle *Z*).

Proof

In triangle *HIZ*, exterior angle $a = 2$ interior opposite angles, $b + Z$ 1.

In triangle *HIE*, exterior angle $2a = 2b + E$ 2.

Multiply equation 1 by 2. Then $2a = 2b + 2Z$

Therefore, $\cancel{2b} + E = \cancel{2b} + 2Z$

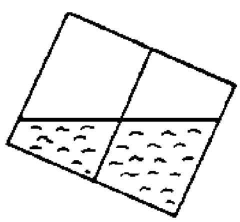
That is, the angle between first and last rays (*E*) equals twice the angle between the mirrors (*Z*).

Errors and Adjustments of the Sextant

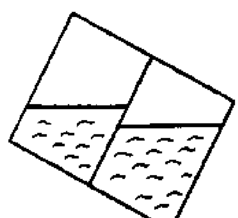
There are three main errors which are likely to exist in a sextant. These can be corrected by turning the appropriate adjustment screw.

Perpendicularity. The first error of the sextant is caused by the index glass not being truly perpendicular to the plane of the instrument. This error can be recognized by the following procedure. The observer holds the sextant horizontally at arm's length. With the arc away from the observer and set at about 35 degrees, he looks down into the index glass at a fine angle. If the reflection of the arc does not coincide with the arc itself, the error of perpendicularity exists. This error is corrected by turning the first adjustment screw on the back of the index mirror until the arc and its reflection do coincide.

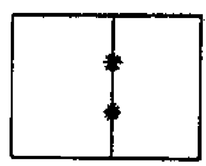
Side Error. The second error of the sextant, which is known as side error, is due to the horizon glass not being truly perpendicular to the plane of the sextant. This error is found by holding the sextant obliquely with the arc at zero and observing the true and reflected images of a clear horizon. If the object and its image are not in a continuous line side error exists. This error can also be found by rotating the micrometer screw back and forth each side of zero while looking at a star. If the reflected star does not pass directly over the true star, then side error exists. This error is corrected by turning the second adjustment screw on the back of the horizon glass until coincidence is effected.



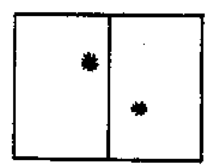
NO ERROR



SIDE ERROR

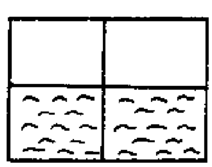


NO ERROR

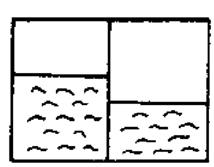


SIDE ERROR

Error of Parallelism. The third adjustable error of the sextant, the error of parallelism, is caused by the index mirror and horizon glass not being truly parallel when the arc is set at zero. This error is discovered by setting the arc at zero and observing a clear horizon or a star which is not too bright with the sextant held vertically. If the true object and its reflected image do not coincide, then the error of parallelism exists. This error can be corrected by turning the third adjustment screw, which is located on the back of the horizon glass nearest to the plane of the instrument.



NO ERROR



ERROR OF PARALLELISM

Index Error. Side error and the error of parallelism are interrelated in that the correction of one error may induce the other. Adjustment for these two errors should be made alternately a number of times. Any error of parallelism remaining which cannot be removed without inducing side error is called index error. The index error must then be applied to every angle that is taken. When a larger arc reading than the true angle results, the index error is subtracted and termed *on the arc*. When the sextant gives a smaller angle than the true angle, then obviously the index error is to be added and is termed so many minutes *off the arc*. Index error is often zero and usually no more than two or three minutes plus or minus.

Four Unadjustable Errors. There are four errors to the sextant that are not adjustable and which can only be corrected by the sextant manufacturer.

Collimation error exists when the axis of the telescope is not exactly parallel to the plane of the sextant. This error causes the measured altitude to be greater than the real altitude.

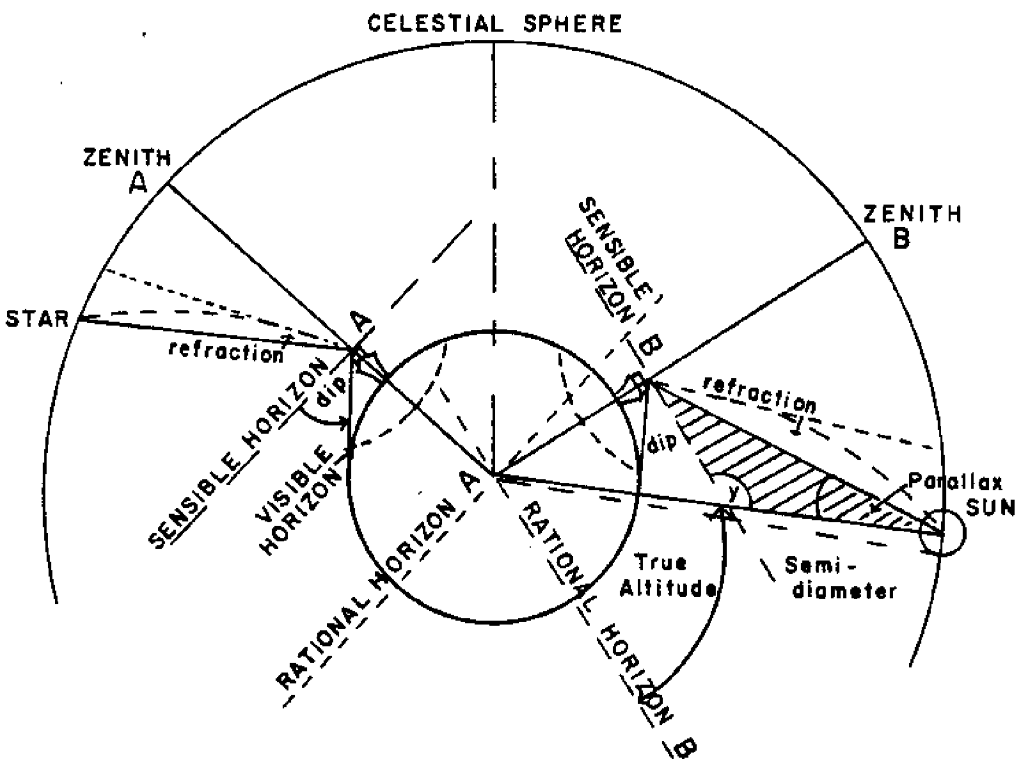
Graduation error exists when the arc, micrometer or vernier are incorrectly calibrated.

Shade error is caused by the faces of shade glasses not being ground parallel. This error is found by comparing the angle between two objects one time with the shade up and another with the shade down.

Centering error exists when the index arm is not pivoted at the arc's true center of curvature.

SEXTANT ALTITUDE CORRECTION

In order to calculate the observer's position, the sextant altitude of a body above the visible horizon at sea must have certain corrections applied to it in order for it to give the altitude of the body above the observer's rational horizon.



Observer's Visible Horizon

The visible horizon is that bounding the observer's view at sea. The visible horizon of an observer with a height of eye (H.E.) 30 feet above sea level would be at a distance of only about six and one-half miles under normal atmospheric conditions.

The Sensible Horizon

The plane of the sensible horizon passes through the observer's eye and is at right angles to the vertical.

The Rational Horizon

The observer's rational horizon is a great circle, the plane of which is parallel to the sensible horizon and, therefore, at right angles to a line from the Earth's center to the observer's zenith.

Sextant Altitude

The altitude of a body, as observed by sextant, is the angle at the observer between his visible horizon and the body or a limb of the body.

Observed Altitude

The observed altitude of a body is the sextant altitude corrected for any index error which may be present in the sextant.

Dip

The angle of depression of the visible horizon below the sensible horizon is known as dip. Clearly the angle of dip will increase depending on the height of the observer's eye above sea level. A dip table giving values of dip in minutes for the observer's H.E. is contained on the inside cover of the *Nautical Almanac.*

Apparent Altitude

The apparent altitude of a body is the observed altitude corrected for dip. Note that dip will always be *subtracted* from the observed altitude to give the apparent altitude.

Refraction

Rays of light from a body are bent toward the Earth as they pass through layers of varying density in the atmosphere. This tends to make the body appear higher than it actually is; therefore, the correction for refraction is always *subtracted* from the apparent altitude. The correction for refraction diminishes with increased altitude. Values are given, in a correction table for stars and planets, inside the front cover of the *Nautical Almanac.* Altitudes of a body less than about 10 degrees are generally unreliable due to severe refraction.

True Altitude of a Star

The true altitude of a star is the apparent altitude corrected for refraction. The true altitude of any body is, in fact, the angle at the center of the Earth between the observer's rational horizon and the center of the body. In the case of stars, it is only necessary to apply the two corrections of dip and refraction to the observed altitude to obtain the true altitude. With the Sun it is necessary to apply two additional corrections for parallax (see below) and semi-diameter in order to obtain the true altitude.

Parallax

Parallax is the angle at the celestial body subtended by the observer and the Earth's center. The value of parallex becomes smaller the farther away from Earth the body is. In the case of stars, parallax is negligible, but for the Moon it becomes as large as one degree. Parallax decreases with altitude and is greatest in value when the body is on the observer's rational horizon. It reduces to zero at a maximum altitude of 90 degrees. The value of parallax at zero altitude is known as horizontal parallax. For the Sun this is about 15 seconds. Intermediate values of parallax can be found by multiplying the horizontal parallax by the cosine of the altitude.

In the diagram of the celestial sphere, for the shaded triangle *By Sun:*

The exterior angle at y = true altitude

Therefore, True Altitude = angle *B* + angle *Sun* (exterior angle of a triangle equals two interior opposite angles)

Therefore, True Altitude = Apparent Altitude (corrected for refraction) + Parallax

Note that the parallax correction will always be *added* to the apparent altitude.

Semi-diameter

For accuracy and convenience, one measures the altitude of the lower limb of the Sun or Moon rather than attempting to estimate the center of the body. Thus, the semi-diameter must be allowed for in order to give the additional arc to the center of the body. Occasionally, it may be necessary to use the upper limb of a body, in which case the measured angle would be too large and the semi-diameter would be a *negative* correction.

True Altitude of the Sun

The true altitude of the Sun is the angle at the center of the Earth subtended by the observer's rational horizon and the center of the Sun. The apparent altitude of the Sun when corrected for refraction, parallax and semi-diameter will give the true altitude. These three corrections are combined in a total correction table found on the inside front cover of the *Nautical Almanac.*

True Altitude + Zenith Distance = 90°.

1. Sextant Altitude Star	1. Sextant Altitude Sun's Lower Limb	
Index Error ±	Index Error ±	
2. Observed Altitude	2. Observed Altitude	
Dip -	Dip -	
3. Apparent Altitude	3. Apparent Altitude	
Refraction -	Refraction - Parallax + Semi-Diameter +	Total Correction
True Altitude Star	True Altitude Sun	

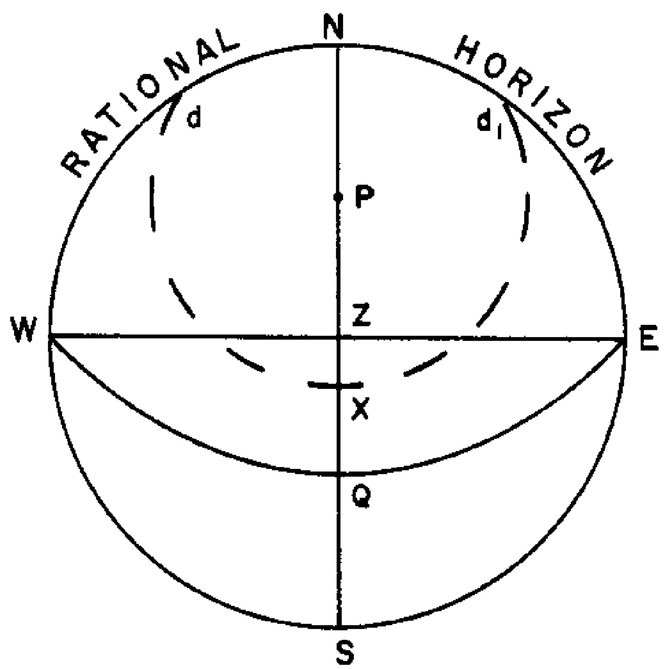
Exercise 3. ALTITUDE CORRECTION

Use the altitude correction tables inside the front cover of the *Nautical Almanac*.

1. Find the true altitude of the Pole Star if its sextant altitude was 27°58'.3 to an observer with a height of eye (H.E.) of 24 feet and the index error (I.E.) was +6'.2.
2. If the sextant altitude of star Rigel was 61°12'.2 to an observer with H.E. 27.3 feet and I.E. was 2'.3 on the arc, calculate the true altitude.
3. Calculate the true altitude of a star with sextant altitude 19°15'.2 if the sextant I.E. was 3'.1 off the arc and observer's H.E. was 29 feet.
4. Find the true altitude of Arcturus if its sextant altitude was 81°15' to an observer with H.E. 32.6 feet and I.E. was 2'.6 off the arc.
5. Find the true altitude of the Sun on March 26, to an observer with H.E. 25.2 feet and I.E. 2'.3 on the arc, if the sextant altitude of $\underline{\odot}$ was 26°31'.5.
6. If the sextant altitude of $\underline{\odot}$ was 63°24'.1 to an observer with H.E. 37 feet and I.E. -2'.1 on January 3, find the true altitude of the Sun.
7. If the sextant altitude of $\underline{\odot}$ was 29°54'.3 to an observer with H.E. 26.1 feet and I.E. was 0'.8 on the arc on June 19, calculate the true altitude of the Sun.
8. The observed altitude of $\overline{\odot}$ was 32°12'.1 to an observer with H.E. 22 feet on January 18. Find the Sun's true altitude.
9. If the sextant altitude of $\underline{\odot}$ was 49°11'.2 to an observer with H.E. 21.6 feet and I.E. was +1'.6, calculate the true altitude if the date was December 12.
10. Find the true altitude of the Sun if the sextant altitude of $\overline{\odot}$ was 36°18' to an observer with H.E. 32 feet and I.E. was 2'.2 on the arc on May 2. State the true zenith distance of the Sun.

PART 2. Practical Navigation

Latitude by Meridian Altitude



QX	declination
ZX	zenith distance
QZ	latitude
SX	true altitude
P	pole
Q	equator
Z	observer's zenith
X	body

As the Earth rotates on its axis each day, any given meridian will come into line with the Sun, Moon, and various stars and planets. It will appear to an observer that these bodies are crossing his meridian from east to west. The apparent path of a body is indicated in the diagram by the dotted circumpolar line dd_1. This is easily drawn with its center at the pole and its radius equal to the complement of the declination (co. dec.). If the exact time of culmination (meridianal passage) of a body is known, then its declination (dec.) can be extracted from the *Nautical Almanac*, and thus the body can be located on the observer's meridian relative to the equator.

A true altitude of a body, taken at time of meridian passage, and subtracted from 90 degrees will give the angular zenith distance of the observer's zenith from the body. Thus a combination of zenith distance and declination will give the observer's latitude. Four examples of latitude by meridian altitude follow. *Note that the diagrams are drawn in the plane of the observer's rational horizon with the observer's zenith at the center.* This simple method of latitude determination is commonly used at local apparent noon when the Sun reaches its zenith, crossing the observer's meridian either to his north or to his south.

When a body is on the observer's meridian, its local hour angle (L.H.A.) is zero. The Greenwich hour angle (G.H.A.) of the body can be found by applying the observer's longitude to the zero L.H.A., and the exact time of meridianal passage can then be calculated easily by extracting the Greenwich Mean Time (G.M.T.) that matches this G.H.A. in the *Nautical Almanac.*

The G.M.T. of meridian passage at Greenwich for Sun, Moon, Aries, and the planets is given to the nearest minute at the bottom of each page in the *Nautical Almanac*. The observer's longitude in time is applied to this figure to give the approximate Greenwich time of local meridian passage of the body concerned. Latitude by meridian altitude is used less often for stars because the time of meridian altitude must then coincide with the few minutes of twilight time that both the star and a clear horizon can be seen.

Example 1

Given true meridian altitude of Sun 35°20′ bearing south with declination 20°10′ N., calculate the observer's latitude.

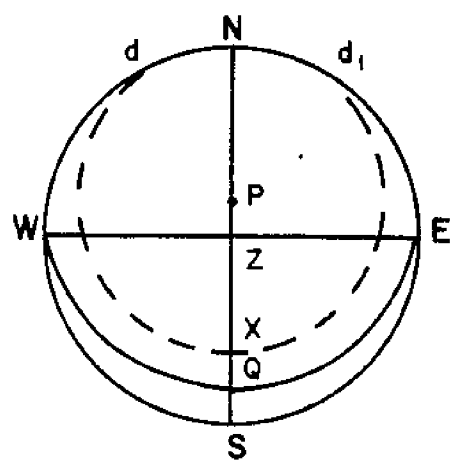
ZX, the zenith distance = 90°-35°20′
= 54°40′

QX, the declination = 20°10′ N

Therefore, equator *Q* is 20°10′ south of the body.

The observer's latitude = *ZQ* = *QX* + *ZX*

Therefore, latitude = 74°50′ N.



Example 2

Given true meridian altitude of Sun 48°18′ bearing north of the observer with declination 3°15′ S, calculate observer's latitude.

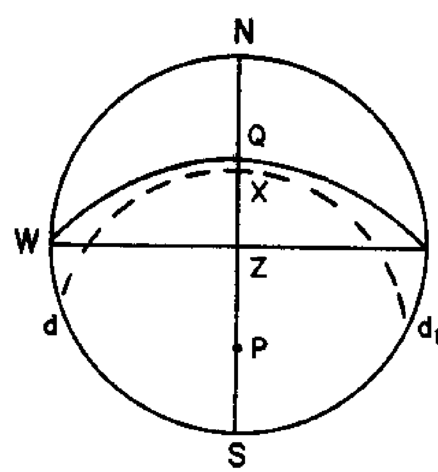
ZX, the zenith distance = 90°-48°18′
= 41°42′

QX, the declination = 3°15′ S

Therefore, equator *Q* is 3°15′ north of the body.

The observer's latitude = *ZQ* = *QX* + *ZX*

Therefore, latitude = 44°57′ S.



Example 3

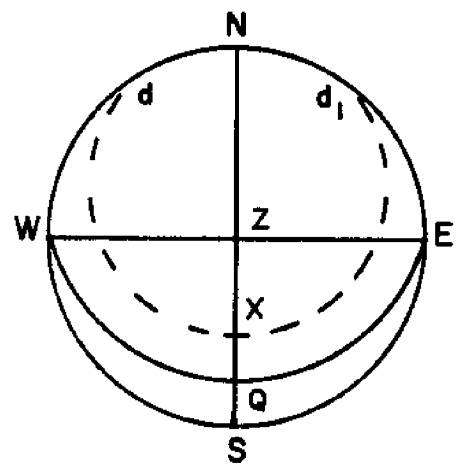
Calculate the latitude of an observer with longitude 0°00′ on August 26, 1968, if the true meridian altitude of the Sun was 43°12′ bearing south.

ZX, the zenith distance = 90°-43°12′
= 46°48′

From *Nautical Almanac* for August 26, G.M.T. of sun's meridian passage at Greenwich is 12h02m and declination is 10°18′ N

From diagram latitude *QZ* = *QX* + *ZX*
= 57°06′ N

Therefore, latitude = 57°06′ N.



Exact time of passage is when L.H.A. = 0.
In this case, L.H.A. = G.H.A. as longitude = zero

G.H.A. = 359°34′.4 at 1200 and, therefore, requires 00°25′.6 until noon.

The increment tables give the equivalent time of 0m 42s.

Therefore, G.M.T. of apparent noon = 12h01m42s.

Example 4

Calculate the latitude of an observer in longitude 22°15′ W if the true meridian altitude of the Sun was 57°05′ bearing north of the observer on August 27, 1968.

ZX, the zenith distance = 90°-57°05′
= 32°55′

From *Nautical Almanac* for August 27, G.M.T. of sun's meridian passage of Greenwich = 1201

Therefore, G.M.T. will be 22°15′ of time later on the observer's meridian

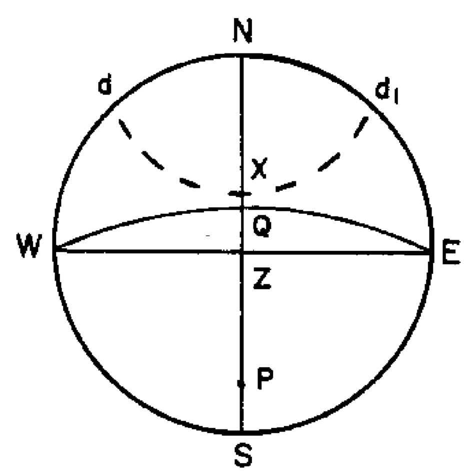
Therefore, G.M.T. of local passage of the Sun = 1201 + 1h29m

G.M.T. of passage = 1330

Therefore, dec. of the Sun = 9°55′.6 *N* = *QX*.

From the diagram, latitude *QZ* = *ZX*-*QX*

Therefore, latitude = 22°59′.4 S.



Exact time of passage is when L.H.A. = 0.
As longitude is 22°15′ W then if L.H.A. = 0, G.H.A. = 22°15′.

G.H.A. = 14°38′.8 at 1300, leaving 22°15′ -14°38′.8 until local noon.

The increment tables give the equivalent time of 30m25s.

Therefore, G.M.T. of apparent noon = 13h30m25s.

Exercise 4. MERIDIAN ALTITUDES

Draw a diagram in the plane of the observer's rational horizon for each question.

1. Determine the latitude of an observer if the sun's true meridian altitude was 67°13′ with a declination of 18°22′S, bearing north of the observer.

2. Find the latitude of an observer when the true altitude of the Sun, bearing north, was 37°22′ with a declination of 7°15′N.

3. The true meridian altitude of the Sun bearing south was 67°22′ when the Sun's declination was 11°18′S. Find the latitude of the observer.

4. The minimum shadow cast by a 6-foot pole was exactly 6 feet on June 21. Find the approximate latitude of the observer without the use of tables or the *Nautical Almanac*. The observer was south of the Sun.

5. Determine the latitude of an observer if the sextant altitude of the ☉ at local apparent noon was 23°28′.5 bearing south. The I.E. was 2′.4 on the arc, H.E. 24 feet, and declination of the Sun, 22°18′S.

6. Calculate the latitude of an observer in longitude 0°00′ on August 25, 1968, if the true meridian altitude of the Sun was 42°22′ bearing south.

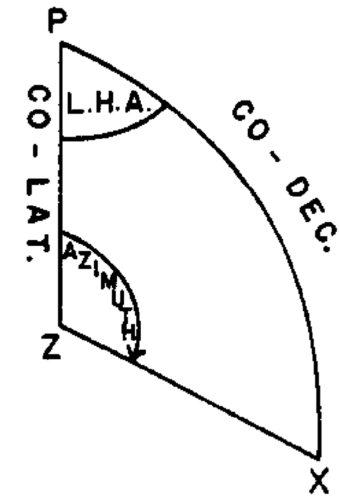
7. Determine the latitude of an observer in longitude 78°20′ W on August 27, 1968, if the true meridian altitude of the Sun was 84°06′ bearing south.

8. Find the latitude of an observer in longitude 62°15′ E if the observed altitude of ☉ was 70°10′.3 on August 26, 1968. The observer's H.E. was 27 feet and the apparent.sun crossed his meridian bearing south.

9. State the exact Greenwich time of meridian passage of the Sun and the observer's latitude if the sun's true altitude at this time was 36°27′ bearing north. The observer's longitude was 38°05′W and the date was August 25, 1968.

10. Calculate the exact local time of meridian passage of the Sun and the observer's latitude if the observed altitude of ☉ at the time was 74°38′.6 bearing south. The observer's longitude was 65°12′E, his H.E. 29 feet and the date was August 27, 1968.

Azimuth and Amplitude



TIME AZIMUTH

The azimuth of a body is the angle at the observer's zenith contained between the observer's meridian and the vertical circle passing through the body concerned.

In the pole-body-zenith spherical triangle (*PZX*) it is possible to calculate the azimuth angle *PZX*, providing two sides and an included angle are known. If Greenwich time is observed upon taking a compass bearing of a body, then figures for an accurate local hour angle and the declination of the body can be extracted from the *Nautical Almanac*. The declination is then either added or subtracted from 90 degrees to give the co-dec, or polar distance, side of the triangle. An estimated latitude, when subtracted from 90 degrees will provide the other side of the triangle and the angle between these two side will be the L.H.A.

Tables based on arguments of latitude, declination and hour angle preclude the necessity for solving the *PZX* triangle by trigonometry, and the true azimuth can be readily extracted from such tables in a few seconds.

Thus, a comparison of the true and compass azimuths of a certain body will yield the compass error at any instant. This method of determining compass error should be regularly practiced and carried out after each alteration of course if possible. The actual compass bearing of the body is obtained by observing the Sun through an azimuth mirror. The azimuth mirror has a ring which is mounted over the compass and is free to turn. The ring bears a glass prism through which a body may be observed while looking at the compass card graduation.

Various azimuth tables are available and the following exercise may be worked with whichever set of tables the student has available. The following example is worked with the A.B.C. tables in *Burton's Nautical Tables.*

Example 1

Calculate the true bearing of the Sun on August 26, 1968, to an observer in latitude 32°24′ N longitude 23°00′ W at 1430 L.M.T. If the compass bearing of the Sun was 260°C at this time, what is the compass error?

Step 1. Determine G.M.T. and extract G.H.A. and dec. from the *Nautical Almanac*.

L.M.T.	14h30m
Long. in time	1h32m
G.M.T.	16h02m
G.H.A. 16h	59°35′.1
Increment 02m	0°30′
G.H.A.	60°05′.1
Dec.	10°14′.5 N

Step 2. Apply longitude to G.H.A. to obtain L.H.A.

G.H.A.	60°	05′.1
Long.	23°	00′ W
L.H.A.	37°	5′.1

NOTE: Longitude west G.H.A. best;
longitude east G.H.A. least.

Step 3. With the three arguments of latitude, declination and L.H.A., enter the A.B.C. tables. Select A to match L.H.A. declination. Combine A and B to give the C factor and enter the C section of the table. Extract the true azimuth where the C factor matches latitude. The bearing quadrant is indicated at the top of the page as to the sign of the C factor, N or S latitude of the observer, and rising or setting state of the body. It is necessary to interpolate throughout the tables for accurate results.

From A table

L.H.A.	37°	37½°
Lat. 32°	.829	.814
Lat. 33°	.862	.846

By interpolation, Lat. 32°24′ and L.H.A. 37°5′.1 yield A = .839 +

From B table

L.H.A.	37°	37½°
Dec. 10°	.293	.290
Dec. 11°	.323	.319

By interpolation, Dec. 10°14′.5 N and L.H.A. 37°5′.1 yield B = .300 -. Combining A and B (+.839, -.300) gives C = .539 +

From C table		
Azimuth	65½°	66°
Lat. 32°	.537	.525
Lat. 33°	.543	.531

By interpolation, Lat. 32°24′ and C .539 yield azimuth = 65½°.

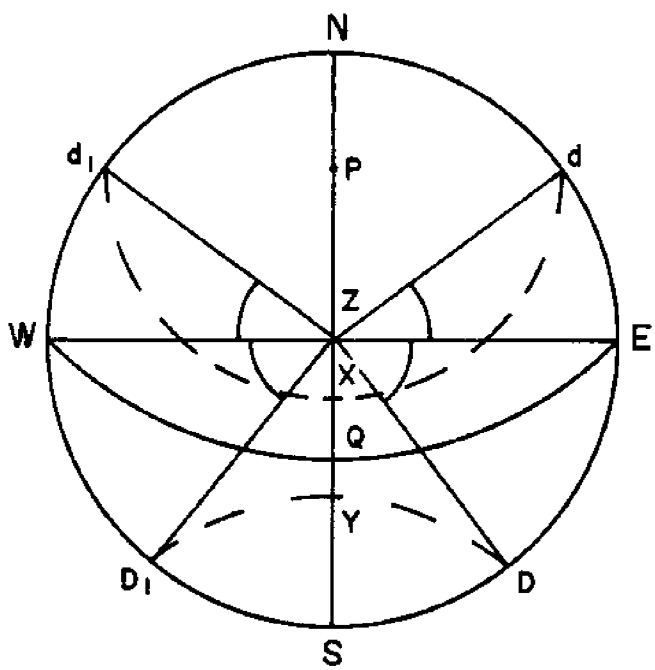
Body is setting (H.A. less than 180°), sign of C is + and Lat. is N; therefore, the body is in the SW quadrant.

Therefore, azimuth is S 65½° W or 245½° T.

The compass error is (260° - 245½°) = 14½ W.

AMPLITUDES

The bearing amplitude of a body is the arc of the horizon contained between east and a rising body or between west and a setting body. The usual 360 degree notation of bearings refers to north; the older quadrantal system refers to north and south, but the amplitude uses east and west as its origin.



The diagram shows the path of body X with a northerly declination, dXd_1, and the path of body Y, with a southerly declination, DYD . Thus, the amplitude of body X rising is angle *EZd* and the amplitude of body Y rising is angle *EZD*. The amplitude of bodies X and Y setting would be angle WZd_1 and angle WZD_1, respectively.

Clearly a body will always rise and set on a northerly bearing when its declination is north, and will rise and set on a southerly bearing when its declination is south. The amplitude, therefore, is always named the same as the body's declination.

Finding the amplitude of the Sun is a quick and simple method of determining compass error. Theoretical sunrise occurs when the center of the Sun is on the observer's rational horizon. However, refraction, which is maximum at zero altitude, makes the Sun appear to rise about 33 minutes above the horizon when it is theoretically at an altitude of zero degrees. This value of 33 minutes roughly corresponds to the diameter of the Sun; to allow for this, amplitudes should be taken when the Sun's center is about the Sun's diameter clear of the horizon. In other words, there should be a clearance about the Sun's semi-diameter between the horizon and the Sun's lower limb at the time of amplitude. A table is provided in Bowditch's *American Practical Navigator* for correction of amplitudes as observed on the visible horizon.

The true amplitude may be calculated from the formula: Sin. amplitude = sin. declination x sec. latitude. This formula is deduced from Napier's Rules for the solution of right-angled spherical triangles. These rules will be discussed later in the text.

The problem is much more readily solved by extracting the true amplitude directly from prepared amplitude tables. Most sets of nautical tables contain an amplitude table which is based on the above formula and which requires no detailed explanation on its use. The table is merely entered with the arguments of latitude and declination to give the amplitude bearing.

The L.M.T. of sunrise and sunset for any observer is listed in the *Nautical Almanac* for certain latitudes at three-day intervals. One must interpolate between the listed latitudes in order to obtain an accurate time of sunrise or sunet in intermediate latitudes.

Exercise 5. TIME AZIMUTHS

1. Determine the true azimuth of the Sun at 0900 G.M.T. to an observer in latitude 59°00′ N longitude 0.00′ on August 26, 1968.

2. What is the true bearing of the Sun at 1130 L.M.T. to an observer in dead reckoning (D.R.) position 36°02′N 14°48′E on August 25, 1968?

3. Calculate the sun's true azimuth if its declination was 12°30′ S to an observer in D.R. position 28°12′N 77°18′W. The G.H.A. of the Sun was 128°57′.

4. Find the true bearing of the Sun at 1740 G.M.T. on August 27, 1968, if the observer's D.R. position was 10°12′S 32°15′W.

5. What was the true azimuth of the Sun at 0940 L.M.T. to an observer in D.R. position 52°10′N 45°18′W on August 25, 1968?

6. Calculate the compass error of a vessel in D.R. position 48°12′ N 37°42′ W if the Sun was bearing 070°C at 0850 G.M.T. on August 26, 1968.

7. What was the true azimuth of star Arcturus on August 25, 1968, at 2030 G.M.T. if the observer was in an estimated position of 45°00′N 8°06′W?

8. An observer in D.R. position 24°12′ S 135°08′ W observed the Sun bearing 090°C. If the declination of the Sun was 8°20′S and its G.H.A. was 94°17′, find the compass error.

9. If the G.H.A. of Aries was 325°18′ to an observer in latitude 46°15′ N longitude 165°12′ E and star Sirius was bearing 195°C, calculate the compass error. The declination of Sirius was 16°40′S and its S.H.A. was 259° 3′.8.

10. Calculate the deviation of the compass at 0720 L.M.T. on August 27, if the Sun was bearing 107°C to an observer in latitude 10°12′ N longitude 57°18′ W. The magnitude variation was 27° W.

Exercise 6. AMPLITUDES

1. Calculate the amplitude of the Sun if it set with a declination of 10°30′ S to an observer in latitude 29°00′N.

2. Calculate the azimuth of the Sun if it rose with a declination of 17°12′ N to an observer in latitude 49°15′N.

3. What would be the setting amplitude of the Sun on June 21, 1968, to an observer in latitude 39°24′ N? If the Sun was bearing 284°C at this time, what was the compass error?

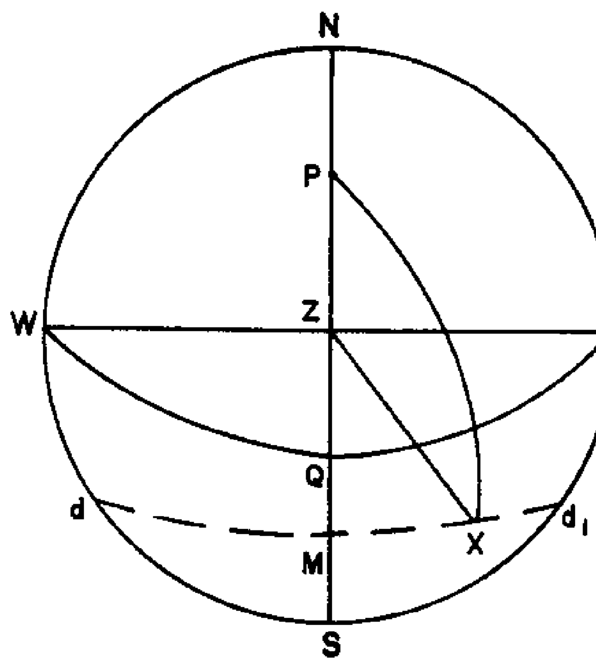
4. Determine the approximate amplitude of the Sun at time of setting on September 22, 1968.

5. Find the compass error if the Sun rose bearing 090°C to an observer in position 45°06′ N 35°10′W on August 26, 1968.

Ex-meridian Altitude

It is not always possible to find the sextant altitude of a body when it is exactly on the meridian. Often the time of meridian passage of a star does not coincide with the twilight time of observation when both the star and a clear horizon are visible. Also, cloudy conditions may prevent a noon sun sight. In these cases, the body can sometimes be observed near the meridian and then its position reduced to the meridian by calculation or by consulting the appropriate tables.

A meridian altitude provides an east-west position line and, therefore, an accurate latitude. An ex-meridian altitude results in a position line perpendicular to the true bearing, or azimuth, of the body. For example, a body bearing 176 degrees T will yield a position line of 086 degrees T/266 degrees T. An accurate latitude will result, providing the observer's dead reckoning longitude is reasonably true.



WNES	observer's rational horizon	Angle *ZPX*	hour angle of the body
NZS	observer's meridian	Angle *PZX*	azimuth of the body
$dMXd_1$	body's declination parallel	*ZM*	meridian zenith distance
X	body at time of sight	*ZX*	zenith distance at time of sight
M	body on the meridian		

We know that the greatest altitude and, thus, the smallest zenith distance occurs when the body is on the observer's meridian. The zenith distance at the time of ex-meridian observation will be larger than the meridian zenith distance (M.Z.D.), and the process of finding the M.Z.D. from the zenith distance at the time of observation is called reduction to the meridian.

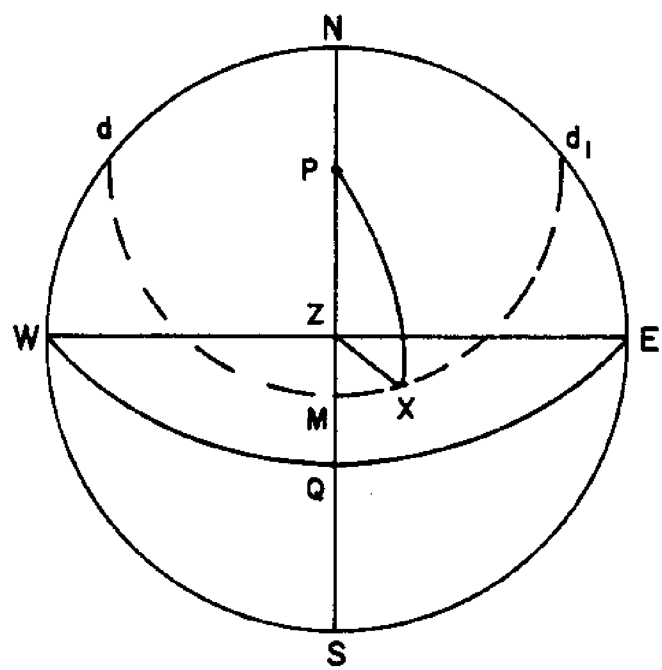
An ex-meridian problem can only be worked when the body is so close to the meridian that *PX* is practically equal to *PM* (See diagram). A dead reckoning latitude must be used in the spherical triangle *PZX* in order to calculate a zenith distance and compare it to the observed zenith distance.

It is assumed that if *PM-PZ* is equal to *ZM* then *PX-PZ* also is equal to *ZM*. Because of this approximation, the problem should be worked a second time with a more accurate D.R. latitude if the calculated latitude differs appreciably from the initial D.R. latitude. This is not usually necessary.

Remember that this method of obtaining a position line is limited to the use of bodies close to the meridian. A general rule is that the hour angle (H.A.) in minutes should be smaller than the number of degrees in the M.Z.D. Some sets of tables give information on the limits of H.A. for a given latitude and declination. Inaccuracies usually result when the H.A. is any more than about ten degrees. It is not intended at this time to examine how to solve this problem by calculation because easily used ex-meridian tables are available, which provide the reduction to the meridian for a large range of combinations of latitudes, declinations and local hour angles.

Excellent ex-meridian tables are provided in *American Practical Navigator* by Bowditch and in *A Set of Nautical Tables* by Burton. These tables contain a good explanation on their use, and completion of Exercise 7 will show their simple application.

It should be emphasized that solving the ex-meridian problem by reduction to the meridian provides a latitude at time of sight to be coupled with the D.R. longitude. This merely provides the position through which the position line of the body should pass at the time of observation. An example of an ex-meridian problem, as worked from *Burton's Nautical Tables,* follows.



Example

On August 26, 1968, D.R. latitude 42°46′ N longitude 32°15′ W., the observed altitude of ☉ near the meridian was 57°08′ south of the observer, H.E. 26 feet. The chronometer read 14h04m07s and was 3m12s fast of G.M.T. Find latitude, time of observation and direction of position line.

Step 1. Determine L.H.A. and declination. If *necessary subtract* L.H.A. from 360 degrees to obtain angle *P in the diagram.*

Chron.	14h	04m	07s	
Error		3m	12s	
G.M.T.	14h	00m	55s	
G.H.A.	14h	29°	34′.7	
Incr.		00°	13′.8	
G.H.A.		29°	48′.5	
Long.		32°	15′ W	
L.H.A.		357	33′.5	(2°26′.5)
Decl.		10°	16′.3 N	

Step 2. Extract F, the ex-meridian factor, from Table I using arguments of declination and latitude.

Ex-meridian factor = 2.64

Step 3. Enter Table II with H.A. = 2°26′.5 and F = 2.64 and take out the reduction. Burton's Table III provides for a greater range of ex-meridian altitudes.

F 2.0 yields 3.16
F .64 yields 1.01
F 2.64 yields 4′.17
Reduction = 4′.2

Step 4. From the observed altitude determine the true zenith distance (T.Z.D.). Subtract the reduction from T.Z.D. to give M.Z.D.

Obs. Alt.	57°	08′
Dip		-4′.9
App. Alt.	57°	3.1
T. Corr.		+15′.4
T. Alt.	57°	18′.5
T.Z.D.	32°	41′.5
Reduction		-4′.2
M.Z.D.	32°	37.3

Step 5. Combine M.Z.D. and declination to obtain latitude at time of sights.

M.Z.D.	32°	37'.3
Decl.	10°	16'.3N
Lat.	42°	53'.6N

Step 6. Calculate azimuth at time of sights. Position line will be perpendicular to the azimuth through the latitude as found and longitude as determined by dead reckoning.

A.	+ 21.3
B.	- 4.30
C.	17.00
Azimuth	S 4.6° E

Therefore, P/L 085.4° -265.4° through lat. 42°53'.6 N long. 32°15' W.

If the noon position is required, a course and distance run from the time of sights to noon must be applied to the position found by the ex-meridian method.

Exercise 7. EX-MERIDIANS

1. Determine the latitude of an observer with H.E. 32 feet in D.R. position 38°22′ N 28°18′ W if the observed altitude of $\underline{\odot}$ near the meridian was 66°37′. The sun's declination at this time was 15°12′N and its G.H.A. 33°6′.

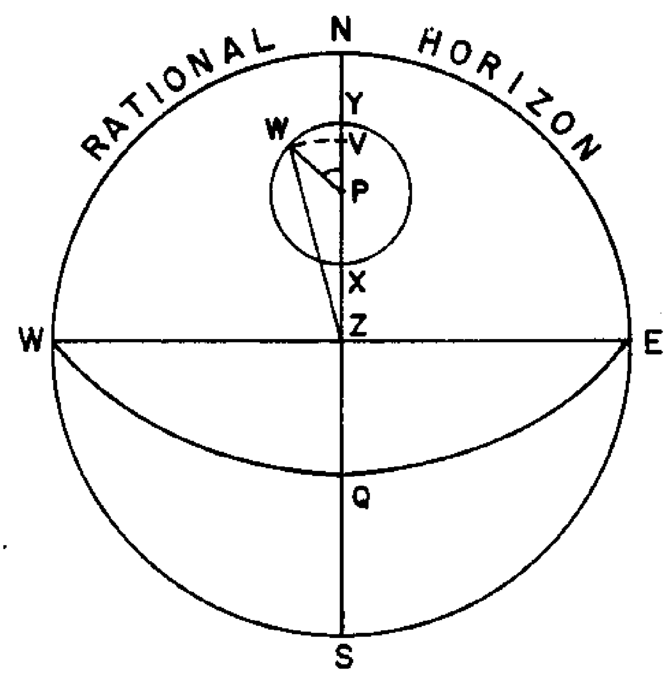
2. Find the latitude of an observer with H.E. 19 feet in D.R. position 47°30′ N 64°11′ W. The sextant altitude of $\underline{\odot}$ near the meridian was 31°18′ and the I.E. was 3′.2 on the arc. The sun's declination was 10°28′S and its G.H.A. 57°32′.

3. On August 25, 1968, in D.R. position 48°12′ N 38°15′ W the observed altitude $\underline{\odot}$ near the meridian was 52°16′ S of the observer, H.E. 17 feet. Time by chronometer was 14h37m12s, and chronometer error was 3m4s slow of G.M.T. Find the latitude by reduction to the meridian.

4. Determine a position line and a position through which to draw it if the sextant altitude of $\underline{\odot}$ was 59°36′ N of the observer and near the meridian. The D.R. position was 22°15′ S 65°17′ E, I.E. 2′.1 off the arc, H.E. 15.0 feet, declination of the Sun 7°18′ N, and its G.H.A., 299°48′.

5. On August 27, 1968, in D.R. position 51°27′ N 63°12′ W, the observed ex-meridian altitude of $\underline{\odot}$ was 48°7′. The observer's H.E. was 14.5 feet, the chronometer which was 2m12s fast of G.M.T. read 15h59m9s. Find the latitude and position line at time of observation.

The Pole Star



In the above diagram, *NP*, the altitude of the celestial pole, will *always* equal *ZQ*, the latitude of the observer. *NZ* equals 90 degrees and *QP* equals 90 degrees; therefore, as the latitude changes, so do the positions of *Q* in the diagram and *P* by the same amount. Thus *NP* equals *ZQ*.

If there were a star situated exactly at the celestial pole, its true altitude would always represent the observer's latitude. Unfortunately, there is no such star, but the star Polaris is sufficiently close to the pole for its altitude to be used to find the observer's latitude after making three minor corrections.

The apparent motion of all stars is circumpolar due to the effect of the rotation of Earth. The Pole Star, as Polaris is known, appears to perform a small circle about the celestial pole, never moving more than about 2 degrees in azimuth east or west of north. As the earth rotates daily, the Pole Star will cross the observer's meridian twice. The *upper meridian passage* occurs when the body is on the meridian between the observer's zenith and the pole (*X* in the diagram). The *lower meridian passage* occurs when the body crosses the observer's meridian on the farther side of the pole at *Y*.

The observer's latitude can be obtained by observation of the Pole Star at any time both the star and a clear horizon are visible. The true altitude of the Pole Star is corrected by an amount equal to *VP* in the diagram where *ZW* equals *ZV* equals the zenith distance.

Thus, true altitude ± correction = latitude.

Clearly, from the diagram, the correction should be *added* to the altitude when the Pole Star is north of an east-west line from the pole and *subtracted* from the altitude when the Pole Star is south of an east-west line from the pole.

However, to avoid confusion the *Pole Star tables* contain a total of 1 degree in constants in order to keep the three necessary corrections always positive. The one degree is subtracted afterwards.

Therefore, from the Pole Star tables

$$\text{Latitude} = \text{True Altitude of Pole Star} + a_0 + a_1 + a_2 - 1^\circ$$

The Pole Star tables are contained in the back of the *Nautical Almanac* and one page of these tables is reproduced in the appendix. (Note the illustration at the bottom of that page.) The following exercise can be worked from this.

Exercise 8. POLE STAR PROBLEMS

1. Calculate the latitude of an observer in longitude 52°15′ W when the sextant altitude of the Pole Star, out of the meridian, was 43°17′. The G.H.A. of Aries was 249°45′, the observer's H.E. 15 feet, I.E. 2′.0 on the arc and the month June.

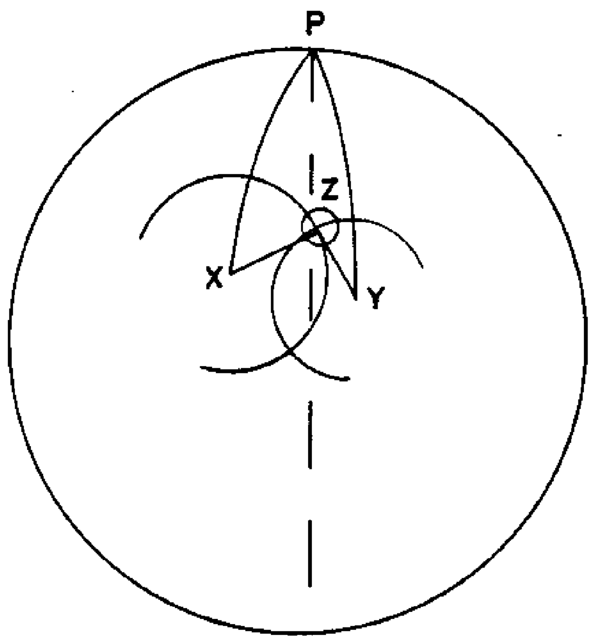
2. On August 25, 1968, at 0540 G.M.T. in longitude 108°14′ E, the sextant altitude of Polaris, out of the meridian, was 34°52′, I.E. 1′.7 off the arc, and H.E. 23 feet. Find the latitude.

3. The observed altitude of the Pole Star, out of the meridian, on August 27, 1968, at 1946 G.M.T., was 53°18′. The observer's longitude was 48°32′ W and his H.E. 23 feet. Calculate the latitude.

4. Determine the latitude of an observer in longitude 137°45′ W, when the sextant altitude of Polaris, out of the meridian, was 47°22′, I.E. 2′.1 on the arc, H.E. 14 feet. The G.H.A. of Aries was 5°23′ and the month was December.

5. On August 26, 1968, at 03h22m13s by chronometer in longitude 99°23′ E, the sextant altitude of the Pole Star, out of the meridian, was 76°52′. The chronometer was 16m15s fast and the observer's H.E. 25 feet. At this time the Pole Star was bearing 348°C. Calculate the latitude of the observer and the compass error.

The Celestial Position Line



The true zenith distance (T.Z.D.) of a body provides the radius of a position circle centered at the geographical position of the body at the instant of observation. Simultaneous observations of two bodies will provide two such position circles, radii *XZ* and *ZY* in the diagram. The observer will be at one of the two intersections of the position circles, and it will be obvious, by consulting the azimuth of the bodies, which intersection is the observer's true position. At position *Z*, as marked in the diagram, body *X* should be bearing about west, while body *Y* will have an azimuth of about south by east.

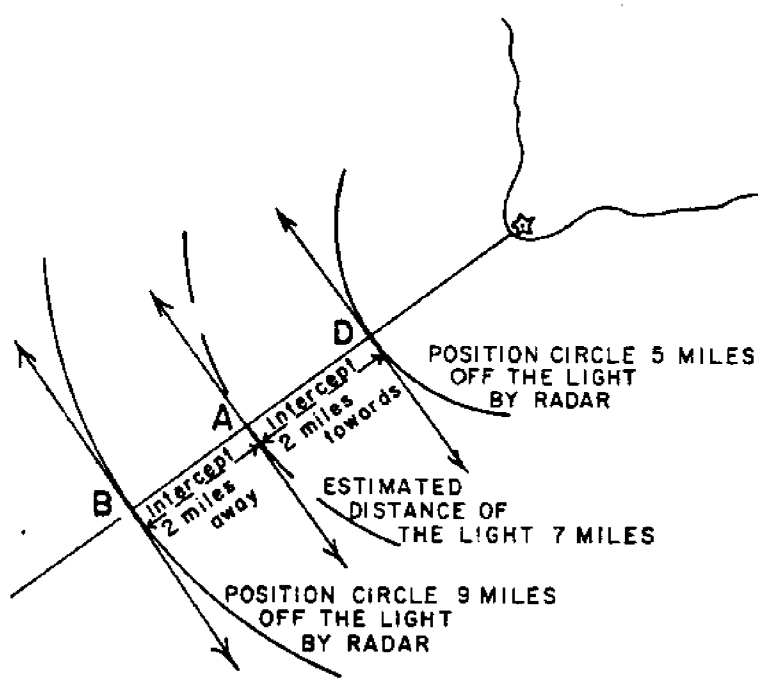
Unfortunately, such a method of plotting a vessel's position does not provide sufficiently accurate results to be of practical use. The chart plotting sheet would need to be very small scale to take such large position circles, and small errors in construction would result in large discrepancies in positions.

Only the small section of arc in the vicinity of the observer's predicted position needs to be drawn when determining it from position circles. If the zenith distance is not too small, this arc can be considered a straight line with no appreciable error. It can easily be seen from the preceding diagram that such a straight position line will be perpendicular to the azimuth of the body. The problem is to be able to plot this position line without working from its center of origin. To do this, the altitude intercept method of plotting celestial position lines is most commonly used.

THE ALTITUDE INTERCEPT METHOD

To plot the celestial position line by the intercept method, the celestial triangle *PZX* is solved for an assumed latitude and longitude to give a calculated zenith distance for these conditions. Any difference between the calculated zenith distance (C.Z.D.) and the T.Z.D., as observed, will be the error of the assumed position in a direction either toward or away from the bearing of the body.

This will perhaps be more easily understood by comparing a result from the intercept method with an estimate of a distance off a light corrected by an accurate distance off by radar.



In the above diagram, the lighthouse is bearing northeast and the skipper assumes his distance offshore to be seven miles by "guesstimate" only. If the radar indicates that he is in fact only five miles off the lighthouse, his actual position can be found by measuring off an intercept *AD* two miles toward the lighthouse along the bearing. If the radar had indicated a distance offshore of nine miles, the position could be located by measuring off an intercept *AB* of two miles away from the assumed position.

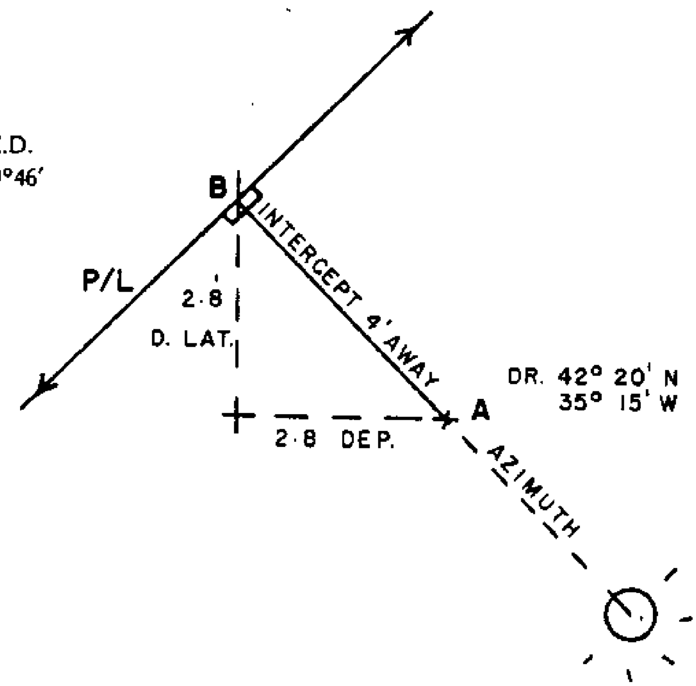
This system is employed with celestial position lines. If the T.Z.D. is *smaller* than the C.Z.D., the intercept is laid off by that amount *toward* the bearing of the body and the position line drawn through this point perpendicular to the bearing. If the T.Z.D. is *larger* than the C.Z.D., then the intercept is laid off by that amount *away* from, or directly opposite the bearing of the body, and the position line is drawn through this point perpendicular to the bearing.

Example

A vessel in D.R. 42°20′ N 35°15′ W observes the Sun with a True Altitude of 60°10′ bearing southeast. The Sun's C.Z.D. as calculated from the spherical *PZX* triangle, using the above D.R. position, was 29°46′. State a position through which to draw the position line.

T.Alt. = 60°10′
T.Z.D. = (90° -60°10′)
T.Z.D. = 29°50′

Intercept = T.Z.D. - C.Z.D.
Intercept = 29°50′ - 29°46′
Intercept = 4′ away



1. The intercept *AB* is constructed to scale, away from the D.R. position, in the direction opposite the azimuth.

2. The position line is laid off at the end of the intercept, perpendicular to the line of bearing.

3. The departure (dep.) and difference of latitude (d. lat.) of position *B* from position *A* are determined by scale measurement.

4. Dep. is converted to difference of longitude (d. long.) by the traverse tables or by the formula:

$$\text{Dep.} = \text{D.Long} \times \text{Cosine Mean Lat.}$$

5. D. lat. and d. long. are applied to the D.R. position to give the intercept terminal position *B*. In the diagram the 4′ Intercept is drawn away from the bearing toward the northwest. The position line is drawn from northeast to southwest through the intercept terminal position (I.T.P.) The d.lat. 2′.8 and dep. 2′.8 are taken off as to the scale and the dep. is converted to d. long. to give 3′.8.

D.R. Position	42°	20′ N	35°	15′ W
	D.Lat.	2′.8 N	D.Long.	3′.8 W
I.T.P.	42°	22′.8 N	35°	18′.8 W

Therefore, to be able to use the intercept method of plotting position lines we require T.Z.D., azimuth and C.Z.D. The T.Z.D. is easily found by subtracting the true altitude from 90 degrees (see Exercise 3), and the azimuth is found by tables (see Exercise 5). Calculation of the C.Z.D. is more complicated but is easily done with a little practice.

Solving the *PZX* triangle to yield the C.Z.D. can be accomplished in three different ways: (1) trigonometric calculation, (2) short method tables, or (3) inspection tables. Theory and examples on the first two methods follow, but we will be more concerned with the practical use of the third.

Three Methods of Sight Reduction

BY CALCULATION

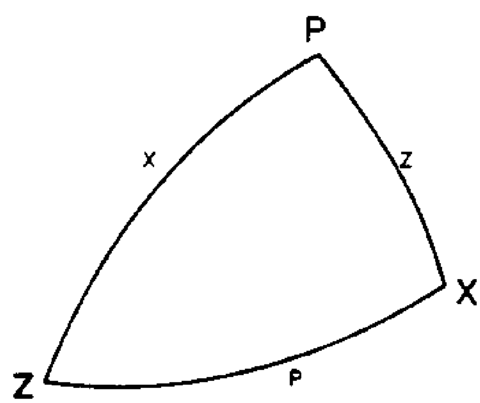
In 1875 the *altitude intercept method* of determining a position line was introduced by a French naval commander named Marq St.-Hilaire. For about 100 years prior to this, position lines were obtained by the longitude-by-time method made possible by the invention of the chronometer. Various applications of longitude by time or longitude by chronometer are still used by some navigators, possibly due more to habit than to any other reason. However, most navigators prefer the altitude intercept method, and some of these obtain the necessary C.Z.D. by the longer route of calculation rather than by short method tables.

The most popular formula used for solving the *PZX* triangle to obtain C.Z.D. is the cosine-haversine formula. An explanation of this method follows for interested students.

The Cosine-Haversine Formula

The sides of a spherical triangle are great circles and are expressed as the number of degrees, minutes and seconds of arc that they subtend at the center of the sphere.

The sum of the three angles of a spherical triangle will be between 180 degrees and 540 degrees. The sum of the three sides will be less than 360 degrees.



The cosine formula is the basic formula for calculating a side of a spherical triangle when the other two sides and the angle between them are known.

The angles in the celestial triangle are given as *P*, *Z*, and *X*, and the sides opposite them *p*, *z*, and *x*, respectively. The C.Z.D., *p*, is required and the complement of latitude (co-lat.), *x*, the co-dec., *z*, and H.A., *P*, are known.

Then by the cosine formula:

$$\cos p = \cos P \sin x \sin z + \cos x \cos z$$

The haversine formula is derived directly from the cosine formula:

$$\text{hav}\, p = \frac{1 - \cos p}{2}$$

By the haversine formula:

$$\text{hav}\, p = \text{hav}\, P \sin x \sin z + \text{hav}\,(x \sim z)$$

Because x and z represent co-dec. and co-lat., respectively, it is simpler to use the haversine formula in the following form:

hav.p = hav.P. cos.dec. cos.lat. + hav. (dec.~lat.)

The proof of the spherical cosine formula and the derivation of the haversine formula are contained in various other books, e.g., *Nicholls' Concise Guide*, Volume II, which have a more theoretical approach to the subject.

An example of sight reduction using the haversine formula follows.

Example

On August 26, 1968, at 0930 L.M.T. the sextant altitude of ☉ was 44°8′.0. The vessel was in D.R. position 39°30′ N 71°15′ W and the chronometer, which was 9m12s fast of G.M.T., was reading 14h16m41s. Calculate the direction of the position line and a position through which it passes if the observer's H.E. was 16 feet and I.E. was 0′.8 off the arc.

Step 1. Determine G.M.T. from the chronometer. Check this by making a comparison with local time, having longitude in time applied.

L.M.T.		0930	
Long. in time		445	
Approx. G.M.T.		1415	
Chron.	14h	16m	41s
Error		- 9m	12s
G.M.T.	14h	07m	29s

Step 2. Calculate the L.H.A. by applying longitude to the G.H.A. as extracted from the *Nautical Almanac*. If the L.H.A. exceeds 180 degrees subtract this from 360 degrees to give angle P in the PZX triangle.

G.H.A. 14h	29°	34′.7
Incr.	1°	52′.3
G.H.A.	31°	27′
Long.	71°	15′
L.H.A.	320°	12′
Angle P	39°	48′

Step 3. Extract declination from the *Nautical Almanac* and then obtain the *difference* from D.R. latitude, if they are of the same name, or the *sum*, if they are of unlike names.

Dec.	10°	16′.2N
Lat.	39°	30′ N
(Dec.~Lat.)	29°	13′.8

Step 4. Correct the sextant altitude and subtract the true altitude from 90 degrees to obtain the T.Z.D.

Sex. Alt.	44°	8′.0
I.E.		+ ′.8
Obs. Alt.	44°	8′.8
Dip.		- 3′.9
App. Alt.	44°	4′.9
T. Corr.		+15′.0
T. Alt.	44°	19′.9
T.Z.D.	45°	40′.1

Step 5. Calculate the C.Z.D. using the haversine formula: Hav. p = Hav P. cos.dec. cos.lat. + hav. (dec.~lat.).

Obtain the log. hav. θ and convert this to the nat. hav. θ by means of the haversine tables. Add the nat. hav. (dec.~lat.) to the nat. hav. θ to get nat. hav. C.Z.D.

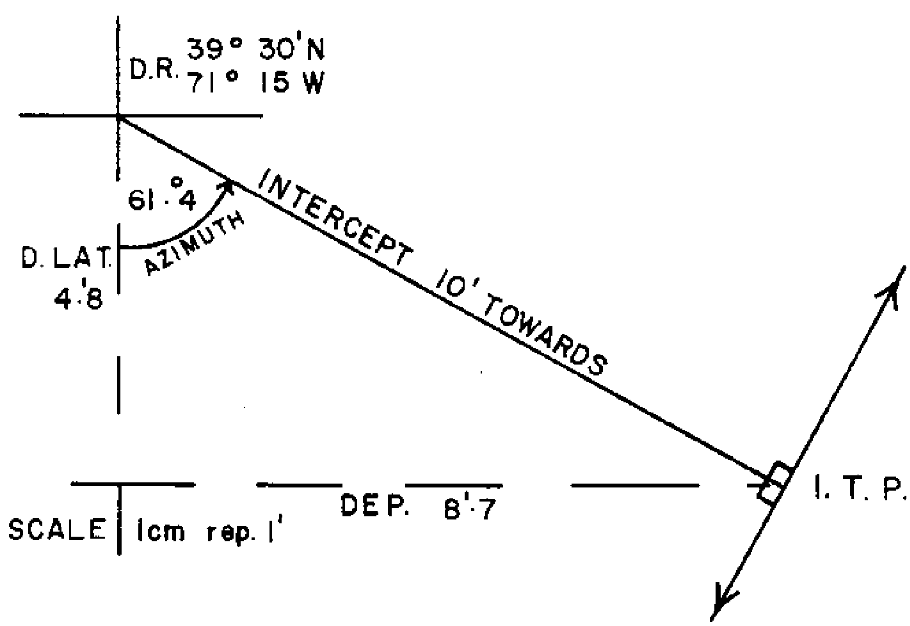
Step 6. Determine the intercept by finding the difference between T.Z.D. and C.Z.D. Remember the intercept is toward if T.Z.D. is smaller than C.Z.D. Calculate the azimuth and the position line will be perpendicular to it.

Log. Hav. Angle P	T.06393
Log. Cos. Lat.	T.99299
Log. Cos. Dec.	T.88741
Log. Hav.θ	2.94433
Nat. Hav. θ	08797
Hav. (Dec. Lat.)	06367
Hav. C.Z.D.	15164
C.Z.D.	45°50'.1
T.Z.D.	45°40'.1
Intercept	10'.0 towards
A	.99 +
B	.283-
C	.707+
Azimuth	S61°4 E
P.L.	028°.6/208°.6

Step 7. Plot the intercept and azimuth from the D.R. position using some suitable scale. Convert to d.long and apply d.lat. and d.long. to the D.R. position to obtain the I.T.P.

From the traverse tables, 8'.7 dep. in latitude 39°30' N gives 11'.3 d. long.

D. R. Position	39°30' N	71°15' W
d. lat	4'.85 d. long	11'.3 E
I.T.P.: Lat. 39°25'.2 N Long.		71°3'.7 W P./L. 028°.6/208°.6



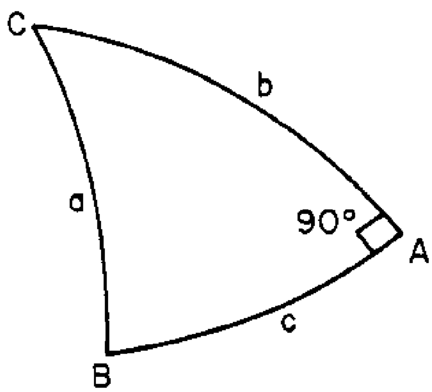
BY SHORT METHOD TABLES

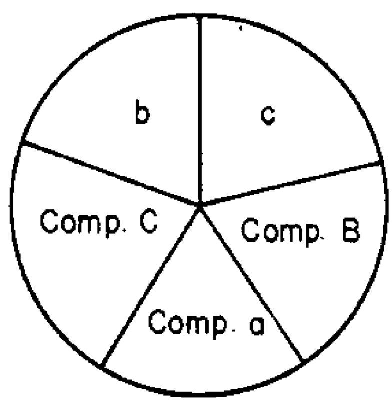
Professional navigators are constantly seeking more rapid and efficient methods of sight reduction. Over the last 50 years many and varied short methods of sight reduction, both mathematical and mechanical, have been devised to satisfy these requirements. Some of these methods are highly complicated and in some cases take just as much time as a calculation. Other methods have been accepted and are used by a few navigators in preference to the inspection method mainly due to the compactness of the tables used.

Some of the more efficient of the short methods divide the celestial *PZX* triangle into two right-angled spherical triangles by having a perpendicular dropped from one angle to the opposite side. Napier's Rules for the solving of right-angled spherical triangles can then be used for sight reduction, and the short method tables can be established, based on a range of conditions already worked and solved by Napier's Rules and presented in tabulated form. A simple explanation of Napier's Rules follows for interested students.

Napier's Rules

The 90-degree angle is omitted, and the other two angles and the three sides are represented in rotation by a five-part diagram as follows. (Note that the complement of the two angles and the complement of the hypotenuse side are used.)





Providing that the values of any two of the five sections in the diagram are known, the other three can be found by the following formulae:

Sine of a Part = Product of Tangents of Adjacent Parts

Sine of a Part = Product of Cosines of Opposite Parts

Adjacent part means the one next to, and opposite part means the one past adjacent. If, for example, side c and angle *B* are known and C is required

sine comp. C = cos. c × cos. comp. *B*.,

cosine C = cos. c × sine. *B*.

If a is required

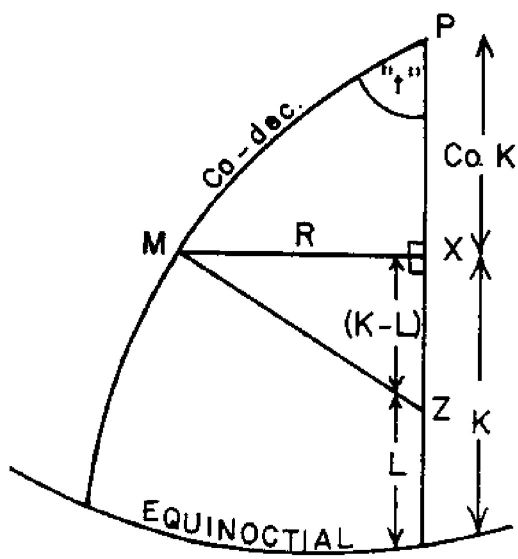
sine comp. *B* = tan. *c* × tan. comp. *a*., and transposing

cotan *a* = cosine *B* × cotan c.

Care must be taken with the *rule of signs* and various special cases which arise using this method. For a detailed explanation refer to *Spherical Trigonometry* by J. H. Clough-Smith.

Following is the only short method table to be examined in this text with a brief explanation and example of the Ageton method, using publication H.O. No. 211, *Ageton's Short Method Tables.*

Ageton Short Method Table



The celestial triangle, in this case triangle *PMZ*, is divided into two right-angled spherical triangles by dropping a perpendicular from the body *M* to meet the observer's meridian at *X*.

In the right triangle *PMX*, "*t*", the L.H.A., and *PM*, the co-dec. of the body, are both known. Therefore, *R* and Co.*K*, easily found by Napier's Rules, are then links, or auxiliaries, for use in the second right triangle *MXZ*.

The *D.R.* latitude *L* is subtracted from *K* to give the side (*K-L*) in right triangle *MXZ*. Thus, finding the value of *R* and (*K-L*), through Napier's Rules, side *MZ* in the triangle *MXZ* can be found. *MZ* is the calculated zenith distance which, when subtracted from the true zenith distance, provides the intercept.

To make these tables easier to use, all formulae are converted to the form of secants and cosecants which are multiplied by 100,000 for simplified presentation.

This method provides the result of the sight reduction as the calculated altitude (*HC*), which, when subtracted from the true altitude (H_O), gives the intercept. Of course, in this case, the intercept is measured *toward* the azimuth from the D.R. position because *Ho* is larger than H_C. A number of examples are provided in the front of H.O. 211, *Ageton*, but how to use the table will become apparent after working a few examples. The previous example of sight reduction by calculation is worked below by the Ageton method.

Example

On August 26, 1968, at 0930 L.M.T. the sextant altitude of ☉ was 44°8'.0. The vessel was in D.R. position 39°30' N 71°15' W and the chronometer, which was 9m12s fast of G.M.T., was reading 14h16m41s. Calculate the direction of the position line and a position through which it passes if the observer's H.E. was 16 feet and I.E. 0'.8 off the arc.

Step 1. Determine G.M.T. from the chronometer. Check this by making a comparison with local time, having longitude in time applied.

L.M.T.		0930	
Long. in time		445	
Approx. G.M.T.		1415	
Chron.	14h	16m	41s
Error		- 9m	12s
G.M.T.	14h	07m	29s

Step 2. Calculate the L.H.A. by applying longitude to the G.H.A. as extracted from the *Nautical Almanac*. If the L.H.A. exceeds 180 degrees subtract this from 360 degrees to give angle *P* in the *PZX* triangle.

G.H.A. 14h	29°	34'.7
Incr.	1°	52'.3
G.H.A.	31°	27'
Long.	71°	15'
L.H.A.	320°	12'
Angle P	39°	48'

Step 3. Correct the sextant altitude.

Dec.	10°	16'.2N
Sex. Alt.	44°	08'
I.E.		+ 0'.8
Obs. Alt.	44°	8'.8
Dip.		- 3'.9
App. Alt.	44°	4'.9
T. Corr.		+15'
T. Alt.	44°	19'.9

		Add	Subtract	Add	Subtract
Angle P	39°48'	A 19,375			
Dec.	10°16'.2N	B 701	A 74,888		
R.		A 20,076	B 10,974	B 10,974	A 20,076
	K 13°16'.2N		A 63,914		
D.R. Lat.	39°30' N				
(K~L)	26°13'.8			B 4,721	
Hc	44° 9'.8			A 15,695	B 14,427
Ho	44°19'.9				A 5,649
Int.	10'.1 To.				Azimuth S.61°24' E

Step 4. Enter tables with angle *P* and take out number from column *A*. Enter tables with declination and take out number from column *B*. Add these two numbers to give the *R* value.

Step 5. Look up the number obtained from step 4 in column *A* and take out the number beside it in column *B*. Subtract this from the *A* function of declination as extracted from the tables.

Step 6. With the number obtained from step 5, take out the nearest tabulated value of *K* from the tables and give it the same name as the declination. Combine *K* with the D.R. latitude to obtain ($K \sim L$). *Add* *K* and *L* if their names are *different*; *subtract* the smaller from the larger if they are *alike*.

Step 7. Enter table with ($K \sim L$) value and extract the *B* value from the table. Add this to the *B* function of *R*. Enter the table with this number and extract H_C. The difference between *Hc* (computed altitude) and *Ho* (true altitude) will be the intercept. The intercept will be *toward* the body from the D.R. position when H_O is larger than H_C.

Step 8. Look up H_C in the *B* column and subtract this from the *A* function of *R* which was determined in step 1. With this number enter the *A* column and take out the azimuth to the nearest minute. The intercept and azimuth are then plotted in exactly the same manner as used in the calculation method.

NOTE. The short method of sight reduction may appear as anything but that when first examined by the student. However, when the tables have been used a number of times and a few obvious shortcuts have been applied it will be found to be somewhat quicker to use than the calculation method.

BY INSPECTION TABLES

Inspection tables are lists of altitudes and azimuths computed from *PZX* celestial triangles at standard intervals. It is obvious that as the latitude, the hour angle or the declination of a celestial triangle changes, then so must the azimuth and the altitude. Modern inspection tables provide instant readouts of azimuth and altitude when entered with the three arguments of latitude, declination and local hour angle. The latitude and hour angle are listed for every single degree and the declination, for each half degree. Intermediate arguments between degrees can be used by interpolation. A table is provided to facilitate this.

Tables of Computed Altitude and Azimuth was first published during the Second World War by the U.S. Navy Hydrographic Office as H.O. Pub. No. 214. The need for such tables had long been obvious, but it was not until the advent of the computer that their construction became feasible. H.O. Pub. No. 214 consists of nine volumes each covering a range of 10 degrees of latitude. It is only because of the relative unwieldiness of nine volumes that some mariners prefer using some of the more compact short method tables.

H.O. Pub. No. 214 is to be replaced as of December 31, 1975, by H.O. Pub. No. 229. The new publication entitled *Sight Reduction Tables for Marine Navigation* will consist of six volumes which are said to provide the navigator with a method of more precise sight reduction and positioning than ever before possible. These inspection tables provide the quickest and simplest method of sight reduction and are recommended to the budding navigator.

Two stages of interpolation can be avoided by using an assumed position rather than a D.R. position. The assumed position will always have a latitude to the nearest whole degree of the D.R. latitude and a longitude that, when applied to the G.H.A., will provide an L.H.A. rounded off to degrees only. Thus the only argument requiring interpolation is that of declination. It is emphasized that this feature is made possible by use of the altitude intercept method of determining a position line; due care should be taken when plotting the position line from assumed position.

The previous example is now reworked, using H.O. Pub. 214. An exercise follows. Worked examples and description of the tables are in the front of each volume of H.O. Pub. 214.

Example

On August 26, 1968, at 0930 L.M.T. the sextant altitude of ☉ was 44°8′.0. The vessel was in D.R. position 39°30′ N 71°15′ W and the chronometer, which was 9m12s fast of G.M.T., was reading 14h16m41s. Calculate the direction of the position line and a position through which it passes if the observer's H.E. was 16 feet and the I.E. was 0′.8 off the arc.

Step 1. Determine G.M.T. from the chronometer. Check this by making a comparison with local time, having longitude in time applied.

L.M.T.		0930	
Long. in time		445	
Approx. G.M.T.		1415	
Chron.	14h	16m	41s
Error		- 9m	12s
G.M.T.	14h	07m	29s

Step 2. Calculate the L.H.A. by applying longitude to the G.H.A. as extracted from the *Nautical Almanac*. If the L.H.A. exceeds 180 degrees subtract this from 360 degrees to give angle *P* in the *PZX* triangle.

G.H.A. 14h	29°	34′.7
Incr.	1°	52′.3
G.H.A.	31°	27′
Long.	71°	15′
L.H.A.	320°	12′
Angle P	39°	48′

Step 3. Correct the sextant altitude.

Dec.	10°	16′.2N
Sex. Alt.	44°	08′
I.E.		+ 0′.8
Obs. Alt.	44°	8′.8
Dip.		- 3′.9
App. Alt.	44°	4′.9
T. Corr.		+15′
T. Alt.	44°	19′.9

Step 4. Assume a latitude to the nearest whole degree of the D.R. Assume a longitude such as to render the L.H.A. in degrees only. Assume a declination to the nearest half-degree.

Assumed Lat.	39°	00′
G.H.A.	31°	27′
Assumed Long.	71°	27′W
L.H.A.	320°	00′
Angle P	40°	

Step 5. Enter H.O. 214 with argument 39° latitude, 40° H.A., and 10°30′ declination and extract altitude.

Alt.	44°	25′.8

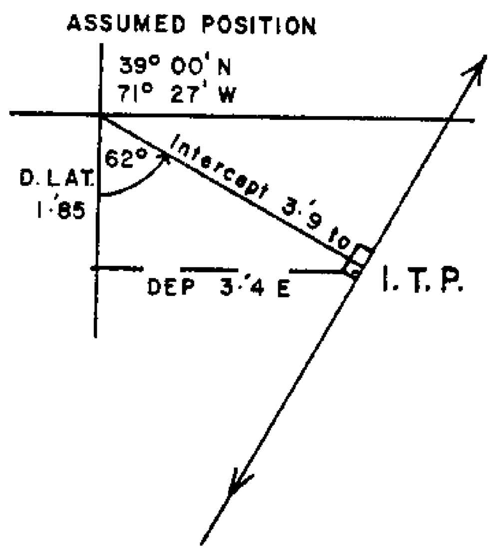
Step 6. Extract triangle *d* which is rate of change of altitude for 1 minute change in declination. Multiply this by the difference between the actual declination and that used. Use multiplication table on inside back cover of tables. Apply this correction to altitude extracted from the tables to get the calculated true altitude.

Triangle *d*	.71
Diff.	13'.8
Corr.	9'.8
Calc. Alt.	44°16'.0
T. Alt.	44°19'.9

Step 7. Determine the intercept by taking the difference between calculated and true altitudes and lay the intercept off from the assumed position to the azimuth extracted from the tables.

Intercept		3'.9 To.
Azimuth	118°	

Assumed Pos.	39°00' N		71°27' W	
D.Lat.	1'.8 S	D.Long.	4'.4 E	
I.T.P.	Lat. 38°58'.2N	Long. 71°22.6W		



Obviously this is not the same I.T.P. found by calculation or by the Ageton method. However, because this position line is another section of the same line, it will pass almost exactly through the I.T.P. found by the other two methods if it is extended. This can be checked by plotting or by using traverse tables.

Exercise 9. SIGHT REDUCTION

Work each problem by all three methods to learn the advantage and disadvantages of each method.

1. A vessel in D.R. position 38°18′ N 42°15′ W had a corrected chronometer reading of 11h 08m 15s on August 27, 1968, when the true altitude of the Sun was 32°57′.4. Calculate the direction of the position line and a position through which it passes.

2. Calculate the direction of the position line and a position through which it passes if the observed altitude of the $\underline{\odot}$ was 41°26′ when the L.H.A. of the Sun was 23°18′ with a declination of 10°47′ S. The vessel was in D.R. position 32°08′ N 73°46′ W with an H.E. of 15 feet.

3. The true altitude of the Sun was 31°10′ to a vessel in D.R. position 36°20′ N 52°36′ W when the G.H.A. of the Sun was 117°23′. Calculate the direction of the position line and a position through which it passes if the Sun's declination at this time was 18°54′ N.

4. On August 25, 1968, the true altitude of the Sun was 38°11′.6 to a vessel in D.R. position 35°18′ N 64°57′ W. The chronometer, which was 7m 12s fast of G.M.T., read 13h 06m 48s. Find the direction of the position line and a position through which it passes.

5. Determine the direction of the position line and a position through which it passes if the observed altitude of $\underline{\odot}$ was 21°01′ to an observer in D.R. position 31°02′ N 125°07′ E. The G.H.A. of the Sun was 285°15′ and its declination was 17°54′ S. The observer's H.E. was 19 feet.

6. On August 26, 1968, at 1000 L.M.T. the sextant altitude $\underline{\odot}$ was 53°08′. The vessel was in D.R. position 33°42′ N 47°18′ W and the chronometer, which was 6m 42s slow of G.M.T., was reading 13h 00m 14s. Calculate the direction of the position line and a position through which it passes if the observer's H.E. was 17 feet and the I.E. was 1′.2 on the arc.

7. The true altitude of star Sirius was 19°35′ when its L.H.A. was 52°17′. Calculate the direction of the position line and a position through which it passes if the observer was in D.R. position 32°47′ N 48°18′ W.

8. On August 27, 1968, the observed altitude of star Pollux was 37°25′.5 when the corrected chronometer reading was 08h 08m 40s. Determine the direction of the position line and a position through which it passes if the vessel was in D.R. position 37°14′ N 44°48′ W. The observer's H.E. was 20 feet.

9. Calculate the direction of the position line and a position through which it passes if star Capella had a sextant altitude of 60°14′.5 to an observer in D.R. position 30°45′ N 72°12′ W when its L.H.A. was 32°58′. The observer's H.E. was 22 feet and the I.E. 2′.5 off the arc.

10. On August 26, 1968, at 0500 L.M.T. the star Aldebaron had a sextant altitude of 38°58′. If the chronometer was reading 17h 58m 42s at this time with an error of 6m 48s slow on G.M.T., calculate the direction of the position line and a position through which it passes. The observer was in D.R. position 31°42′ S 165°15′ E with H.E. 21 feet and I.E. 2′.8 on the arc.

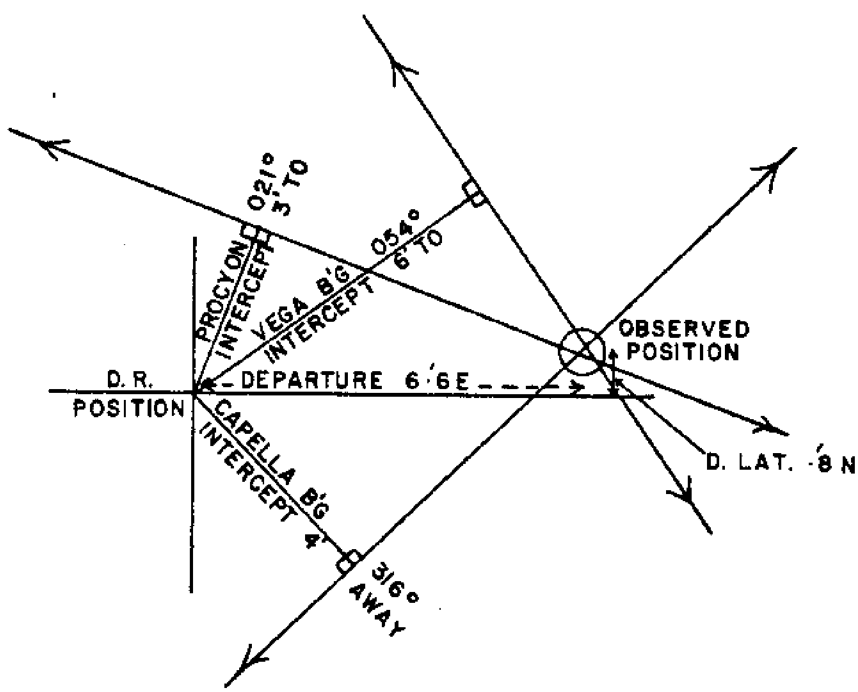
Fixing Position

USING CELESTIAL POSITION LINES

A single position line alone is insufficient to actually fix the vessel's position. Obviously further information, such as a second position line or a sounding, is required to isolate one spot on the first position line.

There are a number of different ways of using celestial position lines to obtain a fix. The most reliable and trusted method of obtaining a vessel's position by celestial observation is to use a number of star position lines taken simultaneously. However, as previously mentioned, star sights may only be taken when both the star and a reasonably clear horizon are visible. This confines star sights to a few minutes of twilight time during morning and evening. Bubble sextants with an artificial horizon have never proved effective at sea, mainly due to irregularities introduced by a ship's motion at sea.

A typical illustration of simultaneous star sights follows.

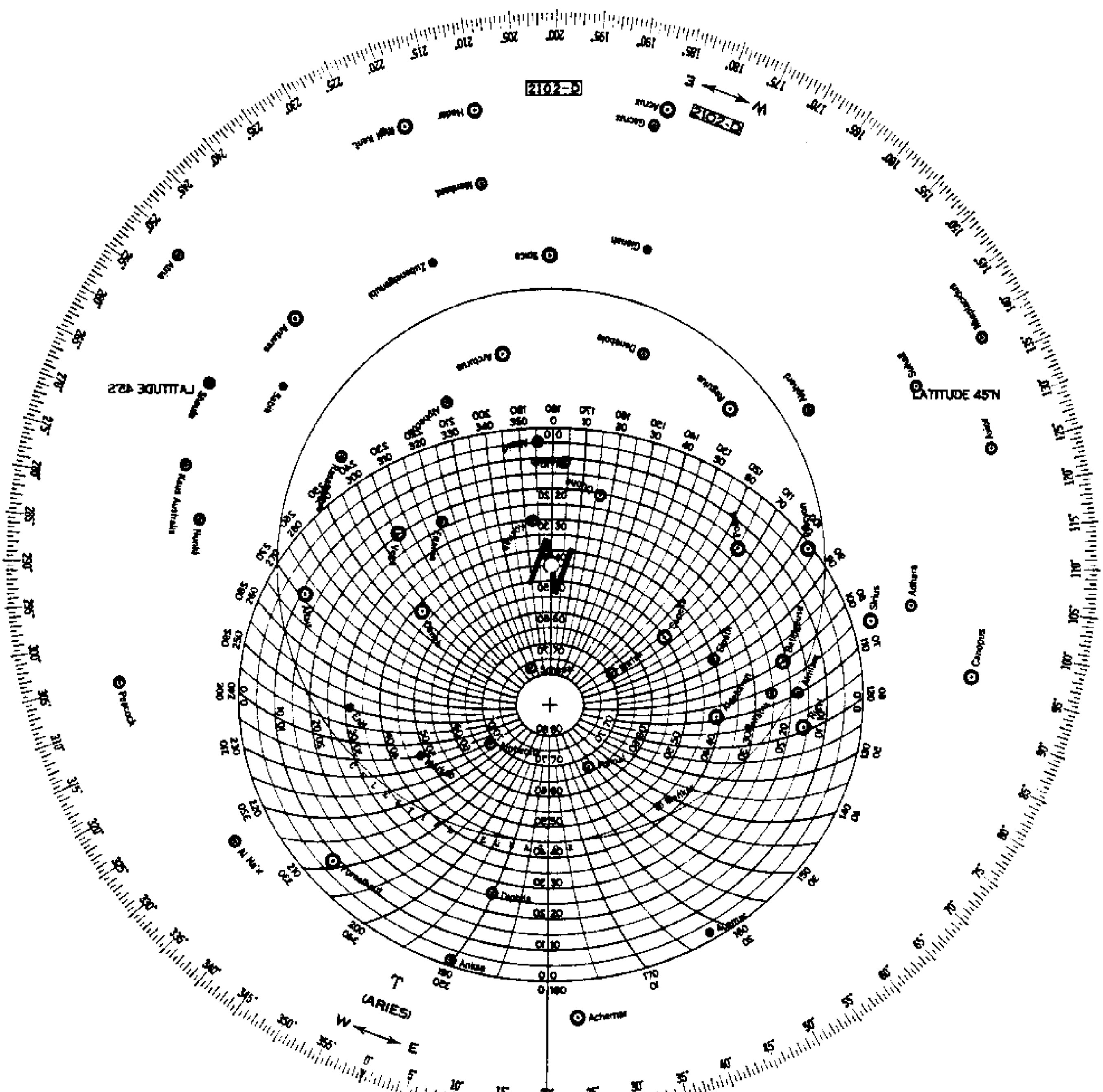


SIMULTANEOUS STAR SIGHTS

STAR IDENTIFICATION

The approximate time of star sights can be extracted from the *Nautical Almanac*; it is usually about midway between sunrise or sunset and civil twilight. The approximate altitudes and azimuths of various stars may be precomputed so the navigator can set the approximate altitude on his sextant and scan the horizon in the vicinity of the precomputed azimuth. This pre-computing procedure is rather laborious and most navigators prefer to use some kind of star identifier. Probably the most popular device for star finding is No. 2102-D, produced by the U.S. Navy, and known to seamen the world over as the *Rude identifier.*

This star finder is very easy to use. The navigator simply selects one of the nine altitude azimuth plastic templates corresponding most nearly to his latitude and places this over the star base. Both templates and base have one side marked for the northern hemisphere and the other for the southern hemisphere. Take care to use the correct side. An arrow on the template is set to L.H.A. Aries on the star base and the approximate altitudes and aximuths of the principal navigational stars can be read off.



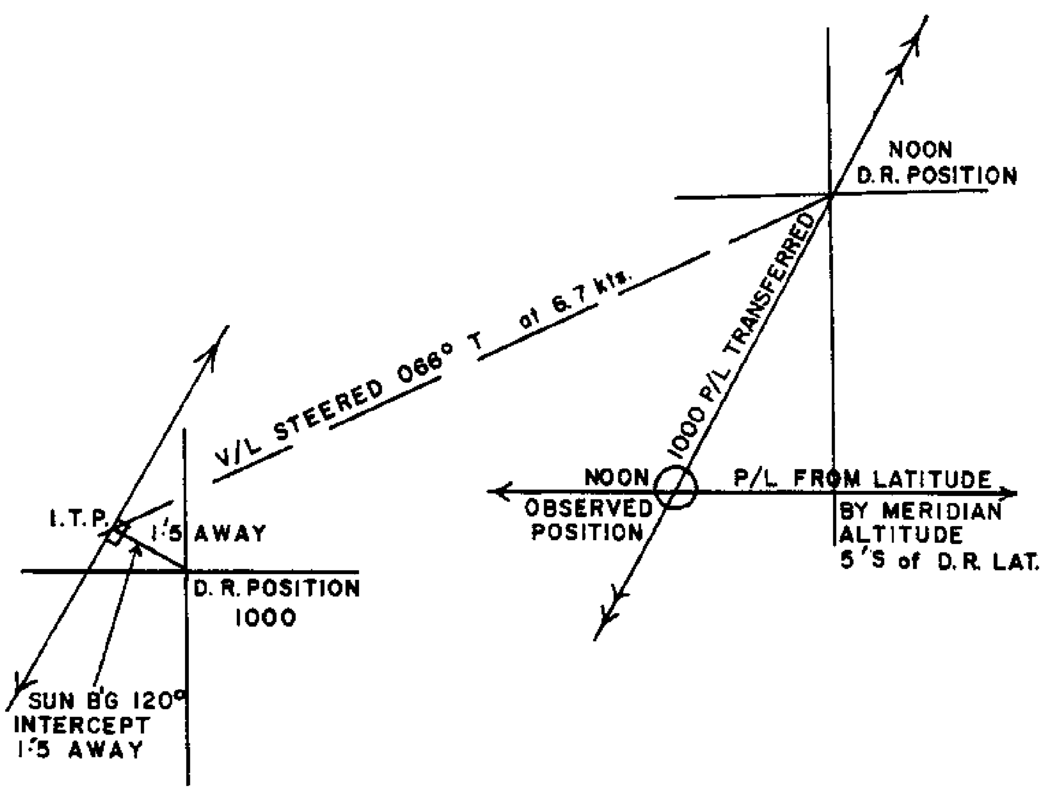
STAR IDENTIFIER SET FOR LAT. 45° N, L.H.A. ARIES 20°

TRANSFERRED POSITION LINE

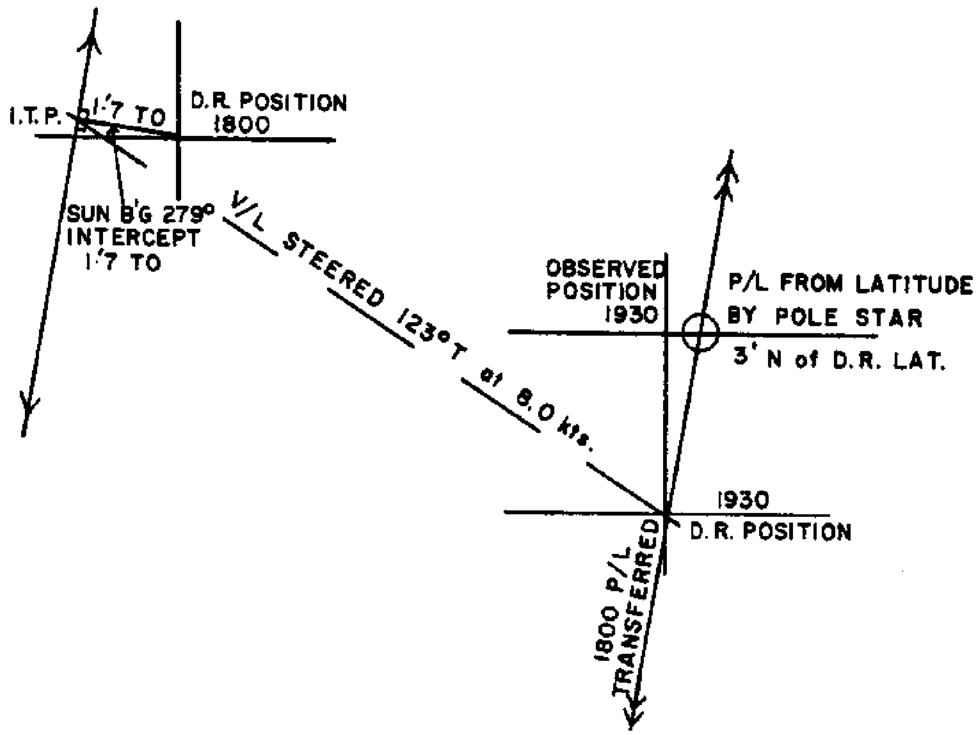
During the daylight hours, the navigator is dependent on the Sun for position lines. Occasionally planets can be observed during daylight hours if their altitude and azimuth are precomputed and they are searched for by sextant telescope. However, at this stage we will confine ourselves to obtaining separate position lines from observations of the Sun.

It is common practice to take morning sun sights when the Sun is at least ten degrees in altitude and as near east as possible. The resulting position line can then be transferred up to noon and crossed with the latitude position line obtained by meridian or ex-meridian altitude in the same way as a running fix is determined for coastal chartwork.

Similarly, the Sun may be observed in the evening as near west as possible with an altitude of at least ten degrees. The position line can be transferred up to a latitude position line obtained by observation of the Pole Star.



MORNING SUN SIGHT POSITION LINE TRANSFERRED
TO GIVE OBSERVED POSITION AT NOON



EVENING SUN SIGHT POSITION LINE TRANSFERRED
TO GIVE OBSERVED POSITION AT TIME
OF POLE STAR OBSERVATION

Step 1. The first position line is constructed from the D.R. position, and the I.T.P. is found by plotting or by using traverse tables.

Step 2. The D.R. position at the time of the second observation is calculated by projecting the estimated course and speed made good away from the I.T.P. The first position line is transferred through this D.R. position.

Step 3. The intercept and azimuth, at the time of the new D.R. position, are laid off and the new position line is constructed. Where the new position line intersects the transferred position line is the observed position. The entire procedure can be carried out by plotting or can be done by calculation, using the traverse tables.

There are many different combinations of position lines, both transferred and instantaneous, terrestrial and celestial, which can lead to a fix. When the basic principles of navigation are understood and practiced regularly, these more sophisticated applications will become obvious. It is a most satisfying accomplishment to reduce observations of a number of bodies to a final position in a short time when thousands of miles out at sea. Celestial navigation will remain for a long time to come one of the finest arts practiced by seamen.

Exercise 10. FIXING BY CELESTIAL POSITION LINES

1. A vessel in D.R. position 34°57′ N 46°22′ W took simultaneous star sights of the following stars: Altair, bearing 242° T, intercept 1′ away; Vega, bearing 292° T, intercept 2½′ away, and Fomalhaut, bearing 172° T, intercept 2′ towards. Calculate the vessel's observed position.

2. The following simultaneous star sights were observed by a vessel in D.R. position 36°22′ N 68°13′ W: Deneb, bearing 060° T, intercept 2½′ away; Altair, bearing 120° T, intercept ½′ towards, and Antares, bearing 195° T, intercept 3′ towards. Calculate the vessel's observed position.

3. At 0830 the Sun bearing 097° T gave an intercept of 4′ towards to a vessel in D.R. position 46°22′ N 56°18′ W. The vessel steered 135° T at 10 knots until noon when the latitude by meridian altitude was 45°51′ N. State the observer's noon longitude.

4. In D.R. longitude 69°13′ W the latitude by meridian altitude of the Sun was 29°18′ N. The vessel ran 250° T at 11 knots from noon until 1700 when the Sun bearing 235° T gave an intercept of 8′ towards. Calculate the observer's position at 1700.

5. An observation of the Sun bearing 270° T gave an intercept of 3′ towards at 1730 hours in D.R. position 32°45′ N 17°12′ W. The vessel then steered 050° T at 15 knots until 1930 hours when latitude by observation of the Pole Star was 33°08′. Calculate the observer's longitude at 1930 hours.

Answers to Exercises

EXERCISE 1

Time

1. 0230
2. 1132
3. 02h16m48s
4. 23h10m17s
5. 13h02m20s
6. 12h54m31s
7. 21h38m06s
8. 09h08m09s
9. July 19, 2140
10. Dec. 21, 21h06m32s

EXERCISE 2

Hour Angles

1. 138°14′
2. 37°36′
3. 31°19′
4. 339°04′
5. 44°36′ W
6. 20°54′ W
7. 15 hr. 31 min.
8. 323°56′
9. 262°32′
10. 30°40′

EXERCISE 3

Altitude Correction

1. 27°57′.9
2. 61°4′.3
3. 19°10′.3
4. 81°11′.9
5. 26°38′.5
6. 63°31′.8
7. 30°2′.8
8. 31°48′.8
9. 49°23′.7
10. 35°52′
    54°8′

EXERCISE 4

Meridian Altitudes

1. 41°09′ S
2. 45°23′ S
3. 11°20′ N
4. 21½° S
5. 44°06′.7 N
6. 58°16′.9 N
7. 15°46′.4 N
8. 30°00′.7 N
9. 14h34m18s, 42°56′.3 S
10. 12h01m25s, 25°11′.7 N

EXERCISE 5

Time Azimuths

1. 125.4° T.
2. 162.1° T.
3. 238.1° T.
4. 289.4° T.
5. 132.6° T.
6. 16.3° E
7. 263.1° T.
8. 15° W
9. 15½° E
10. 3° E

EXERCISE 6

Amplitudes

1. W. 12° S
2. 063° T.
3. W. 31° N, 17° E
4. W.
5. 14.7° W

EXERCISE 7

Ex-meridians

1. 38°02′ N
2. 47°48′.5 N
3. 48°08′.3 N
4. 080° -260°, 22°27′ S, 65°17′ E
5. 51°27′ N, 083½° -263½°

EXERCISE 8

Pole Star

1. 44°02′ N
2. 35°26′ N
3. 54°4′.2 N
4. 48°5′.6 N
5. 66°47′.4 N, 10° E

EXERCISE 9

Sight Reduction

1. 014½°/194½°, 38°18′.6 N 42°17′.8 W or
   014°/194°, 37°56′.5 N 42°24′.7 W
2. 121.2°/301.2°, 32°09′ N 73°45′.2 W or
   121°/301°, 32°13′.8 N 73°54′.2 W
3. 001½°/181½°, 36°20′.1 N 52°40′.6 W or
   001.8°/181.8°, 36°00′.5 N 52°41′.0 W
4. 015½°/195½°, 35°15′.8 N 64°47′ W or
   015°/195°, 34°53′.4 N 64°54′.3 W
5. 141.8°/321.8°, 31°03′ N 125°08′.5 E or
   141.6°/321.6°, 31°10′.8 N 125°01′.2 E
6. 032.2°/212.2°, 33°39′ N 47°12′.2 W or
   032.6°/212.6°, 33°51′.4 N 47°02′.7 W
7. 143½°/323½°, 32°51′.9 N 48°10′ W or
   143.4°/323.4°, 33°07′.2 N 48°23′.7 W
8. 170½°/350½°, 37°14′.6 N 44°43′ W or
   170.3°/350.3°, 37°03′ N 44°40′.5 W
9. 040.2°/220.2°, 30°40′.5 N 72°5′.7 W or
   039.9°/219.9°, 30°50′.2 N 71°56′.3 W
10. 112.2°/292.2°, 31°46′ S 165°13′.1 E or
    112.3°/292.3°, 31°45′.7 S 165°12′.3 E

EXERCISE 10

Fixing by Celestial Position Lines

1. 34°55′.3 N 46°19′.6 W
2. 36°19′.1 N 68°14′.4 W
3. 55°37′ W
4. 28°59′.2 N 70°23′.3 W
5. 16°48′ W

SUN

OCT.–MAR. App. Alt.	Lower Limb	Upper Limb	APR.–SEPT. App. Alt.	Lower Limb	Upper Limb
° ′	′	′	° ′	′	′
9 34	+10·8	−22·7	9 39	+10·6	−22·4
9 45	+10·9	−22·6	9 51	+10·7	−22·3
9 56	+11·0	−22·5	10 03	+10·8	−22·2
10 08	+11·1	−22·4	10 15	+10·9	−22·1
10 21	+11·2	−22·3	10 27	+11·0	−22·0
10 34	+11·3	−22·2	10 40	+11·1	−21·9
10 47	+11·4	−22·1	10 54	+11·2	−21·8
11 01	+11·5	−22·0	11 08	+11·3	−21·7
11 15	+11·6	−21·9	11 23	+11·4	−21·6
11 30	+11·7	−21·8	11 38	+11·5	−21·5
11 46	+11·8	−21·7	11 54	+11·6	−21·4
12 02	+11·9	−21·6	12 10	+11·7	−21·3
12 19	+12·0	−21·5	12 28	+11·8	−21·2
12 37	+12·1	−21·4	12 46	+11·9	−21·1
12 55	+12·2	−21·3	13 05	+12·0	−21·0
13 14	+12·3	−21·2	13 24	+12·1	−20·9
13 35	+12·4	−21·1	13 45	+12·2	−20·8
13 56	+12·5	−21·0	14 07	+12·3	−20·7
14 18	+12·6	−20·9	14 30	+12·4	−20·6
14 42	+12·7	−20·8	14 54	+12·5	−20·5
15 06	+12·8	−20·7	15 19	+12·6	−20·4
15 32	+12·9	−20·6	15 46	+12·7	−20·3
15 59	+13·0	−20·5	16 14	+12·8	−20·2
16 28	+13·1	−20·4	16 44	+12·9	−20·1
16 59	+13·2	−20·3	17 15	+13·0	−20·0
17 32	+13·3	−20·2	17 48	+13·1	−19·9
18 06	+13·4	−20·1	18 24	+13·2	−19·8
18 42	+13·5	−20·0	19 01	+13·3	−19·7
19 21	+13·6	−19·9	19 42	+13·4	−19·6
20 03	+13·7	−19·8	20 25	+13·5	−19·5
20 48	+13·8	−19·7	21 11	+13·6	−19·4
21 35	+13·9	−19·6	22 00	+13·7	−19·3
22 26	+14·0	−19·5	22 54	+13·8	−19·2
23 22	+14·1	−19·4	23 51	+13·9	−19·1
24 21	+14·2	−19·3	24 53	+14·0	−19·0
25 26	+14·3	−19·2	26 00	+14·1	−18·9
26 36	+14·4	−19·1	27 13	+14·2	−18·8
27 52	+14·5	−19·0	28 33	+14·3	−18·7
29 15	+14·6	−18·9	30 00	+14·4	−18·6
30 46	+14·7	−18·8	31 35	+14·5	−18·5
32 26	+14·8	−18·7	33 20	+14·6	−18·4
34 17	+14·9	−18·6	35 17	+14·7	−18·3
36 20	+15·0	−18·5	37 26	+14·8	−18·2
38 36	+15·1	−18·4	39 50	+14·9	−18·1
41 08	+15·2	−18·3	42 31	+15·0	−18·0
43 59	+15·3	−18·2	45 31	+15·1	−17·9
47 10	+15·4	−18·1	48 55	+15·2	−17·8
50 46	+15·5	−18·0	52 44	+15·3	−17·7
54 49	+15·6	−17·9	57 02	+15·4	−17·6
59 23	+15·7	−17·8	61 51	+15·5	−17·5
64 30	+15·8	−17·7	67 17	+15·6	−17·4
70 12	+15·9	−17·6	73 16	+15·7	−17·3
76 26	+16·0	−17·5	79 43	+15·8	−17·2
83 05	+16·1	−17·4	86 32	+15·9	−17·1
90 00			90 00		

STARS AND PLANETS

App. Alt.	Corrn
° ′	′
9 56	−5·3
10 08	−5·2
10 20	−5·1
10 33	−5·0
10 46	−4·9
11 00	−4·8
11 14	−4·7
11 29	−4·6
11 45	−4·5
12 01	−4·4
12 18	−4·3
12 35	−4·2
12 54	−4·1
13 13	−4·0
13 33	−3·9
13 54	−3·8
14 16	−3·7
14 40	−3·6
15 04	−3·5
15 30	−3·4
15 57	−3·3
16 26	−3·2
16 56	−3·1
17 28	−3·0
18 02	−2·9
18 38	−2·8
19 17	−2·7
19 58	−2·6
20 42	−2·5
21 28	−2·4
22 19	−2·3
23 13	−2·2
24 11	−2·1
25 14	−2·0
26 22	−1·9
27 36	−1·8
28 56	−1·7
30 24	−1·6
32 00	−1·5
33 45	−1·4
35 40	−1·3
37 48	−1·2
40 08	−1·1
42 44	−1·0
45 36	−0·9
48 47	−0·8
52 18	−0·7
56 11	−0·6
60 28	−0·5
65 08	−0·4
70 11	−0·3
75 34	−0·2
81 13	−0·1
87 03	0·0
90 00	

1968

App. Alt.	Additional Corrn
VENUS	
Jan. 1–Dec. 14	
0°	
42	+0·1
Dec. 15–Dec. 31	
0°	
47	+0·2
MARS	
Jan. 1–Dec. 31	
0°	
60	+0·1

DIP

Ht. of Eye	Corrn	Ht. of Eye	Corrn
ft.	′	ft.	′
1·1	−1·1	44	−6·5
1·4	−1·2	45	−6·6
1·6	−1·3	47	−6·7
1·9	−1·4	48	−6·8
2·2	−1·5	49	−6·9
2·5	−1·6	51	−7·0
2·8	−1·7	52	−7·1
3·2	−1·8	54	−7·2
3·6	−1·9	55	−7·3
4·0	−2·0	57	−7·4
4·4	−2·1	58	−7·5
4·9	−2·2	60	−7·6
5·3	−2·3	62	−7·7
5·8	−2·4	63	−7·8
6·3	−2·5	65	−7·9
6·9	−2·6	67	−8·0
7·4	−2·7	68	−8·1
8·0	−2·8	70	−8·2
8·6	−2·9	72	−8·3
9·2	−3·0	74	−8·4
9·8	−3·1	75	−8·5
10·5	−3·2	77	−8·6
11·2	−3·3	79	−8·7
11·9	−3·4	81	−8·8
12·6	−3·5	83	−8·9
13·3	−3·6	85	−9·0
14·1	−3·7	87	−9·1
14·9	−3·8	88	−9·2
15·7	−3·9	90	−9·3
16·5	−4·0	92	−9·4
17·4	−4·1	94	−9·5
18·3	−4·2	96	−9·6
19·1	−4·3	98	−9·7
20·1	−4·4	101	−9·8
21·0	−4·5	103	−9·9
22·0	−4·6	105	−10·0
22·9	−4·7	107	−10·1
23·9	−4·8	109	−10·2
24·9	−4·9	111	−10·3
26·0	−5·0	113	−10·4
27·1	−5·1	116	−10·5
28·1	−5·2	118	−10·6
29·2	−5·3	120	−10·7
30·4	−5·4	122	−10·8
31·5	−5·5	125	−10·9
32·7	−5·6	127	−11·0
33·9	−5·7	129	−11·1
35·1	−5·8	132	−11·2
36·3	−5·9	134	−11·3
37·6	−6·0	136	−11·4
38·9	−6·1	139	−11·5
40·1	−6·2	141	−11·6
41·5	−6·3	144	−11·7
42·8	−6·4	146	−11·8
44·2		149	

App. Alt. = Apparent altitude = Sextant altitude corrected for index error and dip.

For daylight observations of Venus, see page 260.

G.M.T.	SUN G.H.A.	SUN Dec.	MOON G.H.A.	v	MOON Dec.	d	H.P.
d h	° ′	° ′	° ′	′	° ′	′	′
25 00	179 28·2	N10 49·2	166 55·3	13·2	N 8 16·0	15·6	57·7
01	194 28·3	48·4	181 27·5	13·2	8 00·4	15·6	57·7
02	209 28·5	47·5	195 59·7	13·2	7 44·8	15·7	57·7
03	224 28·7	· · 46·7	210 31·9	13·2	7 29·1	15·8	57·8
04	239 28·8	45·8	225 04·1	13·2	7 13·3	15·7	57·8
05	254 29·0	44·9	239 36·3	13·3	6 57·6	15·8	57·8
06	269 29·2	N10 44·1	254 08·6	13·2	N 6 41·8	15·9	57·8
07	284 29·4	43·2	268 40·8	13·3	6 25·9	15·9	57·8
08	299 29·5	42·3	283 13·1	13·2	6 10·0	16·0	57·9
S 09	314 29·7	· · 41·5	297 45·3	13·3	5 54·0	15·9	57·9
U 10	329 29·9	40·6	312 17·6	13·3	5 38·1	16·0	57·9
N 11	344 30·0	39·7	326 49·9	13·3	5 22·1	16·1	57·9
D 12	359 30·2	N10 38·9	341 22·2	13·3	N 5 06·0	16·1	57·9
A 13	14 30·4	38·0	355 54·5	13·3	4 49·9	16·1	58·0
Y 14	29 30·6	37·1	10 26·8	13·3	4 33·8	16·2	58·0
15	44 30·7	· · 36·3	24 59·1	13·3	4 17·6	16·1	58·0
16	59 30·9	35·4	39 31·4	13·3	4 01·5	16·2	58·0
17	74 31·1	34·5	54 03·7	13·3	3 45·3	16·3	58·0
18	89 31·3	N10 33·7	68 36·0	13·3	N 3 29·0	16·2	58·1
19	104 31·4	32·8	83 08·3	13·4	3 12·8	16·3	58·1
20	119 31·6	31·9	97 40·7	13·3	2 56·5	16·4	58·1
21	134 31·8	· · 31·1	112 13·0	13·2	2 40·1	16·3	58·1
22	149 31·9	30·2	126 45·2	13·3	2 23·8	16·3	58·1
23	164 32·1	29·3	141 17·5	13·3	2 07·5	16·4	58·2
26 00	179 32·3	N10 28·5	155 49·8	13·3	N 1 51·1	16·4	58·2
01	194 32·5	27·6	170 22·1	13·3	1 34·7	16·4	58·2
02	209 32·6	26·7	184 54·4	13·2	1 18·3	16·5	58·2
03	224 32·8	· · 25·9	199 26·6	13·3	1 01·8	16·4	58·2
04	239 33·0	25·0	213 58·9	13·2	0 45·4	16·5	58·2
05	254 33·2	24·1	228 31·1	13·2	0 28·9	16·4	58·3
06	269 33·3	N10 23·3	243 03·3	13·2	N 0 12·5	16·5	58·3
07	284 33·5	22·4	257 35·5	13·2	S 0 04·0	16·5	58·3
08	299 33·7	21·5	272 07·7	13·1	0 20·5	16·5	58·3
M 09	314 33·9	· · 20·6	286 39·8	13·2	0 37·0	16·5	58·3
O 10	329 34·0	19·8	301 12·0	13·1	0 53·5	16·5	58·4
N 11	344 34·2	18·9	315 44·1	13·1	1 10·0	16·5	58·4
D 12	359 34·4	N10 18·0	330 16·2	13·1	S 1 26·5	16·5	58·4
A 13	14 34·6	17·2	344 48·3	13·1	1 43·0	16·5	58·4
Y 14	29 34·7	16·3	359 20·4	13·0	1 59·5	16·5	58·4
15	44 34·9	· · 15·4	13 52·4	13·0	2 16·0	16·5	58·4
16	59 35·1	14·5	28 24·4	13·0	2 32·5	16·5	58·5
17	74 35·3	13·7	42 56·4	12·9	2 49·0	16·5	58·5
18	89 35·5	N10 12·8	57 28·3	12·9	S 3 05·5	16·5	58·5
19	104 35·6	11·9	72 00·2	12·9	3 22·0	16·5	58·5
20	119 35·8	11·0	86 32·1	12·9	3 38·5	16·5	58·5
21	134 36·0	· · 10·2	101 04·0	12·8	3 55·0	16·5	58·5
22	149 36·2	09·3	115 35·8	12·8	4 11·5	16·4	58·6
23	164 36·3	08·4	130 07·6	12·7	4 27·9	16·4	58·6
27 00	179 36·5	N10 07·5	144 39·3	12·7	S 4 44·3	16·5	58·6
01	194 36·7	06·7	159 11·0	12·7	5 00·8	16·4	58·6
02	209 36·9	05·8	173 42·7	12·6	5 17·2	16·4	58·6
03	224 37·0	· · 04·9	188 14·3	12·6	5 33·6	16·3	58·6
04	239 37·2	04·0	202 45·9	12·6	5 49·9	16·4	58·6
05	254 37·4	03·2	217 17·5	12·5	6 06·3	16·3	58·7
06	269 37·6	N10 02·3	231 49·0	12·4	S 6 22·6	16·3	58·7
07	284 37·8	01·4	246 20·4	12·4	6 38·9	16·3	58·7
08	299 37·9	10 00·5	260 51·8	12·4	6 55·2	16·2	58·7
T 09	314 38·1	9 59·6	275 23·2	12·3	7 11·4	16·2	58·7
U 10	329 38·3	58·8	289 54·5	12·3	7 27·6	16·2	58·7
E 11	344 38·5	57·9	304 25·8	12·2	7 43·8	16·2	58·7
S 12	359 38·7	N 9 57·0	318 57·0	12·1	S 8 00·0	16·1	58·8
D 13	14 38·8	56·1	333 28·1	12·1	8 16·1	16·1	58·8
A 14	29 39·0	55·2	347 59·2	12·1	8 32·2	16·1	58·8
Y 15	44 39·2	· · 54·4	2 30·3	12·0	8 48·3	16·0	58·8
16	59 39·4	53·5	17 01·3	11·9	9 04·3	16·0	58·8
17	74 39·6	52·6	31 32·2	11·9	9 20·3	15·9	58·8
18	89 39·8	N 9 51·7	46 03·1	11·8	S 9 36·2	15·9	58·8
19	104 39·9	50·8	60 33·9	11·8	9 52·1	15·8	58·8
20	119 40·1	50·0	75 04·7	11·7	10 07·9	15·8	58·9
21	134 40·3	· · 49·1	89 35·4	11·6	10 23·7	15·8	58·9
22	149 40·5	48·2	104 06·0	11·5	10 39·5	15·7	58·9
23	164 40·7	47·3	118 36·5	11·5	10 55·2	15·7	58·9
	S.D. 15·9	d 0·9	S.D. 15·8		15·9		16·0

Lat.	Twilight Naut.	Twilight Civil	Sunrise	Moonrise 25	Moonrise 26	Moonrise 27	Moonrise 28
°	h m	h m	h m	h m	h m	h m	h m
N 72	////	01 39	03 30	05 39	07 57	10 19	13 09
N 70	////	02 23	03 49	05 49	07 57	10 07	12 34
68	////	02 51	04 03	05 57	07 56	09 57	12 10
66	01 25	03 12	04 15	06 04	07 56	09 49	11 52
64	02 04	03 29	04 25	06 10	07 55	09 43	11 37
62	02 31	03 42	04 33	06 15	07 55	09 37	11 24
60	02 50	03 54	04 40	06 19	07 55	09 32	11 14
N 58	03 06	04 03	04 47	06 22	07 54	09 28	11 05
56	03 19	04 12	04 52	06 26	07 54	09 24	10 57
54	03 31	04 19	04 57	06 29	07 54	09 21	10 50
52	03 40	04 26	05 02	06 31	07 54	09 18	10 44
50	03 49	04 31	05 06	06 34	07 54	09 15	10 38
45	04 06	04 44	05 15	06 39	07 53	09 09	10 26
N 40	04 20	04 54	05 22	06 44	07 53	09 04	10 16
35	04 31	05 02	05 28	06 47	07 53	08 59	10 08
30	04 40	05 09	05 34	06 51	07 53	08 56	10 01
20	04 54	05 20	05 43	06 56	07 52	08 49	09 48
N 10	05 05	05 30	05 51	07 01	07 52	08 44	09 37
0	05 13	05 37	05 58	07 06	07 52	08 38	09 27
S 10	05 20	05 45	06 06	07 11	07 52	08 33	09 17
20	05 25	05 51	06 13	07 16	07 51	08 28	09 06
30	05 30	05 58	06 22	07 21	07 51	08 22	08 54
35	05 32	06 02	06 27	07 24	07 51	08 18	08 47
40	05 34	06 05	06 33	07 28	07 51	08 14	08 39
45	05 35	06 09	06 39	07 32	07 51	08 10	08 30
S 50	05 37	06 14	06 47	07 37	07 51	08 04	08 20
52	05 37	06 16	06 50	07 40	07 51	08 02	08 15
54	05 37	06 18	06 54	07 42	07 51	07 59	08 09
56	05 38	06 21	06 59	07 45	07 50	07 56	08 03
58	05 38	06 23	07 03	07 48	07 50	07 53	07 56
S 60	05 38	06 26	07 09	07 51	07 50	07 49	07 49

Lat.	Sunset	Twilight Civil	Twilight Naut.	Moonset 25	Moonset 26	Moonset 27	Moonset 28
°	h m	h m	h m	h m	h m	h m	h m
N 72	20 29	22 16	////	20 04	19 28	18 46	17 41
N 70	20 12	21 36	////	20 00	19 32	19 02	18 18
68	19 57	21 08	////	19 57	19 36	19 14	18 44
66	19 46	20 48	22 30	19 54	19 40	19 24	19 04
64	19 37	20 32	21 54	19 52	19 43	19 32	19 20
62	19 29	20 19	21 29	19 50	19 45	19 40	19 34
60	19 21	20 08	21 10	19 49	19 47	19 46	19 46
N 58	19 15	19 59	20 55	19 47	19 49	19 52	19 56
56	19 10	19 50	20 42	19 46	19 51	19 57	20 05
54	19 05	19 43	20 31	19 44	19 53	20 02	20 13
52	19 01	19 37	20 22	19 43	19 54	20 06	20 20
50	18 57	19 31	20 13	19 42	19 56	20 10	20 26
45	18 48	19 19	19 56	19 40	19 58	20 18	20 40
N 40	18 41	19 09	19 43	19 38	20 01	20 25	20 52
35	18 35	19 01	19 32	19 36	20 03	20 31	21 02
30	18 29	18 54	19 23	19 35	20 05	20 36	21 10
20	18 20	18 43	19 09	19 32	20 08	20 45	21 25
N 10	18 12	18 34	18 59	19 30	20 11	20 53	21 39
0	18 05	18 26	18 50	19 28	20 14	21 01	21 51
S 10	17 58	18 19	18 44	19 25	20 16	21 09	22 04
20	17 50	18 13	18 38	19 23	20 19	21 17	22 17
30	17 42	18 06	18 34	19 20	20 23	21 26	22 33
35	17 37	18 02	18 32	19 19	20 24	21 32	22 42
40	17 31	17 59	18 30	19 17	20 27	21 38	22 52
45	17 25	17 55	18 29	19 15	20 29	21 45	23 04
S 50	17 17	17 50	18 28	19 12	20 32	21 54	23 19
52	17 14	17 48	18 28	19 11	20 33	21 58	23 26
54	17 10	17 46	18 27	19 09	20 35	22 03	23 34
56	17 06	17 44	18 27	19 08	20 37	22 08	23 43
58	17 01	17 41	18 27	19 06	20 38	22 13	23 53
S 60	16 56	17 39	18 27	19 04	20 41	22 20	24 04

Day	SUN Eqn. of Time 00h	SUN Eqn. of Time 12h	SUN Mer. Pass.	MOON Mer. Pass. Upper	MOON Mer. Pass. Lower	Age	Phase
	m s	m s	h m	h m	h m	d	
25	02 08	01 59	12 02	13 17	00 54	02	
26	01 51	01 43	12 02	14 03	01 40	03	
27	01 34	01 26	12 01	14 50	02 26	04	

G.M.T. d	h	ARIES G.H.A.	VENUS −3·3 G.H.A.	VENUS Dec.	MARS −2·0 G.H.A.	MARS Dec.	JUPITER −1·2 G.H.A.	JUPITER Dec.	SATURN +0·5 G.H.A.	SATURN Dec.
		° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
25	00	333 19·1	162 09·5	N 5 15·4	198 07·2	N18 09·8	168 28·0	N 7 33·0	308 54·9	N 7 17·9
	01	348 21·6	177 09·1	14·2	213 08·0	09·3	183 29·9	32·8	323 57·4	17·8
	02	3 24·1	192 08·8	12·9	228 08·9	08·9	198 31·9	32·6	339 00·0	17·8
	03	18 26·5	207 08·4	. . 11·7	243 09·7	. . 08·4	213 33·9	. . 32·4	354 02·5	. . 17·8
	04	33 29·0	222 08·0	10·4	258 10·6	08·0	228 35·8	32·2	9 05·0	17·7
	05	48 31·5	237 07·7	09·1	273 11·4	07·6	243 37·8	32·0	24 07·6	17·7
	06	63 33·9	252 07·3	N 5 07·9	288 12·3	N18 07·1	258 39·8	N 7 31·8	39 10·1	N 7 17·6
	07	78 36·4	267 07·0	06·6	303 13·1	06·7	273 41·7	31·6	54 12·6	17·6
	08	93 38·9	282 06·6	05·4	318 14·0	06·2	288 43·7	31·4	69 15·2	17·6
S	09	108 41·3	297 06·2	. . 04·1	333 14·8	. . 05·8	303 45·7	. . 31·2	84 17·7	. . 17·5
U	10	123 43·8	312 05·9	02·9	348 15·7	05·3	318 47·6	31·0	99 20·2	17·5
N	11	138 46·3	327 05·5	01·6	3 16·5	04·9	333 49·6	30·8	114 22·8	17·5
D	12	153 48·7	342 05·2	N 5 00·4	18 17·4	N18 04·5	348 51·5	N 7 30·6	129 25·3	N 7 17·4
A	13	168 51·2	357 04·8	4 59·1	33 18·2	04·0	3 53·5	30·4	144 27·8	17·4
Y	14	183 53·6	12 04·5	57·9	48 19·1	03·6	18 55·5	30·2	159 30·4	17·3
	15	198 56·1	27 04·1	. . 56·6	63 19·9	. . 03·1	33 57·4	. . 30·0	174 32·9	. . 17·3
	16	213 58·6	42 03·7	55·4	78 20·6	02·7	48 59·4	29·8	189 35·4	17·3
	17	229 01·0	57 03·4	54·1	93 21·6	02·2	64 01·4	29·6	204 38·0	17·2
	18	244 03·5	72 03·0	N 4 52·9	108 22·5	N18 01·8	79 03·3	N 7 29·4	219 40·5	N 7 17·2
	19	259 06·0	87 02·7	51·6	123 23·3	01·4	94 05·3	29·1	234 43·0	17·2
	20	274 08·4	102 02·3	50·3	138 24·2	00·9	109 07·3	28·9	249 45·6	17·1
	21	289 10·9	117 02·0	. . 49·1	153 25·0	. . 00·5	124 09·2	. . 28·7	264 48·1	. . 17·1
	22	304 13·4	132 01·6	47·8	168 25·9	18 00·0	139 11·2	28·5	279 50·6	17·0
	23	319 15·8	147 01·2	46·6	183 26·7	17 59·6	154 13·2	28·3	294 53·2	17·0
26	00	334 18·3	162 00·9	N 4 45·3	198 27·6	N17 59·1	169 15·1	N 7 28·1	309 55·7	N 7 17·0
	01	349 20·7	177 00·5	44·1	213 28·4	58·7	184 17·1	27·9	324 58·2	16·9
	02	4 23·2	192 00·2	42·8	228 29·3	58·2	199 19·1	27·7	340 00·8	16·9
	03	19 25·7	206 59·8	. . 41·6	243 30·2	. . 57·8	214 21·0	. . 27·5	355 03·3	. . 16·8
	04	34 28·1	221 59·5	40·3	258 31·0	57·4	229 23·0	27·3	10 05·9	16·8
	05	49 30·6	236 59·1	39·0	273 31·9	56·9	244 24·9	27·1	25 08·4	16·8
	06	64 33·1	251 58·8	N 4 37·8	288 32·7	N17 56·5	259 26·9	N 7 26·9	40 10·9	N 7 16·7
	07	79 35·5	266 58·4	36·5	303 33·6	56·0	274 28·9	26·7	55 13·5	16·7
	08	94 38·0	281 58·1	35·3	318 34·4	55·6	289 30·8	26·5	70 16·0	16·7
M	09	109 40·5	296 57·7	. . 34·0	333 35·3	. . 55·1	304 32·8	. . 26·3	85 18·5	. . 16·6
O	10	124 42·9	311 57·4	32·7	348 36·1	54·7	319 34·8	26·1	100 21·1	16·6
N	11	139 45·4	326 57·0	31·5	3 37·0	54·2	334 36·7	25·9	115 23·6	16·5
D	12	154 47·8	341 56·6	N 4 30·2	18 37·8	N17 53·8	349 38·7	N 7 25·6	130 26·2	N 7 16·5
A	13	169 50·3	356 56·3	29·0	33 38·7	53·3	4 40·7	25·4	145 28·7	16·5
Y	14	184 52·8	11 55·9	27·7	48 39·5	52·9	19 42·6	25·2	160 31·2	16·4
	15	199 55·2	26 55·6	. . 26·5	63 40·4	. . 52·4	34 44·6	. . 25·0	175 33·8	. . 16·4
	16	214 57·7	41 55·2	25·2	78 41·3	52·0	49 46·5	24·8	190 36·3	16·3
	17	230 00·2	56 54·9	23·9	93 42·1	51·5	64 48·5	24·6	205 38·8	16·3
	18	245 02·6	71 54·5	N 4 22·7	108 43·0	N17 51·1	79 50·5	N 7 24·4	220 41·4	N 7 16·3
	19	260 05·1	86 54·2	21·4	123 43·8	50·6	94 52·4	24·2	235 43·9	16·2
	20	275 07·6	101 53·8	20·2	138 44·7	50·2	109 54·4	24·0	250 46·5	16·2
	21	290 10·0	116 53·5	. . 18·9	153 45·5	. . 49·7	124 56·4	. . 23·8	265 49·0	. . 16·1
	22	305 12·5	131 53·1	17·6	168 46·4	49·3	139 58·3	23·6	280 51·5	16·1
	23	320 15·0	146 52·8	16·4	183 47·2	48·9	155 00·3	23·4	295 54·1	16·1
27	00	335 17·4	161 52·4	N 4 15·1	198 48·1	N17 48·4	170 02·3	N 7 23·2	310 56·6	N 7 16·0
	01	350 19·9	176 52·1	13·8	213 49·0	48·0	185 04·2	23·0	325 59·2	16·0
	02	5 22·3	191 51·7	12·6	228 49·8	47·5	200 06·2	22·8	341 01·7	15·9
	03	20 24·8	206 51·4	. . 11·3	243 50·7	. . 47·1	215 08·1	. . 22·6	356 04·2	. . 15·9
	04	35 27·3	221 51·0	10·1	258 51·5	46·6	230 10·1	22·3	11 06·8	15·9
	05	50 29·7	236 50·7	08·8	273 52·4	46·2	245 12·1	22·1	26 09·3	15·8
	06	65 32·2	251 50·3	N 4 07·5	288 53·2	N17 45·7	260 14·0	N 7 21·9	41 11·9	N 7 15·8
	07	80 34·7	266 50·0	06·3	303 54·1	45·2	275 16·0	21·7	56 14·4	15·7
T	08	95 37·1	281 49·6	05·0	318 55·0	44·8	290 18·0	21·5	71 16·9	15·7
U	09	110 39·6	296 49·3	. . 03·7	333 55·8	. . 44·3	305 19·9	. . 21·3	86 19·5	. . 15·7
E	10	125 42·1	311 48·9	02·5	348 56·7	43·9	320 21·9	21·1	101 22·0	15·6
S	11	140 44·5	326 48·6	4 01·2	3 57·5	43·4	335 23·9	20·9	116 24·6	15·6
D	12	155 47·0	341 48·2	N 3 59·9	18 58·4	N17 43·0	350 25·8	N 7 20·7	131 27·1	N 7 15·5
A	13	170 49·4	356 47·9	58·7	33 59·2	42·5	5 27·8	20·5	146 29·6	15·5
Y	14	185 51·9	11 47·6	57·4	49 00·1	42·1	20 29·7	20·3	161 32·2	15·5
	15	200 54·4	26 47·2	. . 56·2	64 01·0	. . 41·6	35 31·7	. . 20·1	176 34·7	. . 15·4
	16	215 56·8	41 46·9	54·9	79 01·8	41·2	50 33·7	19·9	191 37·3	15·4
	17	230 59·3	56 46·5	53·6	94 02·7	40·7	65 35·6	19·7	206 39·8	15·3
	18	246 01·8	71 46·2	N 3 52·4	109 03·5	N17 40·3	80 37·6	N 7 19·5	221 42·4	N 7 15·3
	19	261 04·2	86 45·8	51·1	124 04·4	39·8	95 39·6	19·3	236 44·9	15·3
	20	276 06·7	101 45·5	49·8	139 05·3	39·4	110 41·5	19·0	251 47·4	15·2
	21	291 09·2	116 45·1	. . 48·6	154 06·1	. . 38·9	125 43·5	. . 18·8	266 50·0	. . 15·2
	22	306 11·6	131 44·8	47·3	169 07·0	38·5	140 45·5	18·6	281 52·5	15·1
	23	321 14·1	146 44·4	46·0	184 07·8	38·0	155 47·4	18·4	296 55·1	15·1
Mer. Pass.		h m 1 42·5	v −0·4	d 1·3	v 0·9	d 0·4	v 2·0	d 0·2	v 2·5	d 0·0

STARS

Name	S.H.A.	Dec.
	° ′	° ′
Acamar	315 43·7	S 40 25·4
Achernar	335 51·2	S 57 23·4
Acrux	173 48·2	S 62 55·6
Adhara	255 39·4	S 28 55·3
Aldebaran	291 28·3	N 16 27·0
Alioth	166 50·4	N 56 08·0
Alkaid	153 25·5	N 49 28·4
Al Na'ir	28 25·5	S 47 06·8
Alnilam	276 20·9	S 1 13·0
Alphard	218 29·7	S 8 31·1
Alphecca	126 39·7	N 26 49·3
Alpheratz	358 18·4	N 28 55·1
Altair	62 41·0	N 8 47·1
Ankaa	353 48·5	S 42 28·3
Antares	113 07·8	S 26 22·0
Arcturus	146 26·7	N 19 20·8
Atria	108 40·1	S 68 58·7
Avior	234 32·6	S 59 24·2
Bellatrix	279 08·4	N 6 19·6
Betelgeuse	271 38·1	N 7 24·4
Canopus	264 11·5	S 52 40·2
Capella	281 24·6	N 45 58·1
Deneb	49 54·3	N 45 10·1
Denebola	183 08·4	N 14 45·0
Diphda	349 29·5	S 18 09·3
Dubhe	194 33·3	N 61 55·4
Elnath	278 55·6	N 28 35·1
Eltanin	91 01·6	N 51 29·7
Enif	34 20·1	N 9 43·9
Fomalhaut	16 00·6	S 29 47·2
Gacrux	172 39·5	S 56 56·4
Gienah	176 27·5	S 17 22·0
Hadar	149 36·7	S 60 13·6
Hamal	328 38·9	N 23 19·0
Kaus Aust.	84 28·4	S 34 24·3
Kochab	137 18·3	N 74 17·2
Markab	14 11·8	N 15 02·2
Menkar	314 50·4	N 3 58·3
Menkent	148 47·9	S 36 13·1
Miaplacidus	221 48·2	S 69 35·0
Mirfak	309 29·0	N 49 45·0
Nunki	76 40·0	S 26 20·4
Peacock	54 11·7	S 56 50·4
Pollux	244 09·3	N 28 06·4
Procyon	245 35·3	N 5 18·6
Rasalhague	96 37·7	N 12 34·9
Regulus	208 19·8	N 12 07·5
Rigel	281 44·7	S 8 13·9
Rigil Kent.	140 38·5	S 60 42·6
Sabik	102 51·3	S 15 41·4
Schedar	350 19·1	N 56 21·9
Shaula	97 07·8	S 37 05·2
Sirius	259 03·8	S 16 40·0
Spica	159 07·2	S 10 59·8
Suhail	223 17·9	S 43 18·1
Vega	81 01·7	N 38 45·3
Zuben'ubi	137 43·1	S 15 54·8

	S.H.A.	Mer. Pass.
	° ′	h m
Venus	187 42·6	13 12
Mars	224 09·3	10 46
Jupiter	194 56·8	12 41
Saturn	335 37·4	3 20

POLARIS (POLE STAR) TABLES, 1968

FOR DETERMINING LATITUDE FROM SEXTANT ALTITUDE AND FOR AZIMUTH

L.H.A. ARIES	120°–129°	130°–139°	140°–149°	150°–159°	160°–169°	170°–179°	180°–189°	190°–199°	200°–209°	210°–219°	220°–229°	230°–239°
	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	0 59·0	1 08·1	1 17·0	1 25·2	1 32·7	1 39·2	1 44·4	1 48·3	1 50·7	1 51·5	1 50·8	1 48·5
1	0 59·9	09·0	17·8	26·0	33·4	39·7	44·9	48·6	50·8	51·5	50·6	48·2
2	1 00·8	09·9	18·7	26·8	34·1	40·3	45·3	48·9	51·0	51·5	50·4	47·8
3	01·7	10·8	19·5	27·6	34·8	40·9	45·7	49·2	51·1	51·4	50·2	47·5
4	02·7	11·7	20·4	28·3	35·4	41·4	46·1	49·4	51·2	51·4	50·0	47·1
5	1 03·6	1 12·6	1 21·2	1 29·1	1 36·1	1 41·9	1 46·5	1 49·7	1 51·3	1 51·3	1 49·8	1 46·8
6	04·5	13·5	22·0	29·8	36·7	42·5	46·9	49·9	51·4	51·2	49·6	46·4
7	05·4	14·4	22·8	30·6	37·3	43·0	47·3	50·1	51·4	51·1	49·3	46·0
8	06·3	15·2	23·6	31·3	38·0	43·5	47·6	50·3	51·5	51·0	49·1	45·6
9	07·2	16·1	24·4	32·0	38·6	43·9	48·0	50·5	51·5	50·9	48·8	45·2
10	1 08·1	1 17·0	1 25·2	1 32·7	1 39·2	1 44·4	1 48·3	1 50·7	1 51·5	1 50·8	1 48·5	1 44·7
Lat.	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1
°	′	′	′	′	′	′	′	′	′	′	′	′
0	0·1	0·2	0·2	0·3	0·4	0·4	0·5	0·6	0·6	0·6	0·6	0·5
10	·2	·2	·3	·3	·4	·5	·5	·6	·6	·6	·6	·5
20	·3	·3	·3	·4	·4	·5	·5	·6	·6	·6	·6	·5
30	·4	·4	·4	·4	·5	·5	·6	·6	·6	·6	·6	·6
40	0·5	0·5	0·5	0·5	0·5	0·6	0·6	0·6	0·6	0·6	0·6	0·6
45	·5	·5	·5	·5	·6	·6	·6	·6	·6	·6	·6	·6
50	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6
55	·7	·7	·7	·7	·6	·6	·6	·6	·6	·6	·6	·6
60	·8	·8	·8	·7	·7	·7	·6	·6	·6	·6	·6	·6
62	0·9	0·9	0·8	0·8	0·7	0·7	0·7	0·6	0·6	0·6	0·6	0·6
64	0·9	0·9	0·9	·8	·8	·7	·7	·6	·6	·6	·6	·7
66	1·0	1·0	1·0	0·9	·8	·7	·7	·6	·6	·6	·6	·7
68	1·1	1·1	1·0	1·0	0·9	0·8	0·7	0·6	0·6	0·6	0·6	0·7
Month	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2
	′	′	′	′	′	′	′	′	′	′	′	′
Jan.	0·6	0·6	0·6	0·6	0·6	0·5	0·5	0·5	0·5	0·5	0·5	0·5
Feb.	·8	·7	·7	·7	·7	·6	·6	·6	·6	·5	·5	·5
Mar.	·9	·9	0·9	0·8	·8	·8	·8	·7	·7	·6	·6	·5
Apr.	0·9	0·9	1·0	1·0	0·9	0·9	0·9	0·9	0·8	0·7	0·7	0·6
May	·9	·9	1·0	1·0	1·0	1·0	1·0	1·0	0·9	0·9	·8	·8
June	·8	·8	0·9	0·9	1·0	1·0	1·0	1·0	1·0	1·0	0·9	0·9
July	0·6	0·7	0·8	0·8	0·9	0·9	1·0	1·0	1·0	1·0	1·0	1·0
Aug.	·4	·5	·6	·6	·7	·8	0·8	0·9	0·9	1·0	1·0	1·0
Sept.	·3	·4	·4	·5	·5	·6	·7	·7	·8	0·8	0·9	0·9
Oct.	0·2	0·2	0·3	0·3	0·3	0·4	0·5	0·5	0·6	0·7	0·7	0·8
Nov.	·2	·2	·2	·2	·2	·3	·3	·3	·4	·5	·5	·6
Dec.	0·3	0·2	0·2	0·2	0·2	0·2	0·2	0·2	0·3	0·3	0·4	0·5
Lat.	AZIMUTH											
°	°	°	°	°	°	°	°	°	°	°	°	°
0	359·1	359·2	359·2	359·3	359·4	359·5	359·6	359·8	359·9	0·1	0·2	0·4
20	359·1	359·1	359·1	359·2	359·3	359·5	359·6	359·7	359·9	0·1	0·2	0·4
40	358·9	358·9	359·0	359·1	359·2	359·3	359·5	359·7	359·9	0·1	0·3	0·5
50	358·6	358·7	358·8	358·9	359·0	359·2	359·4	359·6	359·9	0·1	0·3	0·6
55	358·5	358·5	358·6	358·8	358·9	359·1	359·4	359·6	359·9	0·1	0·4	0·6
60	358·2	358·3	358·4	358·6	358·8	359·0	359·3	359·5	359·8	0·1	0·4	0·7
65	357·9	358·0	358·1	358·3	358·6	358·8	359·1	359·5	359·8	0·2	0·5	0·8

ILLUSTRATION

On 1968 January 22 at G.M.T. $22^h\ 17^m\ 50^s$ in longitude W. 55° 19′ the corrected apparent altitude of *Polaris* was 49° 31′·6.

From the daily pages:	° ′
G.H.A. Aries (22^h)	91 19·3
Increment ($17^m\ 50^s$)	4 28·2
Longitude (west)	−55 19
L.H.A. Aries	40 29

	° ′
Corr. App. Alt.	49 31·6
a_0 (argument 40° 29′)	0 06·9
a_1 (lat. 50° approx.)	0·6
a_2 (January)	0·7
Sum − 1° = Lat. =	48 39·8